Resonant matrix elements from lattice QCD

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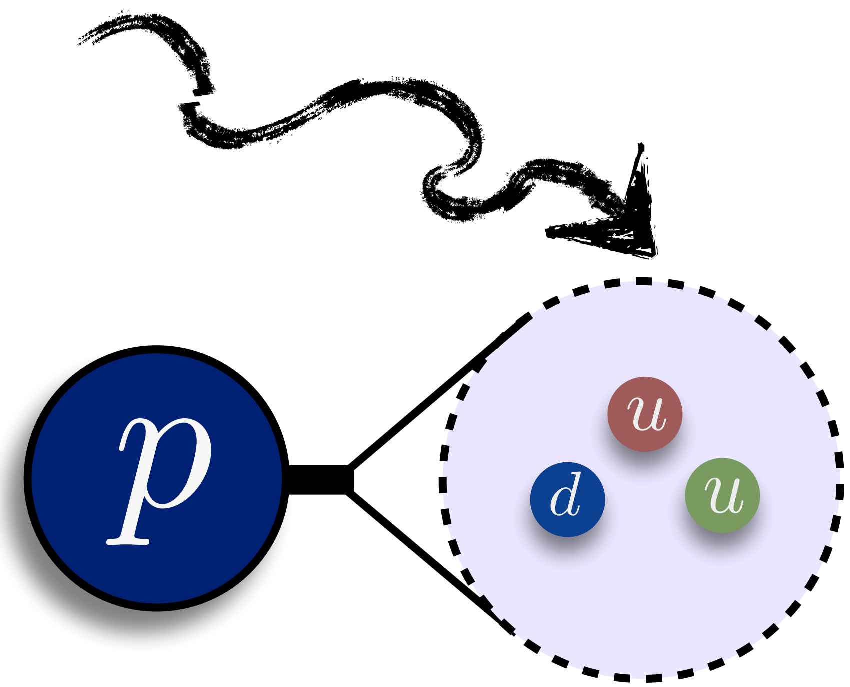
INT, Seattle
November 2015

Resonant matrix elements

(e.g., meson photo/electro-production)



Resonant matrix elements

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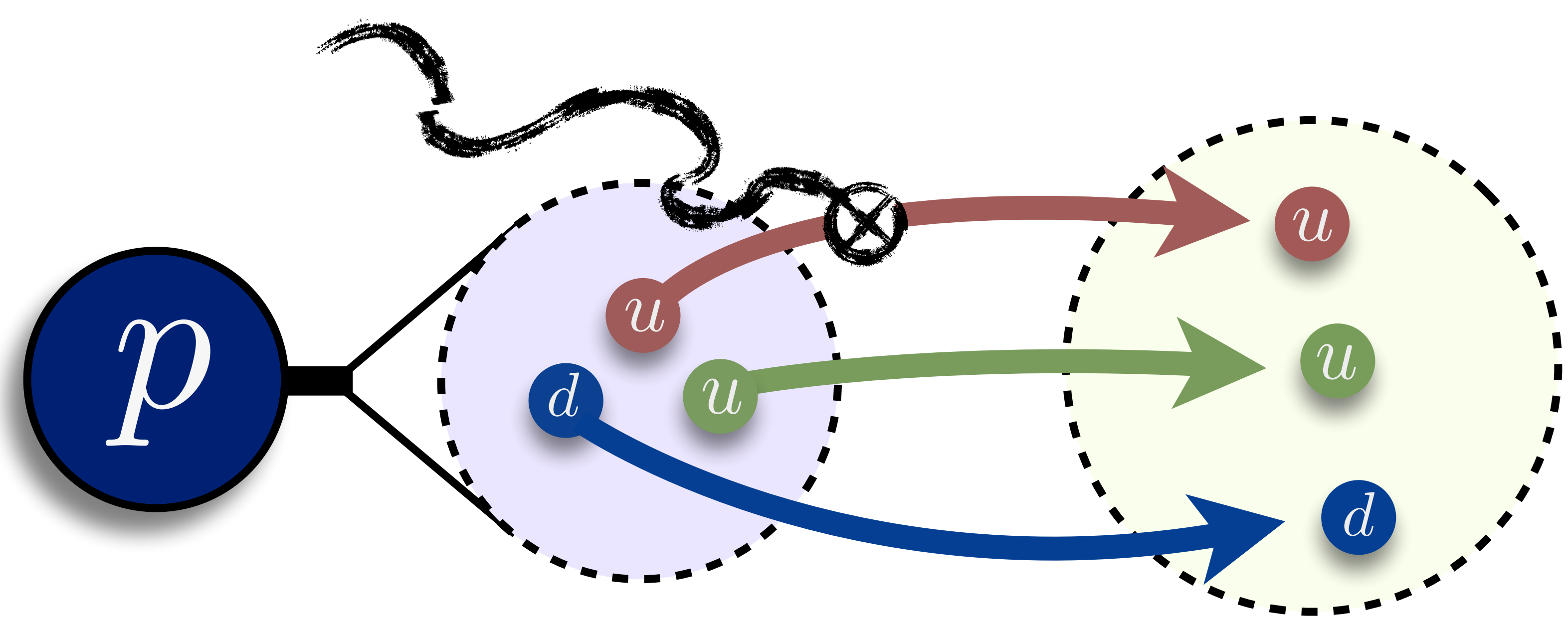
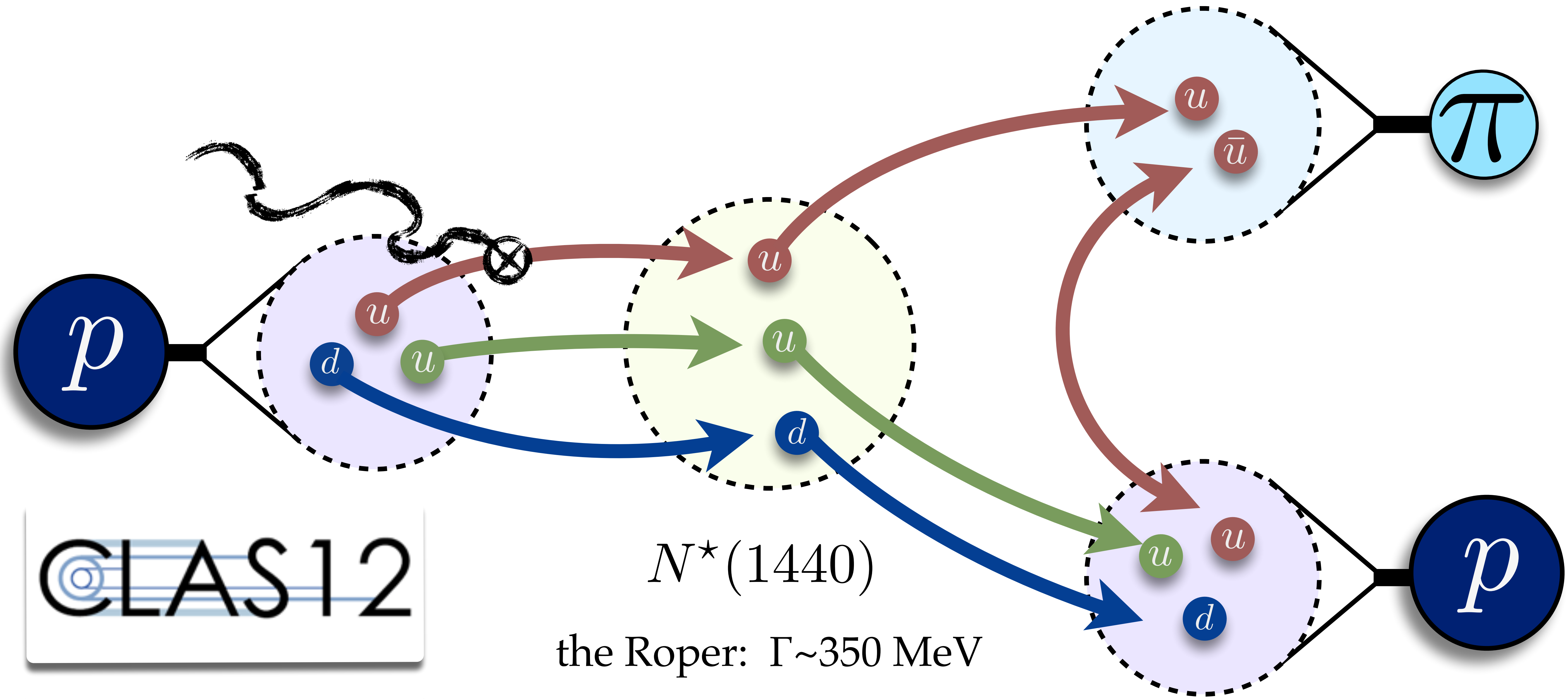


$N^{\star}(1440)$

the Roper: Γ~350 MeV

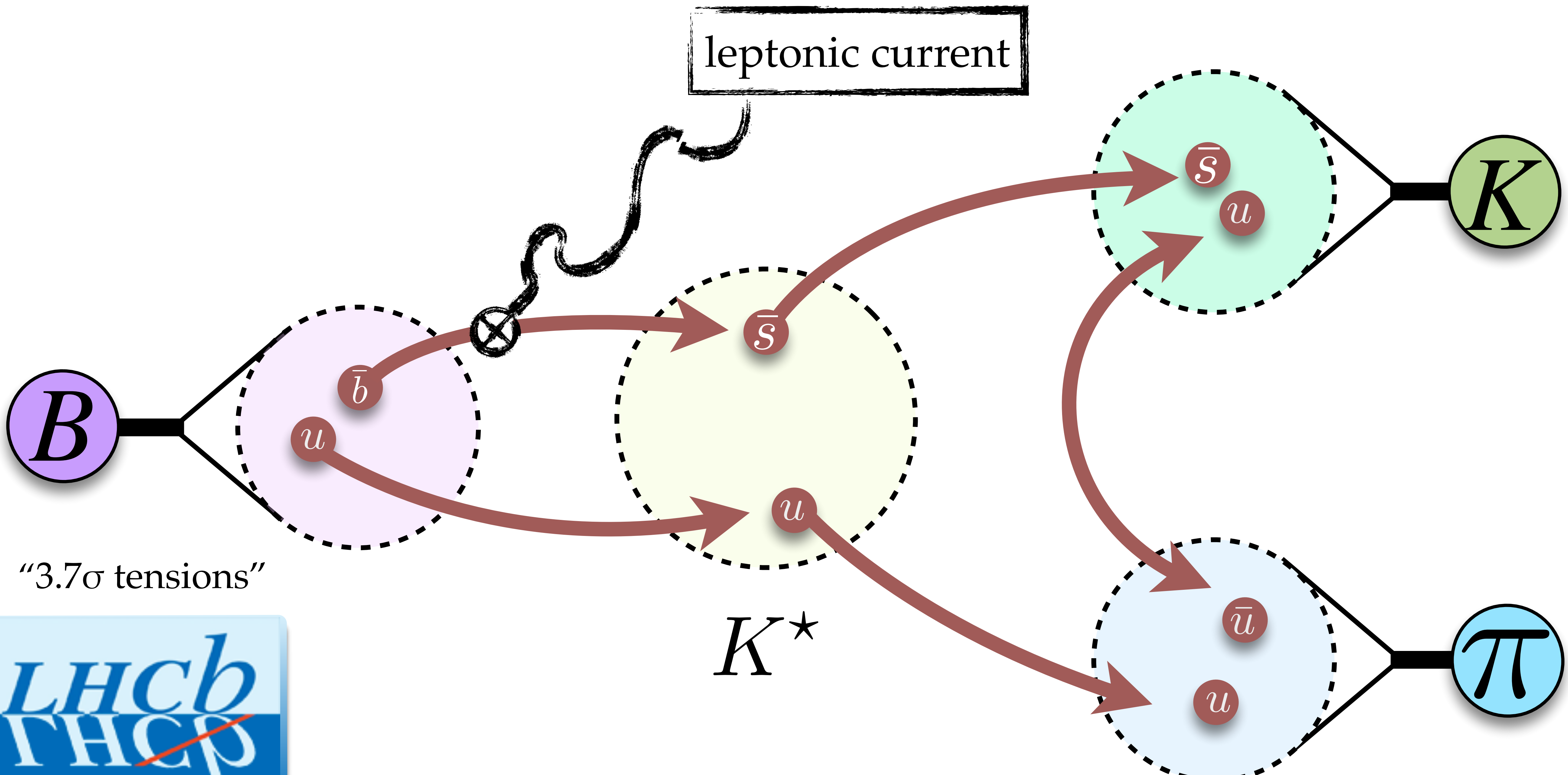
Resonant matrix elements

(e.g., meson photo/electro-production)



Resonant matrix elements

(e.g., rare weak decays)



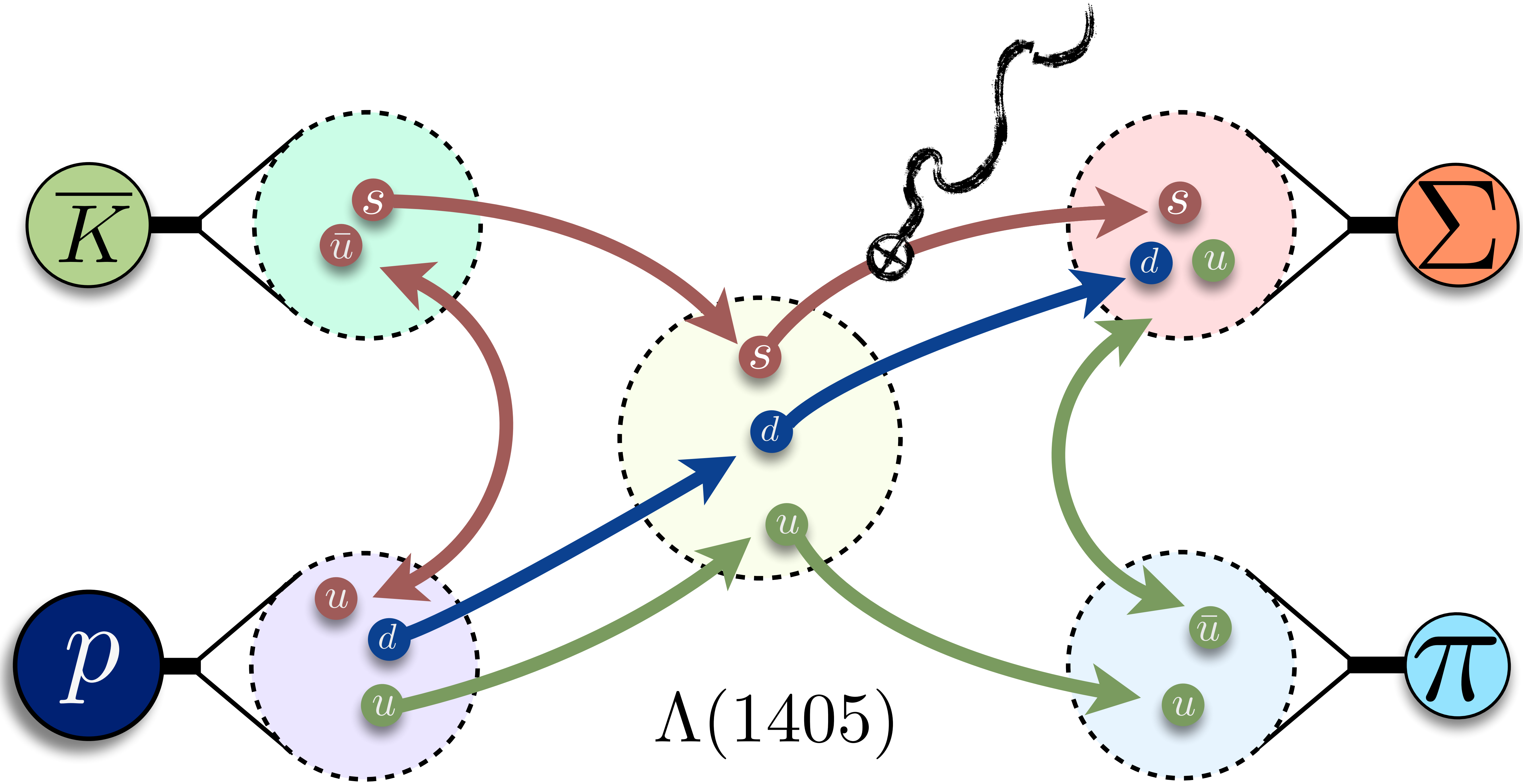
"3.7σ tensions"

LQCD efforts:
Horgan, Liu, Meinel, Wingate (2013)

"testing the standard model or testing inability to understand it?"

Resonant matrix elements

(e.g., form factors of resonance)

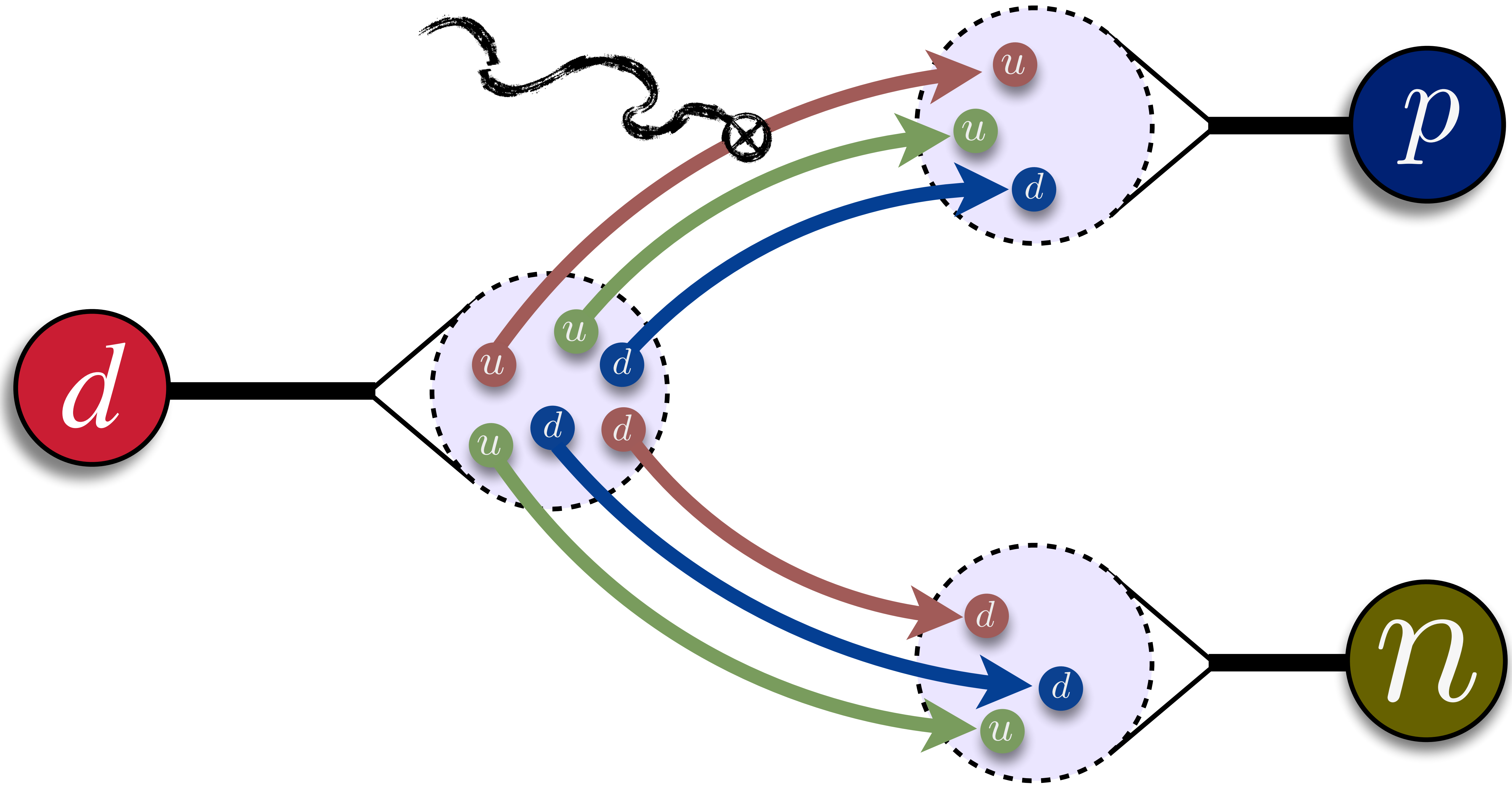


"substantiating the molecular nature of composite states"

Hall, Kamleh, Leinweber, Menadue, Owen, Thomas, Young (2014)

Non-resonant matrix elements

(e.g., deuteron elastic/inelastic form factors)



Transition processes

Importance of transition processes:

- Probe the inner structure and shape of hadrons
- Access the excited spectrum of QCD
- Test our understanding of QCD
- Test the limits of the standard model
- …

Transition processes

Lattice QCD is a theoretical tool that

- is non-perturbative in QCD
- generates resonating states dynamically
- allows resonances to decay in accordance to QCD
- includes quark-core, two-body, three-body, …, n-body effects
- treats electroweak effects perturbatively (or non-perturbatively)
- …

Check list

- ☐ Formalism
  (i.e., do we know what we need to study?)

- ☐ Code development
  (i.e., can we perform said calculation?)

- ☐ Implementation & analysis
  (i.e., what are you waiting for? do it!)

Check list

☐ Formalism

(i.e., do we know what we need to study?)

$1\to2$ and $0\to2$ processes:
RB, Hansen & Walker-Loud (2014)
RB & Hansen (Feb 2015)
$2\to2$
RB & Hansen (Sept 2015)

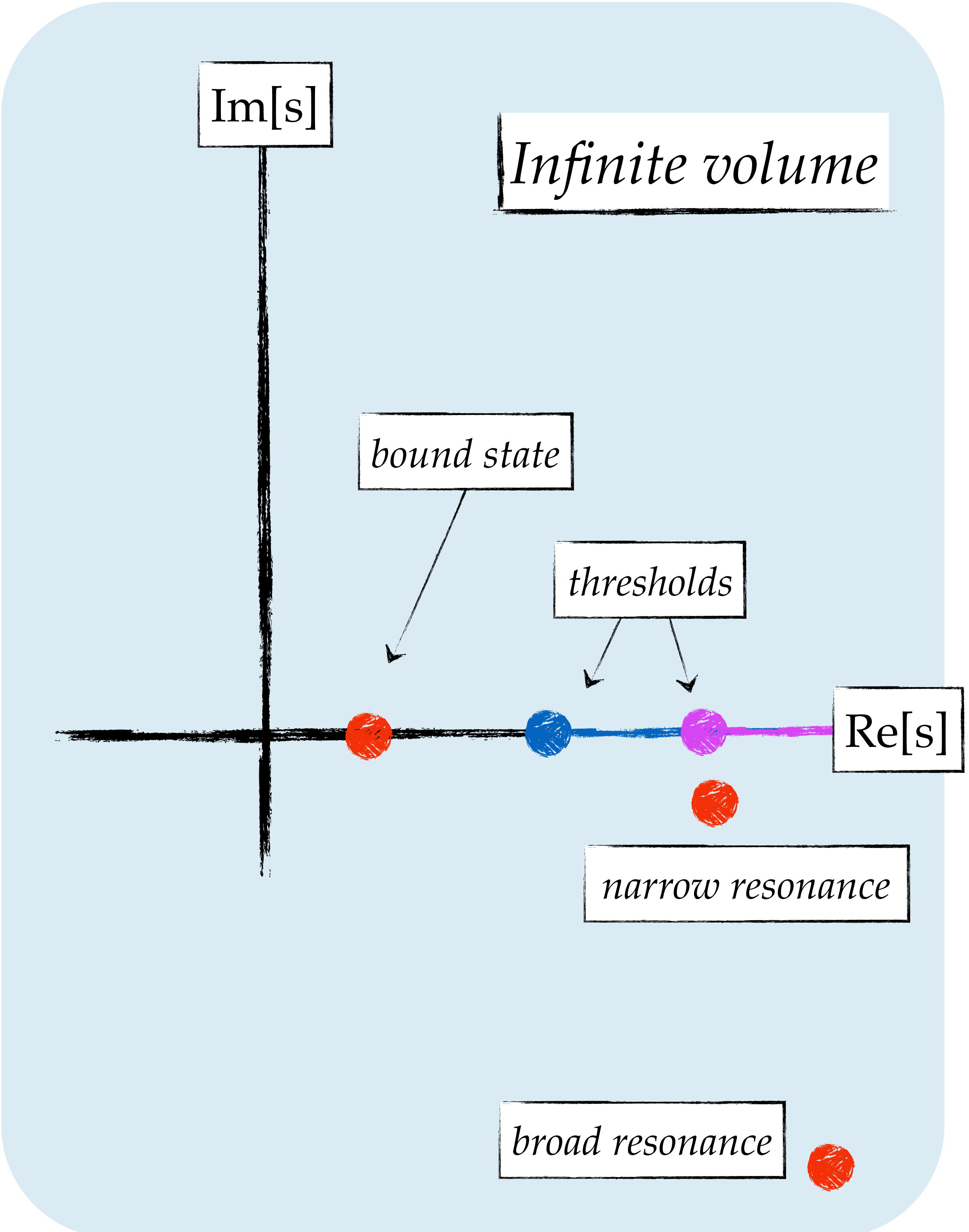

Hansen


Walker-Loud

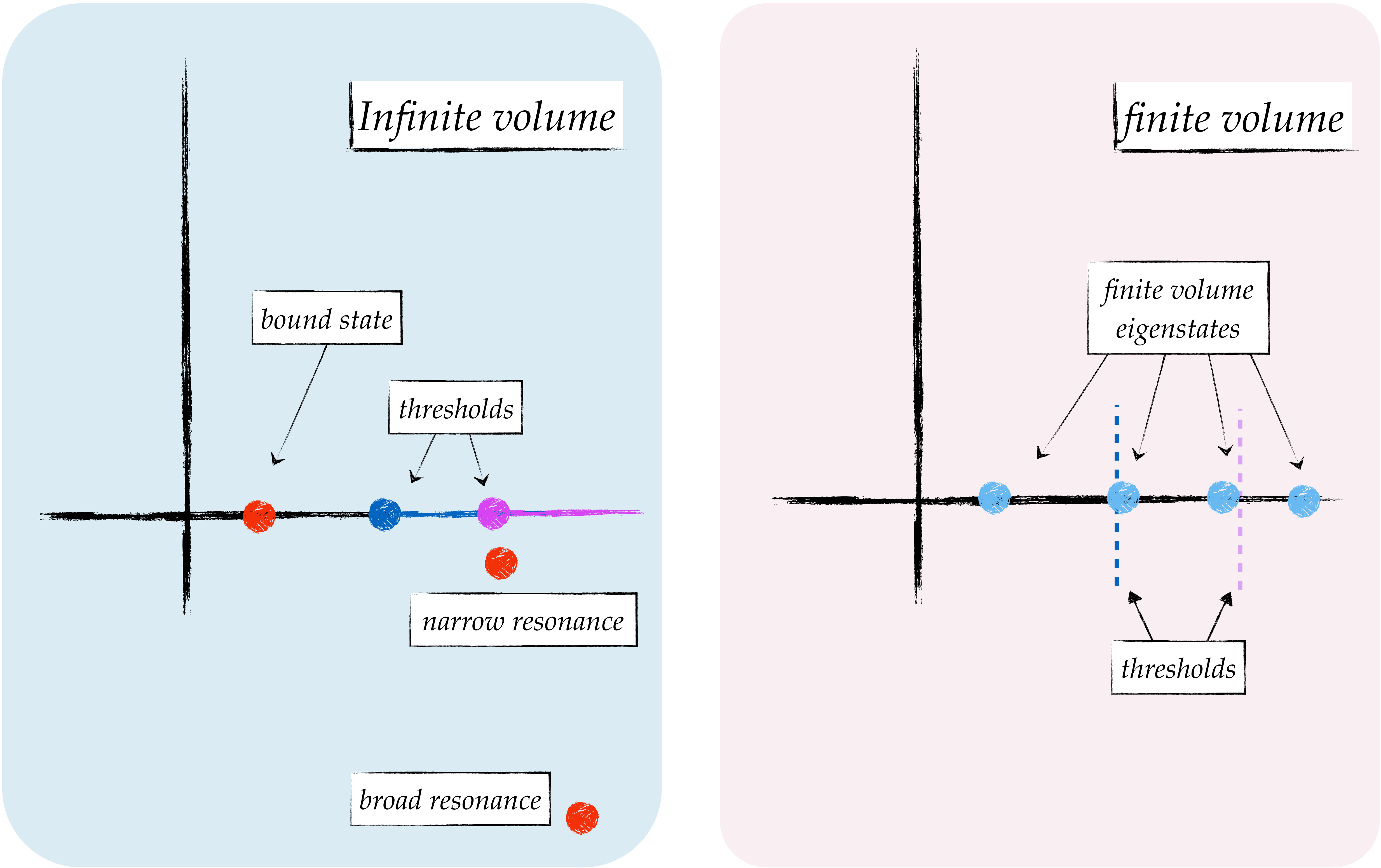
Everyone’s dream

“one can only hope”

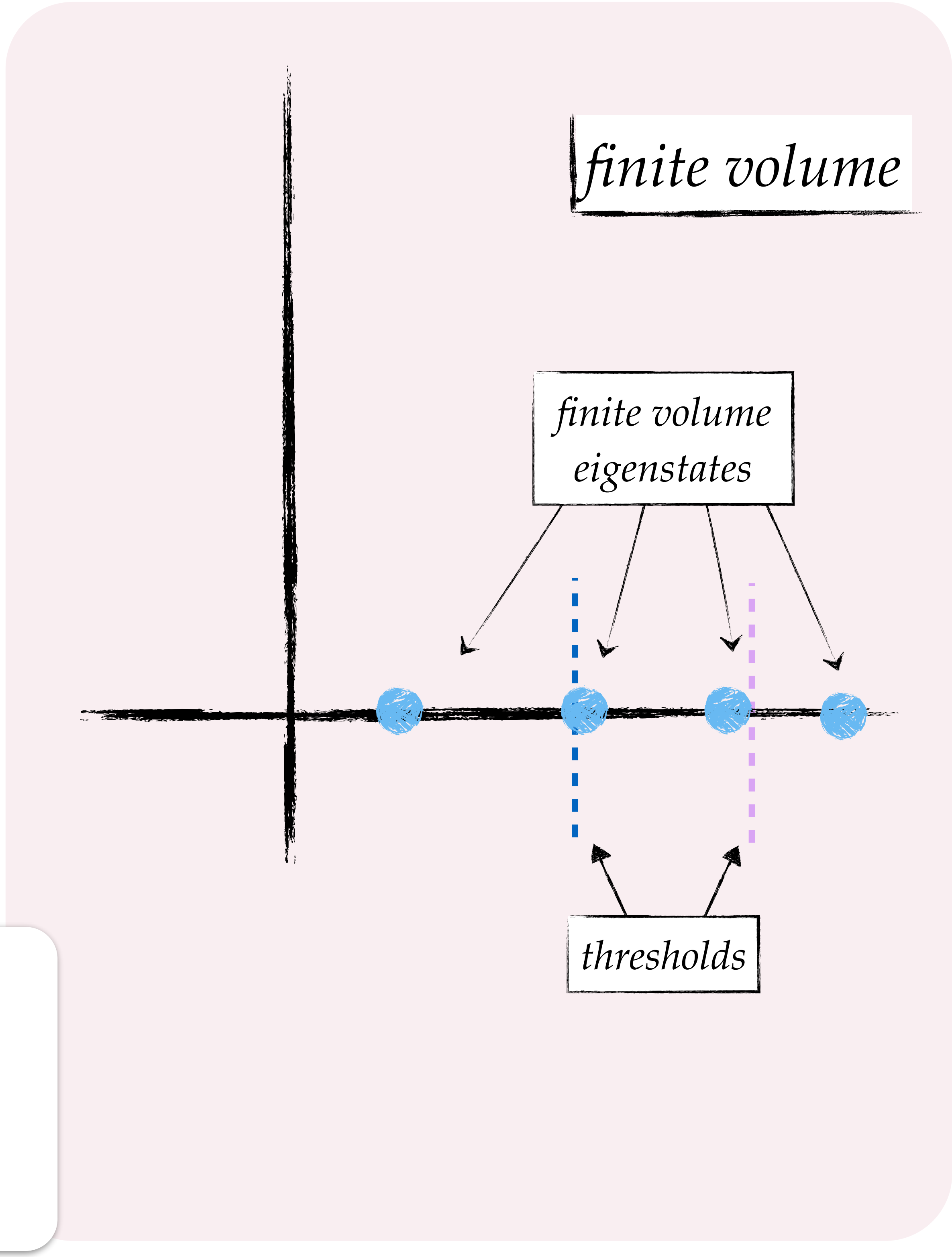
Finite vs. infinite volume spectrum



Finite vs. infinite volume spectrum



Finite vs. infinite volume spectrum



- Finite volume states are not resonance!
- Must do better!
- What about scattering?

Scattering in finite volume: *impossible!*

Finite volume - a necessity for lattice QCD

- No asymptotic states, i.e., no scattering, resonances, etc.
- Challenging, but ***not*** an limitation
- Finite volume effects allow us to determine the S-matrix

Huang & Yang (1957)
Lüscher (1986)
Lellouch & Lüscher (2000)

Lellouch-Lüscher formalism

- Lellouch & Lüscher (2000)
- Lin, G. Martinelli, C. T. Sachrajda (2001)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)
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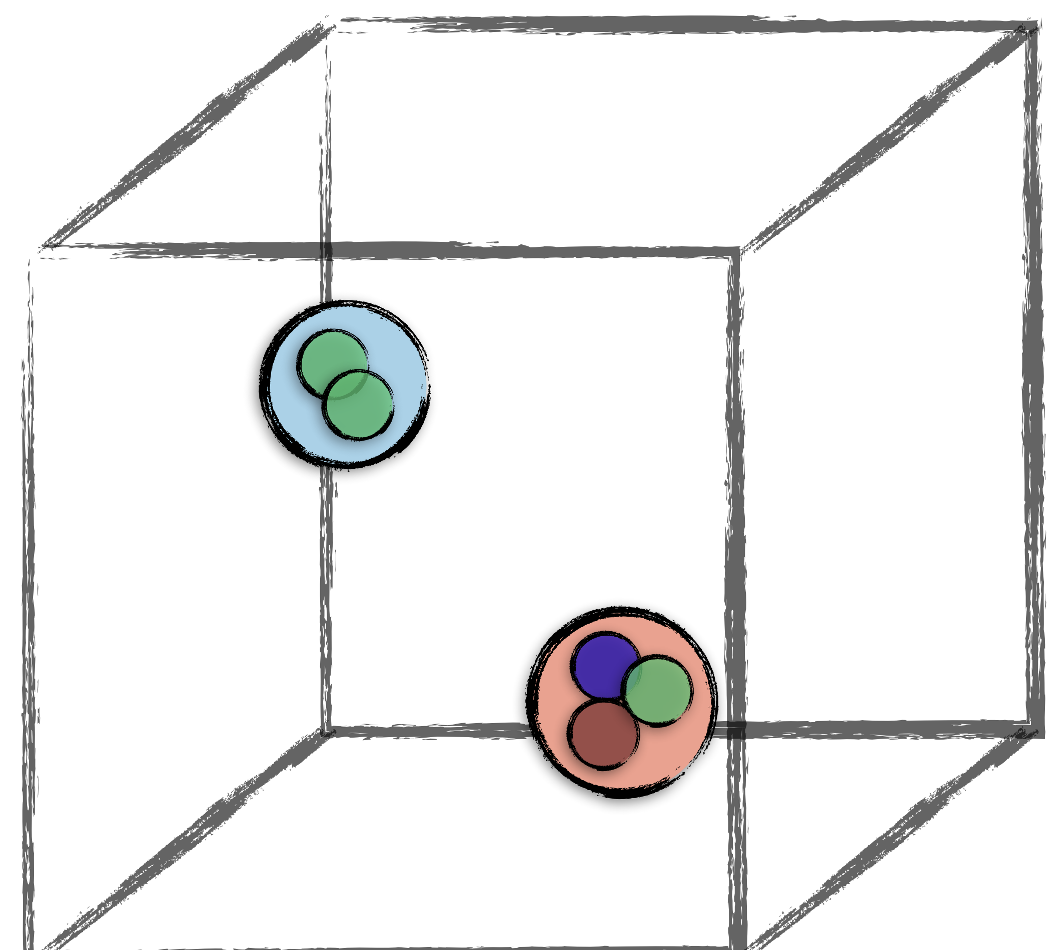
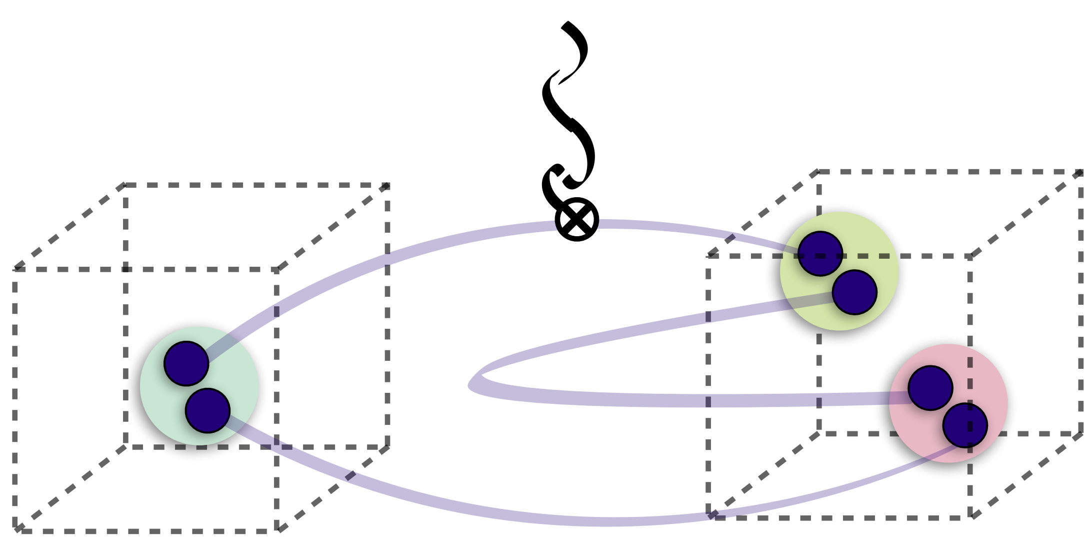


Correlation functions

Three-point functions: $C^{3pt.}_{i\to f\mathcal{J}} = \langle 0|T\mathcal{O}_f(\delta t)\mathcal{J}(t)\mathcal{O}_i^\dagger(0)|0\rangle_L$



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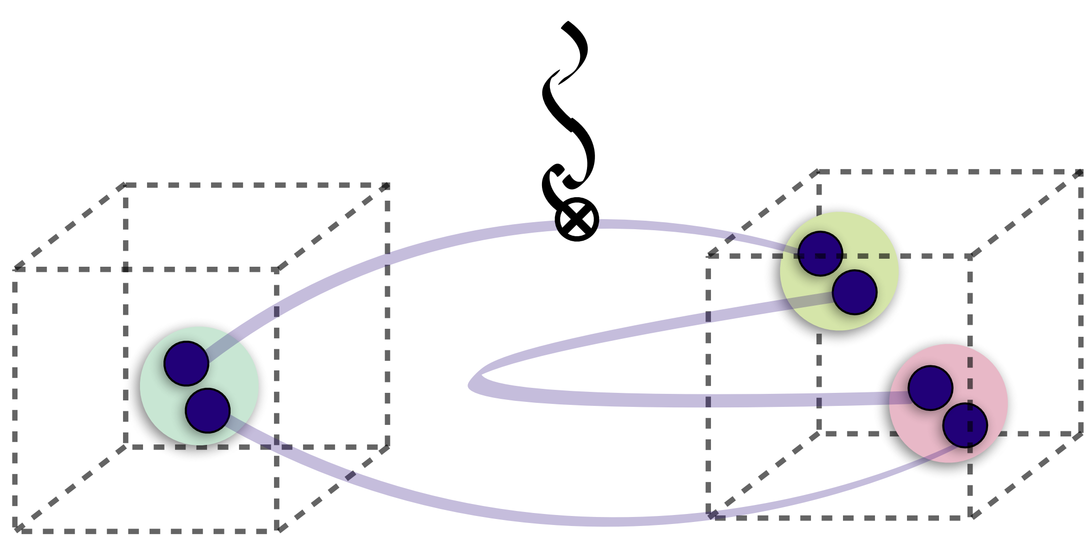
Definition #1:

Complete set of finite volume (L) state: $1 = \sum_n |n, L\rangle\langle n, L|$

Hadrons in a box: the energy and states are those of IR degrees of freedom of the finite volume QCD Hamiltonian



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$$C^{3pt.}_{i\to f\mathcal{J}} = \sum_{n,n'} Z_{n,f} Z^*_{n',i} e^{-(\delta t - t)E_n} e^{-tE_{n'}} \langle n, L|\mathcal{J}|n', L\rangle$$

$E_n \longleftrightarrow$ scattering

$\langle n, L|\mathcal{J}|n', L\rangle \longleftrightarrow ?$

Jo's talk

Field theory

What?: Relativistic quantum field theory

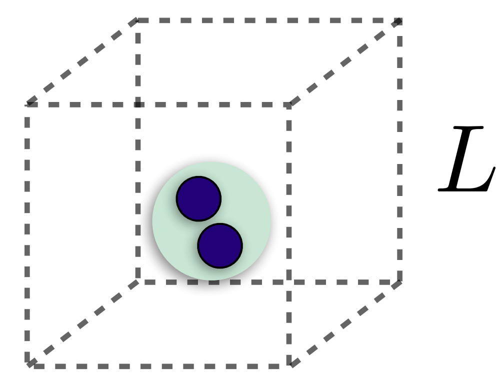
Why?: to give meaning to correlation functions

Where?: a finite Euclidean spacetime?

How?: Non-perturbatively, or to all order in perturbation theory

$$\mathbf{p} = 2\pi \mathbf{n}/L, \text{ where } \mathbf{n} \in Z^3$$

$$p^2 = p_0^2 + \mathbf{p}^2$$

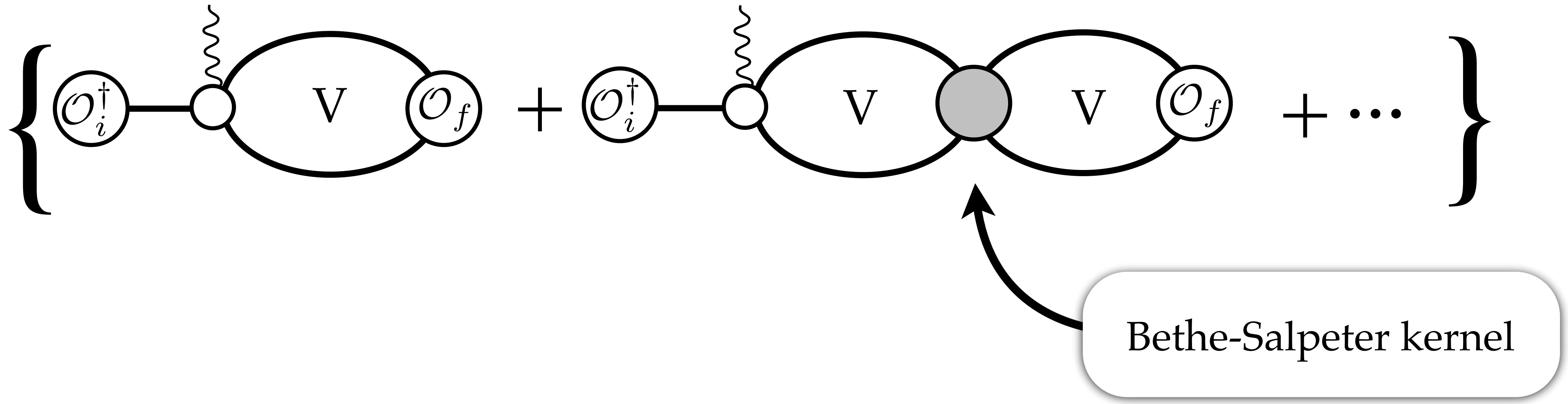


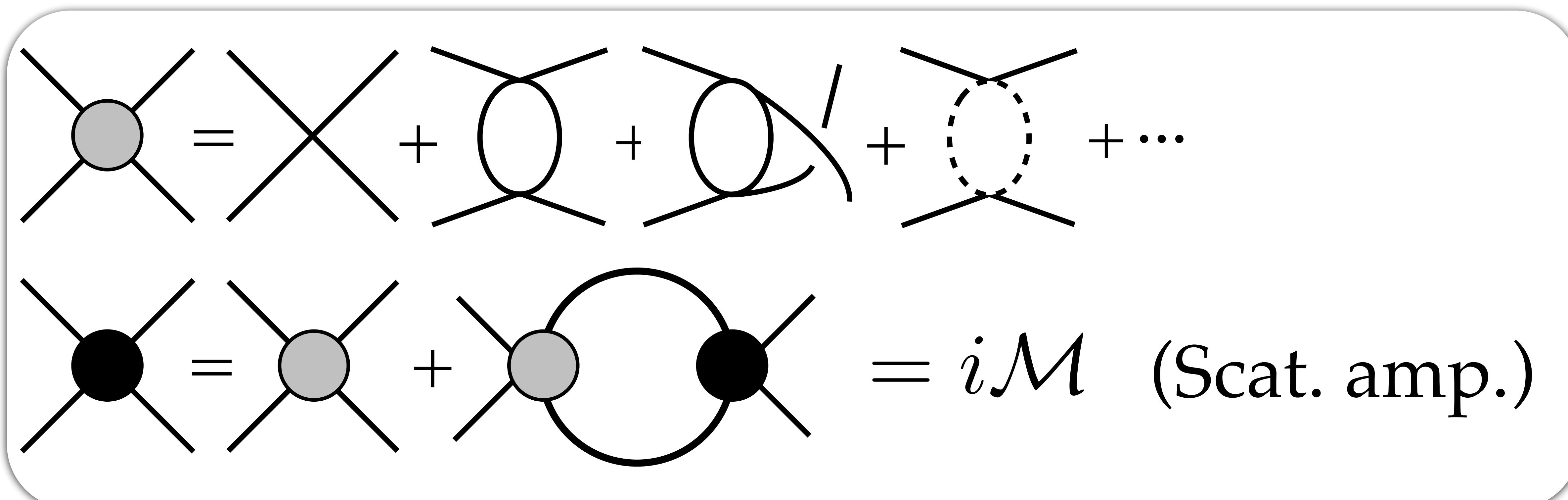
$$C^{3pt.}_{i \to f \mathcal{J}} = \text{ F.T. [sum over finite volume diagram with a single current insertion]}$$

Correlation functions

Definition #2: [e.g., 1→2 processes, below the 3body thresholds]

$$C^{3pt.}_{i\to f\mathcal{J}} = \text{F.T.}$$





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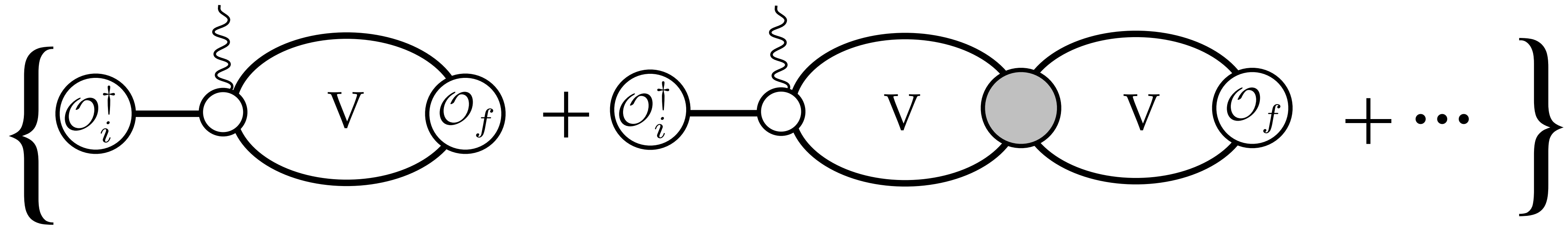

Using techniques developed by Kim, Sachrajda, and Sharpe (2005)

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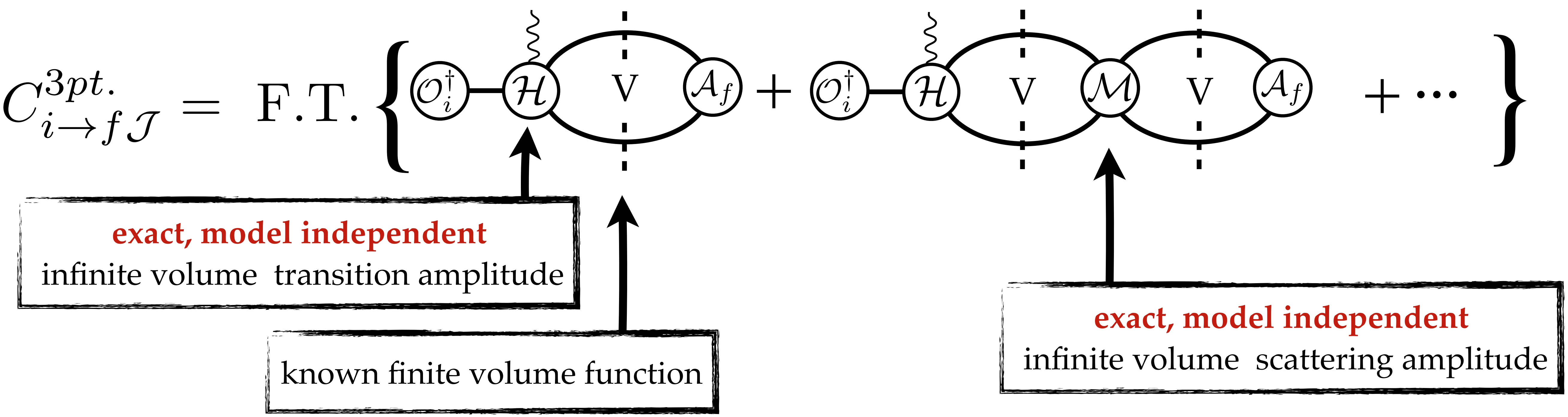

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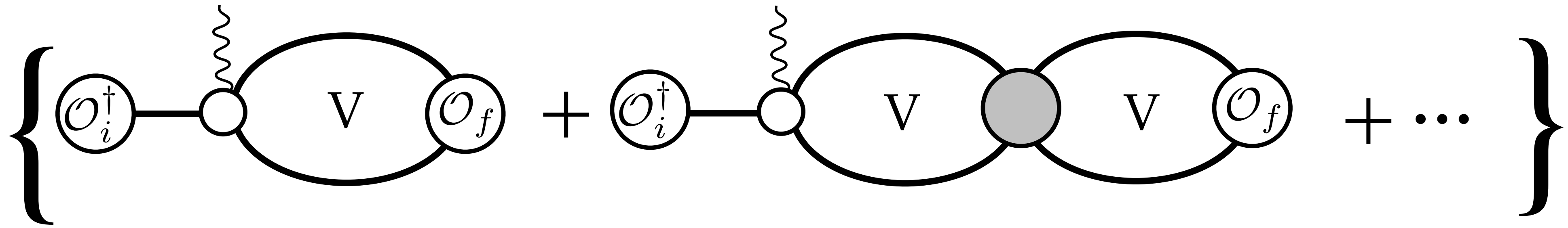
$$C^{3pt.}_{i\to f\mathcal{J}} = \text{F.T.}$$ 

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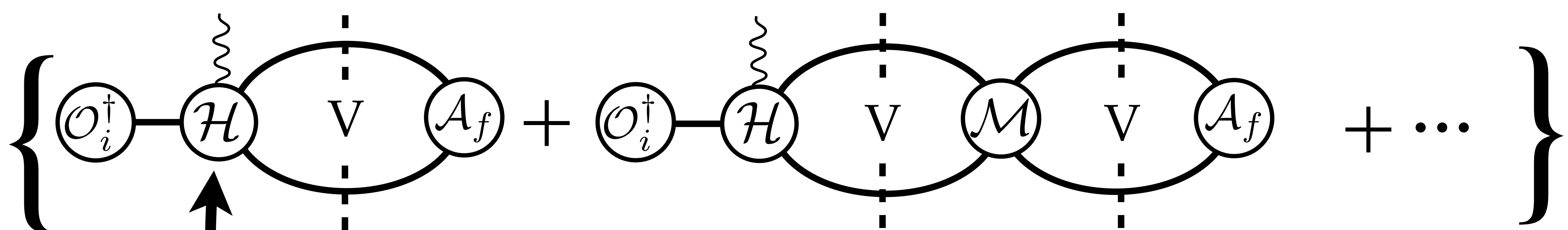
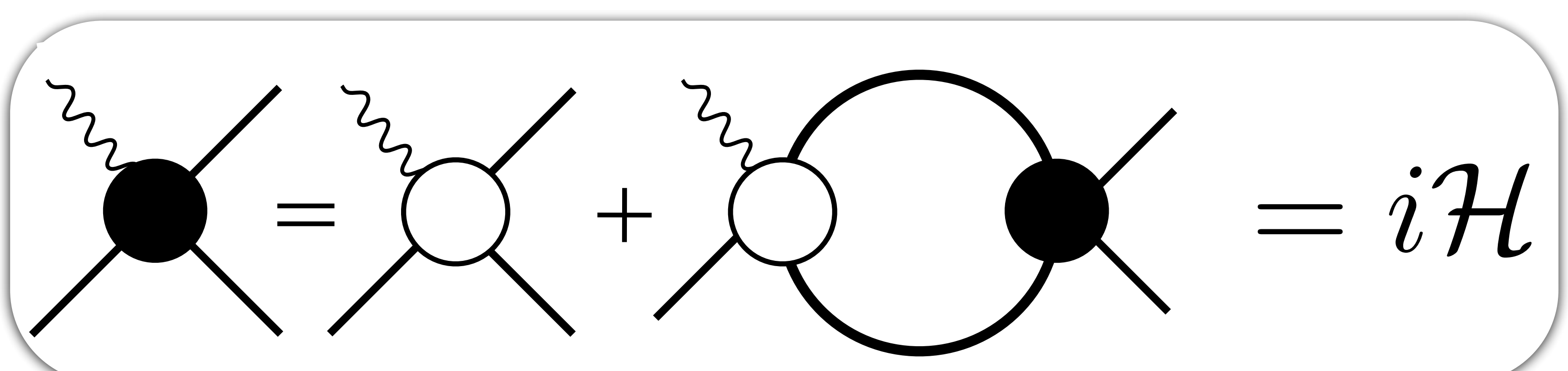
$$C^{3pt.}_{i\to f\mathcal{J}} = \text{F.T.}$$ 

exact, model independent
infinite volume transition amplitude



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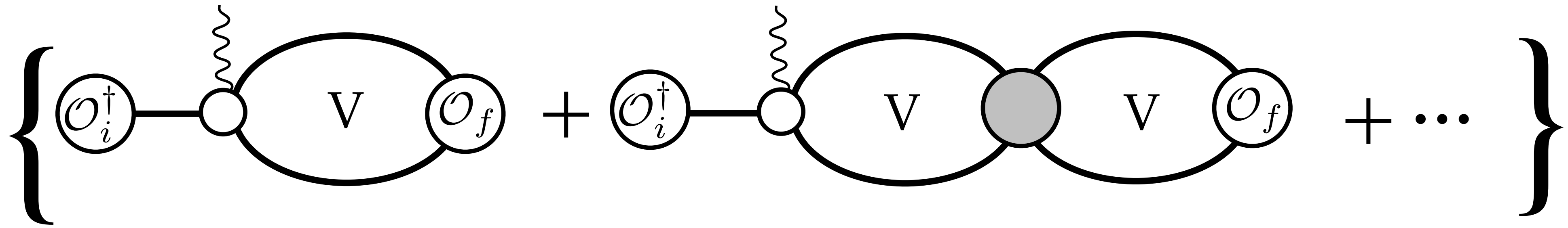

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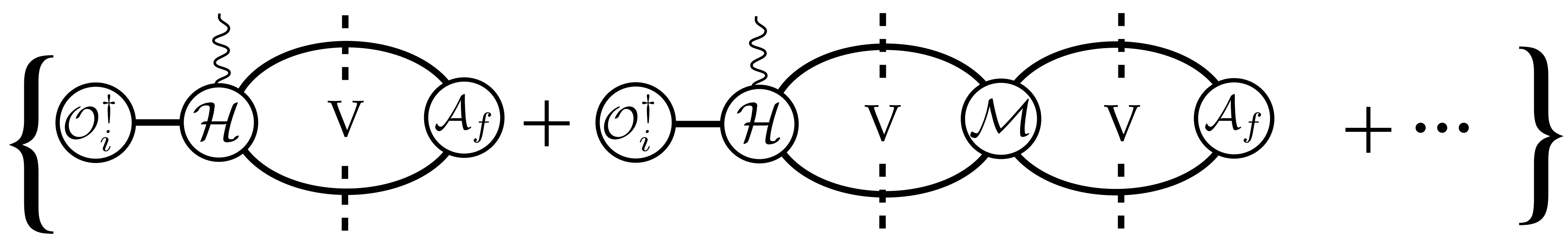


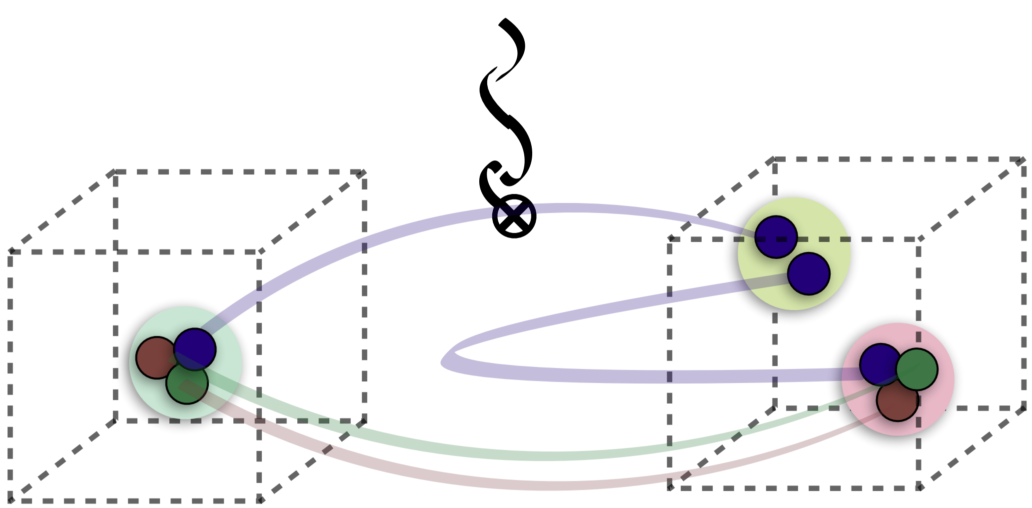
<u>Take home message:</u> finite volume correlation functions can be written in terms of on-shell, infinite volume quantities!

Finite volume matrix elements

By equating the two definitions and after some algebra, we find:

1) 1→2 processes:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\frac{1}{2E_1}}\sqrt{\mathcal{H}^{\text{in}}\ \mathcal{R}\ \mathcal{H}^{\text{out}}}$$



RB, Hansen & Walker-Loud (2014)

RB & Hansen (2015)

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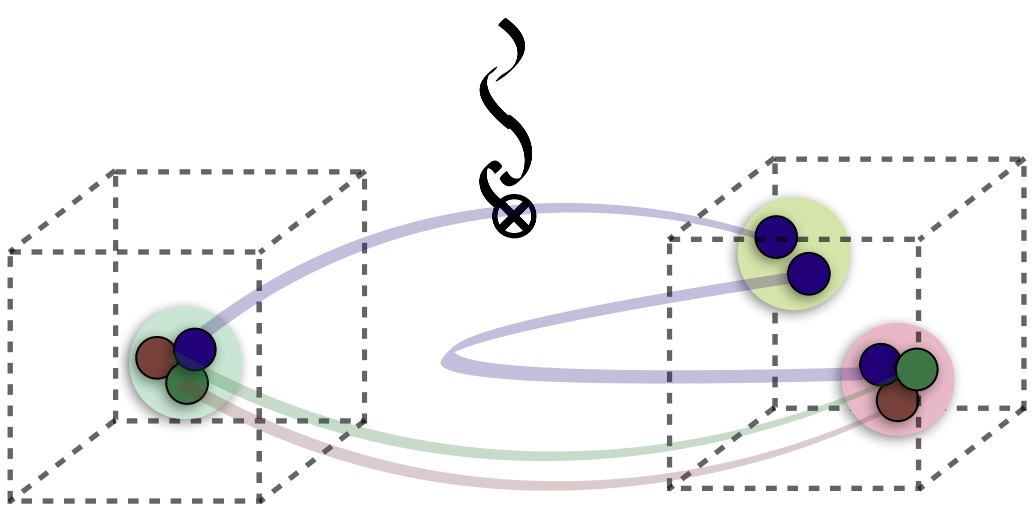
finite volume matrix element

known finite volume function

exact, model independent
infinite volume transition amplitude



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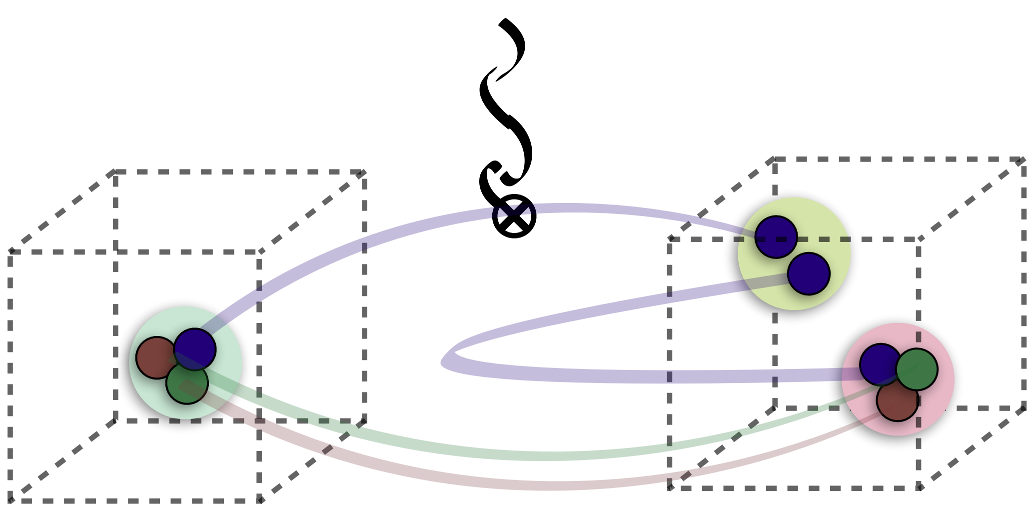
known finite volume function

$$\mathcal{R}\left(E_{\mathbf{2}}, L, \delta, \frac{\partial\delta}{\partial E_{\mathbf{2}}}\right)$$



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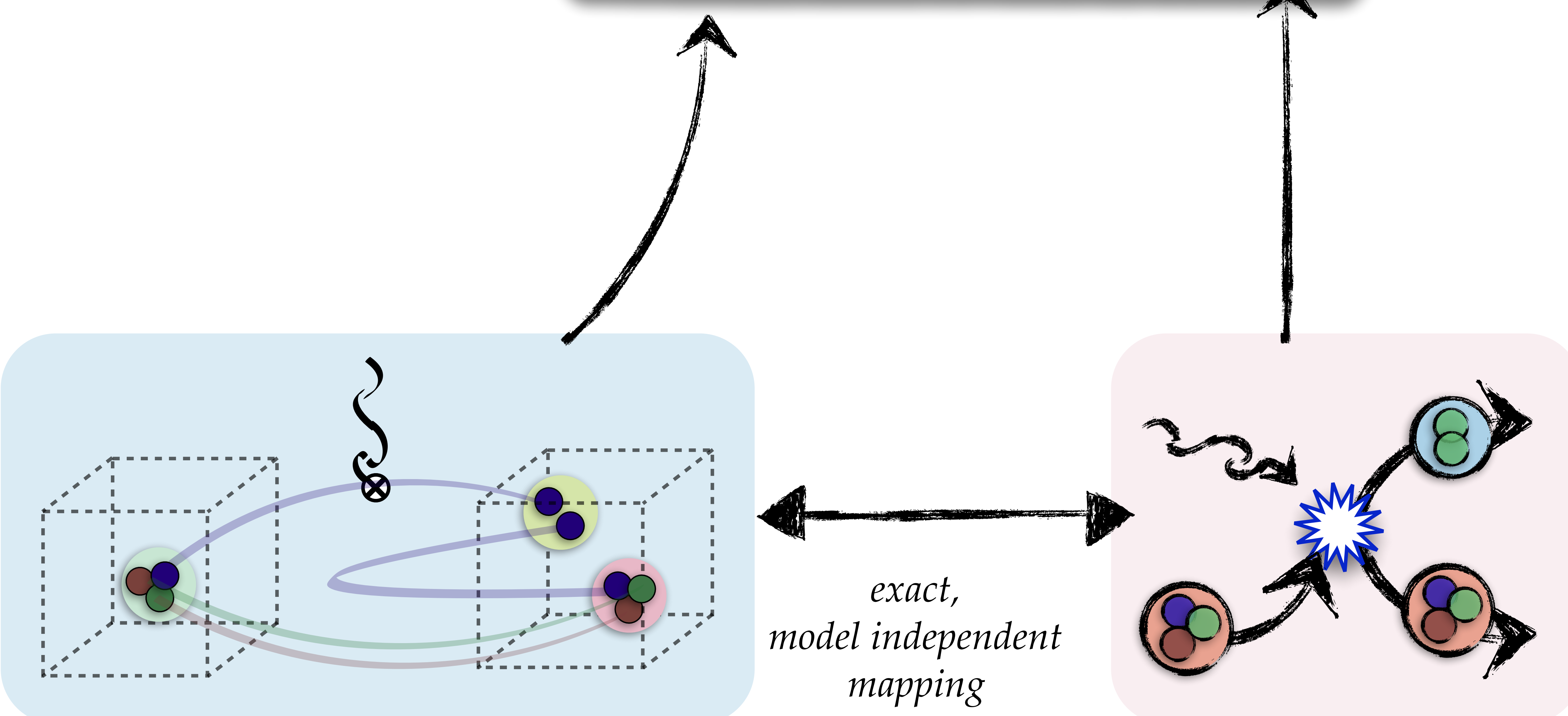

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$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right| = \sqrt{\frac{1}{2E_1}}\sqrt{\mathcal{H}^{\text{in}}\ \mathcal{R}\ \mathcal{H}^{\text{out}}}$$

summarizes everything previously done and more!

Lellouch-Lüscher formalism

- Lellouch & Lüscher (2000)
- Lin, G. Martinelli, C. T. Sachrajda (2001)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)
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- …

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- **Holds below three-particle thresholds**
- **On-going efforts to address these limitation**
  - **Hansen & Sharpe (2014-2015)**

Finite volume matrix elements

By equating the two definitions and after some algebra, we find:

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$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L| = \sqrt{\frac{1}{2E_1}}\sqrt{\mathcal{H}^{\rm in}\ \mathcal{R}\ \mathcal{H}^{\rm out}}$$

RB, Hansen & Walker-Loud (2014)
RB & Hansen (Feb 2015)

2) 0→2 processes:

$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{0}\rangle_L| = \sqrt{L^3}\sqrt{\mathcal{V}^{\rm in}\ \mathcal{R}\ \mathcal{V}\mathcal{H}^{\rm out}}$$

RB & Hansen (Feb 2015)

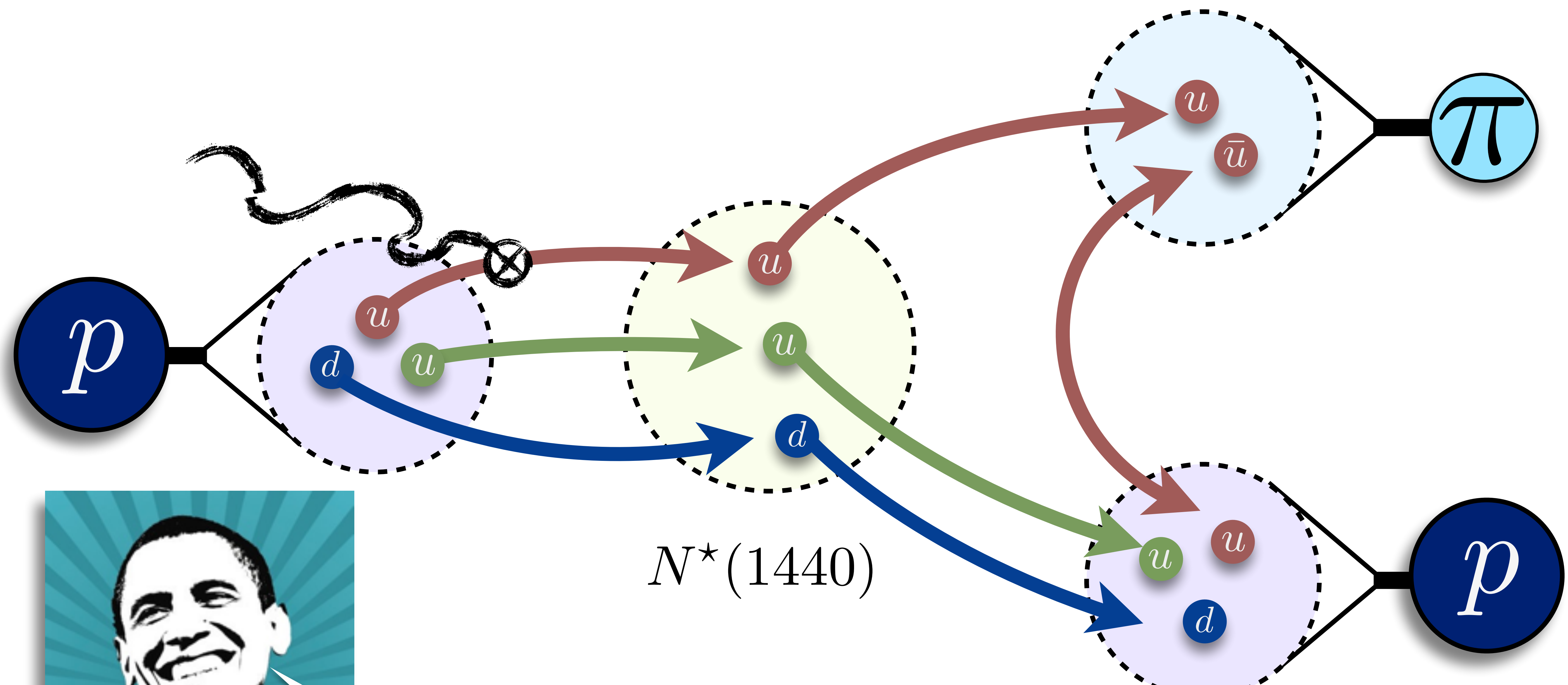
3) 2→2 processes:

$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\ \mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\right]}$$

RB & Hansen (Sept 2015)

Resonant matrix elements

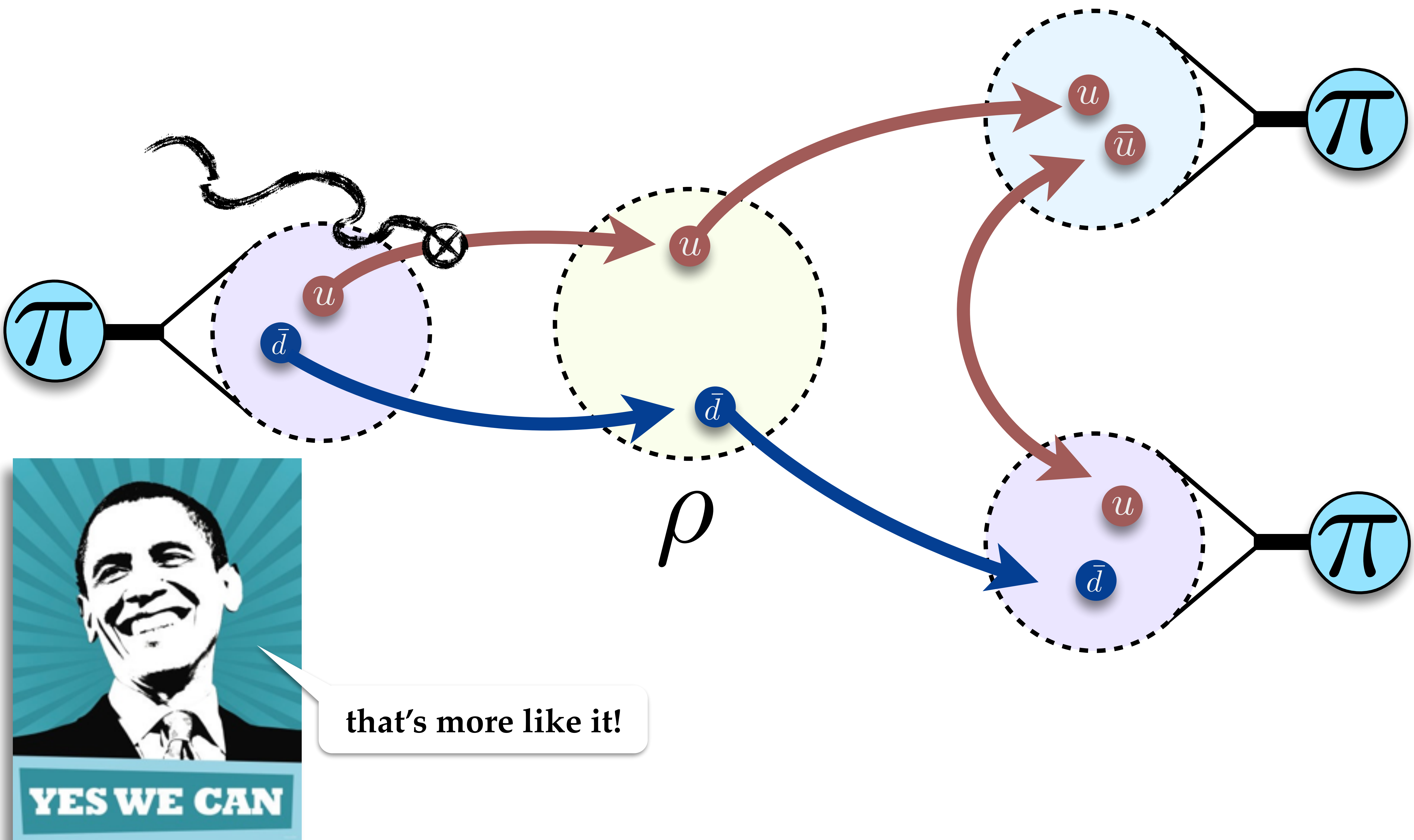
(e.g., meson photo/electro-production)



Well, at least at heavy pion masses, where the Nππ threshold is above the Roper.

Resonant matrix elements

(e.g., meson photo/electro-production)



Check list

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  (i.e., do we know what we need to study?)

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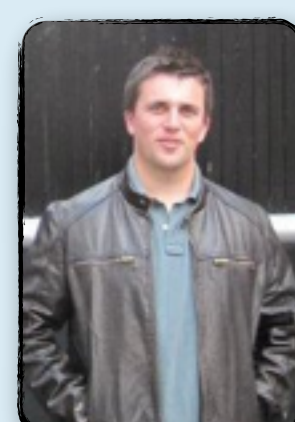
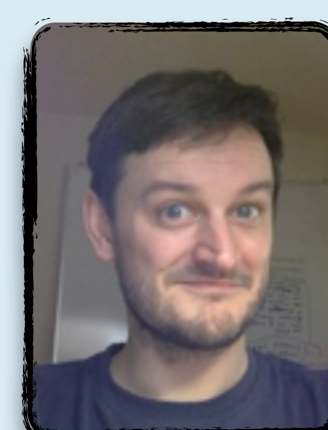
HadSpec Collaboration

Shultz, Dudek & Edwards (2014)



Shultz



Dudek

Edwards

Radiative transitions

[m_π=700 MeV]

Nearly everything is stable:

- analysis is relatively simple
- perfect place to test code

The basic idea is to replace:

$$C^{3pt.}_{i\to f\mathcal{J}} = \langle 0|\mathcal{O}_f(\delta t)\mathcal{J}(t)\mathcal{O}_i^\dagger(0)|0\rangle_L = \sum_{n,n'} Z_{n,f} Z^*_{n',i} e^{-(\delta t - t)E_n} e^{-tE_{n'}} \langle n, L|\mathcal{J}|n', L\rangle$$

with:

$$C^{3pt.}_{i\to f\mathcal{J}} = \langle 0|\Omega_{f,n_f}(\delta t)\mathcal{J}(t)\Omega^\dagger_{i,n_i}(0)|0\rangle_L = Z_{n_f,f} Z^*_{n_i,i} e^{-(\delta t - t)E_{n_f}} e^{-tE_{n_i}} \langle n_f, L|\mathcal{J}|n_i, L\rangle + \cdots$$

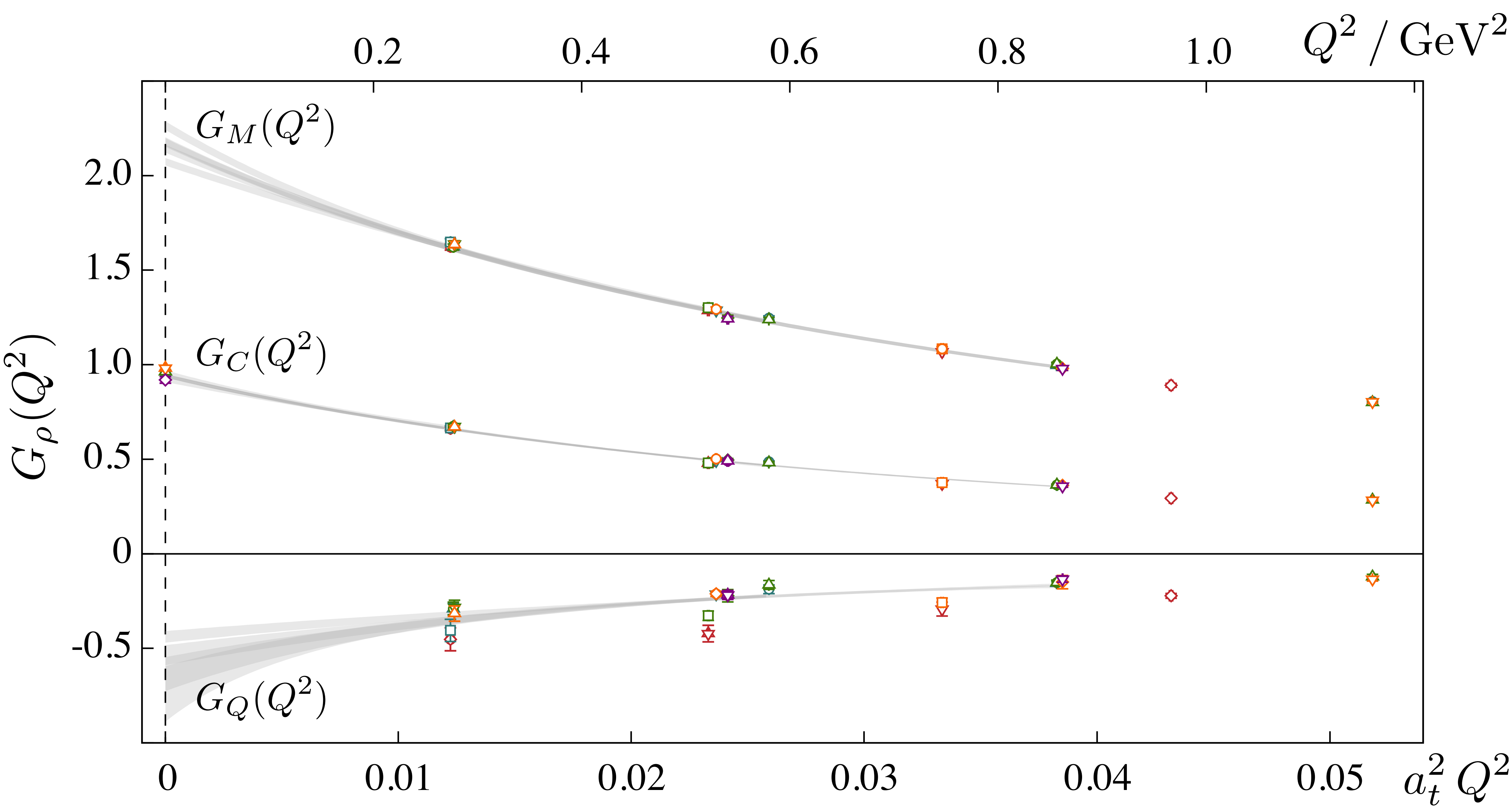
optimized operators:
typically, a linear combination of 10-30 operators

Benefits to using optimized operators:

- excited state contamination is suppressed
- can also access excited state matrix elements

Elastic form factors

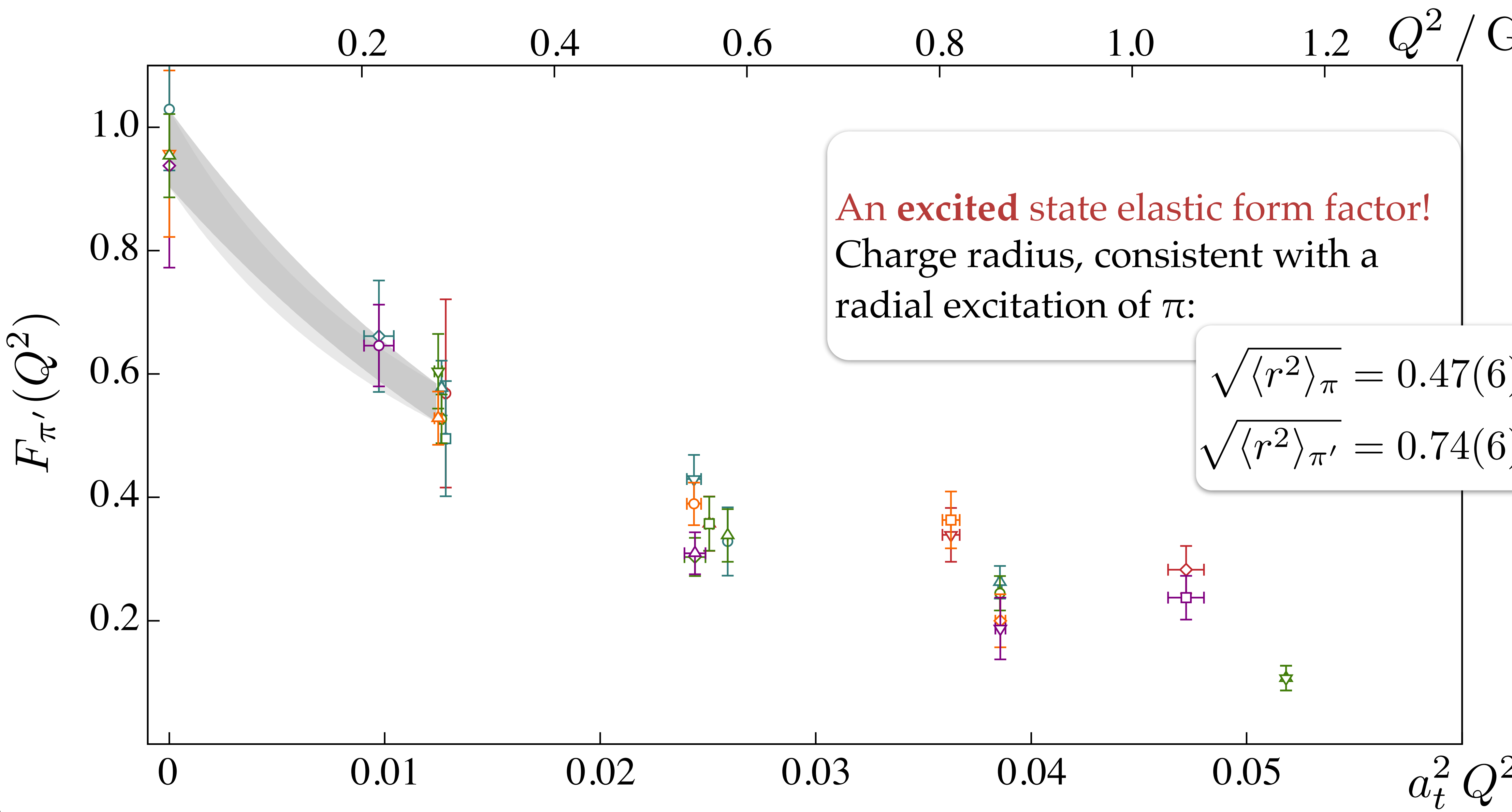
[ϱ form factors @ m_π=700 MeV]



Shultz, Dudek & Edwards (2014)

Elastic form factors

[π' form factor @ m_π=700 MeV]



An **excited** state elastic form factor!

Charge radius, consistent with a radial excitation of π:

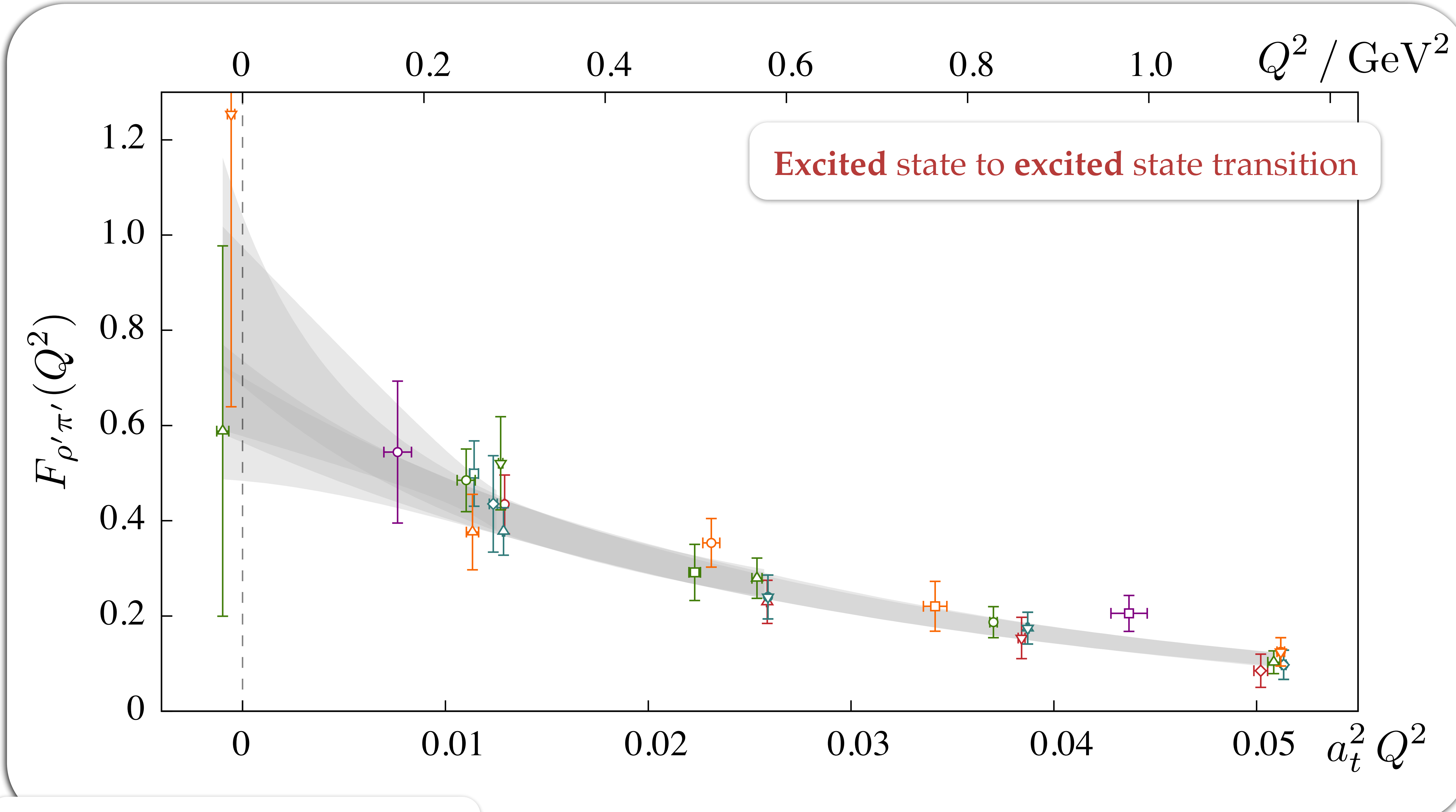
$$\sqrt{\langle r^2 \rangle_\pi} = 0.47(6)\ \text{fm}$$

$$\sqrt{\langle r^2 \rangle_{\pi'}} = 0.74(6)\ \text{fm}$$

Shultz, Dudek & Edwards (2014)

Transition form factors

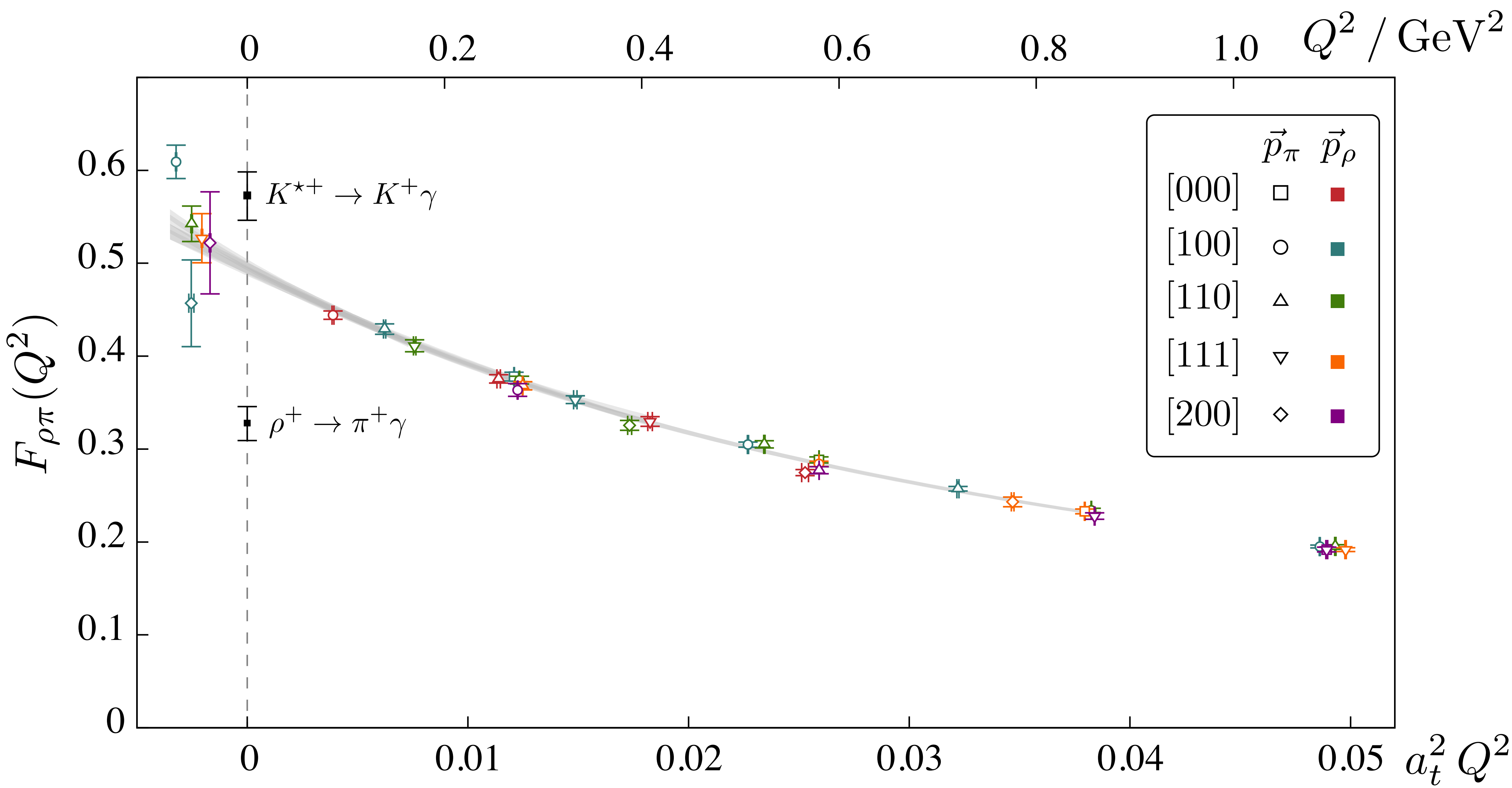
[ϱ′π′ form factors @ m_π=700 MeV]



Shultz, Dudek & Edwards (2014)

Transition form factors

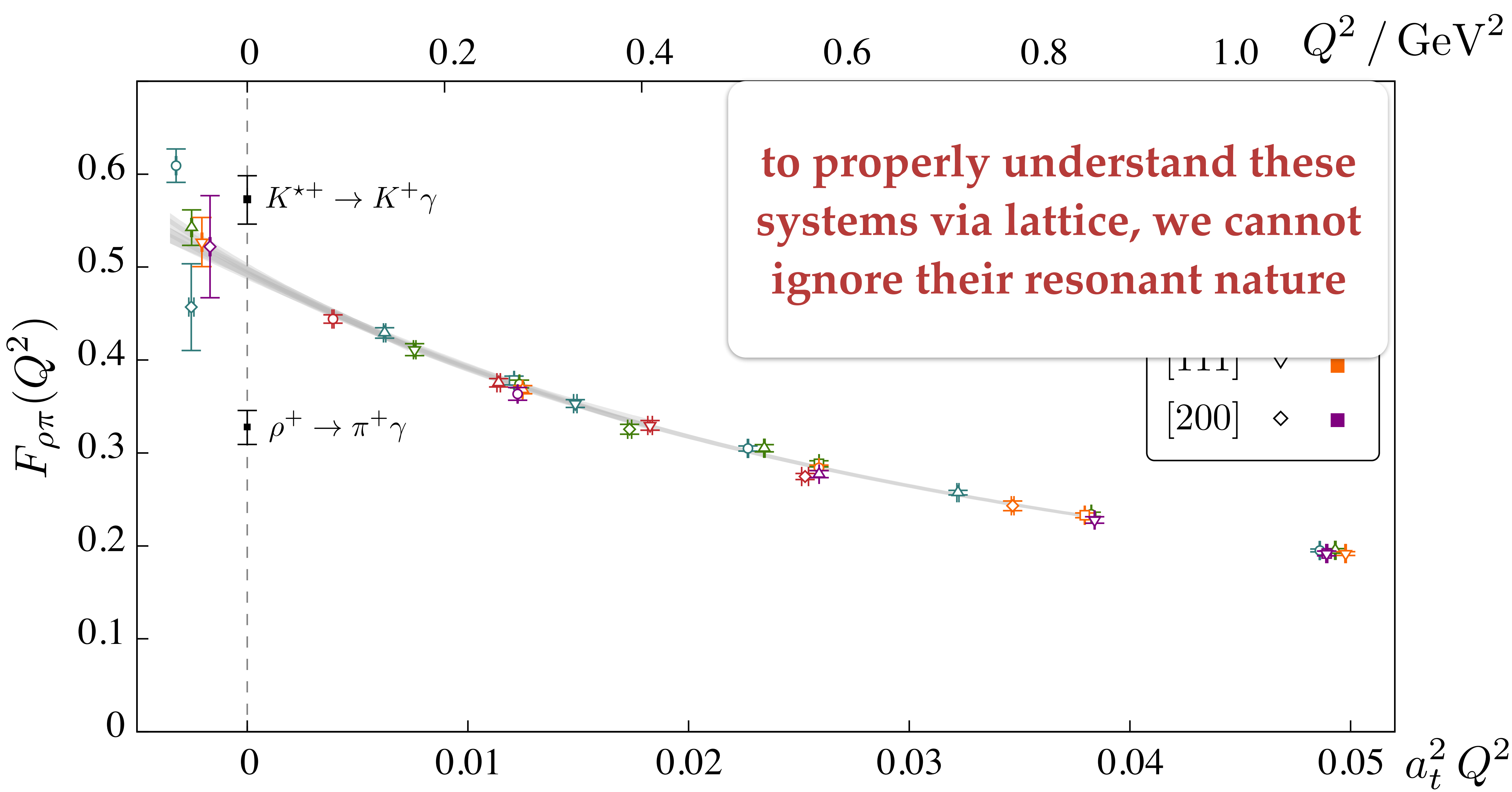
[ϱπ form factors @ m_π=700 MeV]



Shultz, Dudek & Edwards (2014)

Transition form factors

[ϱπ form factors @ m_π=700 MeV]



Shultz, Dudek & Edwards (2014)

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HadSpec Collaboration

Wilson

Shultz

Thomas

Dudek

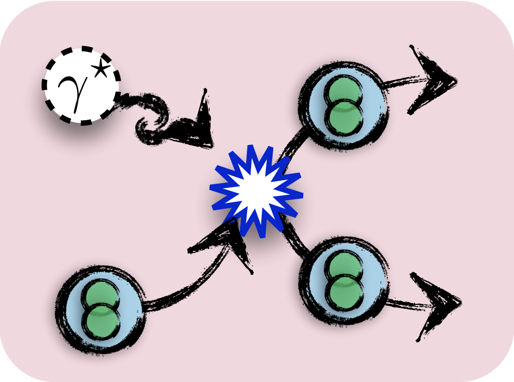
Edwards

RB, Dudek, Edwards, Shultz, Thomas & Wilson **[Accepted to PRL]** (2015)

Implementation & analysis

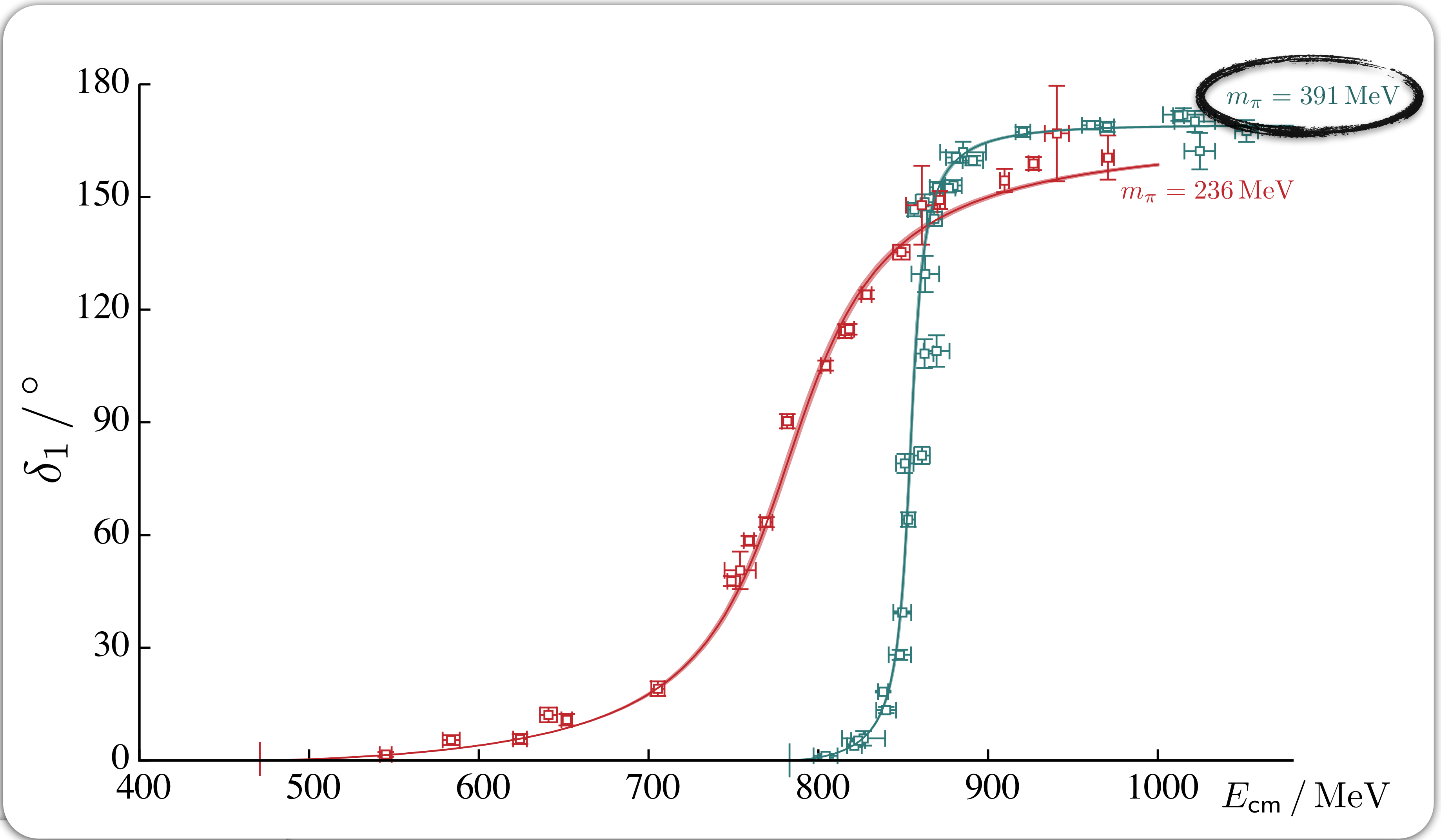
(i.e., what are you waiting for? do it!)

$\pi\gamma^*$-to-$\pi\pi$



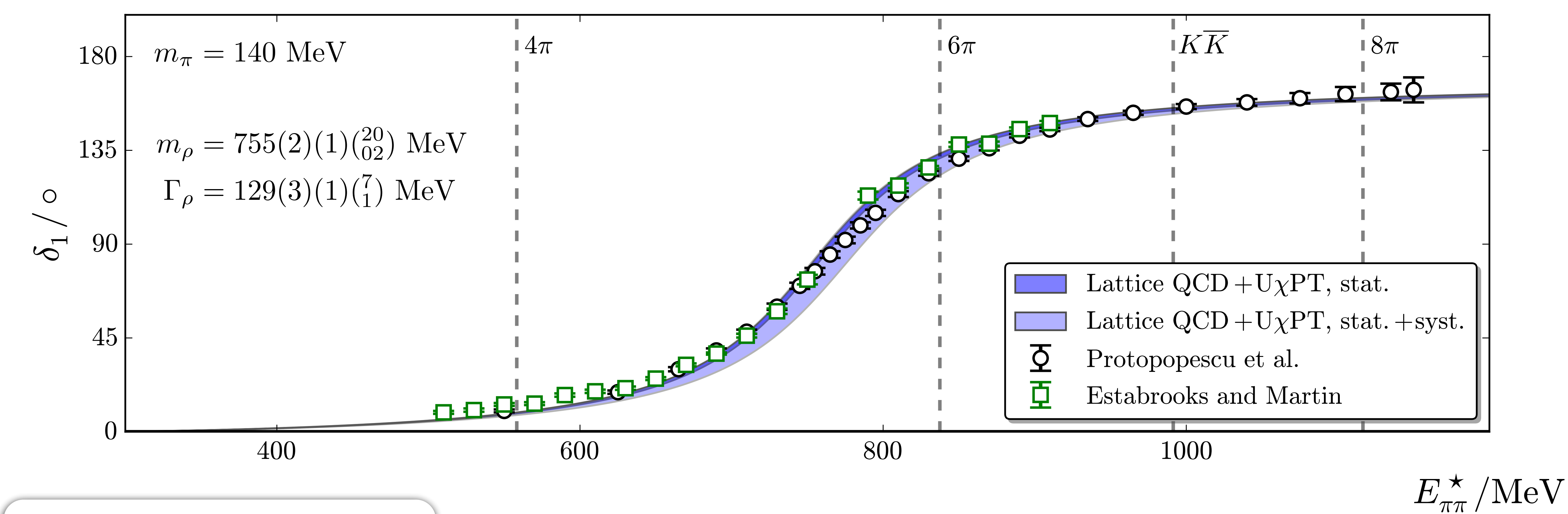
Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- m_π~400MeV
- Matrix element determined in **48** kinematic point: $(E_{\pi\pi}, Q^2)$



Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparing with experiment



Bolton, RB & Wilson (2015)

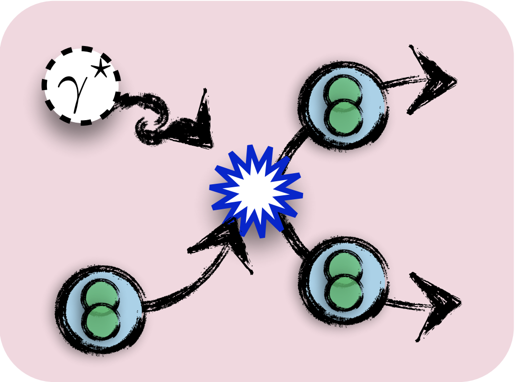
cute, but aren't experiments performed using m_π=140 MeV?

Extrapolation performed using Unitarized χPT

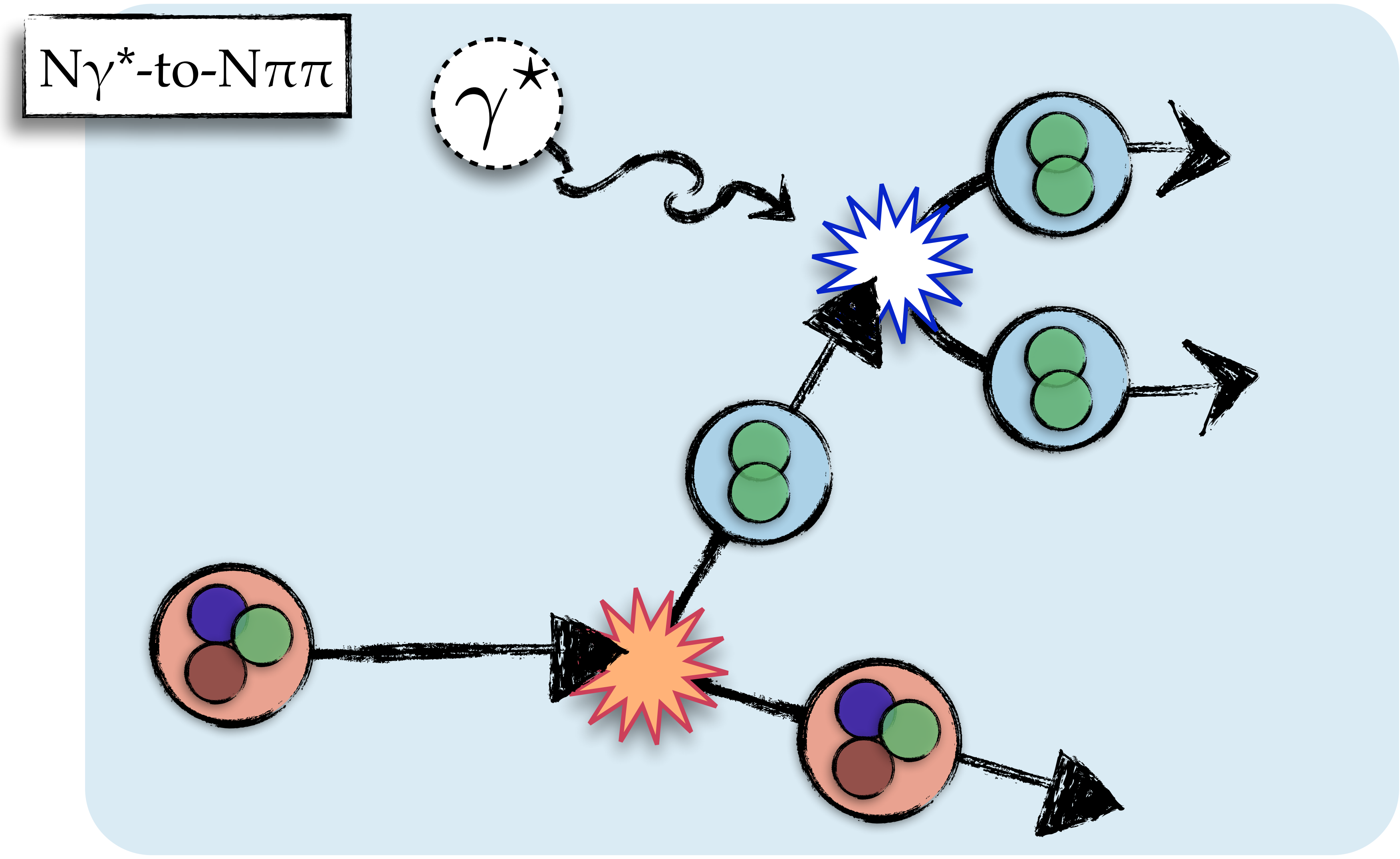
- Weinberg (1966)
- Gasser & Leutwyler (1983-85)
- Dobado and Pelaez (1997)
- Oller, Oset, and Pelaez (1998)

$\pi\gamma^*$-to-$\pi\pi$

(some more motivation)

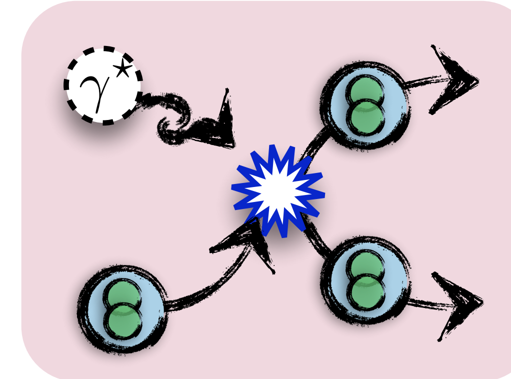


1. Building block of Nγ^*-to-N$\pi\pi$

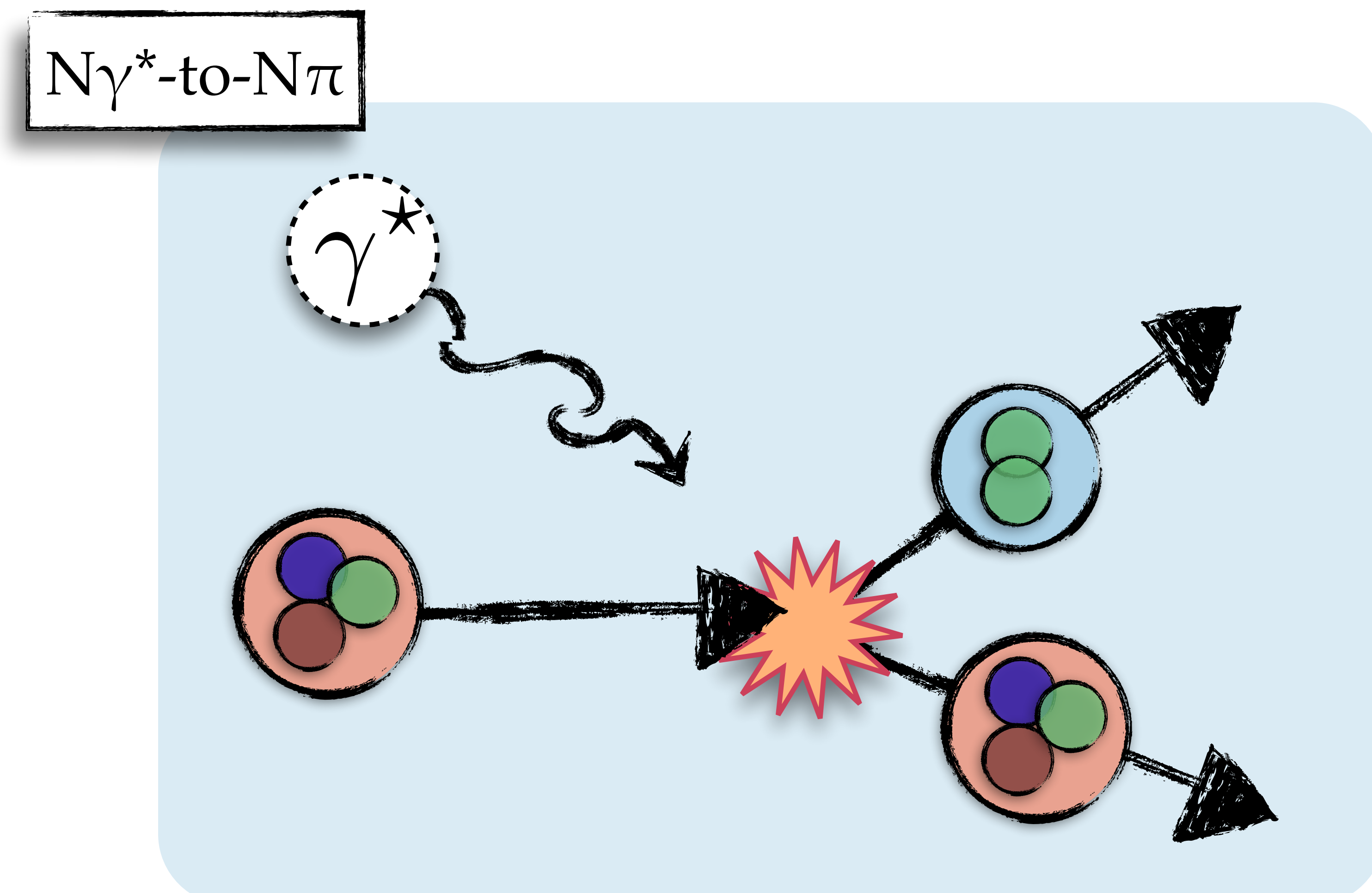


$\pi\gamma^*$-to-$\pi\pi$

(some more motivation)

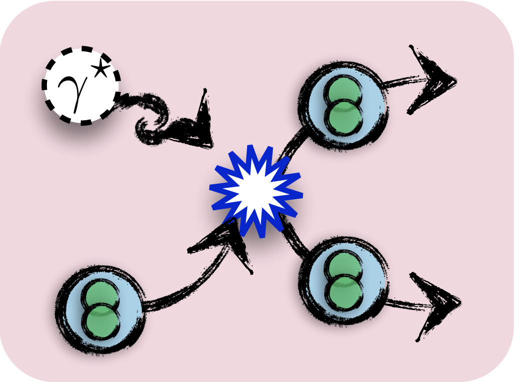


1. Building block of Nγ^*-to-N$\pi\pi$
2. Testing ground for more challenging processes



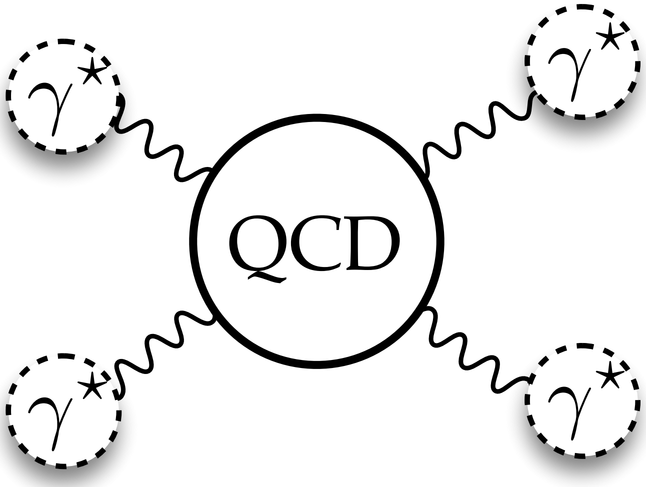
$\pi\gamma^*$-to-$\pi\pi$

(some more motivation)



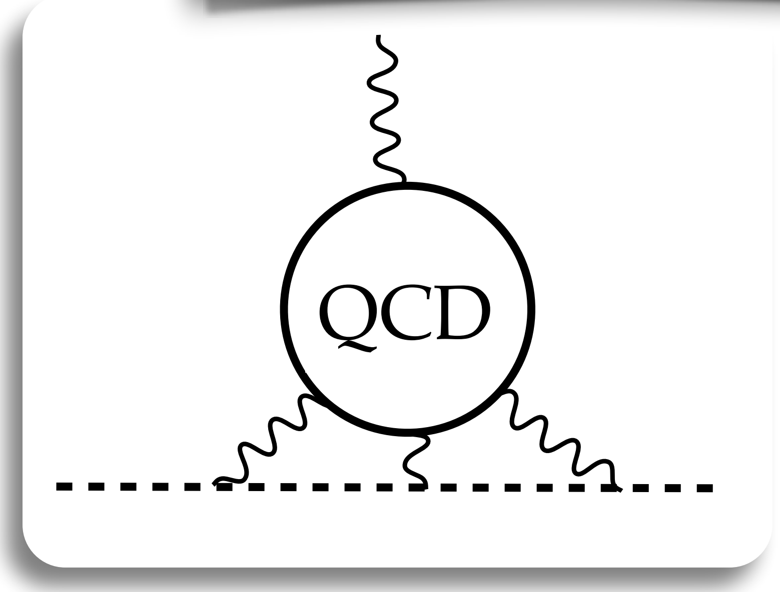
1. Building block of $N\gamma^*$-to-$N\pi$
2. Testing ground for more challenging processes
3. g_μ-2

Building blocks for hadronic light-by-light:



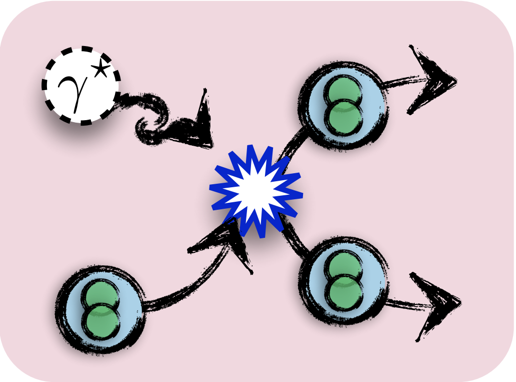
Muon anomalous magnetic moment: $a_\mu = \frac{g_\mu - 2}{2}$

light-by-light contribution to muon magnetic moment



$\pi\gamma^*$-to-$\pi\pi$

(some motivation)



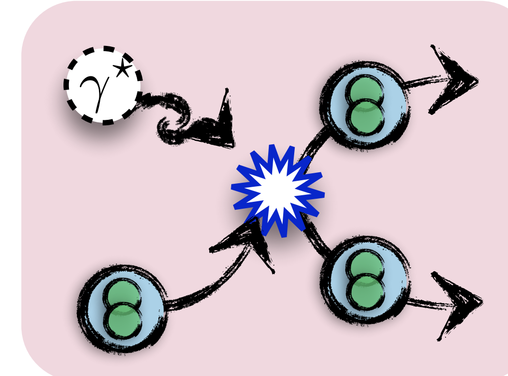
1. Building block of $N\gamma^*$-to-$N\pi$
2. Testing ground for more challenging processes
3. g_μ-2
4. ρ-to-$\pi\gamma^*$ decay
5. chiral anomaly

⋮

First resonating 1-to-2 calculation!

$\pi\gamma^*$-to-$\pi\pi$

(a sketch)



On the lattice we calculate: $_L\langle \pi; P_\pi | \mathcal{J}^\mu_{x=0} | \pi\pi; P_{\pi\pi} \rangle_L$

Electromagnetic current: $\mathcal{J}^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$

This can be *mapped* to : $\mathcal{H}^\mu_{\pi\pi,\pi\gamma^\star} = \langle \mathrm{out}; \pi, \mathrm{P}_\pi | \mathcal{J}^\mu_{\mathrm{x}=0} | \mathrm{in}; \pi\pi, \mathrm{P}_{\pi\pi}, \ell = 1 \rangle$

RB, Hansen & Walker-Loud (2014)

This gives us:

- energy-dependent π-to-ρ form factor
- $\pi\gamma^*$-to-$\pi\pi$ amplitude for arbitrary virtuality
- $\pi\gamma^*$-to-$\pi\pi$ cross section

} not independent

$\pi\gamma^*$-to-$\pi\pi$

(more details)

Lorentz decomposition:

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

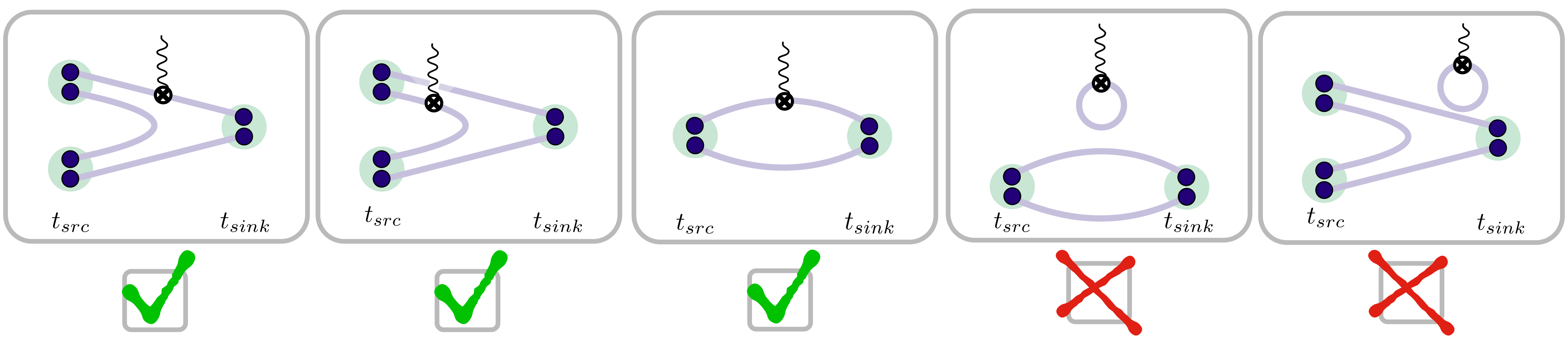
$\pi\pi/\rho$ polarization | *$\pi\pi/\rho$ helicity* | *Lorentz scalar*

Approximations:

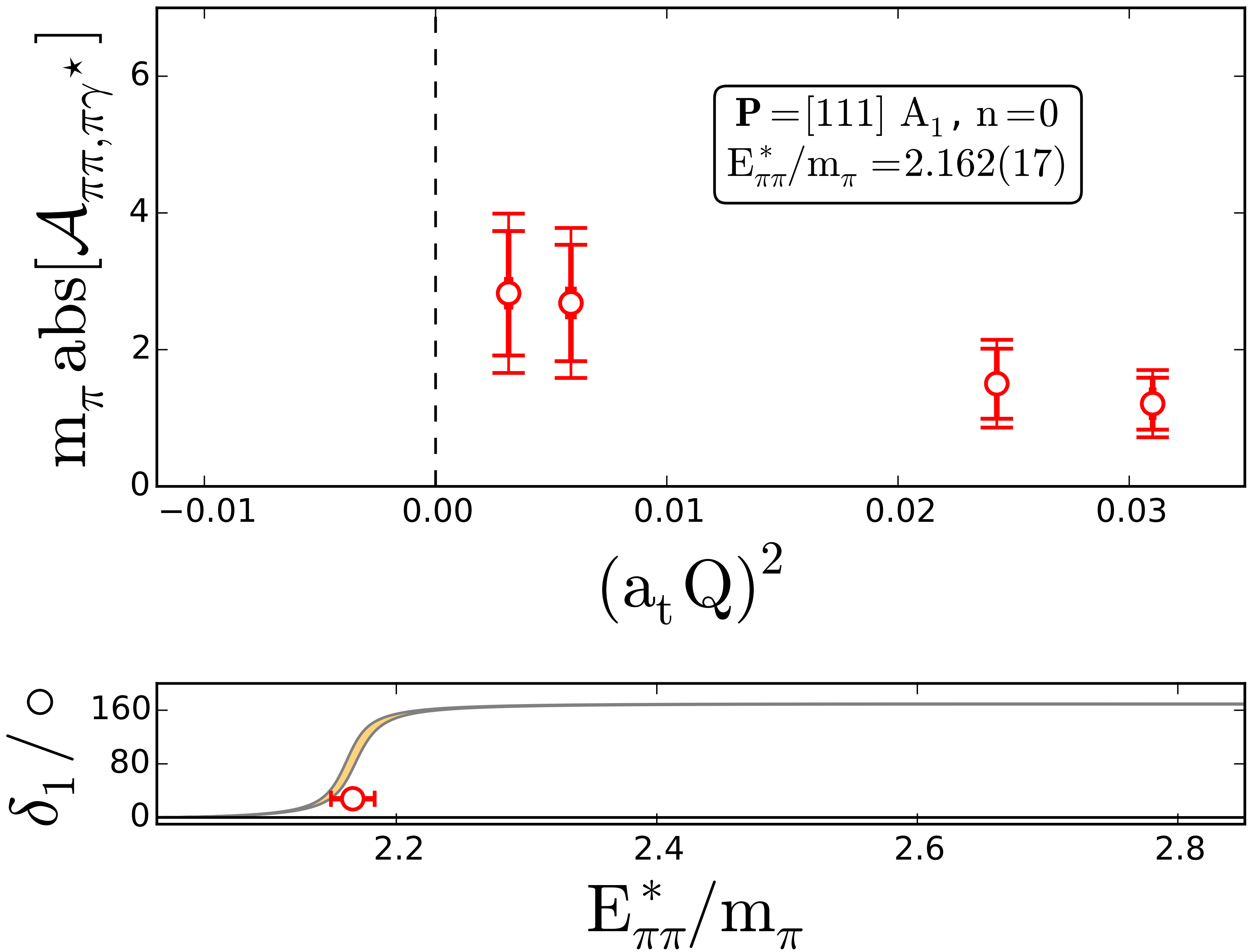
F-wave $\pi\gamma^*$-to-$\pi\pi$ is ignored

kinematically and dynamically suppressed

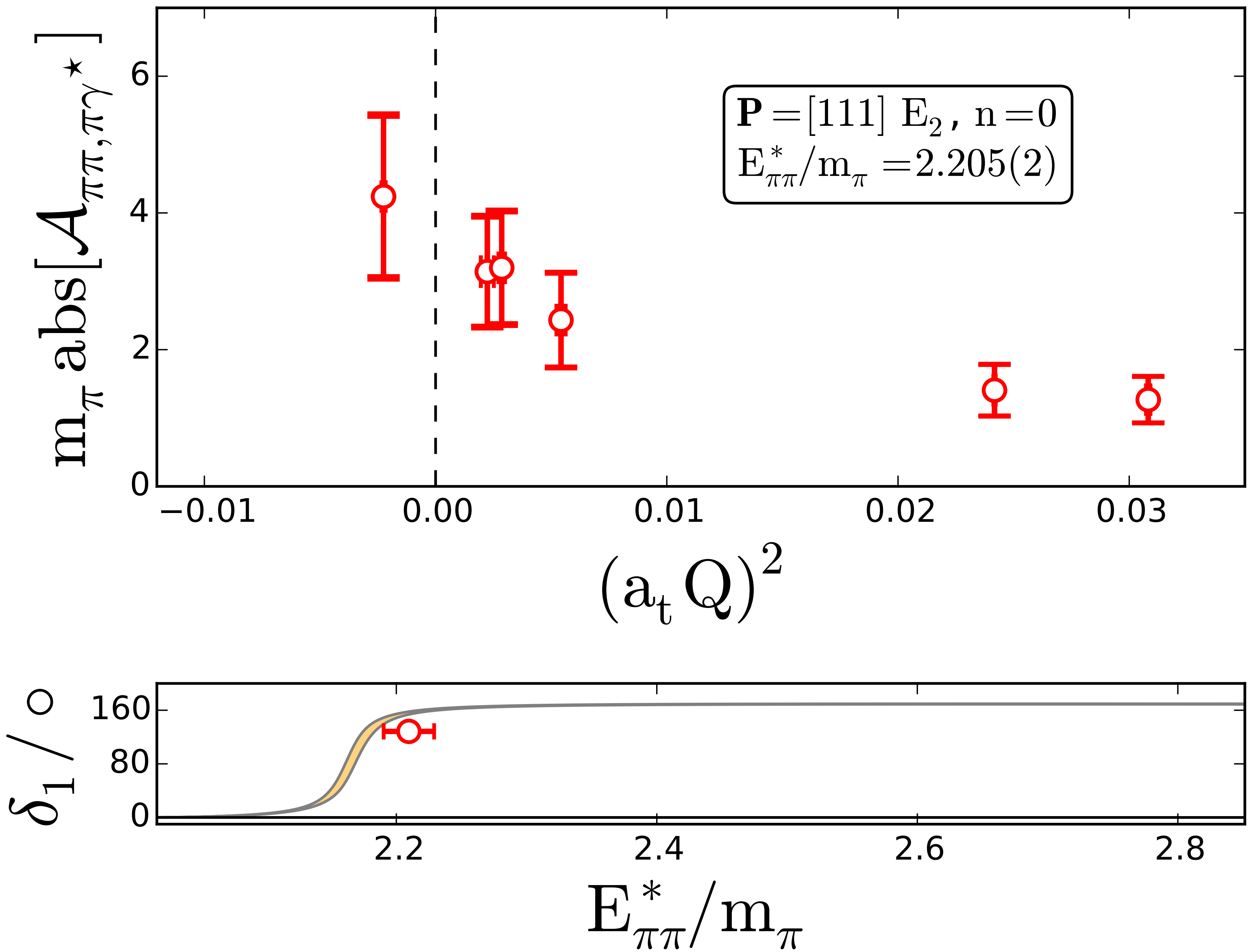
contractions:



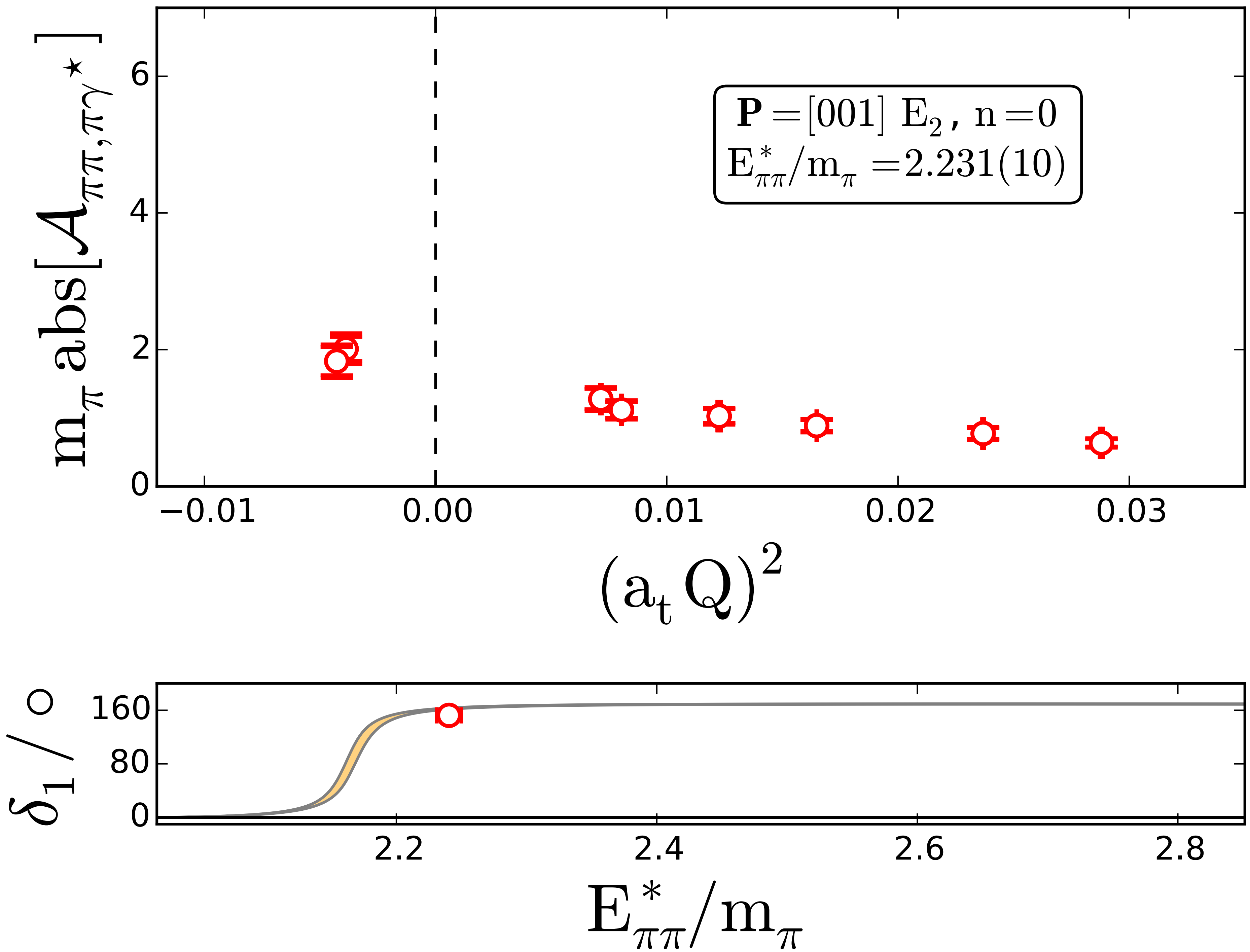
$\pi\gamma^*$-to-$\pi\pi$ amplitude



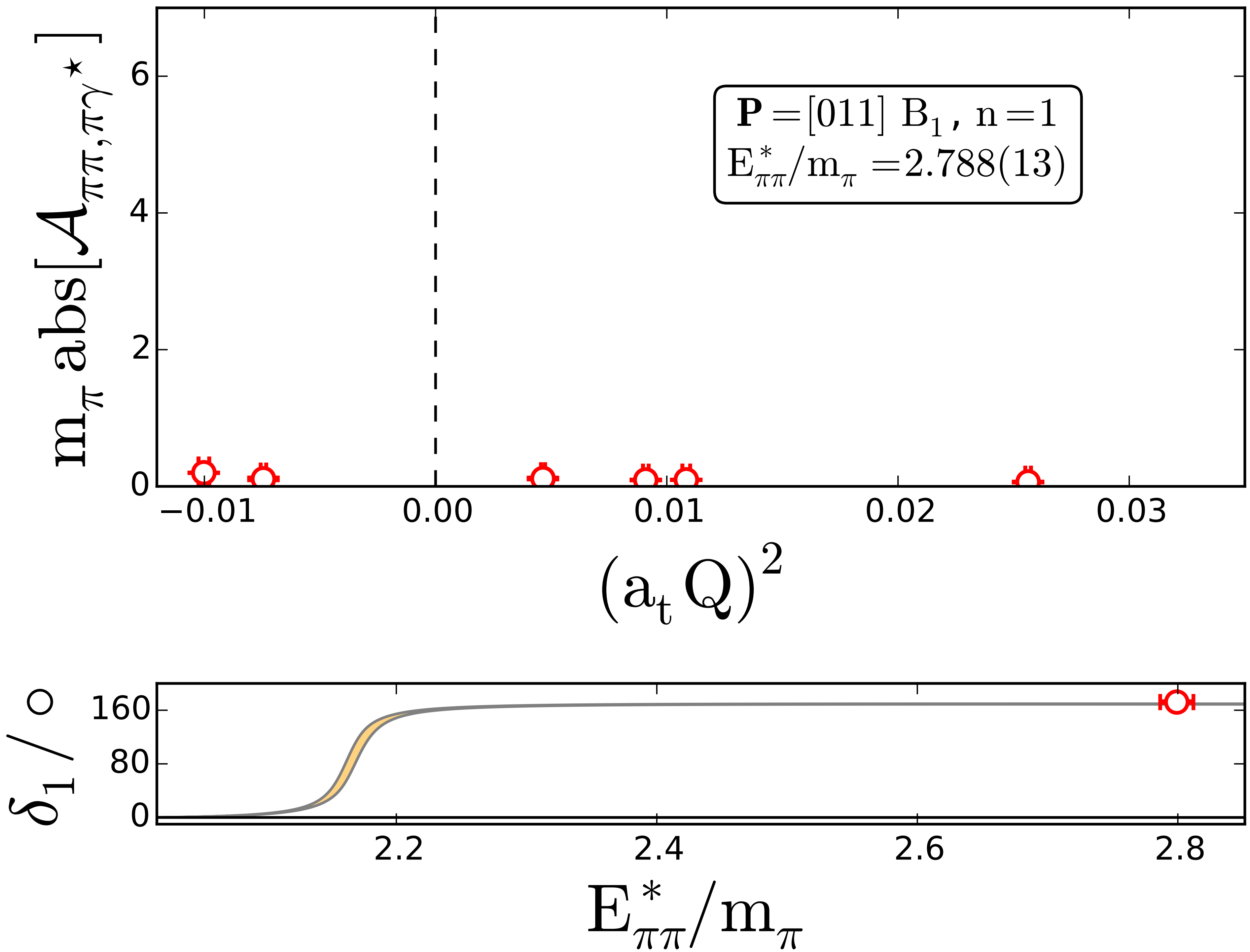
$\pi\gamma^*$-to-$\pi\pi$ amplitude



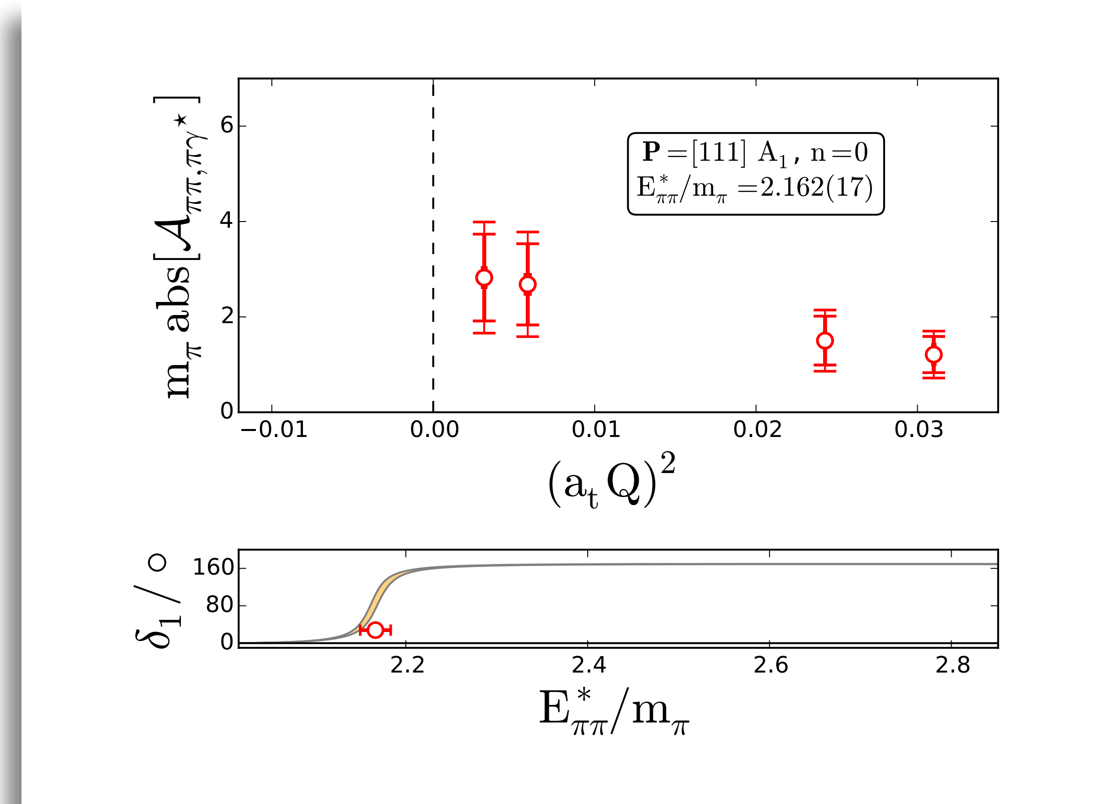
$\pi\gamma^*$-to-$\pi\pi$ amplitude

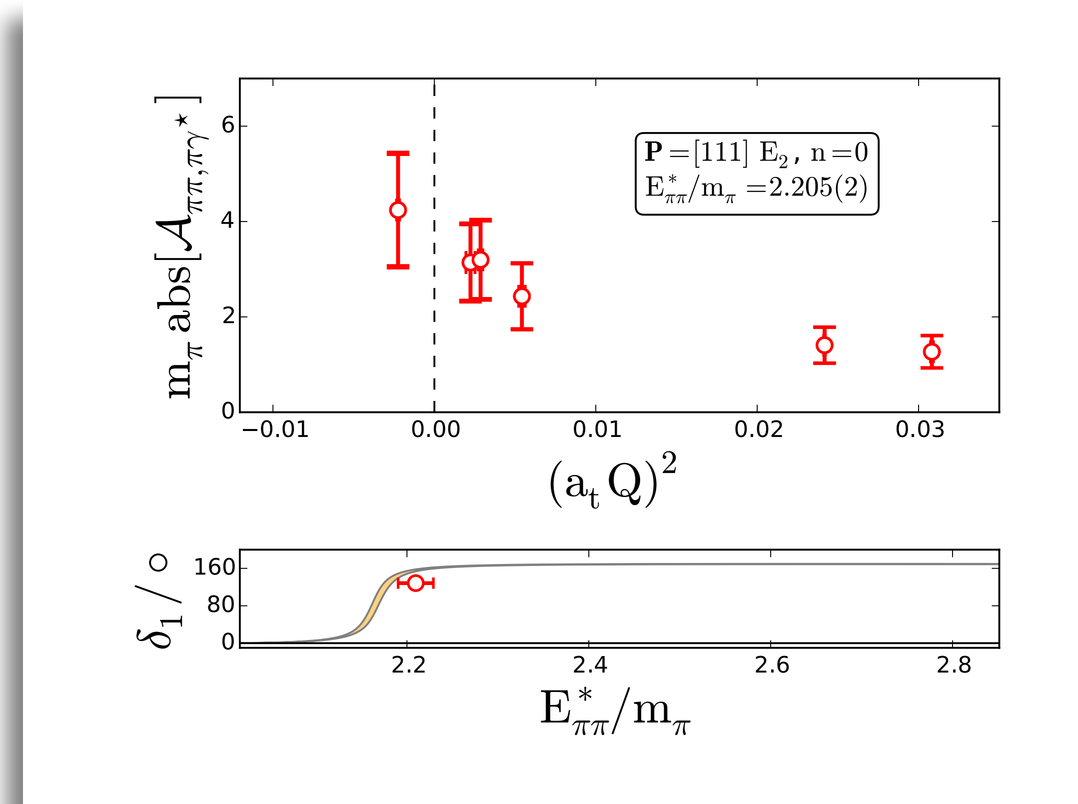


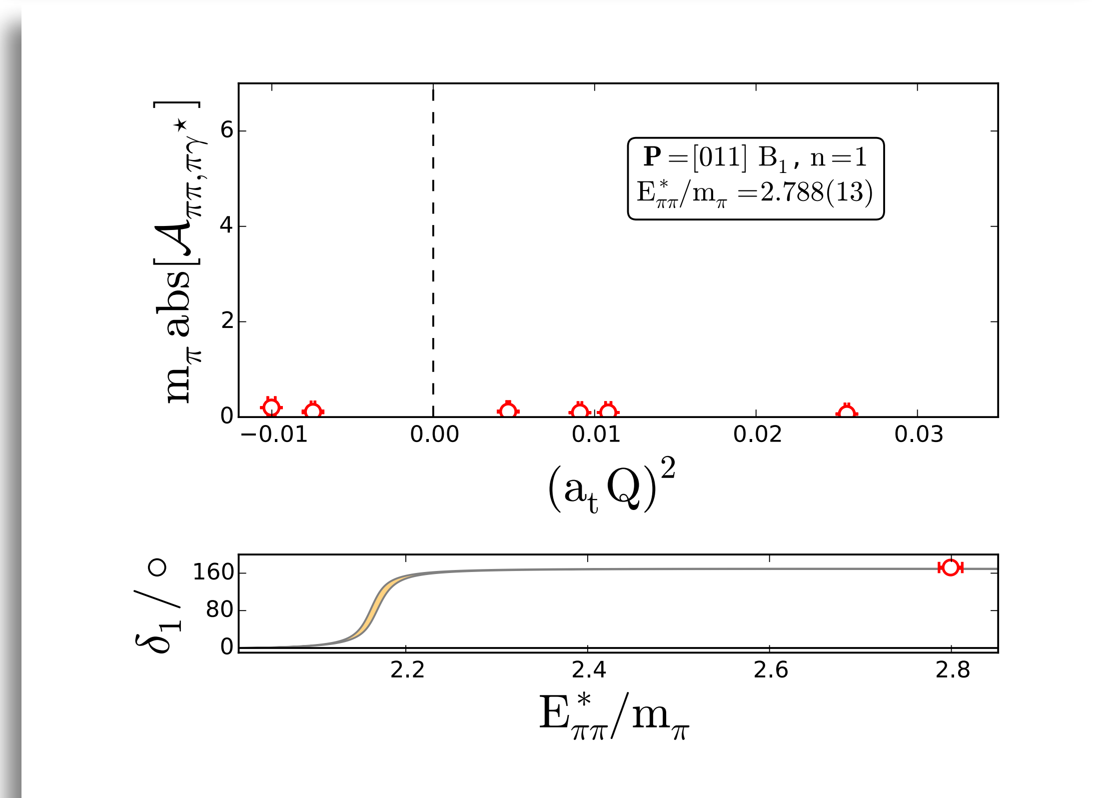
$\pi\gamma^*$-to-$\pi\pi$ amplitude

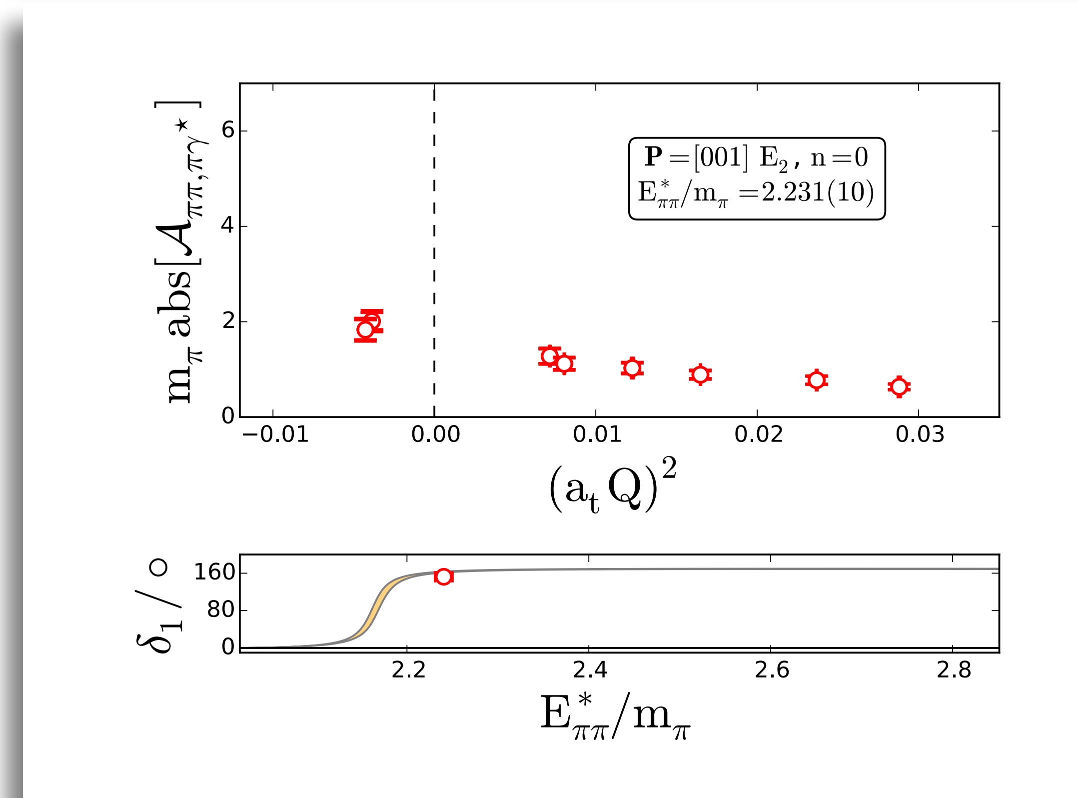


$\pi\gamma^*$-to-$\pi\pi$ amplitude









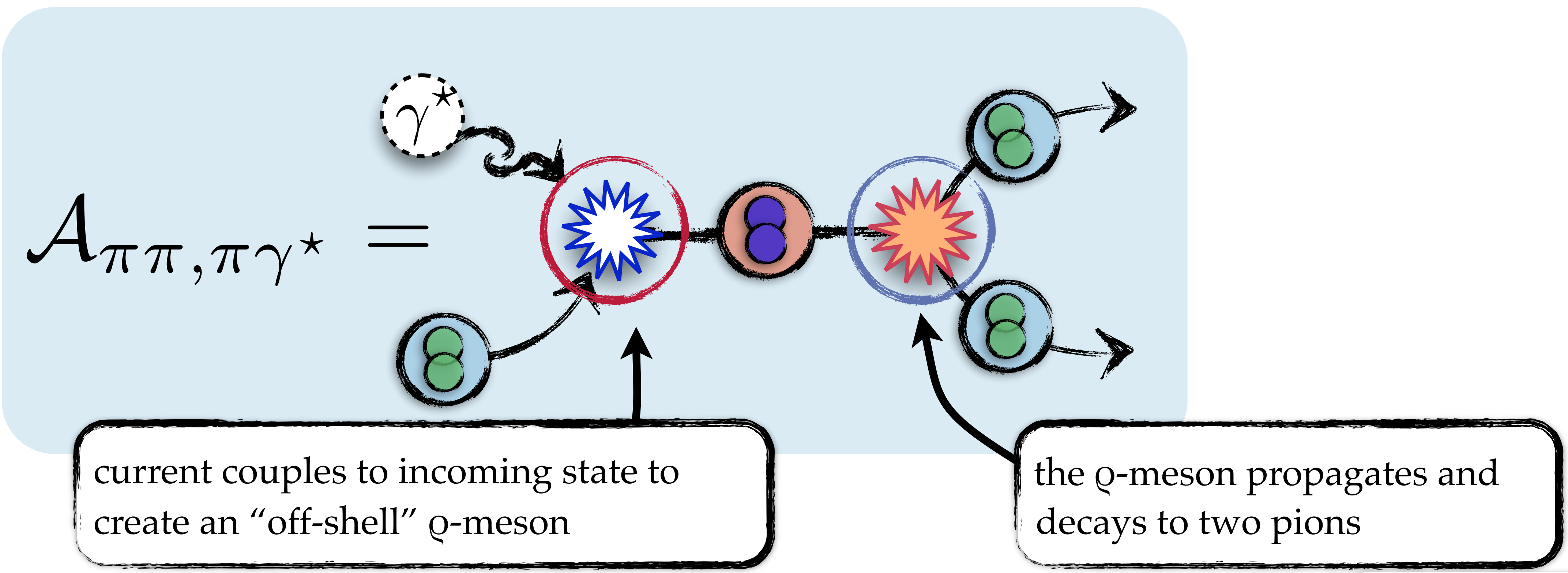
Amplitude vs. form factor

Amplitude near the resonance is dominated by $\pi\pi$ rescattering

Form factor definition:

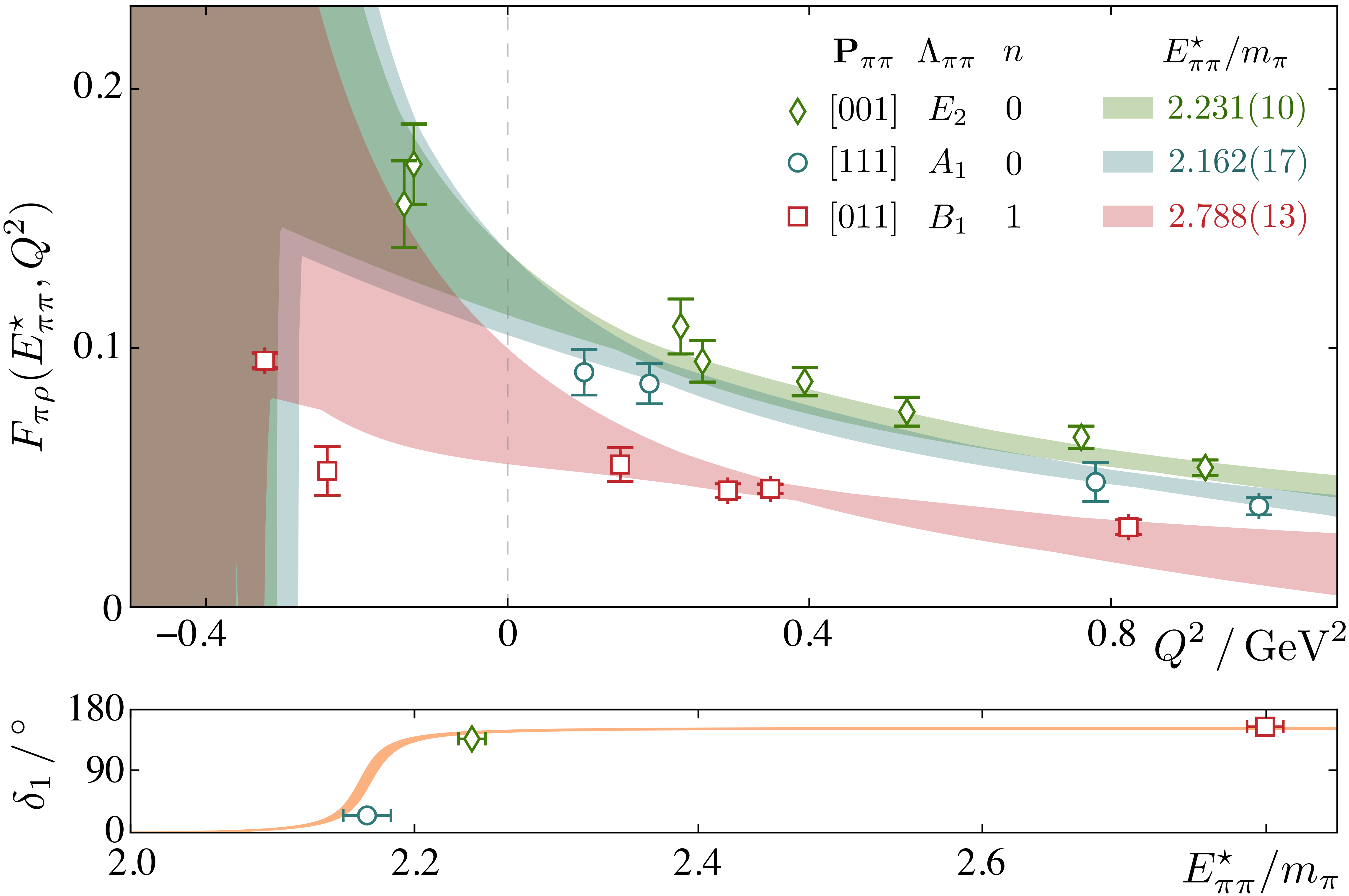
$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi}, Q^2) = F_{\pi\rho}(E^\star_{\pi\pi}, Q^2)\sqrt{\frac{8\pi}{q^\star_{\pi\pi}\,\Gamma_1(E^\star_{\pi\pi})}}\;\sin\delta_1(E^\star_{\pi\pi})\,e^{i\delta_1(E^\star_{\pi\pi})}$$

Intuitive picture:



RB & Hansen (2015)

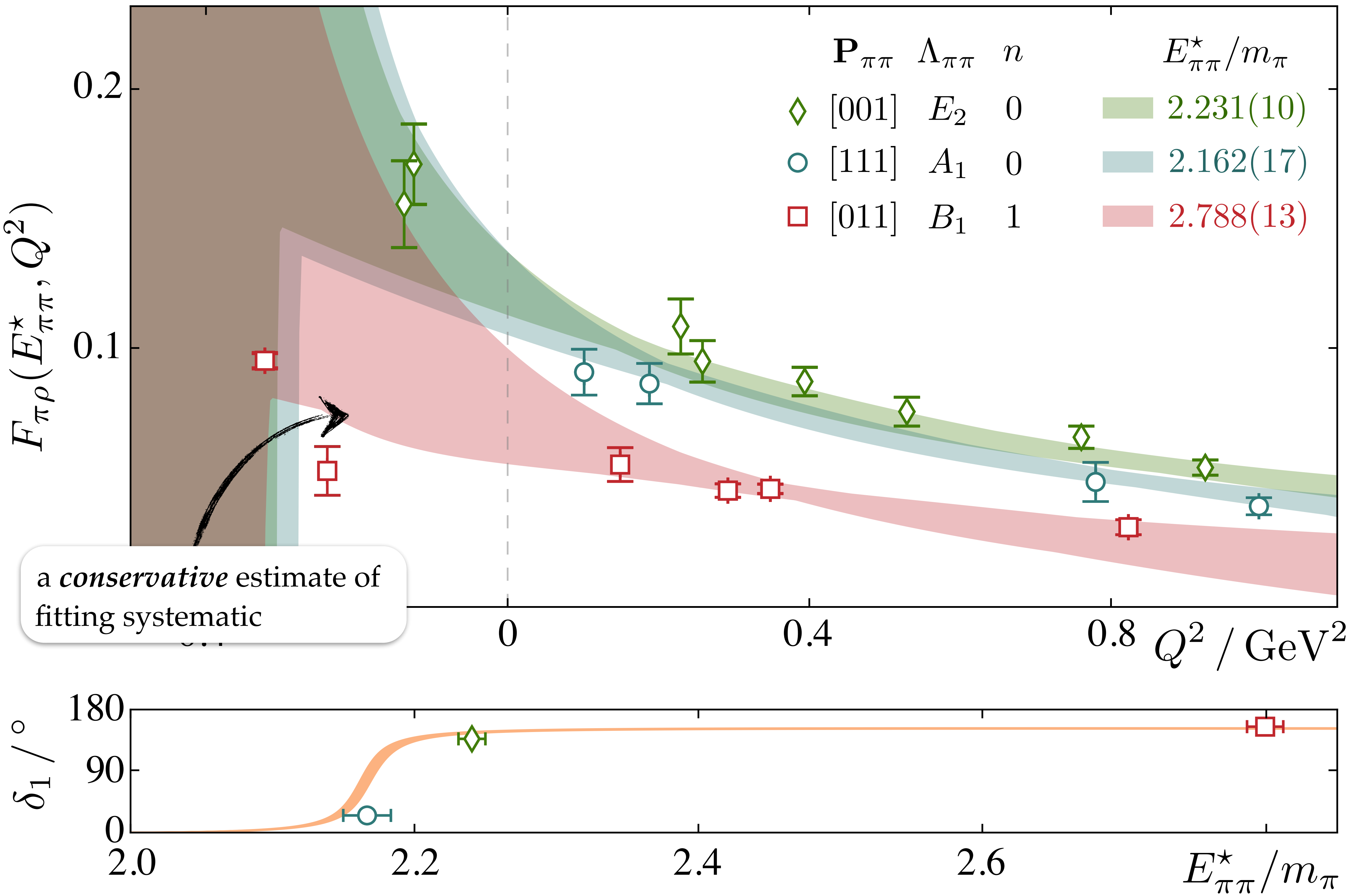
Energy-dependent form factor



RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

$\pi\pi$ cm energy

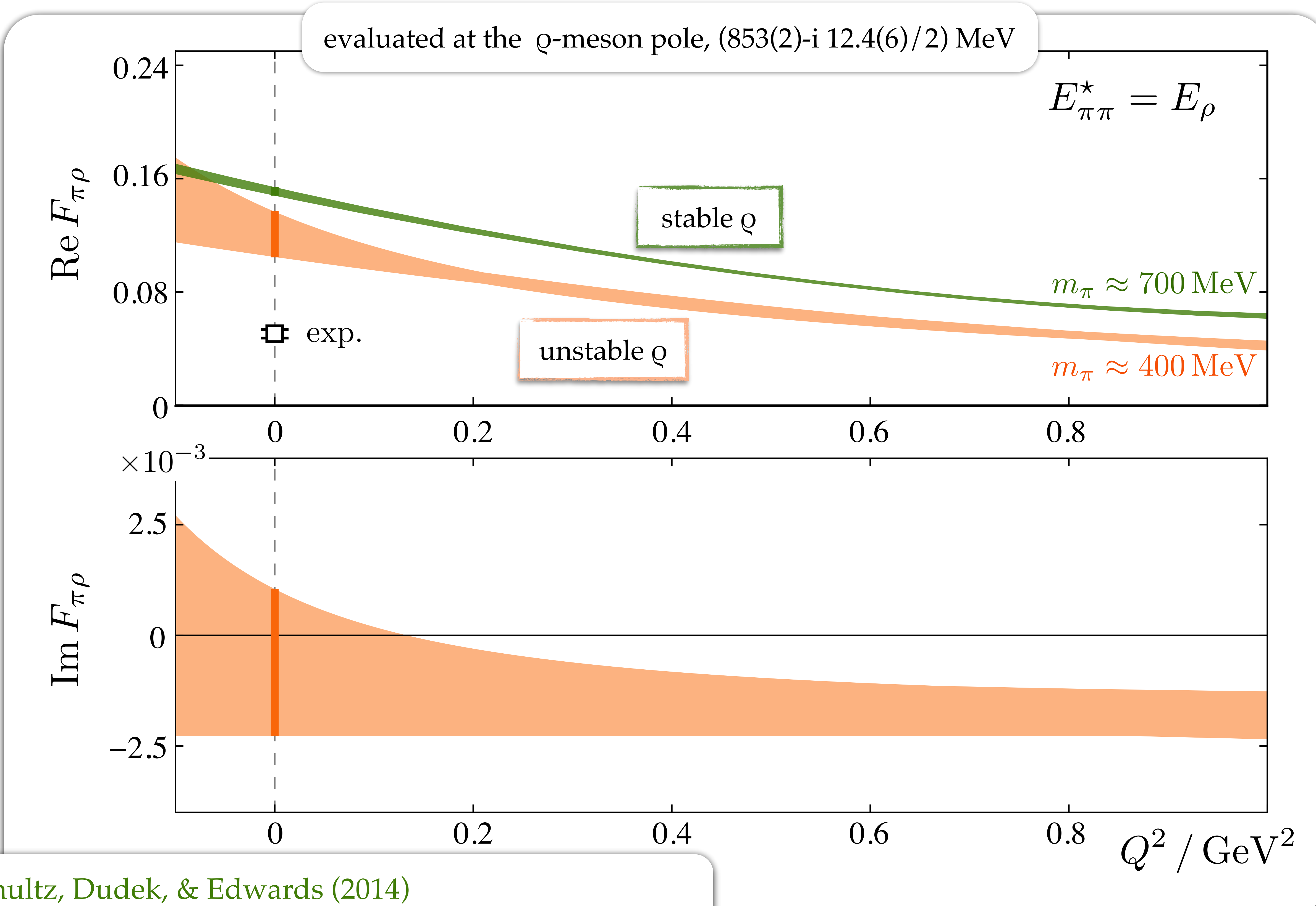
Energy-dependent form factor



ππ cm energy

RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

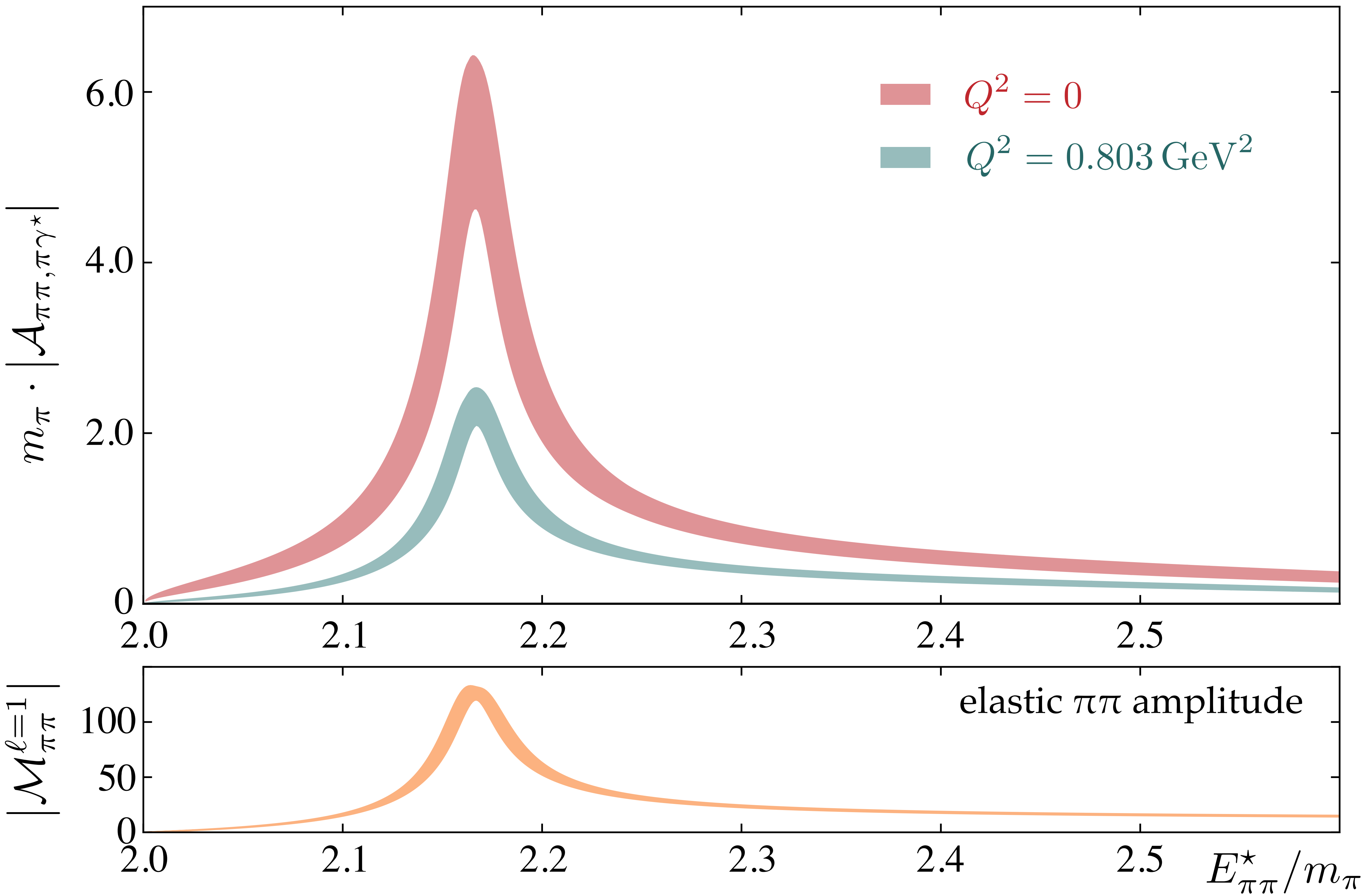
Form factor at ϱ pole



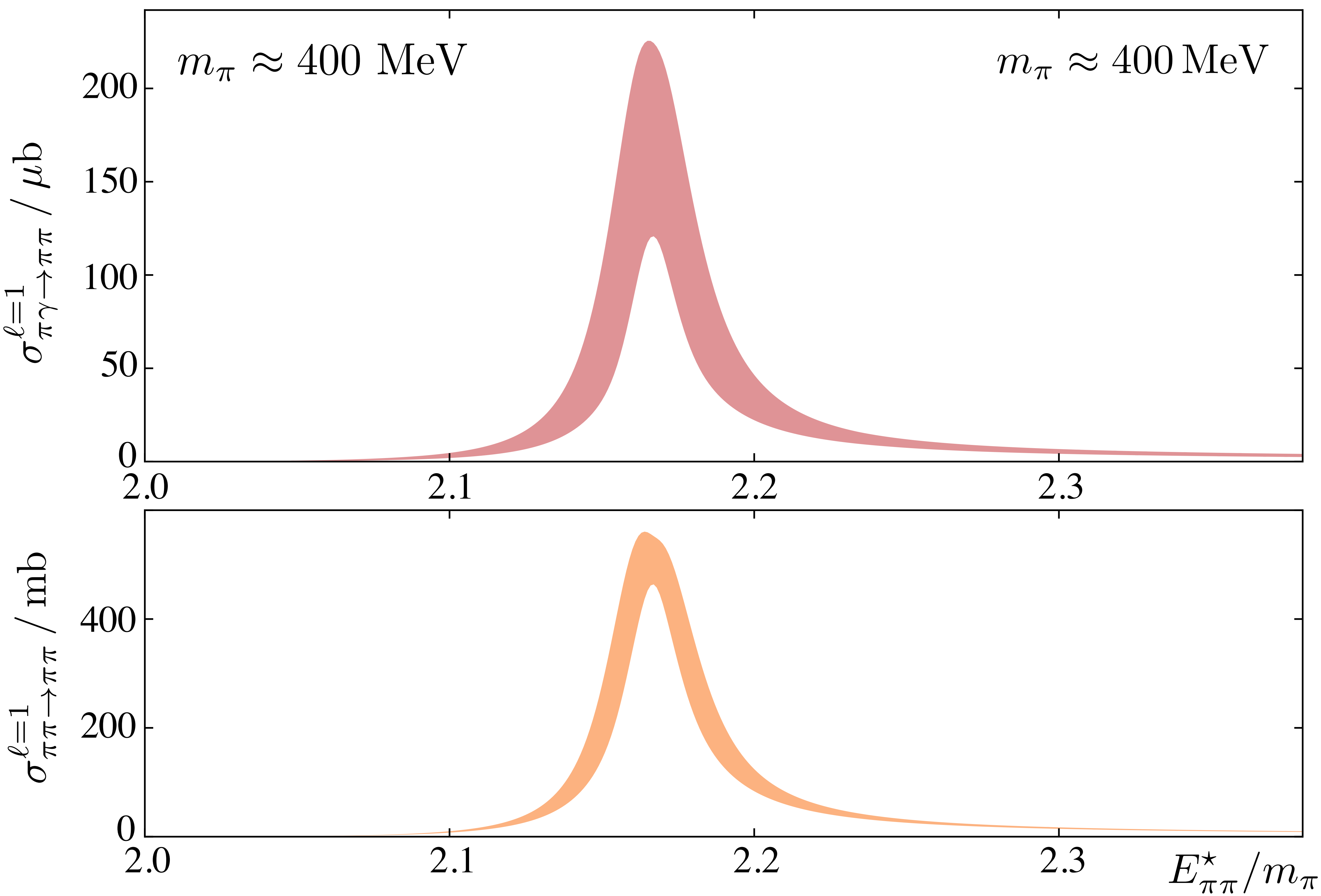
Shultz, Dudek, & Edwards (2014)

RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

$\pi\gamma$*-to-$\pi\pi$ amplitude



$\pi\gamma$-to-$\pi\pi$ cross section



Check list

- [x] Formalism
  (i.e., do we know what we need to study?)
- [x] Code development
  (i.e., can we perform said calculation?)
- [x] Implementation & analysis
  (i.e., what are you waiting for? do it!)

The *exotic* future

Outlook of the future
can sometimes be overly optimistic…

2015 as predicted in 1989

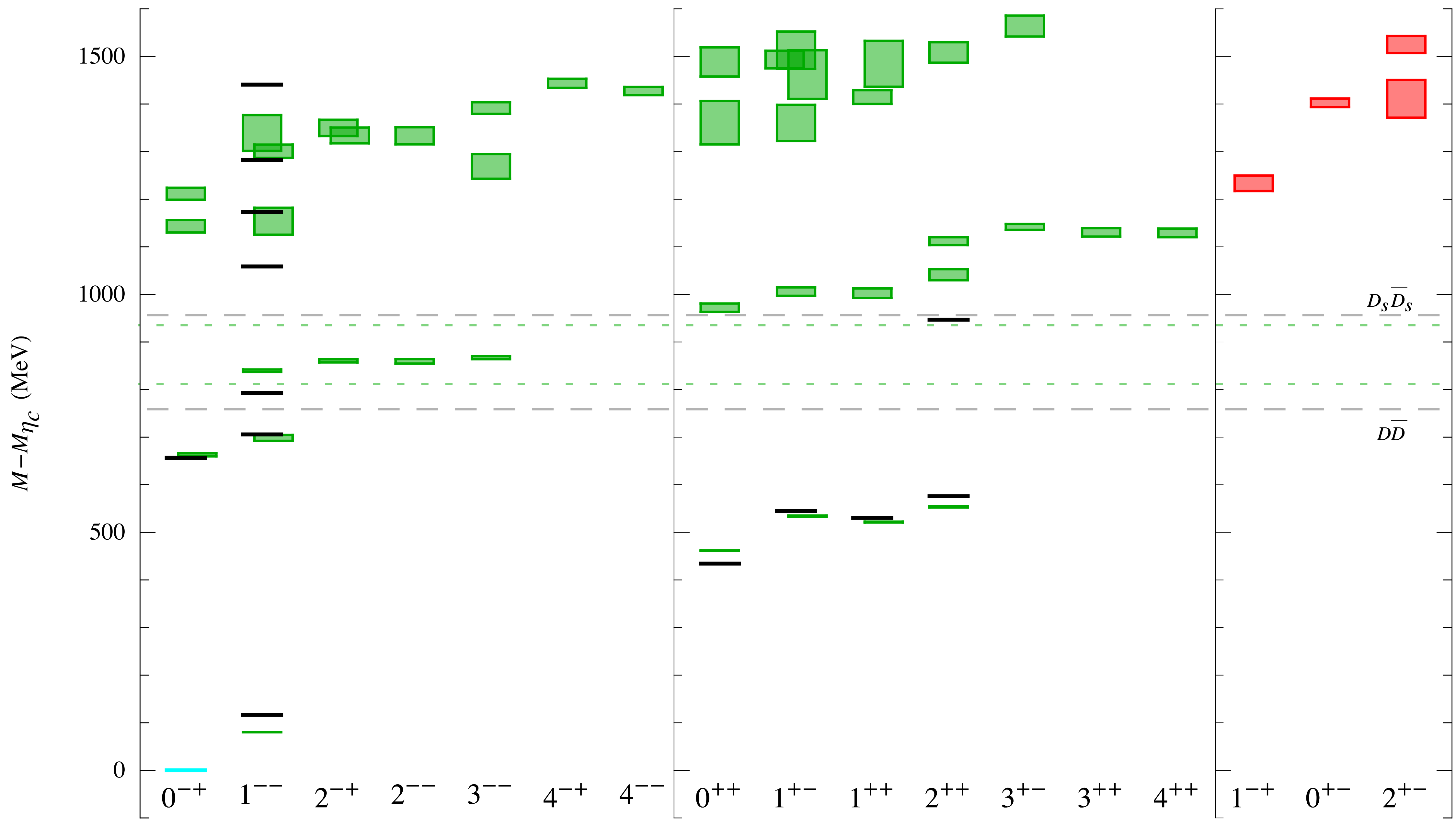
2015

…and at times just right.

The exotic frontier!

(in the light sector)

HadSpec



Liu, Moir, Peardon, Ryan, Thomas, Vilaseca, Dudek, Edwards, Joó, Richards (2012)

The exotic frontier!

(check list)

- Formalism

  (i.e., do we know what we need to study?)

- Code development

  (i.e., can we perform said calculation?)

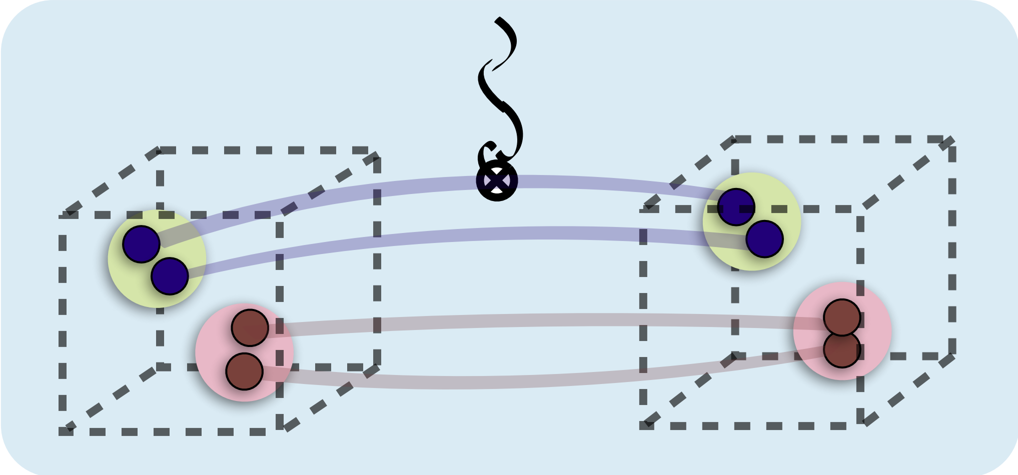
- Implementation & analysis

  (i.e., what are you waiting for? do it!)

The exotic frontier!

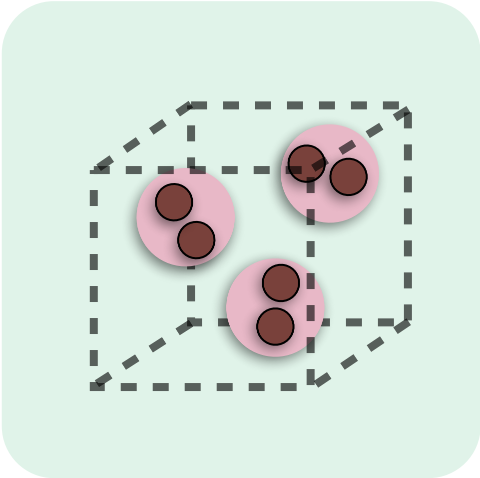
(back to the drawing board)

A relatively clear pathway forward:



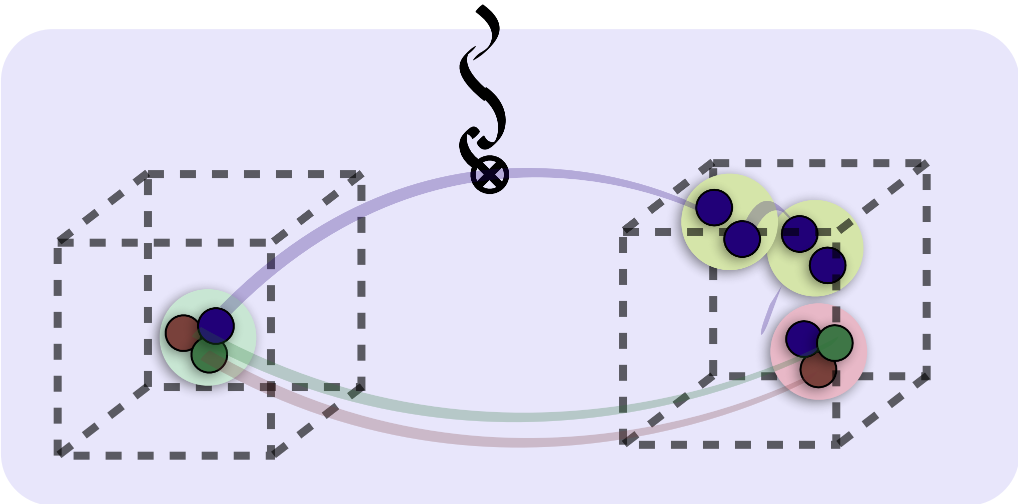
- 2-to-2 transitions:

RB & Hansen (2015)
Bernard, Hoja , Meissner & Rusetsky (2012)
RB & Davoudi (2012)



- 3/2 particles in a box:

Hansen & Sharpe (2014-2015)
RB & Davoudi (2013)
Polejaeva & Rusetsky (2012)



- 1-to-3, 2-to-3, 3-to-3 transitions:

In need of a thesis project?

Collaborators

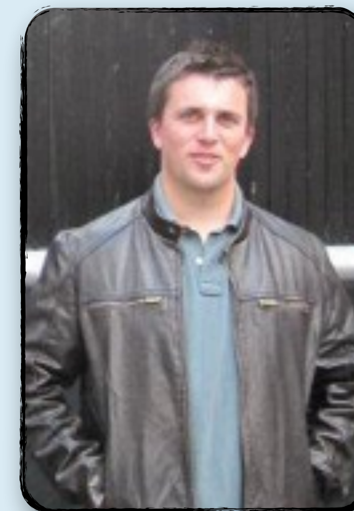
formalism


Hansen


Walker-Loud

LQCD calculations [HadSpec]


Wilson


Shultz


Thomas


Dudek


Edwards

RB, Hansen & Walker-Loud (PRD, 2014)
RB & Hansen (PRD accepted, Feb 2015)
RB & Hansen (arXiv, Sept 2015)

Wilson, RB, Dudek, Edwards & Thomas (PRD accepted, 2015)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (PRL accepted, 2015)