

Ab initio many-body calculations of single nucleon transfer reactions (Application to ${}^7\text{Li}(d,p){}^8\text{Li}$ reaction)

Reactions and structure of exotic nuclei
March 2 – March 13, 2015

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In collaboration with:

G. Hupin, P. Navrátil, S. Quaglioni

INT Workshop
March 3, 2015
Seattle, US

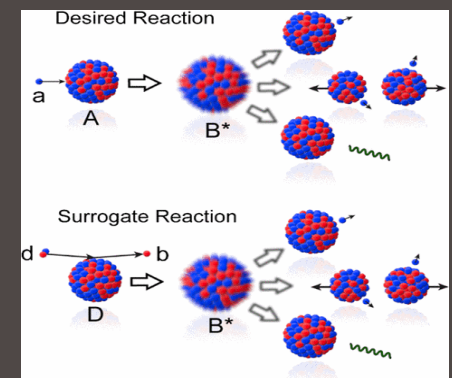
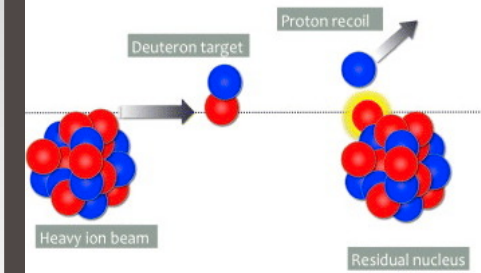


Figure 1 from Kate L Jones 2013 Phys. Scr. 2013 014020

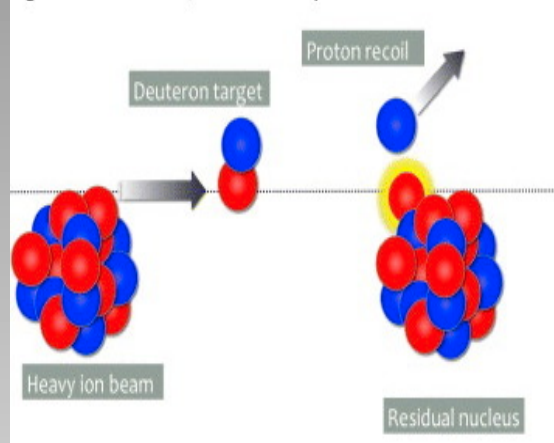


Outline

- ✓ Motivations for *ab initio* study of transfer reactions
- ✓ Interest of ${}^7\text{Li}(d,p){}^8\text{Li}$ reaction
- ✓ Results on the first resonance of ${}^9\text{Be}$ above d - ${}^7\text{Li}$ threshold
- ✓ Recent progress on inclusion of 3N forces in transfer reaction (s-shell nuclei)
(on behalf of: G. Hupin, P. Navrátil, S. Quaglioni)
- ✓ Conclusions

Deuteron-nucleus reaction: experimental motivations

Figure 1 from Kate L. Jones 2013 Phys. Scr. 2013 014020



(*d,p*) reaction in
inverse kinematics

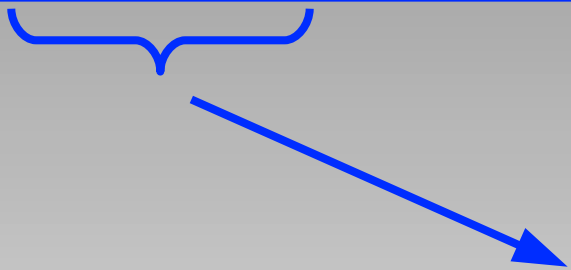
Intense experimental activity (now with exotic nuclei beams in RIB facilities):

- Structure and spectroscopy of nuclei ($^{11}\text{Be}(d,p)^{12}\text{Be}$ at ISAC - TRIUMF)
- Nucleosynthesis and nuclear fusion applications ($^3\text{H}(d,n)^4\text{He}$ reaction)
- Surrogate for (p/n) capture reactions ($^{14}\text{C}(d,p)^{15}\text{C}$ as surrogate of $^{14}\text{C}(n,\gamma)^{15}\text{C}$)

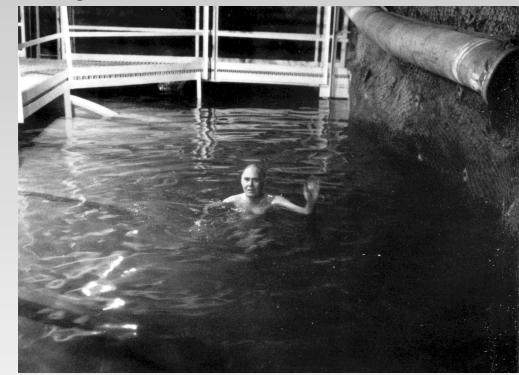
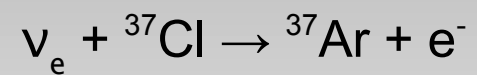
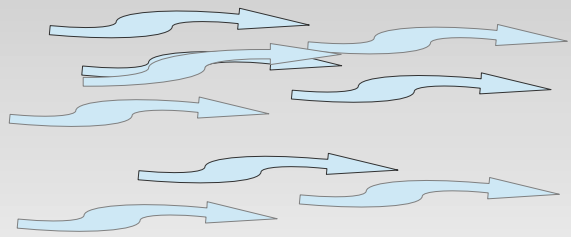
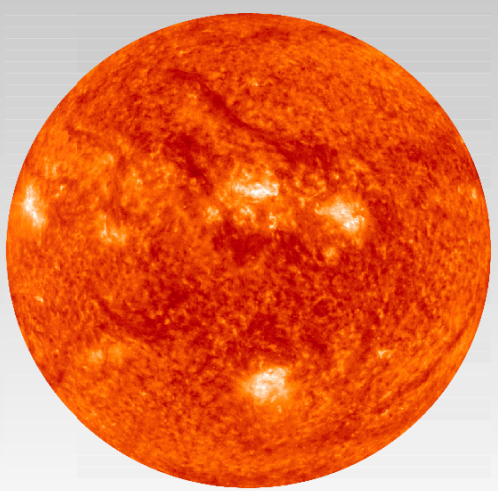
${}^7\text{Li}(d,p){}^8\text{Li}$ transfer reaction



Calibration reaction for astrophysical process: ${}^7\text{Li}(d,p){}^8\text{Li}$ as target calibration for ${}^7\text{Be}(p,\gamma){}^8\text{B}$



Solar neutrino problem:

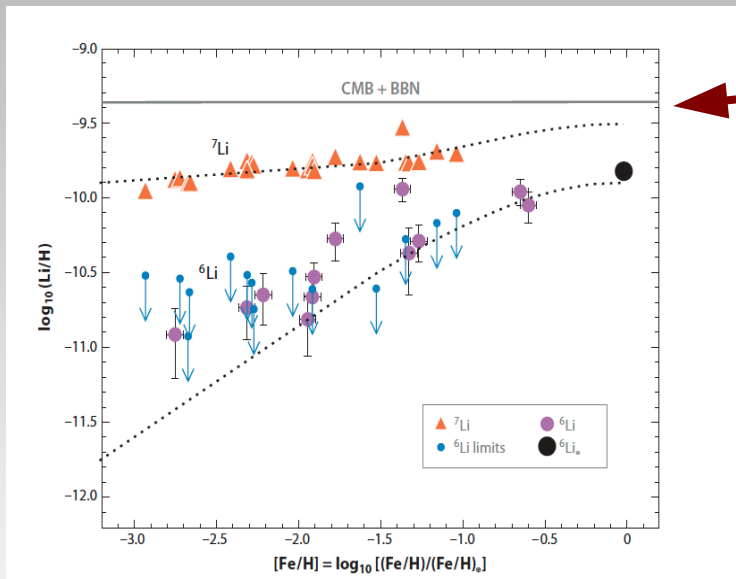


R. Davis Jr takes a dip
At Homestake Mine (1971)

${}^7\text{Li}(d,p){}^8\text{Li}$ transfer reaction

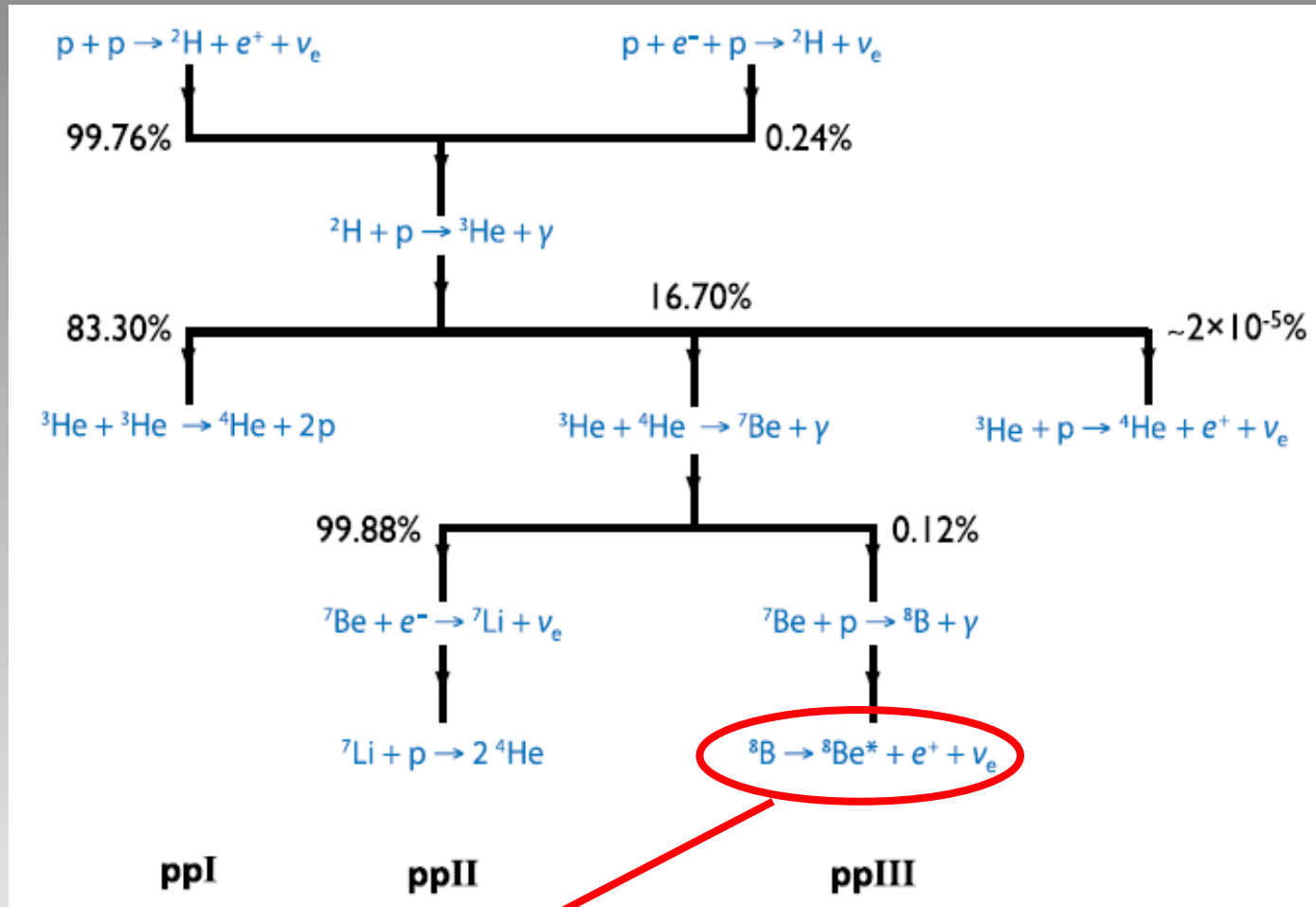
◆ Calibration reaction for astrophysical process: ${}^7\text{Li}(d,p){}^8\text{Li}$ as target calibration for ${}^7\text{Be}(p,\gamma){}^8\text{B}$

◆ Possible mechanism of destruction of ${}^7\text{Li}$ in the context of baryon-inhomogeneous models of the primordial nucleosynthesis



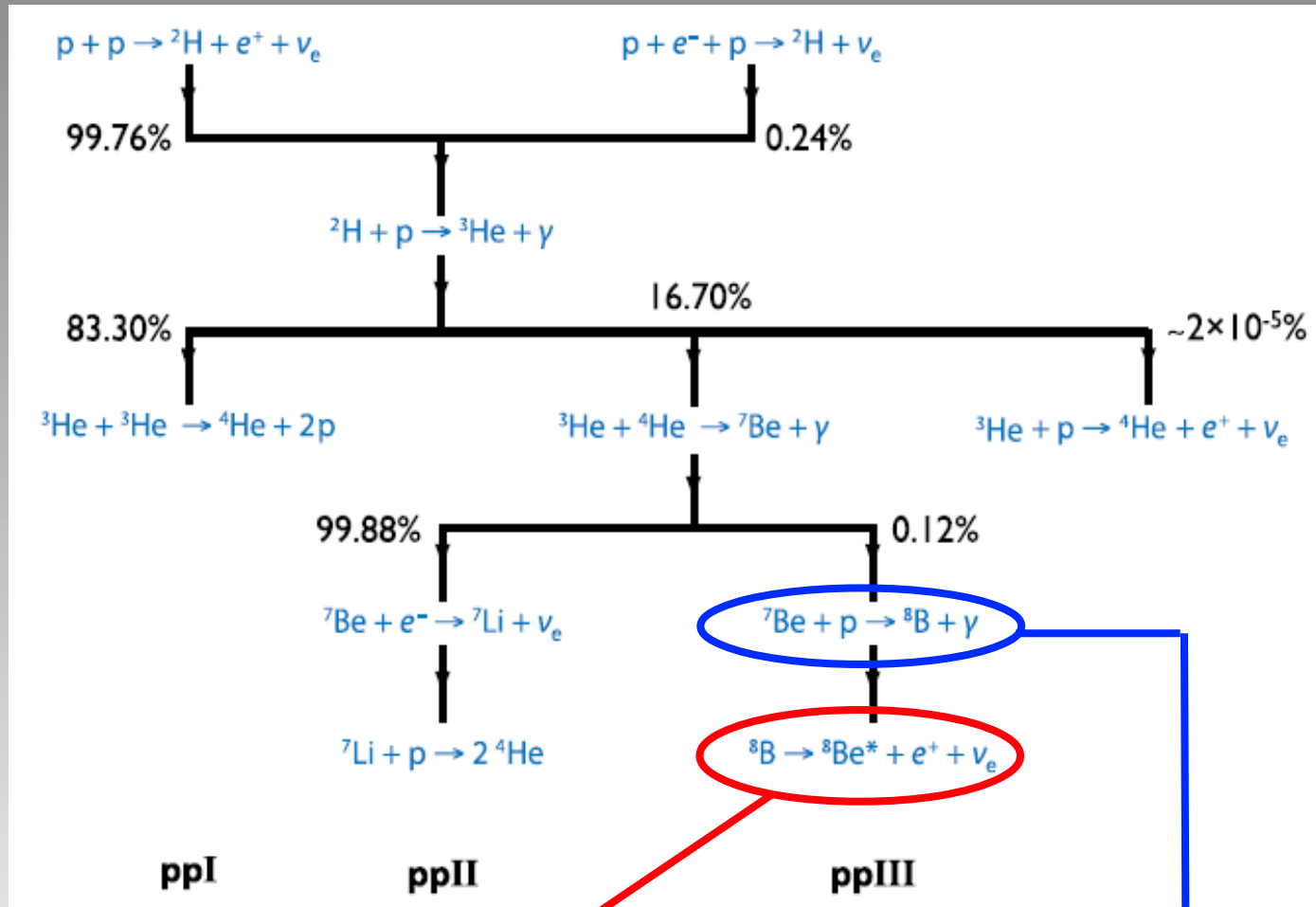
- Primordial Lithium problem:
- 4-5 σ discrepancy between observed and calculated (CMB+BBN) abundance of ${}^7\text{Li}$
 - Nuclear solution to the problem: d - ${}^7\text{Li}$ destruction mechanism is ruled out (but only in a standard BBN scenario PRC 47, 2369 1993)

Solar p-p chain



High-energy solar neutrinos
(E_ν up to 15 MeV)

Solar p-p chain



$S_{17}(0)$ crucial input to estimate solar neutrino flux

High-energy solar neutrinos “Prudent conservative range” $S_{17}(0) = 20.8(2.1) \text{ eV b}$
 (E_ν up to 15 MeV) (Rev Mod Phys 83, 1 (2011))

${}^7\text{Be}(p,\gamma){}^8\text{B}$ direct measurements

Solar fusion cross sections (Rev Mod Phys 70, 4, 1265 (1998))

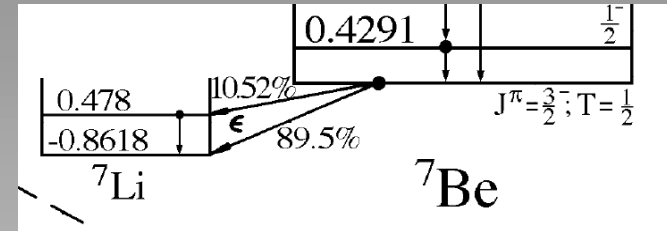
- At low energy, it is an external, direct-capture process ($Q=137.5 \pm 1.2$ KeV)
- Experimental uncertainties:
 - Need to be extrapolated to astrophysical relevant regime ($E_p < 134$ KeV)
 - ${}^7\text{Be}$ target is radioactive: Areal density of ${}^7\text{Be}$ difficult to estimate (required for normalization of the cross section)
- Two possible strategies:
 - Direct: counting 478 KeV photons of ${}^7\text{Li}^*$ after ${}^7\text{Be}$ decay (Filippone 1983)
 - Indirect: measuring the ${}^7\text{Li}(d,p){}^8\text{Li}$ yield on the resonance ($\Gamma \approx 0.2$ MeV) at $E_d = 0.78$ MeV) (Kavanagh 1960, Parker 1966, Filippone 1982, Weissman 1998)

${}^7\text{Li}(d,p){}^8\text{Li}$ calibration reaction (Normalization procedure)

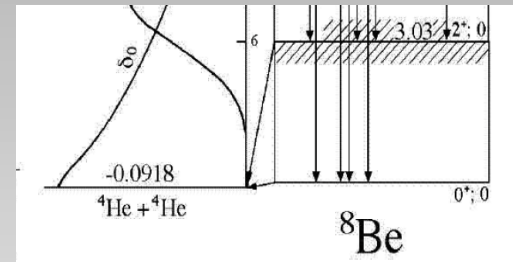
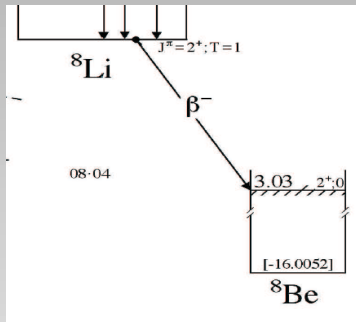
1) Buildup of ${}^7\text{Li}$ in ${}^7\text{Be}$

$$N_{7\text{Li}}(t) = N_{7\text{Be}}(0)(1 - e^{-\lambda t}) + N_{7\text{Li}}(0)$$

N_i areal density at time t



2) Counting ${}^7\text{Li}$ through the yield of (d,p) reaction



Beta-delayed
alphas detected

3) Areal density of Li from total σ

$$\sigma(E_d) \sim \frac{Y_\alpha(E)\beta({}^8\text{Li})}{N_d N_{7\text{Li}}(t)}$$

$Y_\alpha(E)$ α -particle yield

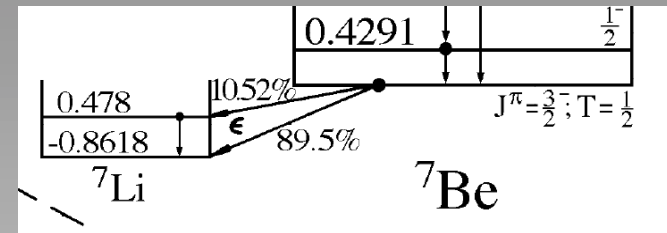
$\beta({}^8\text{Li})$ depends on decay constant of ${}^8\text{Li}$

${}^7\text{Li}(d,p){}^8\text{Li}$ calibration reaction (Normalization procedure)

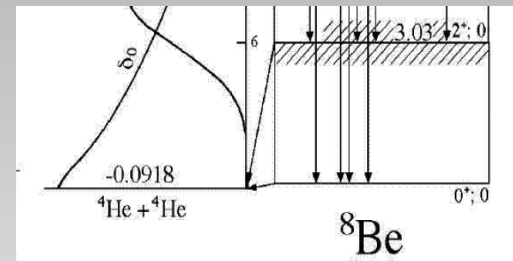
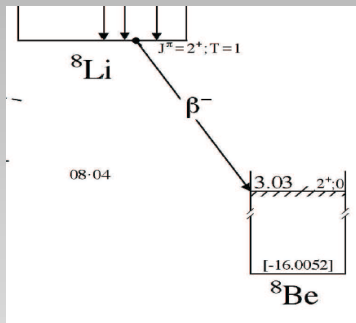
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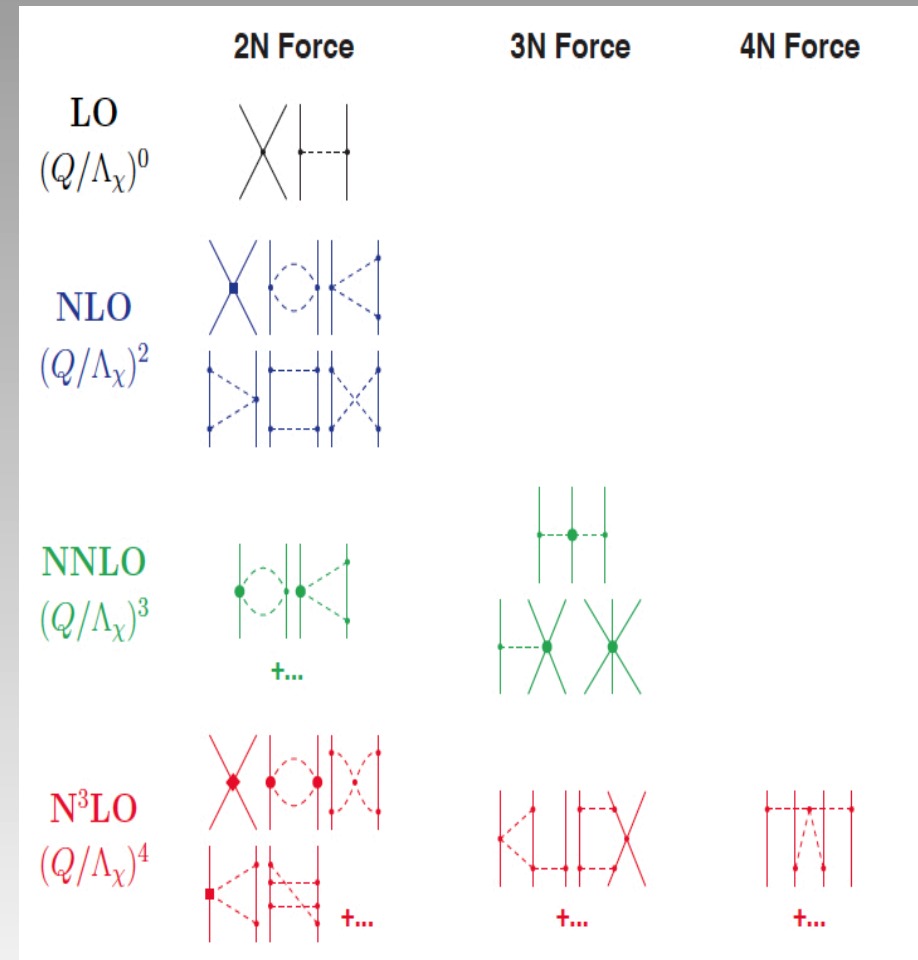
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Ab initio calculation of transfer reactions: theoretical motivations

Transfer reactions from
first principles:

- Inter-nucleon forces from chiral effective field theory:
 - 1) Nucleons and pions as degrees of freedom
 - 2) Based on QCD symmetries
 - 3) Organized as low-momentum expansion
 - 4) Hierarchy and consistency
 - 5) Low energy constants fitted to 2- and 3-body systems
 - 6) Evolution with SRG methods

Entem-Machleidt NN force at N³LO
(cutoff 500 MeV)



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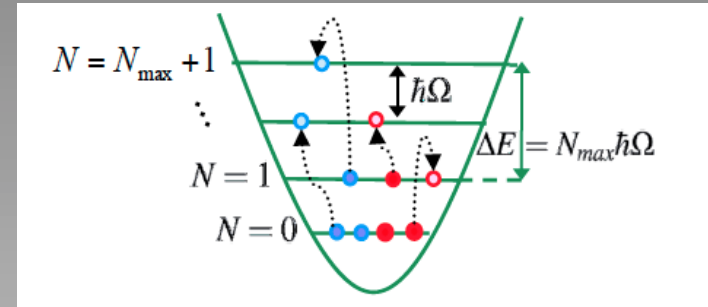
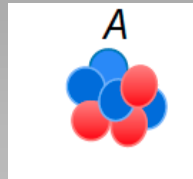


No-core shell model combined with the resonating group method (NCSM-RGM)

No-core shell model (NCSM):

- A-nucleon wave function expansion in the harmonic-oscillator (HO) basis
- Short- and medium-range correlations
- No continuum

$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni} \Phi_{Ni}^A$$



P. Navrátil et al. PRL **84**, 5728 (2000)

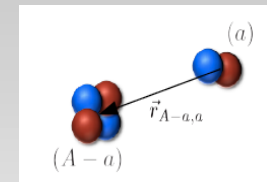
Resonating group method (RGM):

- Microscopic approach to describe the scattering of clusters
- Long range correlations (relative motion of clusters)

$$\Psi^{A-a,a} = \sum_{\nu} \int d\vec{r} \phi_{\nu}(\vec{r}) \mathcal{A} \Phi_{\nu\vec{r}}^{(A-a,a)}$$



$$\psi_{1\nu}^{(A-a)} \psi_{2\nu}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$



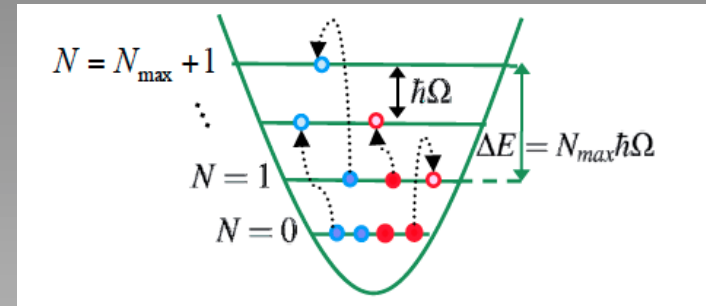
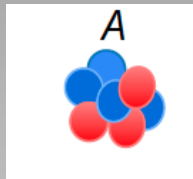
K. Wildermuth, Y.C. Tang
A unified theory of the nucleus 1977

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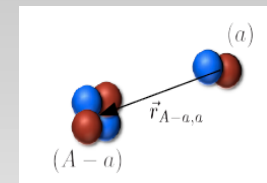


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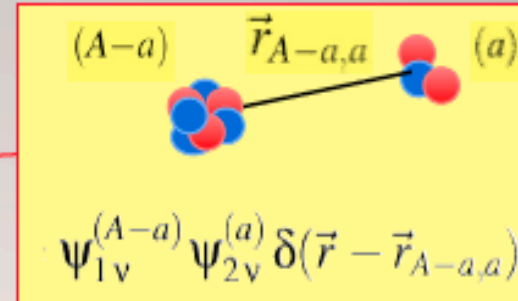
$$\Psi^{A-a,a} = \sum_{\nu} \int d\vec{r} \phi_{\nu}(\vec{r}) \mathcal{A} \Phi_{\nu\vec{r}}^{(A-a,a)} \rightarrow \psi_{1\nu}^{(A-a)} \psi_{2\nu}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$$



Ab initio NCSM/RGM: Combines both approaches
Ab initio nuclear Hamiltonian, **consistent** cluster wave functions,
correct asymptotic expansion, **Pauli principle** and **translational invariance**

The *ab initio* NCSM-RGM equation

- Ansatz: $\Psi^{(A)} = \sum_{\mathbf{v}} \int d\vec{r} \phi_{\mathbf{v}}(\vec{r}) \hat{\mathcal{A}} \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)}$



eigenstates of $H_{(A-a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$T_{\text{rel}}(r) + \mathcal{V}_{\text{rel}} + \vec{V}_{\text{Coul}}(r) + H_{(A-a)} + H_{(a)}$$

realistic nuclear Hamiltonian

$$\sum_{\mathbf{v}} \int d\vec{r} \left[\mathcal{H}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) - E \mathcal{N}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) \right] \phi_{\mathbf{v}}(\vec{r}) = 0$$

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$$

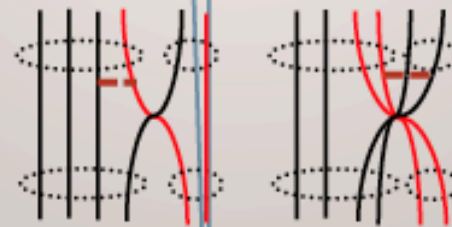
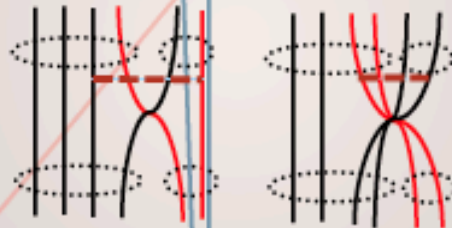
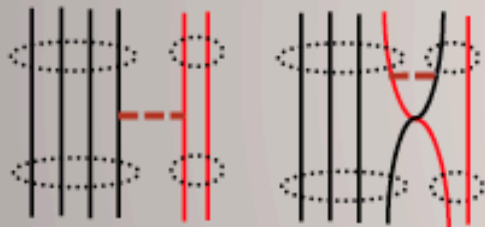
Hamiltonian kernel

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$$

Norm kernel

The deuteron projectile: Hamiltonian kernel

$$\begin{aligned}
 & \left\langle \begin{array}{c} (1, \dots, A-2) \\ \text{---} \\ (A-1, A) \end{array} \right| \left[\begin{aligned} & 2(A-2)V_{A-1,A-2}(1-P_{A-1,A-2}) - 2(A-2)V_{A,A-2}\hat{P}_{A-2,A-1} \\ & - 2(A-2)(A-3)V_{A,A-3}(1-\hat{P}_{A,A-3})\hat{P}_{A-2,A-1} \\ & - 2(A-2)(A-3)V_{A-1,A-3}\hat{P}_{A-2,A-1} \\ & + (A-2)(A-3)(A-4)V_{A-1,A-4}\hat{P}_{A-2,A}\hat{P}_{A-3,A-1} \end{aligned} \right] \left| \begin{array}{c} (1, \dots, A-2) \\ \text{---} \\ (A-1, A) \end{array} \right\rangle
 \end{aligned}$$



$$\text{SD} \left\langle \psi_{\mu_1}^{(A-2)} \left| a^+ a \right| \psi_{\nu_1}^{(A-2)} \right\rangle_{\text{SD}}$$

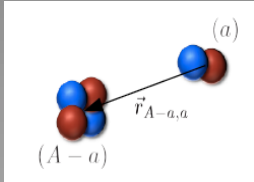
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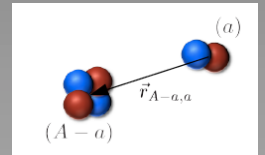
... and 'offdiagonal' coupling kernels

$$\left\langle \begin{array}{c} (A-1) \\ \text{---} \\ (A-\alpha) \end{array} \right| \left[V \hat{P} \right] \left| \begin{array}{c} (A-1) \\ \text{---} \\ (A-1) \end{array} \right\rangle$$

Three-body density Hamiltonian kernel



$$\left\langle \Phi_{k'_{ab}}^{J^\pi T} \left| \left(V_{A,A-4} \hat{P}_{A-2,A-1} \hat{P}_{A-3,A} \right) \right| \Phi_{k_{ab}}^{J^\pi T} \right\rangle$$



$$\underbrace{\text{SD} \langle A-2\alpha' | \hat{a}_{\beta_{A-4}}^\dagger \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_{b'} \hat{a}_{\beta_{A-3}} \hat{a}_{\beta_{A-4}} | A-2\alpha \rangle}_{\text{Three-body density matrix}} \text{SD}_a \underbrace{\langle \beta_{A-4}, a' | V_{A,A-4} | \beta'_{A-4} \beta'_{A-3} \rangle_a}_{\text{Two-body potential}}$$

Three-body density matrix

Two-body potential

- Number of three-body matrix elements increases fast with basis size
- Optimized numerical algorithm: On-the-fly calculation of the needed three-body matrix elements for a given target nucleus in the m-scheme

Memory size issue in kernels computation

Dimension of the RGM kernels affects the calculation:

- Memory of a node could be insufficient to handle kernels files
- R-matrix solver is slowed down by the reading-in of huge kernel files

Three different 'representations' of the three-body kernels:

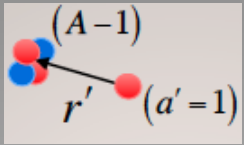
$$1) \text{SD} \langle A-2 \alpha'_1 I'_1 M'_1 T'_1 M'_{T_1} | \hat{a}_{\beta_{A-4}}^\dagger \hat{a}_{n_a \ell_a j_a m_{j_a} \frac{1}{2} m_{t_a}}^\dagger \hat{a}_{n_b \ell_b j_b m_{j_b} \frac{1}{2} m_{t_b}}^\dagger \hat{a}_{n'_b \ell'_b j'_b m'_{j'_b} \frac{1}{2} m'_{t'_b}} \hat{a}_{\beta'_{A-3}} \hat{a}_{\beta'_{A-4}} | A-2 \alpha_1 I_1 M_1 T_1 M_{T_1 a} \rangle_{\text{SD}}$$

$$2) \left\langle \Phi_{k'_{ab}}^{J\pi T} \left| \left(V_{A,A-4} \hat{P}_{A-2,A-1} \hat{P}_{A-3,A} \right) \right| \Phi_{k_{ab}}^{J\pi T} \right\rangle$$

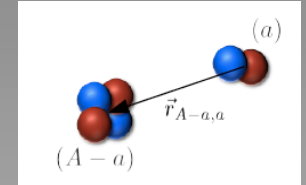
$$3) \left\langle \Phi_{\nu' r'}^{J\pi T} \left| \left(V_{A,A-4} \hat{P}_{A-2,A-1} \hat{P}_{A-3,A} \right) \right| \Phi_{\nu r}^{J\pi T} \right\rangle$$

Nmax (single-channel $d\text{-}^4\text{He}$)	1) # 3-body density matrix	2) # Kernels in SD-basis	3) # Kernels in physical basis
2	$\sim 10^5$	$\sim 10^2$	20
4	$\sim 10^7$	$\sim 10^3$	70
6	$\sim 10^9$	$\sim 10^4$	167
8	$\sim 10^{10}$	$\sim 10^5$	325

Coupling Hamiltonian kernel



$$\left\langle \Phi_{k'_a}^{J^\pi T} \left| \left(V_{A-3, A-2} \hat{P}_{A-2, A} \right) \right| \Phi_{k_{ab}}^{J^\pi T} \right\rangle$$



$$\underbrace{\text{SD} \langle A-1 \alpha' | \hat{a}_{\beta_{A-3}}^\dagger \hat{a}_b^\dagger \hat{a}_a^\dagger \hat{a}_{\beta'_{A-2}} \hat{a}_{\beta'_{A-3}} | A-2 \alpha \rangle}_{\text{"Not-diagonal" density matrix}} \text{SD}_a \langle \beta_{A-3, a'} | V_{A-3, A-2} | \beta'_{A-3} \beta'_{A-2} \rangle_a \underbrace{\hspace{10em}}_{\text{Two-body potential}}$$

"Not-diagonal" density matrix

Two-body potential

(A-2,2) mass partition in the entrance channel

(A-1,1) mass partition in the exit channel

- Inclusion of coupling kernel describing the dominant channel in a (d,p/n) transfer reaction

cluster overlaps

$N_{\max} = 6$

- Understand cluster structure of ⁹Be

Cluster overlaps from NCSM eigenstates

$$g(r) = \langle \Psi^{(A)} | \mathcal{A} \Psi^{(A-a)} \Psi^{(a)} \delta_{r, r_{A, A-a}} \rangle$$

Large overlap of mass partitions considered with compound nucleus in the $5/2^+$ partial wave

$$\langle {}^9\text{Be} | p + {}^8\text{Li} \rangle \quad E_{\text{thr}} = -37.60 \text{ MeV}$$

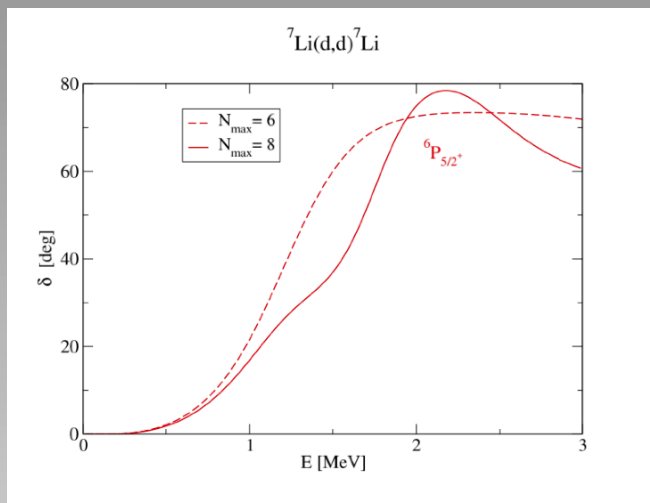
$J^\pi T$	E (⁹ Be) [MeV]	(2s,l)	S
$(3/2^+ 1/2)_5$	4.49	$(3,0)^*$	0.356
$(5/2^- 1/2)_6$	3.87	$(5,1)^*$	0.052
$(5/2^+ 1/2)_4$	3.46	$(5,0)$	0.593

$$\langle {}^9\text{Be} | d + {}^7\text{Li} \rangle \quad E_{\text{thr}} = -38.16 \text{ MeV}$$

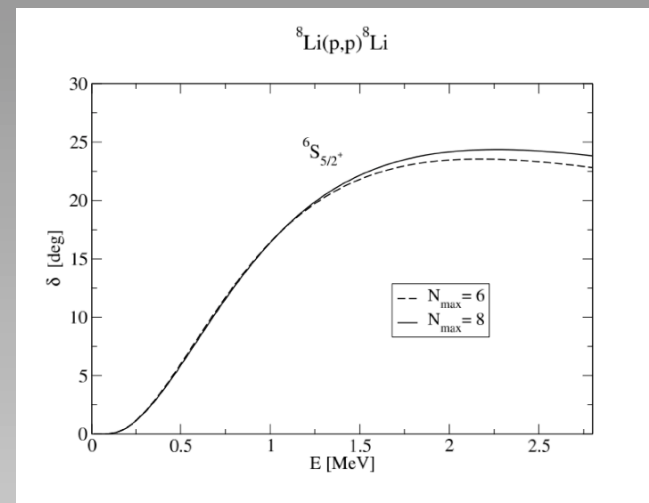
$J^\pi T$	E (⁹ Be) [MeV]	(2s,l)	S
$(1/2^- 1/2)_4$	2.98	$(1,0)^*$	0.140
$(5/2^- 1/2)_6$	4.43	$(5,0)^*$	0.153
$(5/2^+ 1/2)_4$	3.46	$(5,1)$	0.371

Phase shifts analysis (NCSM-RGM)

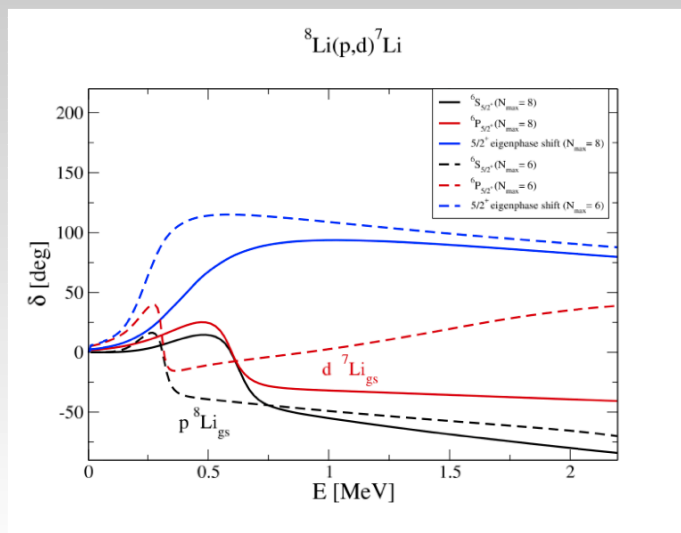
Analysis of the partial wave phase shifts: spin and parity (J^π) of the resonance



$J^\pi = 5/2^+$
dominant
resonance
at low energy



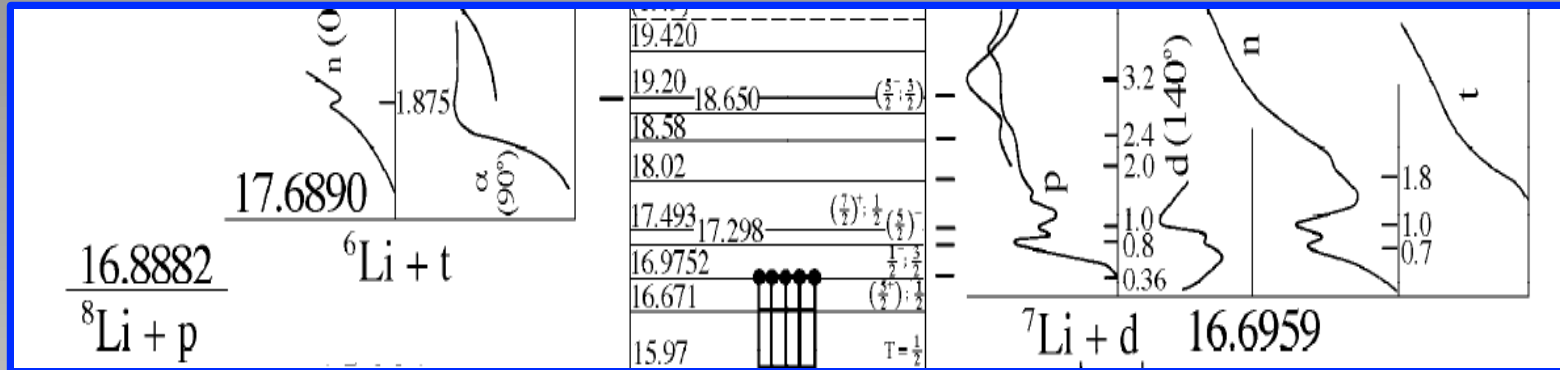
Effect of the coupling:
contribution from
'offdiagonal' elements of
the collision matrix



Eigenphase shifts
convey informations on
all the matrix elements
of the collision matrix

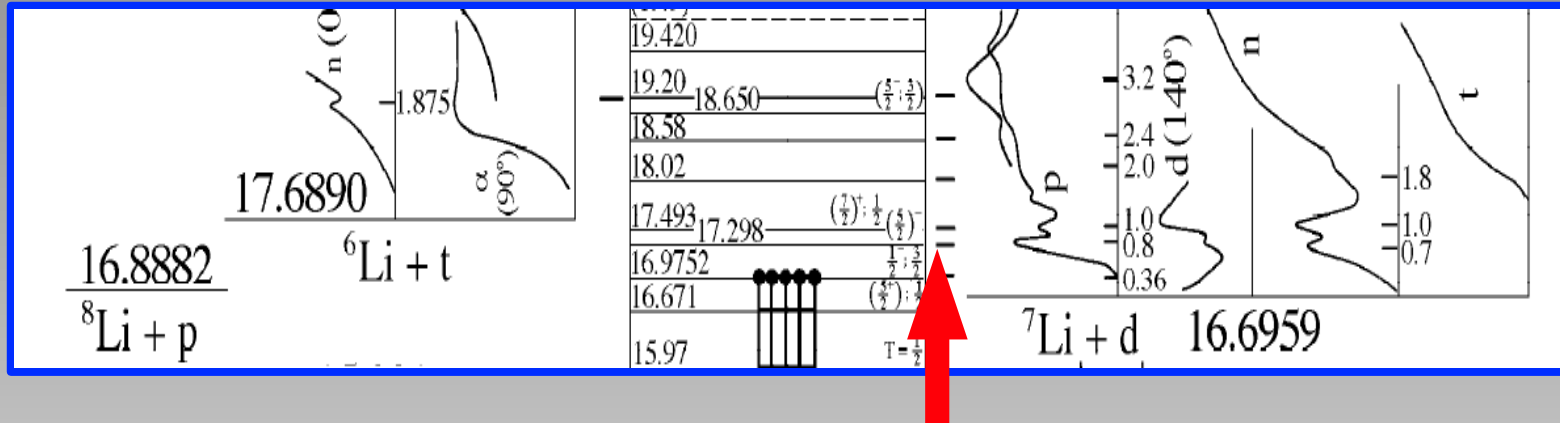
Spin-parity assignment of 0.78 MeV resonance of ${}^9\text{Be}$

${}^9\text{Be}$ spectrum above d - ${}^7\text{Li}$ threshold



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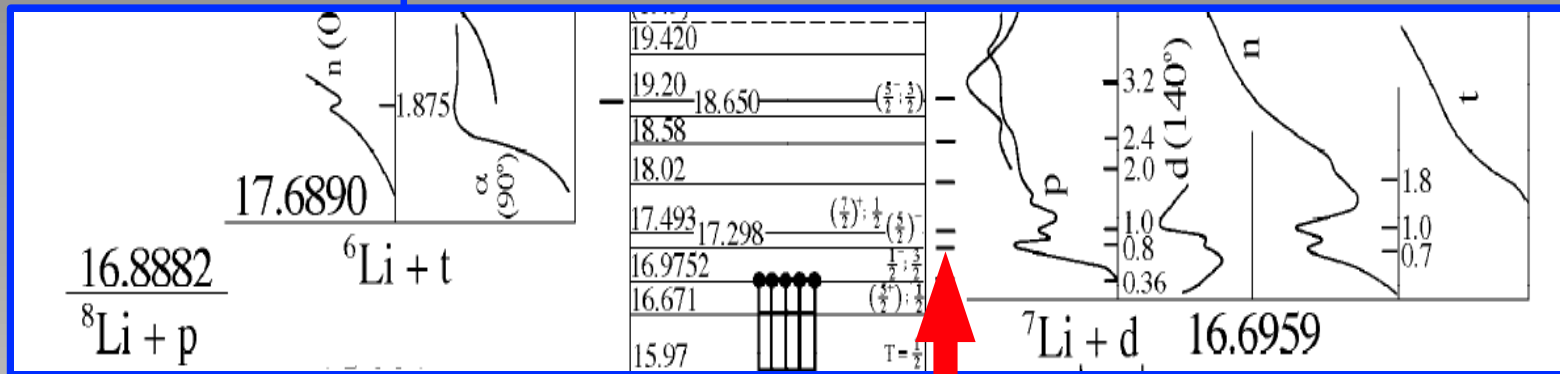
${}^9\text{Be}$ spectrum above d - ${}^7\text{Li}$ threshold



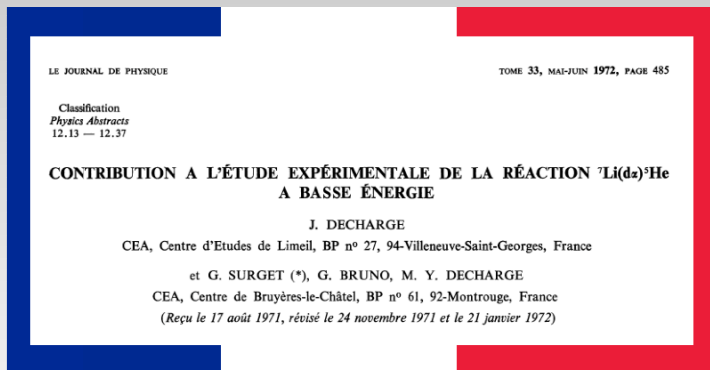
Low peak in the experimental total cross section:
 $E(5/2^-) \sim 0.78 \text{ MeV}$ above the threshold
 (Uncertain spin-parity assignment)

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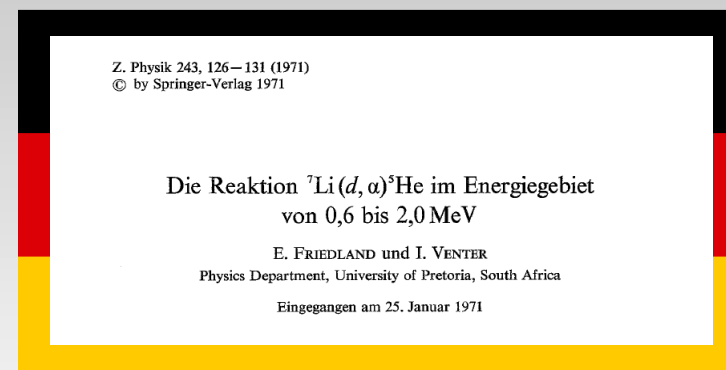
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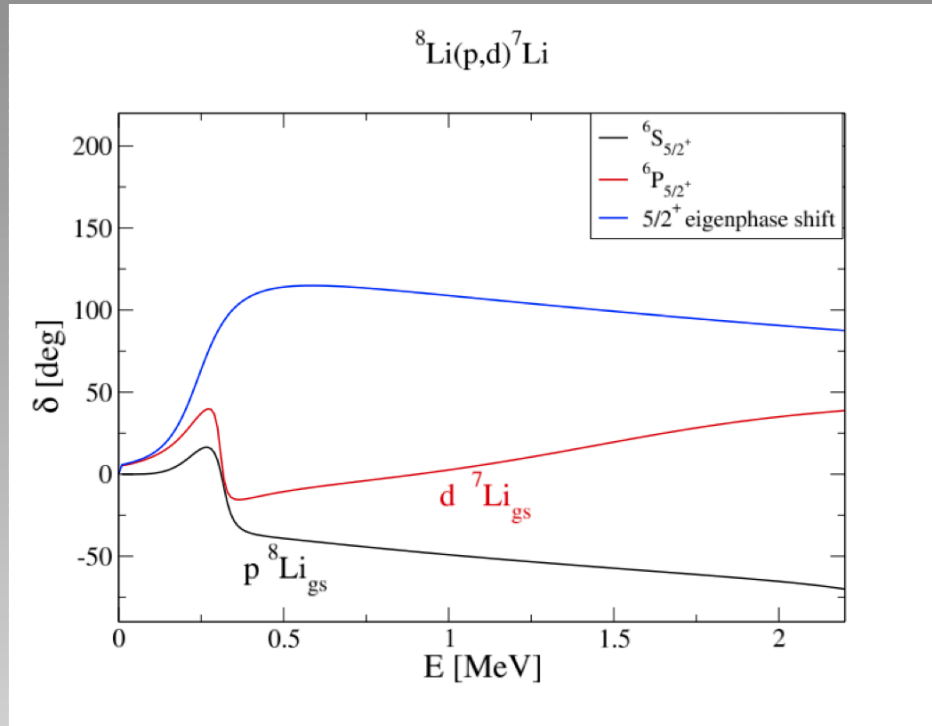


$E(\sim 17.3 \text{ MeV}) \rightarrow 3/2^-$



$E(\sim 17.3 \text{ MeV}) \rightarrow 3/2^+, 5/2^+, 7/2^+$

Reaction mechanism



$d - {}^7\text{Li}$
(p wave)

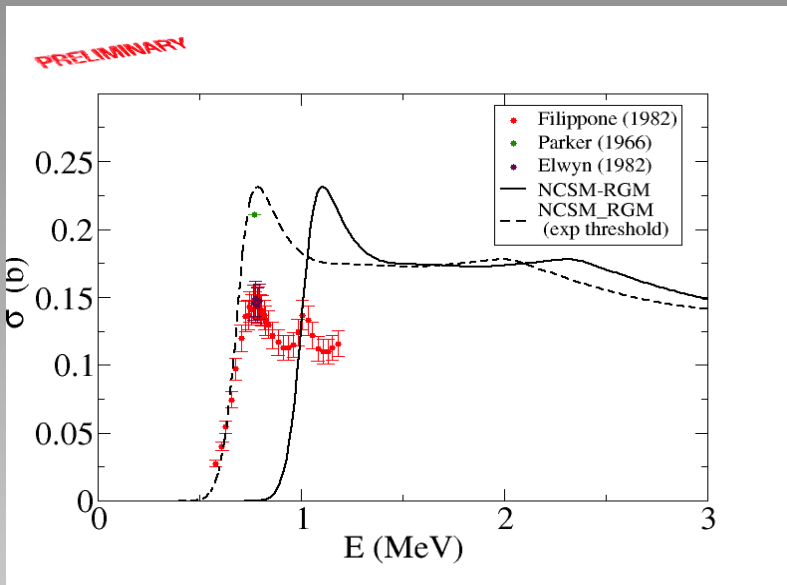


${}^9\text{Be}$
($5/2^+$)



$p - {}^8\text{Li}$
(s wave)

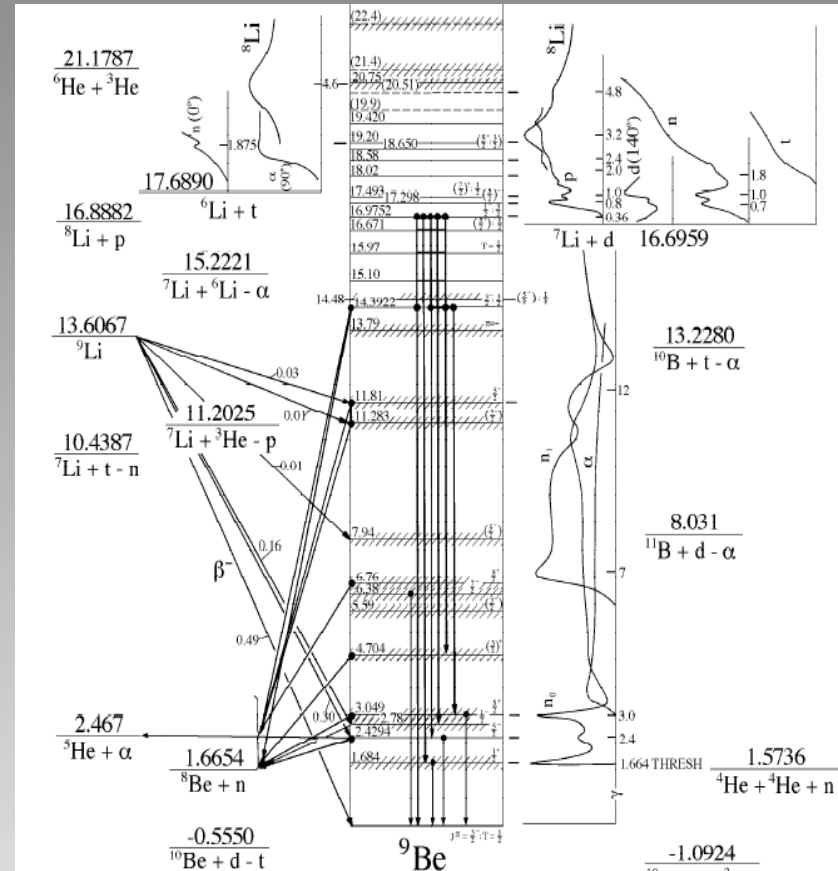
${}^7\text{Li}(d,p){}^8\text{Li}$ scattering results (NCSM-RGM)



Included channels:

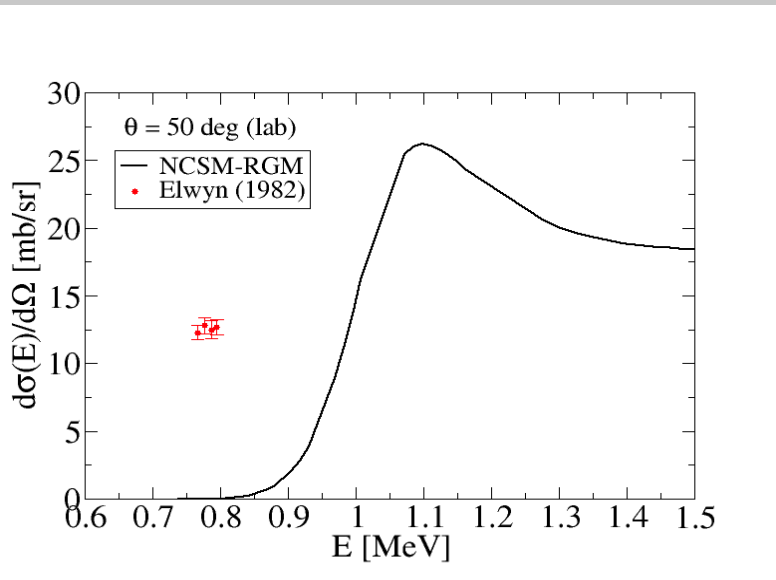
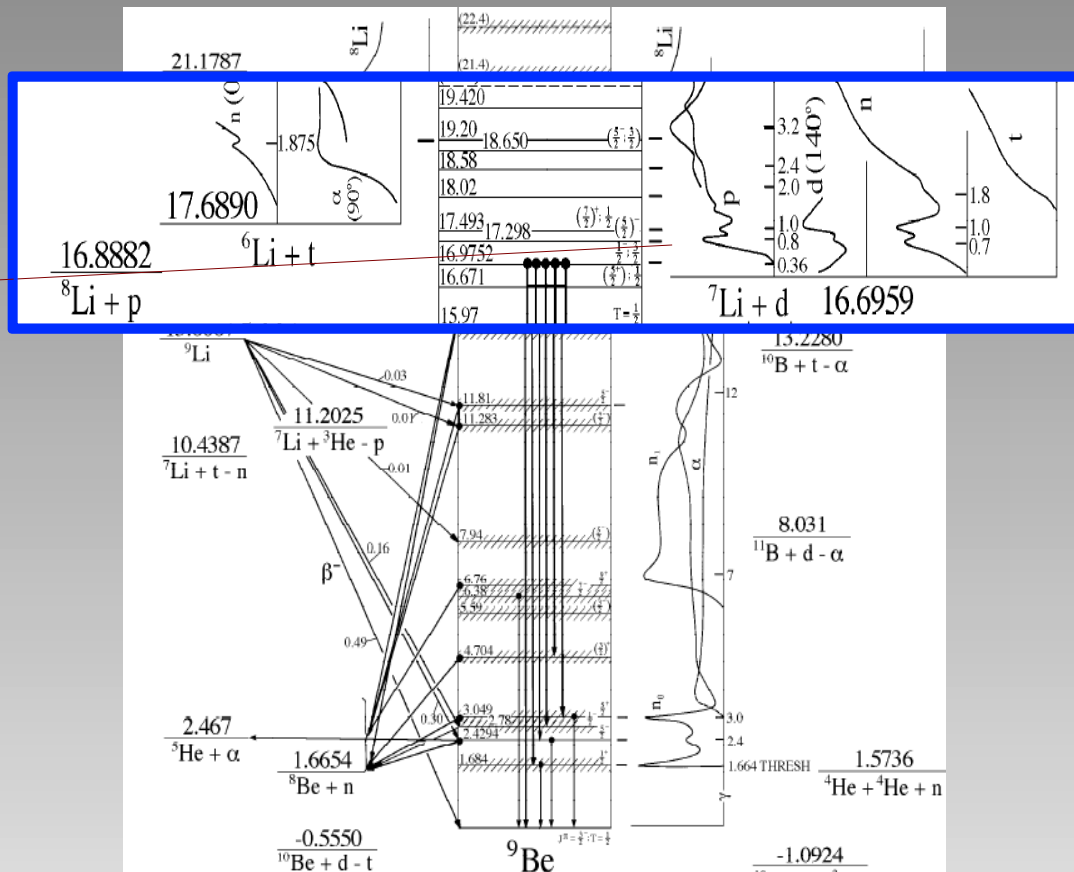
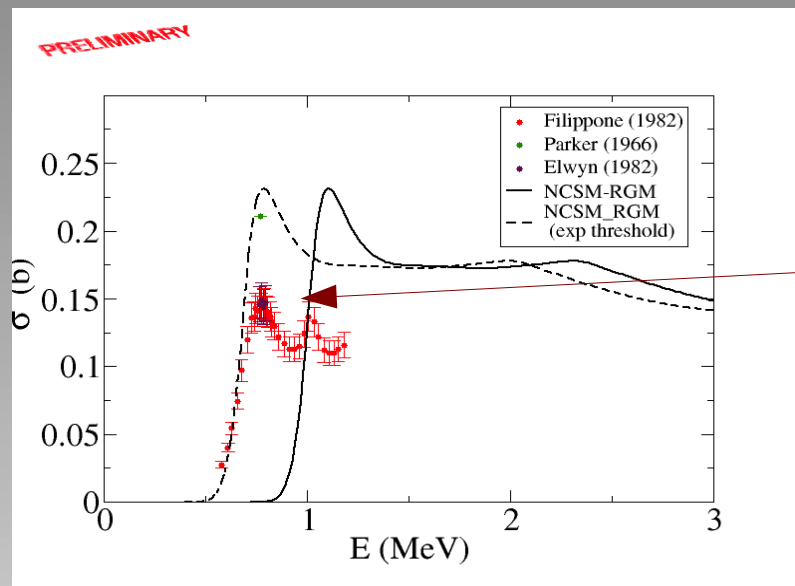
- (1) $p, {}^8\text{Li}$
- (2) $d, {}^7\text{Li}$
- (3) coupling (d,p)
- (4) virtual breakup of d

Not-included channels: (1) ${}^8\text{Be}, n$ (2) ${}^6\text{Li}, t$



NCSM-RGM calculations with chiral N^3LO NN potentials ($\text{SRG } \lambda=2.02 \text{ fm}^{-1}$)
 4 eigenstates of ${}^8\text{Li}$, 2 eigenstates of ${}^7\text{Li}$ and 5 pseudostates of deuteron
 Preliminary: $N_{\text{max}}=6$, $h\Omega=20 \text{ MeV}$

${}^7\text{Li}(d,p){}^8\text{Li}$ scattering results (NCSM-RGM)



Lowest peak in the experimental total cross section:
 $(E(5/2^-) \sim 0.8 \text{ MeV above the threshold})$

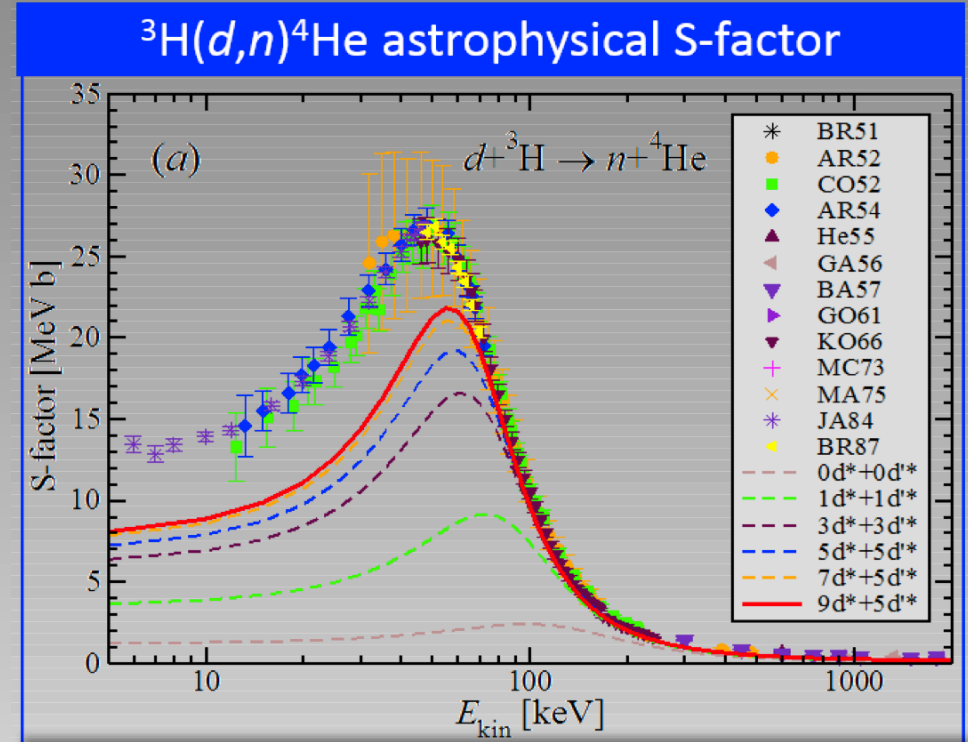
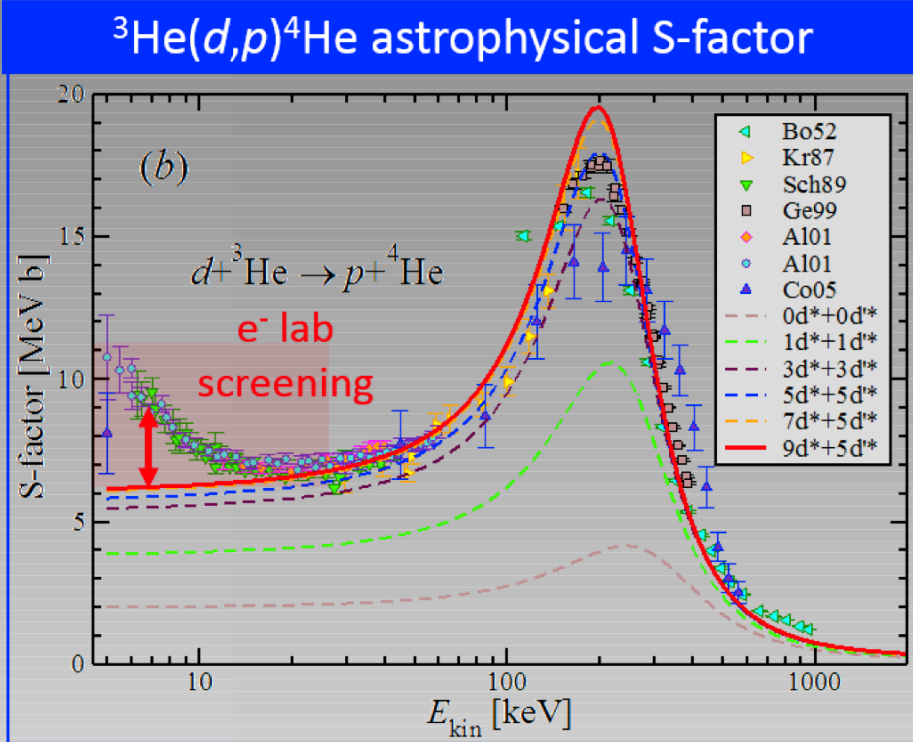
(Uncertain spin-parity assignment)

${}^3\text{H}(d,n){}^4\text{He}$ fusion with 3N forces

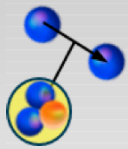
Recent progress on inclusion of 3N forces in transfer reactions (s-shell nuclei)

G. Hupin (CEA DAM, Bruyères-le-châtel),
P. Navrátil (TRIUMF, Vancouver),
S. Quaglioni (LLNL, US)

First step towards *ab initio* calculation transfer reactions (Navrátil, Quaglioni PRL108, 2012)



NCSM/RGM results for the $^3\text{He}(d,n)^4\text{He}$ astrophysical S-factor compared to beam-target measurements.



Deuterium breakup:

Calculated S-factors converge with the inclusion of virtual breakup of the Deuterium, obtained by means of excited $^3\text{S}_1$ - $^3\text{D}_1(d^*)$ and $^3\text{D}_2(d^*)$ pseudostates

Incomplete nuclear interaction: Requires 3N forces (SRG-induced + "real")

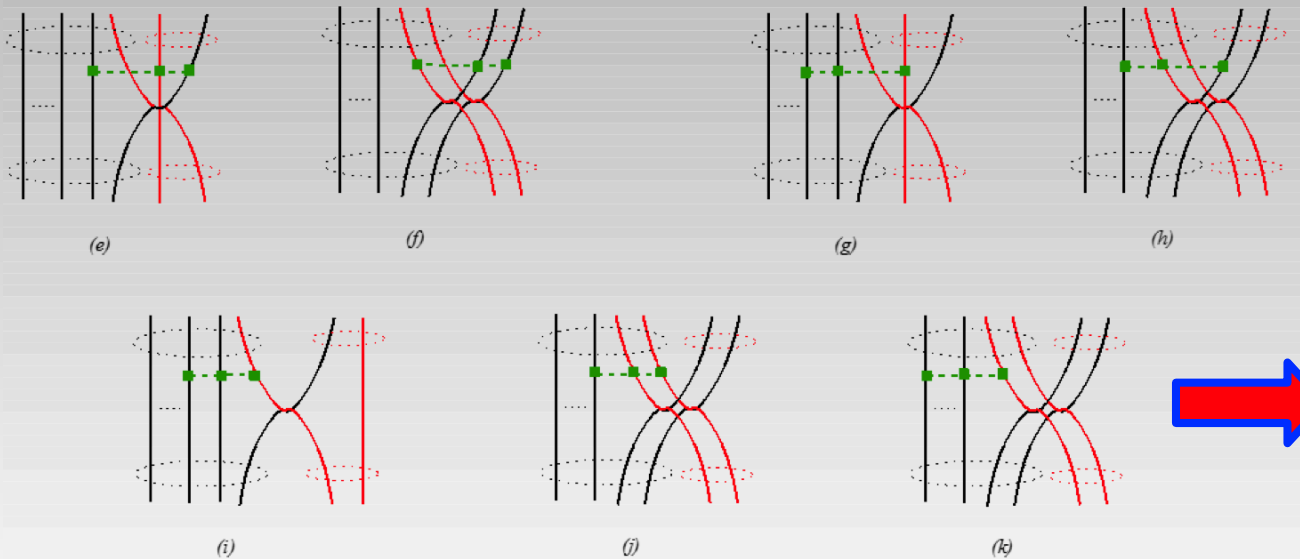
Including 3N force into the description of deuteron-nucleus scattering

$$\langle \Phi_{v'r'}^{J\pi T} | \hat{A}_{v'} V^{NNN} \hat{A}_v | \Phi_{vr}^{J\pi T} \rangle = \left\langle \begin{array}{c} (A-2) \\ \vec{r}' \\ (a'=2) \end{array} \right| V^{NNN} \left(1 - \sum_{i=1}^{A-2} \sum_{k=A-1}^A \hat{P}_{ik} + \sum_{i<j=1}^{A-2} \hat{P}_{iA-1} \hat{P}_{jA} \right) \left| \begin{array}{c} (A-2) \\ \vec{r} \\ (a=2) \end{array} \right\rangle$$

Direct



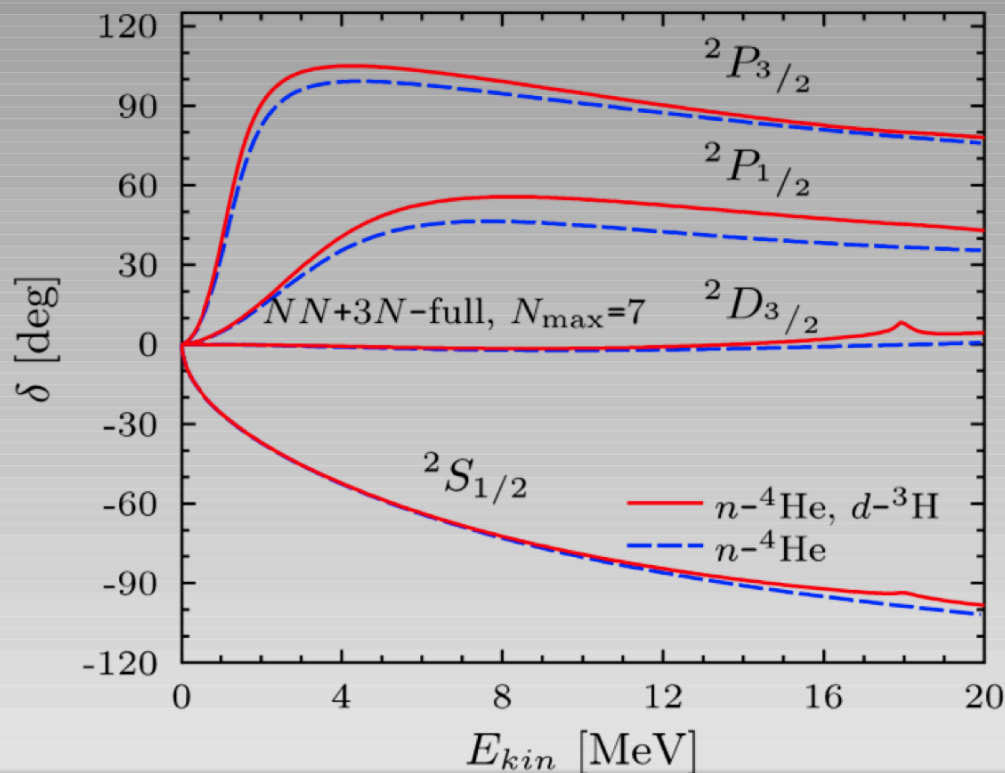
Exchange



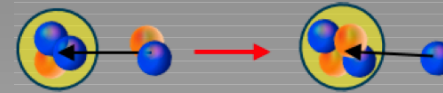
Four body density

NCSM-RGM calculation of transfer reaction ${}^3\text{H}(d,n){}^4\text{He}$ with 3N forces

n - ${}^4\text{He}$ with the d - ${}^3\text{H}$ channel with the chiral two- and three-nucleon force (preliminary)



n - ${}^4\text{He}$ (g.s.) phase shifts with $NN+3N$ potential, $\lambda=2.0$ fm^{-1} , no ${}^5\text{He}$ eigenstates, with/o coupling to d - t .

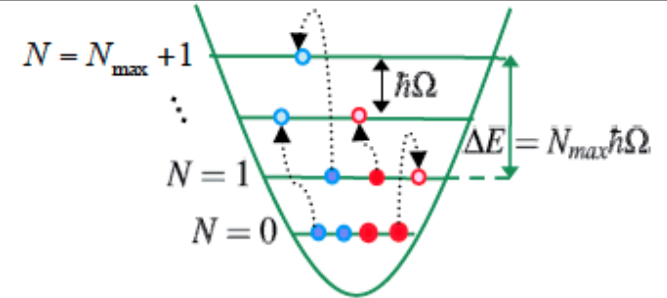


${}^4\text{He}$ fusion

- Perspective to provide accurate $t(d,n){}^4\text{He}$ fusion cross-section for the effort toward earth-based fusion energy generation
- The d - t transfer is known to be very sensitive to the spin-orbit and tensor part of the nuclear interaction
- 7 diagrams (+ exchange) arising from the coupling of n - α with d - t through the three-nucleon force.
- Next step: Coupling to the compound ${}^5\text{He}$ eigenstates

No-core shell model with continuum

- No-core shell model (NCSM)
 - A -nucleon wave function expansion in the harmonic-oscillator (HO) basis
 - short- and medium range correlations
 - Bound-states, narrow resonances
- NCSM with Resonating Group Method (NCSM/RGM)
 - cluster expansion
 - proper asymptotic behavior
 - long-range correlations



S. Baroni, P. Navratil, S. Quaglioni
PRL 110, 022505 (2013)

The most efficient:
No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left[\begin{array}{c} \text{NCSM eigenstates} \\ \left(\begin{array}{c} (A) \\ \text{Nucleon cluster} \end{array}, \lambda \right) \end{array} \right] + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left[\begin{array}{c} \text{NCSM/RGM channel states} \\ \left(\begin{array}{c} (A-a) \text{ cluster} \\ (a) \text{ cluster} \end{array}, \nu \right) \end{array} \right]$$

Unknowns

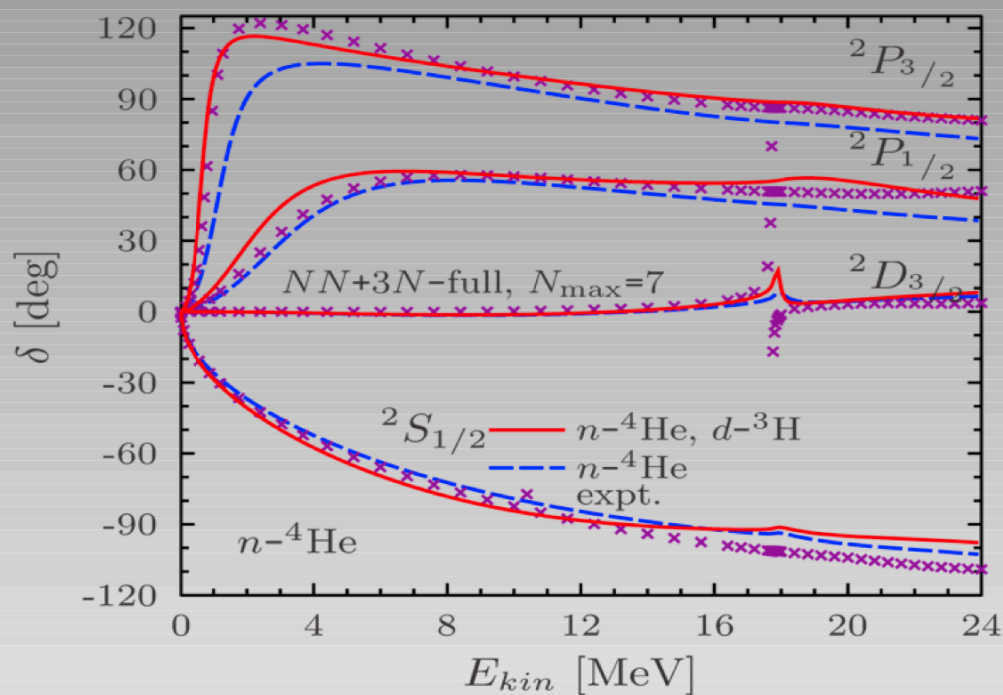
Coupled NCSMC equations

$$\begin{array}{c}
 \begin{array}{c}
 \boxed{E_{\lambda}^{NCSM} \delta_{\lambda\lambda'}} \\
 \downarrow
 \end{array}
 \quad
 \begin{array}{c}
 \boxed{\langle (A) \left| H \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow
 \end{array}
 \quad
 \begin{array}{c}
 \boxed{\delta_{\lambda\lambda'}} \\
 \downarrow
 \end{array}
 \quad
 \begin{array}{c}
 \boxed{\langle (A) \left| \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow
 \end{array}
 \\
 \left(\begin{array}{cc}
 H_{NCSM} & h \\
 h & H_{RGM}
 \end{array} \right)
 \begin{pmatrix}
 \textcircled{C} \\
 \textcircled{\gamma}
 \end{pmatrix}
 = E
 \left(\begin{array}{cc}
 1_{NCSM} & g \\
 g & N_{RGM}
 \end{array} \right)
 \begin{pmatrix}
 \textcircled{C} \\
 \textcircled{\gamma}
 \end{pmatrix}
 \\
 \begin{array}{c}
 \uparrow \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_v H \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \quad
 \begin{array}{c}
 \uparrow \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_v \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \end{array}$$

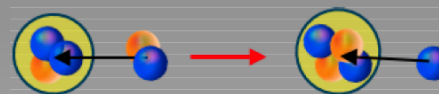
Scattering matrix (and observables) from matching solutions to known asymptotic with microscopic R -matrix on Lagrange mesh

NCSMC calculation of transfer reaction ${}^3\text{H}(d,n){}^4\text{He}$ with 3N forces

${}^4\text{He}$ fusion with NCSMC and the chiral two- and three-nucleon force (preliminary)



$n+{}^4\text{He}$ (g.s.) phase shifts with $NN+3N$ potential, $\lambda=2.0 \text{ fm}^{-1}$, with eigenstates of ${}^5\text{He}$.



${}^4\text{He}$ fusion

- Perspective to provide accurate $t(d,n){}^4\text{He}$ fusion cross-section for the effort toward earth-based fusion energy generation
- The d - t transfer is known to be very sensitive to the spin-orbit and tensor part of the nuclear interaction

- Preliminary, small $N_{MAX} = 7$

Conclusions & Perspectives

First application of the NCSM-RGM for deuteron-projectile and p-shell nucleus as target:

- Inclusion of the “elastic” and coupling channel in the description of transfer reactions

Analysis of the ${}^9\text{Be}$ resonances above d - ${}^7\text{Li}$ threshold:

- Discussion of the spin-parity assignment of 0.78 MeV resonance

NCSM/RGM and NCSMC results on d -t and d - ${}^3\text{He}$ transfer reaction with 3N force

To be done:

- Complete the calculation ($N_{\text{max}}=8$) of the ${}^7\text{Li}(d,p){}^8\text{Li}$ transfer reaction in NCSM-RGM and NCSMC
- Include 3N force also for p-shell nuclei

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Thank you