hadron scattering & resonances from QCD

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QCD and hadron physics

- any sufficiently accurate approach to QCD will reproduce all the complexities of hadron amplitudes determined in experiments

  - bumps, shoulders, thresholds, “resonant”, “non-resonant” …

  - ultimately want to
    reproduce effects & then understand them

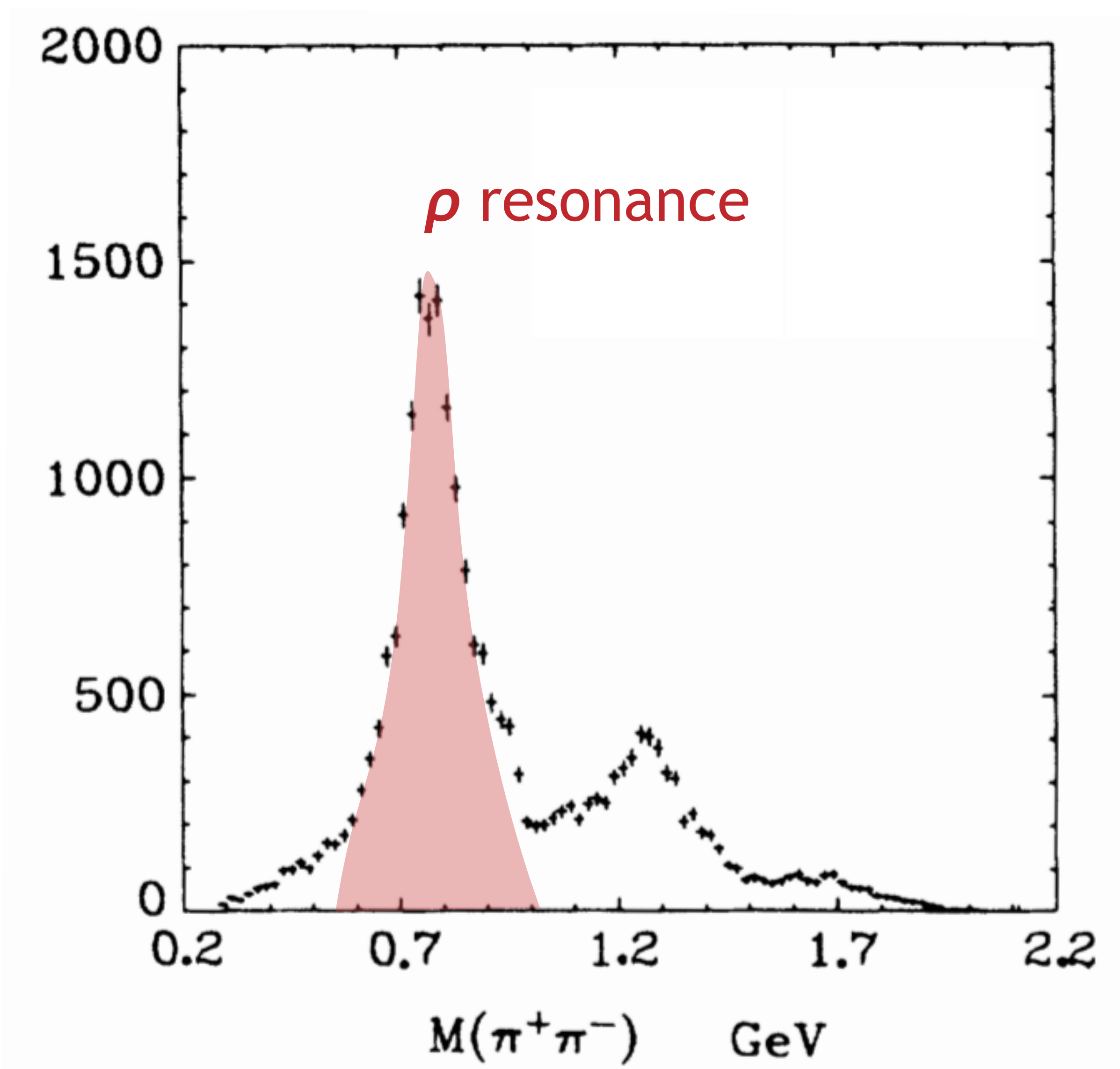
let’s start with something simple,
a well-known elastic resonance …

excited hadrons are resonances

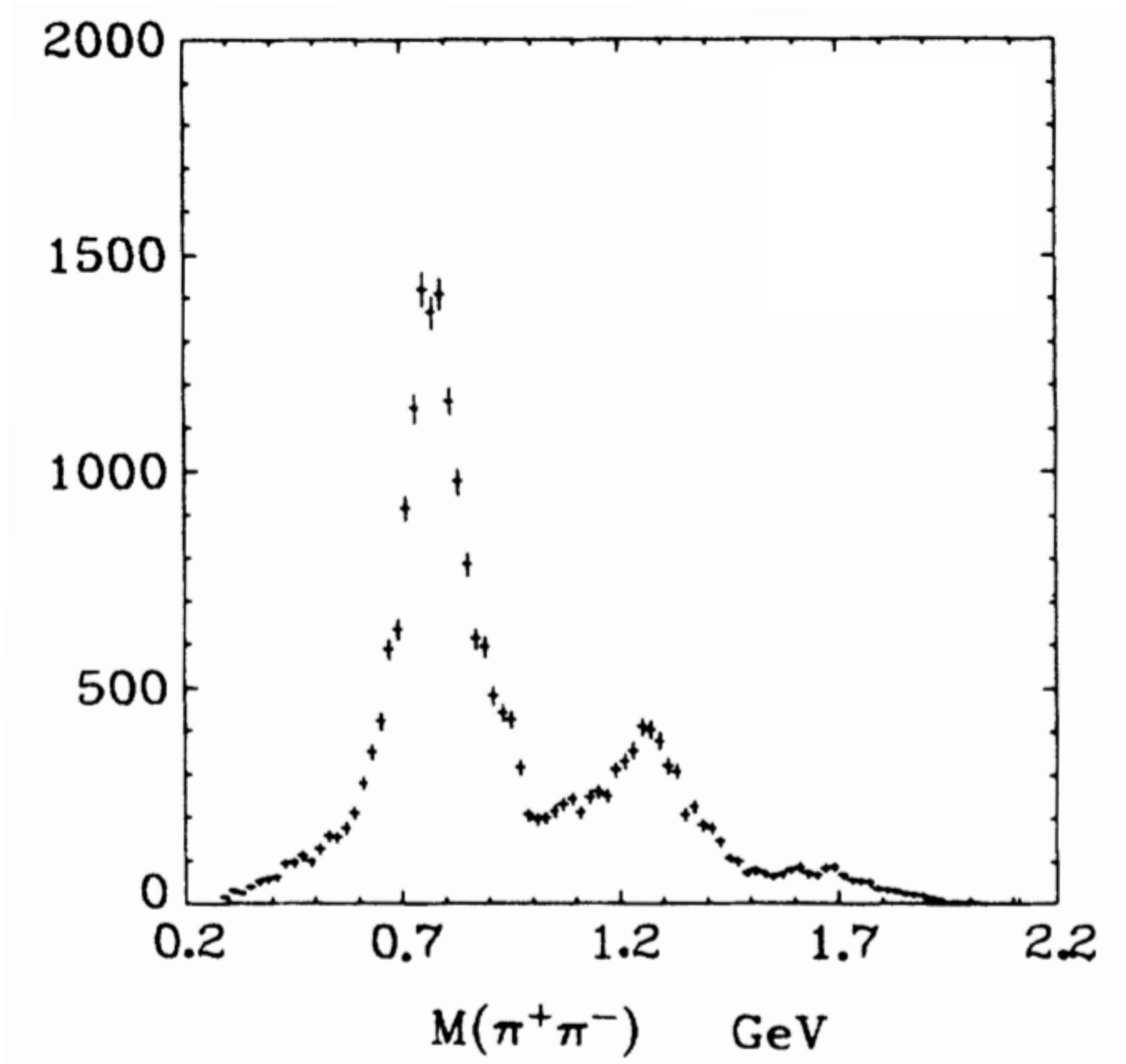
PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,∥ and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 25 September 1972)



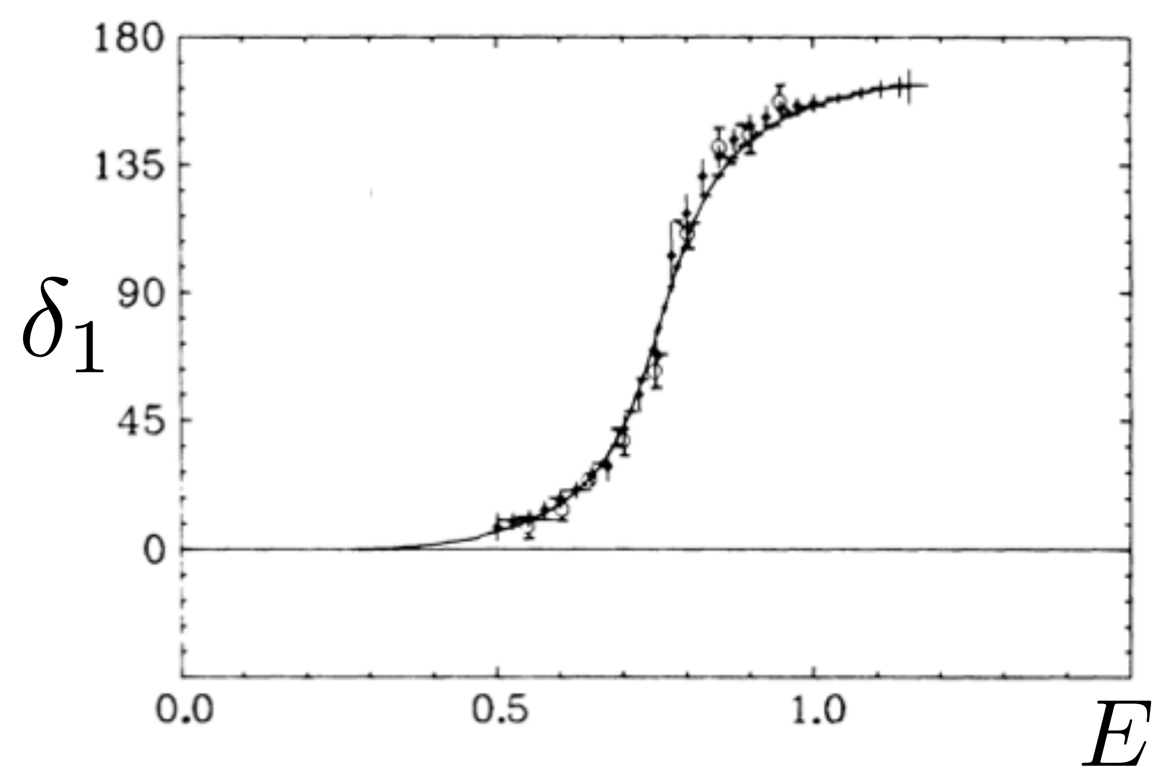
excited hadrons are resonances



PARTIAL WAVE AMPLITUDE

$$f_\ell(E) = \frac{1}{2i}\left(e^{2i\,\delta_\ell(E)} - 1\right)$$

RESONANT PHASE SHIFT

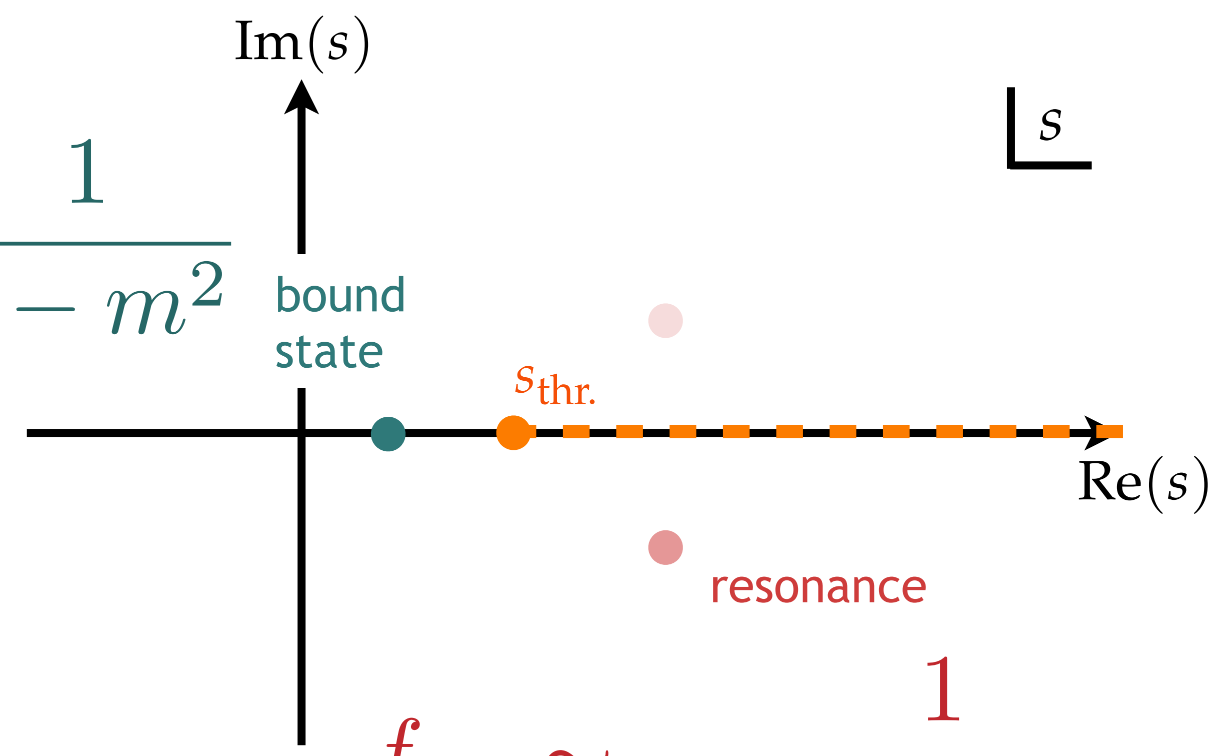


resonances are pole singularities

- pole singularities in complex $s = E^2$

COMPLEX ENERGY PLANE

$$f_{\text{b.s.}} \sim \frac{1}{s - m^2}$$

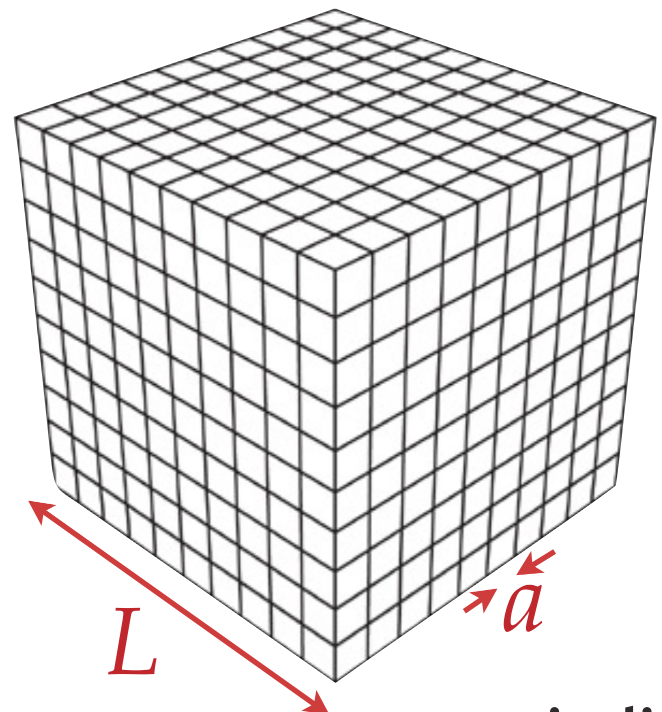


$$f_{\text{res}} \sim \frac{1}{s - m^2 + im\Gamma}$$

lattice QCD

- fields on a finite cubic lattice in Euclidean space time

CUBIC LATTICE



periodic boundary conditions

- compute correlation functions

e.g. $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\,\bar{\psi}\Gamma\psi(0) \; e^{-\int d^4x\, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$

'sum' 'field correlation' 'probability weight'

Monte Carlo sample fields

$$C(t) = \langle 0 | \mathcal{O}(t)\mathcal{O}(0) | 0 \rangle$$

$$C(t) = \sum_n e^{-E_n t} \left| \langle 0 | \mathcal{O} | n \rangle \right|^2$$

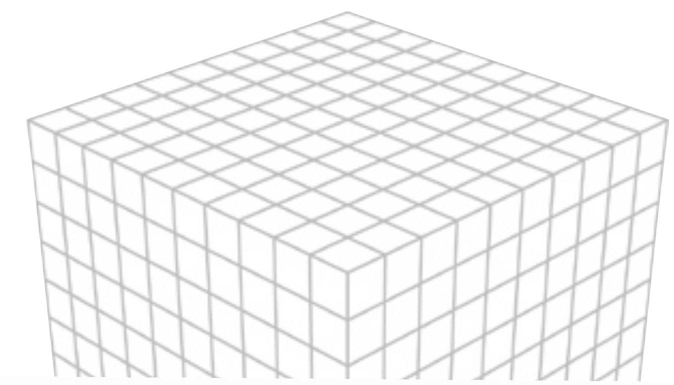
no direct access to scattering amplitudes in Euclidean time

can extract a discrete spectrum

lattice QCD

- fields on a finite cubic lattice in Euclidean space time

CUBIC LATTICE



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Mont
samp

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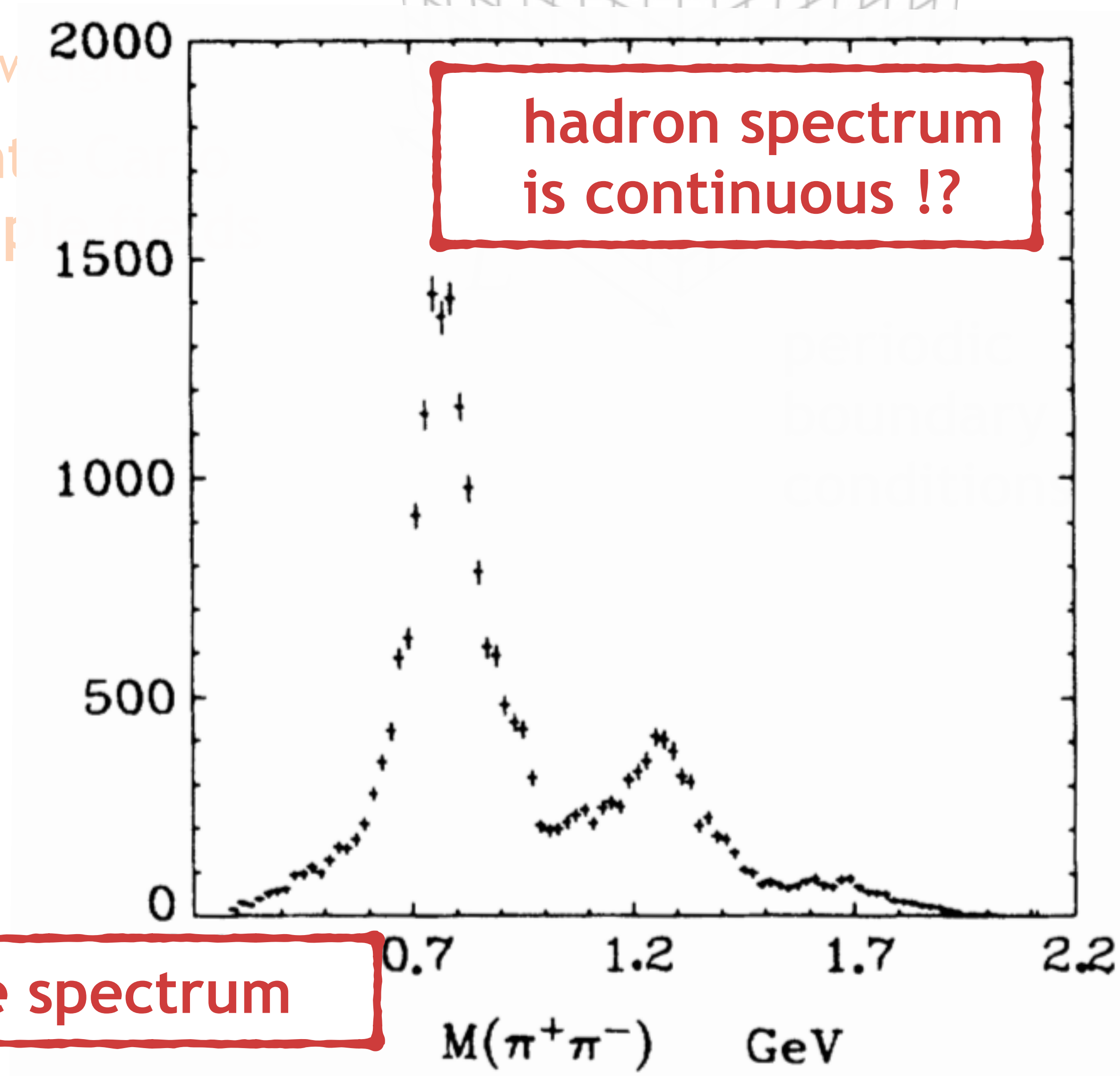
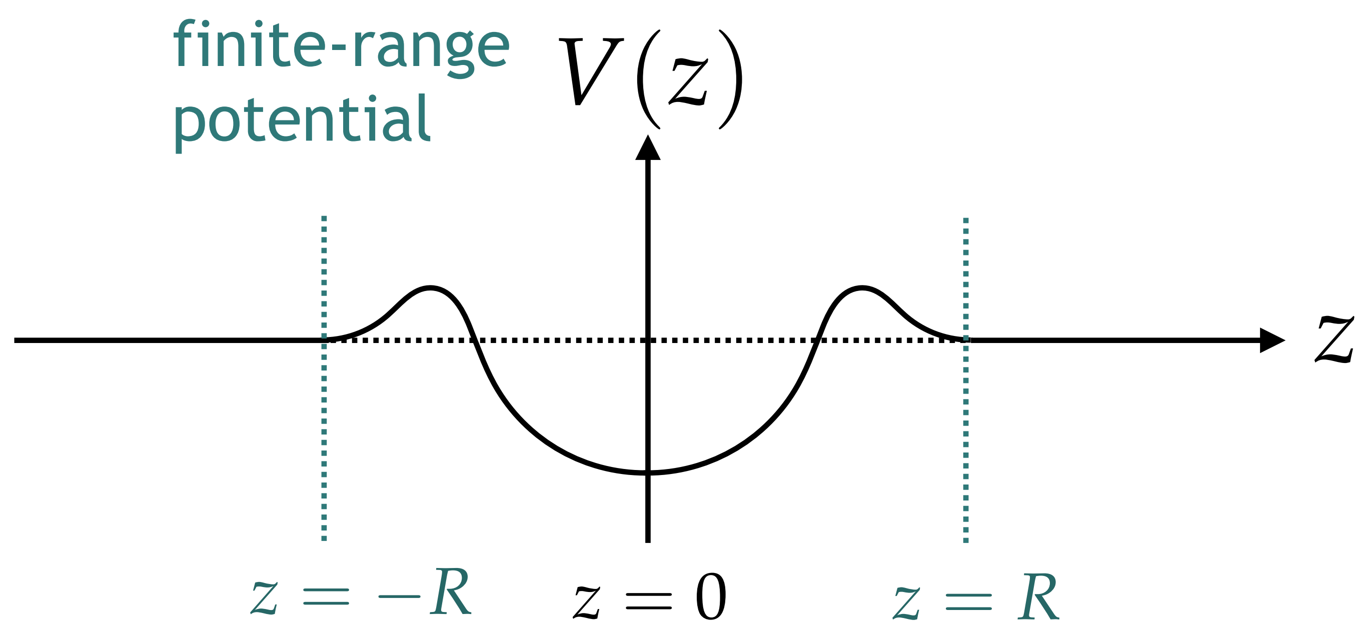
can extract a discrete spectrum



one-dim quantum mechanics

- consider scattering of two identical bosons



outside the well

$$\psi(|z|>R) \sim \cos\left(p|z| + \delta(p)\right)$$

scattering phase-shift

continuous momentum

'scattering' in a finite-volume

- consider scattering of two identical bosons

outside the well

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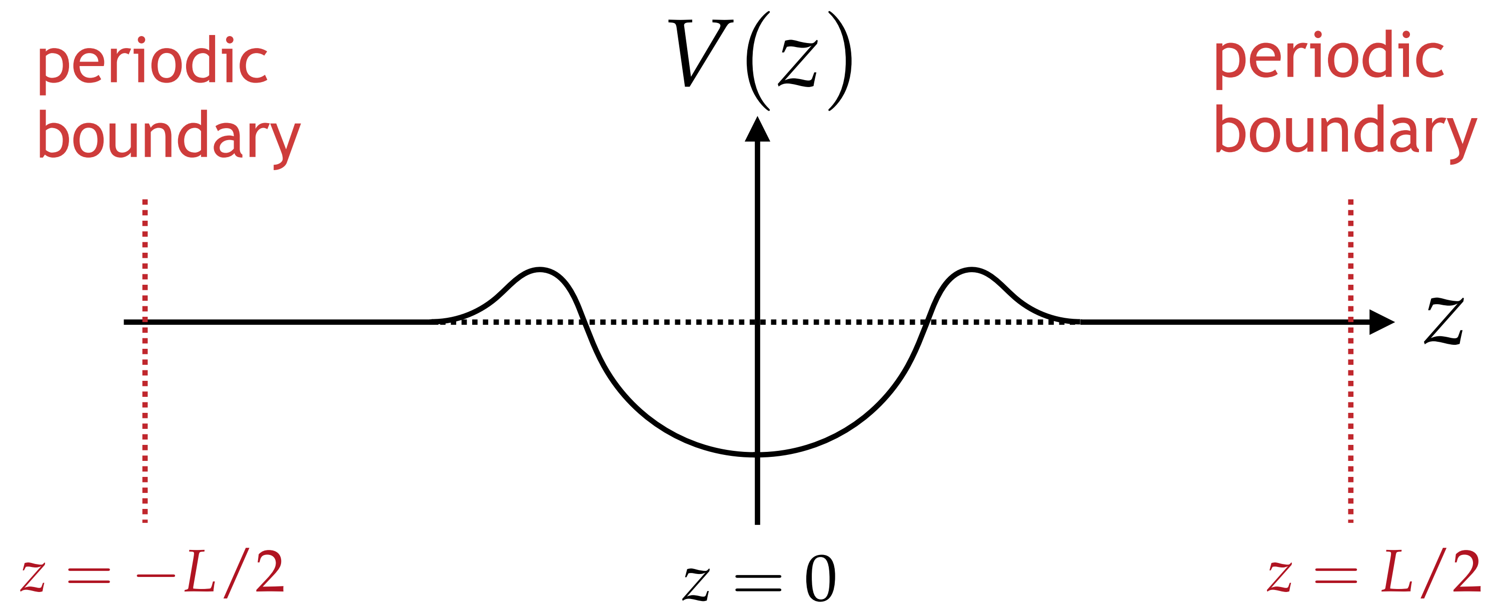


- apply periodic boundary conditions

$$\left.\begin{aligned} \psi(-L/2) &= \psi(L/2) \\ \frac{d\psi}{dz}(-L/2) &= \frac{d\psi}{dz}(L/2) \end{aligned}\right\} \frac{pL}{2} + \delta(p) = n\pi$$

$$\boxed{p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)}$$

discrete energy spectrum

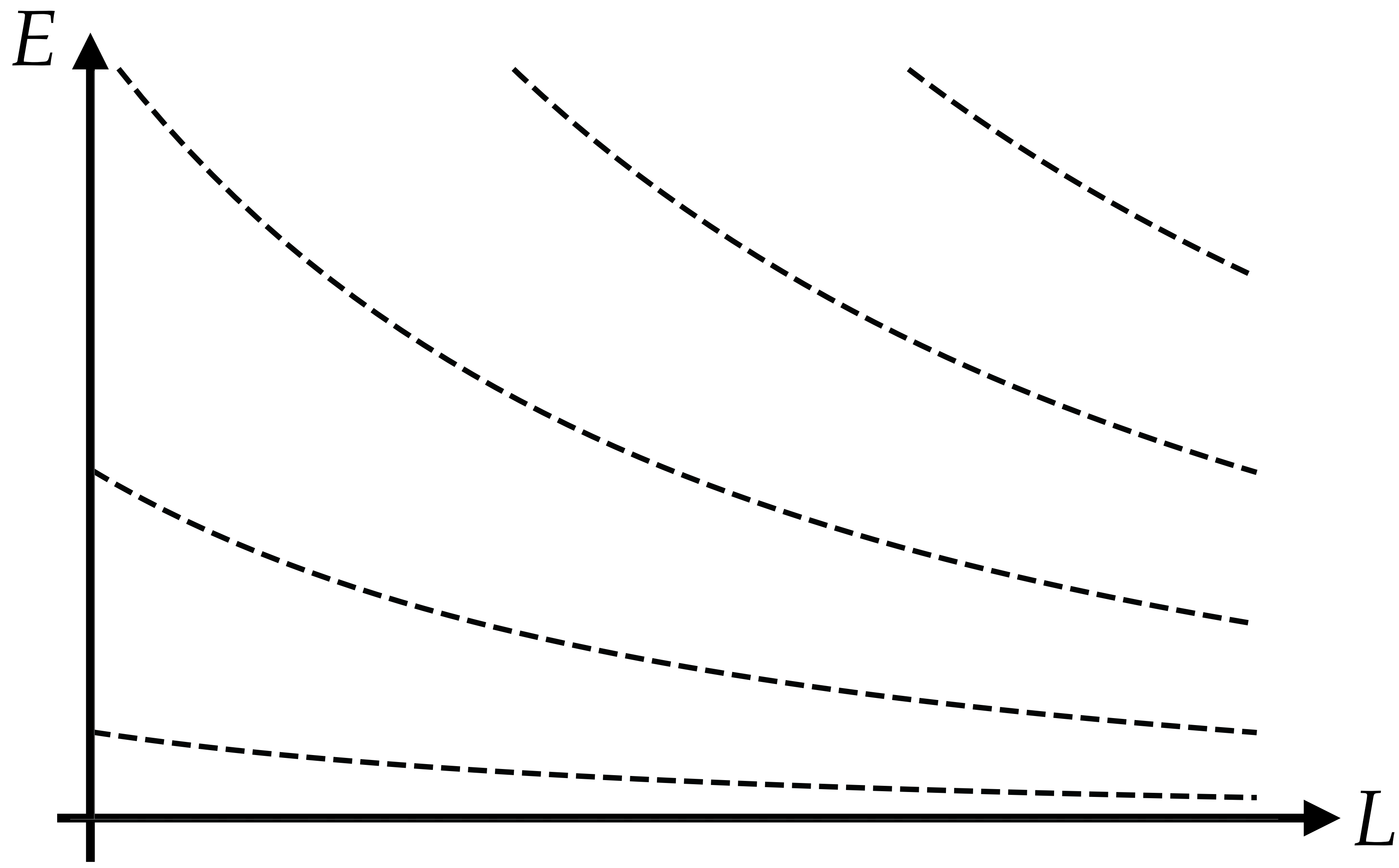
‘scattering’ in a finite-volume

$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. no interaction, $\delta(p) = 0$

$$p = \frac{2\pi}{L} n$$

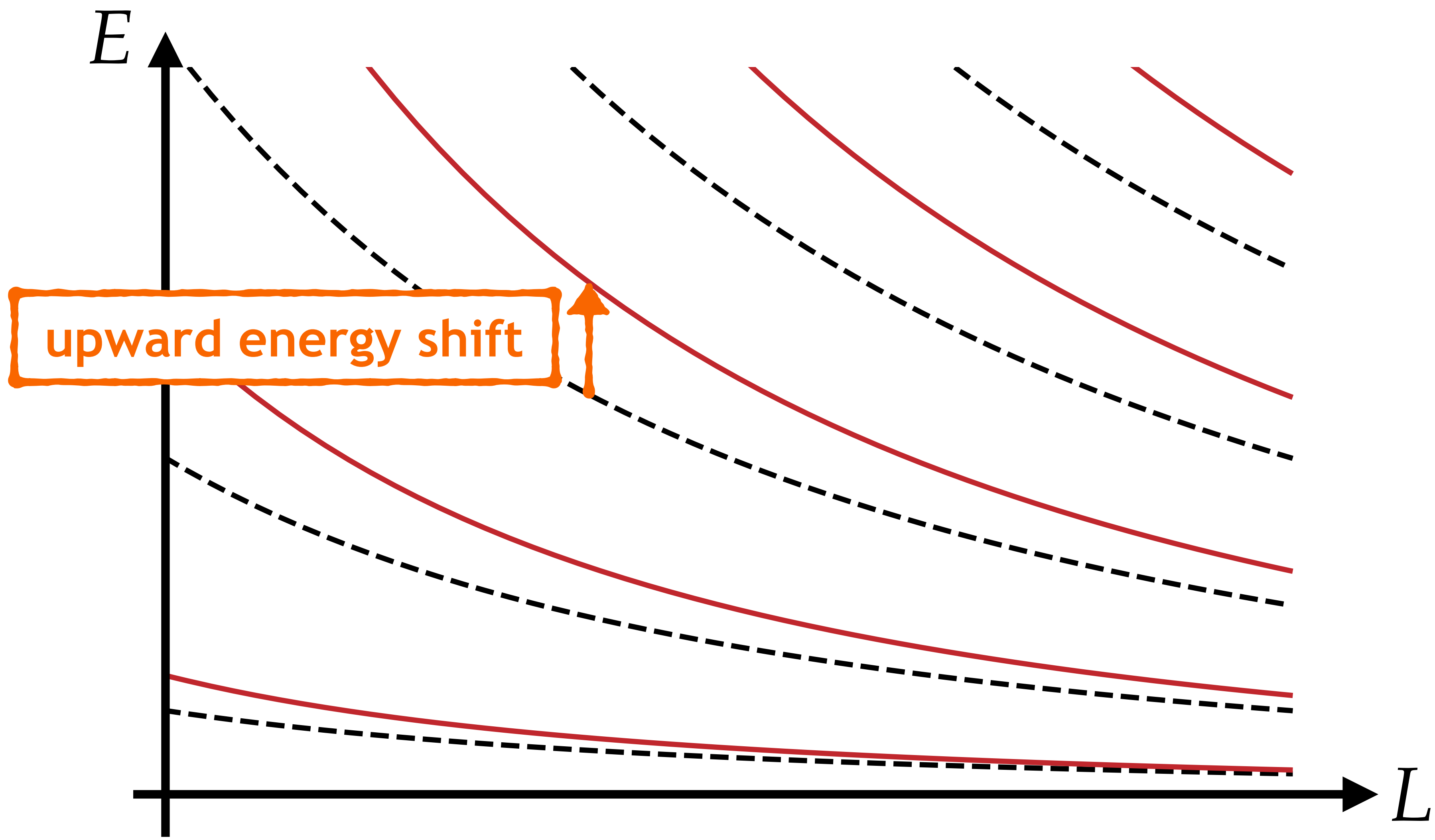


‘scattering’ in a finite-volume

$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a weak repulsive interaction, $\delta(p) = -ap$

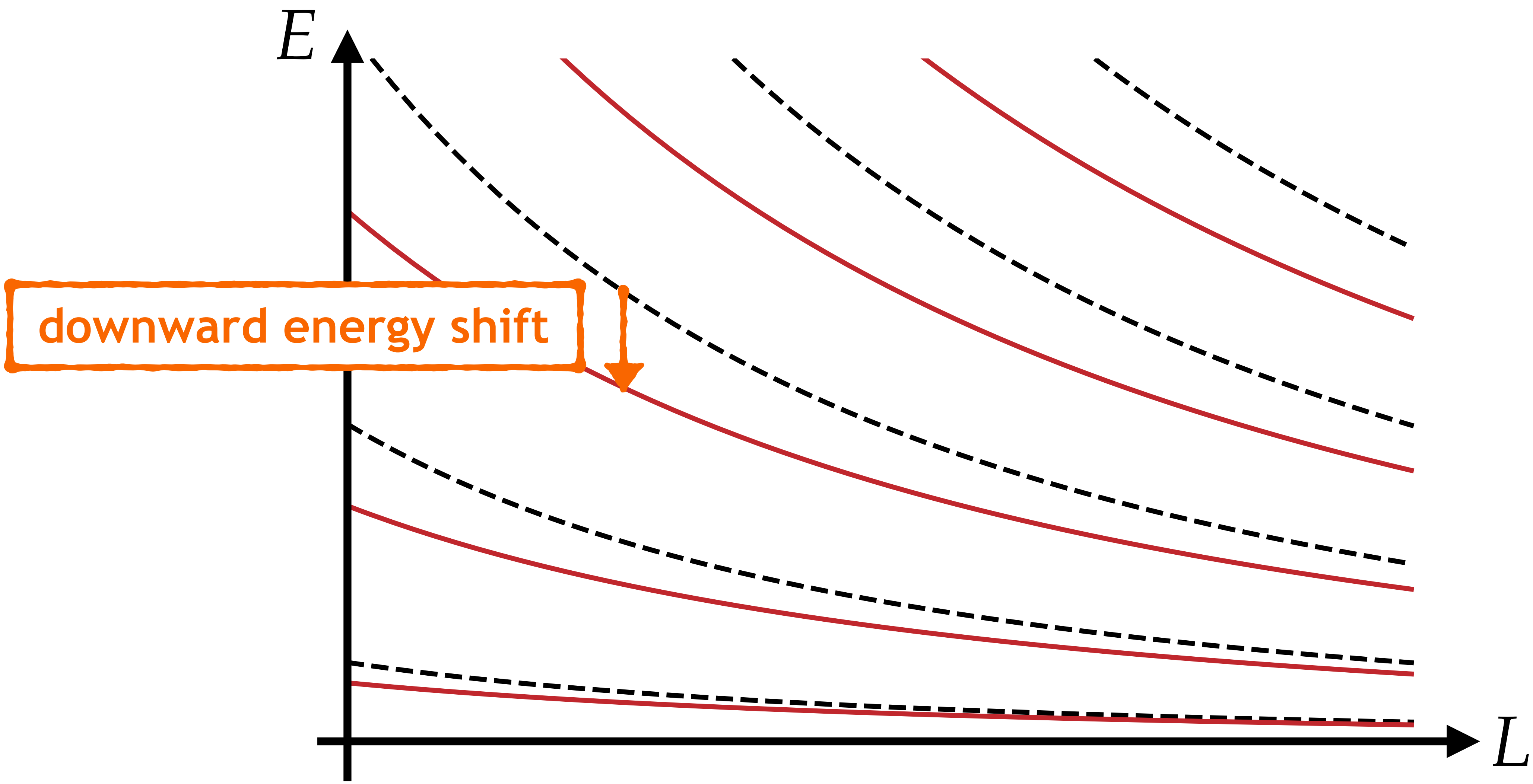


'scattering' in a finite-volume

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a weak attractive interaction, $\delta(p) = ap$

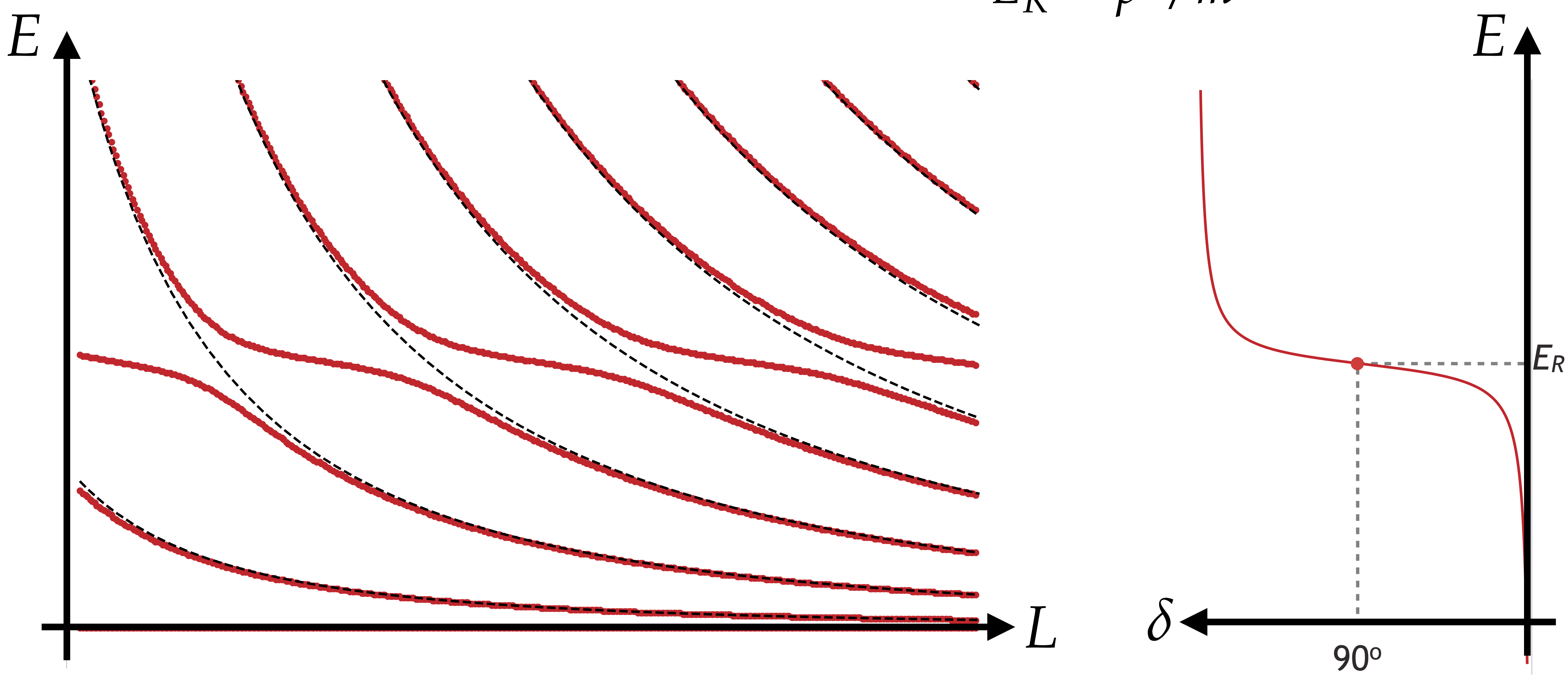


‘scattering’ in a finite-volume

$$p = \frac{2\pi}{L} n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

e.g. a non-rel Breit-Wigner resonance $\tan\delta(p) = \dfrac{\Gamma/2}{E_R - p^2/m}$



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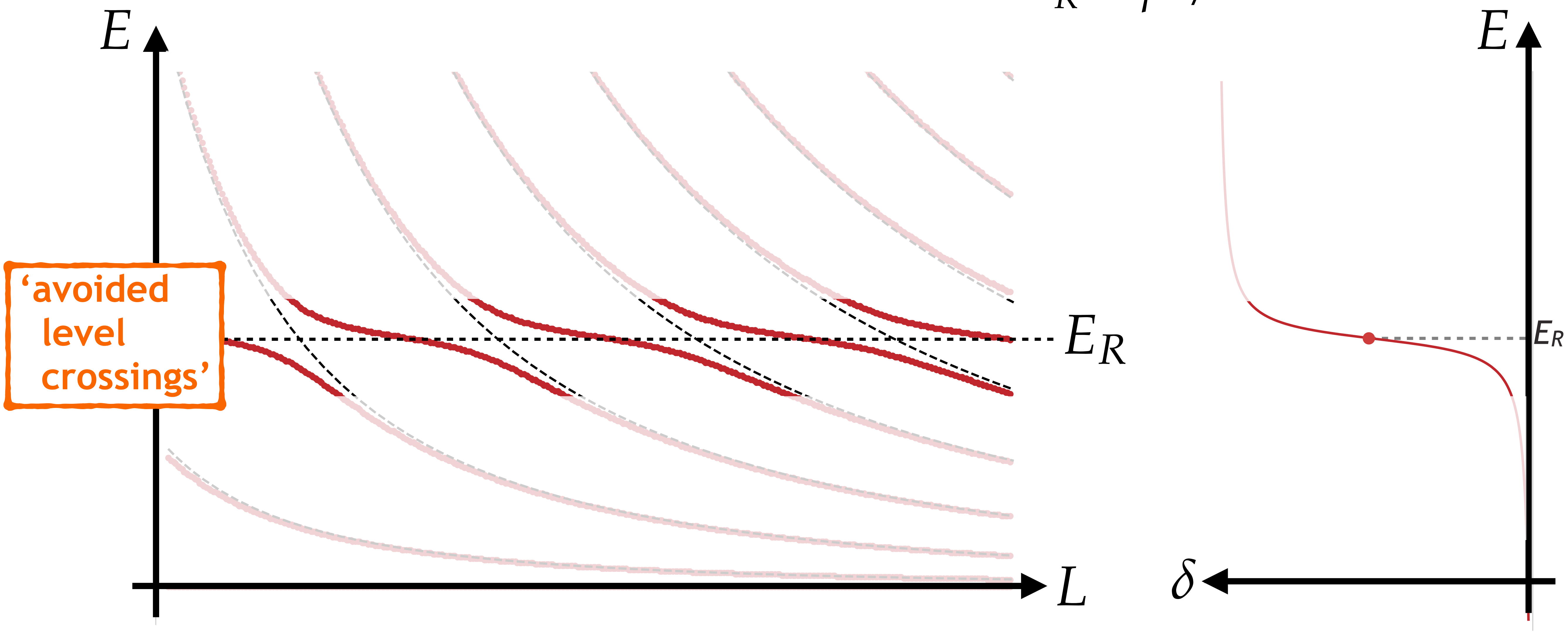


3+1 dim field theory in a cubic volume

TWO-PARTICLE STATES ON A TORUS AND THEIR RELATION TO THE SCATTERING MATRIX

Martin LÜSCHER

Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, 2000 Hamburg 52, Germany

Received 1 November 1990

25 years this week

The energy spectrum of a system of two particles enclosed in a box with periodic boundary conditions is characteristic for the forces between the particles. For box sizes greater than the interaction range, and for energies below the inelastic threshold, the spectrum is shown to be determined by the scattering phases at these energies. Simple exact formulae are derived which can be used to compute the energy levels given the scattering phases or, conversely, to calculate the scattering phases if the energy spectrum is known.

Lüscher:

$$\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$$

[modulo some subtleties regarding ℓ-mixing]

known functions

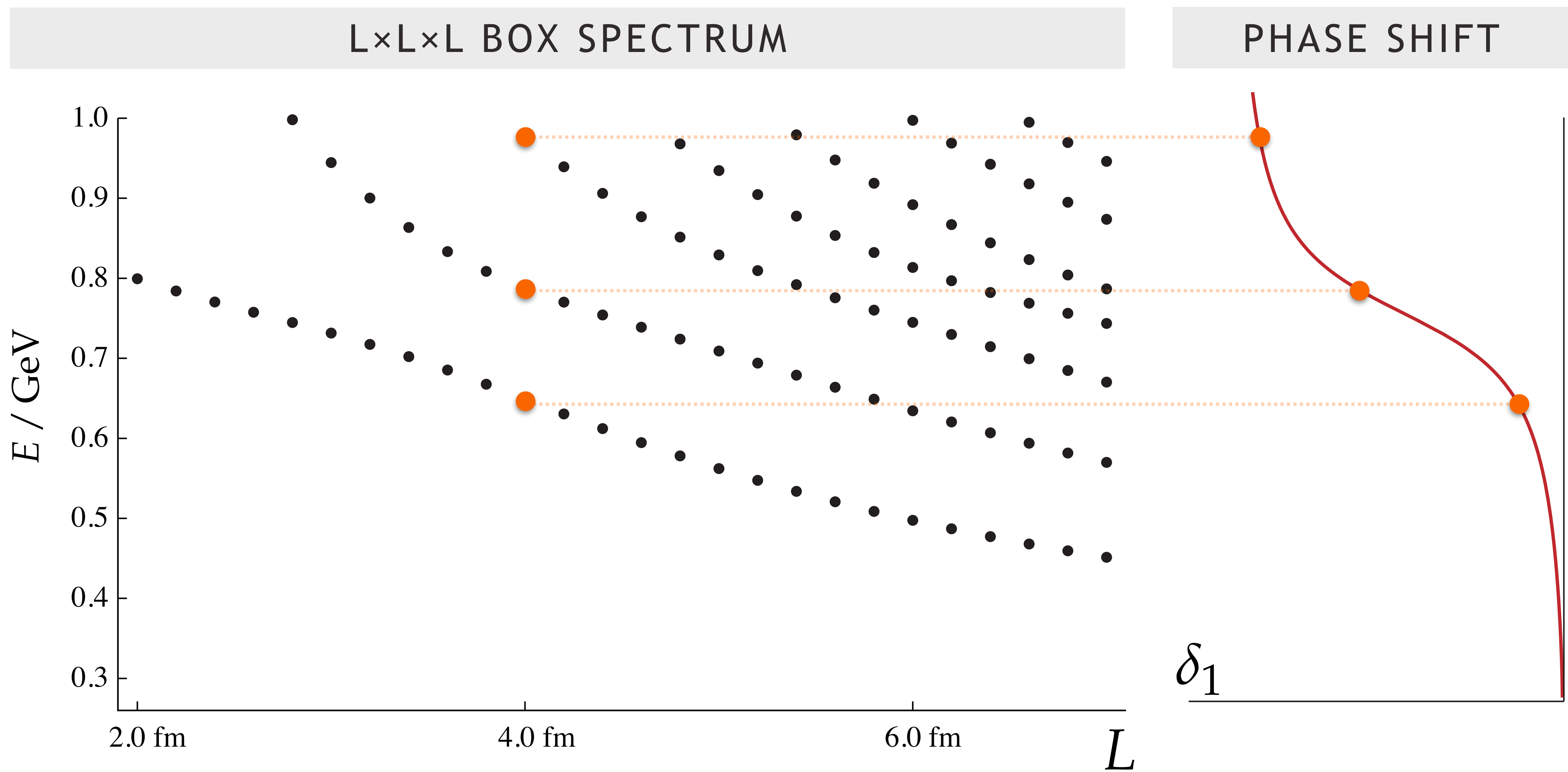
AND MANY EXTENSIONS BY OTHER AUTHORS ...

spectrum → phase-shift

- e.g. the ρ resonance in cubic boxes

$\rho(770)$ [h] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV



spectrum → phase-shift

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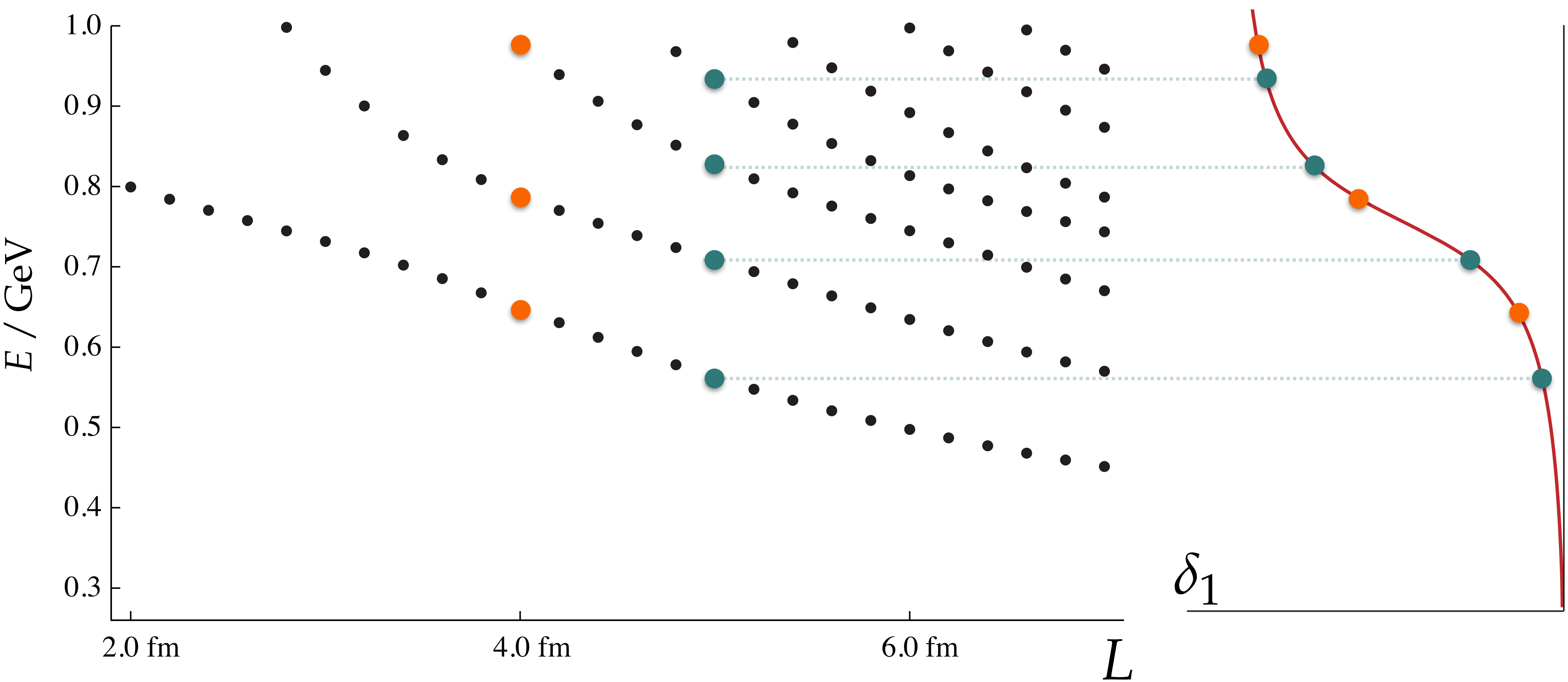
L×L×L BOX SPECTRUM	PHASE SHIFT



obtaining the spectrum

- a large basis of operators

qq̄-like $\bar{\psi}\boldsymbol{\Gamma}\psi = \bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\, \psi$ with vector quantum numbers

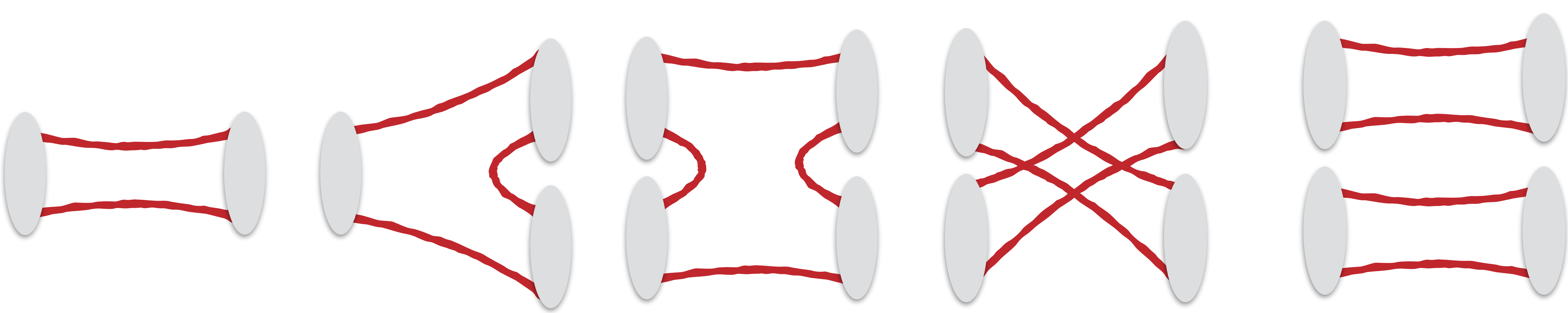
meson-meson-like $$\mathcal{O}^{|\vec{p}|}_{\pi\pi} = \sum_{\hat{p}} C(\hat{p})\, \pi(\vec{p})\pi(-\vec{p})$$

variationally optimized pion operator

$$\pi^+ = \sum_i v_i \left(\bar{u}\boldsymbol{\Gamma}_i d\right)$$

different |p| furnish the basis

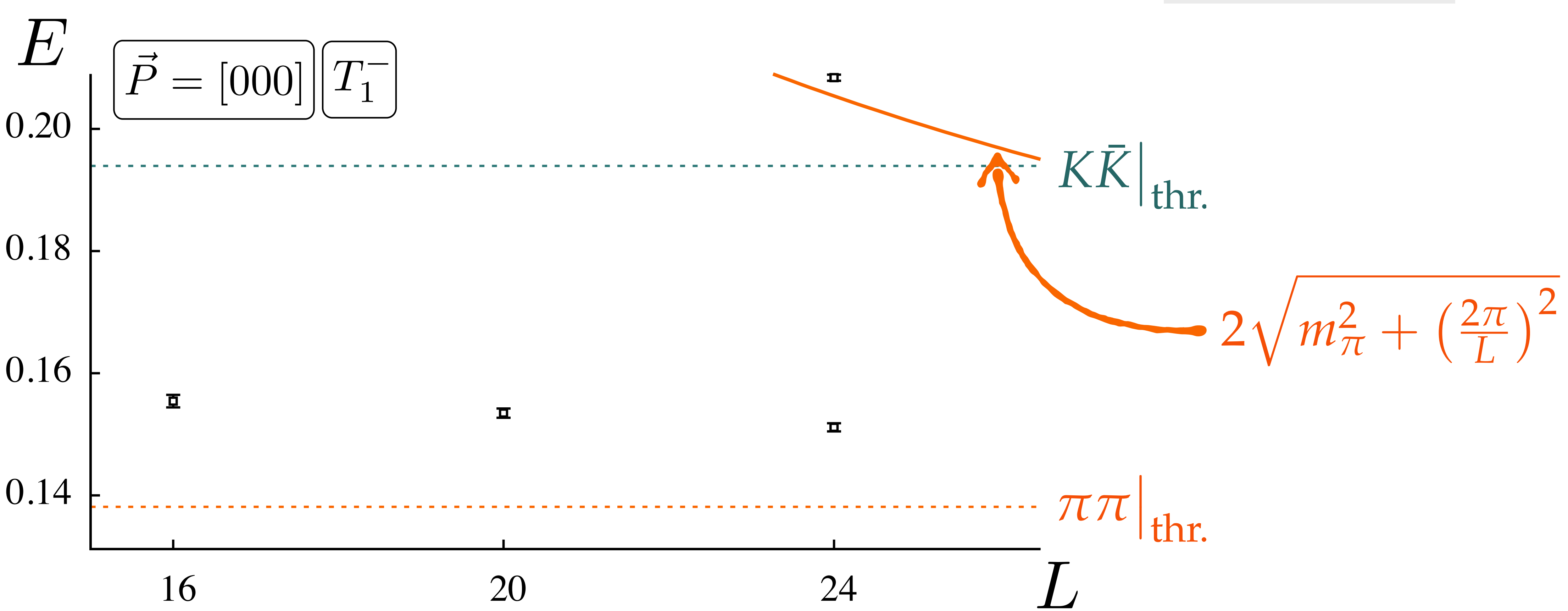
compute the matrix of correlation functions & 'diagonalize'



$\pi\pi$ *P*-wave scattering

- discrete spectrum of states in rest frame on three volumes 16^3, 20^3, 24^3

$m_\pi \sim 391\,\mathrm{MeV}$



ππ *P*-wave scattering

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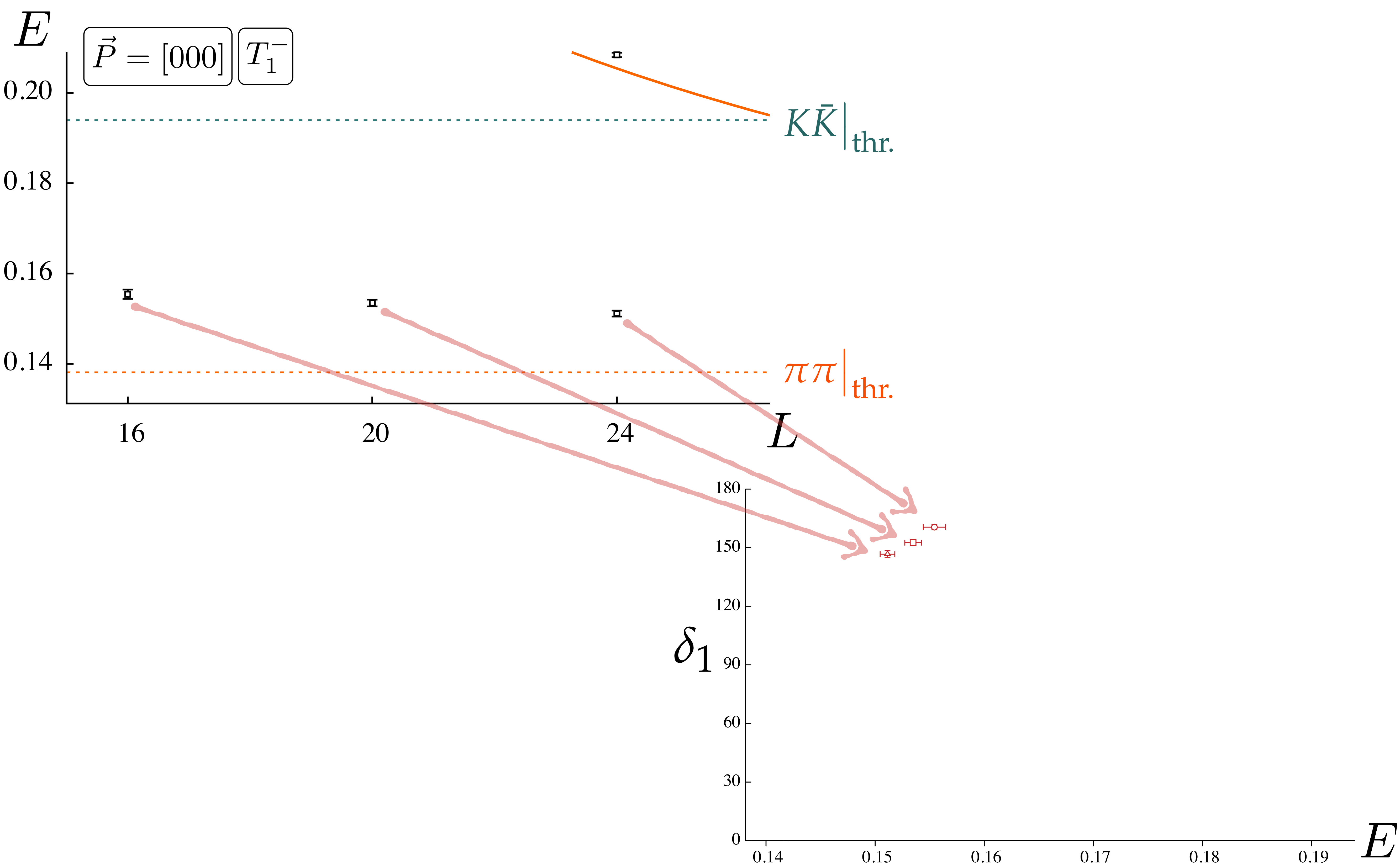
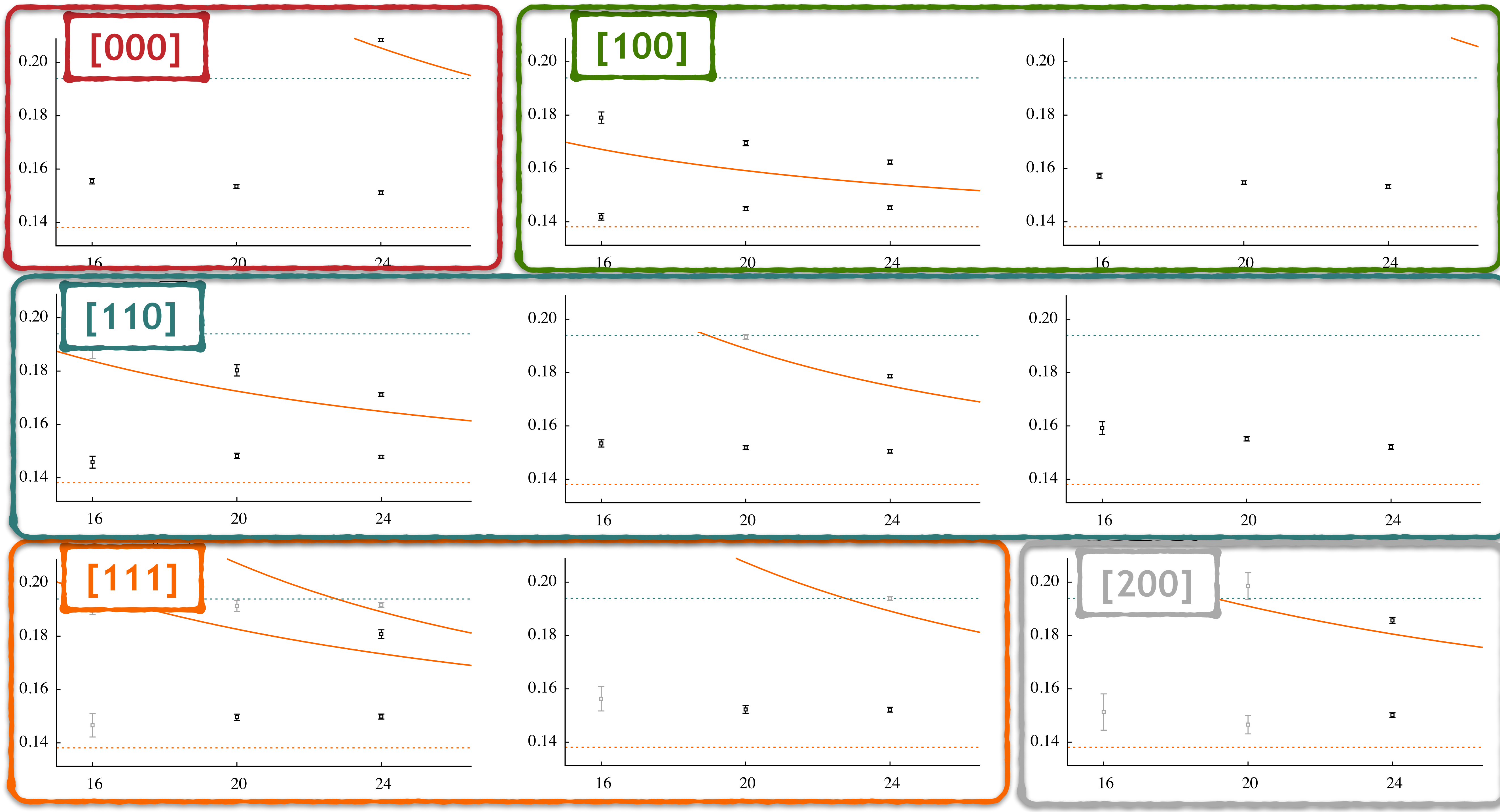
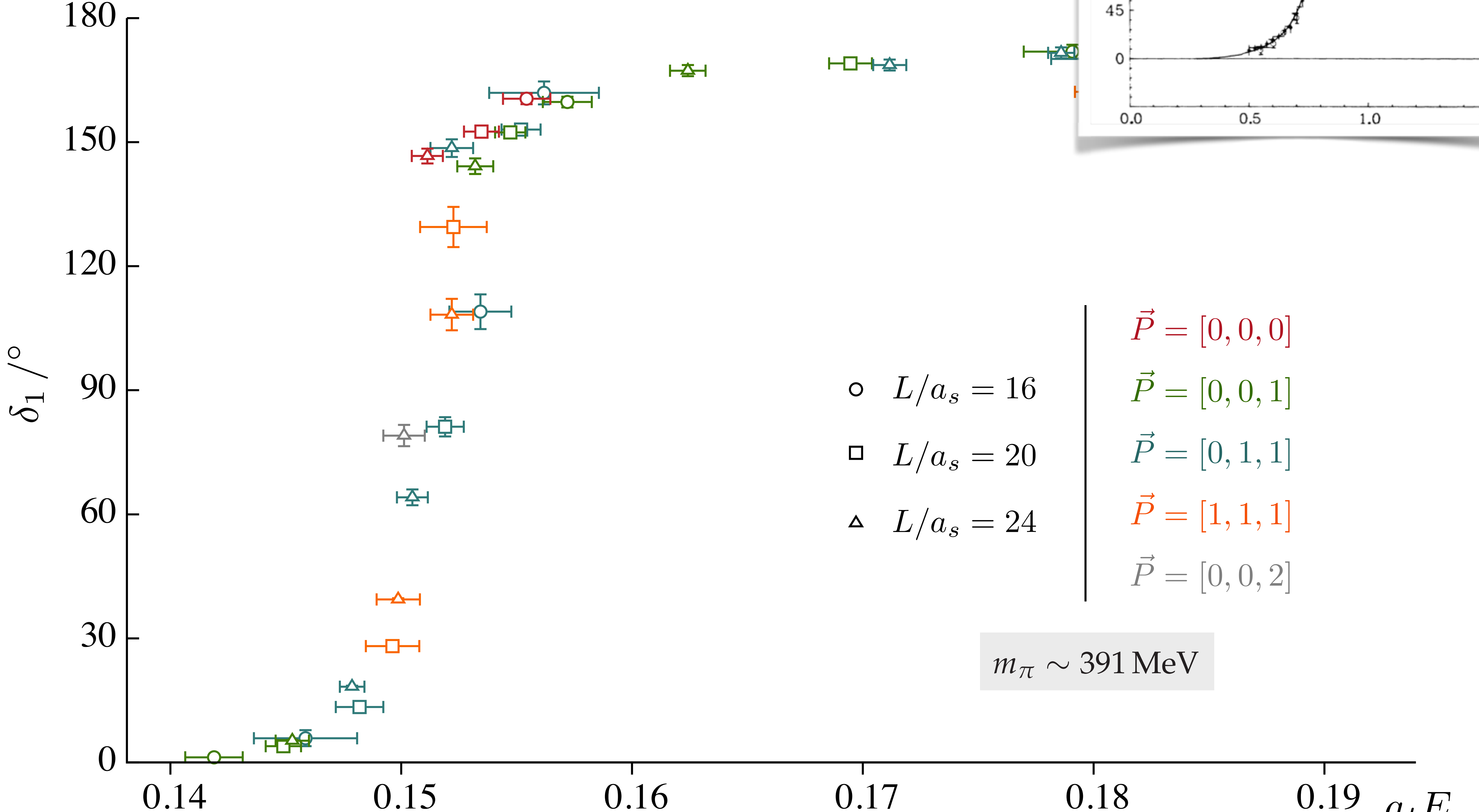
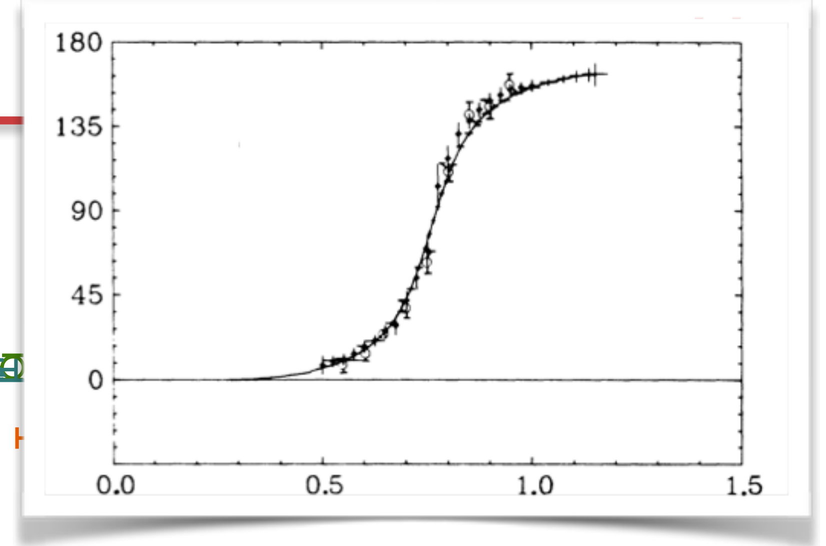


spectra in moving frames



$\pi\pi$ *P*-wave phase-shift

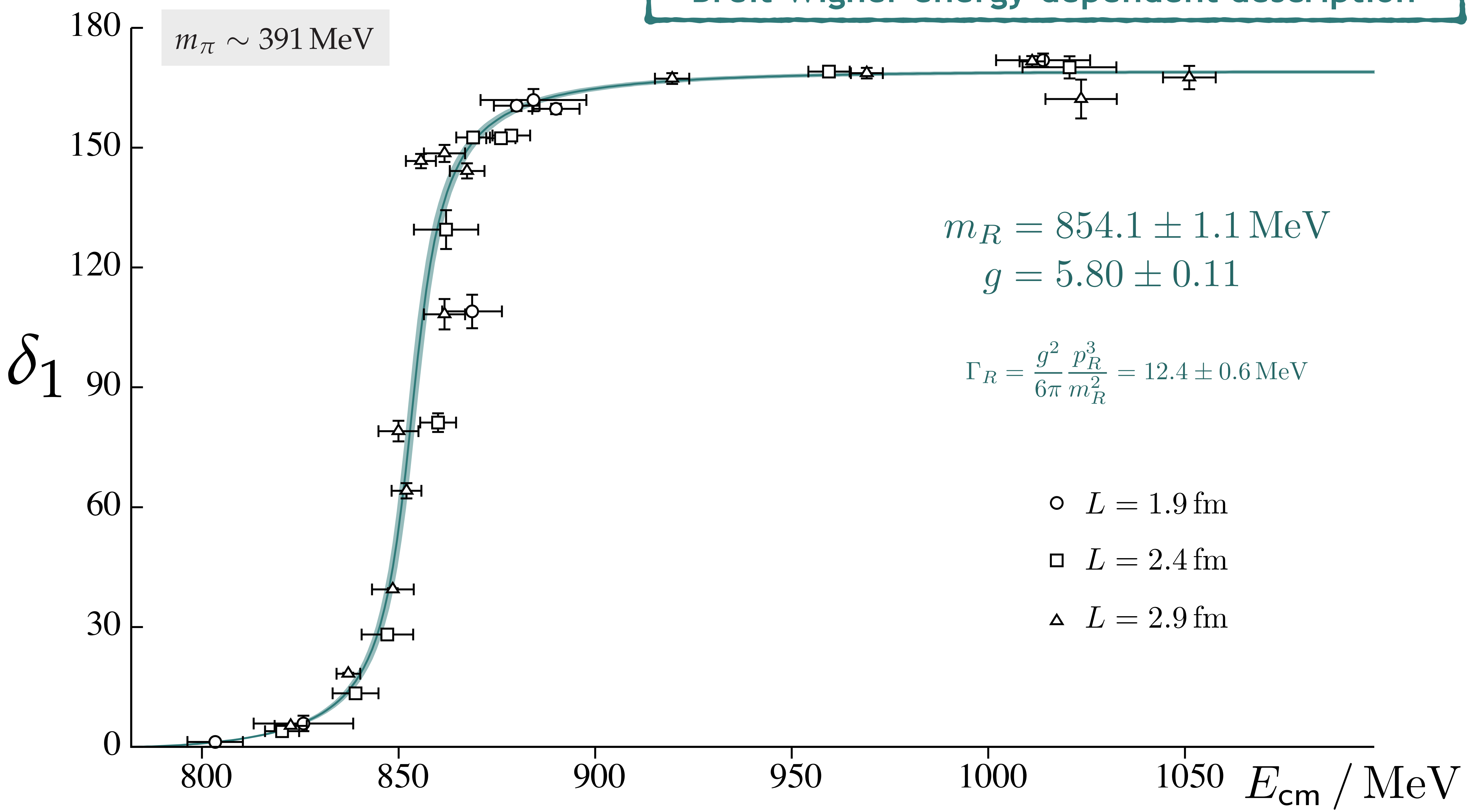




PRD87 034505 (2013)

$\pi\pi$ *P*-wave phase-shift

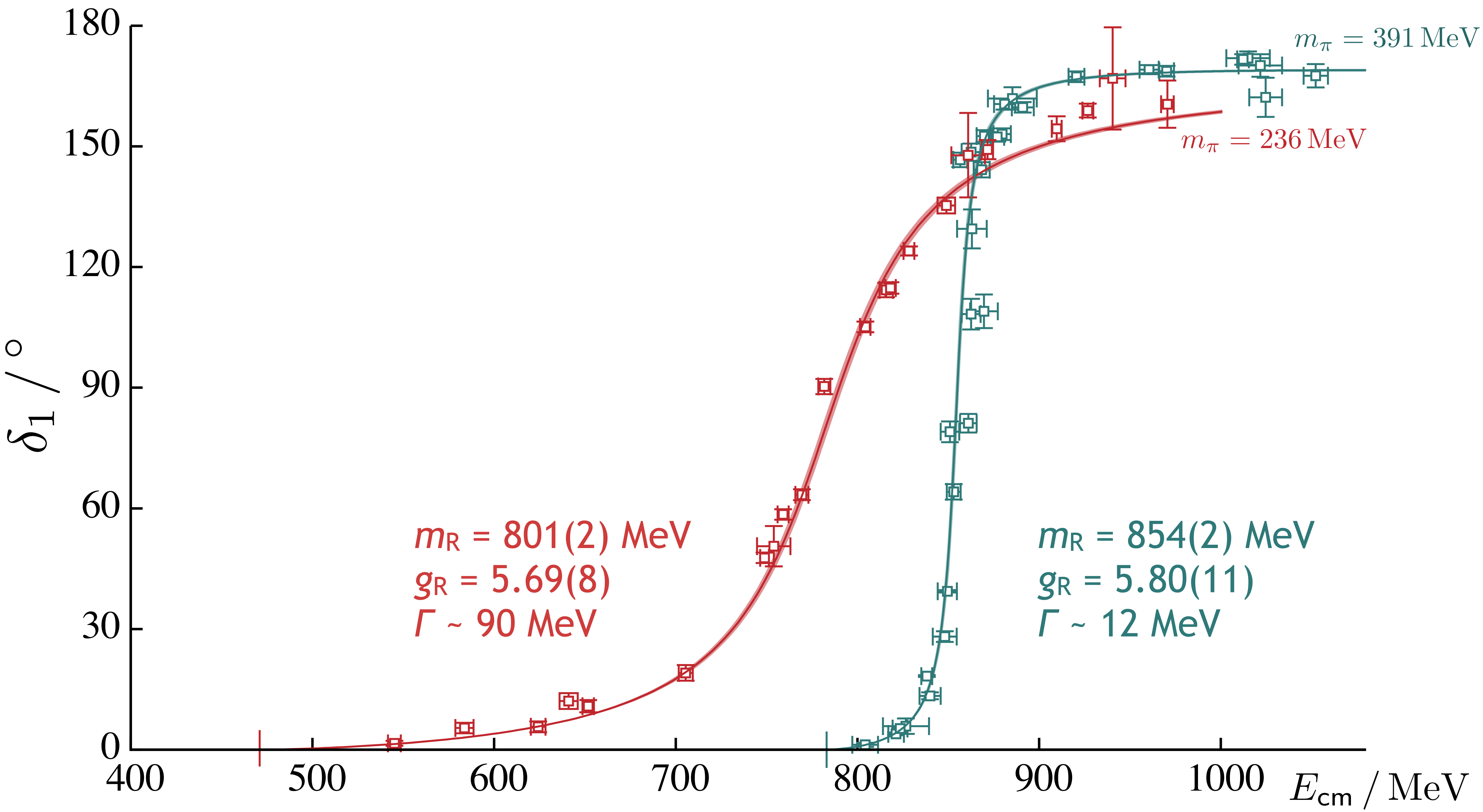
Breit-Wigner energy-dependent description



PRD87 034505 (2013)

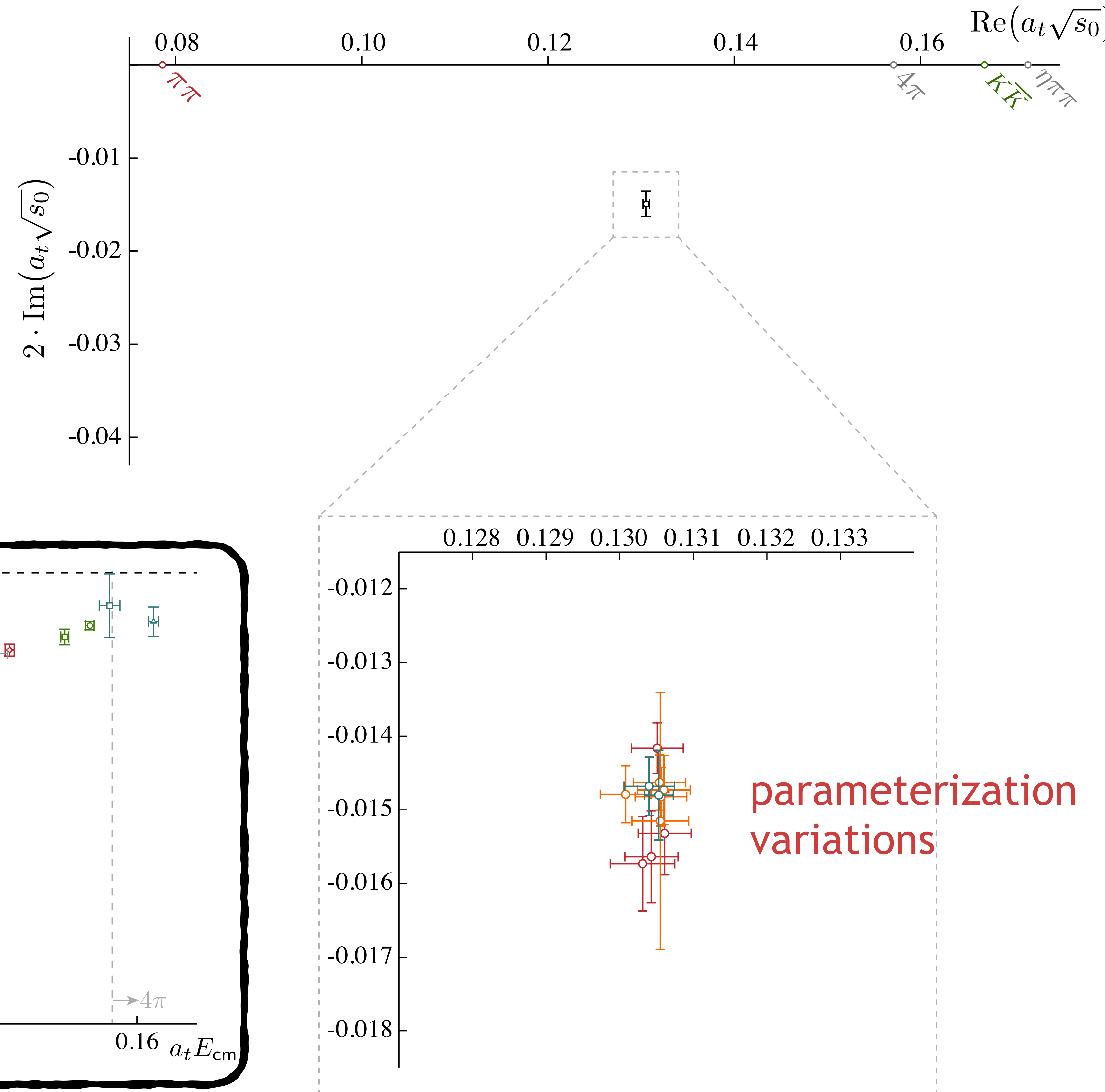
$\pi\pi$ P-wave phase-shift

- reducing the pion mass moves mass, width in the right direction …

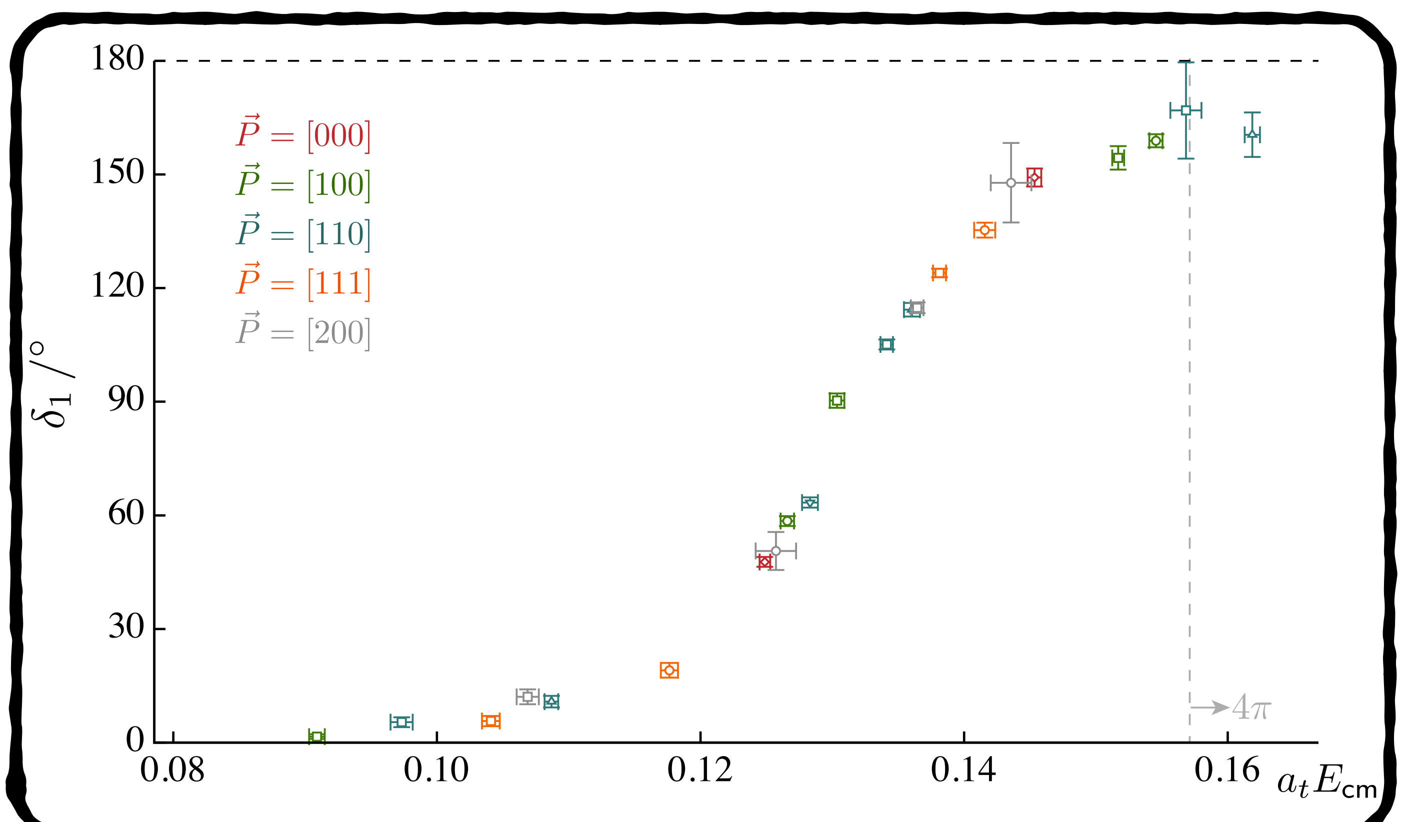


PRD 92, 094502 (2015)

ρ pole at m_π=236 MeV



parameterization variations



coupled-channel case

- most resonances decay to more than one final state or lie near thresholds

  - study the **coupled-channel *S*-matrix**

$$\mathbf{S} = \mathbf{1} + 2i\sqrt{\boldsymbol{\rho}}\,\mathbf{t}\sqrt{\boldsymbol{\rho}}$$

  - find poles [*mass*, *width*] & residues [*couplings*]

$$t_{ij}(s) \sim \frac{g_i g_j}{s_R - s}$$

PHASE-SPACE

$$\rho_i(s) = \frac{2k_i(s)}{\sqrt{s}}$$

2×2 *S*-MATRIX

$$S_{11} = \eta e^{2i\delta_1}$$
$$S_{22} = \eta e^{2i\delta_2}$$
$$S_{12} = i\sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)}$$

coupled-channel in a finite-volume

- the discrete spectrum is again related to scattering amplitudes:

$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

scattering matrix — *phase space* — *known functions*

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051
⋮

- spectrum given by the values of *E* which solve this equation
- we compute the spectrum in lattice QCD to determine $\mathbf{t}(E)$

multiple unknowns for each energy level - can't solve !

parameterize the energy dependence & describe the 'entire' spectrum

unitarity condition

$$\mathrm{Im}\,\mathbf{t}^{-1}(E) = -\boldsymbol{\rho}(E)$$

computing the $\pi K, \eta K$ spectrum

operator basis :

$q\bar{q}$ -like

$$\bar{u}\mathbf{\Gamma}s = \bar{u}\,\Gamma D \ldots D\, s$$

πK-like

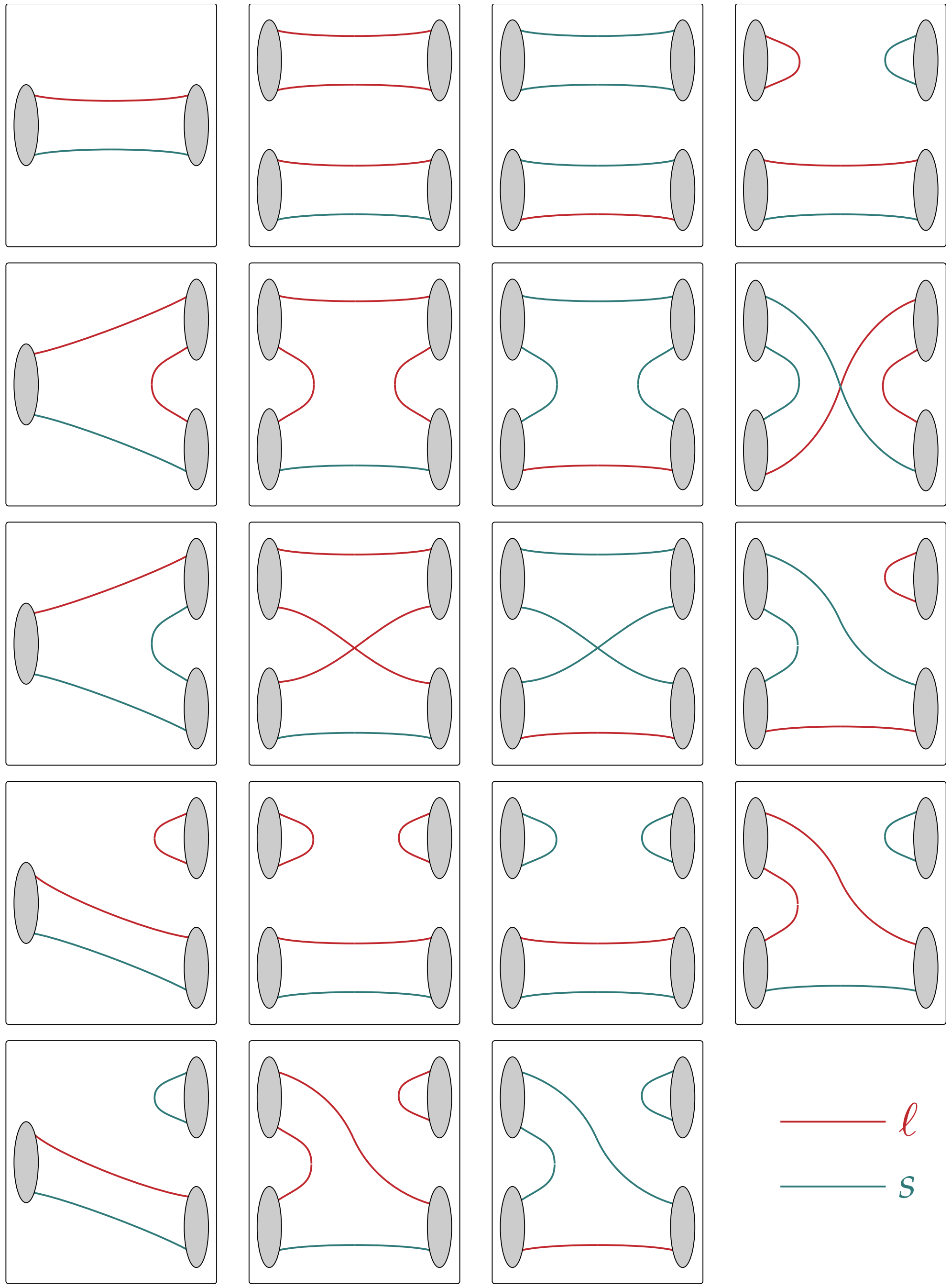
$$\sum_{\hat{p}_1, \hat{p}_2} C(\Lambda, \vec{P}; \vec{p}_1, \vec{p}_2)\, \pi(\vec{p}_1) K(\vec{p}_2)$$

ηK-like

$$\sum_{\hat{p}_1, \hat{p}_2} C(\Lambda, \vec{P}; \vec{p}_1, \vec{p}_2)\, \eta(\vec{p}_1) K(\vec{p}_2)$$

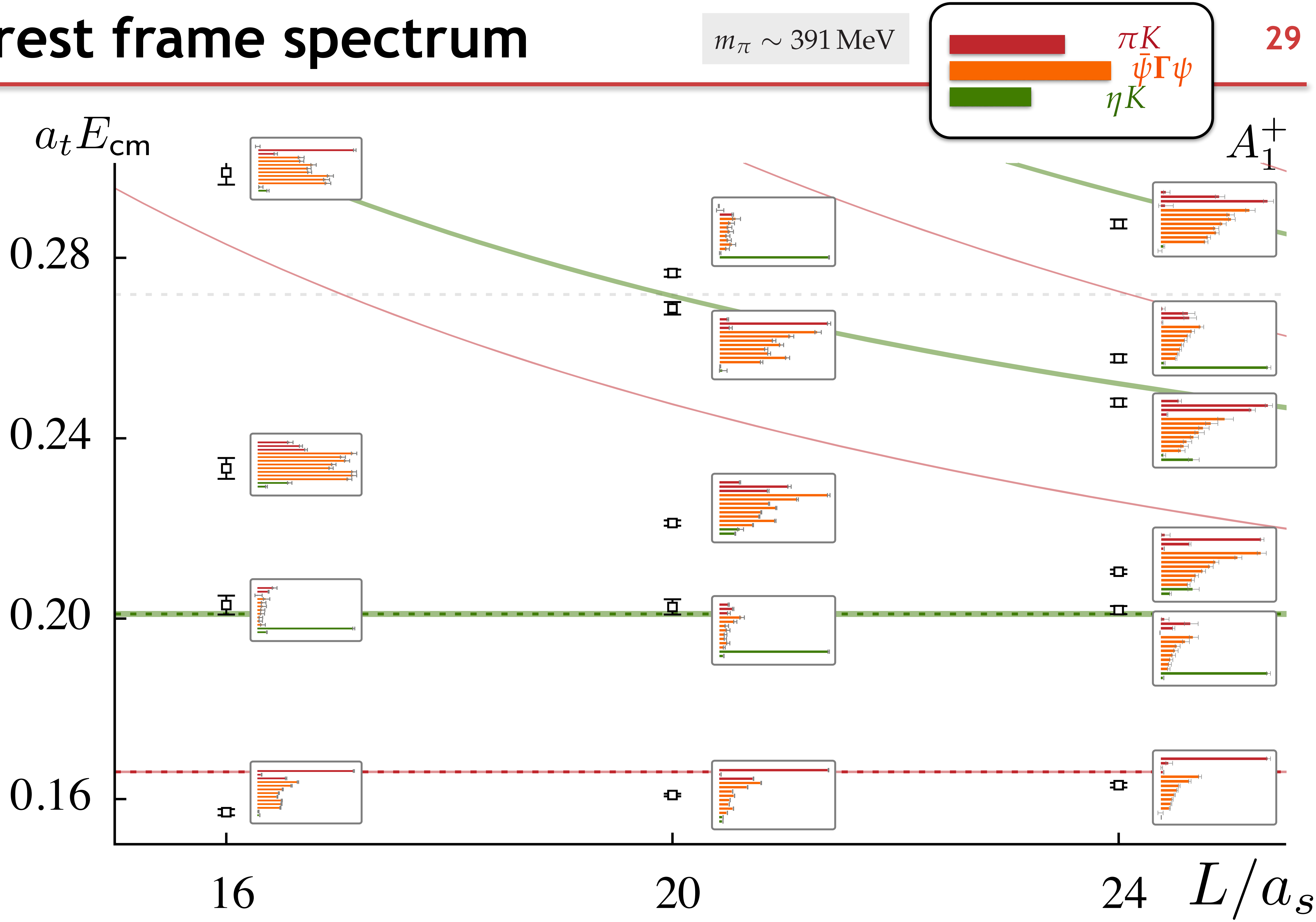
integrate out the quark fields ...

WICK CONTRACTIONS



rest frame spectrum

$m_\pi \sim 391\,\text{MeV}$



$\pi K / \eta K$ lattice QCD spectra

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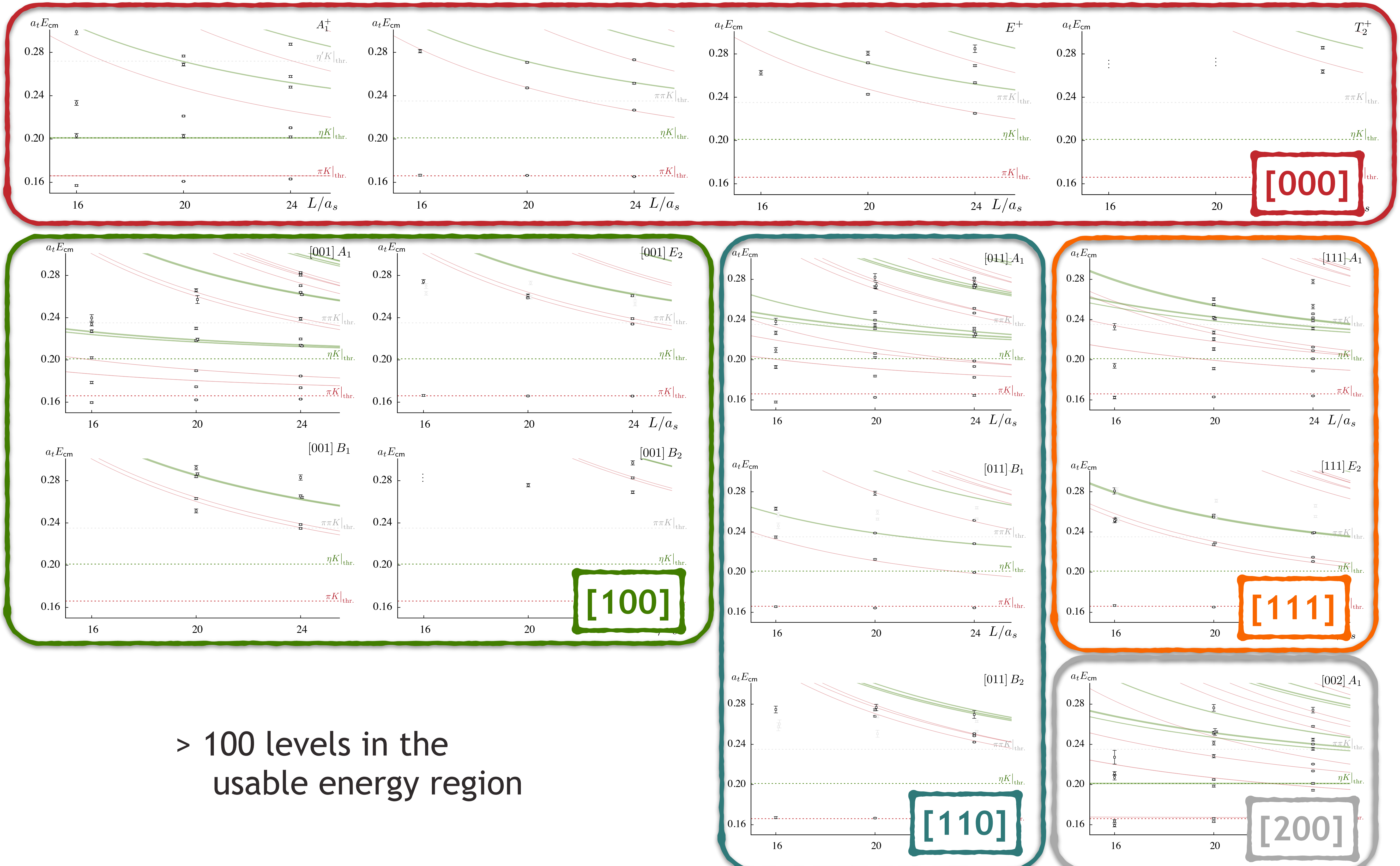

> 100 levels in the usable energy region

$\pi K/\eta K$ lattice QCD spectra

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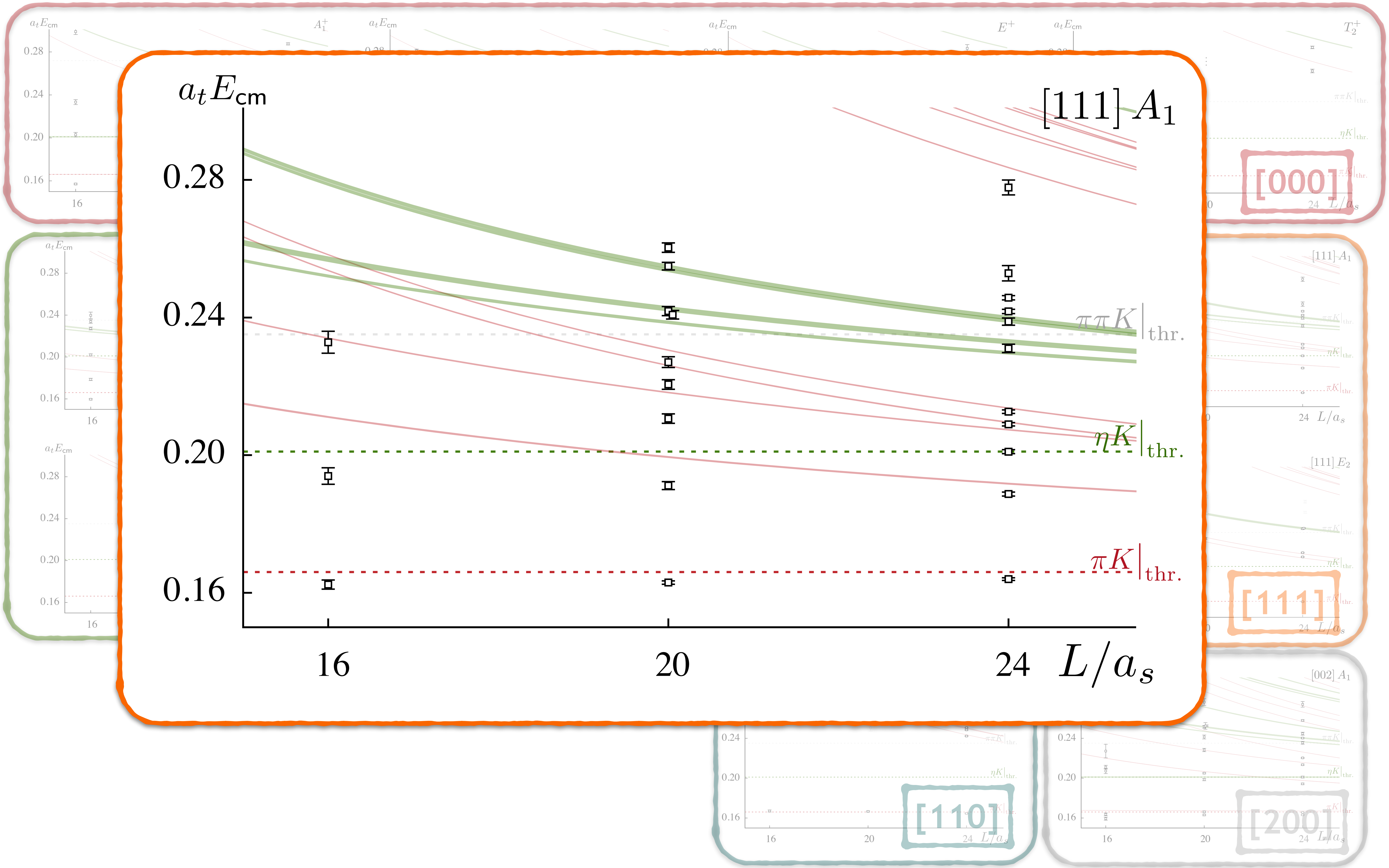

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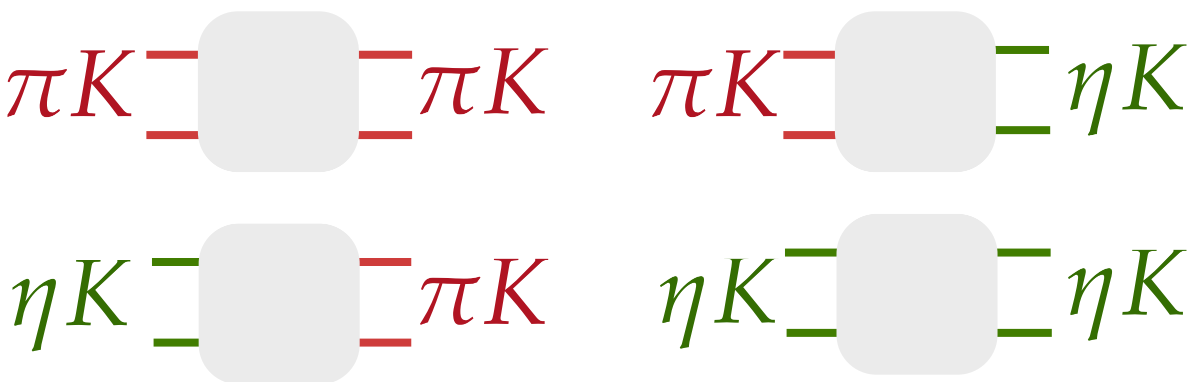
$\pi K/\eta K$ coupled-channel scattering

- parameterize the t-matrix in a unitarity conserving way

unitarity condition

$$\mathrm{Im}\,\mathbf{t}^{-1}(E) = -\boldsymbol{\rho}(E)$$



one example (from many)

$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij}\, I_i(E)$$

$$K_{ij}(E) = \frac{g_i\, g_j}{m^2 - E^2} + \gamma_{ij}$$

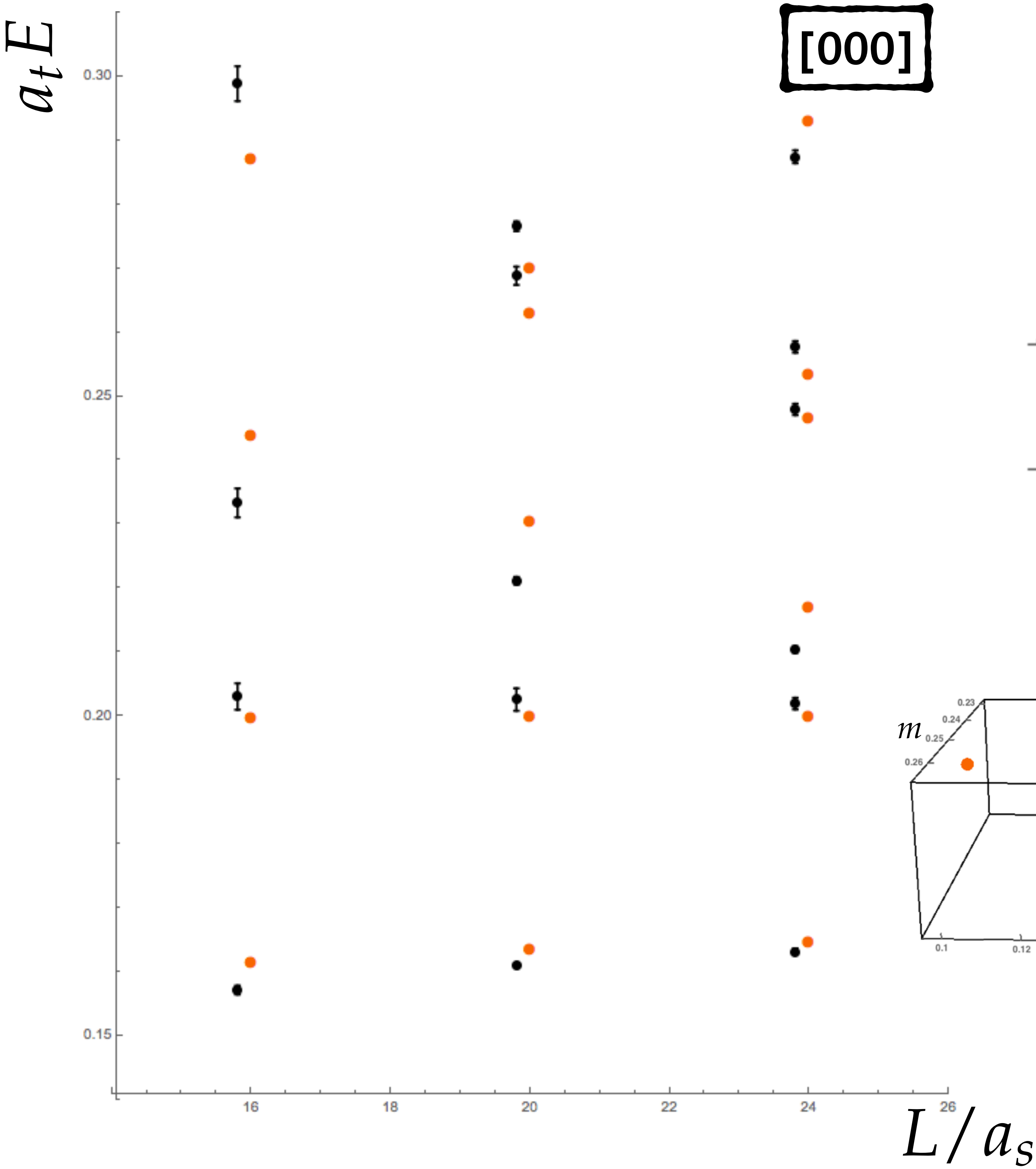
– vary the parameters, solving

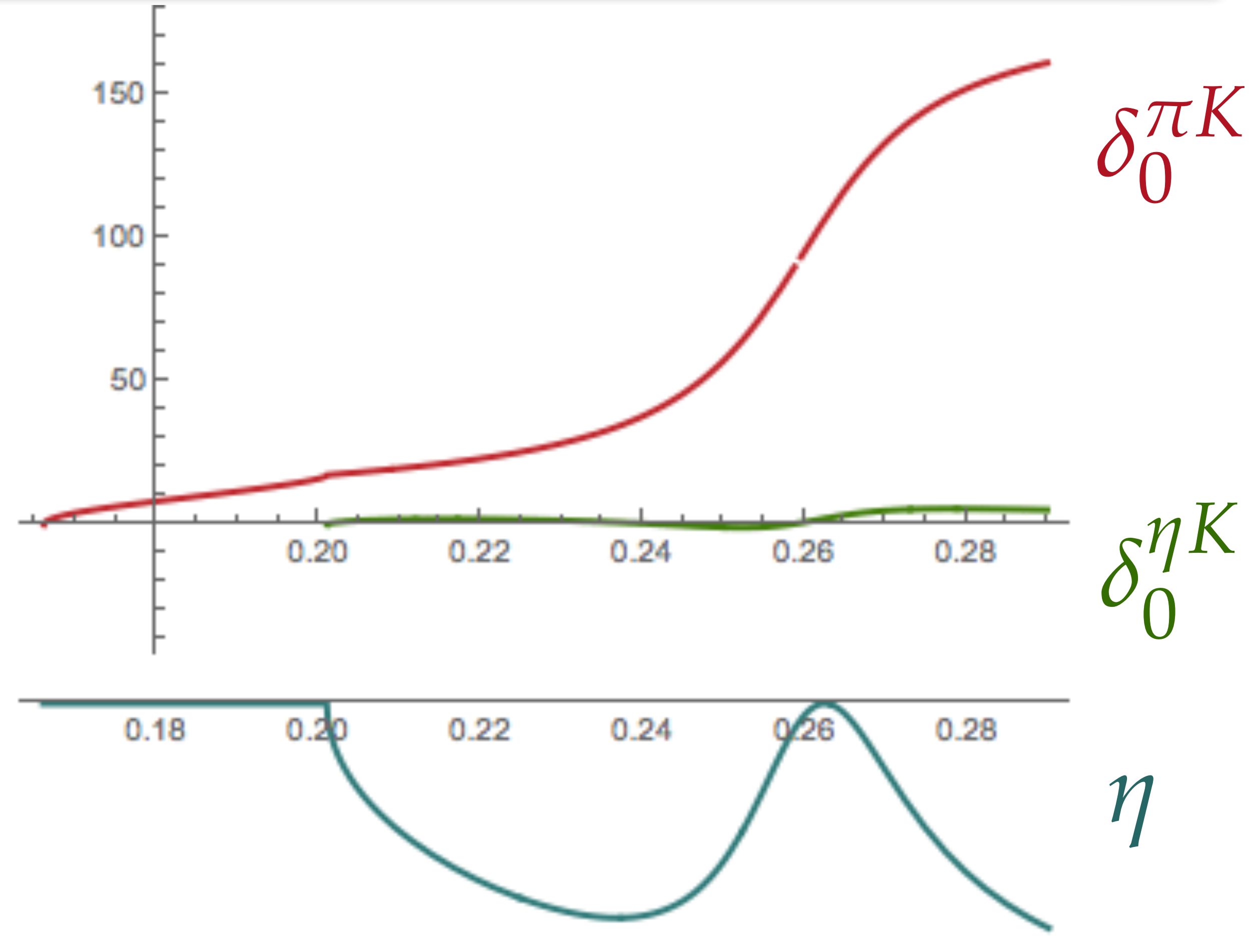
$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

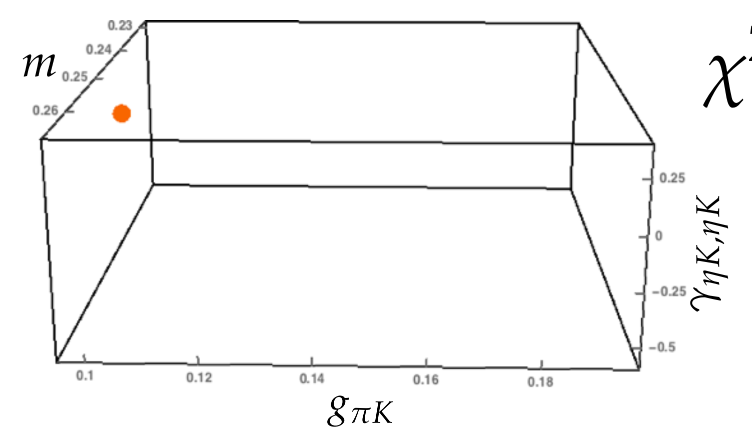
for the spectrum each time

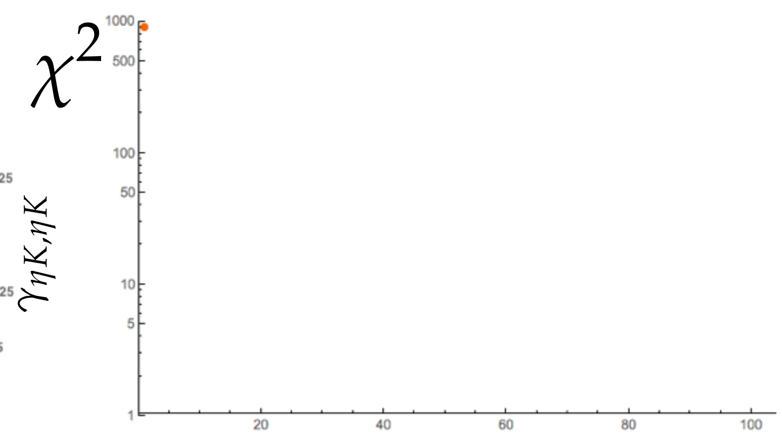
$\pi K/\eta K$ coupled-channel scattering

$m_\pi \sim 391\,\mathrm{MeV}$









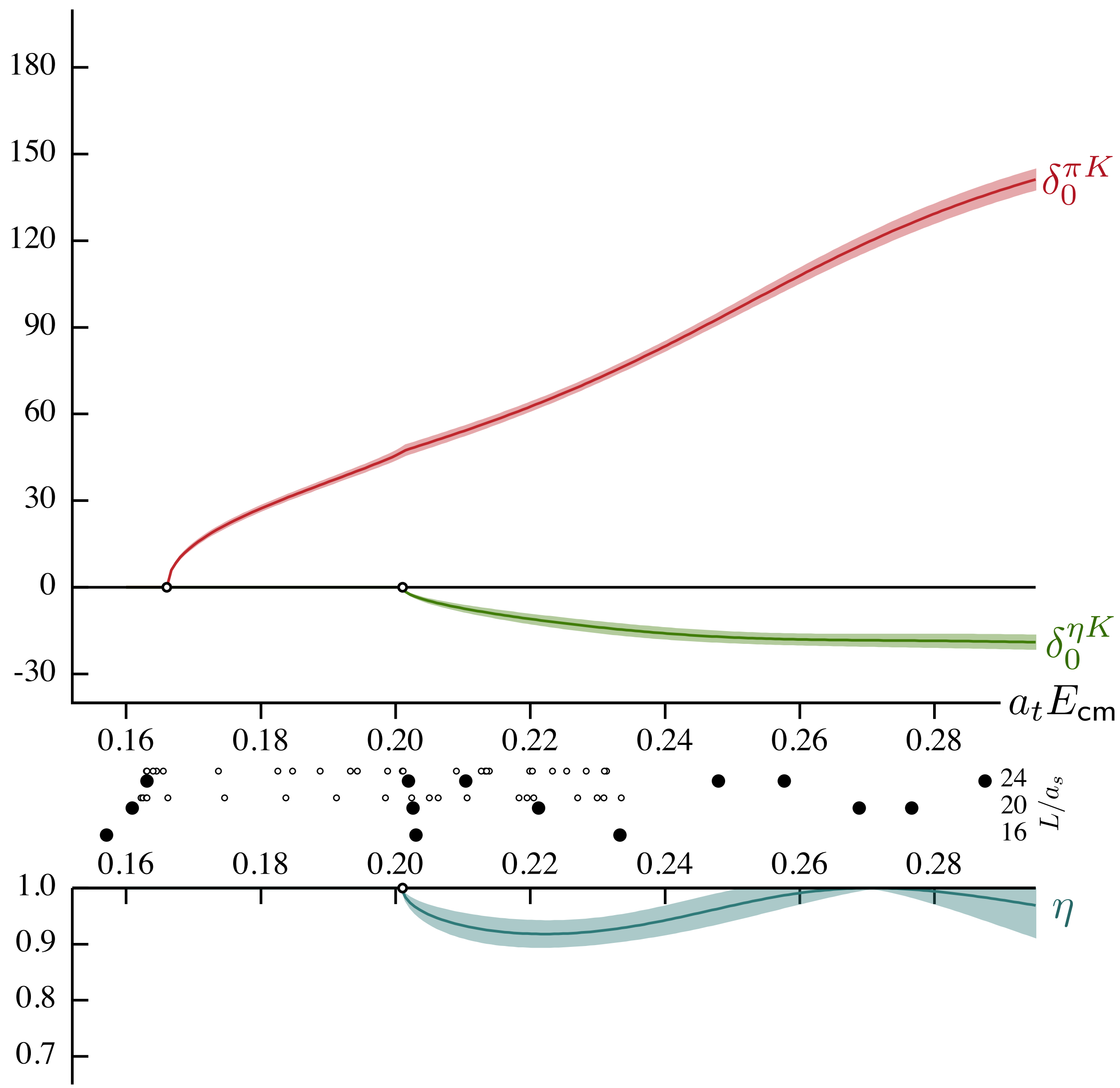
$\pi K/\eta K$ coupled-channel scattering

$m_\pi \sim 391\,\mathrm{MeV}$

- describe all the finite-volume spectra

$$\chi^2/N_{\mathrm{dof}} = \frac{49.1}{61-6} = 0.89$$

S-WAVE $\pi K/\eta K$ SCATTERING



$\pi K/\eta K$ coupled-channel scattering

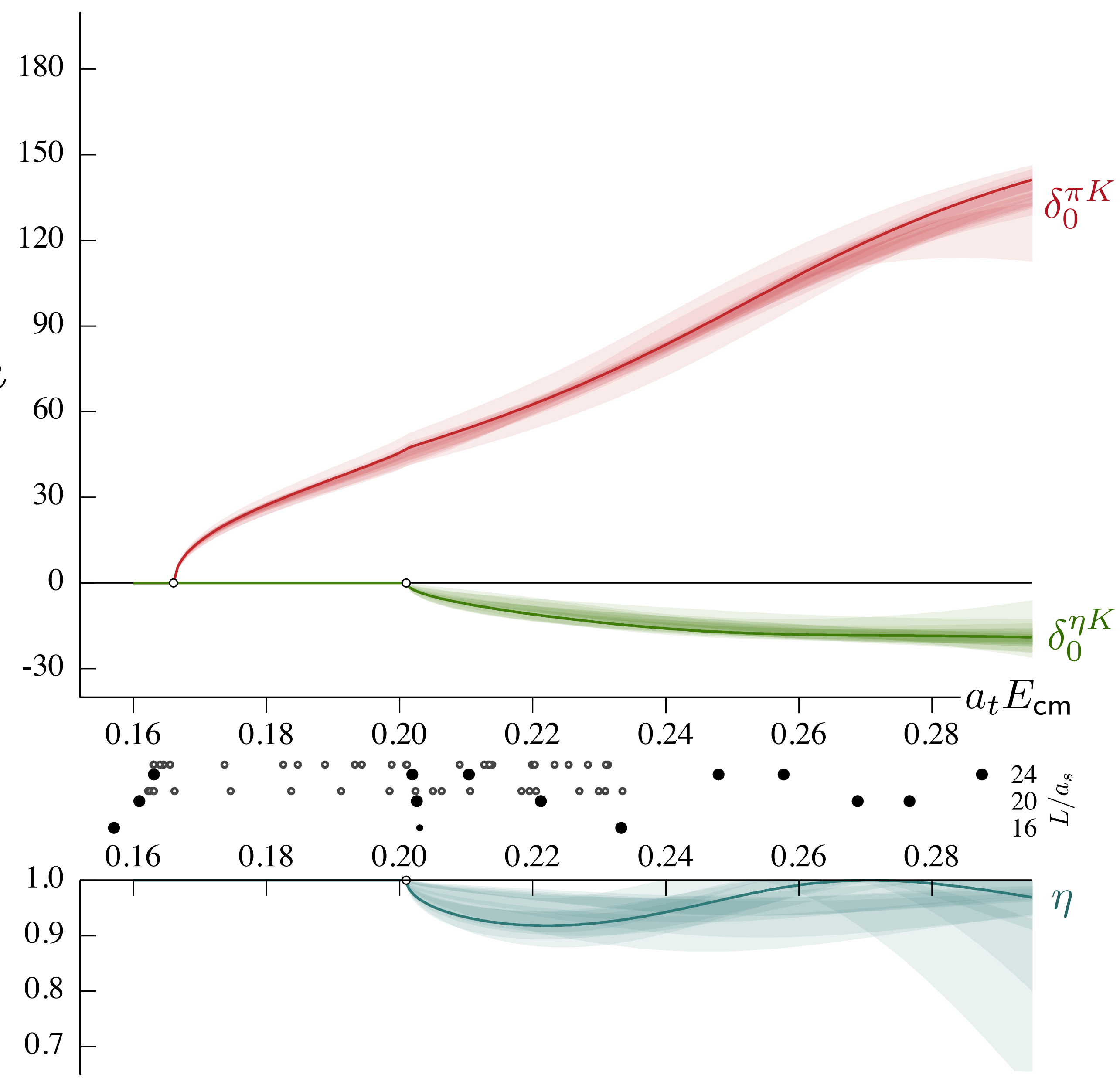
are the result parameterization dependent ?

– try a range of parameterizations ...

$$K^{-1}_{ij}(s) = \sum_{n=0}^{N_{ij}} c^{(n)}_{ij}\, s^n$$

$$K_{ij}(s) = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^2 - s} + \sum_n \gamma^{(n)}_{ij}\, s^n$$

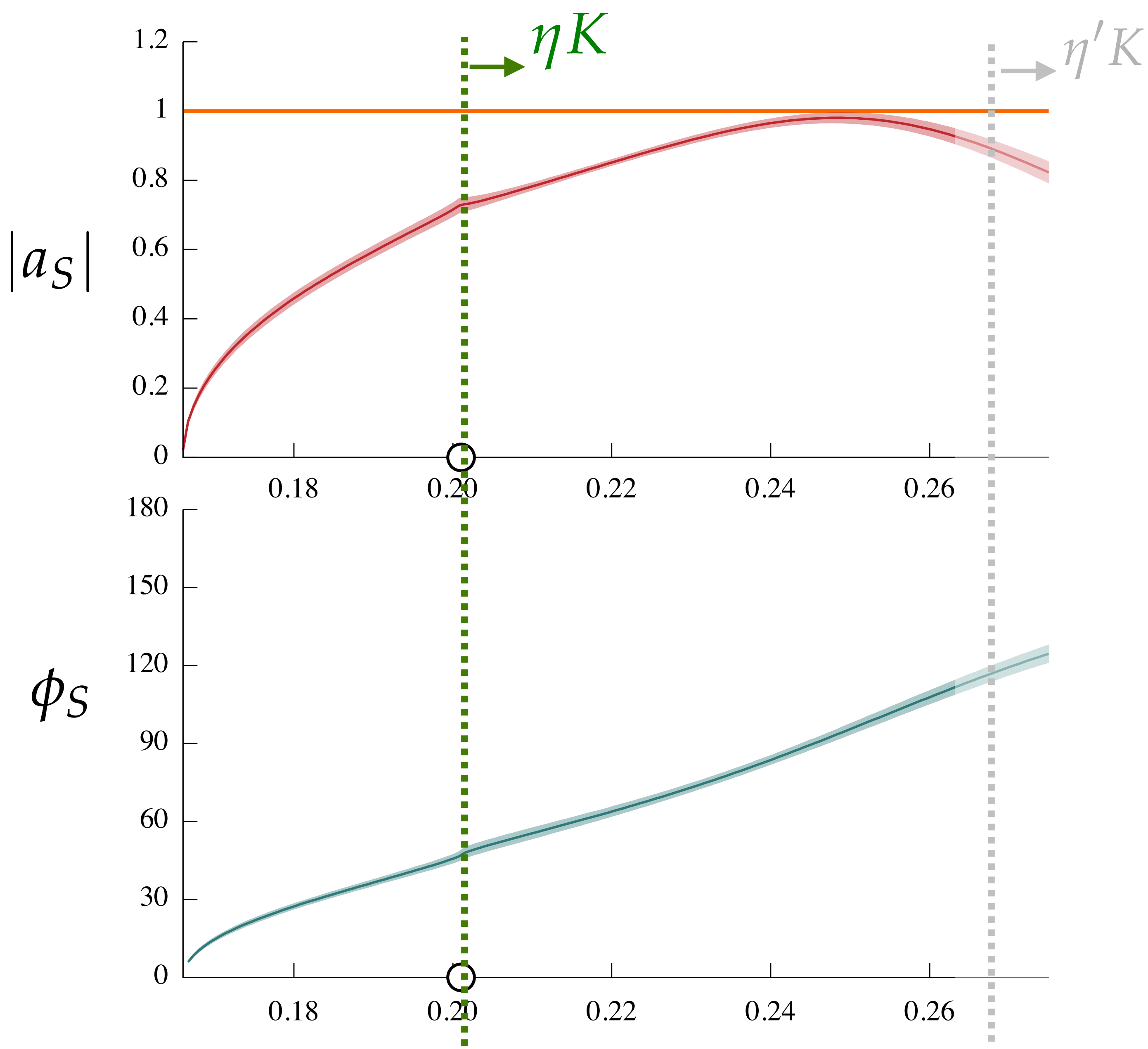
S-WAVE $\pi K/\eta K$ SCATTERING

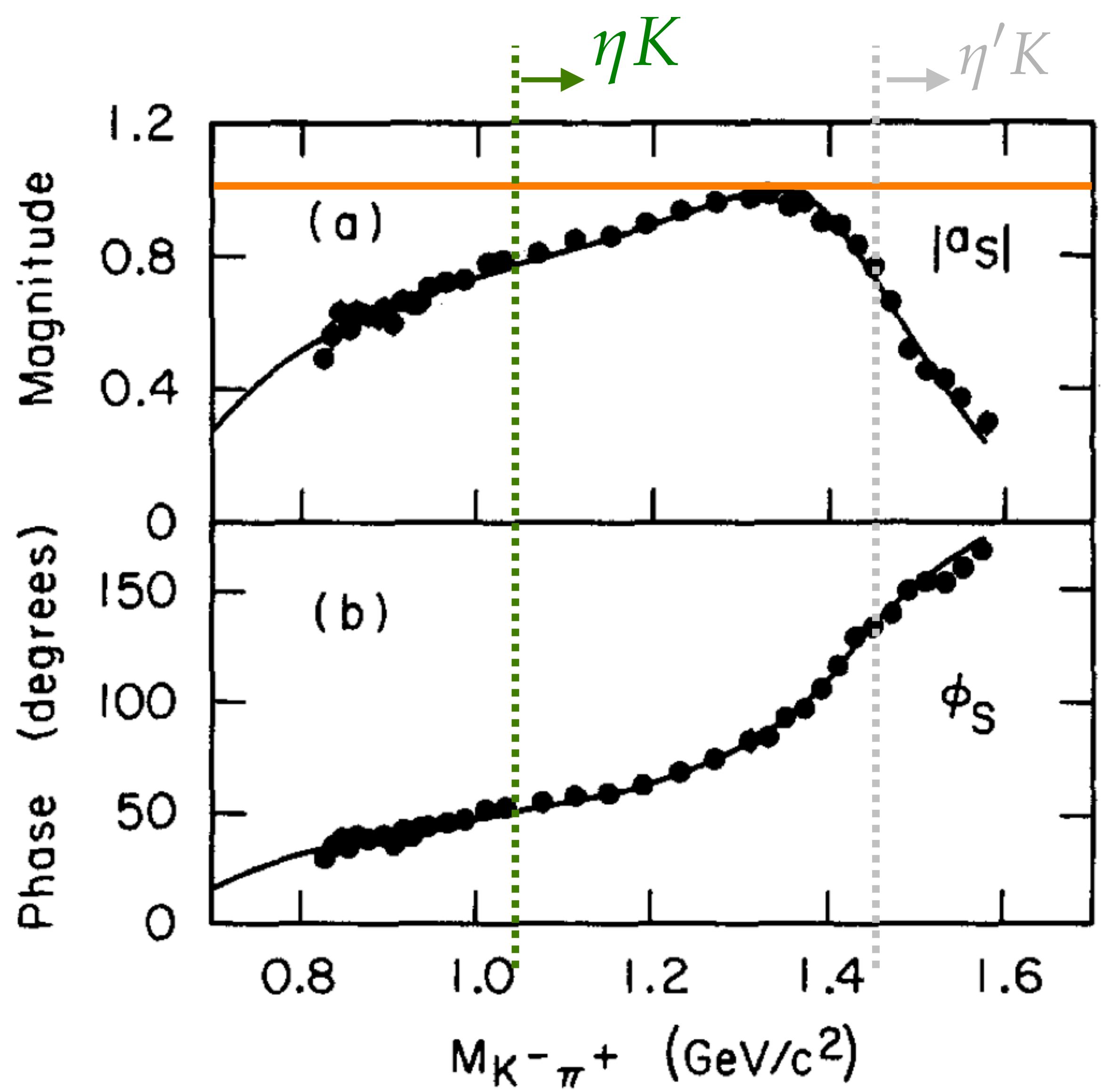


– gross features are robust

versus experimental scattering

S-WAVE $\pi K \to \pi K$ AMPLITUDE



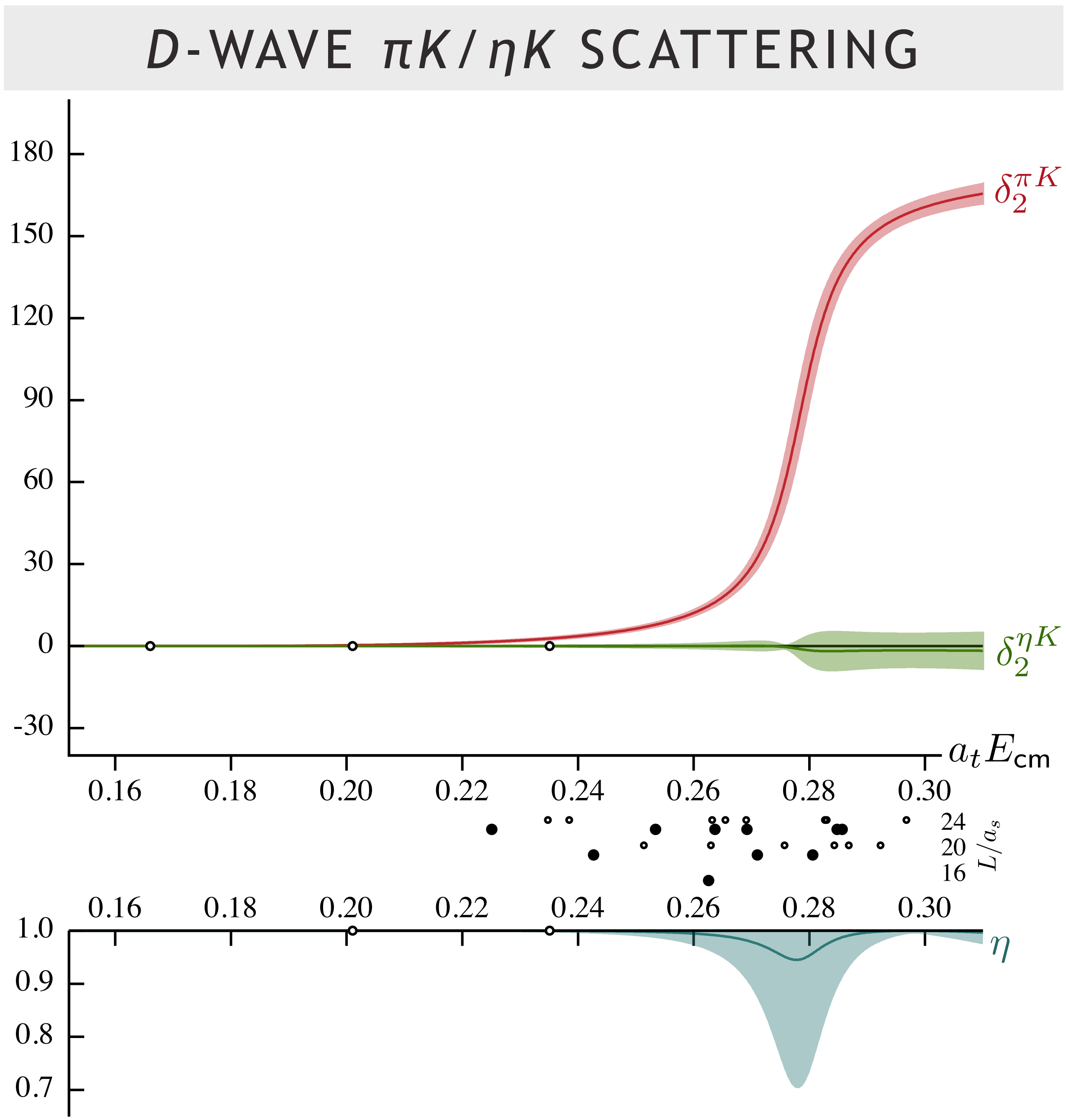


LASS, NPB296 493

$m_\pi \sim 391\,\mathrm{MeV}$

$\pi K/\eta K$ coupled-channel scattering

- clear narrow resonance in *D*-wave scattering

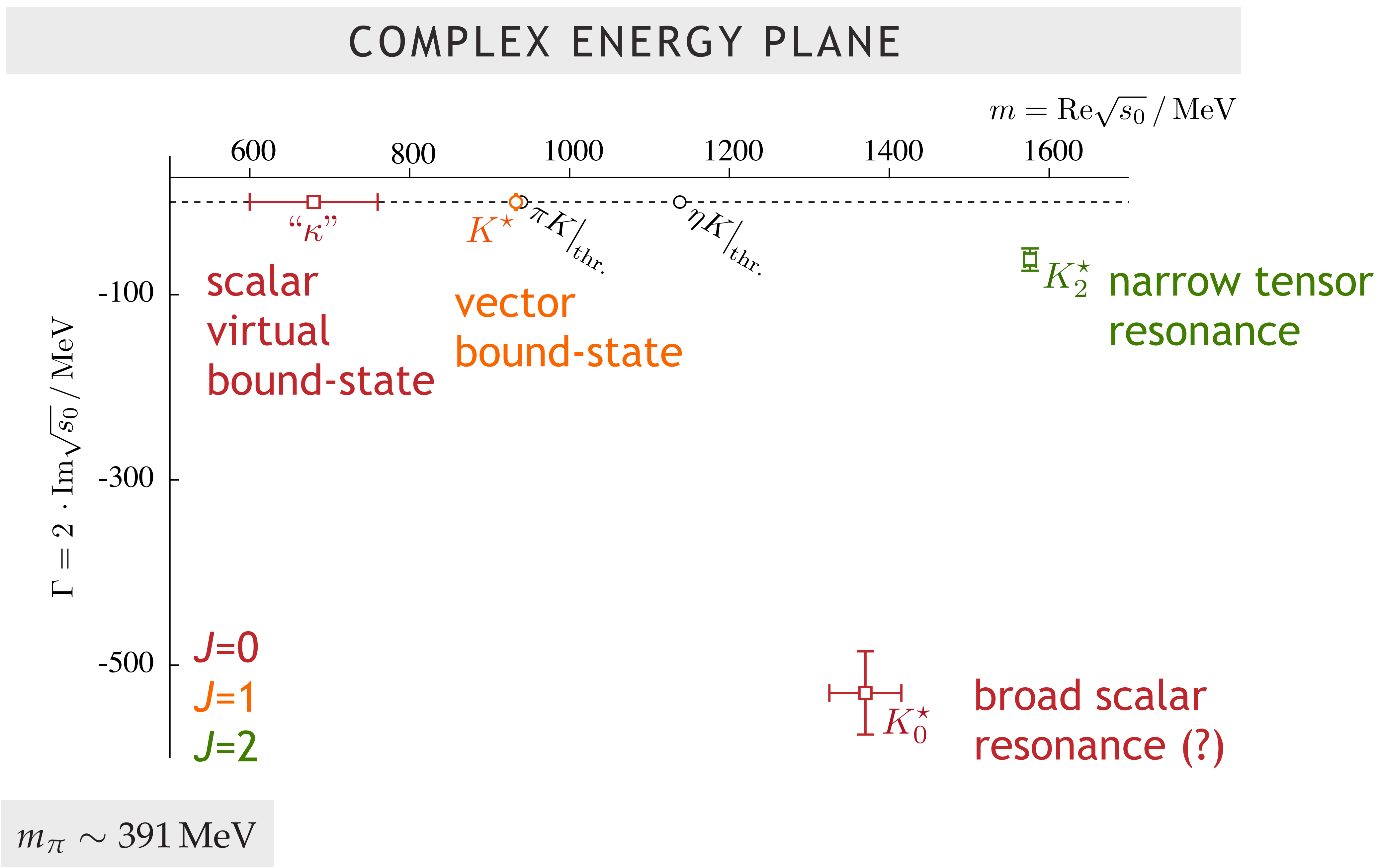


$m_\pi \sim 391\,\mathrm{MeV}$

but you might worry about $\pi\pi K$ …

singularity content

- ***t*-matrix poles** as least model-dependent characterization of resonances



PRL 113 182001
PRD 91 054008

singularity content

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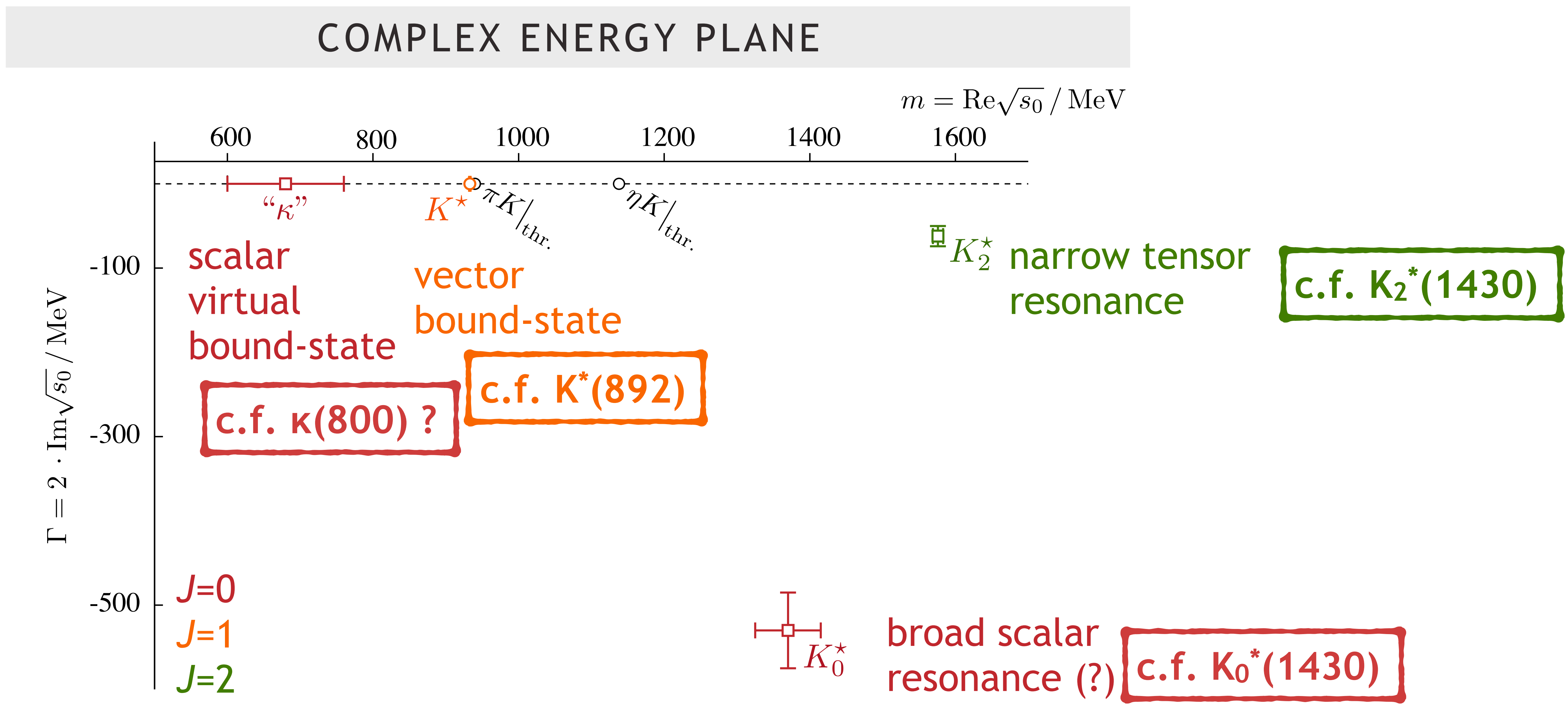

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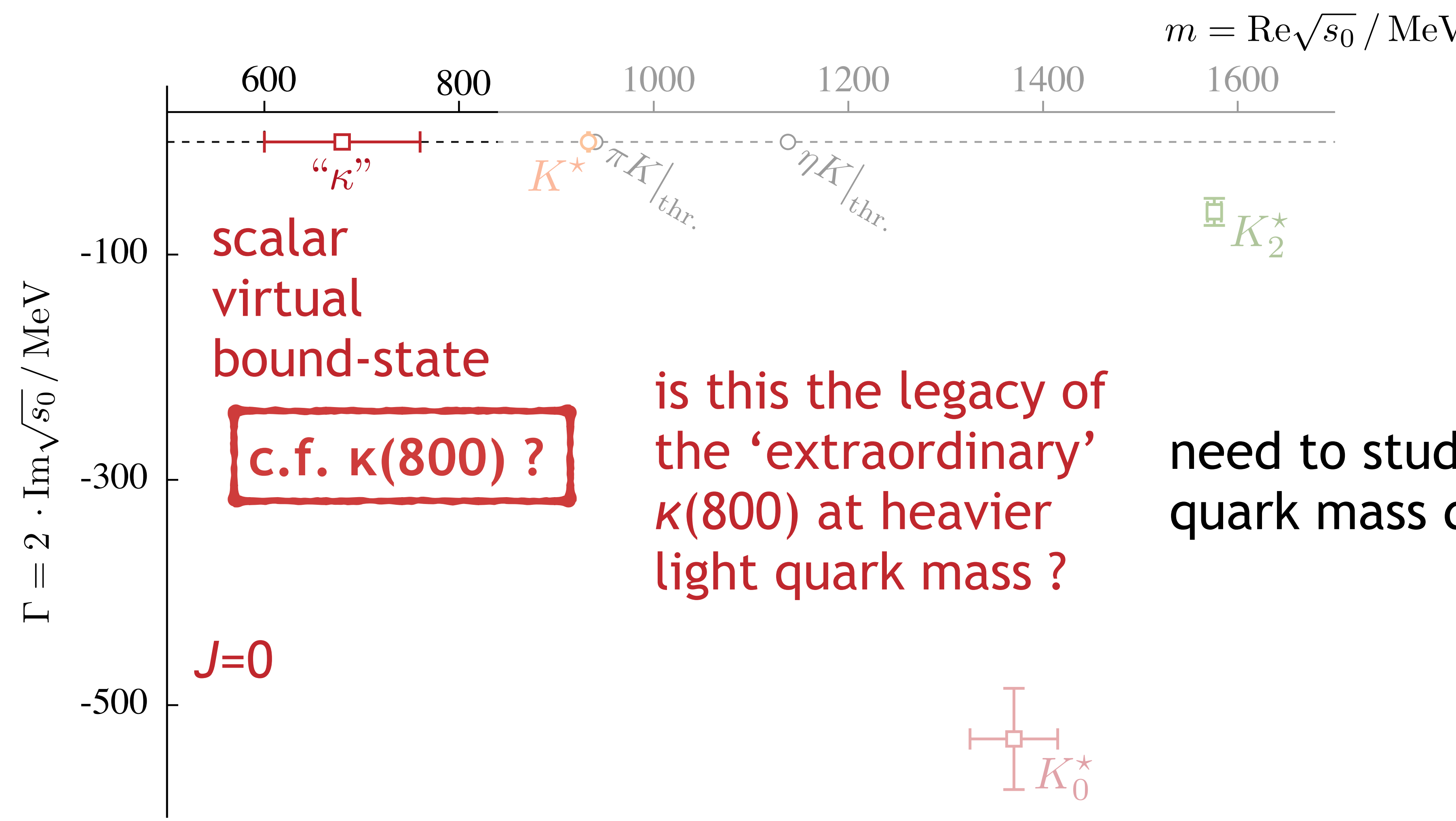
COMPLEX ENERGY PLANE



scalar
virtual
bound-state

c.f. κ(800) ?

J=0

is this the legacy of
the 'extraordinary'
κ(800) at heavier
light quark mass ?

need to study the
quark mass dependence

$m_\pi \sim 391\,\mathrm{MeV}$

PRL 113 182001
PRD 91 054008

other calculations

ongoing, well underway

- $\pi\eta/K\overline{K}$ / $\pi\eta'$ scattering (e.g. $a_0(980)$ …), $\pi\pi/K\overline{K}/\eta\eta$ scattering (σ, $f_0(980)$ …)
  David Wilson, ODU→DAMTP Raul Briceno, JLab/ODU

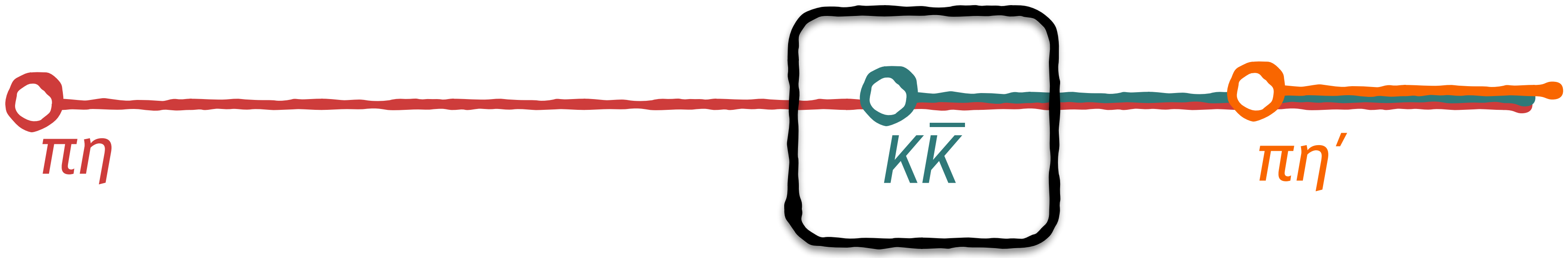
- $D\pi$ scattering, DK scattering (various isospins)
  Christopher Thomas, Graham Moir, DAMTP

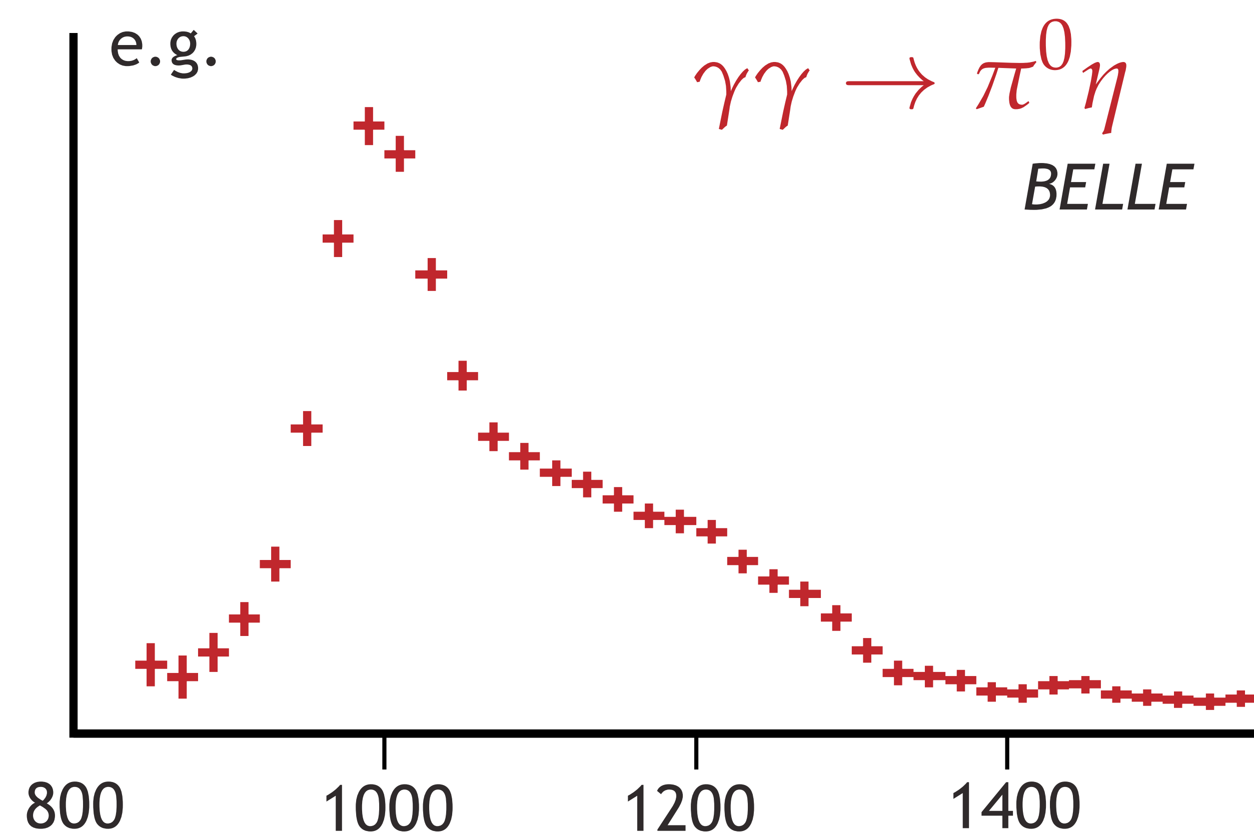
coming up soon

- $D\overline{D}$ scattering
- (correct) consideration of 0^- 1^- and other 'spinning' scattering problems
- implementing a large basis of local tetraquark operators

…

$\pi\eta/K\bar{K}$ scattering & the a_0 resonance





- many outstanding questions about $a_0(980)$

extraordinary ?

  - an ‘extra’ state [$a_0(1450)$ as $q\bar{q}$?]
  - hidden strangeness tetraquark ?
  - relevance of $K\bar{K}$ threshold proximity ?
  - pole structure - which sheets ?

strongly-coupled two-channel problem … ?

applications to new hadrons

- a few papers, but not at the level of analysis I've been showing so far

mostly Prelovsek and collaborators …

Z_c^+ ?

- large basis of meson-meson operators
- plus diquark-antidiquark tetraquark constructions

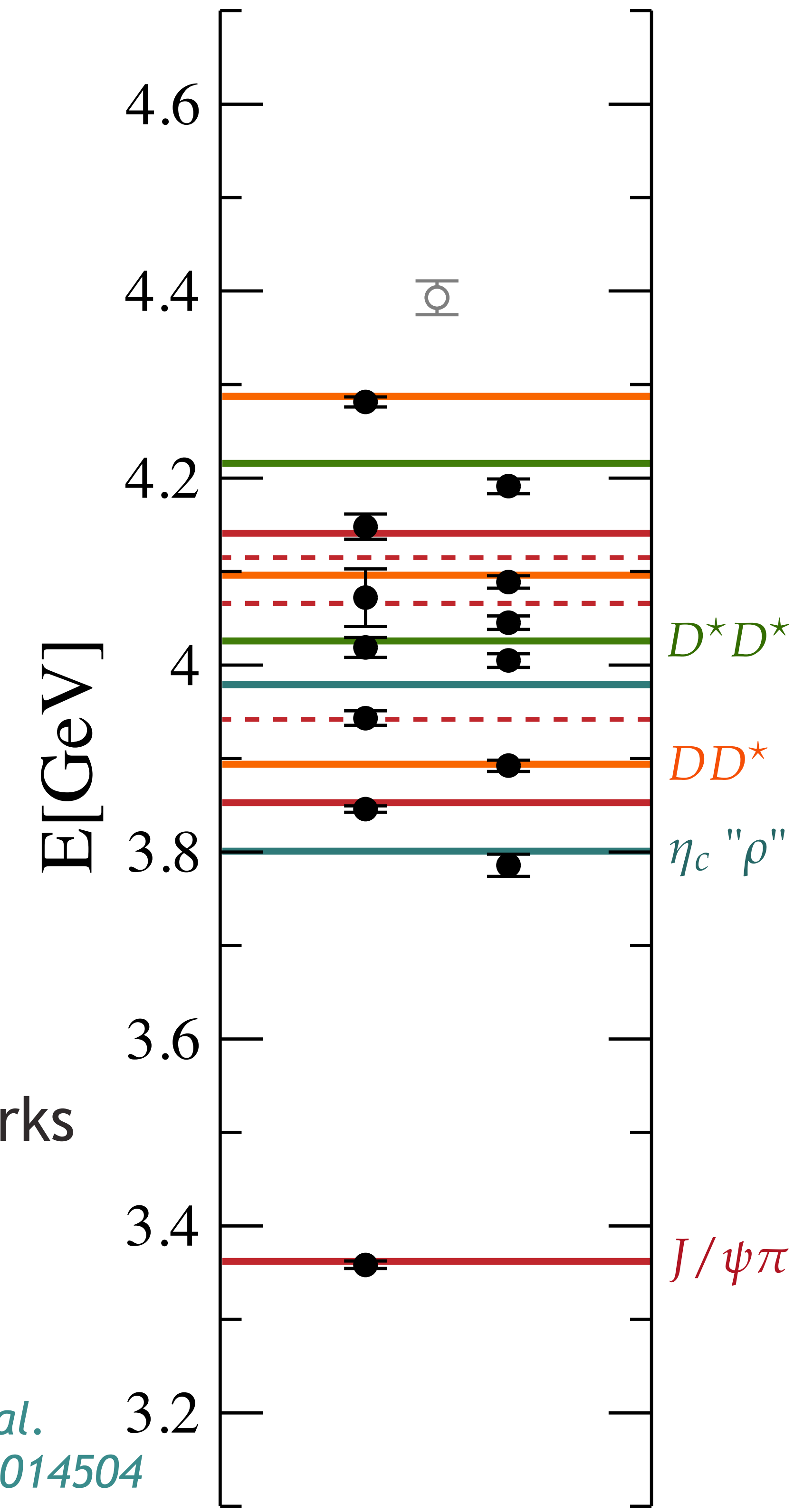
- their argument:
  should get one level per meson-meson operator,
  any excess beyond that is signal for narrow Z_c

m_π~ 266 MeV
no strange quarks

this is rather small

L~2 fm
[$m_\pi L$~ 2.7]

Prelovsek et al.
PhysRevD.91.014504



Z_c^+ ?

- large basis of meson-meson operators

    but probably not big enough to get the 'full' spectrum

    e.g. $\pi[100]\psi[-100]$
    - two operator constructions, only one included

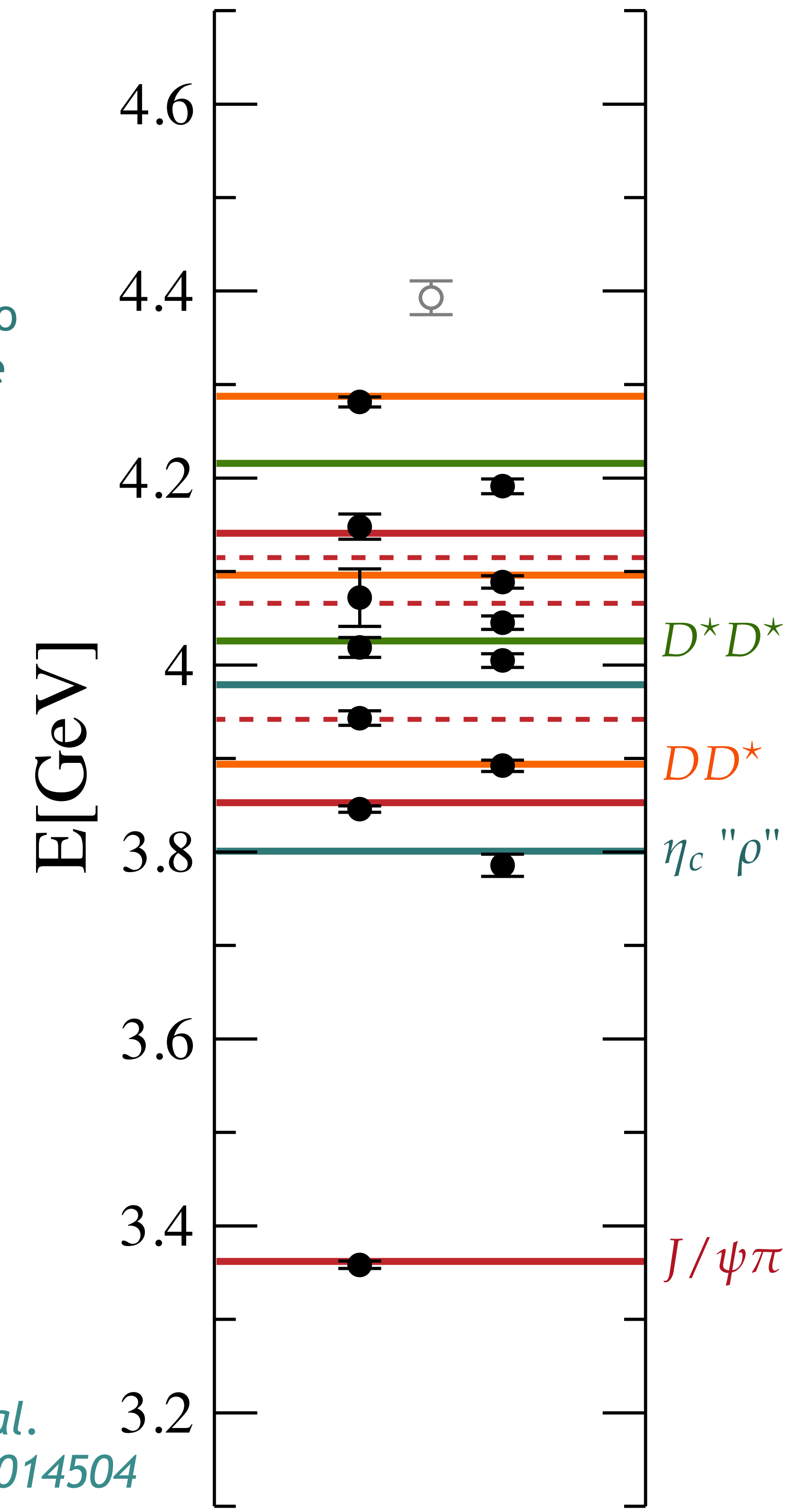
    related to S/D wave

    e.g. no $\pi[100]h_c[-100]$ construction included

    e.g. ρ is unstable into $\pi\pi$

- no attempt to determine scattering amplitudes

    not sure what to conclude from this figure ?



Prelovsek et al.
PhysRevD.91.014504

Z_c^+ - what might be done ?

- current calculations at artificially heavy quark mass

  π more like a K ?

  but $D\bar{D}$ thresholds about right

  implications of experimental absence of a $J/\psi\, K$ enhancement ?

- a nice feature of the lattice calculations:

  can avoid the experimental complication of production in a three-body process
  (π π J/ψ) (π $\bar{D}D^*$) …

  suppose ultimately the lattice calcs find no resonance in this channel

  → kinematic singularity in three-body process ?

  if they determine a resonance, then we need to understand it

  → distribution of the resonance pole across sheets ?

  → matrix elements at the pole ?

  external currents, see Raul's talk

hadron resonances from QCD ?

progress

- new lattice field theory techniques → extraction of many discrete energies
- finite-volume energies can be related to scattering amplitudes
- elastic case well studied ($\rho \rightarrow \pi\pi$)
- first extraction of coupled-channel case now demonstrated ($\pi K, \eta K$)

Mike Peardon

ongoing

- other coupled two-body channels being explored
- and coupling to external currents (*photo-, electro-production, weak decays …*) see Raul's talk
- currently lack a complete formalism for three-body states (e.g. $\pi\pi\pi$) talk to Steve Sharpe

future

- ultimately aim to determine properties of exotic meson resonances

  (*mass, width, branching ratios, coupling to photons …*)

hadron spectrum collaboration

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MESON SPECTRUM

PRL103 262001 (2009)	$I=1$
PRD82 034508 (2010)	$I=1,\ K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	$D,\ D_s$

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\mathrm{hyb}}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
PRD91 094502 (2015)	$\Xi^\star_{cc}$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1,\ \rho$
PRL113 182001 (2014)	$\pi K, \eta K$
PRD91 054008 (2015)	$\pi K, \eta K$
PRD92 094502 (2015)	$\pi\pi, K\bar{K}$

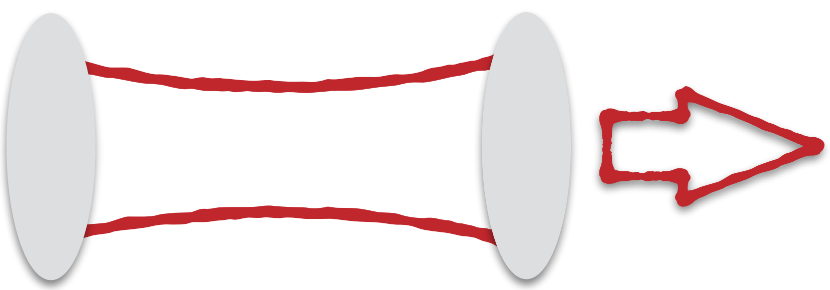
"TECHNOLOGY"

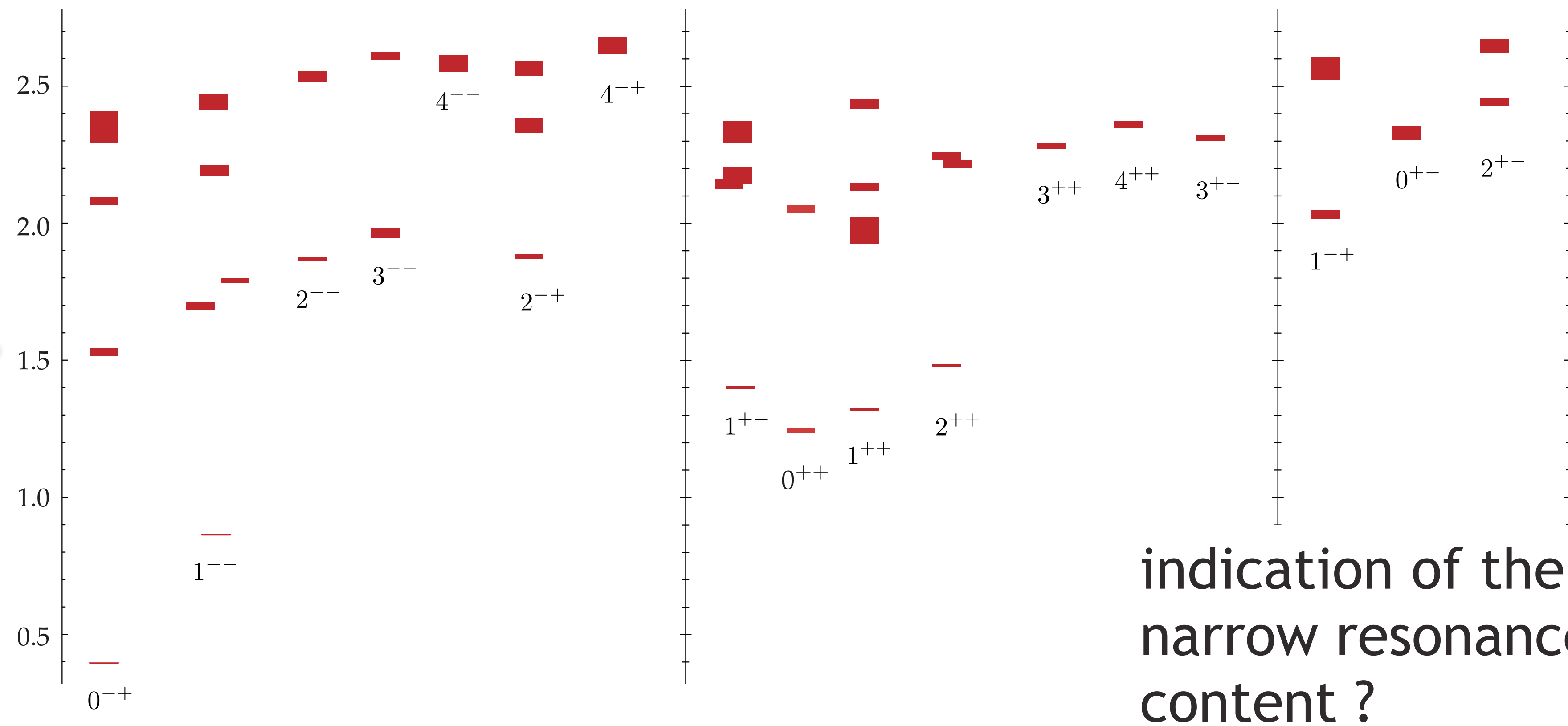
PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p}>0$

MATRIX ELEMENTS

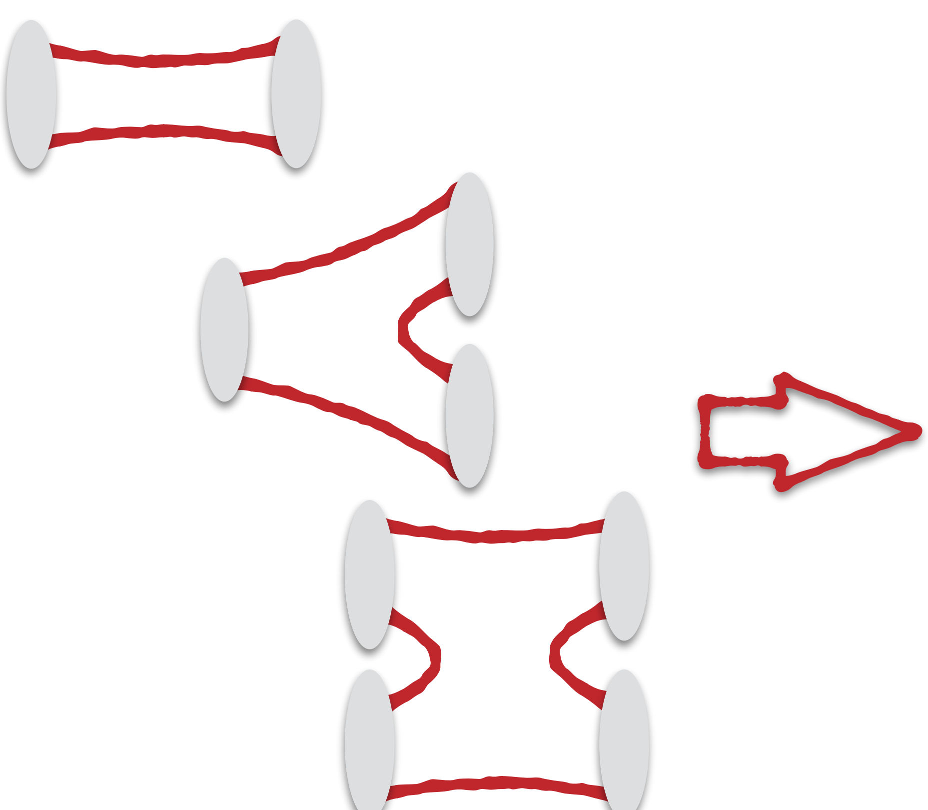
PRD91 114501 (2015)	$M'\to\gamma M$
PRD90 014511 (2014)	$f_{\pi^\star}$
arXiv:1507.06622	$\gamma^\star\pi\to\pi\pi$

‘single-hadron’ spectrum



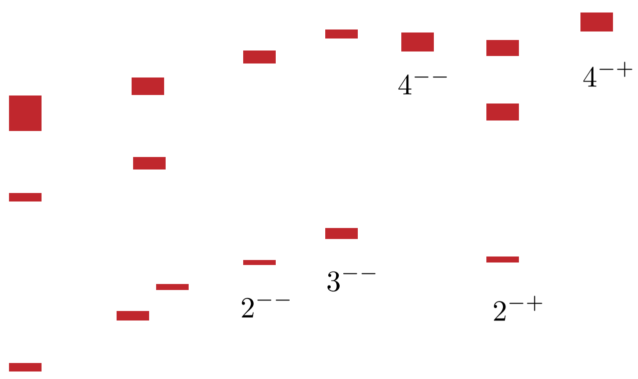


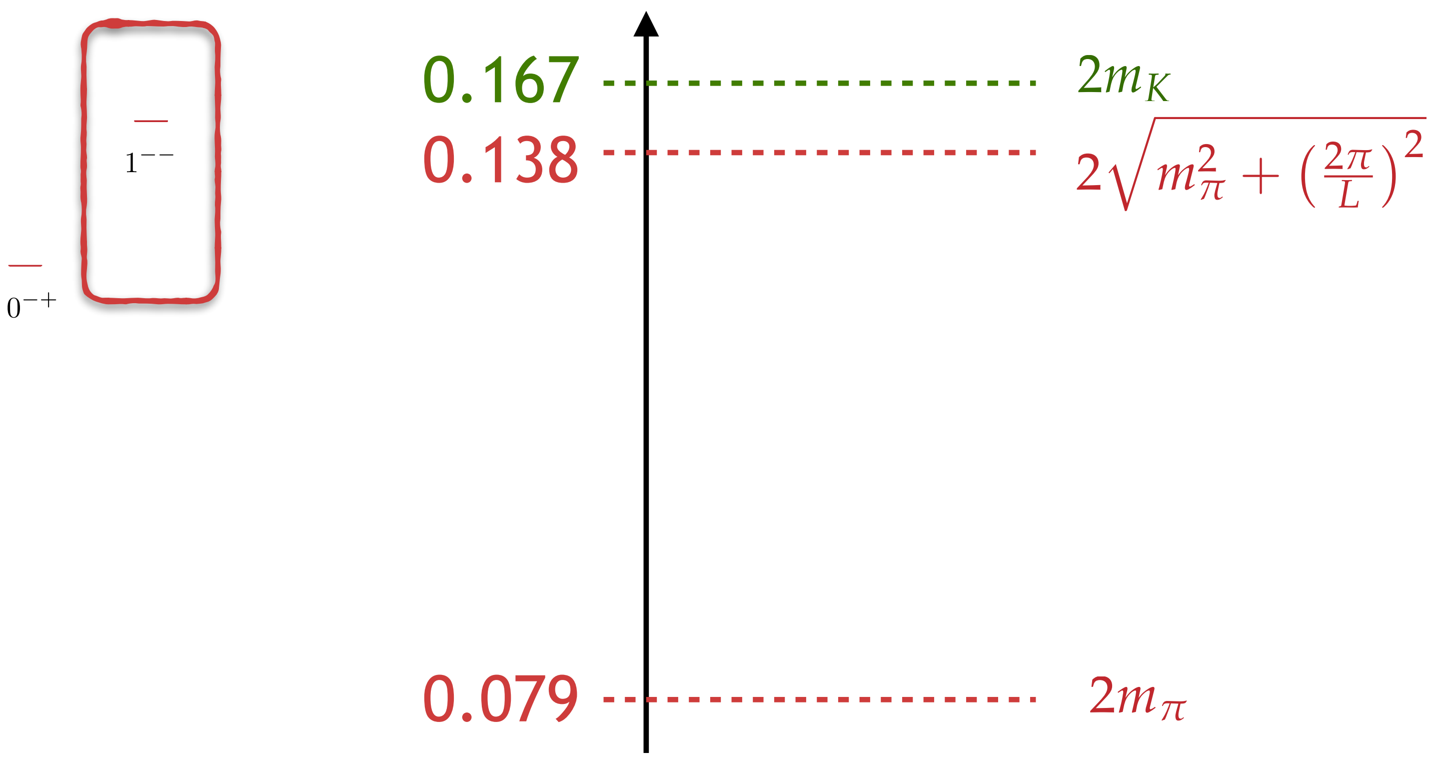
indication of the narrow resonance content ?



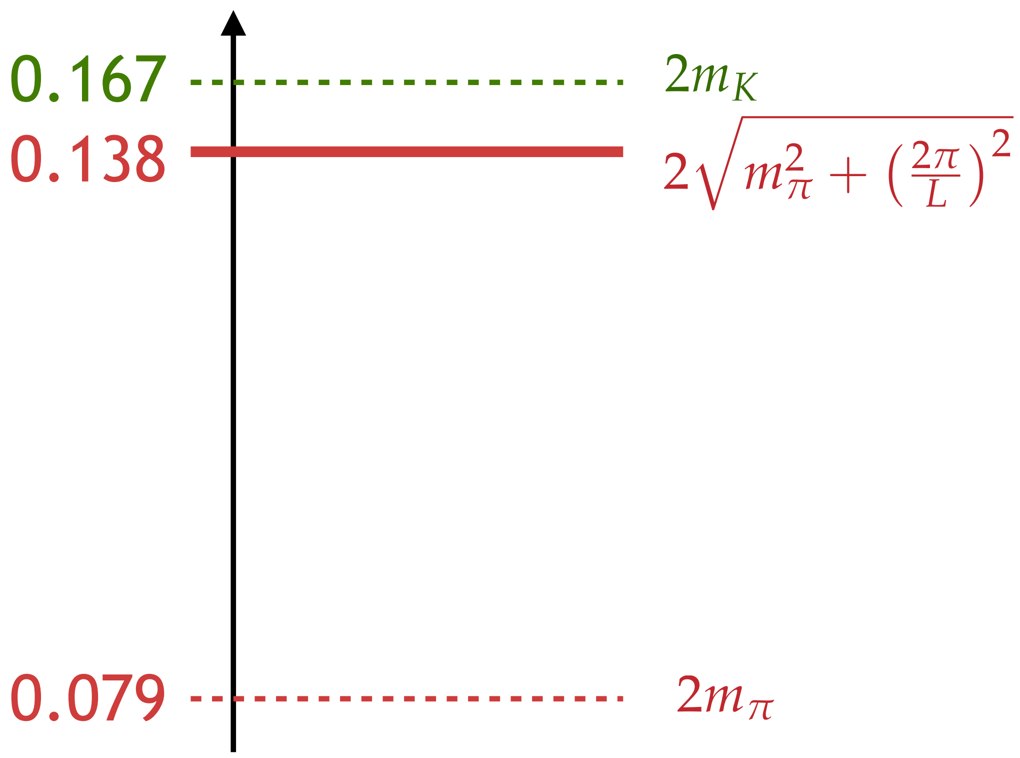
?

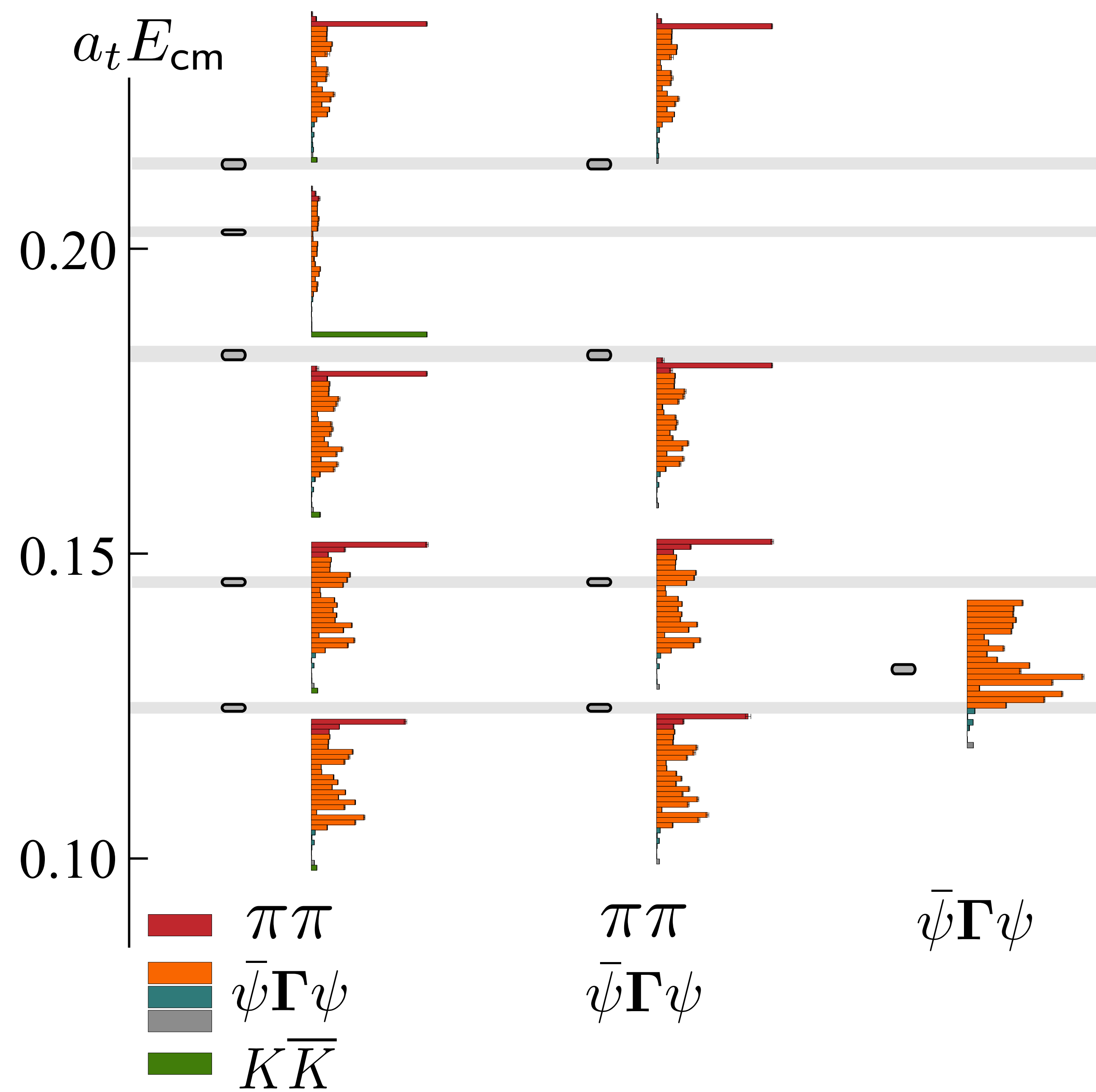
‘single-hadron’ spectrum





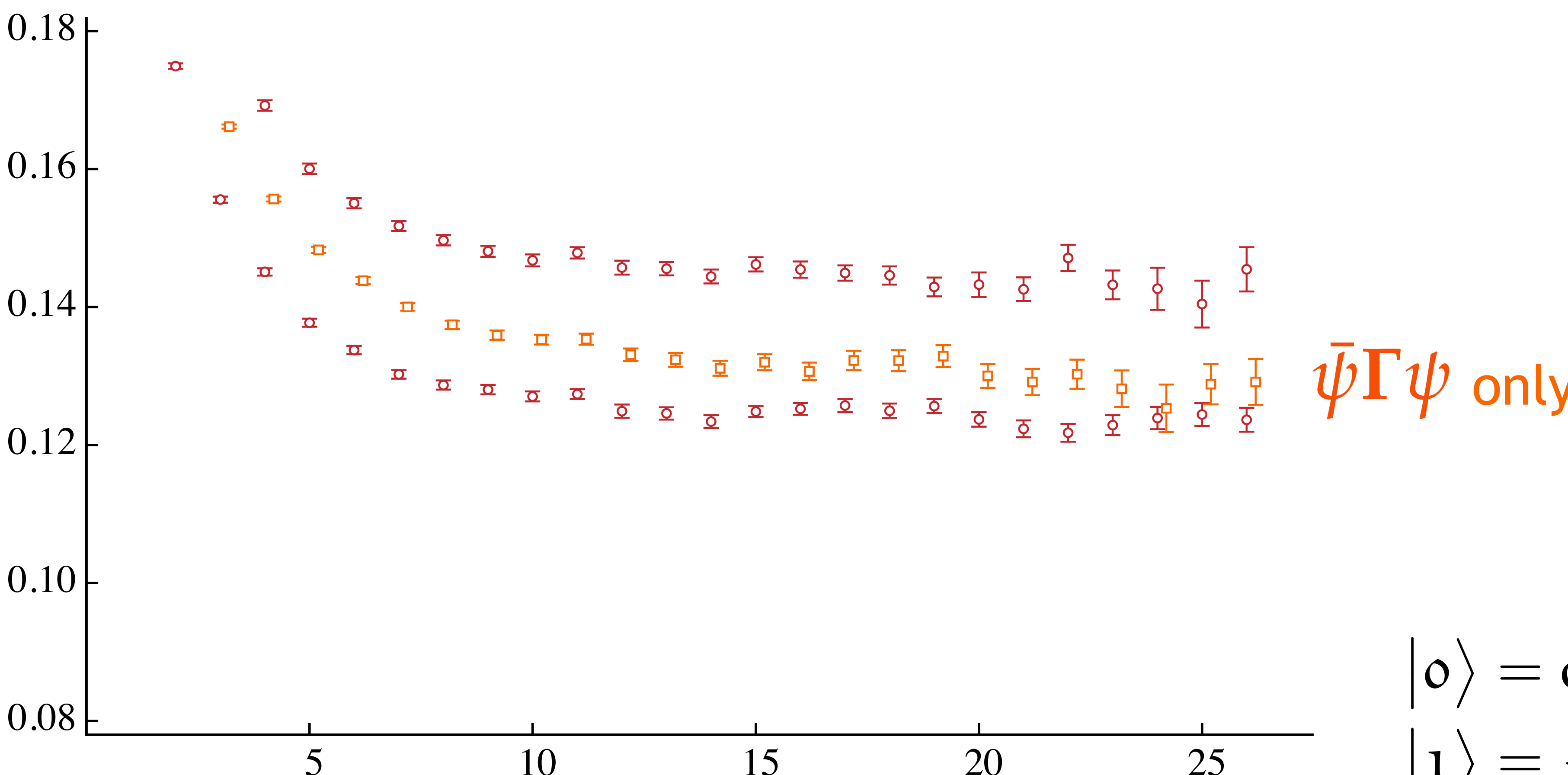
‘single-hadron’ spectrum





‘two-state’ admixture model

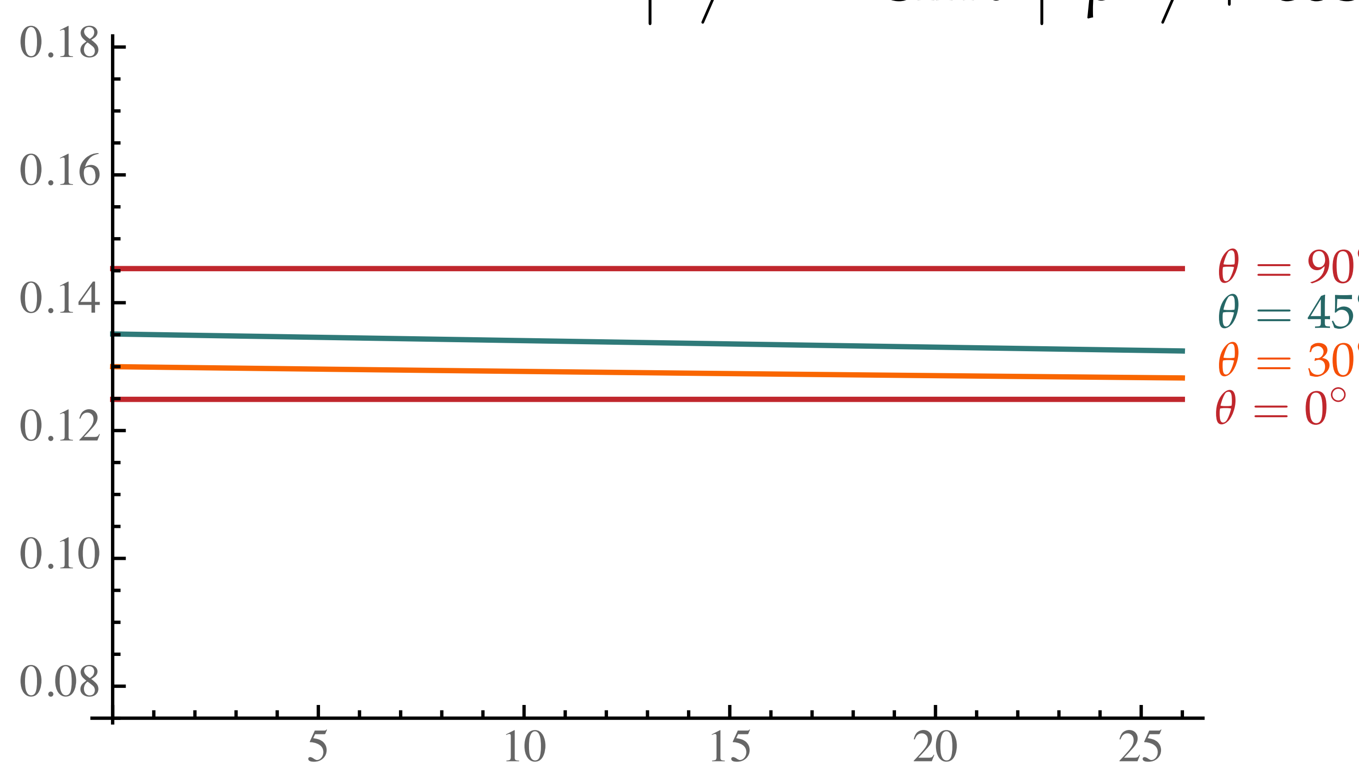
$m_\pi \sim 230\,\mathrm{MeV}$



$$|0\rangle = \cos\theta\,|“\rho”\rangle + \sin\theta\,|“\pi\pi”\rangle$$
$$|1\rangle = -\sin\theta\,|“\rho”\rangle + \cos\theta\,|“\pi\pi”\rangle$$

if we only overlap with the $|“\rho”\rangle$ part

$$\lambda(t) \sim \cos^2\theta\, e^{-E_0 t} + \sin^2\theta\, e^{-E_1 t}$$



just using $q\bar{q}$ operators ?