Institute for Nuclear Theory - Seattle, November 10, 2015

Excited light-quarks and the Z family

Susana Coito

Institute of Modern Physics, Chinese Academy of Sciences

I. Introduction

$Z_c(3900)^{\pm}$	$M = 3888.7 \pm 3.4$, $\Gamma = 35 \pm 7$ [MeV]
	$\pi^{\pm} J/\psi$, $(DD^*)^{\pm}$ (BESIII, Belle, Cleo-c group)
$Z_c(3900)^0$	$M = 3894.8 \pm 2.3 \pm 3.2$, $\Gamma = 29.6 \pm 8.2 \pm 8.2$ [MeV]
	$\pi^0 J/\psi$ (Cleo-c group, BESIII)
	favored $J^P = 1^+$
$Z_c(4020)^{\pm}$	$M = 4022.9 \pm 0.8 \pm 2.7$, $\Gamma = 7.9 \pm 2.7 \pm 2.6$ [MeV]
	$\pi^{\pm} h_c$ (BESIII) $J^P =?$
$Z_c(4025)^{\pm}$	$M = 4026.3 \pm 2.6 \pm 3.7$, $\Gamma = 24.8 \pm 5.6 \pm 7.7$ [MeV]
	$(D^* D^*)^{\pm}$ (BESIII) $J^P =?$
$Z_c(4050)^{\pm}$	$M = 4051 \pm 14^{+20}_{-41}$, $\Gamma = 82^{+21+47}_{-17-22}$ [MeV]
	$\pi^+ \chi_{c1}$ (Belle) $J^P =?$

$Z_b(10610)^{\pm}$	$M_1 = 10607.2 \pm 2.0$, $\Gamma_1 = 18.4 \pm 2.4$ [MeV]
	$\pi^{\pm}\Upsilon(1,2,3S)$ and $\pi^{\pm}h_c(1,2P)$ (Belle)
$Z_b(10610)^0$	$M = 10609 \pm 4 \pm 4$
	$\pi^0\Upsilon(2,3S)$
	favored $J^P = 1^+$
$Z_b(10650)^{\pm}$	$M_2 = 10652.2 \pm 1.5$, $\Gamma_2 = 11.5 \pm 2.2$ [MeV]
	$\pi^{\pm}\Upsilon(1,2,3S)$ and $\pi^{\pm}h_c(1,2P)$ (Belle)
	favored $J^P = 1^+$

Theoretical Models

Z_c family

Tetraquarks	consistency in masses, no width predictions Braaten PRL 111, 162003 - all three states with 1^+ other works - $Z_c(3900)$ 1^+, $Z(4020)$ other q.n.
Molecules	pure molecules desfavored namely, in lattice: PLB 727, 172; 1411.1389 [hep-lat]; PRD 89, 094506
Molecules with light hadrons	possible, namely QCD sum rules require light $< \bar{q}q >$ condensate for OPE convergence PRD 87, 116004; JPG 41, 075003; EPJC 74, 2891
Threshold effects	might be. Swanson PRD 91, 034009

Z_b family

Tetraquarks	masses far below thresholds i.e. , 10.44 and 10.50/10.63 [GeV]
Molecules	pure molecules desfavored disagreement about $^1P_1/^3P_1$ dominance
Molecules with light hadrons	possible, with $^3P_1$. one-pion-exchange
Threshold effects	might be Bugg EPL 96, 11002, Swanson PRD 91, 034009.

Idea

Previous works employed a simple Schrödinger model to study the system $c\bar{c} - D^0 D^{*0}$, which could represent the $X(3872)$ as a highly unquenched $2\,^3P_1$

EPJC 73, 2351(2013); EPJC 75, 26(2015); ZPC 19, 275(1983)

N. Kochelev (private communication)

What if the same model is used to the system $n\bar{n} - D^{\pm/0} D^{*0/\pm}$ to describe the $Z(3900)^{\pm/0}$ with $J^P = 1^+$.

Requirement

A very high radial excitation strongly coupled to the decay channel. Within the harmonic oscillator confining potential with $\omega = 190$ MeV, the closest bare state is unequivocal.

H.O. eigenvalues, $\omega = 190$ MeV

n	l=0	l=1
0	1.097	1.287
1	1.477	1.667
2	1.857	2.047
3	2.237	2.427
4	2.617	2.807
5	2.997	3.187
6	3.377	3.567
7	3.757	3.947
8	4.137	4.327
9	4.517	4.707
10	4.897	5.087
11	5.277	5.467
12	5.657	5.847
13	6.037	6.227
14	6.417	6.607

n	l=0	l=1
15	6.797	6.987
16	7.177	7.367
17	7.557	7.747
18	7.937	8.127
19	8.317	8.507
20	8.697	8.887
21	9.077	9.267
22	9.457	9.647
23	9.837	10.027
24	10.217	10.407
25	10.597	10.787
26	10.977	11.167
27	11.357	11.547
28	11.737	11.927
29	12.117	12.307

Th [MeV]
$KK = 994$
$KK^* = 1389$
$K^*K^* = 1784$
$DD = 3734$
$DD^* = 3876$
$D^*D^* = 4018$
$BB = 10558$
$BB^* = 10604$
$B^*B^* = 10650$

$n = 0$
$a_1(1260)$
$b_1(1235)$
$f_1(1285)$
$h_1(1170)$

II. A coupled-channel Schrödinger model

Coupled-channel system: $q\bar{q} - MM$

$$\begin{pmatrix} h_c & V \\ V & h_f \end{pmatrix} \begin{pmatrix} u_c \\ u_f \end{pmatrix} = E \begin{pmatrix} u_c \\ u_f \end{pmatrix}$$

$$h_c = \frac{1}{2\mu_c}\left(- \frac{d^2}{dr^2} + \frac{l_c(l_c+1)}{r^2}\right) + \frac{\mu_c \omega^2 r^2}{2} + m_q + m_{\bar{q}}$$

$$h_f = \frac{1}{2\mu_f}\left(- \frac{d^2}{dr^2} + \frac{l_f(l_f+1)}{r^2}\right) + m_{M_1} + m_{M_2}$$

$$V = \frac{g}{2\mu_c a}\delta(r-a)$$

$$E = (2\nu + l_c + \frac{3}{2})\,\omega + m_q + m_{\bar{q}}$$

$$\left\{ \frac{1}{2\mu_c} \left(-\frac{d^2}{dr^2} + \frac{l_c(l_c+1)}{r^2} \right) + \frac{1}{2}\mu_c \omega^2 r^2 + m_q + m_{\bar{q}} - E \right\} u_c(r)$$
$$= -\frac{\lambda}{2\mu_c a} \delta(r-a) u_f(r)$$

$$u_c(r) = \begin{cases} A_c F_c(r) & r < a \\ B_c G_c(r) & r > a \end{cases}$$

with the definitions

$$F(r) = \frac{1}{\Gamma(l+3/2)} \, z^{(l+1)/2} \, e^{-z/2} \, \Phi(-\nu, l+3/2, z)$$
$$G(r) = -\frac{1}{2} \Gamma(-\nu) \, z^{l/2} \, e^{-z/2} \, \Psi(-\nu, l+3/2, z)$$

$$\left\{ \frac{1}{2\mu_f} \left(-\frac{d^2}{dr^2} + \frac{l_f(l_f+1)}{r^2} \right) + M_1 + M_2 - E \right\} u_f(r)$$
$$= -\frac{\lambda}{2\mu_c a} \delta(r-a) u_c(r)$$

Bound states

$$u_f(r) = \begin{cases} A_f J_{l_f}(kr) & r < a \\ B_f \left[J_{l_f}(kr) k^{2l_f+1} i - N_{l_f}(kr) \right] & r > a \end{cases}$$

with $J_l(kr) = k^{-l} r j_l(kr),\ N_l(kr) = k^{l+1} r n_l(kr)$.

Resonances

$$u_f(r) = \begin{cases} A_f J_{l_f}(kr) & r < a \\ B_f \left[J_{l_f}(kr) k^{2l_f+1} i + N_{l_f}(kr) \right] & r > a \end{cases}$$

Boundary conditions

$$u_c'(r \uparrow a) - u_c'(r \downarrow a) = -\frac{\lambda}{a} u_f(a)$$
$$u_f'(r \uparrow a) - u_f'(r \downarrow a) = -\frac{\lambda \mu_f}{a \mu_c} u_c(a)$$

$$u_c(r \uparrow a) = u_c(r \downarrow a)$$
$$u_f(r \uparrow a) = u_f(r \downarrow a)$$

Partial amplitudes:

$$A_c \ , \quad B_c = \frac{F_c(a)}{G_c(a)} A_c \ ,$$

$$A_f = \frac{a}{\lambda} \frac{1}{G_c(a) J_{l_f}} A_c \ ,$$

$$B_f = \frac{\lambda}{a} \frac{\mu_f}{\mu_c} J_{l_f}(ka) F_c(a),$$

$$B_f = \frac{a}{\lambda} \frac{1}{G_c(a)[J_{l_f}(ka) k^{2l_f+1} i + N_{l_f}(ka)]} A_c.$$

Poles:

$$J_{l_f}^2(ka) k^{2l_f+1} i + J_{l_f}(ka) N_{l_f}(ka) = \left(\frac{a}{\lambda}\right)^2 \frac{\mu_c}{\mu_f} \frac{1}{F_c(a) G_c(a)}$$

Coupled-channel: $n\ q\bar{q} - m\ MM$

$$\begin{pmatrix} h_c^\nu & V\tilde{g}_j^\nu \\ V\tilde{g}_j^\nu & h_{fj} \end{pmatrix} \begin{pmatrix} u_c^\nu \\ u_{fj} \end{pmatrix} = E \begin{pmatrix} u_c^\nu \\ u_{fj} \end{pmatrix}$$

Boundary conditions

$$u'^{\nu}_c(r \uparrow a) - u'^{\nu}_c(r \downarrow a) = -\frac{\lambda}{a} \sum_j g_j^\nu u_{fj}(a),$$

$$u'_{fj}(r \uparrow a) - u'_{fj}(r \downarrow a) = -\frac{\lambda}{a} \mu_{fj} \sum_\nu \tilde{g}_j^\nu u_c^\nu(a)$$

$$A_c^\nu = \frac{\lambda}{a} G_c^\nu(a) \sum_j g_j^\nu J_{l_f j}(k_j a) A_{fj}$$

$$A_{fj} = \frac{\lambda}{a} \mu_{fj} C_{l_f j}(k_j a) \sum_\nu \tilde{g}_j^\nu F_c^\nu(a) A_c^\nu$$

with $C_{l_f j}(k_j a) = J_{l_f j}(k_j r) k^{2 l_f j + 1} i + N_{l_f j}(k_j r)$, and $\tilde{g}_j^\nu = g_j^\nu / \mu^\nu$

Dividing the above equations by A^a, similarly defined, we get

$$A_c^\nu = \frac{G_c^\nu(a)}{G_c^a(a)} \frac{\sum_j g_j^\nu \mu_{fj} J_{l_f j} C_{l_f j} \sum_\alpha \tilde{g}_j^\alpha F^\alpha(a) A_c^\alpha}{\sum_j g_j^a \mu_{fj} J_{l_f j} C_{l_f j} \sum_\alpha \tilde{g}_j^\alpha F^\alpha(a) A_c^\alpha} A_c^a$$

$$A_{fj} = \frac{a}{\lambda G_c^a(a)} \frac{\mu_{fj} C_{l_f j} \sum_\alpha \tilde{g}_j^\alpha A_c^\alpha}{\sum_j g_j^a \mu_{fj} J_{l_f j} C_{l_f j} \sum_\alpha \tilde{g}_j^\alpha F^\alpha(a) A_c^\alpha} A_c^a$$

If $\alpha = 2$ the amplitudes are fully resolved analytically.

Poles:

$$\left[\left(\frac{\lambda}{a}\right)^2 \frac{G^b F^a}{\mu_a} \sum_i \mu_i J_i C_i g_i^b g_i^a\right]\left[\left(\frac{\lambda}{a}\right)^2 \frac{G^a F^b}{\mu_b} \sum_i \mu_i J_i C_i g_i^a g_i^b\right]$$

$$= \left[1 - \left(\frac{\lambda}{a}\right)^2 \frac{G^a F^a}{\mu_a} \sum_i \mu_i J_i C_i {g_i^a}^2\right]\left[1 - \left(\frac{\lambda}{a}\right)^2 \frac{G^b F^b}{\mu_b} \sum_i \mu_i J_i C_i {g_i^b}^2\right]$$

Model Parameters

Partial couplings: as in *Resonance-Spectrum-Expansion* coupled-channel model, computed by Eef van Beveren

g^2

	l	1^{++}	1^{+-}	n factor
PV	0	1/18	1/36	$n+1$
VV	2	5/72	5/36	$2n/5+1$
PV	0	0	1/36	$n+1$
VV	2	5/24	5/36	$2n/5+1$

Meson masses: experimental

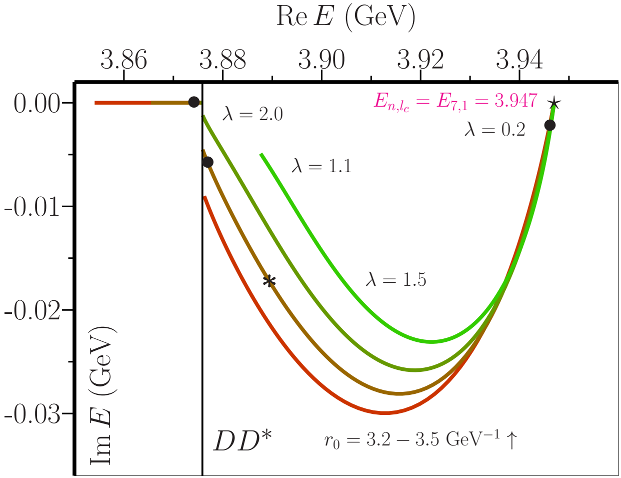
Constituent light quark masses: $m_n = 406$ MeV

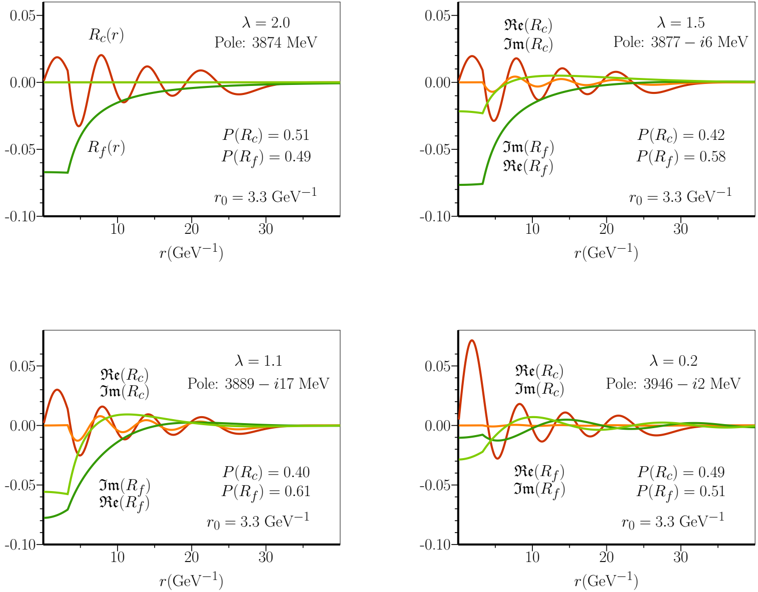
Global coupling λ: free

'String-breaking' distance $a \equiv r_0$: free

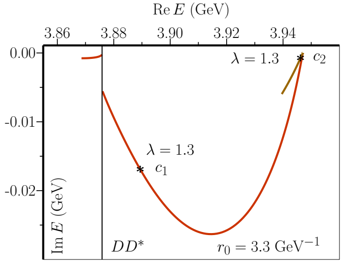
III. In the Z_c region

$n\bar{n} - DD^*$, $n = u, d$ with $l_c = 1$, $l_f = 0$

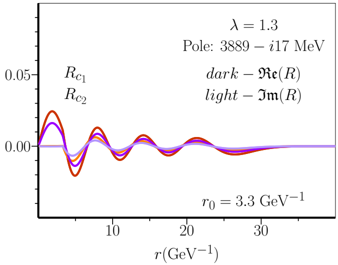


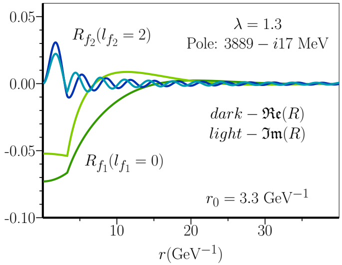

$R_c(r)$
$\lambda = 2.0$
Pole: 3874 MeV
$R_f(r)$
$P(R_c) = 0.51$
$P(R_f) = 0.49$
$r_0 = 3.3$ GeV^{-1}
$\mathfrak{Re}(R_c)$
$\mathfrak{Im}(R_c)$
$\lambda = 1.5$
Pole: $3877 - i6$ MeV
$\mathfrak{Im}(R_f)$
$\mathfrak{Re}(R_f)$
$P(R_c) = 0.42$
$P(R_f) = 0.58$
$r_0 = 3.3$ GeV^{-1}
$\mathfrak{Re}(R_c)$
$\mathfrak{Im}(R_c)$
$\lambda = 1.1$
Pole: $3889 - i17$ MeV
$\mathfrak{Im}(R_f)$
$\mathfrak{Re}(R_f)$
$P(R_c) = 0.40$
$P(R_f) = 0.61$
$r_0 = 3.3$ GeV^{-1}
$\mathfrak{Re}(R_c)$
$\mathfrak{Im}(R_c)$
$\lambda = 0.2$
Pole: $3946 - i2$ MeV
$\mathfrak{Re}(R_f)$
$\mathfrak{Im}(R_f)$
$P(R_c) = 0.49$
$P(R_f) = 0.51$
$r_0 = 3.3$ GeV^{-1}
r(GeV^{-1})

$n\bar{n}_{(^1P_1+^3P_1)} - DD^*_{(l=0+l=2)}$

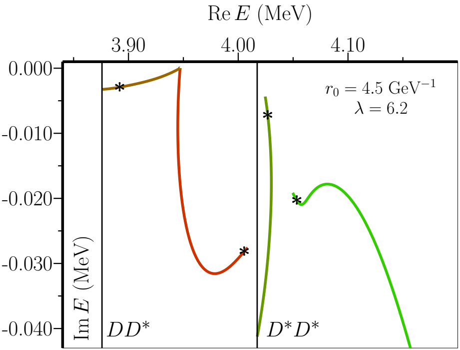


Prob. (%)	$3889 - i17$	$3946 - i0.6$
$P(R_{c1})$	26.3	68.3
$P(R_{c2})$	11.6	29.7
$P(R_{DD^*(l=0)})$	53.9	1.9
$P(R_{DD^*(l=2)})$	8.3	0.2





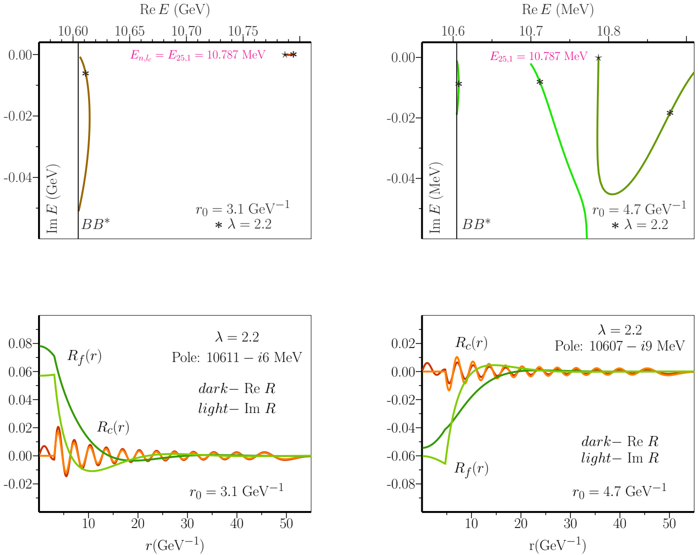
$n\bar{n}_{(^1P_1+^3P_1)} - [DD^* + D^*D^*]_{(l=0+l=2)}$



r_0	λ	*Pole*	Type
3.2	1.0	$3889 - i14$	conf1
		$3942 - i1$	conf2
		$4000 - i163$	dyn1
		$4026 - i176$	dyn2
4.5	6.2	$3890 - i3$	conf2
		$4006 - i28$	conf1
		$4027 - i7$	dyn1
		$4053 - i20$	dyn2

III. In the Z_b region

$n\bar{n} - BB^*$, $n = u, d$ with $l_c = 1$, $l_f = 0$



$n\bar{n} - BB^*$

r_0	λ	Poles	Type
3.1	2.2	$10611 - i6$	dyn
		10794	conf
4.7	2.2	$10607 - i9$	dyn
		$10709 - i6$	dyn
		$10879 - i18$	conf

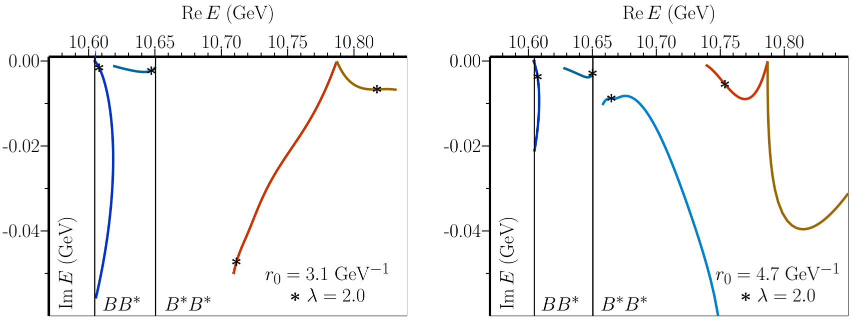
$n\bar{n}_{(^1P_1+\,^3P_1)} - BB^*_{(l=0+l=2)}$

r_0	λ	Poles	Type
3.1	2.0	$10609 - i3$	dyn1
		$10779 - i26$	conf1
		$10791 - i0.6$	conf2
4.7	2.0	$10607 - i5$	dyn1
		$10701 - i3$	dyn2
		$10785 - i0.8$	conf2
		$10891 - i17$	conf1

$n\bar{n}_{(^1P_1+^3P_1)} - BB^* + B^*B^*_{(l=0+l=2)}$

r_0	λ	Poles	Type
3.1	2.0	$10608 - i2$	dyn1
		$10647 - i2$	dyn2
		$10711 - i47$	conf1
		$10817 - i7$	conf2
4.7	2.0	$10607 - i4$	dyn1
		$10650 - i3$	dyn2
		$10665 - i9$	dyn3
		$10754 - i5$	conf2
		$10911 - i7$	conf1

$n\bar{n}_{(^1P_1+^3P_1)} - [BB^* + B^*B^*]_{(I=0+I=2)}$

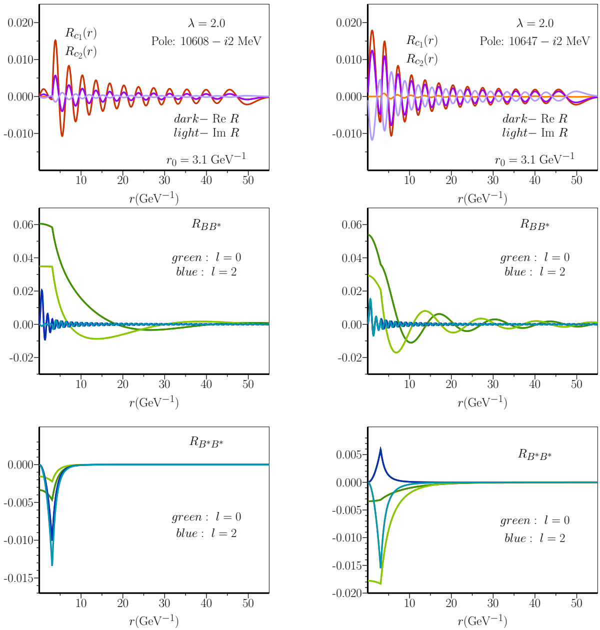


$n\bar{n} - BB^*$ [w.f. Prob. (%)]

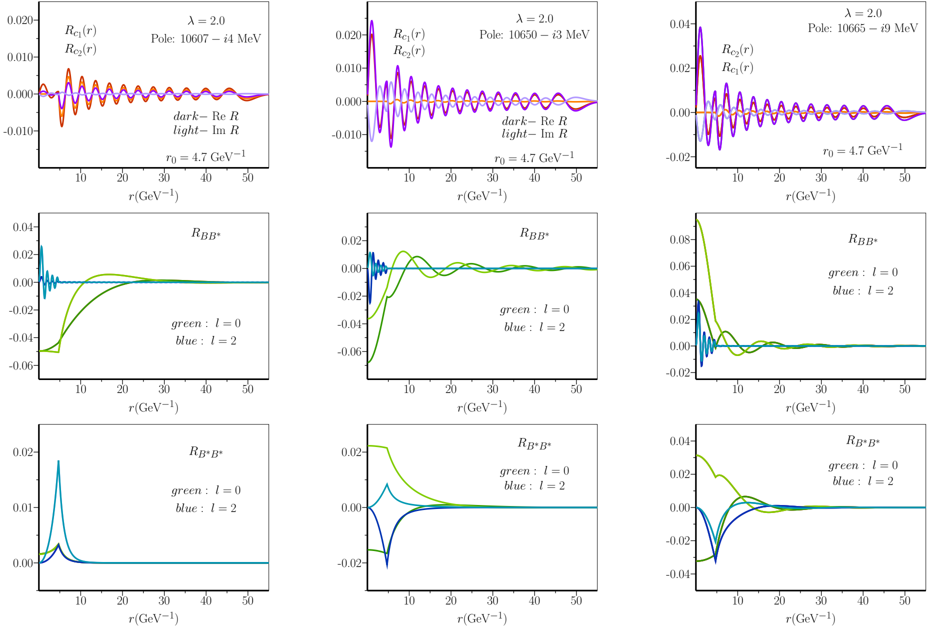
r_0	λ	Poles	Type	$P(R_c)$	$P(R_f)$
3.1	2.2	$10611 - i6$	dyn	41.3	58.7
		10794	conf	11.3	88.7
4.7	2.2	$10607 - i9$	dyn	28.5	71.5
		$10709 - i6$	dyn	18.0	82.0
		$10879 - i18$	conf	14.1	85.9

$n\bar{n}_{(^1P_1+^3P_1)} - [BB^* + B^*B^*]_{(l=0+l=2)}$

r_0	λ	Poles	Type	$P(R_{c1})$	$P(R_{c2})$	$P(R^{l=0}_{BB^*})$	$P(R^{l=2}_{BB^*})$	$P(R^{l=0}_{B^*B^*})$	$P(R^{l=2}_{B^*B^*})$
3.1	2.0	$10608 - i2$	dyn1	15.2	2.0	76.2	6.3	0.1	0.3
		$10647 - i2$	dyn2	14.1	13.0	66.4	4.5	1.6	0.4
		$10711 - i47$	conf1	42.9	50.2	3.2	0.7	0.9	2.2
		$10817 - i7$	conf2	59.7	33.7	0.3	0.7	0.1	5.6
4.7	2.0	$10607 - i4$	dyn1	13.0	1.8	83.9	0.11	0.10	1.1
		$10650 - i3$	dyn2	15.0	27.1	46.5	0.04	9.6	1.7
		$10665 - i9$	dyn3	14.5	36.7	18.2	0.17	20.3	10.1
		$10754 - i5$	conf2	51.9	28.9	3.3	0.06	0.12	15.8
		$10911 - i7$	conf1	4.8	10.1	18.7	16.6	19.3	31.6



0.020
0.010
0.000
-0.010
$R_{c_1}(r)$
$R_{c_2}(r)$
$\lambda = 2.0$
Pole: $10608 - i2$ MeV
dark− Re R
light− Im R
$r_0 = 3.1$ GeV^{-1}
10 20 30 40 50
r(GeV^{-1})
0.020
0.010
0.000
-0.010
$R_{c_1}(r)$
$R_{c_2}(r)$
$\lambda = 2.0$
Pole: $10647 - i2$ MeV
dark− Re R
light− Im R
$r_0 = 3.1$ GeV^{-1}
10 20 30 40 50
r(GeV^{-1})
0.06
0.04
0.02
0.00
-0.02
R_{BB^*}
green : $l = 0$
blue : $l = 2$
10 20 30 40 50
r(GeV^{-1})
0.06
0.04
0.02
0.00
-0.02
R_{BB^*}
green : $l = 0$
blue : $l = 2$
10 20 30 40 50
r(GeV^{-1})
0.000
-0.005
-0.010
-0.015
$R_{B^*B^*}$
green : $l = 0$
blue : $l = 2$
10 20 30 40 50
r(GeV^{-1})
0.005
0.000
-0.005
-0.010
-0.015
-0.020
$R_{B^*B^*}$
green : $l = 0$
blue : $l = 2$
10 20 30 40 50
r(GeV^{-1})



$\lambda = 2.0$
Pole: $10607 - i4$ MeV
$R_{c_1}(r)$
$R_{c_2}(r)$
dark$-$ Re R
light$-$ Im R
$r_0 = 4.7$ GeV^{-1}
$\lambda = 2.0$
Pole: $10650 - i3$ MeV
$R_{c_1}(r)$
$R_{c_2}(r)$
dark$-$ Re R
light$-$ Im R
$r_0 = 4.7$ GeV^{-1}
$\lambda = 2.0$
Pole: $10665 - i9$ MeV
$R_{c_2}(r)$
$R_{c_1}(r)$
$r_0 = 4.7$ GeV^{-1}
R_{BB^*}
green : $l = 0$
blue : $l = 2$
$R_{B^*B^*}$
r(GeV^{-1})

VI. Summary & Conclusions

• We present a description of several exotic resonances Z as systems originating from a highly excited $q\bar{q}$ component, from a nonperturbative spectrum, strongly coupled to the nearby decay channels.

• By varying both real free parameters within a short range, not different than the range considered in other applications of the same type of model, but for lower excitations, we are able to find poles corresponding to the $Z(3900)$, $Z(10610)$, and $Z(10650)$.

• the same model can be used with diferent quantum numbers to study or predict new 'exotic' states.

$\therefore$