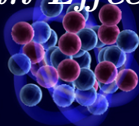




TRIUMF

Canada's National Laboratory for Particle and Nuclear Physics
Laboratoire national canadien pour la recherche en physique
nucléaire et en physique des particules

Light Exotic Nuclei in Effective Field Theory

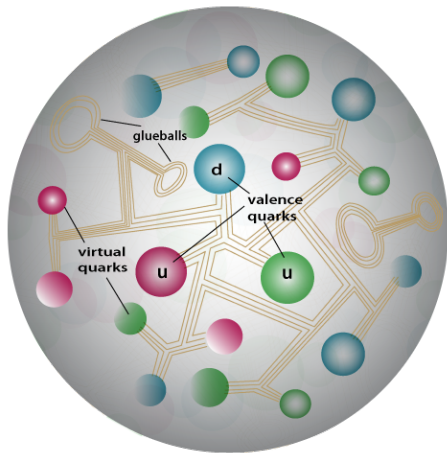


Chen Ji || TRIUMF

Reactions and Structure of Exotic Nuclei
INT, Mar 2-13, 2015

- We study physics at different resolution scales with different effective theories

- describe nucleon structures
 - physics scale: $Q \gtrsim \text{GeV}$
 - d.o.f.: quarks & gluons
 - effective theory: lattice QCD



- We study physics at different resolution scales with different effective theories

- light/medium mass nuclei
 - physics scale: $Q \sim 200$ MeV
 - d.o.f.: nucleons & pions
 - effective theory: chiral EFT
 - use *ab initio* methods



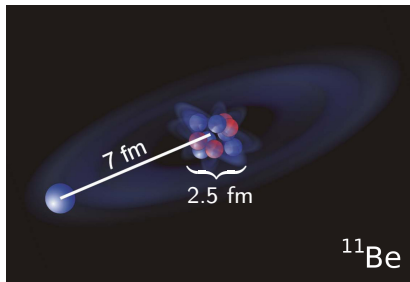
- We study physics at different resolution scales with different effective theories

- very light nuclei ($d, t, {}^3\text{He}, \alpha$)
 - physics scale: $Q \ll m_\pi$
 - d.o.f.: nucleons in contact
 - effective theory: pionless EFT
 - use few-body methods



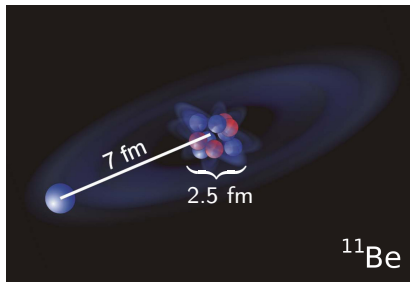
- halo nuclei (core + valence N)
- separation in length scales

$$R_{\text{core}} \ll R_{\text{halo}}$$



- **halo nuclei** (core + valence N)
- **separation in length scales**

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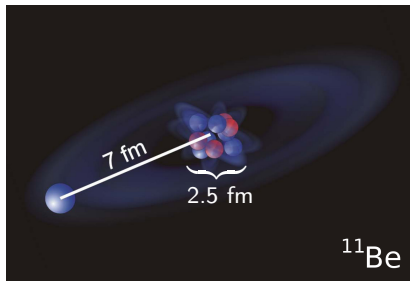


ab initio methods

- capture dynamics inside and outside the core
- numerically expensive for loosely bound systems

- halo nuclei (core + valence N)
- separation in length scales

$$R_{\text{core}} \ll R_{\text{halo}}$$



ab initio methods

- capture dynamics inside and outside the core
- numerically expensive for loosely bound systems

halo effective field theory

- valence nucleon + core d.o.f.
- systematic expansion in $R_{\text{core}}/R_{\text{halo}}$
- capture only clustering mechanism
- numerically simpler
- complimentary to *ab initio* methods
- explain universal correlations in clustering physics

- We adopt EFT with contact interactions to describe clustering in halo nuclei

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \psi + \eta d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d - \frac{g}{\sqrt{2}} \left(d^\dagger \psi \psi + \text{h.c.} \right) + h d^\dagger d \psi^\dagger \psi + \dots$$

... are higher orders in $R_{\text{core}}/R_{\text{halo}}$ expansion

- We adopt EFT with contact interactions to describe clustering in halo nuclei

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... are higher orders in $R_{\text{core}}/R_{\text{halo}}$ expansion

- 2-body contact (LO)** introduce a two-body field

$$\begin{array}{c} \diagup \quad \diagdown \\ \bullet \\ \diagdown \quad \diagup \end{array} = -iC_0 \quad \xrightarrow{C_0 = g^2/\Delta} \quad \begin{array}{c} \diagup \\ \text{---} \\ \diagdown \end{array} = -i\sqrt{2}g$$

g determined by a 2-body observable

- 3-body contact (LO)**


$$\begin{array}{c} \diagup \quad \diagdown \\ \bullet \\ \diagdown \quad \diagup \end{array} = -iD_0 \quad \xrightarrow{D_0 = -3hg^2/\Delta^2} \quad \begin{array}{c} \diagup \quad \diagdown \\ \text{---} \\ \diagdown \quad \diagup \end{array} = ih$$

h determined by a 3-body observable [Bedaque, Hammer, van Kolck PRL '99]

● EFT for $1n$ halo



● ${}^5\text{He}$ shallow resonance ($P_{3/2}$)



$$= \frac{1}{4\pi^2 \mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$


$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1}$

Ardnt et al. NPA '73

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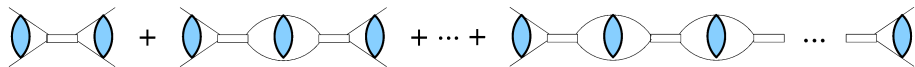
$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1}$
Ardnt et al. NPA '73

- $n\alpha$ p-wave EFT power counting

two fine tunings: Bertulani, Hammer, van Kolck, NPA '02

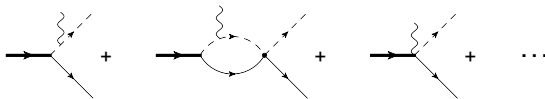
one fine tuning: Bedaque, Hammer, van Kolck, PLB '02

● EFT for $1p$ halo nucleus



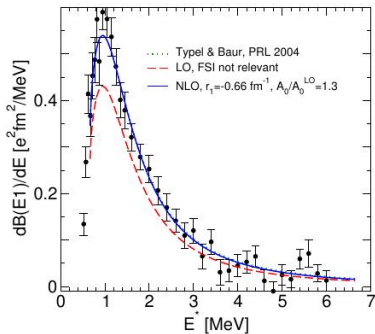
p - α and α - α scattering [Higa, Hammer, van Kolck, NPA '08; Higa, FBS '11]

^{17}F [Ryberg, Forssén, Hammer, Platter, PRC '14]



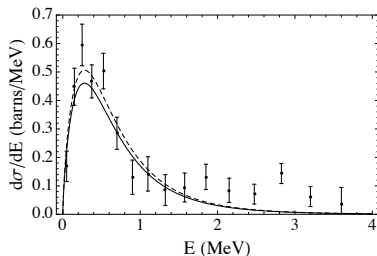
- E1 transition**

^{11}Be photo-dissociation



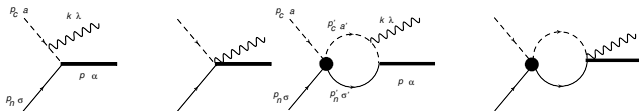
[Hammer, Phillips, NPA '11]

^{19}C photo-dissociation

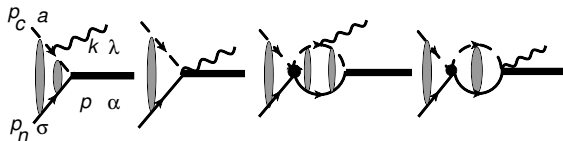


data: Nakamura *et al*, RIKEN,
PRL '99; NPA '03;
theory: Acharya, Phillips, NPA '13

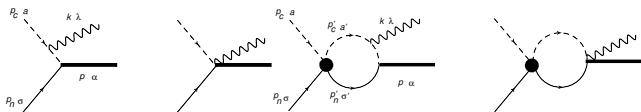
neutron captures



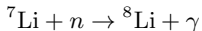
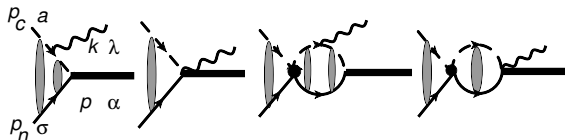
proton captures



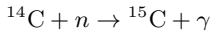
neutron captures



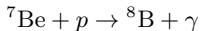
proton captures



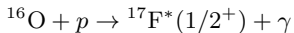
Rupak, Higga, PRL '11; Fernando, Higa, Rupak, EPJA '12;
Zhang, Nollett, Phillips, PRC '14



Rupak, Fernando, Vaghani, PRC '12



Zhang, Nollett, Phillips, PRC '14; Ryberg, *et al.* EPJA '14



Ryberg, Forssén, Hammer, Platter, PRC '14

E1 S-factor for ${}^7\text{Be}(p, \gamma){}^8\text{B}$

- Zhang, Nollett, Phillips, PRC '14

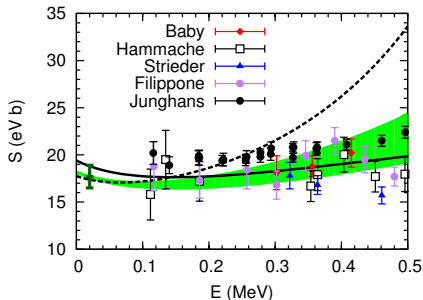
— NSCM-GRM result

[Navratil, Roth, Quaglioni, PLB '11]

---- LO EFT: fit to NSCM-GRM ANC

■ LO EFT: fit to ANC from VMC

VMC [Nollett, Wiringa, PRC '11]



E1 S-factor for ${}^7\text{Be}(p, \gamma){}^8\text{B}$

- Zhang, Nollett, Phillips, PRC '14

— NSCM-GRM result

[Navrátil, Roth, Quaglioni, PLB '11]

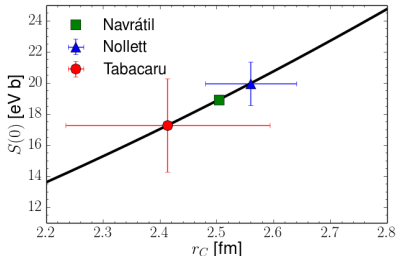
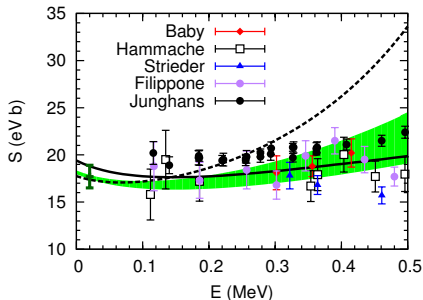
---- LO EFT: fit to NSCM-GRM ANC

■ LO EFT: fit to ANC from VMC

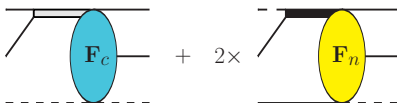
VMC [Nollett, Wiringa, PRC '11]

- Ryberg, Forssén, Hammer, Platter, EPJA '14

correlation btw $S(0)$ and $r_C[{}^8\text{B}]$



- 2n-halo wave functions

$$\Psi_x(p, q) = \text{Diagram 1} + 2 \times \text{Diagram 2}$$


The diagram shows the decomposition of the 2n-halo wave function $\Psi_x(p, q)$. It consists of two terms: a core nucleus F_c (blue oval) and a neutron halo F_n (yellow oval). The core nucleus F_c is shown with a solid line representing the continuum and a dashed line representing the bound state. The neutron halo F_n is shown with a solid line representing the continuum and a dashed line representing the bound state. The term F_n is multiplied by a factor of 2.

- 2n-halo wave functions

$$\Psi_x(p, q) = \text{diagram with } F_c \text{ and } F_n \text{ components}$$

The diagram shows the decomposition of the two-neutron halo wave function $\Psi_x(p, q)$. It consists of two terms: a blue oval labeled F_c and a yellow oval labeled F_n . The F_c term is multiplied by 1, and the F_n term is multiplied by 2. Each term is represented by a set of horizontal lines (representing nucleon states) and a dashed line (representing the core). The F_c diagram shows a core state and two neutron states connected by a solid line. The F_n diagram shows a core state and two neutron states connected by a dashed line.

- Three-body Faddeev equation

$$\begin{aligned} \text{diagram } n-n \text{ and } c \text{ with } F_c &= 2 \times \text{diagram } n-n \text{ and } c \text{ with } F_n \\ \text{diagram } c-n \text{ and } n \text{ with } F_n &= \text{diagram } c-n \text{ and } n \text{ with } F_c + \text{diagram } c-n \text{ and } n \text{ with } F_n \end{aligned}$$

The diagram illustrates the three-body Faddeev equation for two-neutron halo nuclei. It shows two equations. The first equation shows the $n-n$ and c components of the wave function with a blue F_c interaction term equal to twice the $n-n$ and c components with a yellow F_n interaction term. The second equation shows the $c-n$ and n components of the wave function with a yellow F_n interaction term equal to the sum of the $c-n$ and n components with a blue F_c interaction term and the $c-n$ and n components with a yellow F_n interaction term.

- 2n-halo wave functions

$$\Psi_x(p, q) = \text{diagram with } F_c \text{ and } F_n \text{ components}$$

The diagram shows the decomposition of the two-neutron halo wave function $\Psi_x(p, q)$. It consists of two terms: a blue oval labeled F_c (compact core) and a yellow oval labeled F_n (neutron halo). The F_c term is connected to a core state (top line) and a neutron state (bottom dashed line). The F_n term is connected to a core state (top line) and a neutron state (bottom solid line). The F_n term is multiplied by a factor of 2.

- Three-body Faddeev equation

$$\text{diagram } n-n \text{ and } c \text{ connected by } F_c = 2 \times \text{diagram } n-n \text{ and } c \text{ connected by } F_n$$

$$\text{diagram } c-n \text{ and } n \text{ connected by } F_n = \text{diagram } c-n \text{ and } n \text{ connected by } F_c + \text{diagram } c-n \text{ and } n \text{ connected by } F_n + \text{diagram } c-n \text{ and } n \text{ connected by } F_n$$

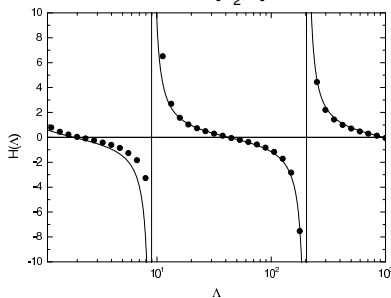
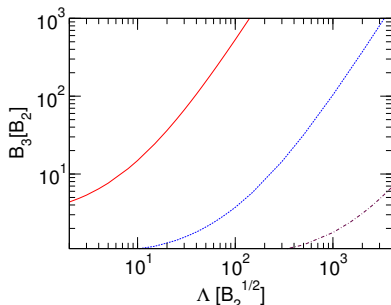
The diagram illustrates the three-body Faddeev equation for two-neutron halo nuclei. It shows the relationship between the core (c) and neutron (n) components of the wave function and the interaction kernels F_c and F_n . The first equation shows the $n-n$ and c components connected by F_c is equal to 2 times the $n-n$ and c components connected by F_n . The second equation shows the $c-n$ and n components connected by F_n is equal to the sum of three terms: the $c-n$ and n components connected by F_c , the $c-n$ and n components connected by F_n (with a dashed line), and the $c-n$ and n components connected by F_n (with a loop).

- Without 3BF:

- 3-body spectrum:
 - cutoff dependent ($\Lambda \sim 1/\ell$)
 - Platter '09

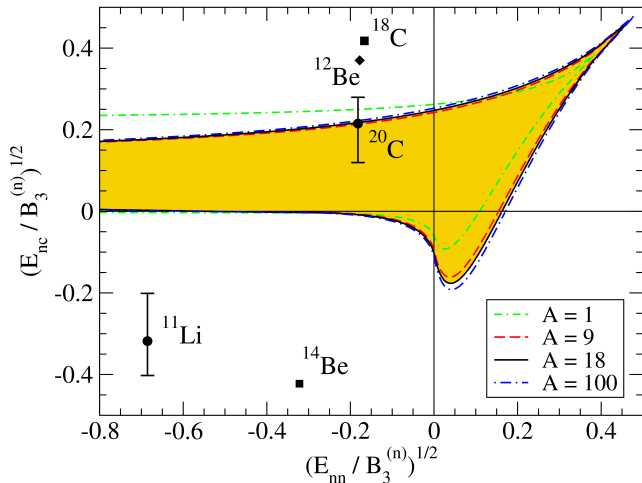
- LO 3BF h :

- tune $H(\Lambda) = \Lambda^2 h/2mg^2$:
 - fix one 3-body observable
- limit cycle:
 - $H(\Lambda)$ periodic for $\Lambda \rightarrow \Lambda(\text{const})^n$
 - Bedaque *et al.* '00
- \rightarrow Efimov physics



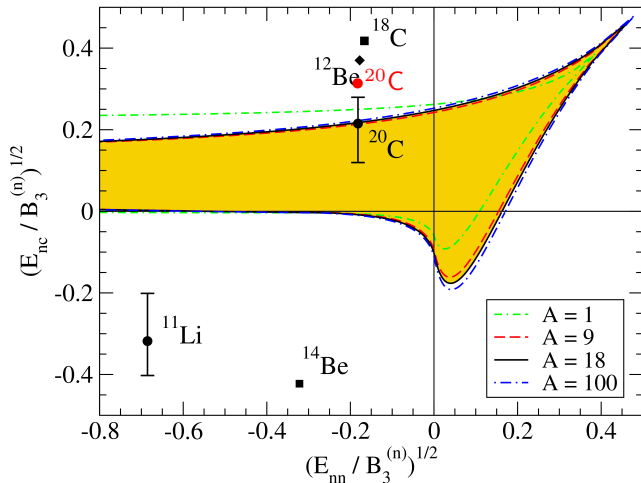
- n -core in s-wave virtual/real bound state:
 - ^{11}Li , ^{12}Be , ^{20}C [Canham, Hammer, EPJA '08, NPA '10]
 - ^{22}C [Yamashita, Carvalho, Frederico, Tomio, PLB '11]
 - ^{22}C Acharya, C.J., Phillips, PLB 723 (2013)
- charge radius of $2n$ s-wave halos [Hagen, Hammer, Platter, EPJA '13]
- heaviest $2n$ s-wave halo:
 - ^{62}Ca [Hagen, Hagen, Hammer, Platter, PRL '13]
fit n - ^{60}Ca scattering length from coupled-cluster calculations
- ^6He : n - α in p-wave resonance
 - EFT + Gamow shell model [Rotureau, van Kolck, FBS '13]
 - EFT + Faddeev Equations C.J., Elster, Phillips, PRC 90, 044004 (2014)

- Implication of excited Efimov halo
for an excited state with $S_{2n} > 0$



Canham, Hammer, EPJA '08

- Implication of excited Efimov halo
for an excited state with $S_{2n} > 0$



Canham, Hammer, EPJA '08

	^{20}C	^{21}C	^{22}C
bound/unbound	bound		
ground state	0^+		
binding/virtual energy [MeV]	S_{2n} : 3.50(24) AME2012		
matter radius r_m [fm]	2.97(5) Ozawa et al. '01		

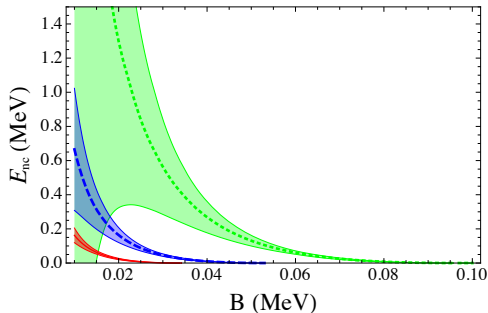
	^{20}C	^{21}C	^{22}C
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matter radius r_m [fm]	2.97(5) Ozawa et al. '01	—	5.4(9) Tanaka et al. '10

	^{20}C	^{21}C	^{22}C
bound/unbound	bound	unbound	bound
ground state	0^+	$S_{1/2}$	0^+
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	AME2012	AME2012 > 2.9	AME2012 S_{2n} : -0.14(46)
		Mosby et al. '13	Gaufrey et al. '12
matter radius	2.97(5)	—	5.4(9)
r_m [fm]	Ozawa et al. '01		Tanaka et al. '10

- Halo EFT [Acharya, C.J., Phillips, PLB **723** 196 (2013)]
we fit to ^{22}C matter radius to constrain:
 - E_{nc} in ^{21}C ($a < 0$)
 - S_{2n} in ^{22}C

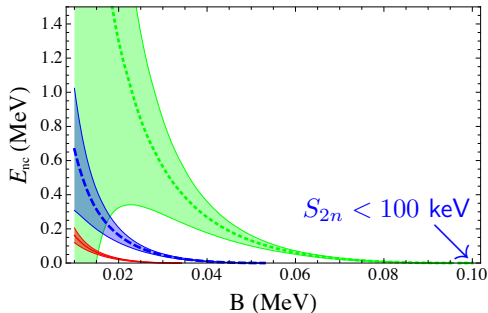
Input: $r_m[^{22}\text{C}] = 5.4^{+0.9}_{-0.9}$ fm



bands: uncertainty from higher-order EFT

Acharya, C.J., Phillips, PLB **723** 196 (2013)

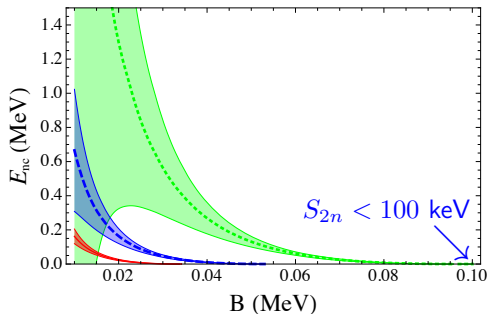
Input: $r_m[^{22}\text{C}] = 5.4^{+0.9}_{-0.9}$ fm



bands: uncertainty from higher-order EFT

Acharya, C.J., Phillips, PLB **723** 196 (2013)

Input: $r_m[^{22}\text{C}] = 5.4^{+0.9}_{-0.9}$ fm



c.f. Horiuchi & Suzuki, PRC '06 (theo)

→ $S_{2n} < 570$ keV

Yamashita et al., PLB '11 (theo)

→ $S_{2n} < 120$ keV

Fortune & Sherr, PRC '12 (theo)

→ $S_{2n} < 220$ keV

AME2012 (expt)

→ $S_{2n} < 170$ keV

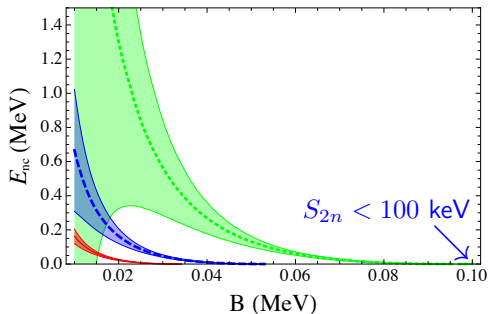
Gaudefroy et al., PRL '12 (expt)

→ $S_{2n} < 320$ keV

bands: uncertainty from higher-order EFT

Acharya, C.J., Phillips, PLB **723** 196 (2013)

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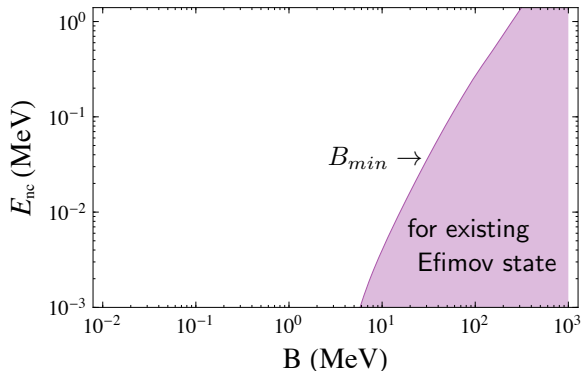
→ $S_{2n} < 320$ keV

Mosby et al., NPA '13 $E_{nc} > 2.9$ MeV

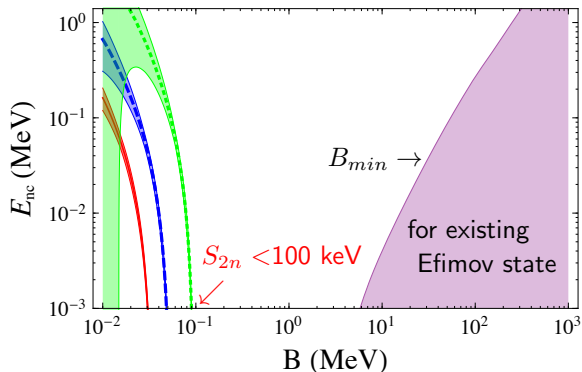
Halo EFT → $S_{2n} < 20$ keV

(inconsistent with other measurements)

- possibility of finding Efimov excited states in ^{22}C
Mazumdar *et al.* PRC '00; Frederico *et al.* PPNP '12; Acharya, C.J., Phillips PLB '13
- an Efimov excited state exists if G.S. $S_{2n} > B_{min}$



- possibility of finding Efimov excited states in ^{22}C
Mazumdar et al. PRC '00; Frederico et al. PPNP '12; Acharya, C.J., Phillips PLB '13
- an Efimov excited state exists if G.S. $S_{2n} > B_{min}$



- The Efimov excited state only occurs in ^{22}C if:
 \rightarrow the virtual energy of ^{21}C $E_{nc} < 1$ keV (unlikely)

AME2012

	^{21}N	^{22}N	^{23}N
S_{1n} [MeV]	4.59(11)	1.28(21)	1.79(36)
S_{2n} [MeV]	6.75(10)	5.87(20)	3.07(31)

AME2012

	^{21}N	^{22}N	^{23}N
S_{1n} [MeV]	4.59(11)	1.28(21)	1.79(36)
S_{2n} [MeV]	6.75(10)	5.87(20)	3.07(31)

- We study ^{23}N in $n + n + ^{21}\text{N}$ cluster model

Zhang, Ren, Lyu, C.J., PRC 91, 024001 (2015)

AME2012

	^{21}N	^{22}N	^{23}N
S_{1n} [MeV]	4.59(11)	1.28(21)	1.79(36)
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- **We study ^{23}N in $n + n + ^{21}\text{N}$ cluster model**
Zhang, Ren, Lyu, C.J., PRC 91, 024001 (2015)
- **Faddeev equation in hyperspherical harmonics expansion**
numerical tool: FaCE [Thompson, Nunes, Danilin, Comp. Phys. Comm. '04]

- realistic nn : Gogny-Pires-De Turreil (GPT)
- phenomenological n - ^{21}N : Wood Saxon

$$V_{n\text{-core}}(r) = -\frac{V_0}{1 + \exp(\frac{r-r_0}{a})} - \frac{V_{\text{so}}}{ra} \frac{\exp(\frac{r-r_0}{a})}{(1 + \exp(\frac{r-r_0}{a}))^2} \mathbf{L} \cdot \mathbf{S}$$

- core-neutrons occupy $(0s_{1/2})^2 (0p_{3/2})^4 (0p_{1/2})^2 (0d_{5/2})^6$ shells
 $\epsilon(0d_{5/2}) = S_{1n}[^{21}\text{N}]$
- valence neutrons occupy either $(1s_{1/2})^2$ or $(0d_{3/2})^2$
 $\epsilon(1s_{1/2}) = S_{1n}[^{22}\text{N}]$

- We tune $V_{n\text{-core}}$ to reproduce
$$^{21}\text{N } S_{1n} = 4.59(11) \text{ MeV}$$
$$^{22}\text{N } S_{1n} = 1.28_{-21}^{+21} \text{ MeV}$$
- We predict S_{2n} and r_m

S_{2n}	r_m	S_{2n}^*	r_m^*
MeV	fm	MeV	fm
4.13	2.969	0.315	4.272
3.64	2.985	0.185	4.358
3.13	3.004	0.069	4.476

Experiment: $S_{2n} = 3.07(31) \text{ MeV}$

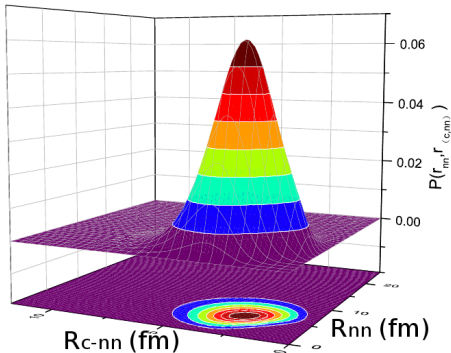
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- We predict S_{2n} and r_m
- add 3BF $V_3(\rho) = W_0 e^{-\rho^2/\rho_0^2}$ to reproduce
$$^{23}\text{N } S_{2n} = 3.07 \text{ MeV}$$
- Predictions in S_{2n} and r_m

S_{2n}	r_m	S_{2n}^*	r_m^*
MeV	fm	MeV	fm
4.13	2.969	0.315	4.272
3.64	2.985	0.185	4.358
3.13	3.004	0.069	4.476

S_{2n}	r_m	S_{2n}^*	r_m^*
MeV	fm	MeV	fm
3.07	3.022	0.195	4.629
3.07	3.019	0.128	4.790
3.07	3.011	0.064	5.011

Experiment: $S_{2n} = 3.07(31) \text{ MeV}$

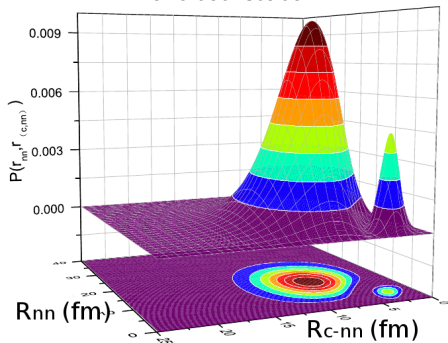
ground state



$$(1s_{1/2})^2 \text{ 95\%}$$

$$(0d_{3/2})^2 \text{ 5\%}$$

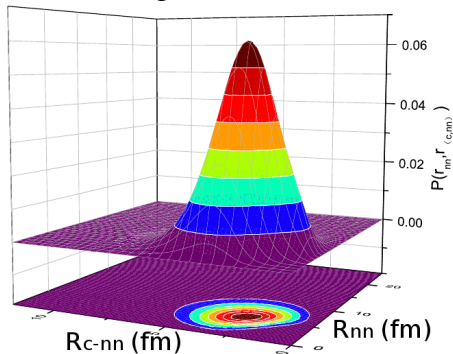
excited state



$$(1s_{1/2})^2 \text{ 77\%}$$

$$(0d_{3/2})^2 \text{ 23\%}$$

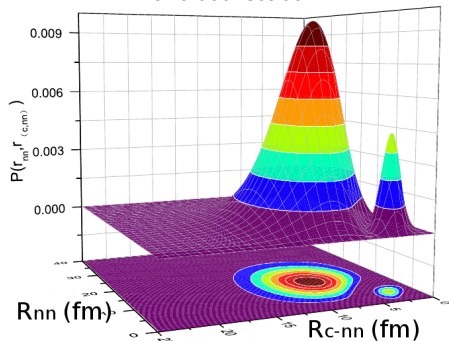
ground state



$$(1s_{1/2})^2 \text{ 95\%}$$

$$(0d_{3/2})^2 \text{ 5\%}$$

excited state



$$(1s_{1/2})^2 \text{ 77\%}$$

$$(0d_{3/2})^2 \text{ 23\%}$$

- future work: Halo EFT analysis of universal correlations in ^{23}N

● experiment in ${}^6\text{He}$

- matter radius Tanihata *et al.* '92, Alkhazov *et al.* '97, Kislev *et al.* '05
- charge radius Wang *et al.* '04, Mueller *et al.* '07
- ${}^6\text{He}$ mass Brodeur *et al.* '12

● cluster model

- separable potential Ghovanlou, Lehman '74
- variational method Funada *et al.* '94
- density-dependent nn contact interaction Esbensen *et al.* '97
- Wood Saxon $n\alpha + \text{GPT } nn$ Danilin, Thompson, Vaagen, Zhukov '98

● *ab initio* calculation

- no-core shell model Navrátil *et al.* '01; Sääf, Forssén '14
- NCSM-RGM Romero-Redondo *et al.* '14
- Green's function Monte Carlo Pieper *et al.* '01; '08
- hyperspherical harmonics (EIHH) Bacca *et al.* '12

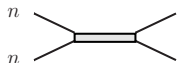
- halo EFT

- explore **universal correlations** in ${}^6\text{He}$
- compare **predictions** with experiments and *ab initio* calculations

Rotureau, van Kolck *Few Body Syst.* **54** 725 2013

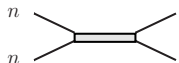
C.J., Elster, Phillips, *PRC* **90**, 044004 (2014)

- nn interaction is dominated by the 1S_0 state


$$= \frac{1}{4\pi^2\mu_{nn}} \frac{1}{-1/a_0 + r_0k^2/2 - ik}$$

$$a_0 = -18.7 \text{ fm}, r_0 = 2.75 \text{ fm} \text{ González Trotter et al. '99}$$

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$$a_0 = -18.7 \text{ fm}, r_0 = 2.75 \text{ fm} \text{ González Trotter et al. '99}$$

- LO EFT: $r_0 \rightarrow 0$

- $n\alpha$ interaction is dominated by the ${}^2P_{\frac{3}{2}}$ state

$$\begin{array}{c} n \\ \alpha \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \text{---} \text{---} \text{---} \begin{array}{c} \diagup \\ \diagdown \end{array} = \frac{1}{4\pi^2\mu_{n\alpha}} \frac{\vec{p} \cdot \vec{q}}{-1/a_1 + r_1 k^2/2 - ik^3}$$

$$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1} \text{ Ardnt et al. '73}$$

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$$a_1 = -62.95 \text{ fm}^3, r_1 = -0.8819 \text{ fm}^{-1} \text{ Ardnt et al. '73}$$

- $r_1 \not\rightarrow 0$ Nishida '12
- keep both a_1 and r_1 in LO EFT

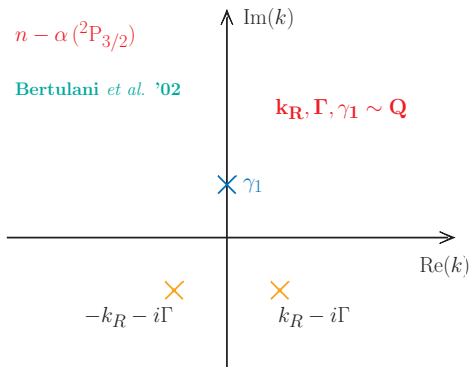
- $n\alpha$ EFT power counting: Bertulani, Hammer, van Kolck NPA '02

- $a_1 \sim 1/(Q^3)$ $r_1 \sim Q$
- two fine tunings at LO

- ${}^2P_{3/2}$:

shallow resonance: $k_R, \Gamma \sim Q$

shallow bound state: $\gamma_1 \sim Q$



adopted by Rotureau, van Kolck Few Body Syst. (2013) for ${}^6\text{He}$

- $n\alpha$ EFT power counting: Bedaque, Hammer, van Kolck PLB '02

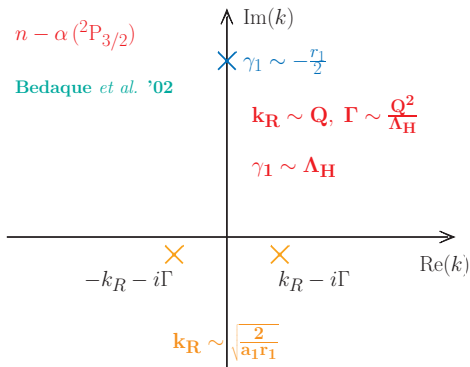
- $a_1 \sim 1/(Q^2 \Lambda_H)$ $r_1 \sim \Lambda_H$
- $Q/\Lambda_H \sim 0.15$
- one fine tuning at LO

- ${}^2P_{3/2}$:

shallow resonance:

$$k_R \sim Q, \quad \Gamma \sim Q^2/\Lambda_H$$

deep bound state: $\gamma_1 \sim \Lambda_H$



adopted by C.J., Elster, Phillips, PRC 90, 044004 (2014) for ${}^6\text{He}$

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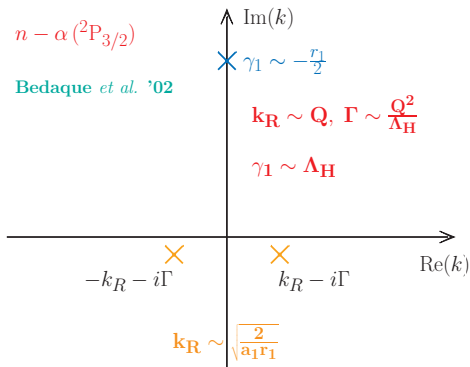
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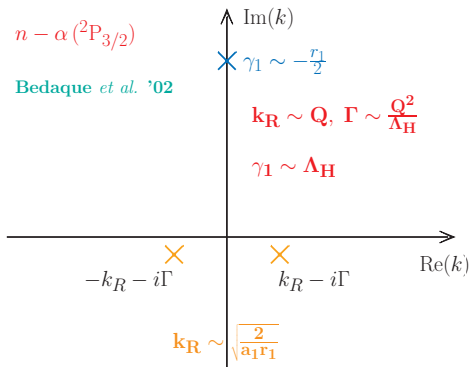
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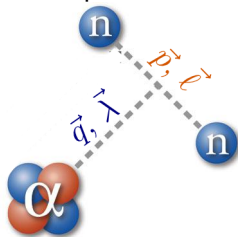


$n - \alpha$ ${}^1S_{1/2}$ and ${}^2P_{1/2} \rightarrow$ beyond LO

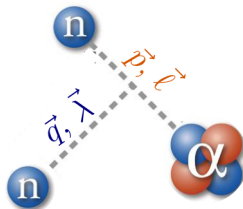
adopted by C.J., Elster, Phillips, PRC 90, 044004 (2014) for ${}^6\text{He}$

- Jacobi-momentum

α spectator



n spectator

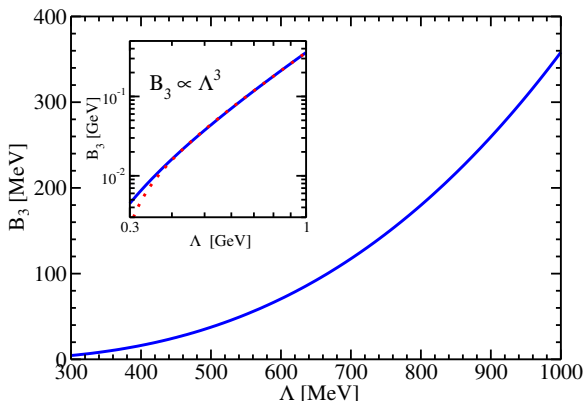


spin-orbit coupling for ${}^6\text{He}$ ($J = 0^+$)

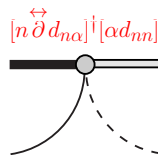
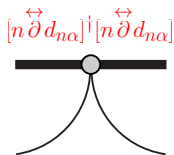
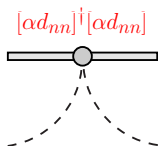
pair, spec	pair	spectator	total L, S	total J
nn, α	$\ell_{nn} = 0, s_{nn} = 0$	$\lambda_{\alpha-nn} = 0, s_{\alpha-nn} = 0$	$L = 0, S = 0$	$J = 0^+$
$n\alpha, n$	$\ell_{n\alpha} = 1, s_{n\alpha} = \frac{1}{2}$	$\lambda_{n-n\alpha} = 1, s_{n-n\alpha} = \frac{1}{2}$	$L = 0, S = 0$ $L = 1, S = 1$	

- without $nn\alpha$ 3-body force:

- S_{2n} is strongly cutoff dependent: $S_{2n} \sim \Lambda^3$ ← need 3body force!



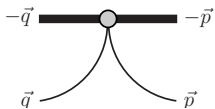
- candidates for $nn\alpha$ counterterms



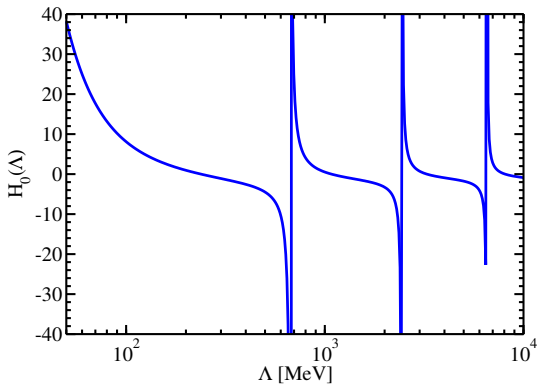
- **only needs** $[n \overleftrightarrow{\partial} d_{n\alpha}]^\dagger [n \overleftrightarrow{\partial} d_{n\alpha}]$ counterterm

- Pauli principle
- A similar p-wave three-body counterterm is discovered by Rotureau, van Kolck *Few Body Syst.* **54** 725 2013

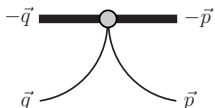
- p-wave 3BF:
 reproduce $S_{2n} = 0.973 \text{ MeV}$



$$= M_n \text{ } qp \frac{H(\Lambda)}{\Lambda^2}$$

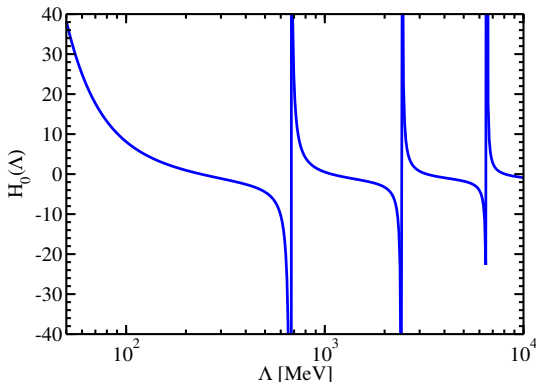
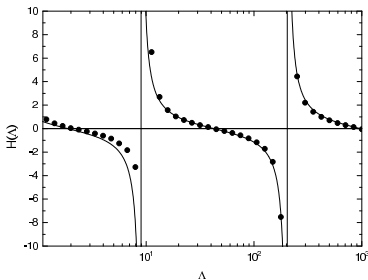


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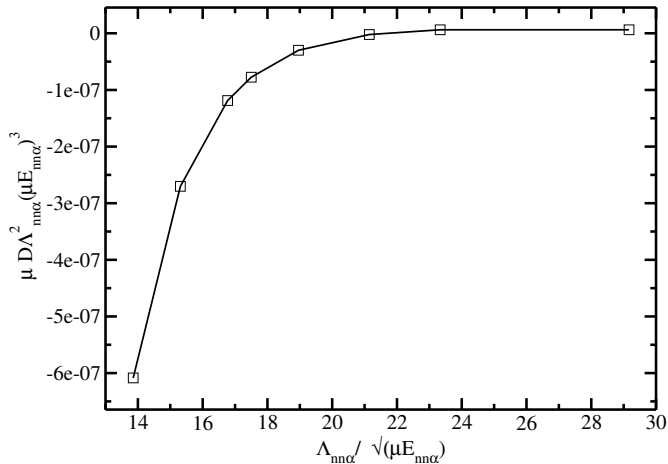


$$= M_n qp \frac{H(\Lambda)}{\Lambda^2}$$

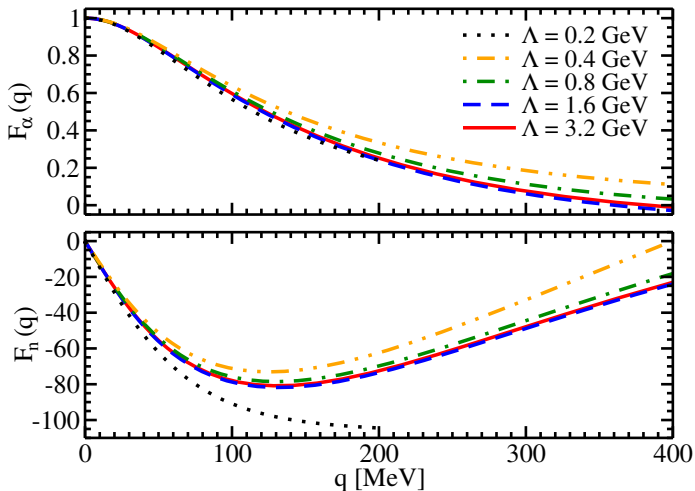
- log oscillation
- No limit cycle
(c.f. 3-body in S-wave)



- Three-body force by Rotureau, van Kolck Few Body Syst. (2013)
reproducing $S_{2n} = 0.973$ MeV

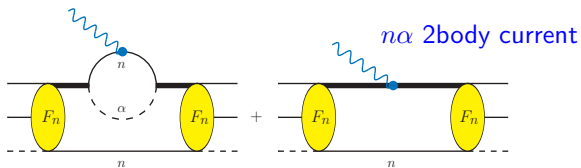


$F_\alpha(\alpha, nn)$ and $F_n(n, \alpha n)$ are cutoff independent

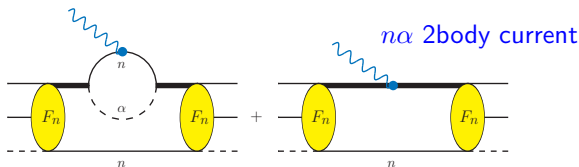


C.J., Elster, Phillips, PRC **90**, 044004 (2014)

- 3-body form factor (with p-wave n -core interactions)

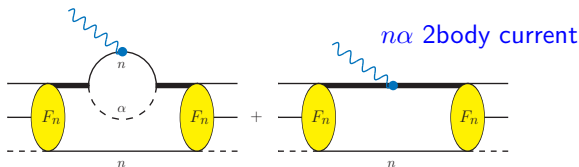


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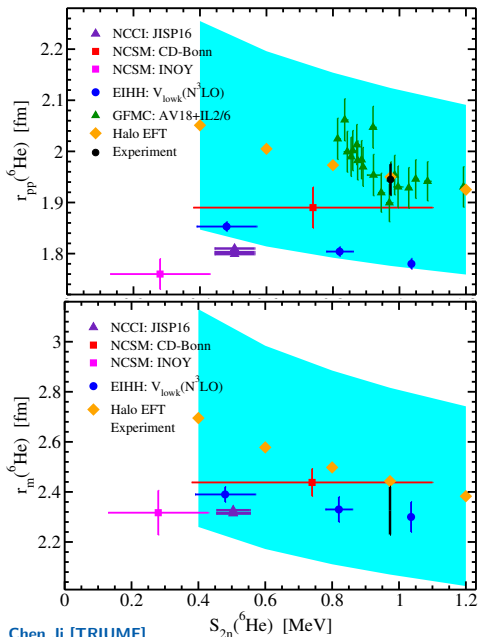
- The $n\alpha$ two-body current counterterm is fixed by r_1 in $n\alpha$ $3/2^-$ state
- It does not require an additional 3-body input

- 3-body form factor (with p-wave n -core interactions)



- The $n\alpha$ two-body current counterterm is fixed by r_1 in $n\alpha$ $3/2^-$ state
- It does not require an additional 3-body input
- matter/charge radii

$$F_{c(m)}(q^2) = 1 - \frac{1}{6} \langle r_{c(m)}^2 \rangle q^2 + \dots$$



[Preliminary]

- He-6 point-proton radius
- He-6 matter radius

compare with

NCCI: Caprio, Maris, Vary, PRC '14

NCSM: Caurier, Navratil, PRC '06

GFMC: Pieper, RNC '08

EIHH: Bacca, Barnea, Schwenk, PRC '12

Halo EFT: preliminary (uncertainty)

- The nuclear charge radius can be extracted from the atomic isotope shifts:

$$\delta_{AA'} = \delta_{AA'}^{MS} + K_{FS} \delta\langle r^2 \rangle_{AA'}$$

- mass shift term $\delta_{AA'}^{MS}$
- charge radii difference $\delta\langle r^2 \rangle_{AA'} = \langle r_A^2 \rangle - \langle r_{A'}^2 \rangle$

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- The nuclear polarization δ_{pol} contribute to the mass shift term $\delta_{AA'}^{MS}$

$$\delta_{pol} = \mathcal{A} \underbrace{\left[\int_0^\infty d\omega \frac{\sigma_\gamma(\omega)}{\omega^2} \right]}_{\propto \alpha_E} + \mathcal{B} \underbrace{\left[\int_0^\infty d\omega \frac{\sigma_\gamma(\omega)}{\omega^2} \ln \frac{2\omega}{m} \right]}_{\propto \alpha_{E \log}} + \dots$$

Pachucki, Moro PRA '07

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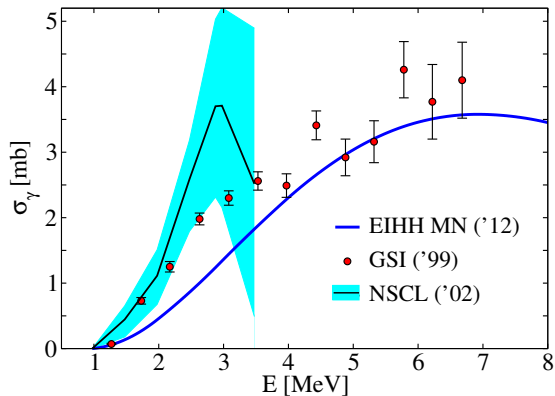
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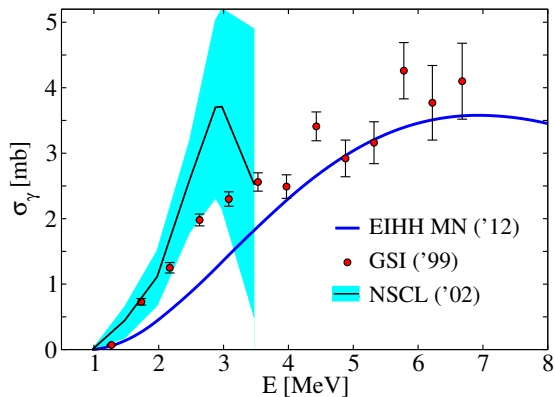
$$\delta_{pol} = \underbrace{\mathcal{A} \left[\int_0^\infty d\omega \frac{\sigma_\gamma(\omega)}{\omega^2} \right]}_{\propto \alpha_E} + \underbrace{\mathcal{B} \left[\int_0^\infty d\omega \frac{\sigma_\gamma(\omega)}{\omega^2} \ln \frac{2\omega}{m} \right]}_{\propto \alpha_{E \log}} + \dots$$

Pachucki, Moro PRA '07

- δ_{pol} is larger in atoms with unstable nuclear isotopes (lower threshold energy)
halo nuclei: δ_{pol} is important for accurately extracting nuclear charge radii



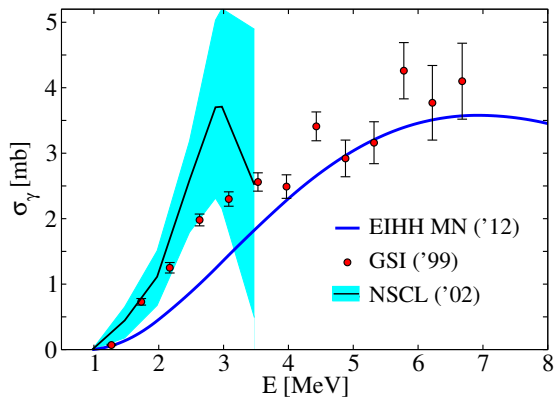
Pic:
Goerke, Bacca, Barnea PRC '12



Pic:

Goerke, Bacca, Barnea PRC '12

σ_γ is dominated by physics at \sim few MeVs

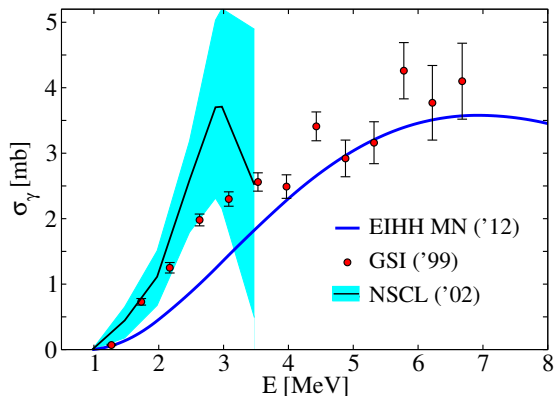


Pic:

Goerke, Bacca, Barnea PRC '12

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current data for $\sigma_\gamma(\omega)$ are not very accurate



Pic:

Goerke, Bacca, Barnea PRC '12

σ_γ is dominated by physics at \sim few MeVs

current data for $\sigma_\gamma(\omega)$ are not very accurate

- *ab initio* methods are computationally expensive for halo systems / continuum
- halo EFT works economically at low energies
- future EFT calculations of σ_γ in ^6He ; δ_{pol} in ^6He isotope shift

- Halo EFT describes structure/reaction in halo nuclei in a systematic expansion of R_{core}/R_{halo}
- Halo EFT rejuvenate cluster models with a systematic uncertainty estimates
- Halo EFT can be complimentary to *ab initio* calculations
 - adopt inputs from *ab initio* results
 - benchmark with *ab initio* calculations
 - explain universal correlations from observables in *ab initio* work

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Bijaya Acharya



Zhongzhou Ren

Liuyang Zhang **Nanjing University**

Mengjiao Lyu



תודה
 Dankie Gracias
 Спасибо شكراً
 Merci Takk
 Köszönjük Terima kasih
 Grazie Dziękujemy Děkojame
 Ďakujeme Vielen Dank Paldies
 Kiitos Täname teid 谢谢
Thank You Tak
 感谢您 Obrigado Teşekkür Ederiz
 Σας Ευχαριστούμ 감사합니다
 ඔබට
 Bedankt Děkujeme vám
 ありがとうございます
 Tack