Towards Faddeev-AGS equations in a Coulomb basis in momentum space

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Focus on deuteron-induced reactions

Deuteron-induced reactions are a point of the theoretical interest

To treat experimental results on the exotic nuclei beams with the deuterated targets in inverse kinematics.

Low energy

E_{lab} from few up to 50 MeV/A.

Broad range of involved nuclei

From He isotopes up to Pb.

Main focus is on (d, p)

But the tools must analyze other three-body reaction channels (e.g. deuteron breakup) on the same footing.

$d + A$ reaction channels

Approximated three-body:

- elastic scattering,
- direct deuteron breakup $A(d, pn)A$ without core excitation,
- direct deuteron stripping (d, p) or (d, n) with nucleon capture without core excitation.

Approximated three-body with excitations:

- direct deuteron breakup and stripping with core excitations,
- non-elastic direct (d, d') scattering with core excitations.

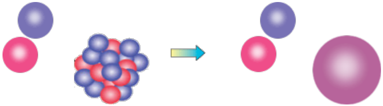
Many-body problems:

- core breakup,
- compound-nucleus reactions. . .

Deuteron-induced reactions as three-body problem

Deltuva *et al.*. // Phys. Rev. C **76**, 064602 (2007)

- Neglect the internal degrees of freedom of the nucleus to get 3-body problem.
- Use Faddeev-AGS approach to treat the problem.



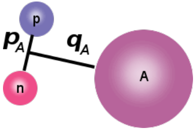
To do later:

Take into account some target degrees of freedom:

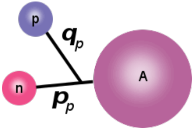
- collective core excitations,
- more complicated internal dynamics.

Formal considerations about a 3-body system

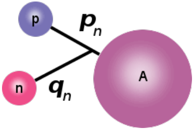
3-body configurations & Jacobi coordinates



$A(pn) = d + A$



$p(nA)$



$n(Ap)$

- Numerical indices: $A \Leftrightarrow 1$, $p \Leftrightarrow 2$, $n \Leftrightarrow 3$.
- 'Odd man out' configuration notation: $a(bc)$ labeled by a.

Denoting the two-body interactions

- $V_a \equiv V_{bc}$, i.e. a is a spectator.

3-body Hamiltonian

$$H = H_0 + V_{np} + U_{nA} + v_{pA}, \qquad H_0 = \mathbf{q}_a/2M_a + \mathbf{p}_a/2\mu_a.$$

Interactions between the particles

$$v_{pA} = V^C_{pA} + v^S_{pA}, \qquad V^C_{pA} = \frac{Z_A\alpha^2}{r}, \qquad v^S_{pA} = U_{pA} + v^{cd}_{pA},$$

- V_{np}: NN-interaction potential, e.g. chiral.
- U_{nA}, U_{pA}: phenomenological optical potentials, e.g. CH89.
- v^{cd}_{pA}: ‘short-range Coulomb interaction’, usually, the charged sphere potential.

Faddeev approach to a three-body problem

- $|\phi\rangle = \sum_{a=1}^{3} |\phi_a\rangle [= |\phi_1\rangle]$ is initial state $[A + (pn)]$.

Faddeev components ψ_a

$$|\Psi\rangle = |\phi\rangle + \sum_{a=1}^{3} g_0^C(E) V_a^S |\Psi\rangle, \quad g_0^C(E) = \left(E - H_0 - V_{pA}^C + i\varepsilon\right)^{-1},$$

$$|\Psi\rangle = \sum_a \underbrace{\delta_{a,1}|\phi_1\rangle + g_0^C(E) V_a^S |\Psi\rangle}_{|\psi_a\rangle} = \sum_a |\psi_a\rangle.$$

Faddeev-type equations

$$\begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) V_1^S \sum_a |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) V_2^S \sum_a |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) V_3^S \sum_a |\psi_a\rangle \end{cases} \Rightarrow \begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) t_1^S \sum_{a\neq 1} |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) t_2^S \sum_{a\neq 2} |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) t_3^S \sum_{a\neq 3} |\psi_a\rangle \end{cases}$$

Coulomb interaction in momentum space

Screening factor (g_0^C becomes g_0, $\widetilde{V}^C$ goes with the other potentials)

$$\widetilde{V}^C = \frac{Z_1 Z_2 \alpha^2}{r} \exp(-\mu r^n).$$

- Various short-range amplitude corrections required.

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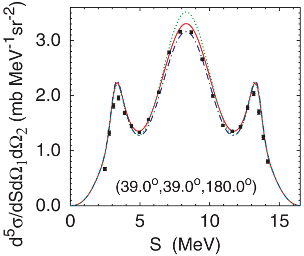
It works for *dp* scattering

- Deltuva // Phys. Rev. C **80**, 064002 (2009).
- $E_{p(lab)} = 9$ MeV.
- The most advanced screening techniques to the date.
- AV18 and AN18+UIX potentials.

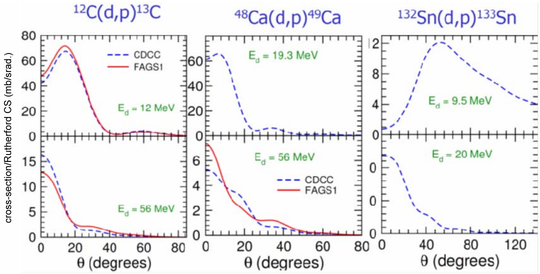


$A(d,p)B$ reaction with screened Coulomb interaction

It works for the nuclei up to ${}^{48}_{20}$Ca (Deltuva)

* Deltuva. // Phys. Rev. C **80**, 064002 (2009).
* Upadhyay, Deltuva, Nunes. // Phys. Rev. C **85**, 054621 (2012).

‘The charge wall’



- Discrepancy between the results of CDCC & Faddeev calculations becomes unreasonably large as the nucleus’ charge climbs up.
- Faddeev-AGS method hits the ‘charge wall’.

Faddeev approach on the way to the highly-charged nuclei

'Total recall'

$$\begin{cases} \widetilde{V}^C = \dfrac{Z_1 Z_2 \alpha^2}{r} \exp(-\mu r^n) \\ g_0 = [E - H_0 + i\varepsilon]^{-1} \end{cases} \Rightarrow \begin{cases} V^C = \dfrac{Z_1 Z_2 \alpha^2}{r} \\ g_0^C = \left[E - H_0 + i\varepsilon - V_{pA}^C\right]^{-1} \end{cases},$$

$$(H_0 = \mathbf{q}_a/2M_a + \mathbf{p}_a/2\mu_a).$$

In terms of the g_0 and g_0^C proper basis states (simplified notation):

$$g_0 = \frac{|\phi\rangle\langle\phi|}{E - H_0 + i\varepsilon}, \quad \Rightarrow \quad g_0^C = \frac{\left|\psi_{\mathbf{k},\eta}^C\right\rangle \left\langle\psi_{\mathbf{k},\eta}^C\right|}{E - H_0 + i\varepsilon},$$

$$\eta = Z_A e^2 \mu_p / k.$$

Coulomb function in momentum space

3D:

$$\left\langle \mathbf{p} \left| \psi^C_{\mathbf{k},\eta} \right.\right\rangle = \psi^C_{\mathbf{k},\eta}(\mathbf{p}) = -\frac{1}{2\pi^2} \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \frac{\left[p^2 + (\gamma - ik)^2\right]^{i\eta}}{\left[\gamma^2 + (\mathbf{p}-\mathbf{k})^2\right]^{1+i\eta}} \right\}.$$

* Guth and Mullin. // Phys. Rev. **83**, 667 (1951).

In partial waves:

$$\psi^C_{l,k,\eta}(p) = -\frac{2\pi e^{\pi\eta/2}}{pk} \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \left[\frac{p^2 - (k+i\gamma)^2}{2pk}\right]^{i\eta} \frac{Q_l^{i\eta}(\zeta)}{(\zeta^2-1)^{i\eta/2}} \right\},$$

$$\eta = Z_1 Z_2 \alpha^2 \mu / k; \qquad \zeta = (k^2 + p^2 + \gamma^2)/2kp.$$

* Dolinskii and Mukhamedzhanov. // Sov. Journ. of Nucl. Phys., vol. 3, No. 2, p. 180 (1966).

The two representations

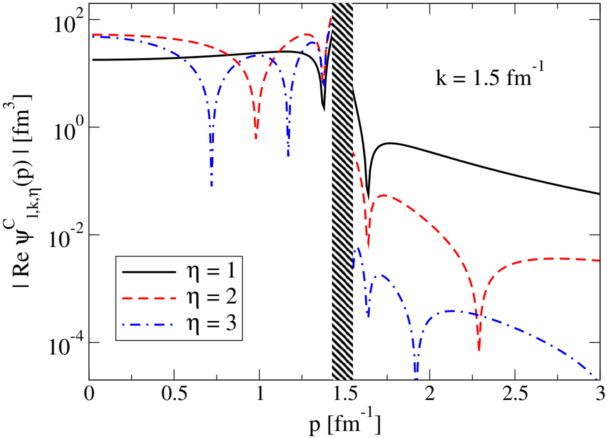
The 'regular' representation ($p \neq k$):

$$\psi^C_{l,k,\eta}(p) = -\frac{4\pi\eta e^{-\pi\eta/2} q (pk)^l}{(p^2+k^2)^{1+l+i\eta}} \lim_{\gamma\to+0} \left[p^2 - (k+i\gamma)^2\right]^{-1+i\eta}$$
$$\times \left[\frac{\Gamma(1+l+i\eta)}{(1/2)_{l+1}}\right] {}_2F_1\left(\frac{2+l+i\eta}{2}, \frac{1+l+i\eta}{2}, l+3/2; \frac{4k^2p^2}{(p^2+k^2)^2}\right).$$

The 'pole-proximity' representation ($p \approx k$):

$$\psi^C_{l,k,\eta}(p) = i\frac{2\pi}{p}\exp(-\pi\eta/2 + i\sigma_l)\left[\frac{(p+k)^2}{4pk}\right]^l \lim_{\gamma\to+0} 2$$
$$\times \operatorname{Im}\left[\frac{\Gamma(1+i\eta)e^{-i\sigma_l}(p+k)^{-1+i\eta}}{(p-k+i\gamma)^{1+i\eta}} {}_2F_1\left(-l, -l-i\eta, 1-i\eta; \frac{(p-k)^2}{(p+k)^2}\right)\right].$$

Coulomb function in momentum space ($l = 0$ plot)



* Eremenko *et al.* // Comp. Phys. Comm. **187**, 195 (2015).

The two-body t-matrix elements in Coulomb basis

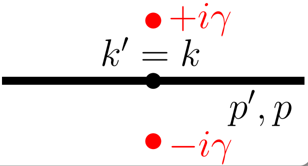
$$g_0^C(E)t^S_{(l)a} = \frac{\left|\psi^C_{l,k',\eta'}\right\rangle \overbrace{\left\langle\psi^C_{l,k',\eta'}\middle|t^S_{(l)a}\middle|\psi^C_{l,k,\eta}\right\rangle}^{t^C_{a,l}(k',k,E)} \left\langle\psi^C_{l,k,\eta}\right|}{E - H_0 + i\varepsilon},$$

$$t^C_{a,l}(k',k,E) = \int dp'\,dp\,\psi^C_{l,k',\eta'}(p')^\dagger\,t^S_{a,l}(p',p,E)\,\psi^C_{l,k,\eta}(p).$$

* Some indices omitted for the simplisity.

Pinch singularity in the elastic channel:

- $E = 2\mu k^2 = 2\mu {k'}^2$.
- Since $\gamma \to +0$, singularities of $\psi^{C\dagger}_{l,k',\eta'}$ and $\psi^C_{l,k,\eta}$ are pinching the integration contour.



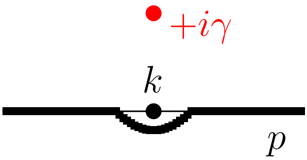
If t has a separable representation

$$t_l^S(k', k, E) = \sum_{zy} u_{l,z}(k')\, \lambda_{l,zy}(E)\, u_{l,y}(k)^\dagger,$$

$$t_l^C(k', k, E) = \sum_{zy} u_{l,z}^C(k')\, \lambda_{l,zy}(E)\, u_{l,y}^C(k)^\dagger,$$

$$u_{l,z}^C(k') = \int \frac{dp'\, {p'}^2}{2\pi^2} u_{l,z}(p') \psi^C_{l,k',\eta'}(p')^\dagger, \quad u_{l,y}^C(k)^\dagger = \int \frac{dp\, p^2}{2\pi^2} u_{l,y}(p)^\dagger \psi^C_{l,k,\eta}(p).$$

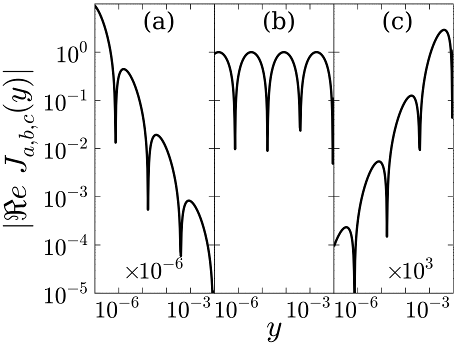
- Two independent integrals over p and p'.
- Cauchy's theorem.
- No pinch singularity!



Gel'fand-Shilov regularization for complex form-factors

$$u_l^C(k) \propto \int_{a<0}^{b>0} \frac{f(y)\,dy}{y^{1+i\eta}} \equiv \int_a^b dy\; J_a(y), \qquad \text{(e.g. } f(y) = y^2 + y + 1\text{).}$$

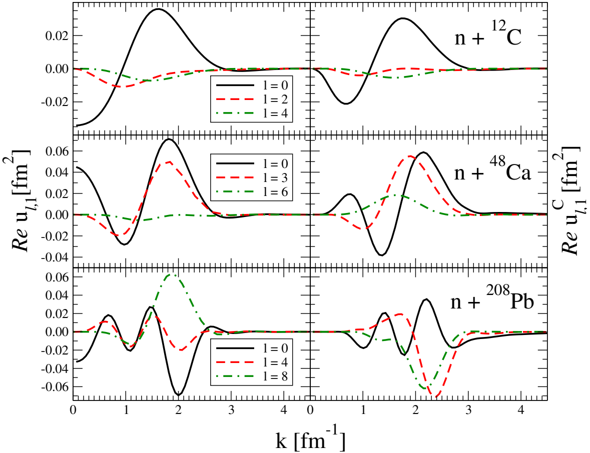
Subtract as many terms of Laurent expansion of $f(y)$ around the integrand's special point $y = 0$ as needed to split the integral and get the regular term, plus the analytically calculated terms.



* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

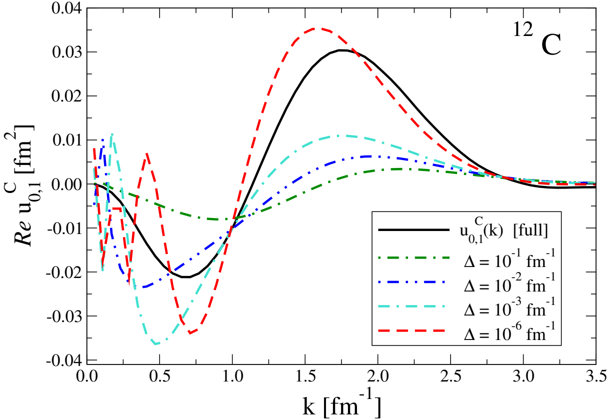
Results: form-factors in Coulomb basis

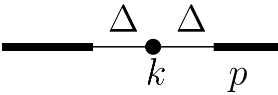
$$u_{l,1}^{C}(k) = \int \frac{dp\, p^2}{2\pi^2} u_{l,1}(p) \psi_{l,k,\eta}^{C}(p)^*.$$



* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

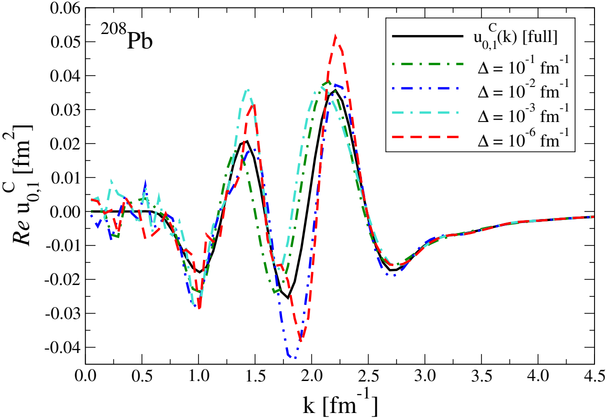
Singularity contribution for $n + {}^{12}\mathrm{C}$

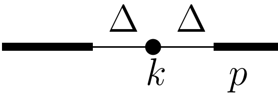




* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

Singularity contribution for $n + {}^{208}\mathrm{Pb}$





* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

Faddeev-AGS equations in Coulomb basis (in progress)

Published abstract formalism

Faddeev-AGS equations in Coulomb basis for real two-body t-matrices and spinless particles.

* Mukhamedzhanov *et al.* // Phys. Rev. C **86**, 034001 (2012).

To do:

- Take spin degrees of freedom into account.
- Deal with the 3-body singularity.
- Generalize the equations for the complex potentials.
- Take care about other implementation details, develop the algorithms and codes to solve the equations.

Objective:

- Calculate scattering observables and make the code available as open source.

Summary & Outlook

- Faddeev formalism is the theoretical tool to study deuteron-induced reactions.
- This formalism treats all possible three-body channels on the same footage.
- Momentum space is preferable due to the boundary conditions.
- Coulomb interaction can be treated properly by using the Coulomb basis in momentum space.
- Pinch singularity is avoided by choosing the two-body interactions in separable form.
- Mathematics and machinery are developed to compute Coulomb functions and matrix elements in Coulomb basis in momentum space.
- Work is in progress to cast the Faddeev-AGS equations in the Coulomb basis taking into account spin degrees of freedom and 3-body singularity in order to solve the equations numerically.