Glueballs, tetraquarks and excited baryons from Dyson-Schwinger equations

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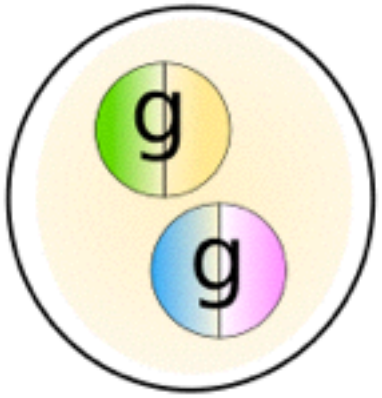
HIC for FAIR Helmholtz International Center

Overview

1. Introduction

2. Gluons and glueballs



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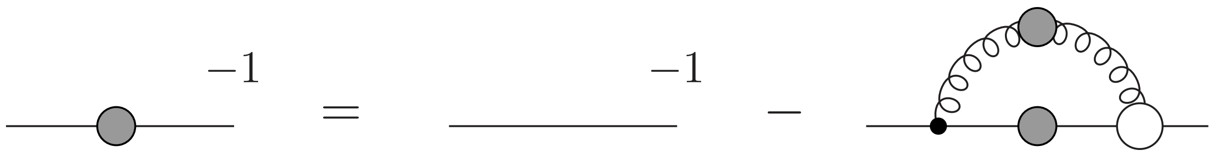

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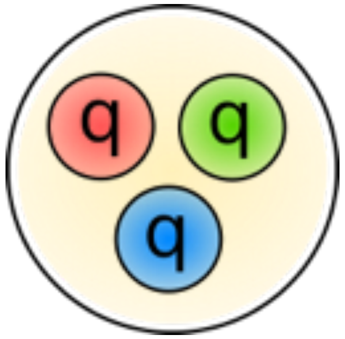
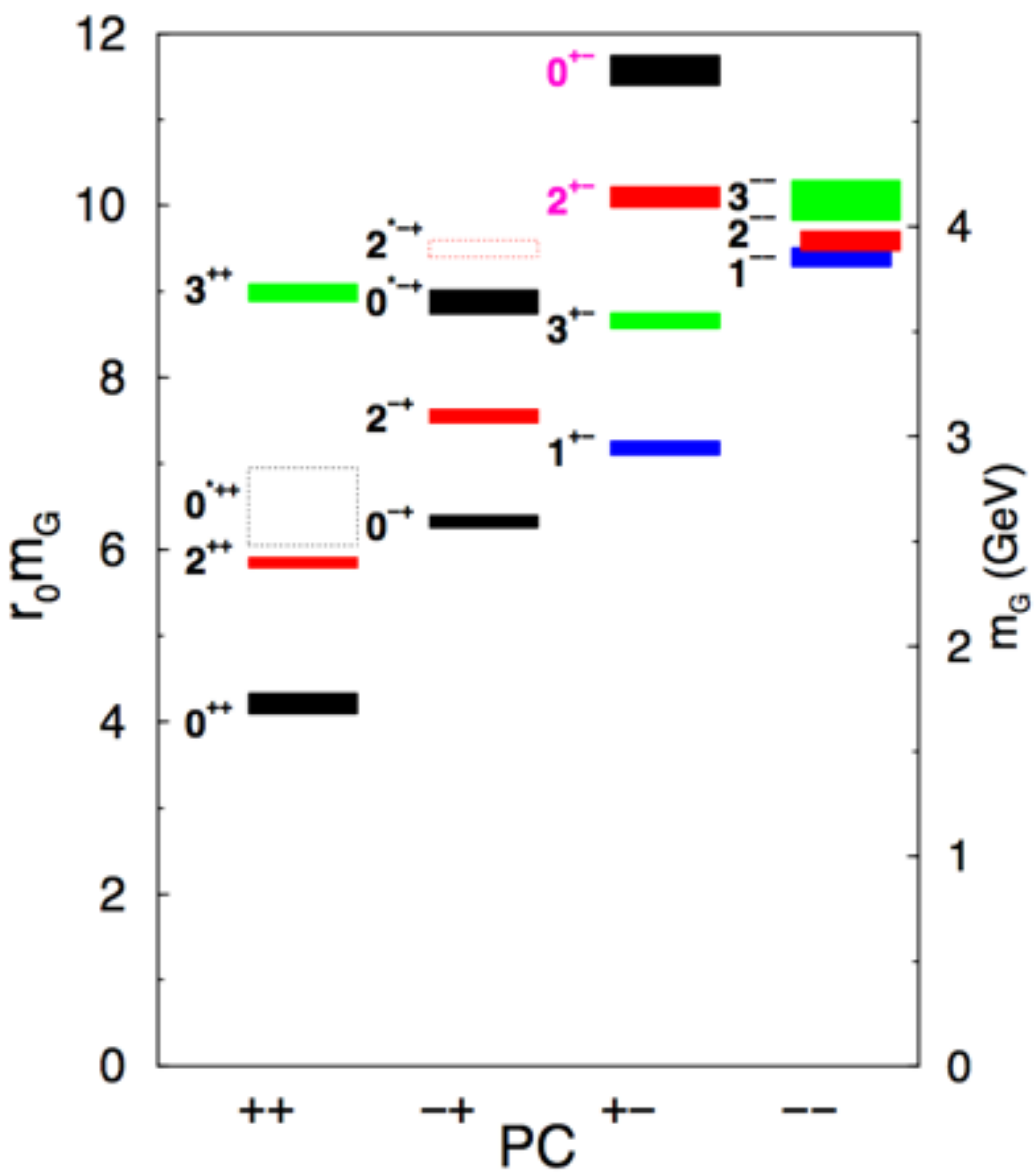


Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

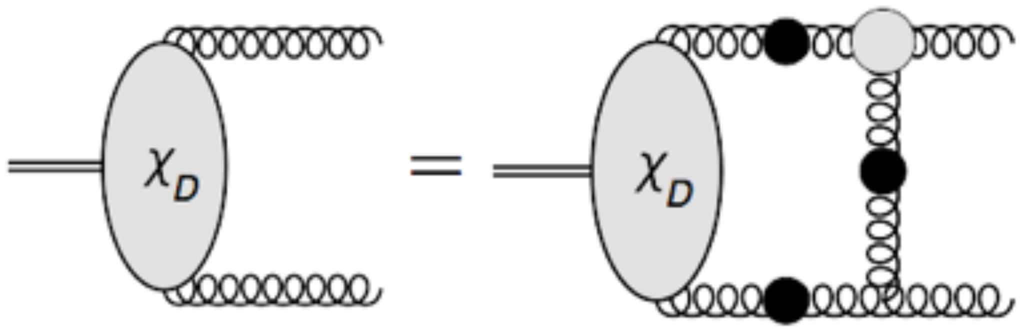
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Preliminary unquenched results: larger masses

Gregory et al., JHEP 1210 (2012) 170

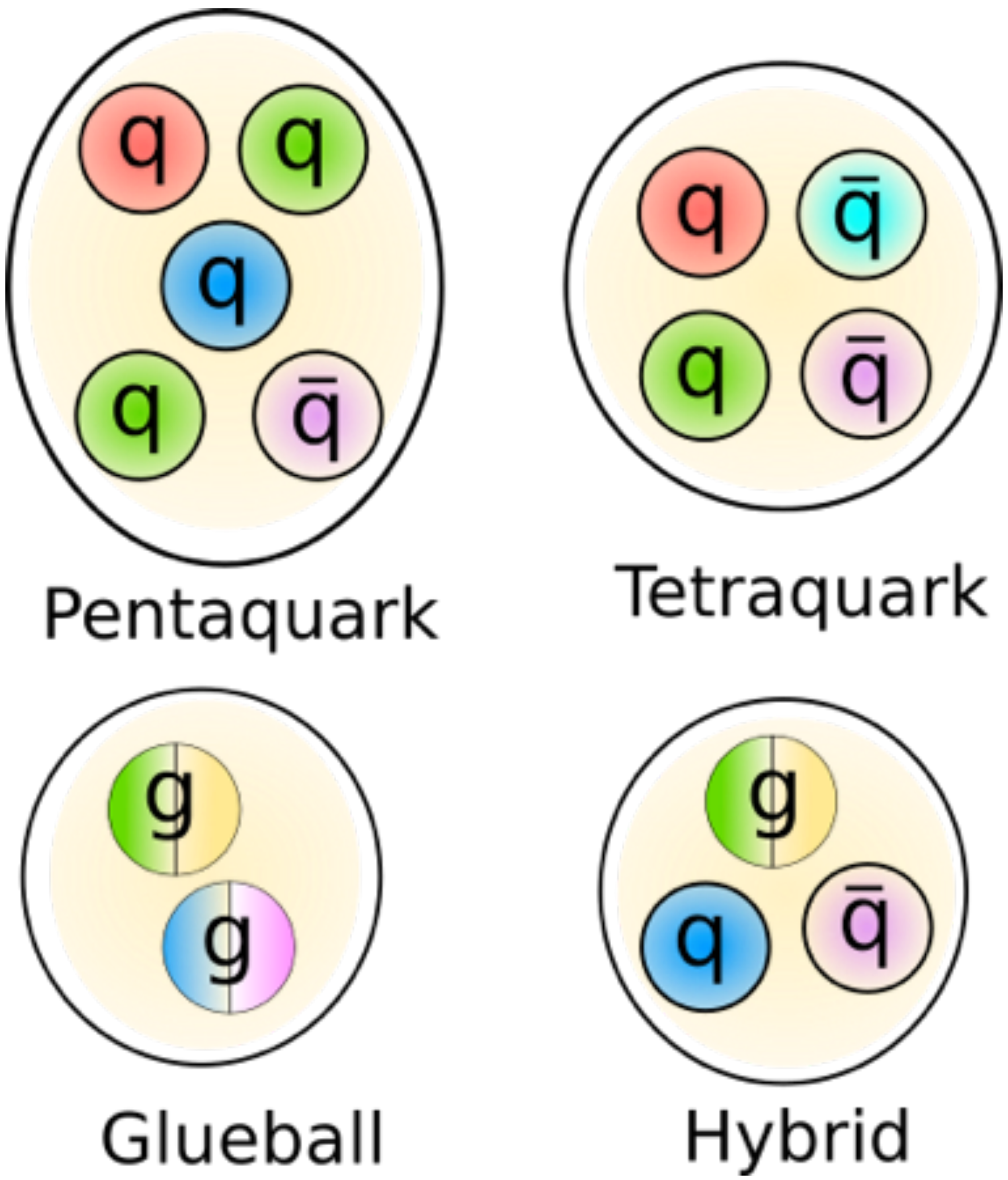
DSE:

- structural information



Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Tetraquarks in the light meson sector



$$f_0(980) \to \pi\pi, K\bar{K}$$
$$a_0(980) \to \pi\eta, K\bar{K}$$

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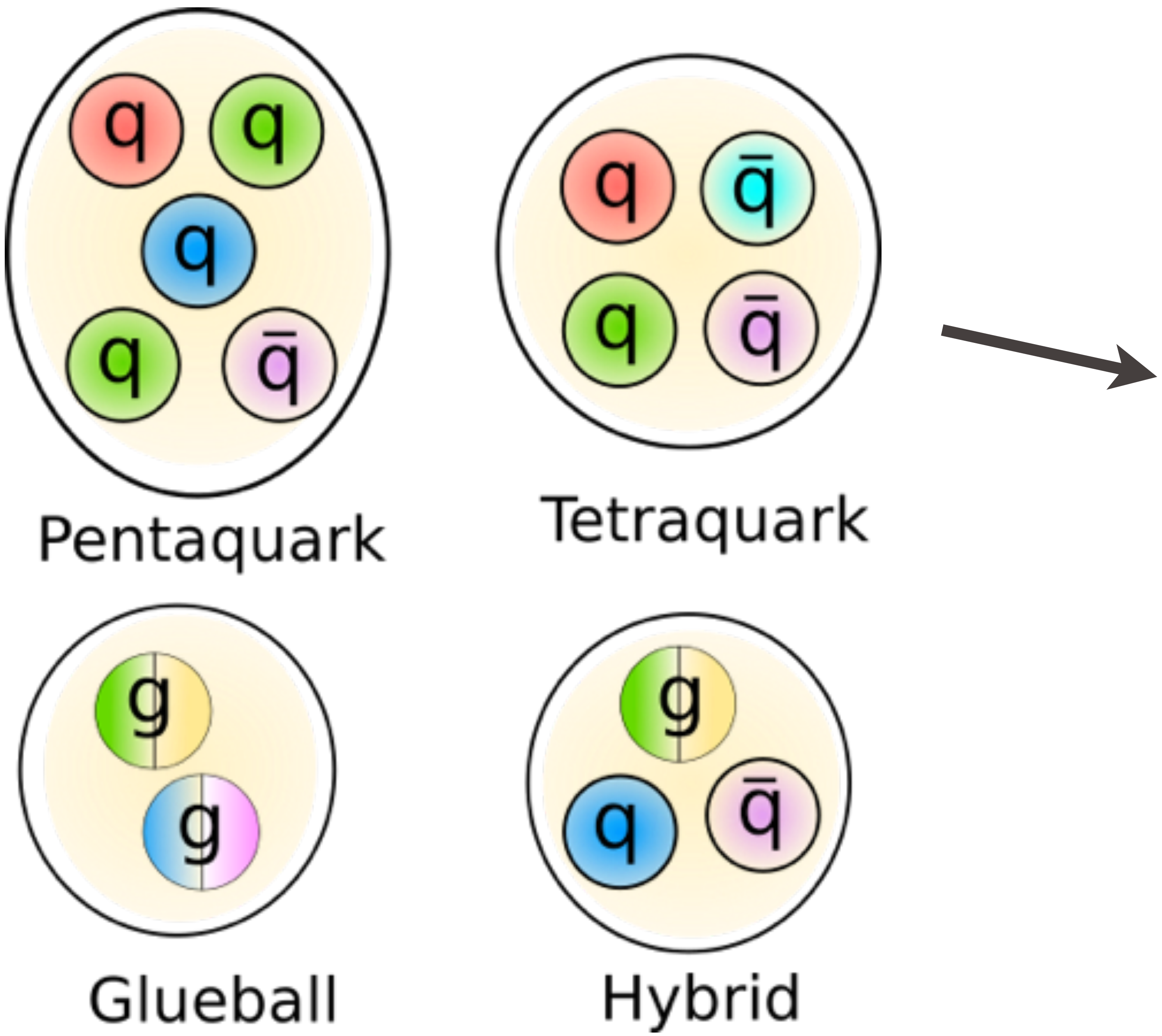
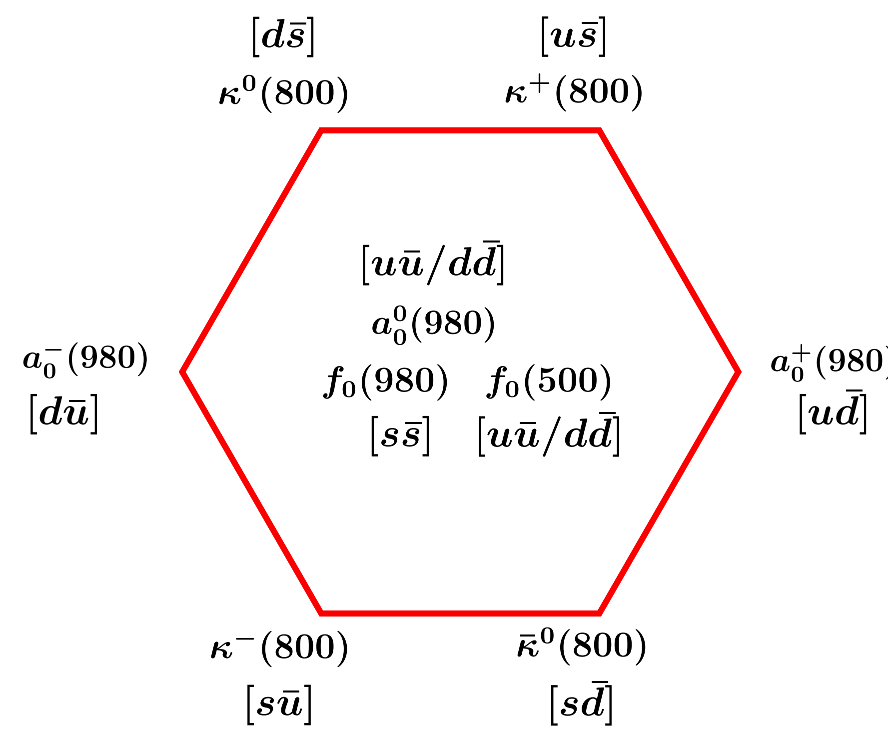


Light meson sector:
Scalars!



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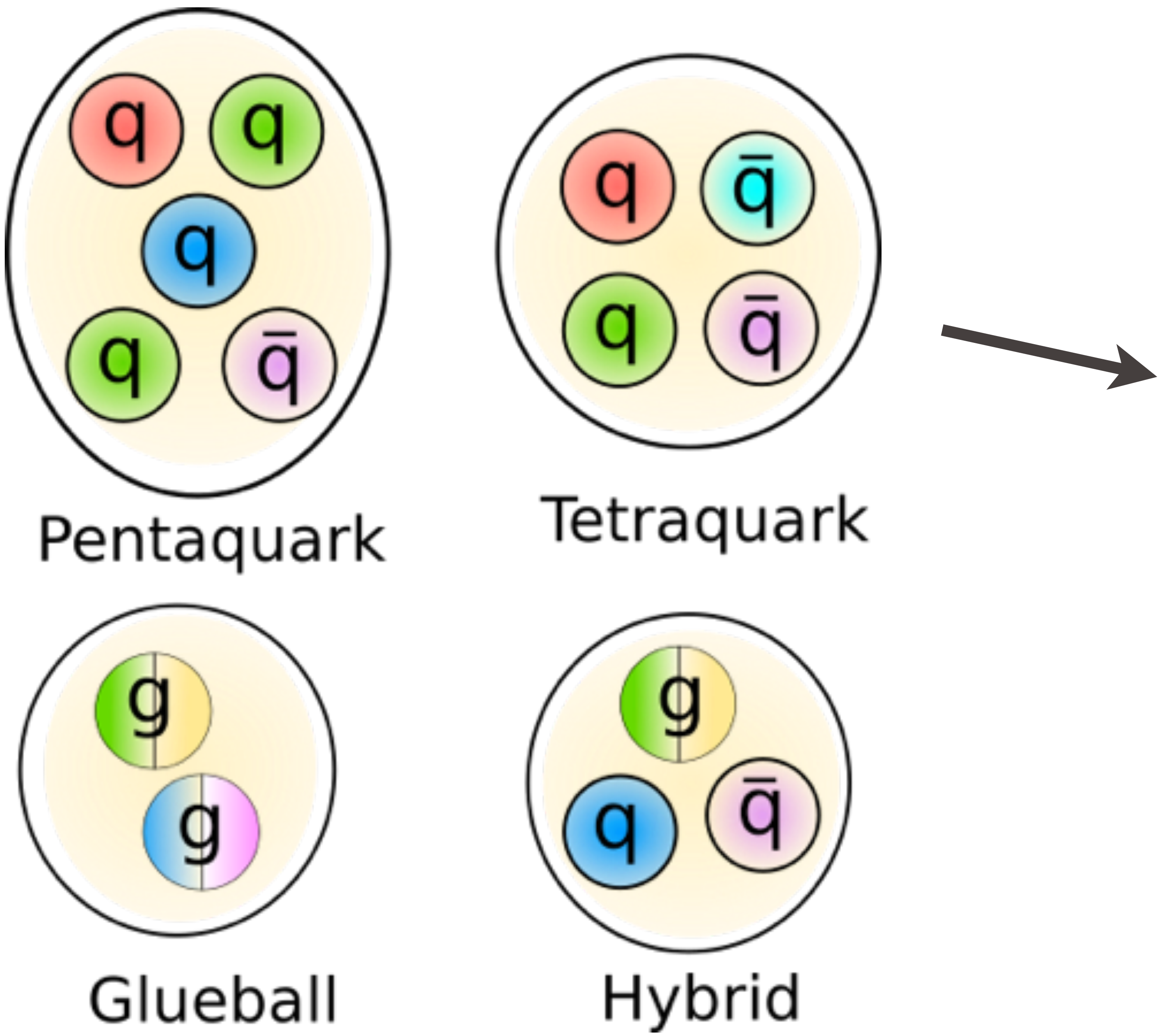
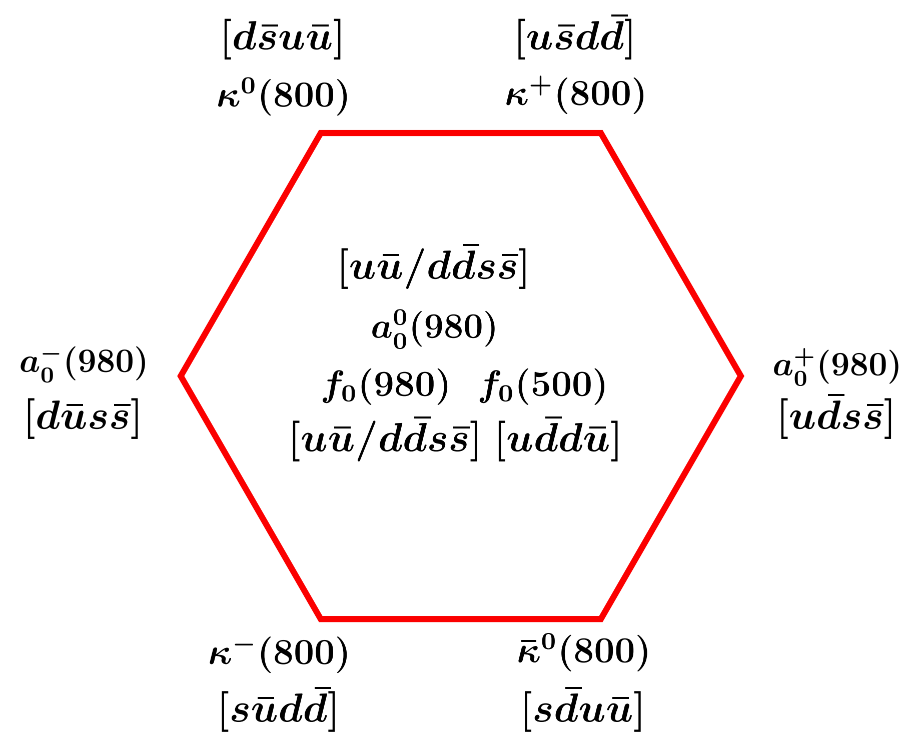


Light meson sector: Scalars!



$$f_0(980) \to \pi\pi, K\bar{K}$$

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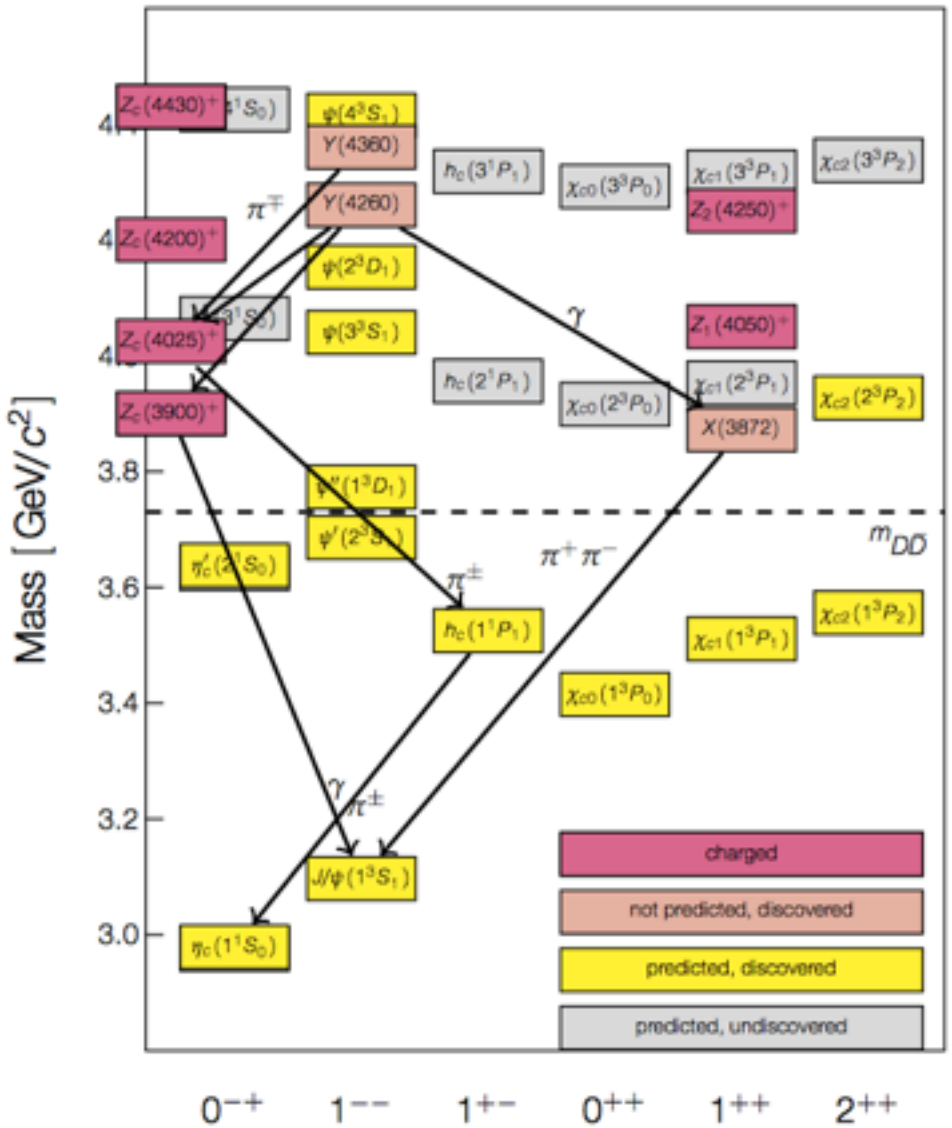
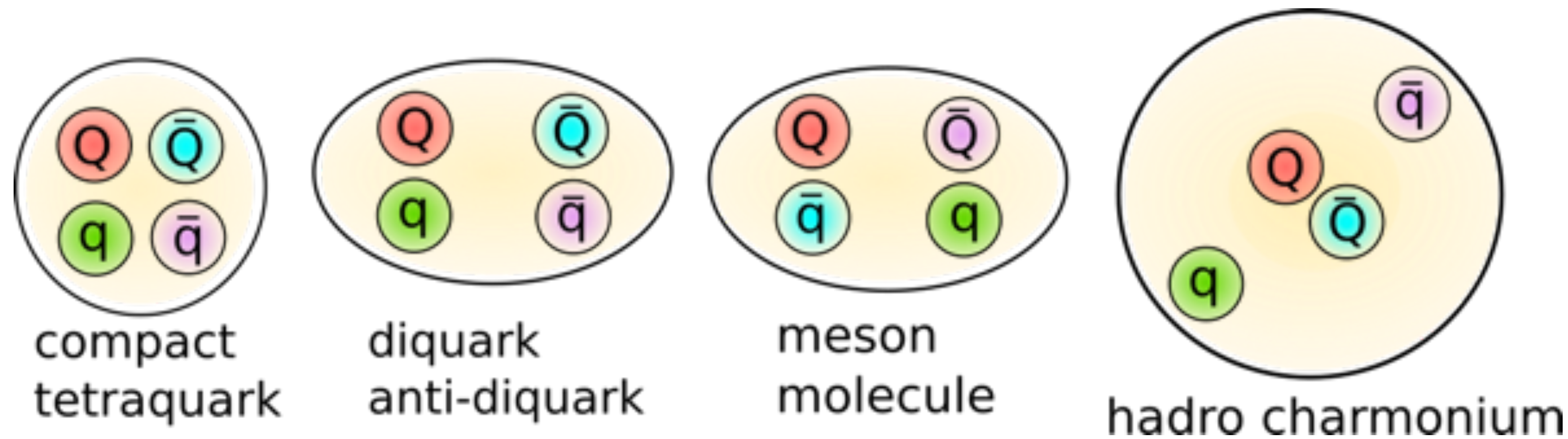
Tetraquark candidates in charmonium region



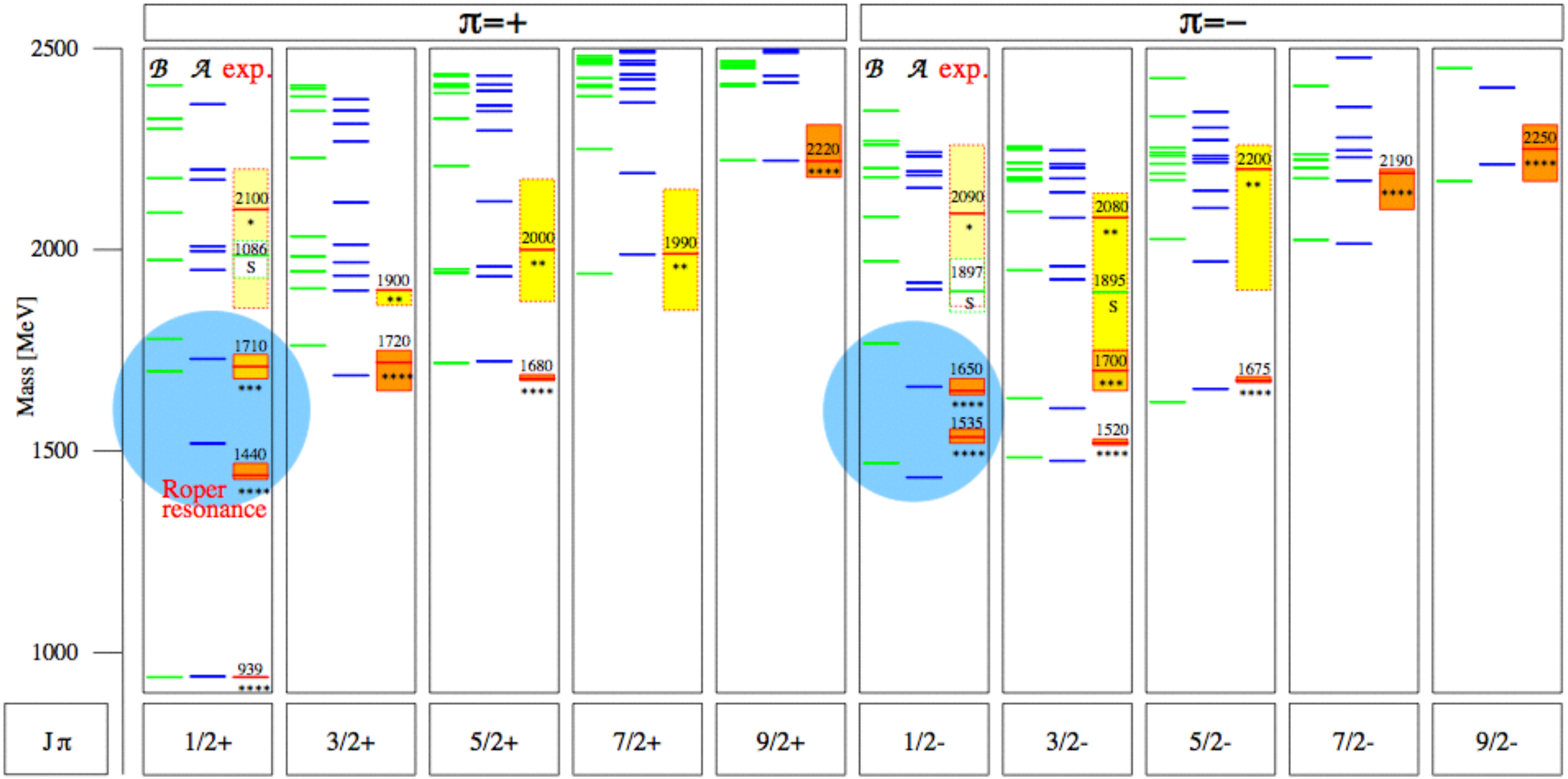
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks

Baryons: quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances' ?!
- parity doubling ?!
- level ordering: $N\frac{1}{2}^{\pm}$ vs. $\Lambda\frac{1}{2}^{\pm}$

Explaining the Roper

- Quark model: p(2S), but generically too large mass

  e.g. Loring, Metsch, Petry, EPJA 10 (2001) 395 and many others…

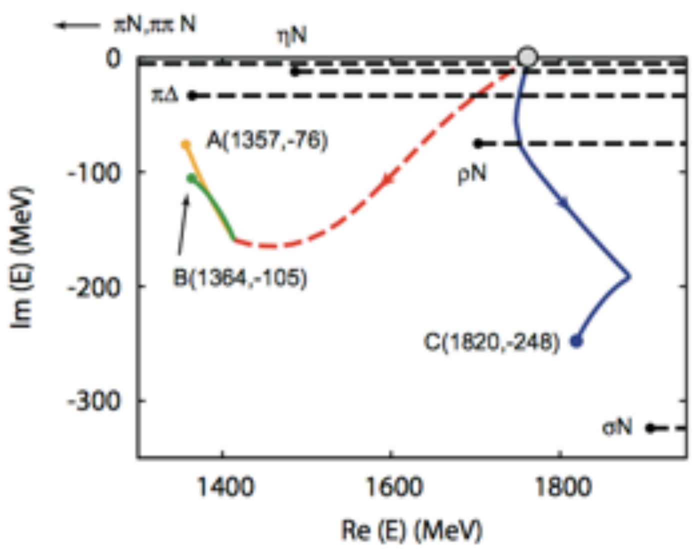
- Hybrid ? Evidence from lattice to the contrary

  Dudek, Edwards, PRD 85 (2012) 054016

- Dynamically generated by coupled channels (no 'bare' state)

  Krehl, Hanhart, Krewald and Speth, PRC C 62 (2000) 025207
  Doring, Hanhart, Huang, Krewald and Meissner, NPA 829 (2009) 170

- Dynamically modified by coupled channels

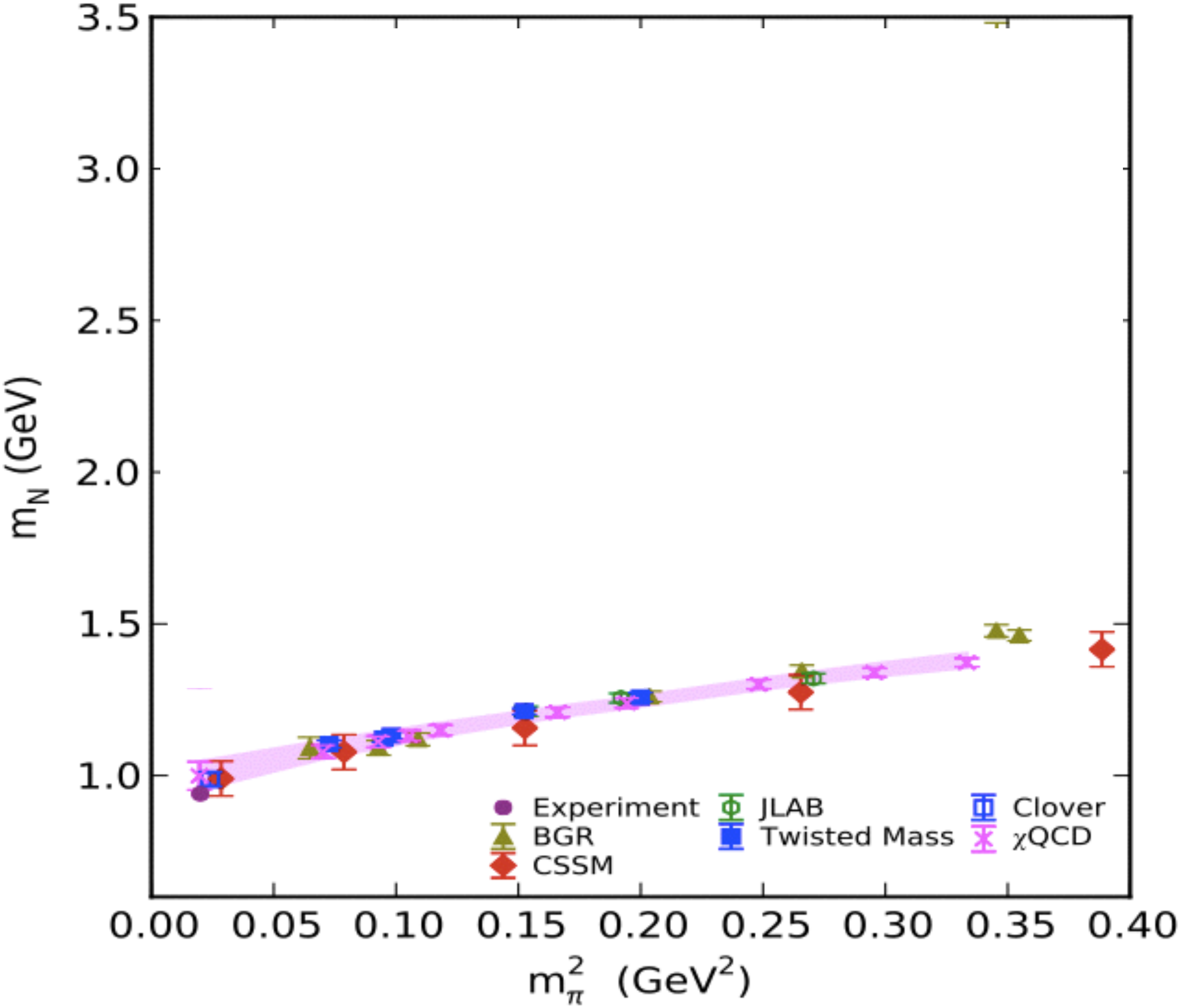


  Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama and Sato, PRL 104 (2010) 042302

- 'bare' state via DSE/Faddeev (NJL, QCD inspired model)

  Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
  Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17

Lattice QCD and the Roper



Liu, Chen, Gong, Sufian, Sun and Li, PoS LATTICE 2013 (2014) 507

$$m_\pi^2 \sim m_q$$

(Gell-Mann-Oakes-Renner)

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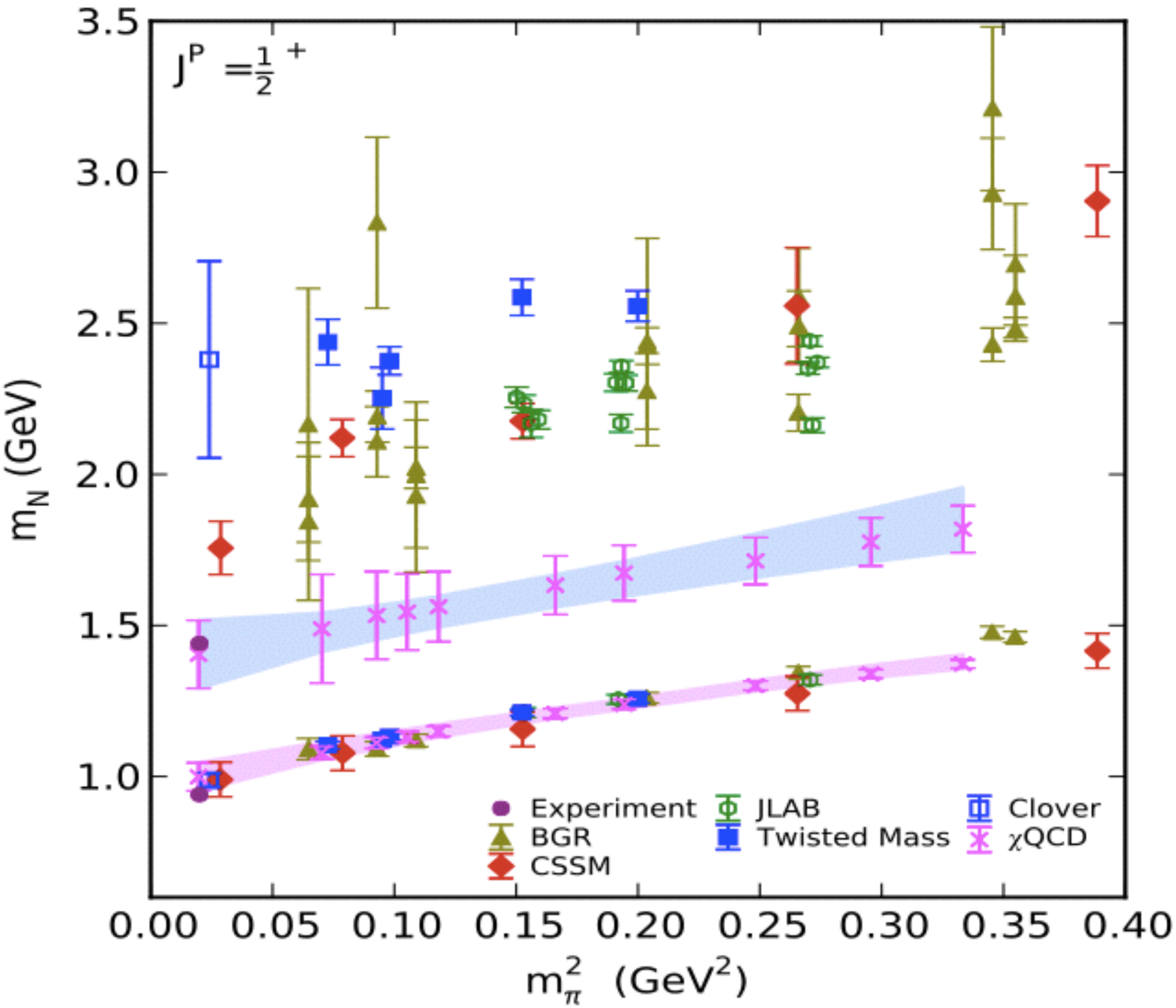


Liu, Chen, Gong, Sufian, Sun and Li, PoS LATTICE 2013 (2014) 507

- different actions
- volume effects
- interpolators
- large discrepancies !

$$m_\pi^2 \sim m_q$$

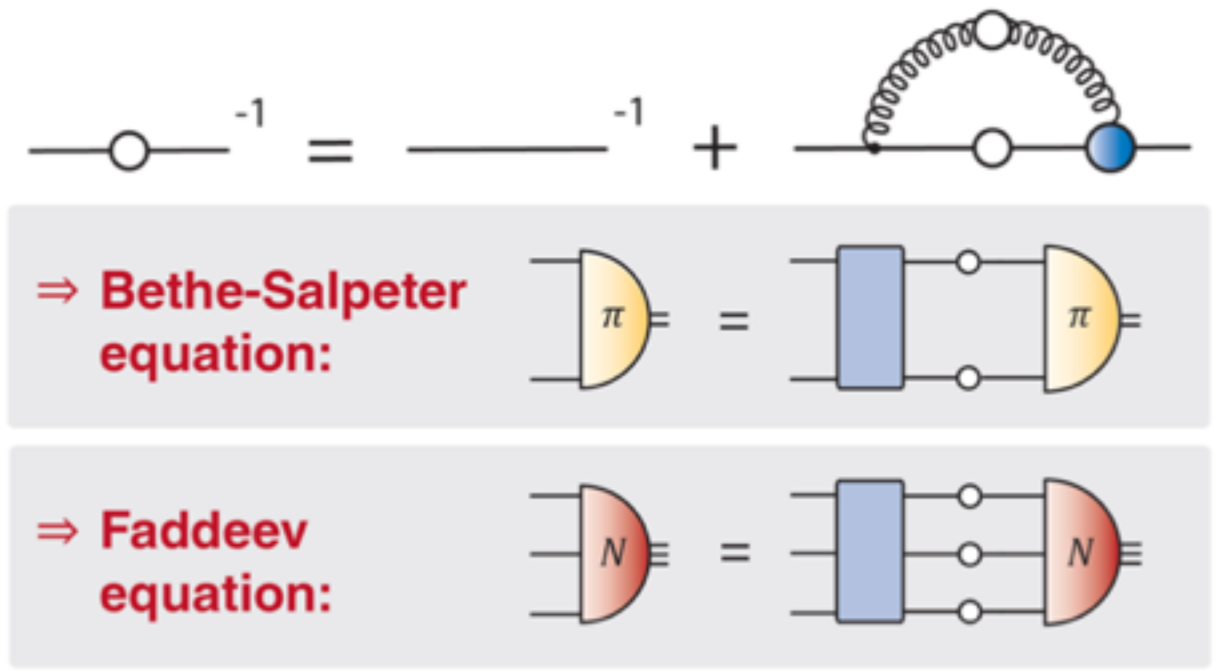
(Gell-Mann-Oakes-Renner)

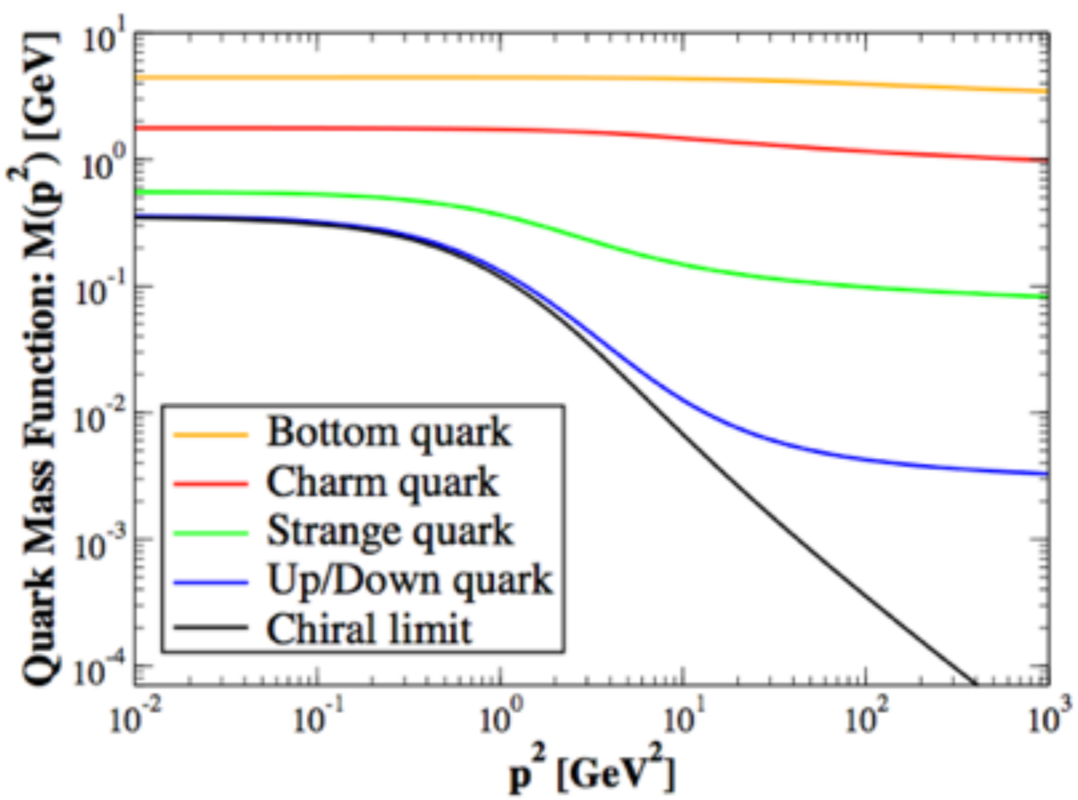
Bound states and resonances from DSEs, BSEs, FEs

General goal:
Experimental observables from nonperturbative quark and gluon structure of QCD

Framework: DSEs, BSEs, FEs

- Dynamics at perturbative and nonperturbative scales
- Dynamical chiral symmetry breaking: connects dynamically generated 'constituent-quark mass' with current quark mass
- Dynamical realization of Goldstone boson nature of pseudoscalar mesons





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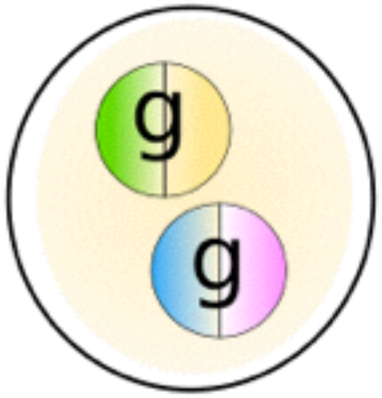

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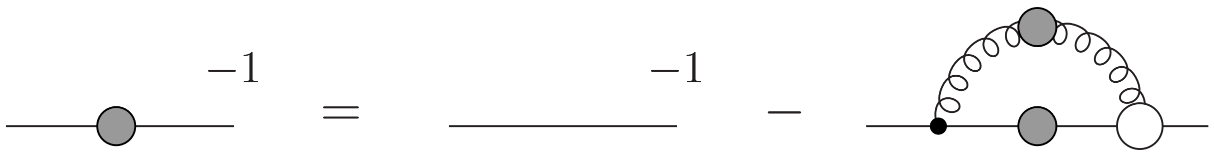

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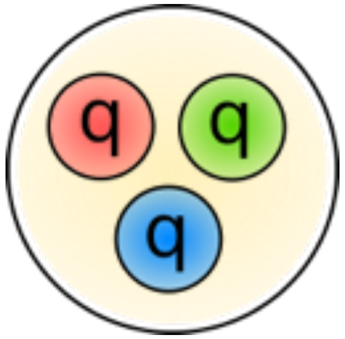
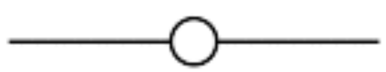


Quarks, gluons and ghosts

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \exp\left\{ -\int d^4x \left(\bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}(F^a_{\mu\nu})^2 + \text{gauge term} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge propagators in momentum space,

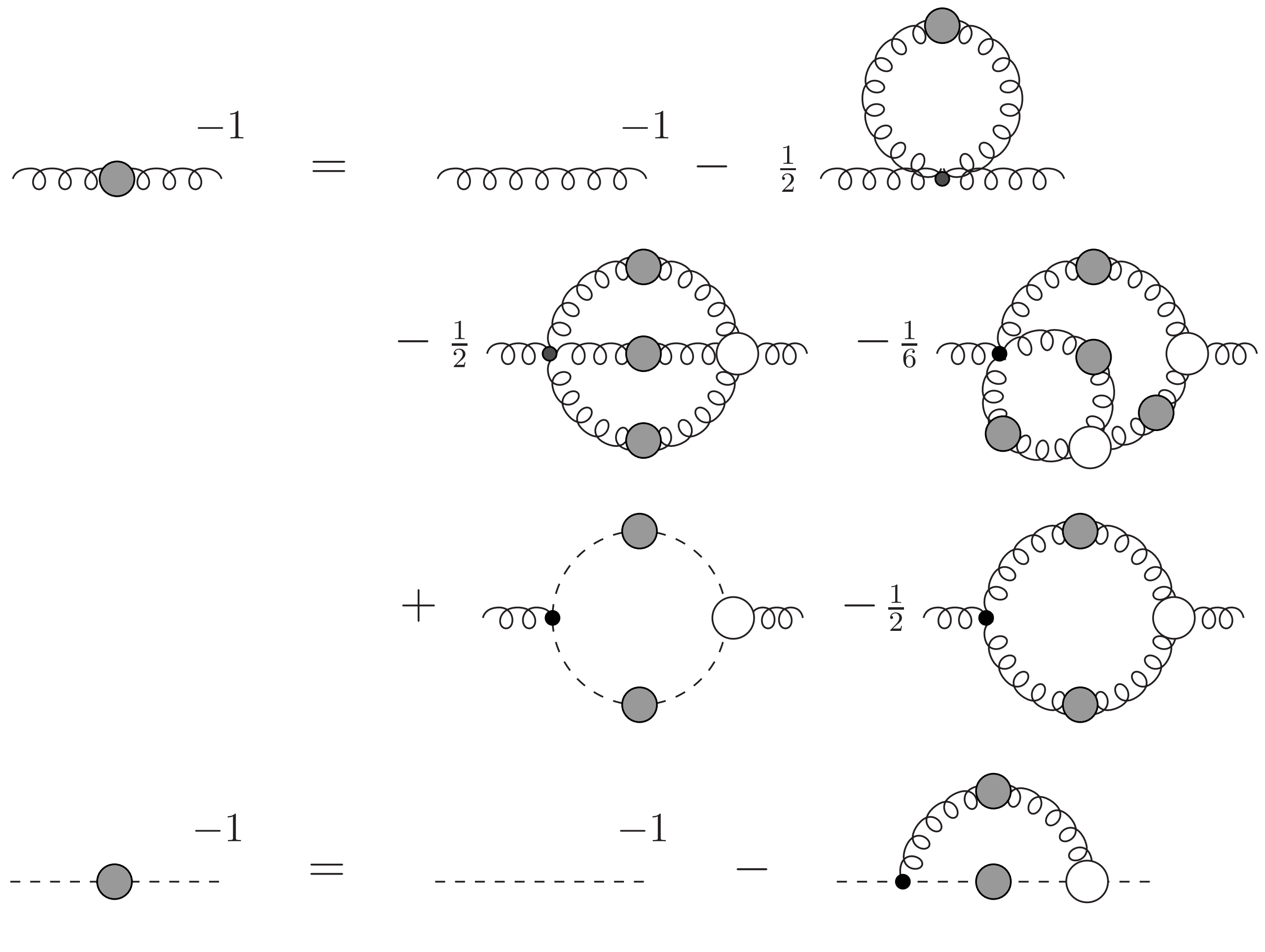
$$D^{Gluon}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$



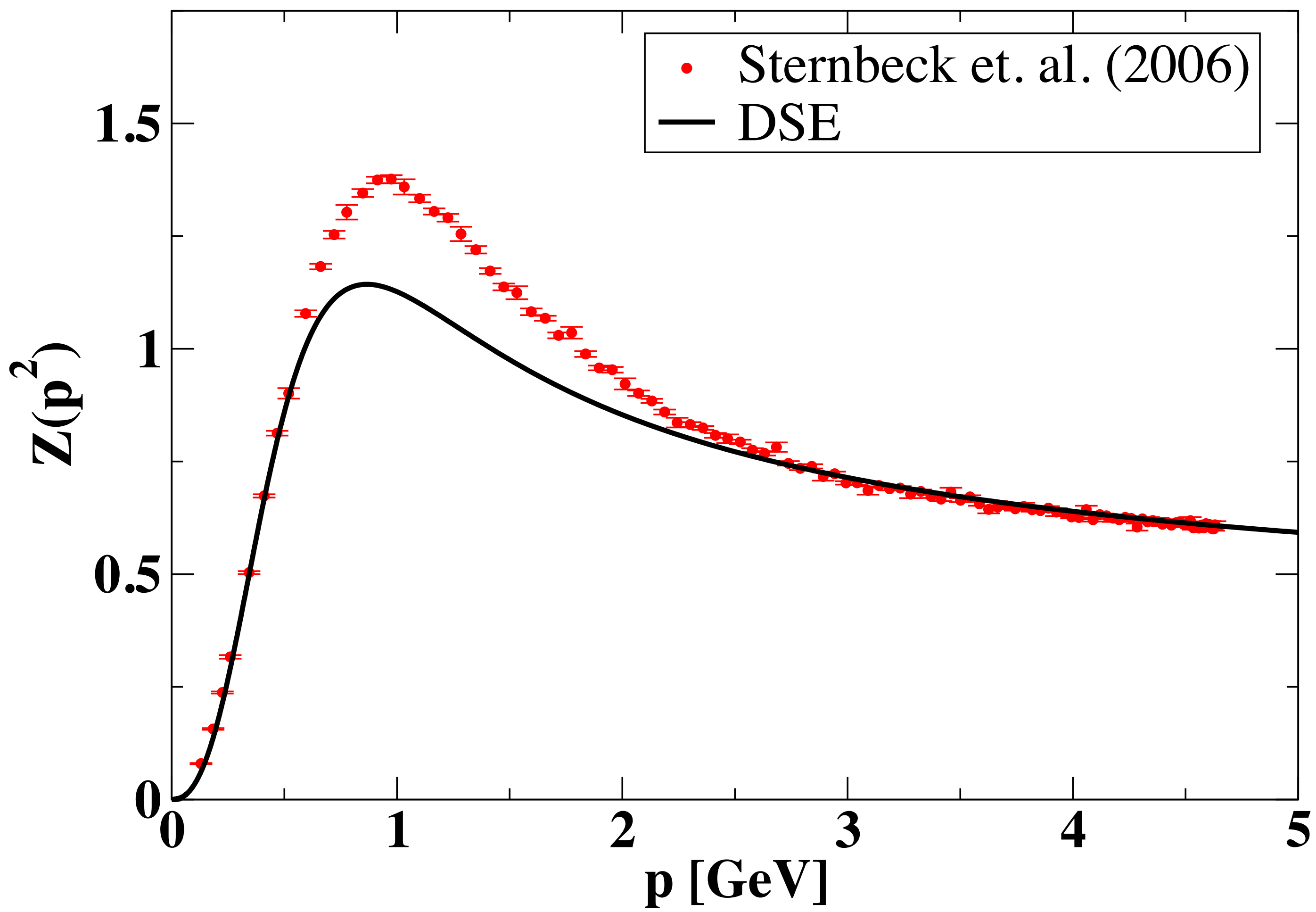
$$S^{Quark}(p) = Z_f(p^2)[-i\not{p} + M(p^2)]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

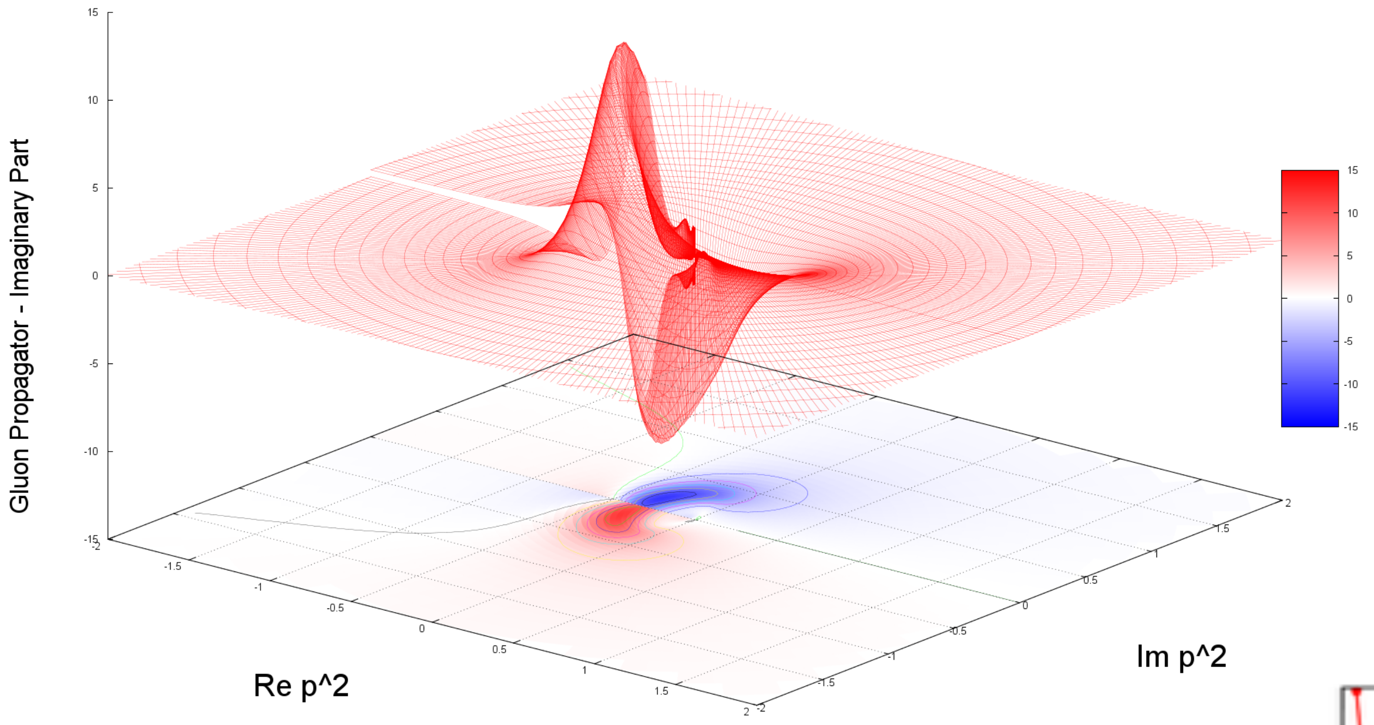


- spacelike momenta:
  good agreement with lattice
- fully dressed gluon is not massless !
- recent improvement: 3g-vertex

Eichmann, Williams, Alkofer, Vujinovic PRD 89, (2014) 10

CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.

Landau gauge gluon propagator

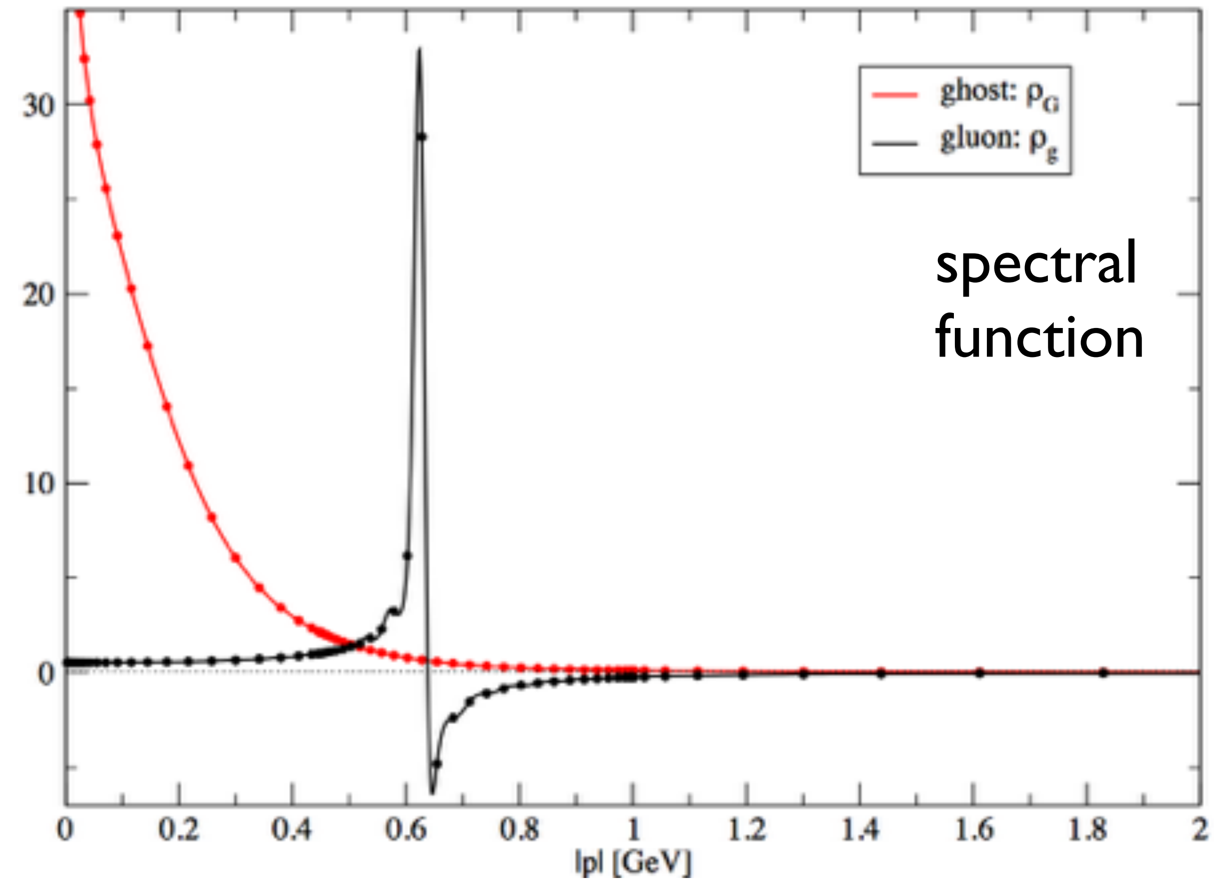


- spectral function: positivity violations

$$600\,\mathrm{MeV} < m_g < 700\,\mathrm{MeV}$$

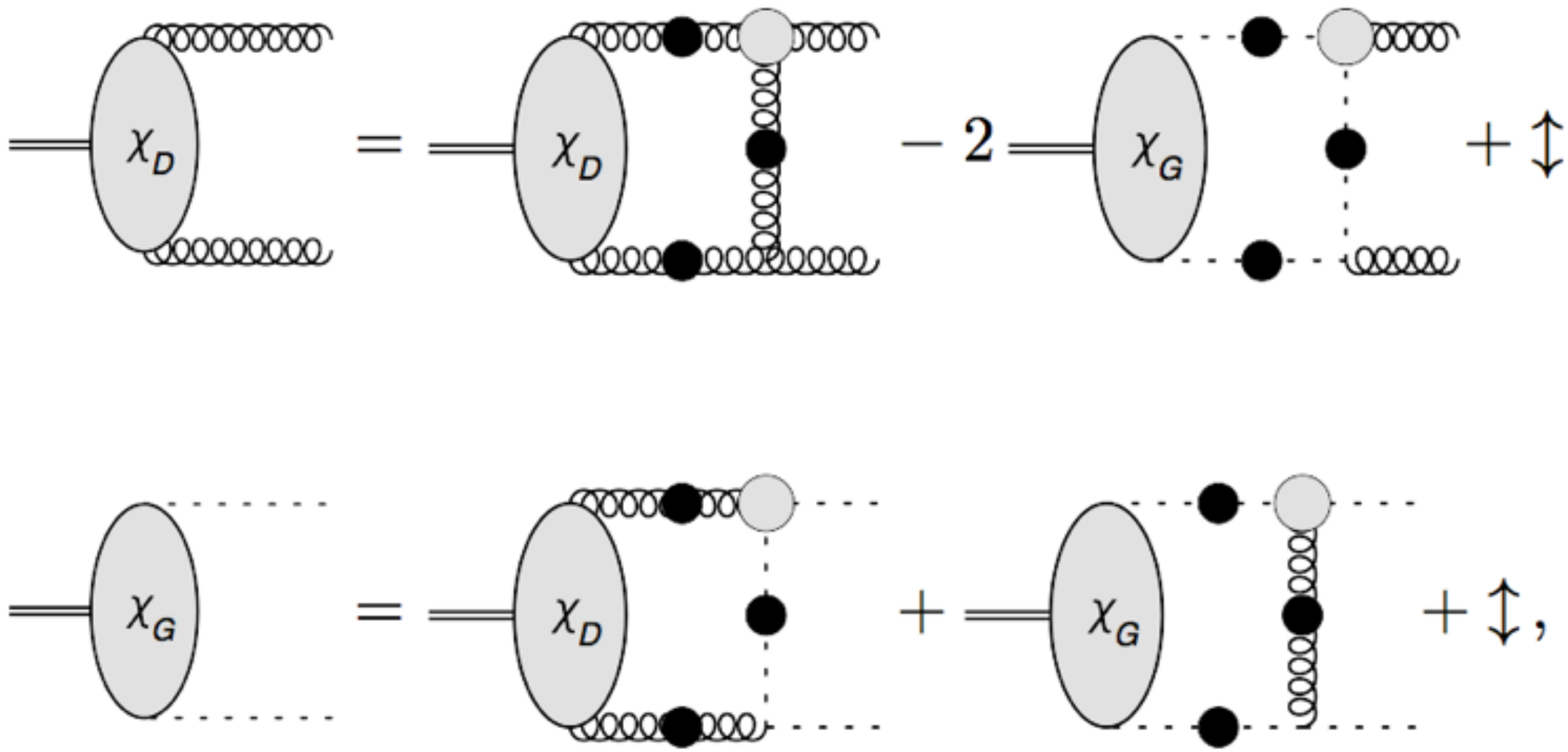
Cornwall, Papavassiliou,...

Gluon cannot appear in detector!



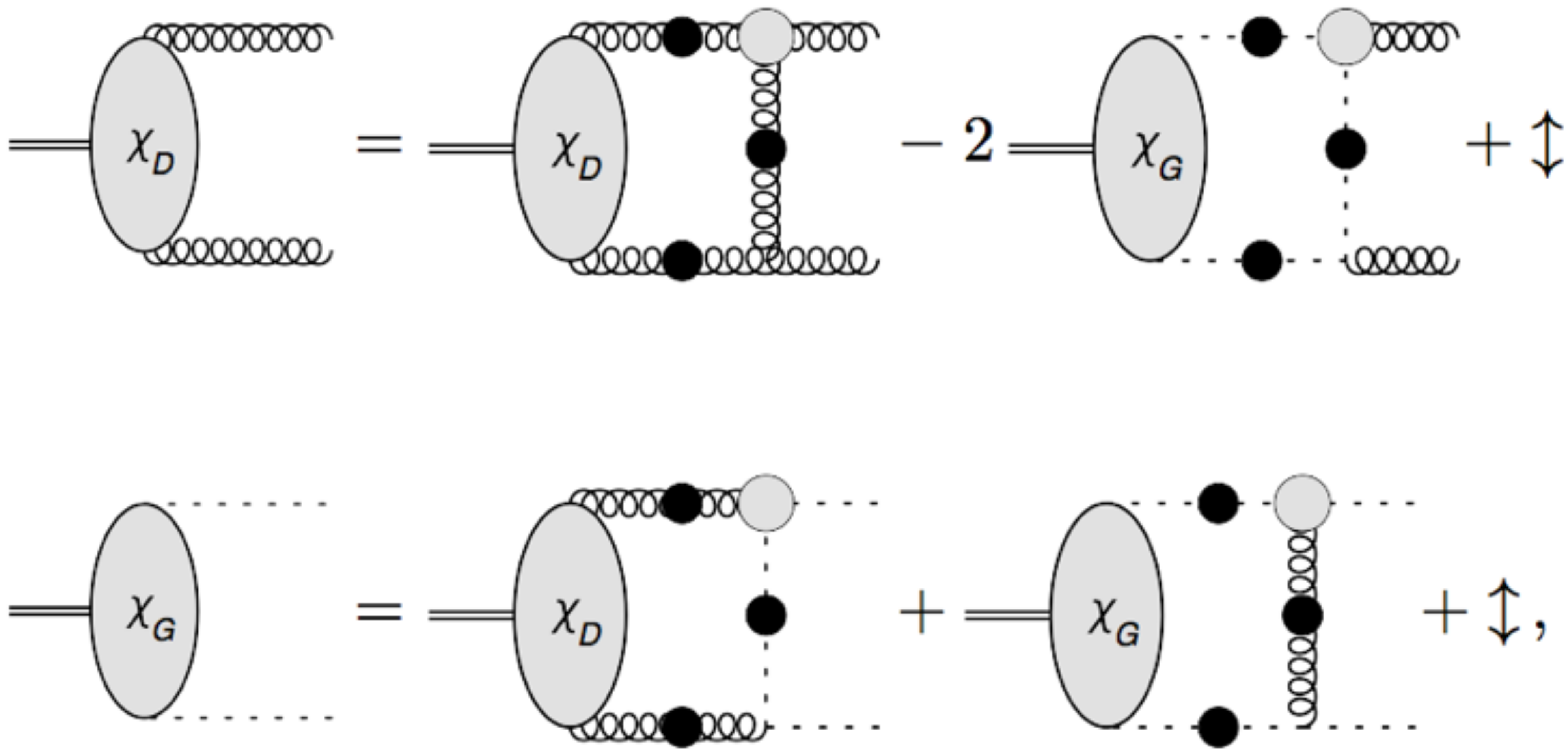
Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Results: $M(0^{++}) = 1.64\,\mathrm{GeV}$

$M(0^{-+}) = 4.53\,\mathrm{GeV}$ ← ghosts do not contribute !

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001
(see also Meyers, Swanson, PRD 87 (2013) 3, 036009)

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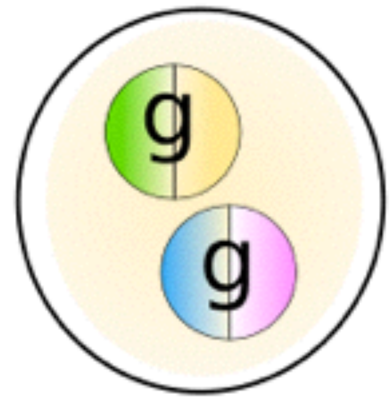

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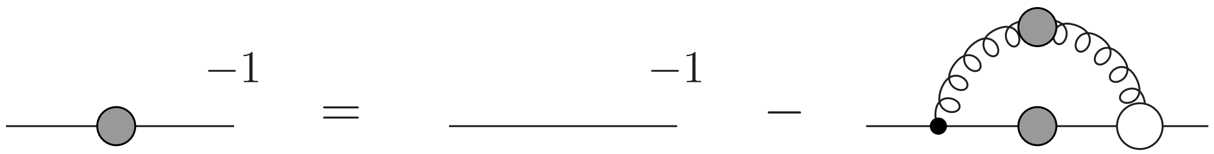

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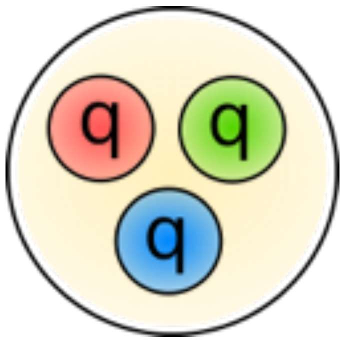
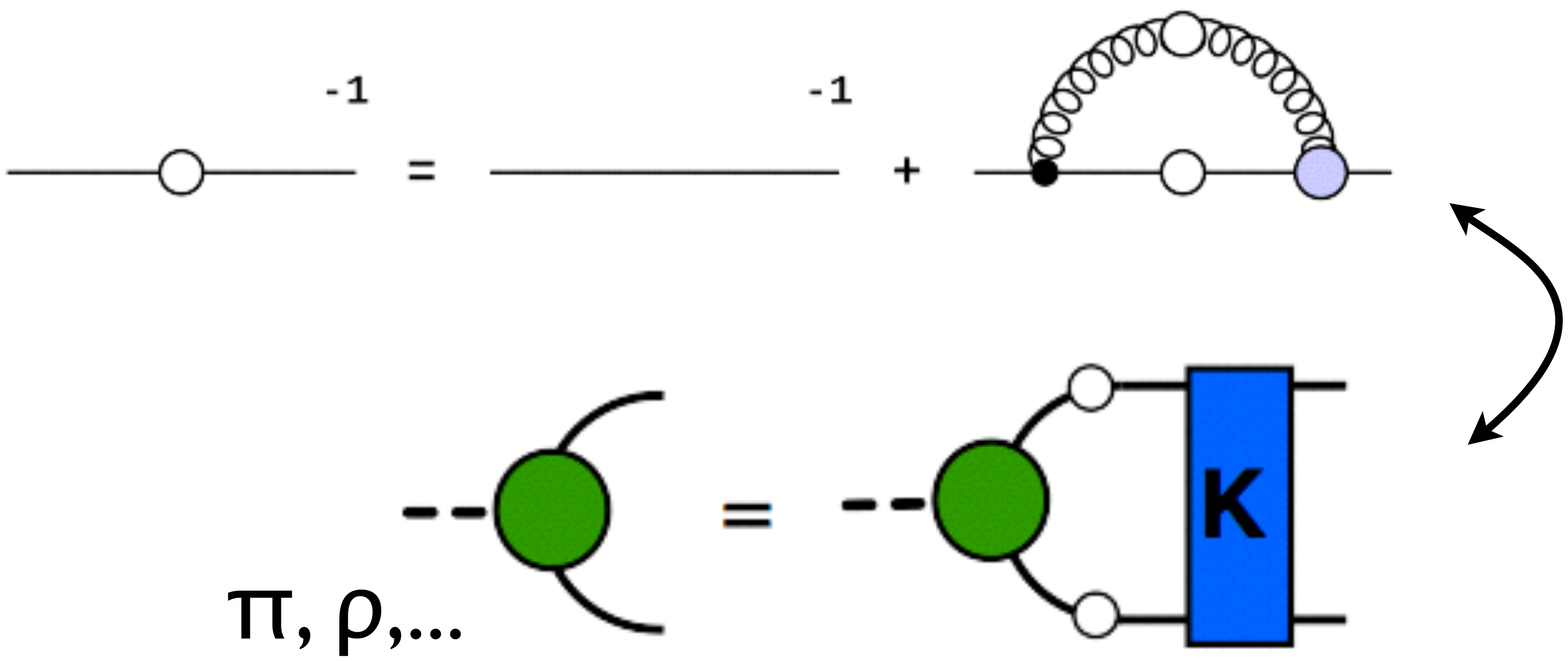


DSEs and Bethe-Salpeter equation



Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

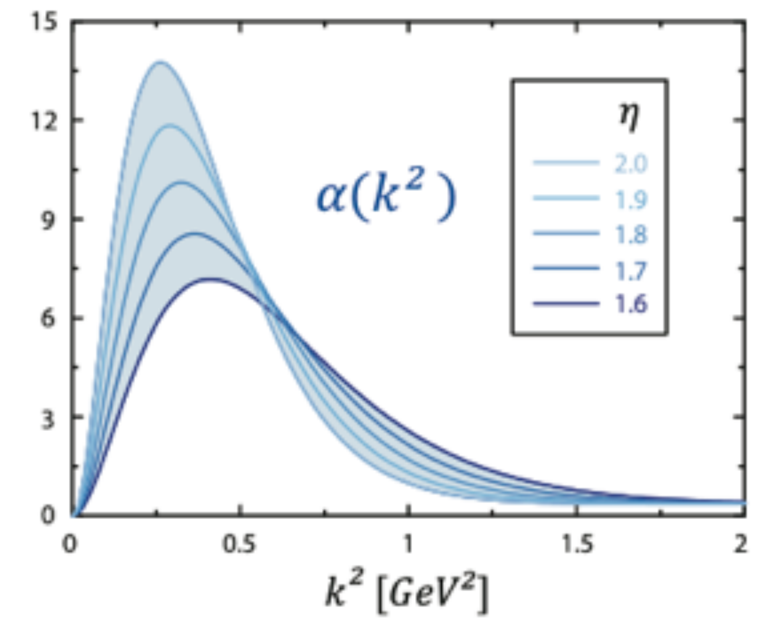
Maris, Roberts, Tandy, PLB 420 (1998) 267

Two strategies:

I. use rainbow-ladder model for quark-gluon interaction
→ok for many phenomenological applications

II. calculate gluon and vertex from their DSEs

Strategie I: Model for quark-gluon interaction



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

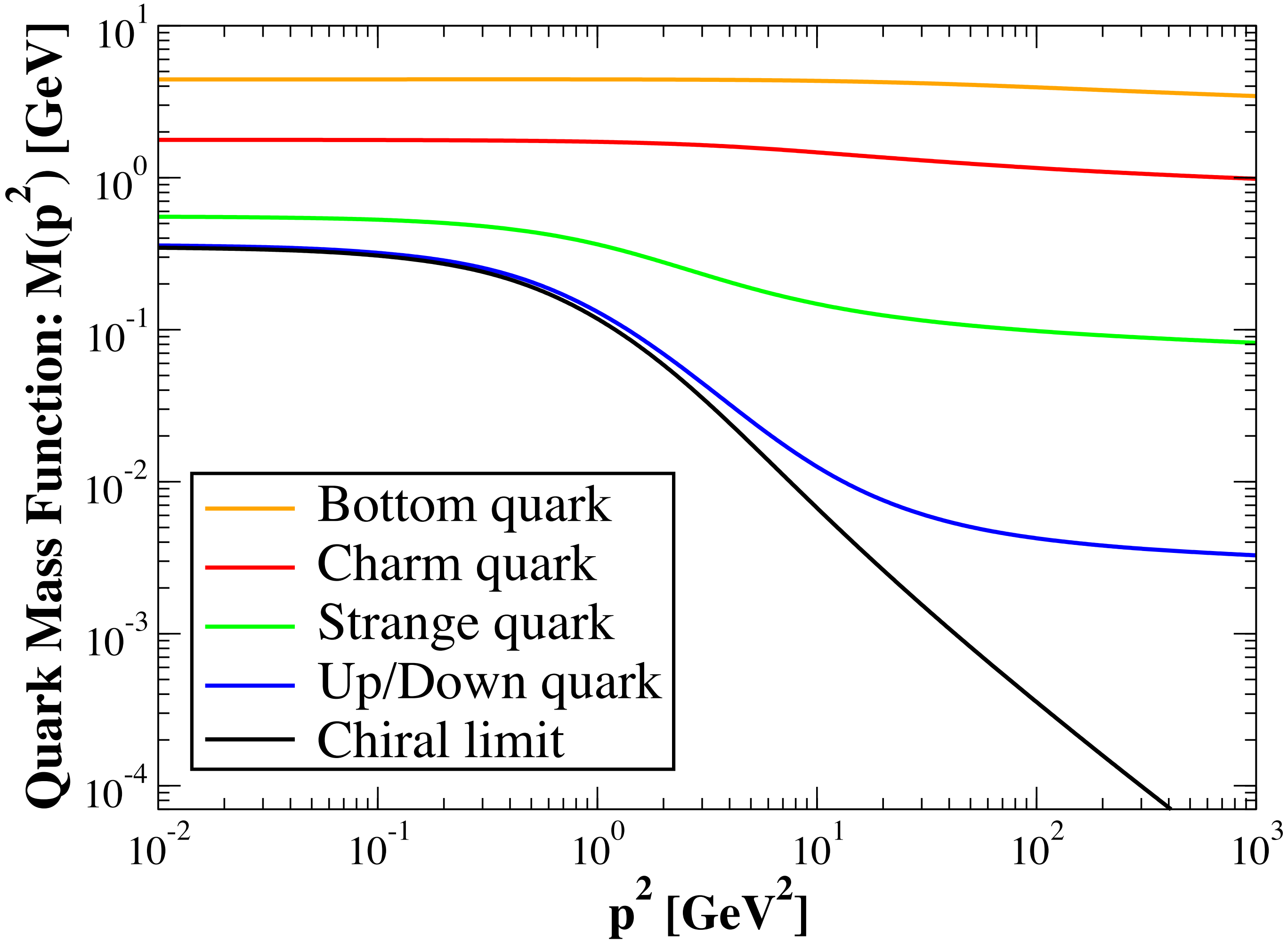
$$\alpha(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- fix Λ from f_π; small dependence of many results on η
- masses $m_u = m_d$, m_s, m_c, from π, K, J/ψ
- Renormalizable and momentum dependent !
- Qualitatively similar to results from explicit calculation

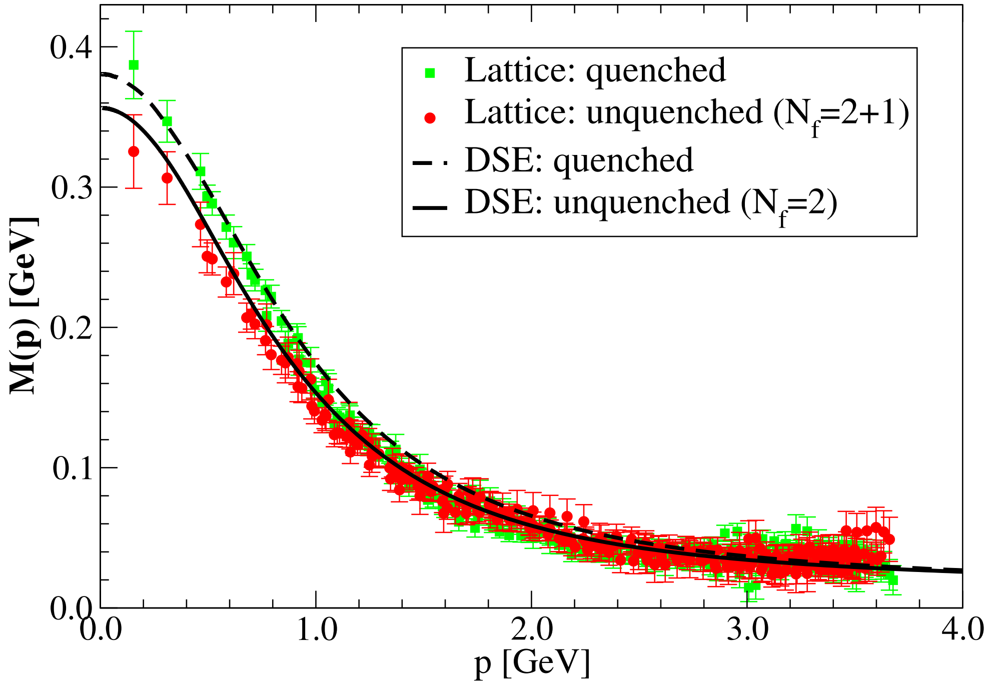
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.
Williams, EPJA 51 (2015) 5, 57.

Quark mass: flavor dependence

Typical solution:



$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$



CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

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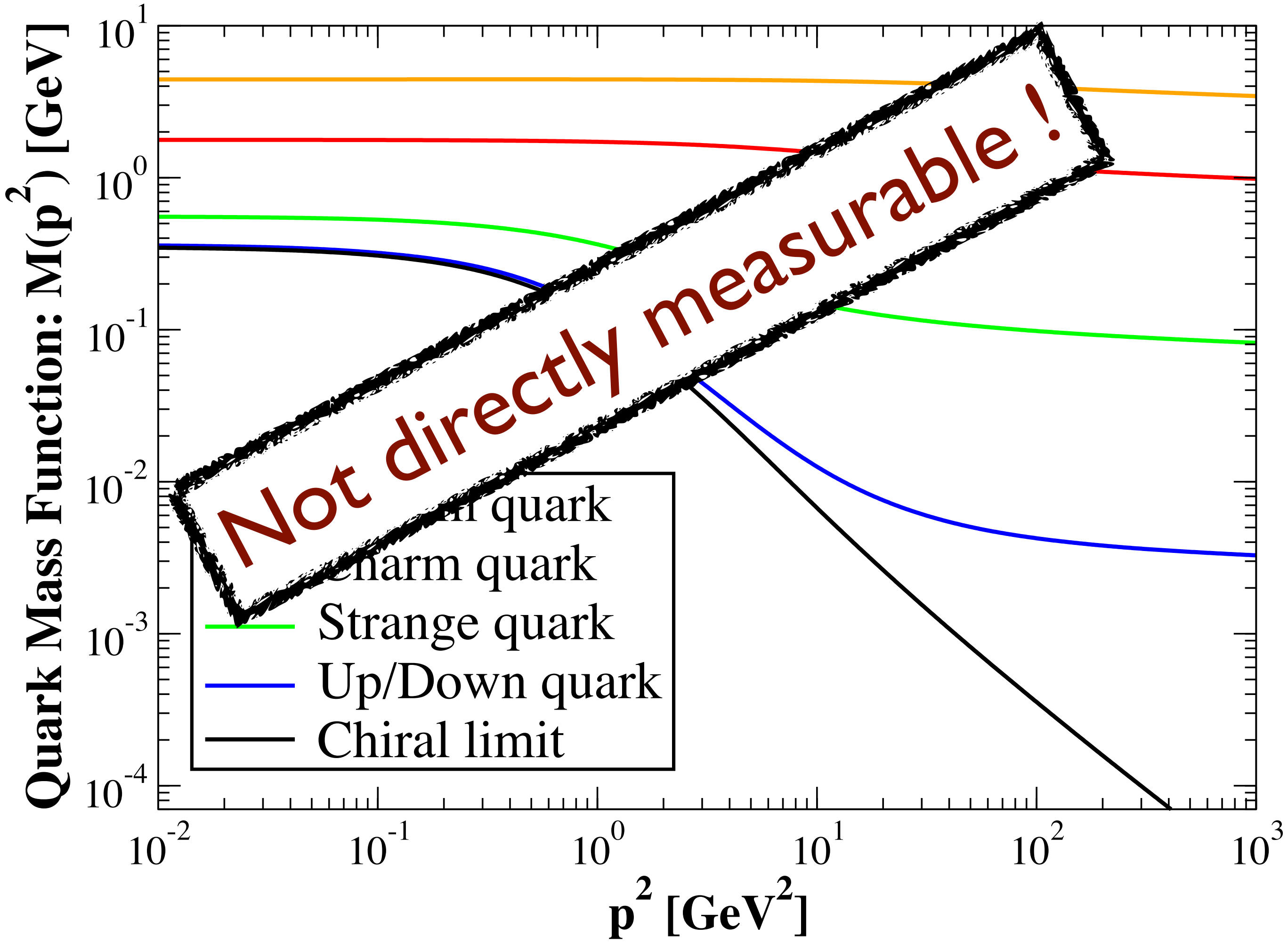

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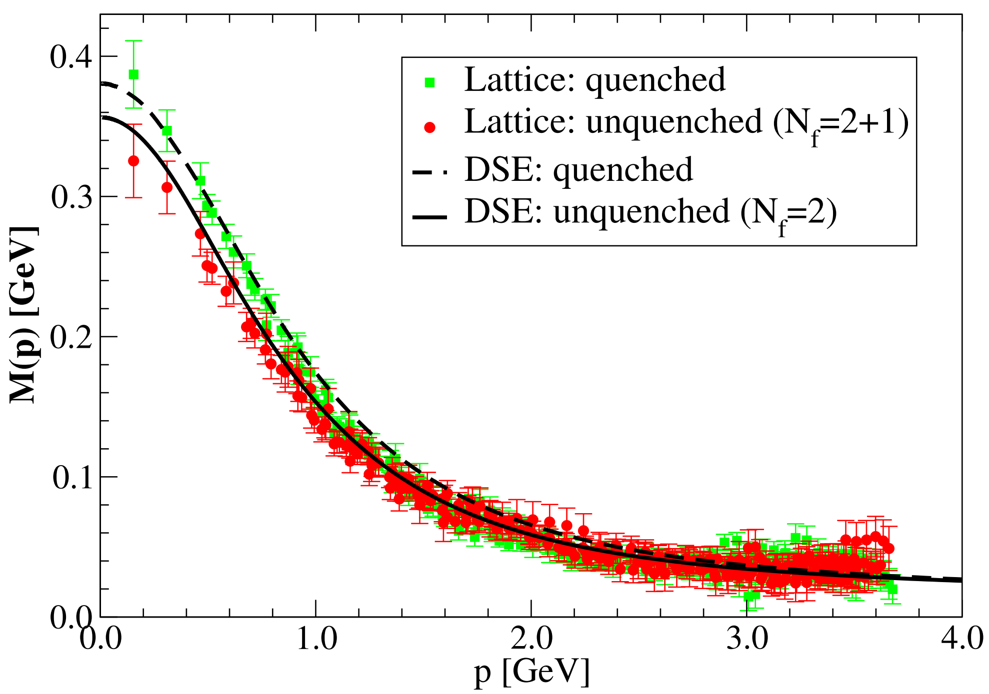

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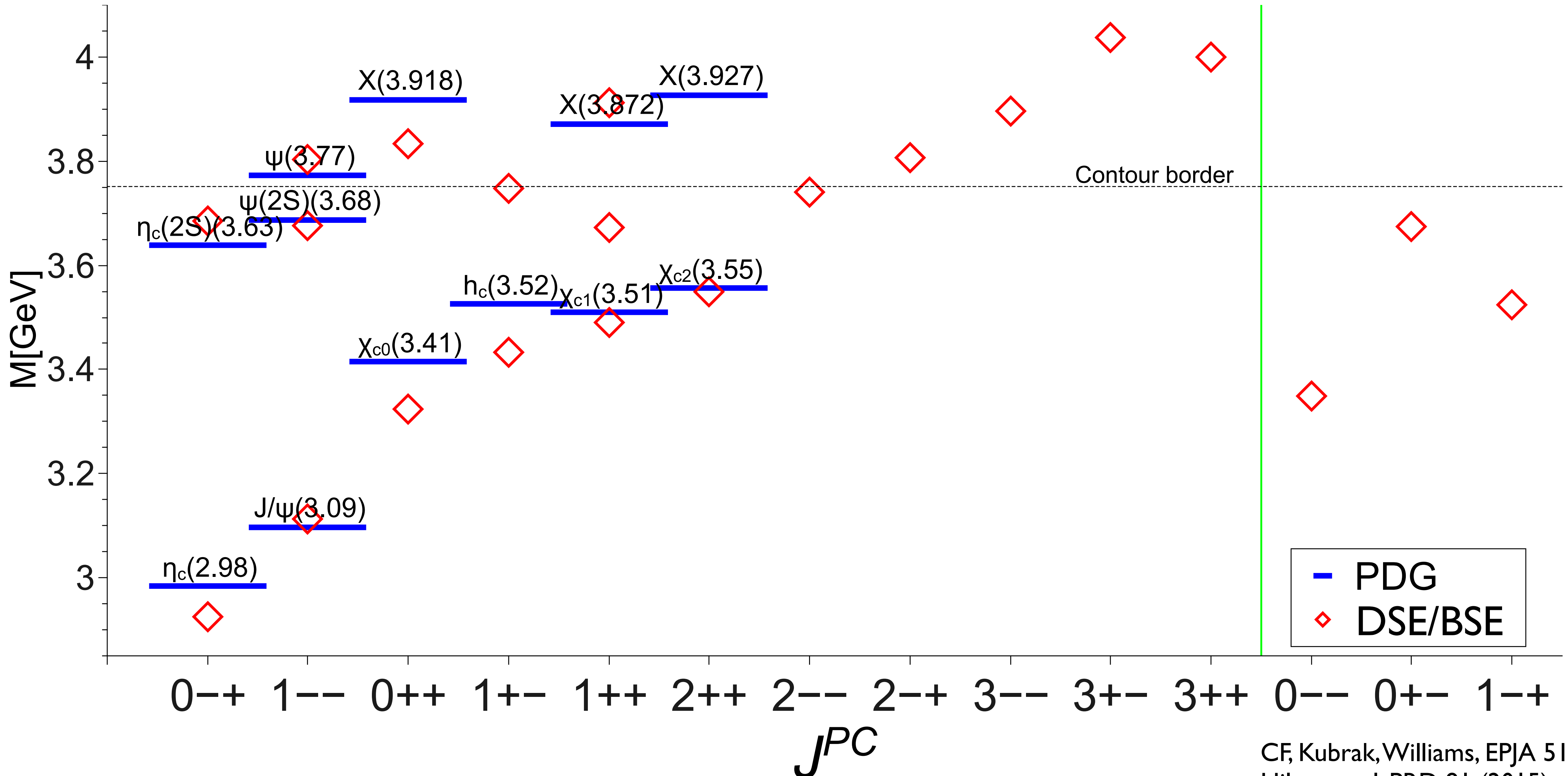
Rainbow-ladder: heavy meson spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$: prediction for tensor state
- acceptable channels : $0^{-+}, 1^{++}, \ldots$
- deficiencies in other channels: 'imbalance' of spin-structure

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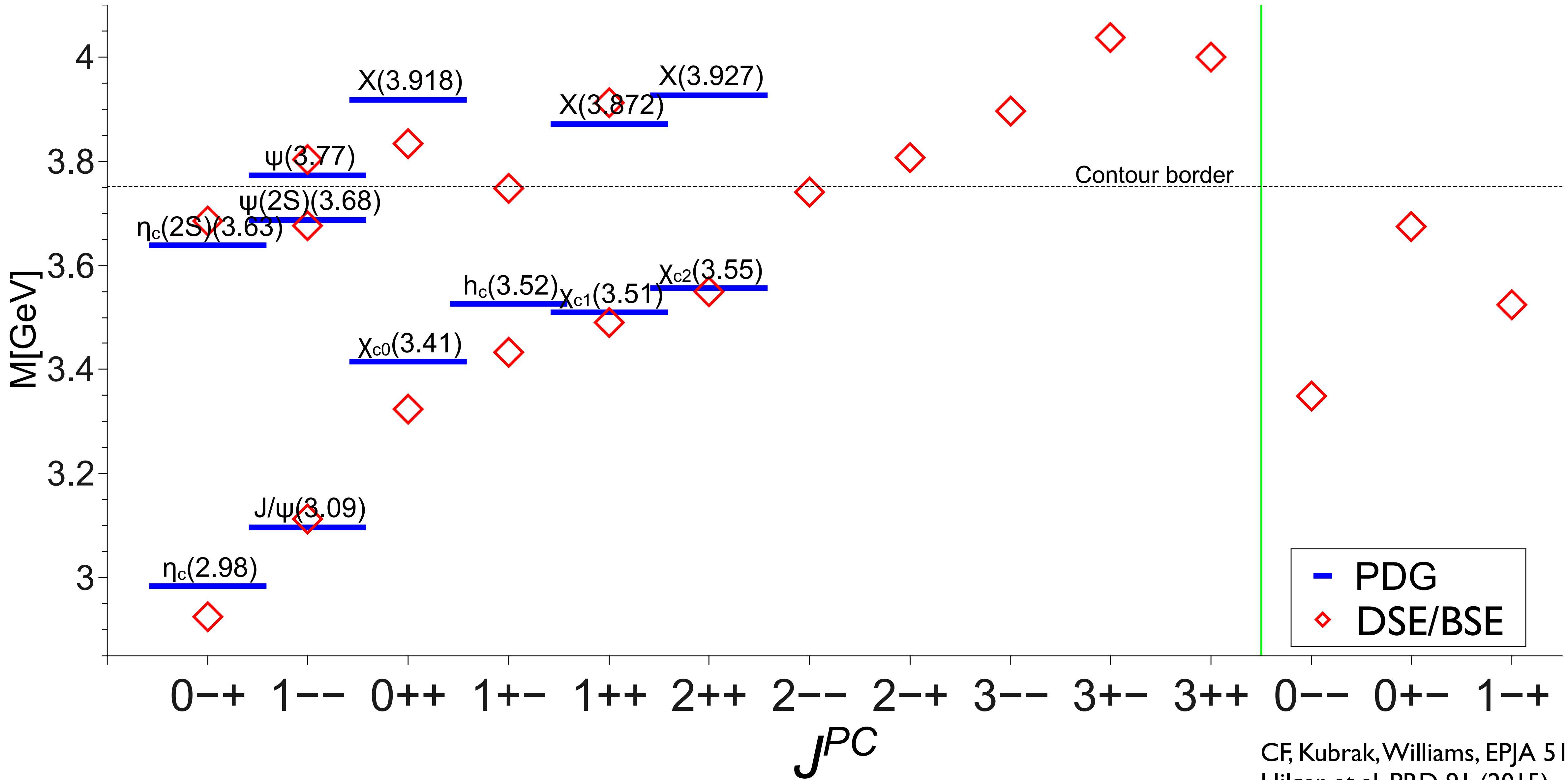

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

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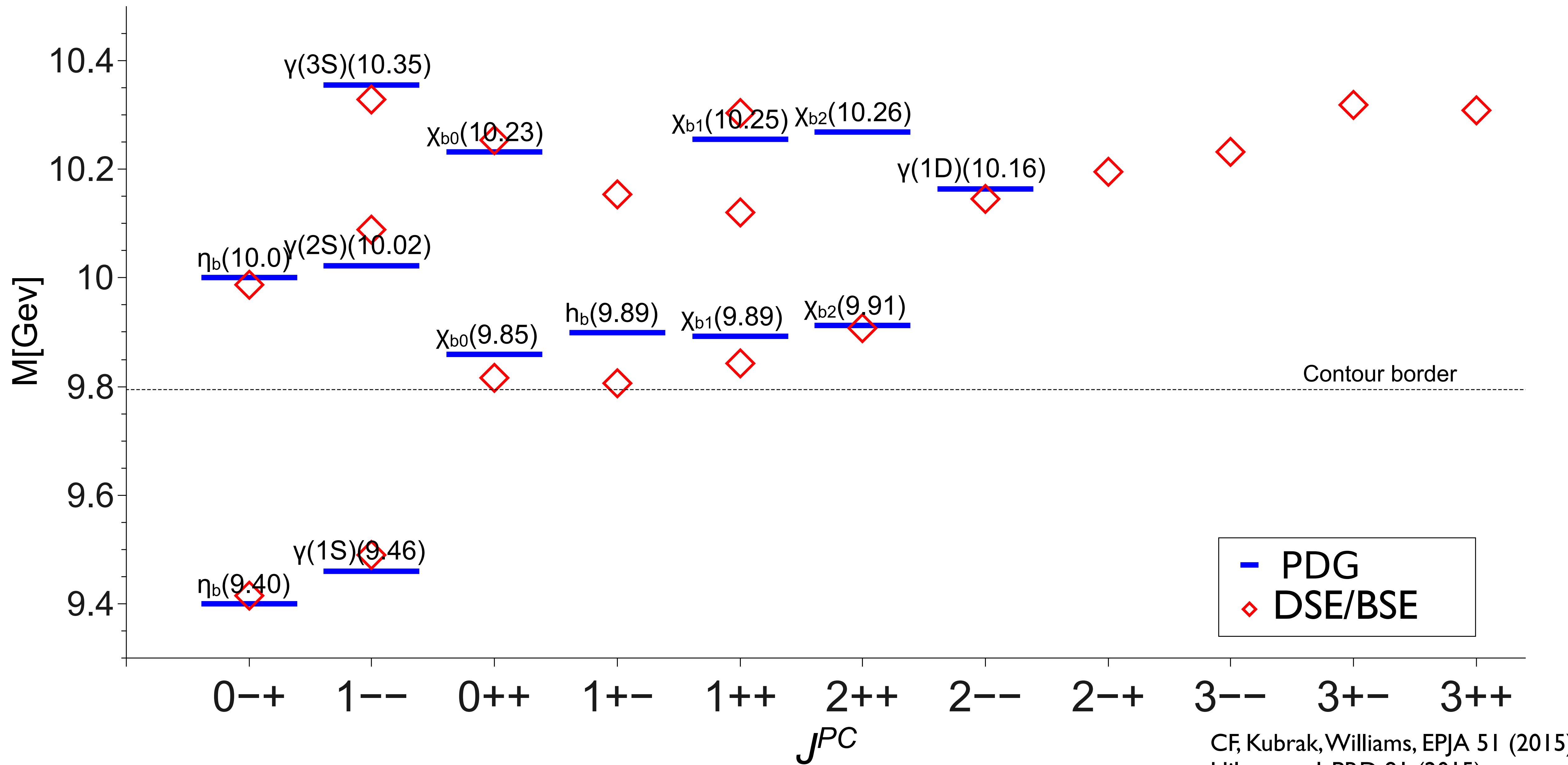

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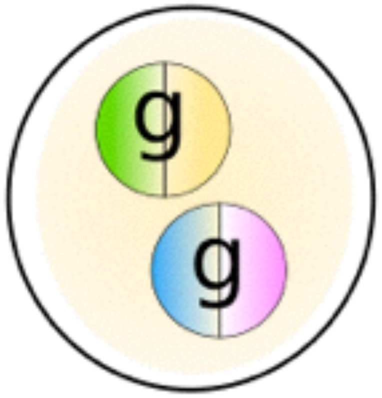

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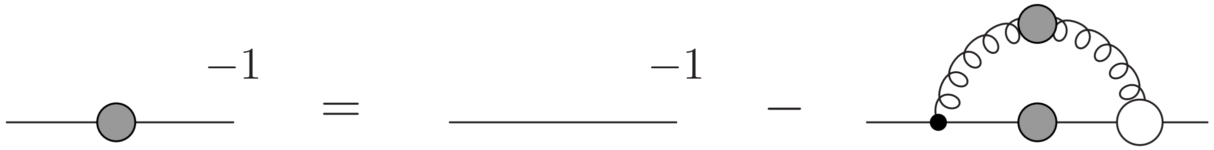

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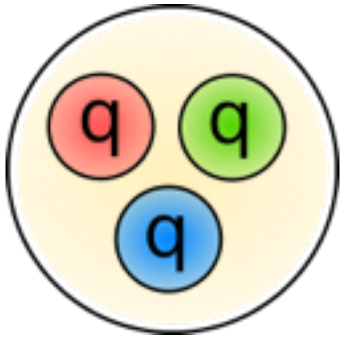
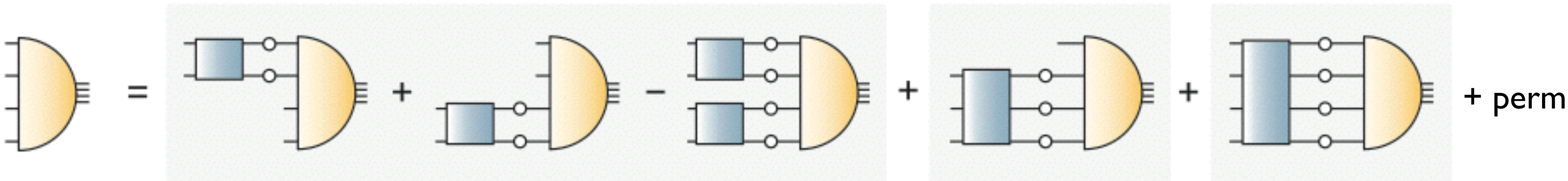


Tetraquarks from the four-body interaction

Exact equation:



Two-body interactions

Three- and four-body interactions

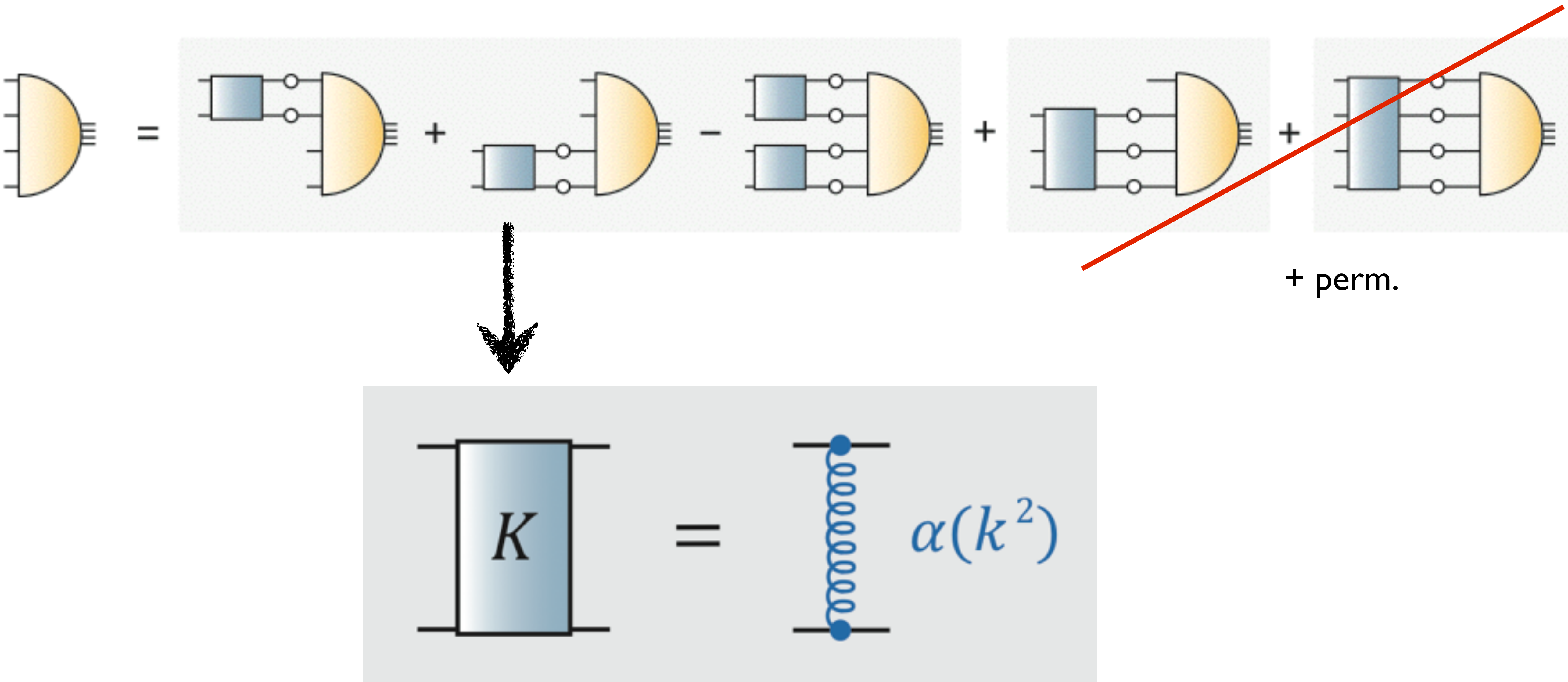
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichman, CF, PLB 718 (2012) 545-549

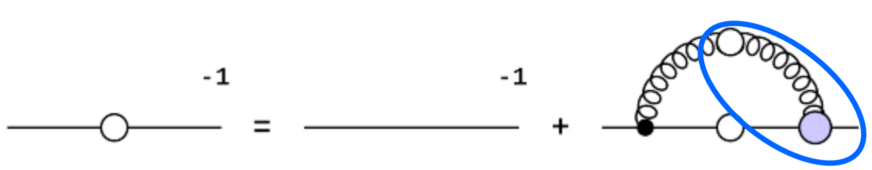
Eichman, CF, Heupel, 1508.07178

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Solving the four-body equation

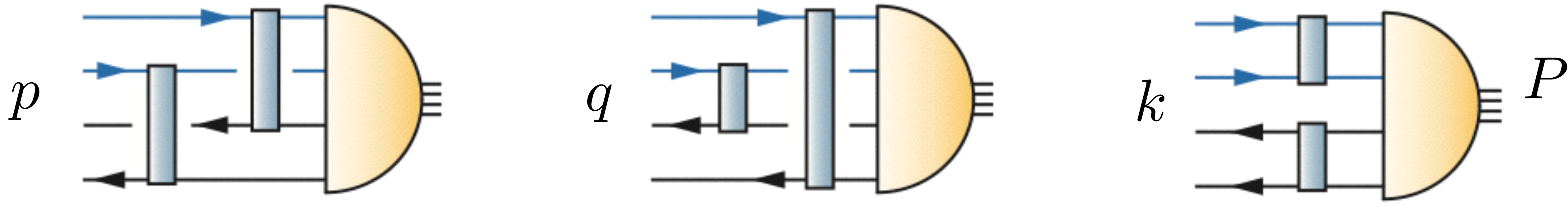


- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi \eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1, ..., s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

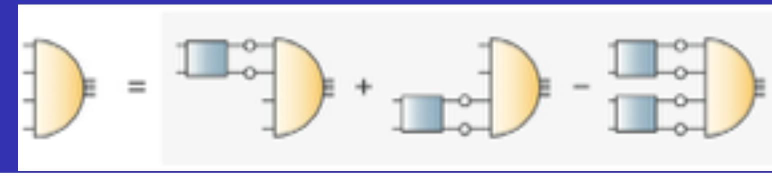
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

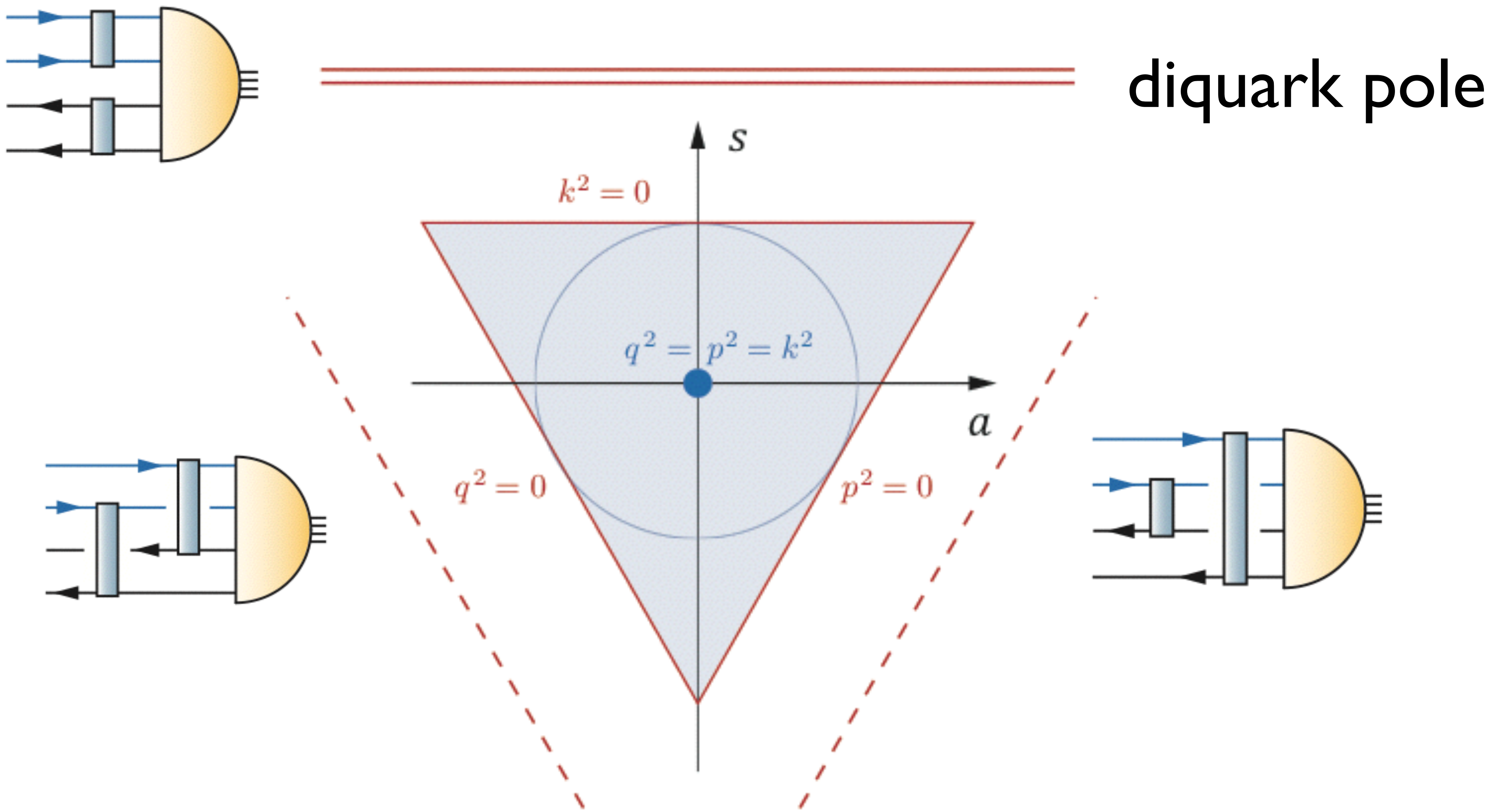
- good approximation: keep s-waves only; 16 tensor structures

Four-body equation:



Organise Dirac-Lorentz-tensors into multiplets of **S4**

- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$, carries overall scale
- Doublet: $a = \sqrt{3}(q^2 - p^2)/(4S_0); \;\; s = (p^2 + q^2 - 2k^2)/(4S_0)$



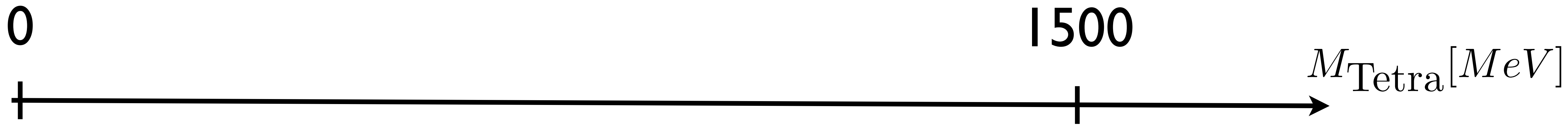
meson poles

Eichman, CF, Heupel, arXiv: 1505.06336

- Two triplets

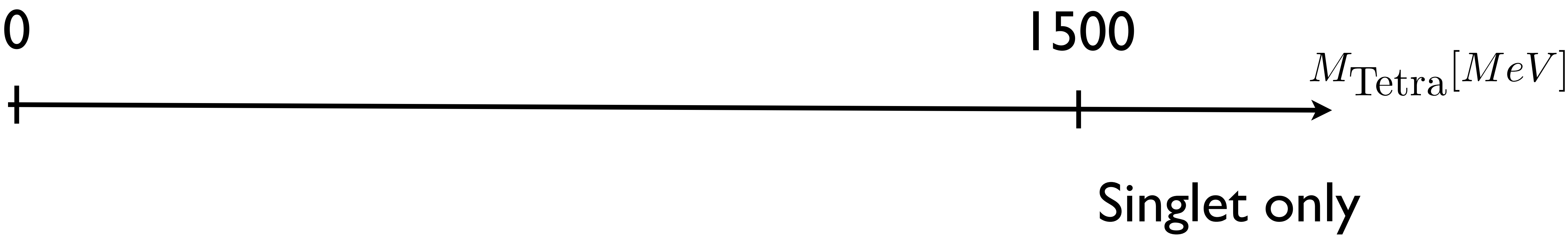
Bound state masses

- Different levels of approximations:



Bound state masses

- Different levels of approximations:



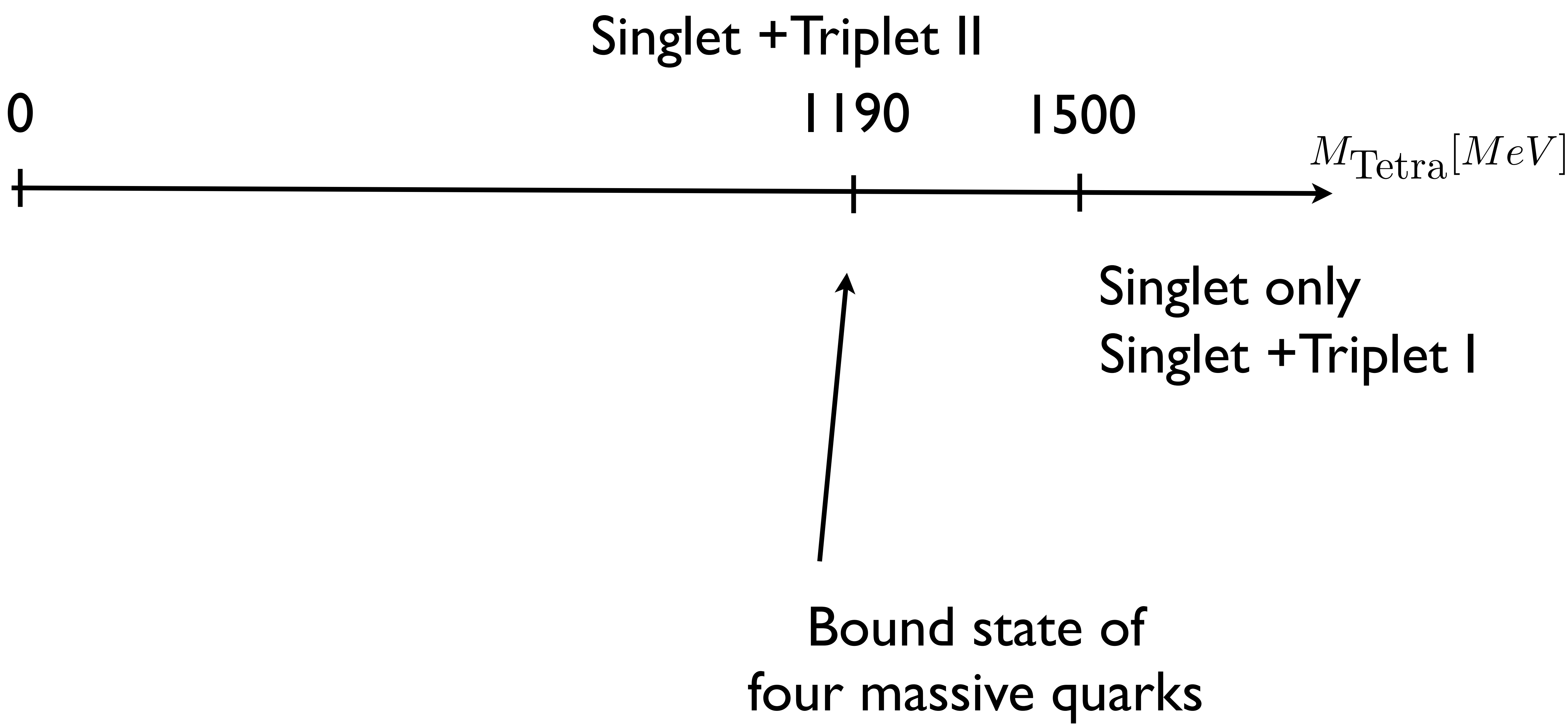
Bound state masses

- Different levels of approximations:

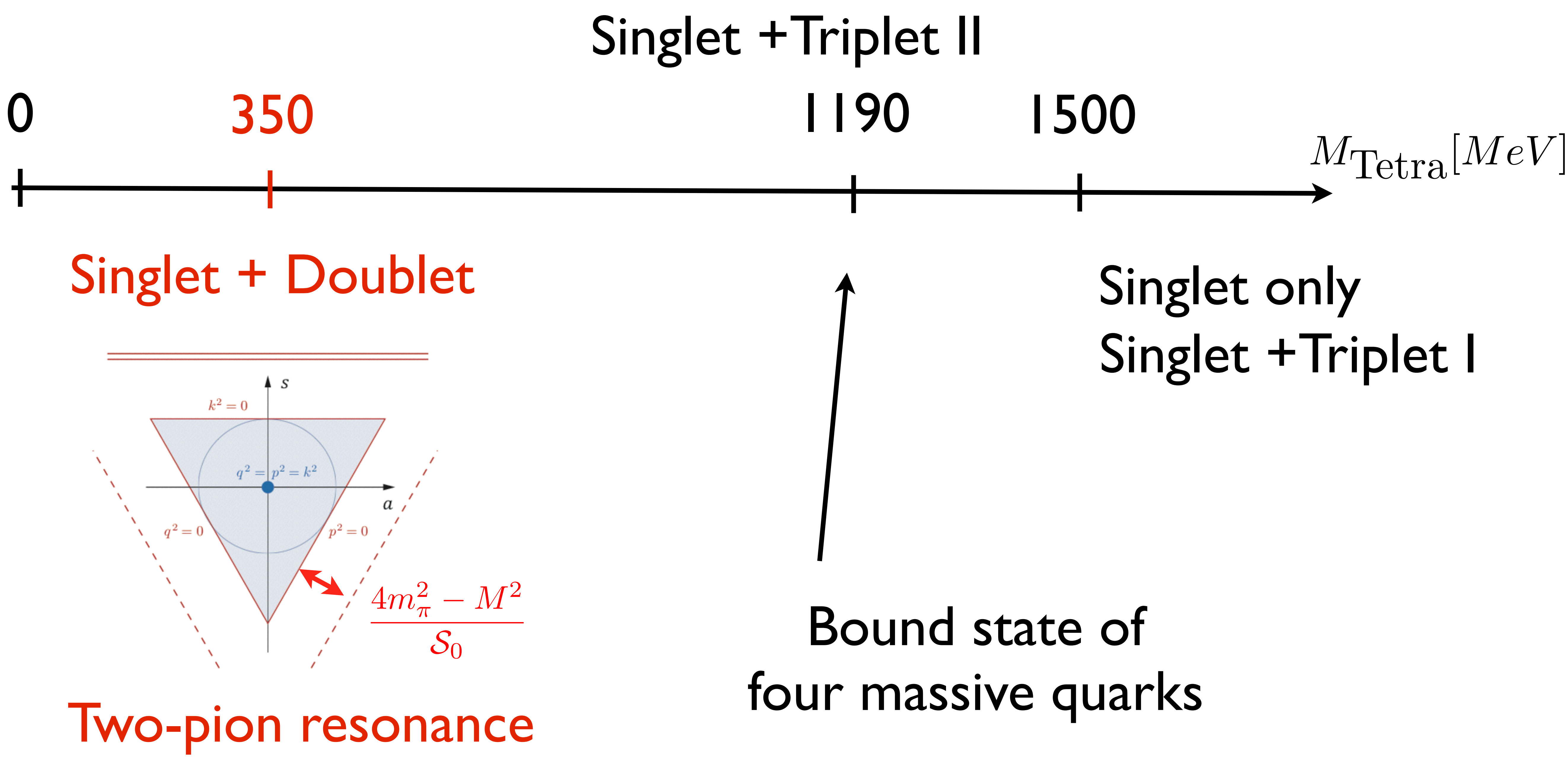
Singlet only
Singlet +Triplet I

Bound state masses

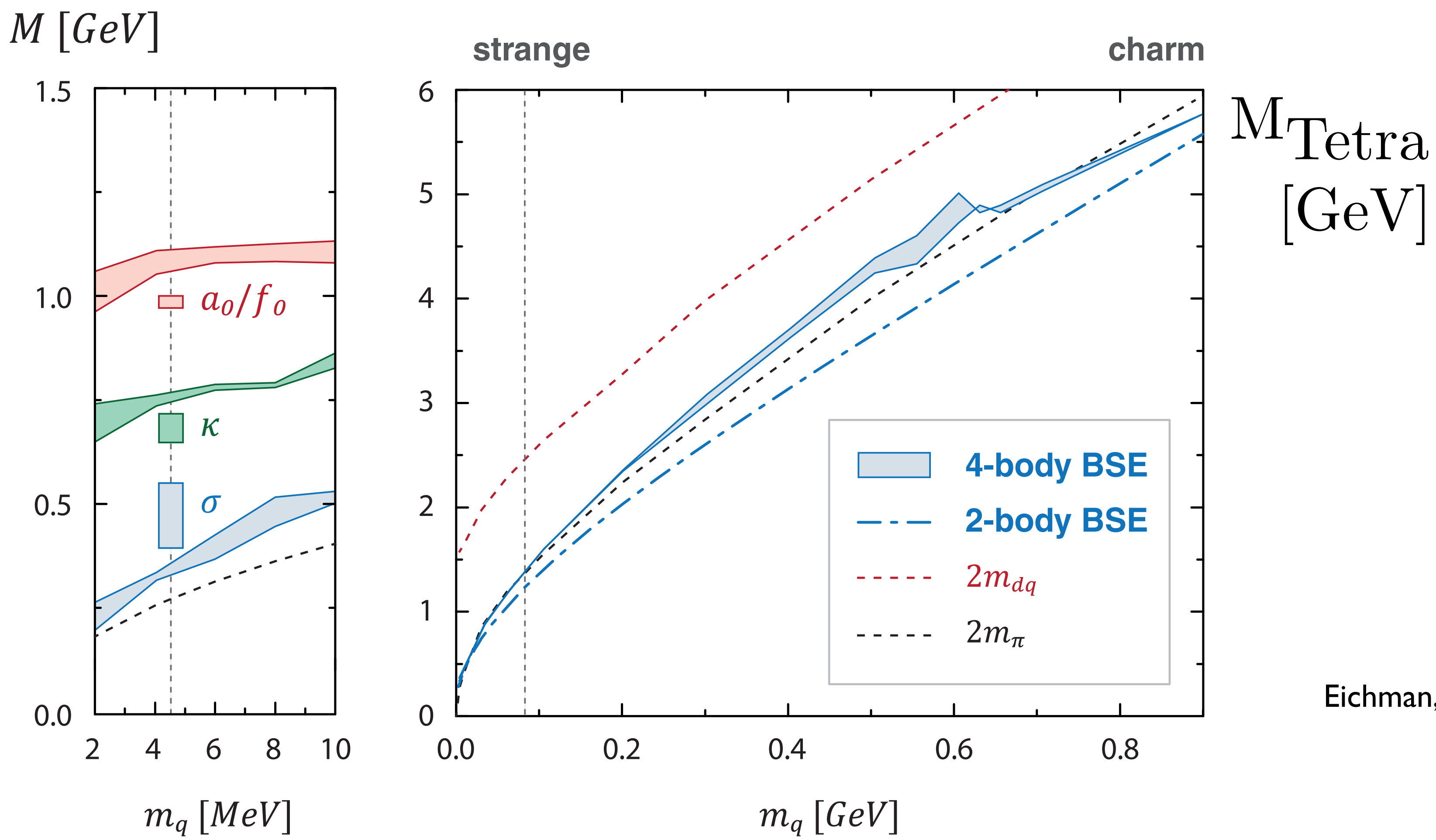
- Different levels of approximations:



- Different levels of approximations:



Mass evolution of tetraquark



Eichman, CF, Heupel, 1508.07178

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\text{MeV}$$

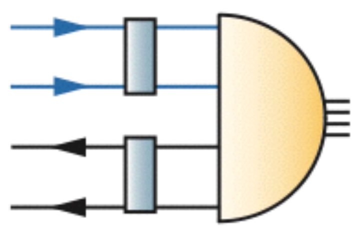
$$m_\kappa \sim 750\,\text{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\text{GeV}$$

$$m_{a_0,f_0} \sim 1080\,\text{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\text{GeV}$$

qualitatively similar to two-body framework

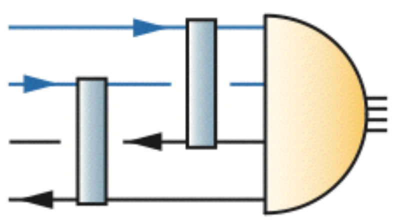
Heupel, Eichman, CF, PLB 718 (2012) 545-549

Dynamical situation in **S4**-doublet:



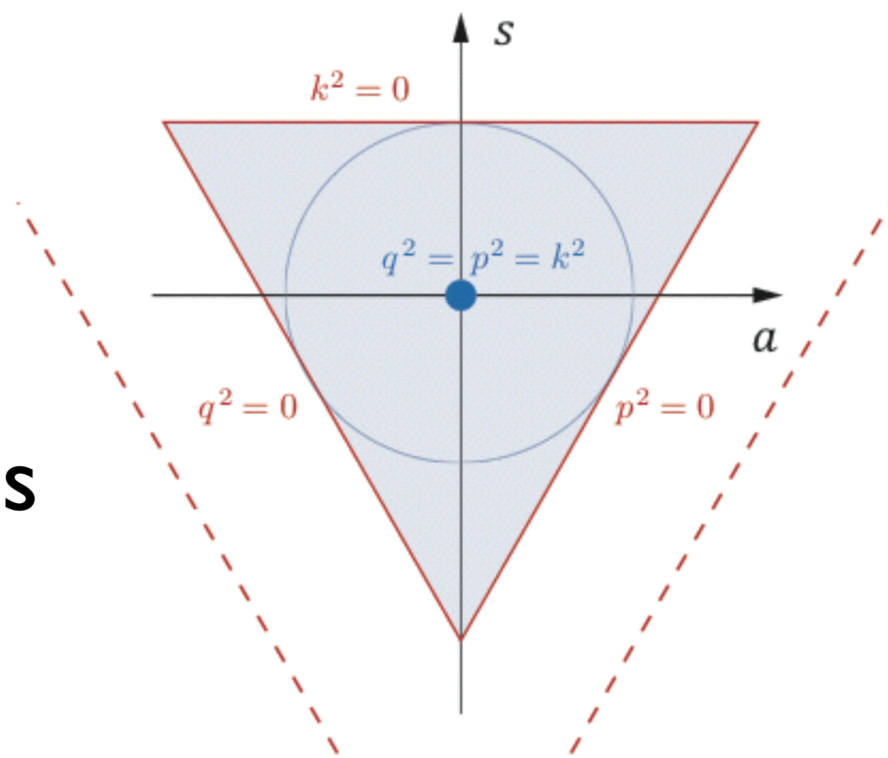
heavy-light diquarks

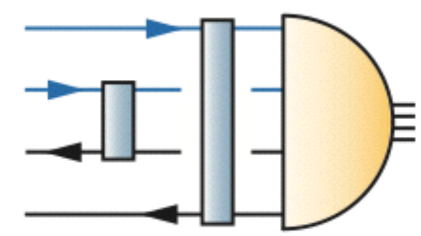
$cq - \bar{c}\bar{q}$



meson-molecules

$c\bar{q} - \bar{c}q$





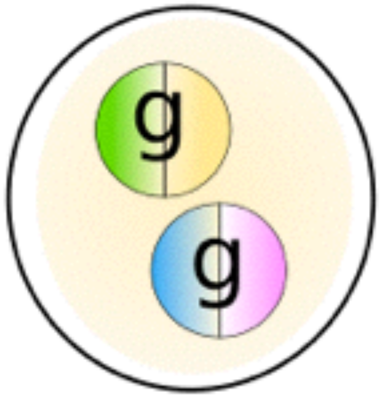
hadro-charmonium

$c\bar{c} - q\bar{q}$

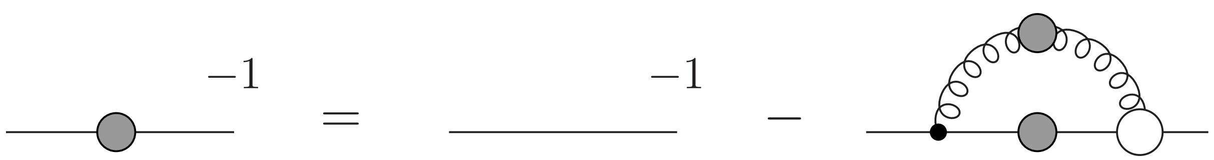
Dynamical decision of most important clustering!

Overview

1. Introduction

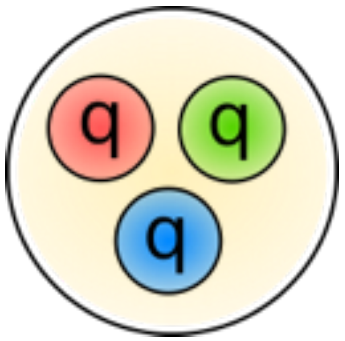


2. Gluons and glueballs



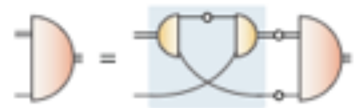
3. Quarks and mesons

4. Tetraquarks



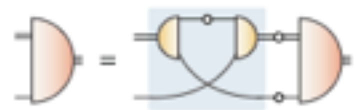
5. Excited baryons

DSE/Faddeev landscape

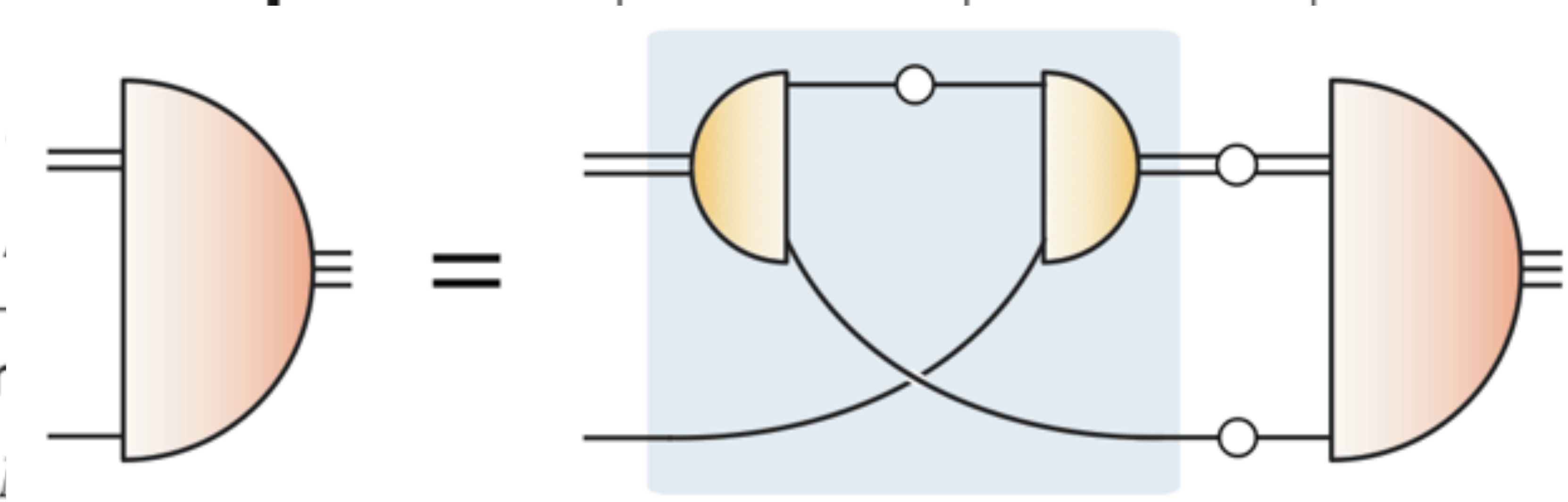
	Quark-diquark			Three-quark		
						
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	$\dots$
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	$\dots$		
Roper	√	√		$\dots$		
$N \to N^*\gamma$	√	√		$\dots$		
$N^*(1535), \dots$	$\dots$	$\dots$		$\dots$	$\dots$	
$N \to N^*\gamma$	$\dots$	$\dots$				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

Eichmann, N*-Workshop, Trento 2015

DSE/Faddeev landscape



	Quark-diquark			Three-quark		
	Contact interaction	**QCD-based model**	**DSE (RL)**	**RL**	**bRL**	**bRL + 3q**
N, Δ					√	…
N, Δ						
$N \to$						
Roper						
$N \to$						
$N^*(1535), \ldots$	…	…		…	…	
$N \to N^*\gamma$	…	…				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	



Eichmann, N*-Workshop, Trento 2015

DSE/Faddeev landscape

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	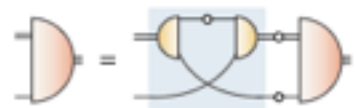					
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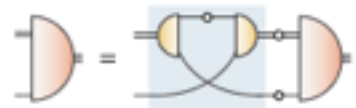
Eichmann, N*-Workshop, Trento 2015

DSE/Faddeev landscape

	Quark-diquark 			Three-quark 		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	$\dots$
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	$\dots$		
Roper	√	√		$\dots$		
$N \to N^*\gamma$	√	√		$\dots$		
$N^*(1535), \dots$	$\dots$	$\dots$		$\dots$	$\dots$	
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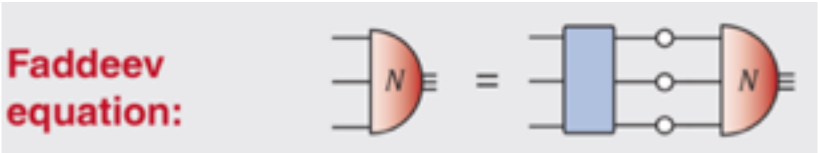
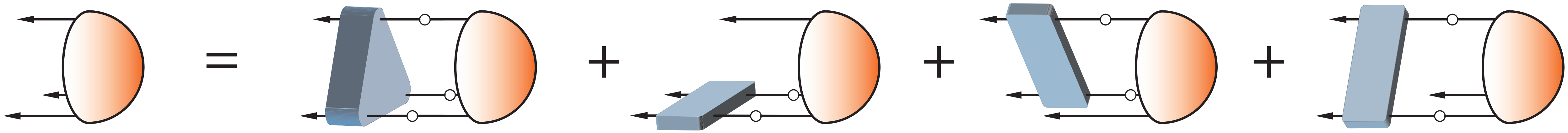
Eichmann, N*-Workshop, Trento 2015

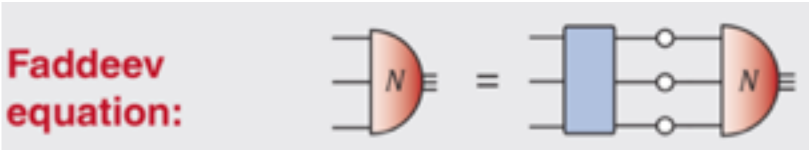
Faddeev - equation

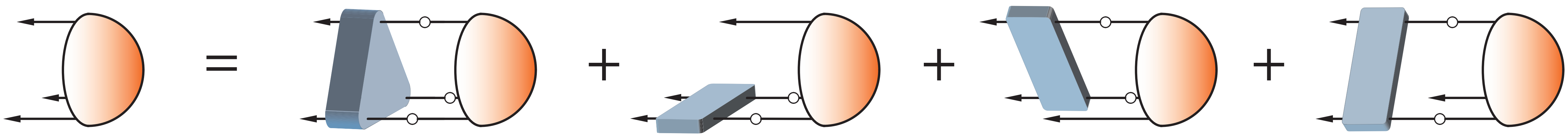




- relativistic bound state:
  - 64 tensor structures for nucleon: s, p, d - wave
  - 128 tensor structures for Delta: s, p, d, f - wave

Faddeev - equation





- irreducible three-body forces

Sanchis-Alepuz, Williams, work in progress…

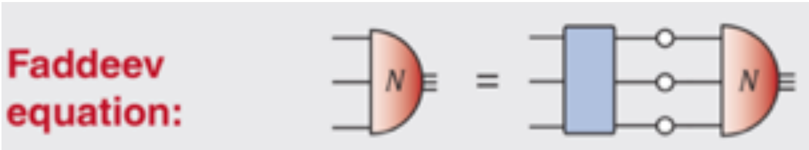
- two-body interactions:
  - non-perturbative gluon exchange

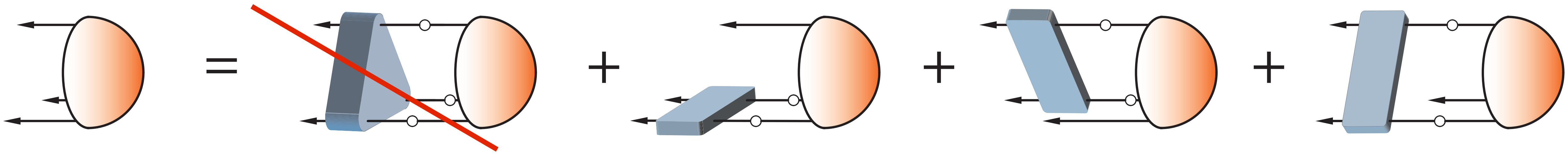
    Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
  - meson exchange

    Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)
  - two-body forces beyond one-particle exchange

    Sanchis-Alepuz, Williams, PLB 749 (2015) 592
- numerically expensive but manageable !

Faddeev - equation





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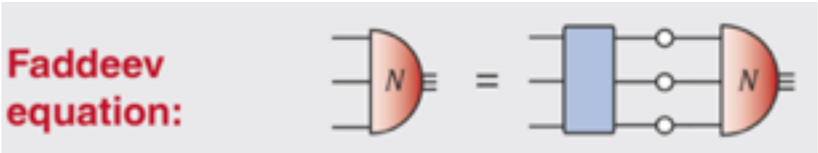

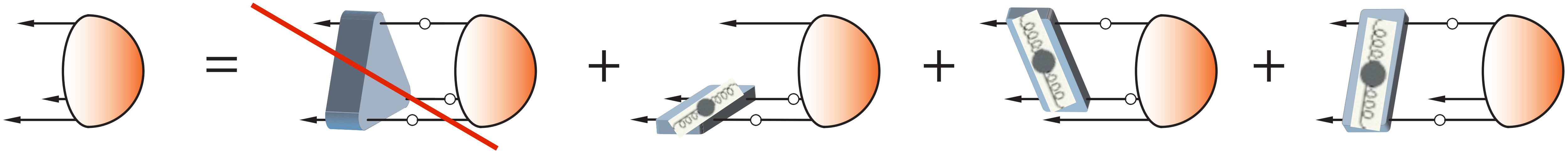

- irreducible three-body forces
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Sanchis-Alepuz, Williams, work in progress…

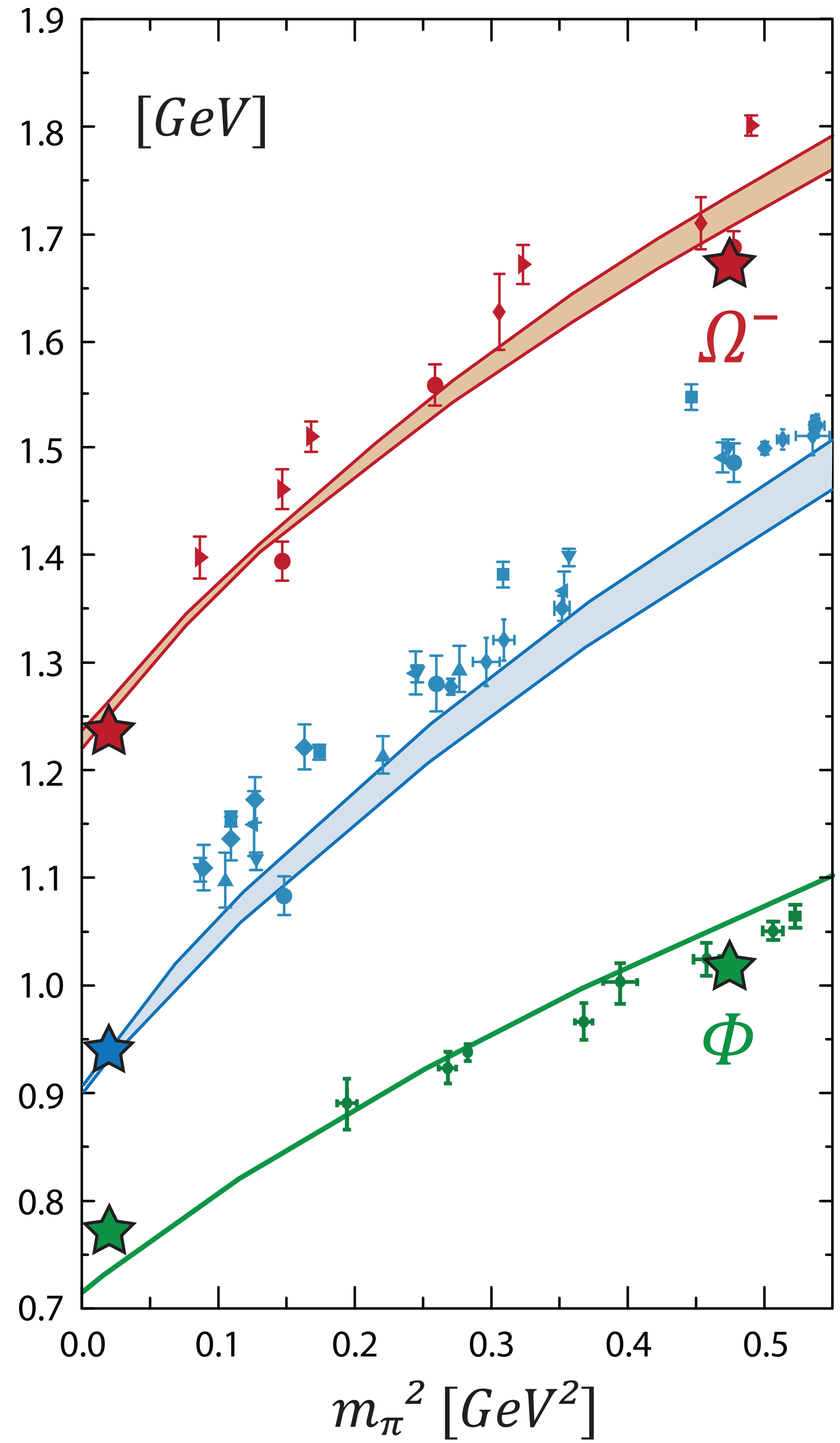
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014)

Sanchis-Alepuz, Williams, PLB 749 (2015) 592

Baryon masses - gluon exchange only

- first covariant three-body calculations !
- grosso modo: consistent description of mesons and baryons
- wave functions contain sizable p-wave contributions



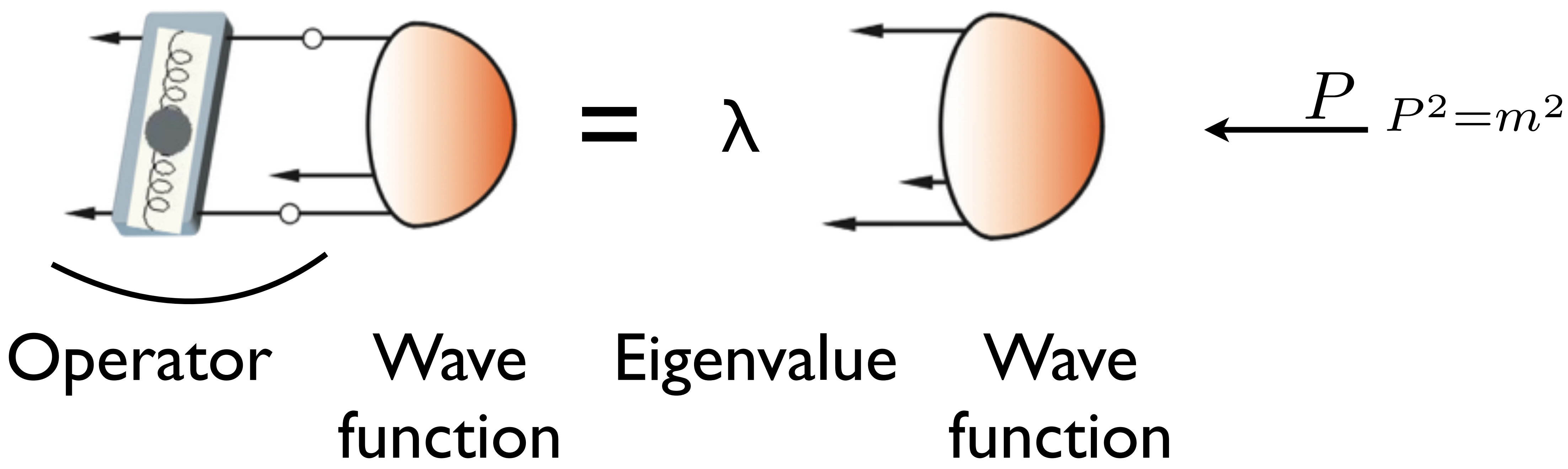
Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

Nucleon:
Eichmann et al., PRL 104 (2010), PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
Eichmann, PRD 84 (2011)
Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Mass of the Roper

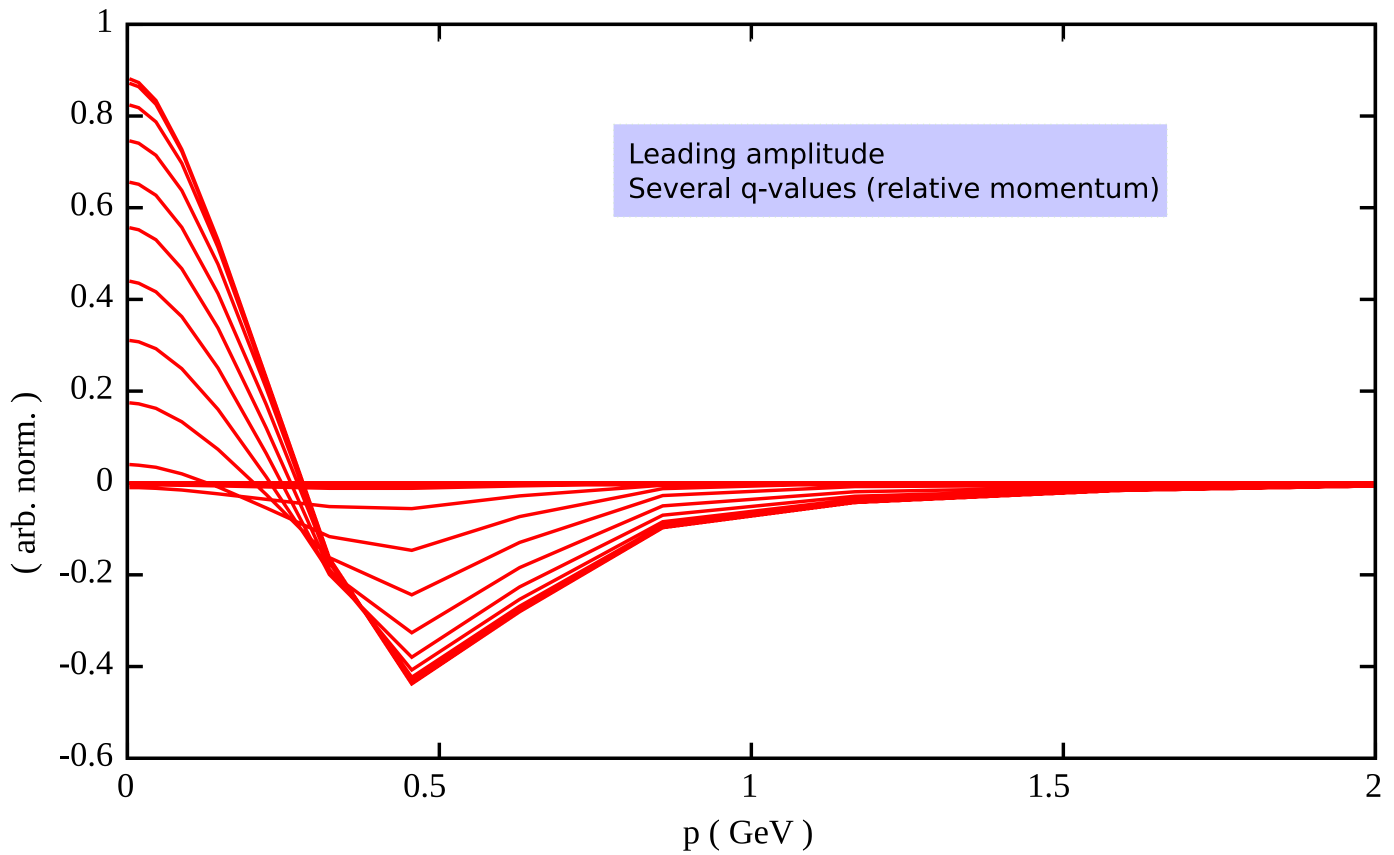


- ground state: largest eigenvalue λ=1
- fake excited state at $m = 1.26\,\mathrm{GeV}$ cf. talk at PANDA meeting
- need full flavor wave function (MS+MA) to eliminate fake
- huge numerical effort
- extraction of wave function very expensive

Properties of the Roper

$$M_{N^*} = 1.45\,\mathrm{GeV}$$

	Nucleon	Roper
s-wave	66%	15%
p-wave	33%	61%
d-wave	1%	24%

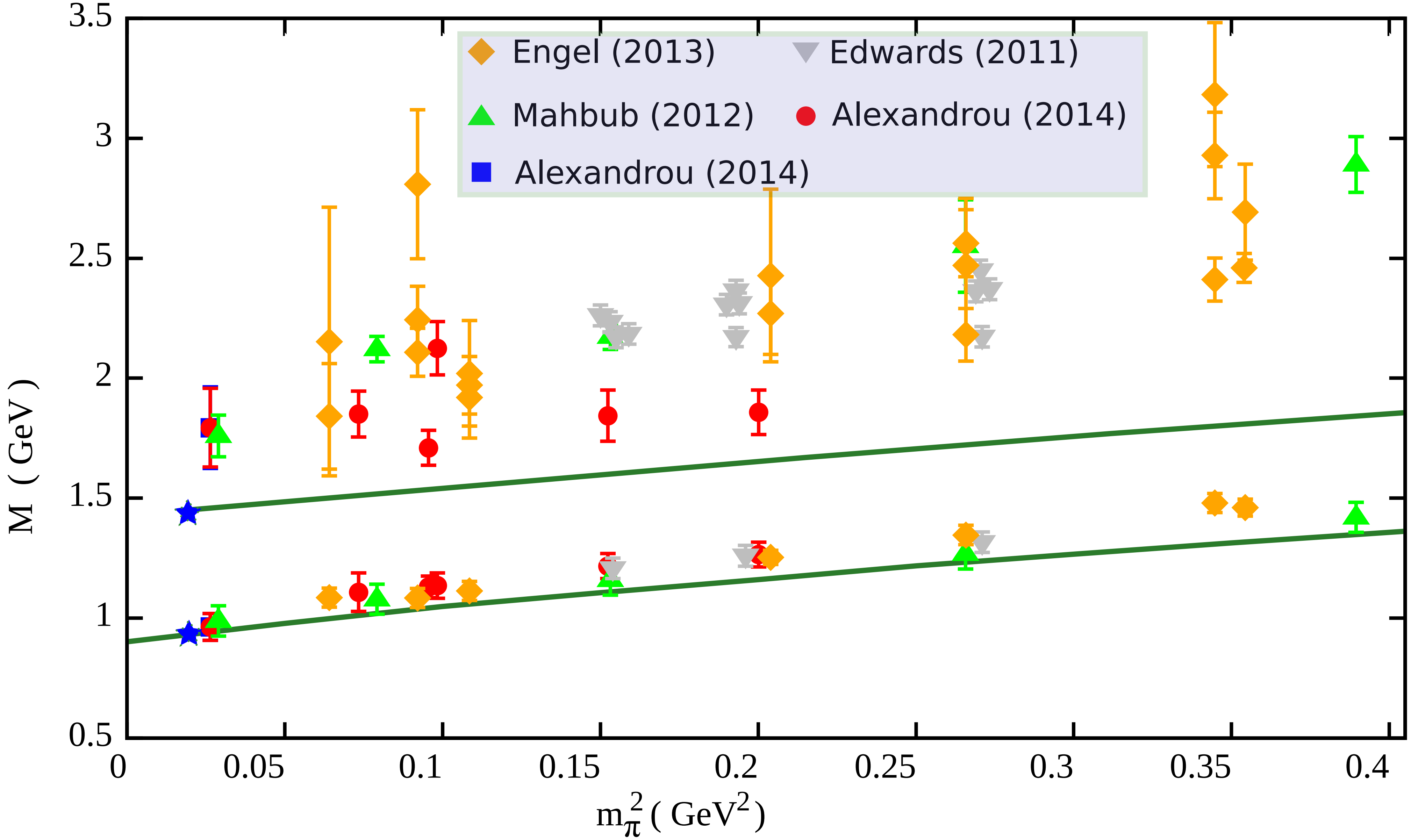


Sanchis-Alepuz, CF in preparation

- different internal structure than nucleon
- zero crossing of wave function: 2s-state

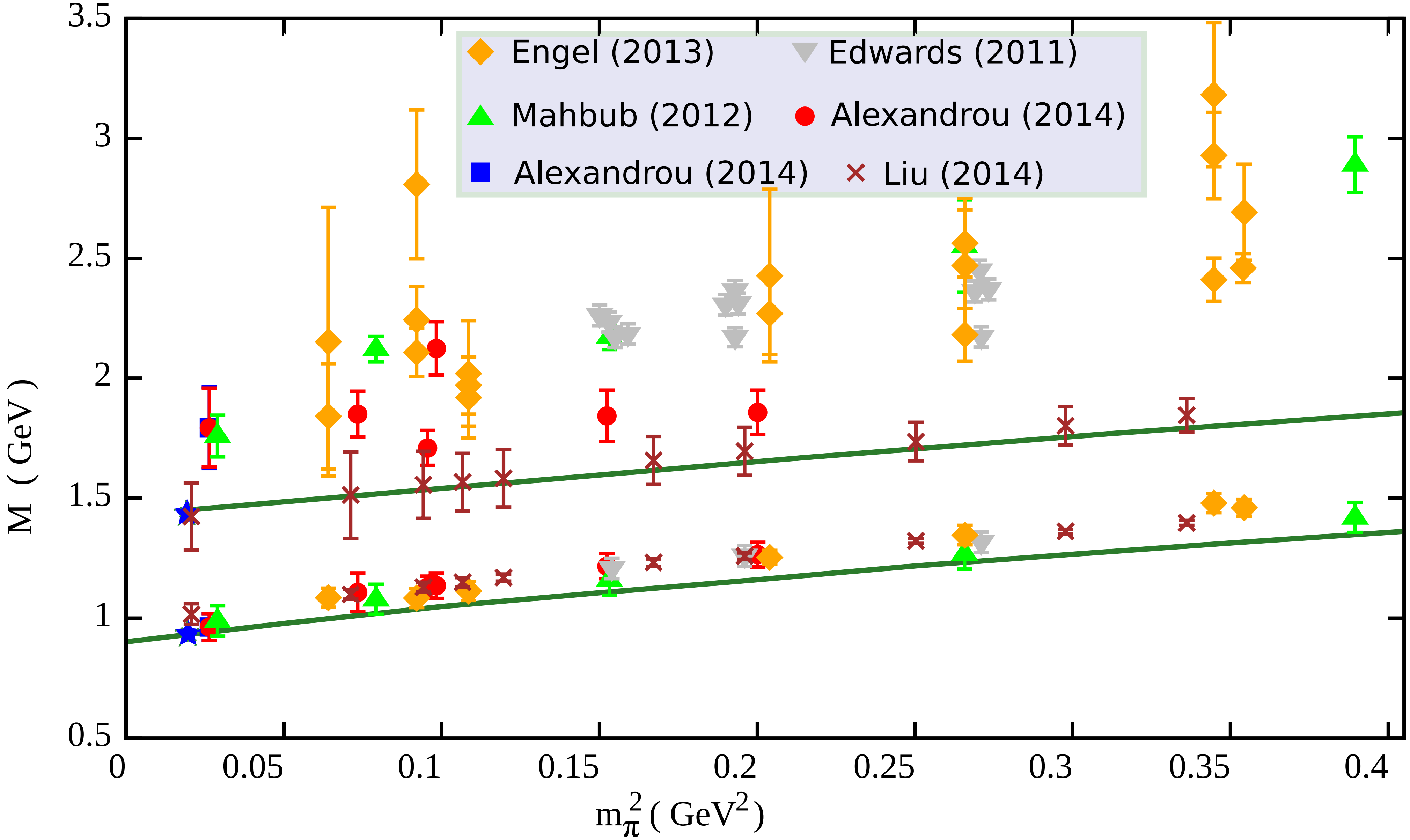
tension with simple models:
Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17

Mass evolution



Sanchis-Alepuz, CF in preparation

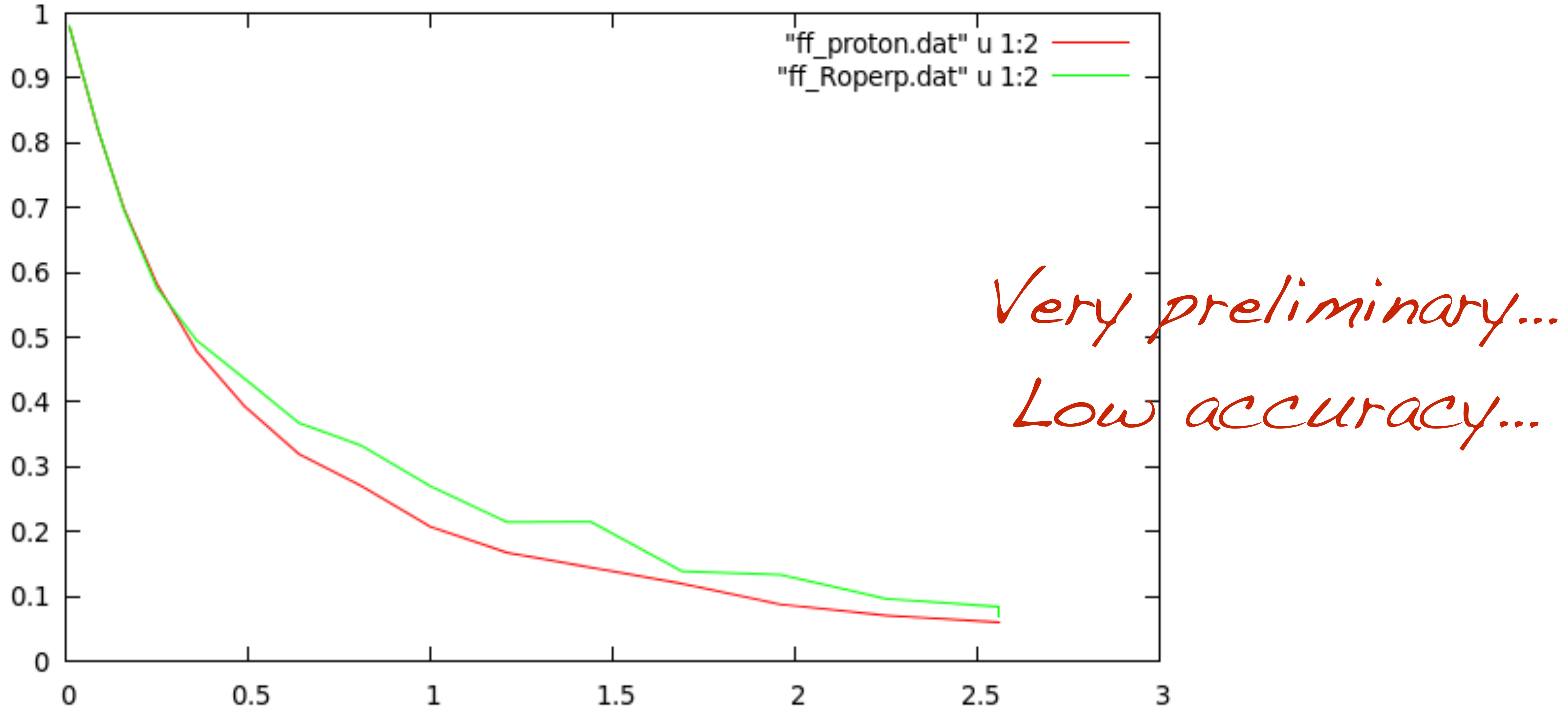
Mass evolution



Sanchis-Alepuz, CF in preparation

- Mass evolution in agreement with Liu (2014)

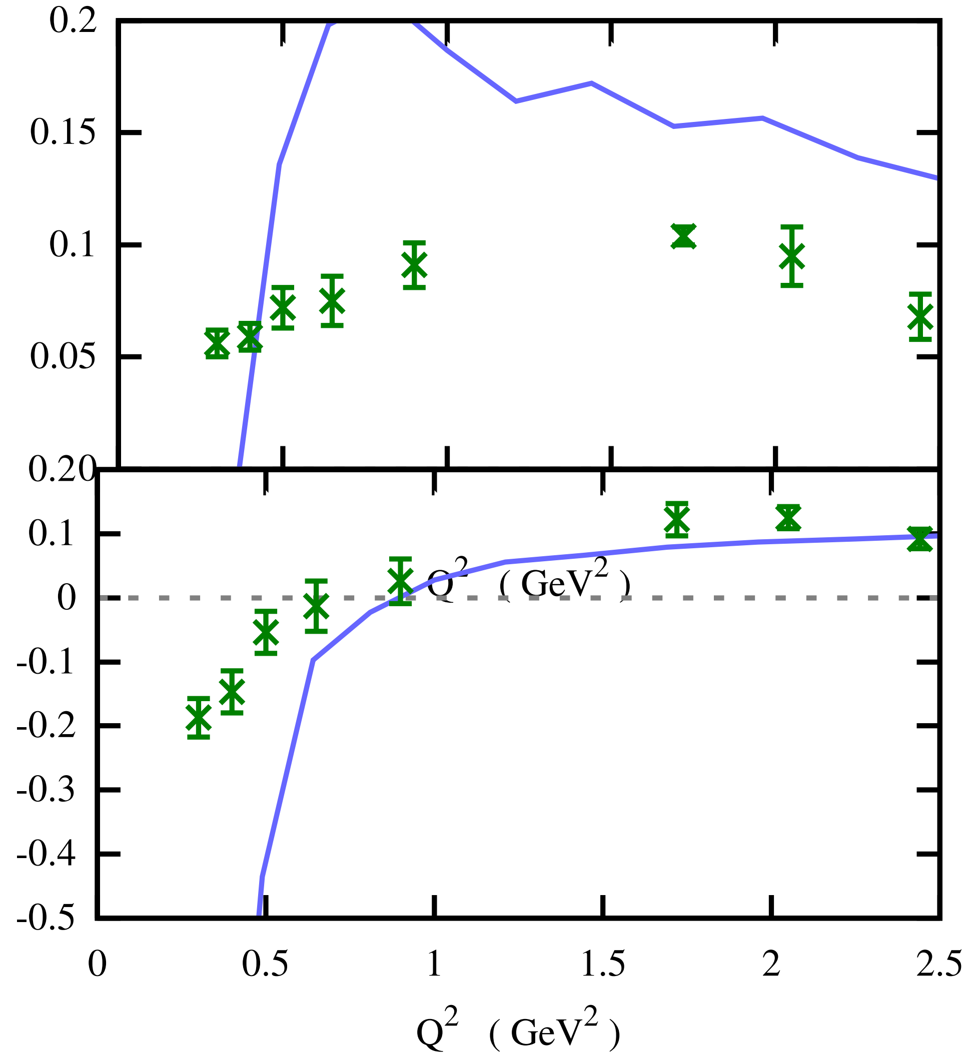
EM form factor



- small differences at larger Q^2

$$r_p = 0.75\,\text{fm}$$
$$r_R = 0.65\,\text{fm}$$

N-Roper transition form factor



Very preliminary...

Low accuracy...

- Numerical problems below 0.5 GeV^2 and above 2.5 GeV^2
- F_2: zero crossing seen !

Summary and outlook

Summary

- Mass gap in YM-theory: scalar glueball mass
- Tetraquarks dominated by internal meson-meson configurations
- Dynamical description of σ as π-π resonance
- First results for Roper in three-body framework

Outlook

- Improve numerical framework: precision, systematics
- Unquench complex gluon propagator
- Tetraquarks: explore heavy-light systems
- Baryons: transition form factors