INPP INSTITUTE OF NUCLEAR & PARTICLE PHYSICS @OHIO UNIVERSITY

Microscopic Folding Potentials and Connections to Structure Description

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Collaborators over time:

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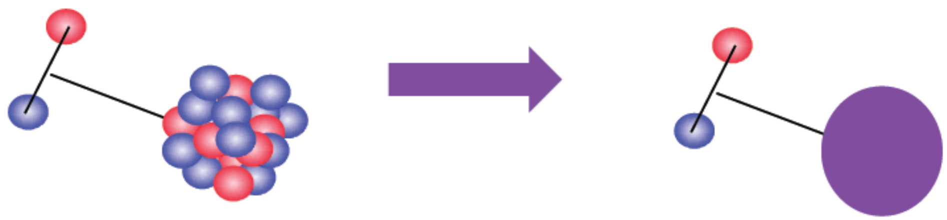
(d,p) reactions are a tool to explore structure of exotic nuclei

A+2 body problem



3-body problem

Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body poblem:

$$\mathbf{H = H_0 + V_{np} + V_{nA} + V_{pA}}$$

A+1 body problem

np interaction

Effective interactions p+A and n+A

Effective Three-Body Problem

Scattering n+A: Lippmann-Schwinger Equation
(Schrödinger Equation in momentum space)

- Hamiltonian: $H = H_0 + V$
- Transition amplitude : $T = V + V\, G_0\, T$
  - Integral equation summing interaction V to all orders
- Free Hamiltonian: $H_0 = h_0 + H_A$
  - h_0: kinetic energy of projectile '0'
  - H_A: target hamiltonian with $H_A |\Phi\rangle = E_A |\Phi\rangle$
- V: interactions between projectile '0' and target nucleons 'i' $V = \Sigma^A_{i=0}\, v_{0i}$
- Propagator is (A+1) body operator

$$G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$$

Ansatz suggested by: Watson & Faddeev-Yakubovsky:

- $T = \Sigma^{A}_{i=0} T_{0i}$
  - with $T_{0i} = v_{0i} + v_{0i} G_0(E) T$

$$
\begin{aligned}
T_{0i} &= v_{0i} + v_{0i} G_0(E) \sum_j T_{0j} \\
&= v_{0i} + v_{0i} G_0(E) T_{0i} + v_{0i} G_0(E) \sum_{j \neq i} T_{0j} \\
(1 - v_{0i} G_0(E)) T_{0i} &= v_{0i} + v_{0i} G_0(E) \sum_{j \neq i} T_{0j}
\end{aligned}
$$

$$
T_{0i} = t_{0i} + t_{0i} G_0(E) \sum_{j \neq i} T_{0j}.
$$

with $t_{0i} = v_{0i} + v_{0i} G_0(E)\, t_{0i}$

Multiple Scattering Problem

Theory developed by
- Watson
- Faddeev & Yakubovsky

Faddeev & Yakubovsky

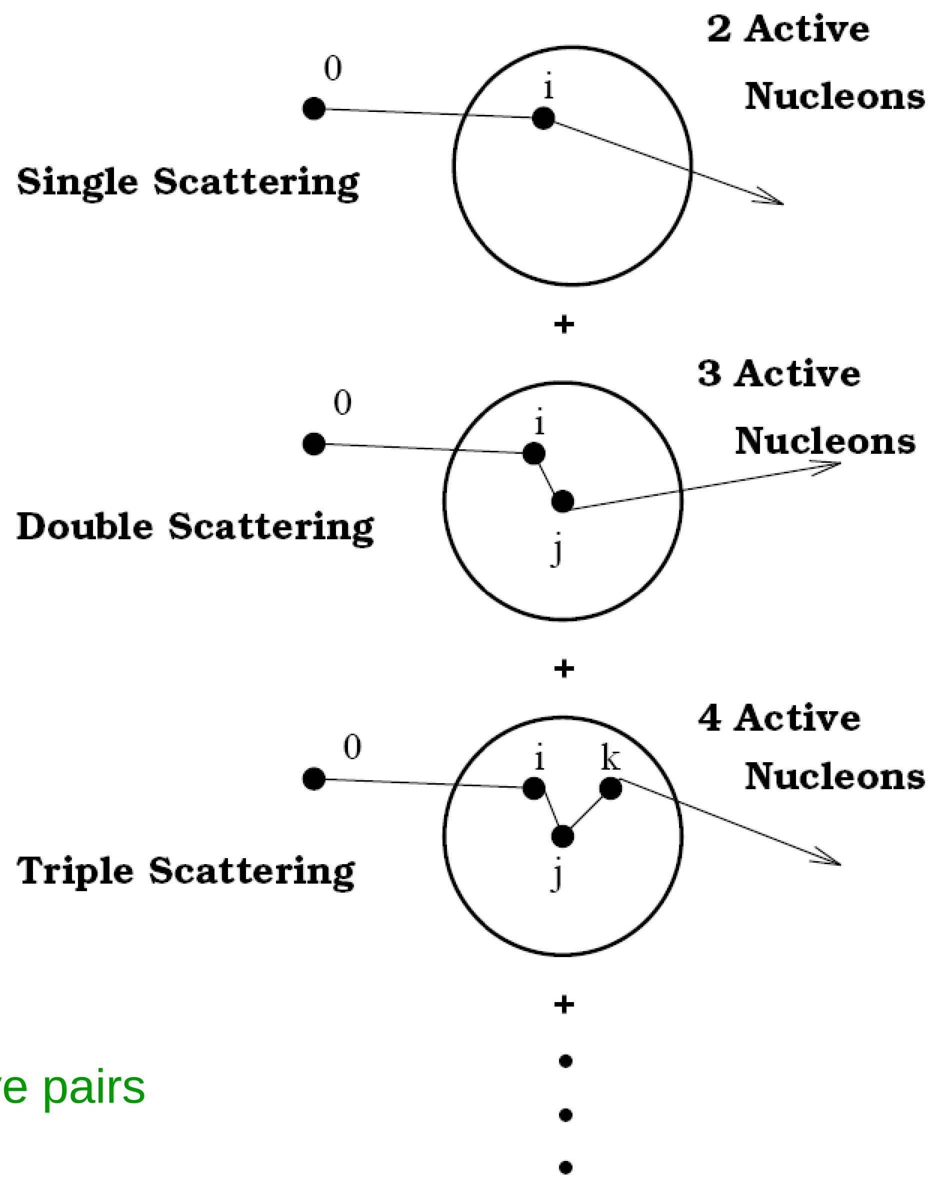
→ Different flavors of equations for practical applications
- Alt-Grassberger-Sandas
- Glöckle

Watson →
- Kerman-McManus-Thaler (KMT)
- Spectator Expansion
- Variations thereof

Multiple scattering expansions are schemes to organize the multiple scattering in terms of two-body (three-body …) scattering amplitudes

Spectator Expansion:

Organize according to active particles



Written down by

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Pauli Principle:

Antisymmetrize in active pairs

Spectator Expansion in equations

$$T = \sum_{i=1}^{A} t_{0i} \quad + \quad \sum_{i<j} (t_{ij} - t_{0i} - t_{0j})$$

Scattering from pairs

$$+ \quad \sum_{i<j<k} (t_{ijk} - t_{ij} - t_{ik} - t_{jk} + t_{0i} + t_{0j} + t_{0k}) + \ldots$$

2nd order term: $t_{ij} = (v_{0i} + v_{0j}) + (v_{0i} + v_{0j}) G_0(E) t_{ij},$

Faddeev amplitudes

Single scattering approximation

Elastic Scattering & optical potential

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state **$P = |\Phi_0\rangle\langle\Phi_0|$**
  - With $1=P+Q$ and $[P,G_0]=0$
- For elastic scattering one needs
- $P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P$
- Or
  - $\mathbf{T = U + U\,G_0(E)\,P\,T}$
  - $\mathbf{U = V + V\,G_0(E)\,Q\,U}$ $\Leftarrow$ **optical potential**

Standard: $\mathbf{U^{(1)}} \approx \Sigma^{\mathbf{A}}_{\mathbf{i=0}}\, \tau_{\mathbf{0i}}$ (1^{st} order)

with $\tau_{0i} = \mathbf{v}_{0i} + \mathbf{v}_{0i}\, \mathbf{G_0(E)}\, \mathbf{Q}\, \tau_{0i}$

$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

  ⟹ (A+1) body operator

  - Standard **“impulse approximation”:**
  - Average over $H_A \Rightarrow E_A$

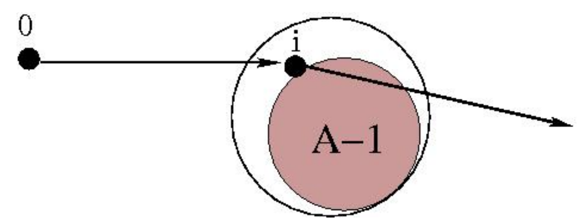
  $\rightarrow G_0(e)$ ==: two body operator

Three-body problem with particles:

o – i – (A-1)-core

o – i : NN interaction

i – (A-1) core : e.g. mean field force



“medium modification”

Chinn, Elster, Thaler, PRC48, 2956 (1993)
Chinn, Elster, Thaler, Weppner, PRC51 1418 (1995)

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$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- Deal with **Q**
  - Define "two-body" operator $\mathbf{t_{0i}^{free}}$ by
  - $t_{0i}^{free} = v_{0i} + v_{0i}\, G_0(e)\, t_{0i}^{free}$
  - and relate via integral equation to τ_{oi}
  - $\tau_{oi} = t_{0i}^{free} - t_{0i}^{free}\, G_0(e)\, \tau_{oi}$ **[integral equation]**
  - keeps iso-spin character of optical potential
  - $$U^{(1)} = \Sigma^{A}_{i=1}\, \tau_{oi} =: N\, \tau_n + Z\, \tau_p$$

Neutron and proton contributions are cleanly separated
Important for $N \neq Z$ nuclei

$t_{pp} \neq t_{np}$ **and** $\rho_p \neq \rho_n$

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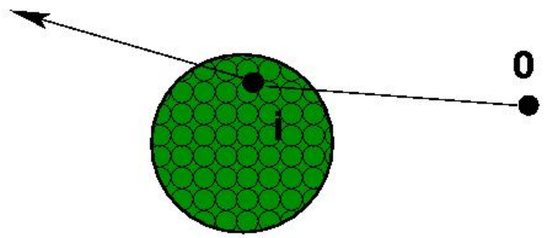
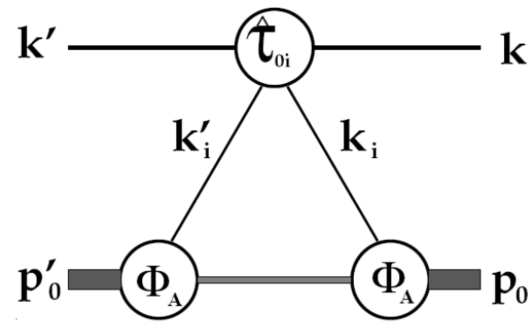
Chinn, Elster, Tandy, Redish, Thaler
Crespo, Johnson, Tostevin
Arrellano, Love

Calculation :

- **Single Scattering Optical Potential** $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$



Single Scattering



$$\langle \mathbf{k}'|\langle\phi_A|PUP|\phi_A\rangle\mathbf{k}\rangle \equiv U_{el}(\mathbf{k}',\mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}'|\langle\phi_A|\hat{\tau}_{0i}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$$

Proton scattering: $U_{el}(\mathbf{k}',\mathbf{k}) = Z\langle\hat{\tau}^{pp}_{01}\rangle + N\langle\hat{\tau}^{np}_{01}\rangle$

Optical Potential is non-local and depends on energy

Off-shell NN t-matrix and nuclear density matrix

NN amplitude $f_{NN}(k'k;E) = C \langle k'|t_{NN}(E)|k\rangle$

Variables $(E,k',k,\varphi) \Rightarrow (E, q, K, \theta)$

with $q = k' - k$

$K = \frac{1}{2} (k' + k)$

Invariant $\mathcal{I}_\alpha$	Amplitude A_α		
$\mathbb{1}$	$A(q, K, \vec{q}\cdot\vec{K})$		
$(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{w}$	$C(q, K, \vec{q}\cdot\vec{K})$		
. . . .	. . . .		
$\vec{\sigma}_1\cdot\vec{w}$ $\vec{\sigma}_2\cdot\vec{w}$	$B(q, K, \vec{q}\cdot\vec{K})$		$M(\ldots)$
$\vec{\sigma}_1\cdot\hat{q}$ $\sigma_2\cdot\hat{q}$	$E(q, K, \vec{q}\cdot\vec{K})$	or	$G(\ldots)$
$\vec{\sigma}_1\cdot\hat{K}$ $\vec{\sigma}\cdot\hat{K}$	$F(q, K, \vec{q}\cdot\vec{K})$		$H(\ldots)$
	(Wolfenstein)		(Hoshizaki)
$(\vec{\sigma}_1\cdot\hat{q}\,\vec{\sigma}_2\cdot\hat{K} + \vec{\sigma}_1\cdot\hat{K}\,\vec{\sigma}_2\cdot\hat{q})\,\vec{q}\cdot\vec{K}$	$D(q, K, \vec{q}\cdot\vec{K}) = 0$ on-shell		

Most general form

Calculate: $\langle\hat{\tau}_{01}\rangle \equiv \langle\mathbf{k}'|\langle\phi_A|\hat{\tau}_{01}(\mathcal{E})|\phi_A\rangle\mathbf{k}\rangle$

e.g. Jacobi Coordinates

$$\langle\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4...\mathbf{k}_A|\phi_A\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4...+\mathbf{k}_A-\mathbf{p}_0)\langle\zeta_\mathbf{1}\zeta_\mathbf{2}\zeta_\mathbf{3}\zeta_\mathbf{4}...\zeta_\mathbf{A-1}|\phi_A\rangle.$$

$$\langle\hat{\tau}_{01}\rangle = \int\prod_{j=1}^{A}d\mathbf{k}'_j\int\prod_{l=1}^{A}d\mathbf{k}_l\ \langle\phi_A|\zeta'_1\zeta'_2\zeta'_3\zeta'_4...\zeta'_{A-1}\rangle\delta(\mathbf{p}'-\mathbf{p}'_0)\ \langle\mathbf{k}'\mathbf{k}'_1|\hat{\tau}_{01}(\mathcal{E})|\mathbf{k}\mathbf{k}_1\rangle$$
$$\prod_{j=2}^{A}\delta(\mathbf{k}'_j-\mathbf{k}_j)\delta(\mathbf{p}-\mathbf{p}_0)\ \langle\zeta_1\zeta_2\zeta_3\zeta_4...\zeta_{A-1}|\phi_A\rangle, \qquad (2.48)$$

With single particle density matrix :

$$\rho(\zeta'_\mathbf{1},\zeta_\mathbf{1}) \equiv \int\prod_{l=2}^{A-1}d\zeta'_\mathbf{l}\int\prod_{j=2}^{A-1}d\zeta_\mathbf{j}\langle\phi_A|\zeta'_1\zeta'_2\zeta'_3\zeta'_4...\zeta'_{A-1}\rangle\ \langle\zeta_1\zeta_2\zeta_3\zeta_4...\zeta_{A-1}|\phi_A\rangle.$$

$$\langle\hat{\tau}_{01}\rangle = \int d\zeta'_\mathbf{1}\int d\zeta_\mathbf{1}\langle\mathbf{k}'\ \zeta'_\mathbf{1}+\frac{\mathbf{p}'_0}{A}|\hat{\tau}_{01}(\mathcal{E})|\mathbf{k}\ \zeta_\mathbf{1}+\frac{\mathbf{p}_0}{A}\rangle\ \rho(\zeta'_\mathbf{1},\zeta_\mathbf{1})$$
$$\delta(\frac{A-1}{A}\mathbf{p}'_0-\zeta'_\mathbf{1}-\frac{A-1}{A}\mathbf{p}_0+\zeta_\mathbf{1}).$$

Calculate:

$$\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle | \mathbf{k} \rangle$$

Jacobi Coordinates

$$\langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 \mathbf{k}_4 \ldots \mathbf{k}_A | \phi_A \rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4 \ldots + \mathbf{k}_A - \mathbf{p}_0) \langle \boldsymbol{\zeta}_1 \boldsymbol{\zeta}_2 \boldsymbol{\zeta}_3 \boldsymbol{\zeta}_4 \ldots \boldsymbol{\zeta}_{A-1} | \phi_A \rangle.$$

$$\langle \hat{\tau}_{01} \rangle = \int \prod_{j=1}^{A} d\mathbf{k}'_j \int \prod_{l=1}^{A} d\mathbf{k}_l \ \langle \phi_A | \zeta'_1 \zeta'_2 \zeta'_3 \zeta'_4 \ldots \zeta'_{A-1} \rangle \delta(\mathbf{p}' - \mathbf{p}'_0) \langle \mathbf{k}' \mathbf{k}'_1 | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \mathbf{k}_1 \rangle$$

$$\prod_{j=2}^{A} \delta(\mathbf{k}'_j - \mathbf{k}_j) \delta(\mathbf{p} - \mathbf{p}_0) \ \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \ldots \zeta_{A-1} | \phi_A \rangle \quad (2.48)$$

With single particle density matrix :

$$\rho(\boldsymbol{\zeta}'_1, \boldsymbol{\zeta}_1) \equiv \int \prod_{i=2}^{A-1} d\boldsymbol{\zeta}'_i \int \prod_{j=2}^{A-1} d\boldsymbol{\zeta}_j \langle \phi_A | \zeta'_1 \zeta'_2 \zeta'_3 \zeta'_4 \ldots \zeta'_{A-1} \rangle \ \langle \zeta_1 \zeta_2 \zeta_3 \zeta_4 \ldots \zeta_{A-1} | \phi_A \rangle.$$

$$\langle \hat{\tau}_{01} \rangle = \int d\boldsymbol{\zeta}'_1 \int d\boldsymbol{\zeta}_1 \langle \mathbf{k}' \ \boldsymbol{\zeta}'_1 + \frac{\mathbf{p}'_0}{A} | \hat{\tau}_{01}(\mathcal{E}) | \mathbf{k} \ \boldsymbol{\zeta}_1 + \frac{\mathbf{p}_0}{A} \rangle \ \rho(\boldsymbol{\zeta}'_1, \boldsymbol{\zeta}_1)$$

$$\delta(\frac{A-1}{A} \mathbf{p}'_0 - \boldsymbol{\zeta}'_1 - \frac{A-1}{A} \mathbf{p}_0 + \boldsymbol{\zeta}_1).$$

Not quite the whole story!

Where is the nucleon spin?

General Single Particle Density Matrix

Wave function ~ $\Phi_0(i) \sim f_l(i)\, Y_l^m(i)\, \chi_S^{(i)}$

Single particle density matrix

$$\rho_{I\,M_I,I\,M_I'}(i,i') \sim \sum_{k_l\,q_l;\,k_s\,q_s;\,k\,q} \begin{bmatrix} I & K & I \\ M_I' & q & M_I \end{bmatrix} \left\langle \Psi_I \,||\, \rho_{kq} \,||\, \Psi_I \right\rangle \chi^{ll'}_{k_l q_l}(i,i')\, f_l(i)\, f^*_{l'}(i')$$
$$\left\langle S\, m_s \,|\, \tau^{(i)}_{k_s q_s}(S) \,|\, S'\, m_s' \right\rangle \begin{bmatrix} k_l & k_s & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} l & l' & k_l \\ s & s & k_s \\ j & j' & k \end{Bmatrix}$$

Auxiliary tensor operator

$$\tau^{(i)}_{00} = 1$$
$$\tau^{(i)}_{k_s,q_s}\left(S=\tfrac{1}{2}\right):\quad \tau^{(i)}_{10} = 2s_z$$
$$\tau^{(i)}_{1,\pm1} = \mp\frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

$$\chi^{ll'}_{k_l\,q_l}(i,i') = \sum_{l\,l'} (-1)^{l'-\lambda'} \begin{bmatrix} l & l' & k_l \\ \lambda & -\lambda' & q_l \end{bmatrix} Y_{l\lambda}(i)\, Y^*_{l'\lambda'}(i')$$

Orbital angular momentum

Case k_s = 0

$$\left\langle \Phi_0 \middle| \mathbf{1}^{(i)} \middle| \Phi_0 \right\rangle$$

s-shell

=1

$$\rho_{0\,0,0\,0}(i,i') \sim \left\langle \Psi_s \,||\, \rho_{00} \,||\, \Psi_s \right\rangle \chi^{00}_{00}(i,i')\, f_s(i)\, f^*_s(i') \left\langle S\, m_s \,\middle|\, \tau^{(i)}_{00}(S) \,\middle|\, S'\, m'_s \right\rangle$$

p-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \sum_{k_l} \left\langle \Psi_p \,||\, \rho_{kq} \,||\, \Psi_p \right\rangle \chi^{11}_{k_l 0}(i,i')\, f_p(i)\, f^*_p(i')$$

=1

$$\left\langle S\, m_s \,\middle|\, \tau^{(i)}_{00}(S) \,\middle|\, S'\, m'_s \right\rangle \begin{Bmatrix} 1 & 1 & k_l \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix}$$

Scalar density

Case k_s = 1 $\left\langle \Phi_0 \middle| \boldsymbol{\sigma}^{(i)} \middle| \Phi_0 \right\rangle$

p-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \sum_{k_l\ q_l;\ 1\ q_s;\ k\ q} \left\langle \Psi_0 \,||\, \rho_k \,||\, \Psi_0 \right\rangle \chi^{11}_{k_l q_l}(i,i')\, f_p(i)\, f_p^*(i') \left\langle S\, m_s \,|\, \tau^{(i)}_{1q_s}(S) \,|\, S'\, m'_s \right\rangle \begin{bmatrix} k_l & 1 & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} 1 & 1 & k_l \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix}$$

with

$$\tau^{(i)}_{10} = 2s_z$$

$$\tau^{(i)}_{1,\pm1} = \mp \frac{2}{\sqrt{2}} (S_x \pm i S_y)$$

For closed shell nuclei this term is zero

NN amplitude $f_{NN}(k'k;E) = C\,\langle k'|t_{NN}(E)|k\rangle$

Variables (E,k',k,φ) $\Rightarrow$ (E, q, K, θ)

with $q = k' - k$
$K = \frac{1}{2}(k' + k)$

Invariant $\mathcal{I}_\alpha$	Amplitude A_α	
$\mathbb{1}$	$A\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}})$	
$(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{w}$	$C\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}})$	
...	...	
$\vec{\sigma}_1\cdot\vec{w}$ $\quad\vec{\sigma}_2\cdot\vec{w}$	$B\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}})$	$M\,(\ldots)$
$\vec{\sigma}_1\cdot\hat{q}$ $\quad\vec{\sigma}_2\cdot\hat{q}$	$E\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}})$ or	$G\,(\ldots)$
$\vec{\sigma}_1\cdot\hat{\mathcal{K}}$ $\quad\vec{\sigma}\cdot\hat{\mathcal{K}}$	$F\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}})$ (Wolfenstein)	$H\,(\ldots)$ (Hoshizaki)
$(\vec{\sigma}_1\cdot\hat{q}\,\vec{\sigma}_2\cdot\hat{\mathcal{K}} + \vec{\sigma}_1\cdot\hat{\mathcal{K}}\,\vec{\sigma}_2\cdot\hat{q})\,\hat{q}\cdot\hat{\mathcal{K}}$	$D\,(q, \mathcal{K}, \hat{q}\cdot\hat{\mathcal{K}}) = 0$ on-shell	

Most general form

Densities used in 1997 for closed shell optical potentials:

- Dirac-Hartree (Horowitz-Serot)

$$\rho_{t_z}(\mathbf{r}', \mathbf{r}) = \sum_{n\nu} \left[\frac{G_{n,\nu,t_z}(r')}{r'} \frac{G_{n,\nu,t_z}(r)}{r} + \frac{F_{n,\nu,t_z}(r')}{r'} \frac{F_{n,\nu,t_z}(r)}{r} \right] \frac{2J+1}{2l+1} \sum_{m_l} Y_l^{*m_l}(\hat{\mathbf{r}}') Y_l^{m_l}(\hat{\mathbf{r}}).$$

- Gogny (collaboration with C.R. Chinn crucial)
- Harmonic Oscillator (home-made for ^{16}O) for tests

$$\rho_s(\mathbf{q}, \mathbf{P}) = \frac{4\pi}{\nu_s} e^{-(P^2/\nu_s + q^2/4\nu_s)},$$

$$\rho_p(\mathbf{q}, \mathbf{P}) = \frac{2}{3} \frac{4\pi}{\nu_p} (P^2/\nu_p - q^2/4\nu_p) e^{-(P^2/\nu_p + q^2/4\nu_p)}$$

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Better Variables for calculation:

$$\mathbf{k} = \mathbf{K} - \frac{1}{2}\mathbf{q} \qquad \zeta_1 = \mathbf{P} + \frac{A-1}{2A}\mathbf{q}$$

$$\mathbf{k}' = \mathbf{K} + \frac{1}{2}\mathbf{q} \qquad \zeta_1' = \mathbf{P} - \frac{A-1}{2A}\mathbf{q}.$$

$$\langle \hat{\tau}_{01} \rangle = \langle \frac{1}{2}(\mathbf{K} - \mathbf{P} + \frac{2A-1}{2A}\mathbf{q} - \frac{\mathbf{p}_0'}{A}) | \hat{\tau}_{01}(\hat{\mathcal{E}}) | \frac{1}{2}(\mathbf{K} - \mathbf{P} - \frac{2A-1}{2A}\mathbf{q} - \frac{\mathbf{p}_0}{A}) \rangle$$

$$\rho(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}). \tag{2.59}$$

$$U_{el}(\mathbf{q}, \mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\, \hat{\tau}_{0i}(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K} - \mathbf{P}), \hat{\mathcal{E}})\ \rho_i(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q})$$

$$\hat{\mathcal{E}} = E_{NA} - \frac{(\mathbf{k} + \mathbf{k}_1)^2}{4m} = E_{NA} - (\frac{(\frac{A-1}{A}\mathbf{K} + \mathbf{P})^2}{4m})$$

Similar energy behavior as in Faddeev calculation

Deuteron pole !

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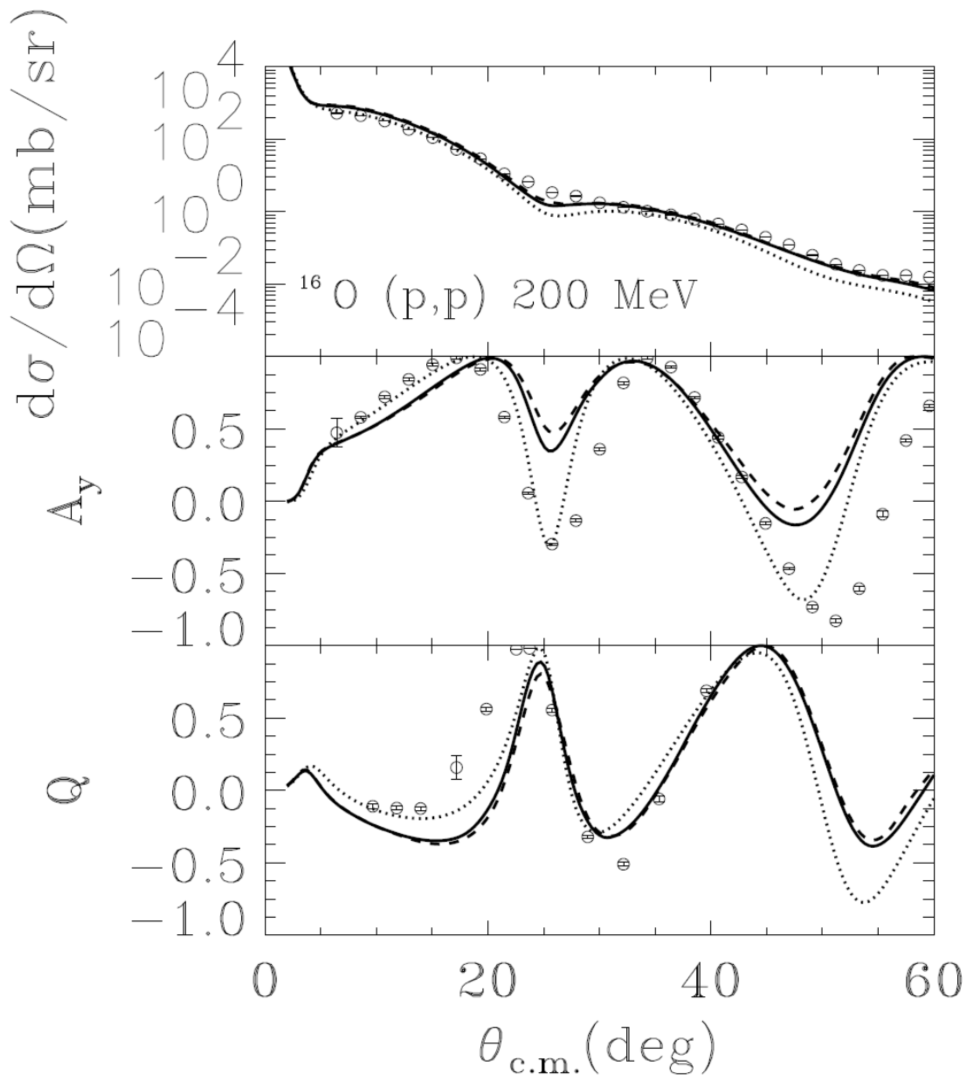


Solid: fixed ε, ⟨H_A⟩=0 MeV

Dashed: integrated ε and ⟨H_A⟩=0 MeV

Dotted: integrated ε and ⟨H_A⟩= -8 MeV

Physics:

Polarization A_y →
Spin dependence out of the scattering plane

Spin rotation parameter Q →
Spin dependence in the scattering plane

$$\tau_{0i} = v_{0i} + v_{0i}\, G_0(E)\, Q\, \tau_{0i}$$

- $G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

  ⟹ (A+1) body operator

  – Standard **"impulse approximation"**:

  – Average over $H_A \Rightarrow E_A$

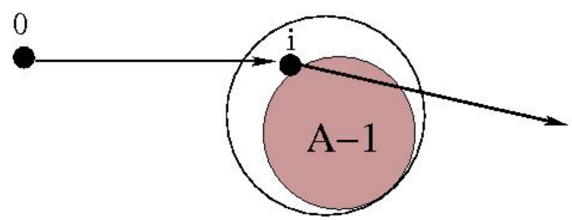
  $\rightarrow G_0(e)$ ==: two body operator

Three-body problem with particles:

o – i – (A-1)-core

o – i : NN interaction

i – (A-1) core : e.g. mean field force



"medium modification"

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Propagator (A+1) body operator:
$G_0(E) = (E - h_0 - H_A + i\varepsilon)^{-1}$

$$H_A = h_i + \sum_{j \neq i}^{A} v_{ij} + H^i$$

Chinn, Elster, Thaler
PRC 48, 2956 (1993)

h_i : kinetic energy of nucleon i

v_{ij} : interaction potential between i and j

H^i : (A-1) Hamiltionian $\rightarrow \langle H^i \rangle \equiv \varepsilon^i \equiv$ c-number

$\langle \Sigma_{j \neq i}^{A} v_{ij} \rangle \equiv U_i \equiv$ mean field

$\varepsilon^i + \varepsilon_i = \langle H_A \rangle = 0$

Two-body operator via construction:

$$G_0(E_i, i)^{-1} = (E - \varepsilon^i) - h_0 - h_i - U_i$$

$$E_i \equiv E - [\varepsilon^i + \varepsilon_i] + \varepsilon_i$$

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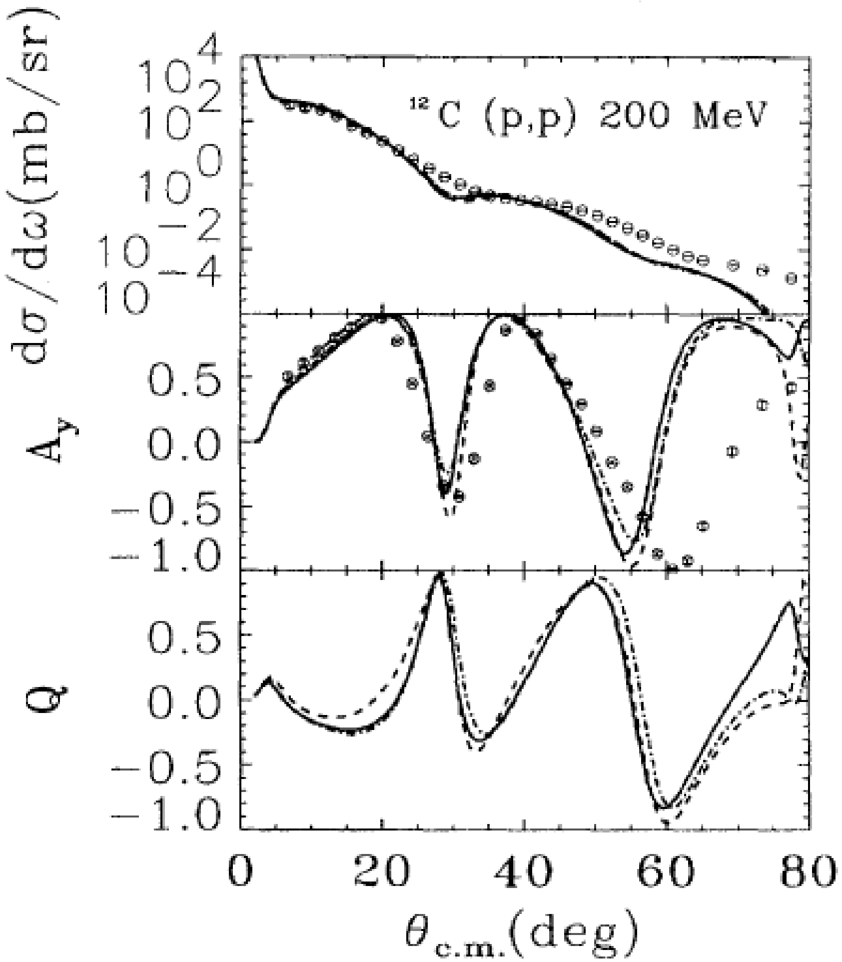
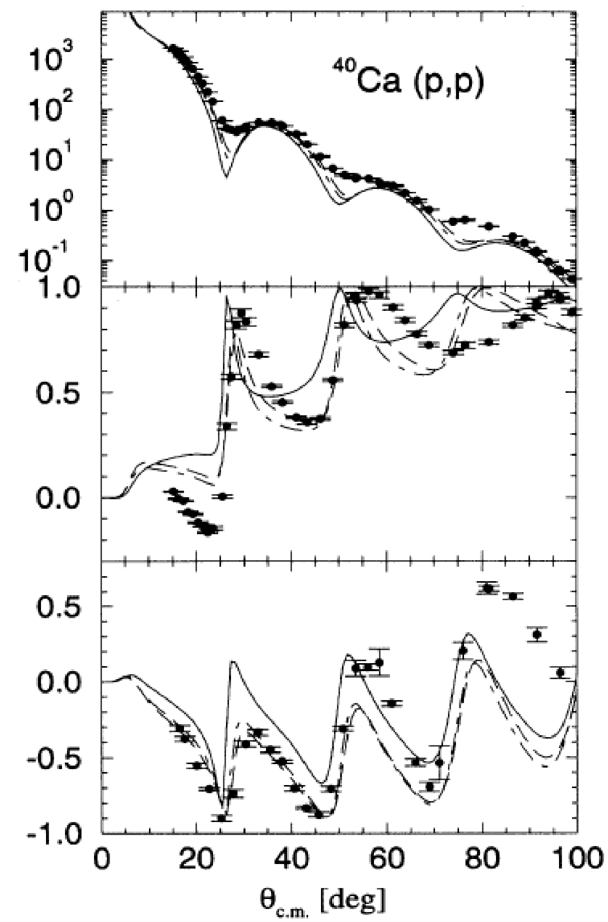
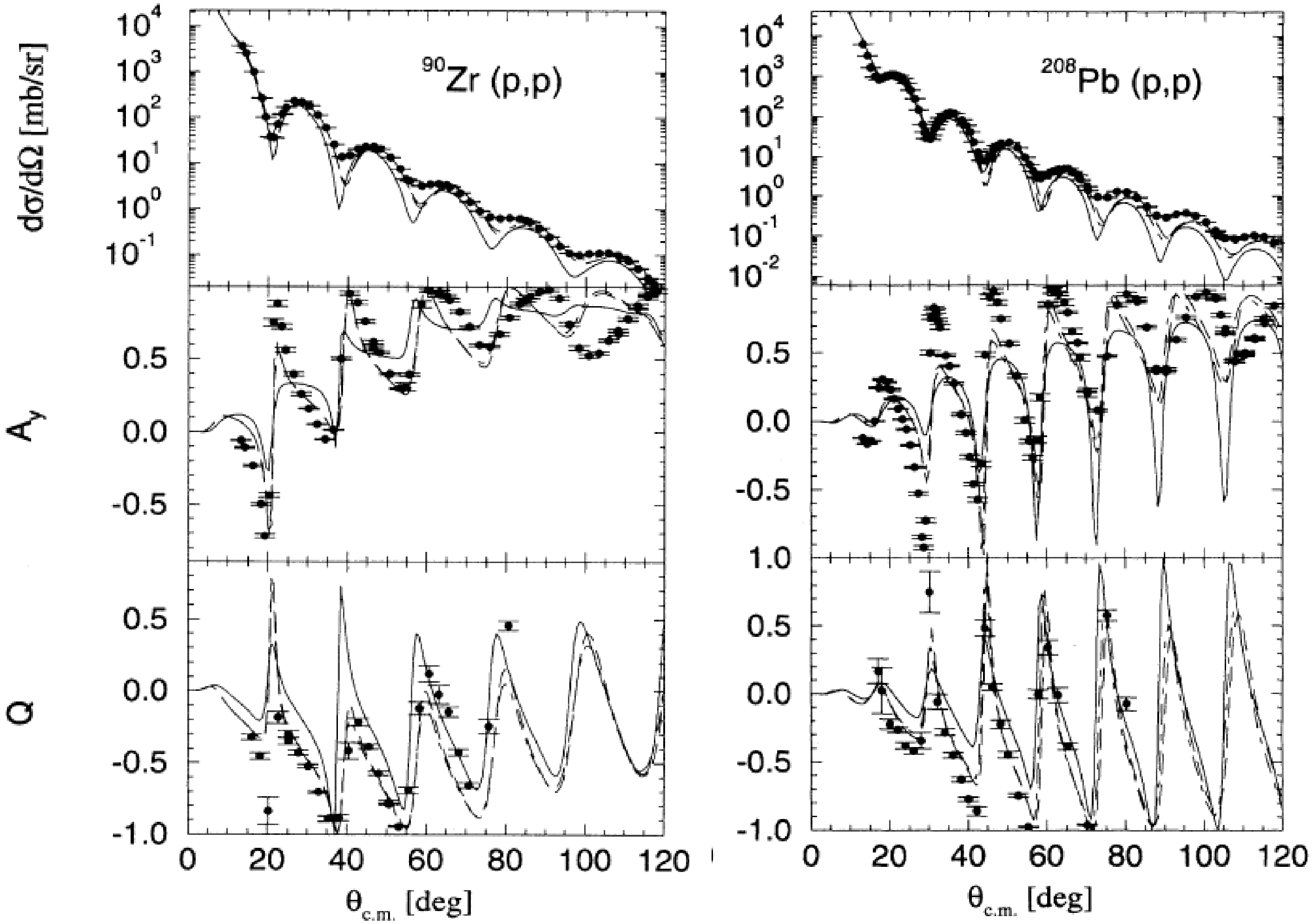
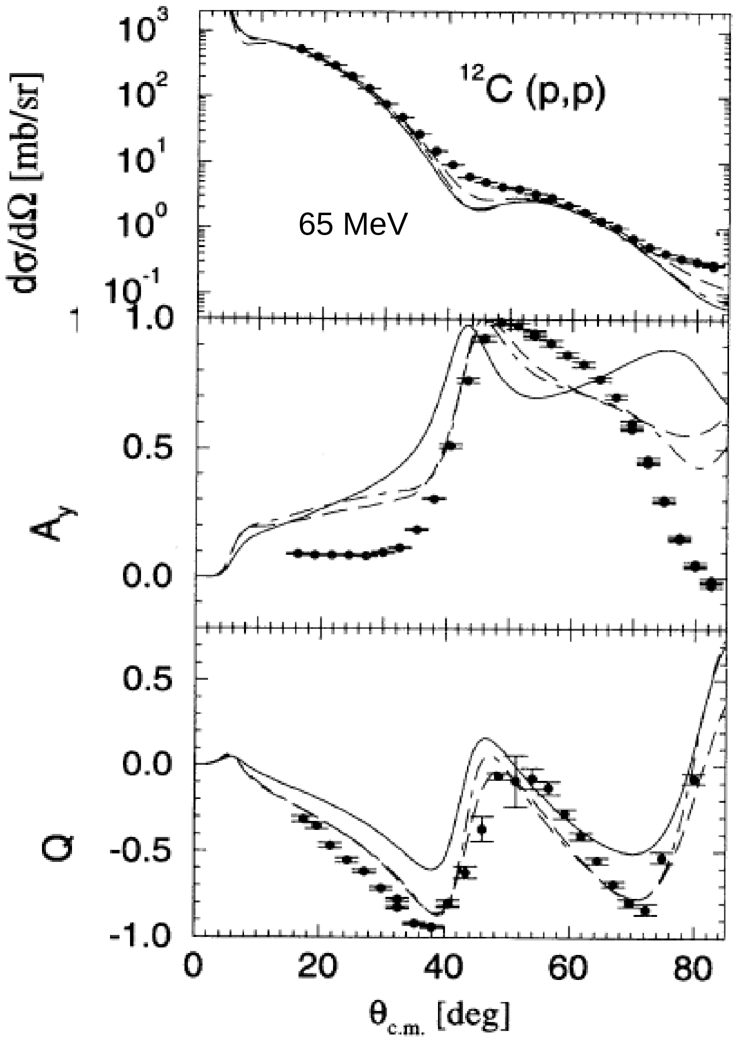

^{12}C (p,p) 200 MeV
dσ/dω(mb/sr)
10^4
10^2
10^0
10^{-2}
10^{-4}
A_y
0.5
0.0
−0.5
−1.0
Q
0.5
0.0
−0.5
−1.0
0
20
40
60
80
$\theta_{c.m.}$(deg)


^{40}Ca (p,p)
10^3
10^2
10^1
10^0
10^{-1}
1.0
0.5
0.0
0.5
0.0
-0.5
-1.0
0
20
40
60
80
100
$\theta_{c.m.}$ [deg]

65 MeV



65 MeV



65 MeV

^{12}C is not closed shell

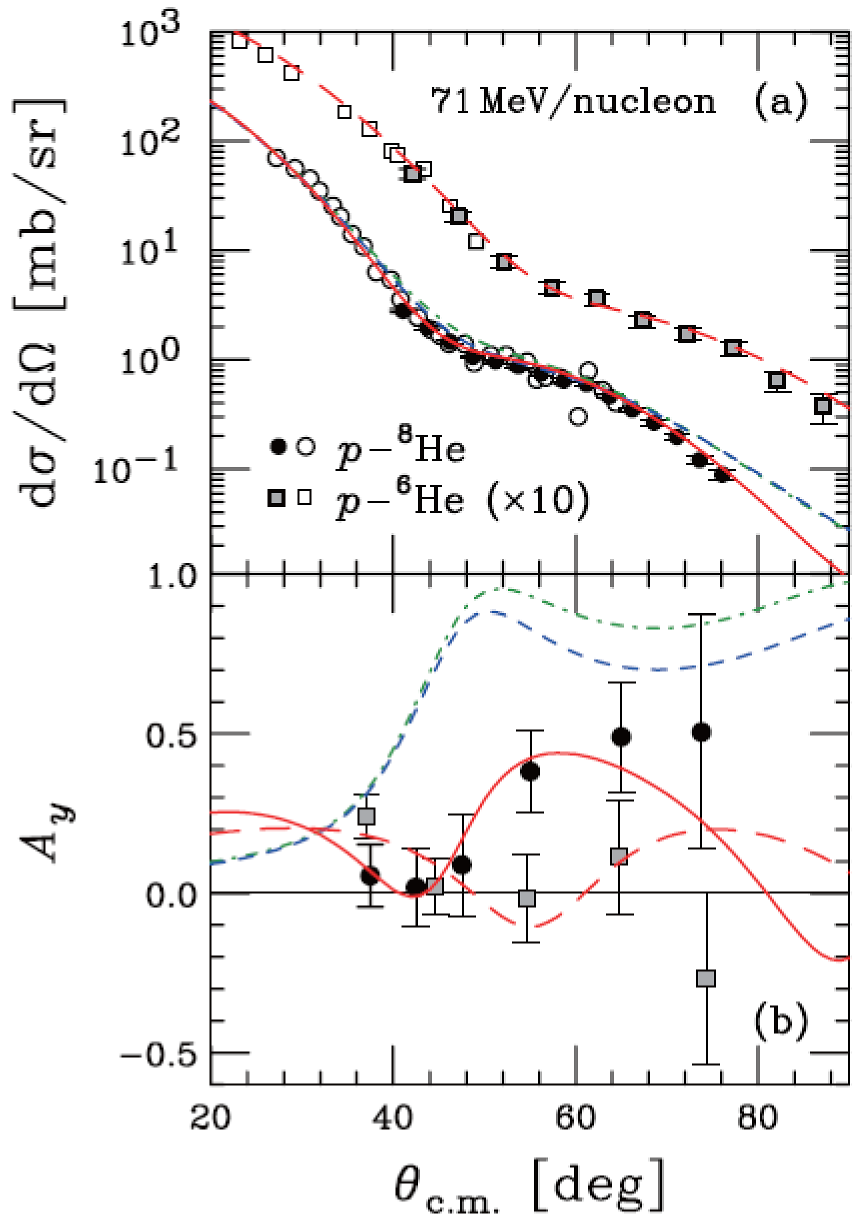
RIKEN:
$^{6}He(p,p)^{6}He$
and
$^{8}He(p,p)^{8}He$

S. Sakaguchi et al.

PRC 87, 021601(R) (2013)

Analyzing Powers of ^{6}He and ^{8}He behave differently!

A new A_y puzzle ?



General Single Particle Density Matrix

Wave function ~ $\Phi_0(i) \sim f_l(i)\, Y_l^m(i)\, \chi_S^{(i)}$

Single particle density matrix

$$\rho_{I\,M_I,I\,M_I'}(i,i') \sim \sum_{k_l\,q_l;\,k_s\,q_s;\,k\,q} \begin{bmatrix} I & K & I \\ M_I' & q & M_I \end{bmatrix} \left\langle \Psi_I \,||\, \rho_{kq} \,||\, \Psi_I \right\rangle \chi_{k_l q_l}^{ll'}(i,i')\, f_l(i)\, f_{l'}^*(i')$$

$$\left\langle S\, m_s \,|\, \tau_{k_s q_s}^{(i)}(S) \,|\, S'\, m_s' \right\rangle \begin{bmatrix} k_l & k_s & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} l & l' & k_l \\ s & s & k_s \\ j & j' & k \end{Bmatrix}$$

Auxiliary tensor operator

$$\tau_{00}^{(i)} = 1$$

$$\tau_{k_s,q_s}^{(i)}\left(S = \tfrac{1}{2}\right):\quad \tau_{10}^{(i)} = 2s_z$$

$$\tau_{1,\pm 1}^{(i)} = \mp \frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

$$\chi_{k_l\,q_l}^{ll'}(i,i') = \sum_{l\,l'} (-1)^{l'-\lambda'} \begin{bmatrix} l & l' & k_l \\ \lambda & -\lambda' & q_l \end{bmatrix} Y_{l\lambda}(i)\, Y_{l'\lambda'}^*(i')$$

Orbital angular momentum

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Case k_s = 1 $\left\langle \Phi_0 \middle| \boldsymbol{\sigma}^{(i)} \middle| \Phi_0 \right\rangle$

p-shell

$$\rho_{0\,0,0\,0}(i,i') \sim \sum_{k_l\,q_l;\,1\,q_s;\,k\,q} \left\langle \Psi_0 \,||\, \rho_k \,||\, \Psi_0 \right\rangle \chi^{11}_{k_l q_l}(i,i')\, f_p(i)\, f_p^*(i')$$

$$\left\langle S\, m_s \,|\, \tau^{(i)}_{1q_s}(S) \,|\, S'\, m'_s \right\rangle \begin{bmatrix} k_l & 1 & k \\ q_l & q_s & q \end{bmatrix} \begin{Bmatrix} 1 & 1 & k_l \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{3}{2} & \frac{3}{2} & k \end{Bmatrix}$$

with

$$\tau^{(i)}_{10} = 2s_z$$

$$\tau^{(i)}_{1,\pm1} = \mp\frac{2}{\sqrt{2}}(S_x \pm iS_y)$$

For closed shell nuclei this term is zero

Reminder: calculate $\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$

NN t-matrix in Wolfenstein representation:

Projectile "*0*" : plane wave basis
Struck nucleon "*i*" : target basis

$$\overline{\mathrm{M}}(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) = A(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, \mathbf{1} + i \cdot C(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})\, (\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}})$$
$$+ (G(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN})$$

$$+ D(\mathbf{q}, \mathbf{K}_{NN}, \mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}}) \right]$$ Off-shell

Couple to k_s=1 in
single particle density matrix

Model Ansatz for ground states of ^{6}He and ^{8}He

HO

$$\text{s-shell: } f_{00\frac{1}{2}}(\boldsymbol{\zeta}) = (2\pi)^{3/2}\sqrt{\frac{4}{\sqrt{\pi}\nu_s^{3/2}}}\, exp\left(-\frac{\zeta^2}{2\nu_s}\right)\,\mathcal{Y}_0^{\frac{1}{2},m_j}$$

$$\text{p-shell: } f_{01\frac{3}{2}}(\boldsymbol{\zeta}) = (2\pi)^{3/2}\sqrt{\frac{4}{\sqrt{\pi}\nu_p^{3/2}}}\sqrt{\frac{2}{3}}\frac{\zeta}{\sqrt{\nu_p}} exp\left(-\frac{\zeta^2}{2\nu_p}\right)\mathcal{Y}_1^{\frac{3}{2},m_j}(\hat{\boldsymbol{\zeta}})$$

With change of variables

$$\mathbf{q} = \mathbf{k}' - \mathbf{k} = \frac{A}{A-1}(\boldsymbol{\zeta} - \boldsymbol{\zeta}')$$

$$\mathbf{P} = \frac{\boldsymbol{\zeta}+\boldsymbol{\zeta}'}{2}; \quad \mathbf{K} = \frac{\mathbf{k}+\mathbf{k}'}{2}$$

$k_s=0$

$$\rho_s(\mathbf{q},\mathbf{P}) = \frac{1}{\pi^{3/2}\nu_s^{3/2}}\, exp\left(-\frac{P^2}{\nu_s} - \left(\frac{A-1}{2A}\right)^2\frac{q^2}{\nu_s}\right),$$

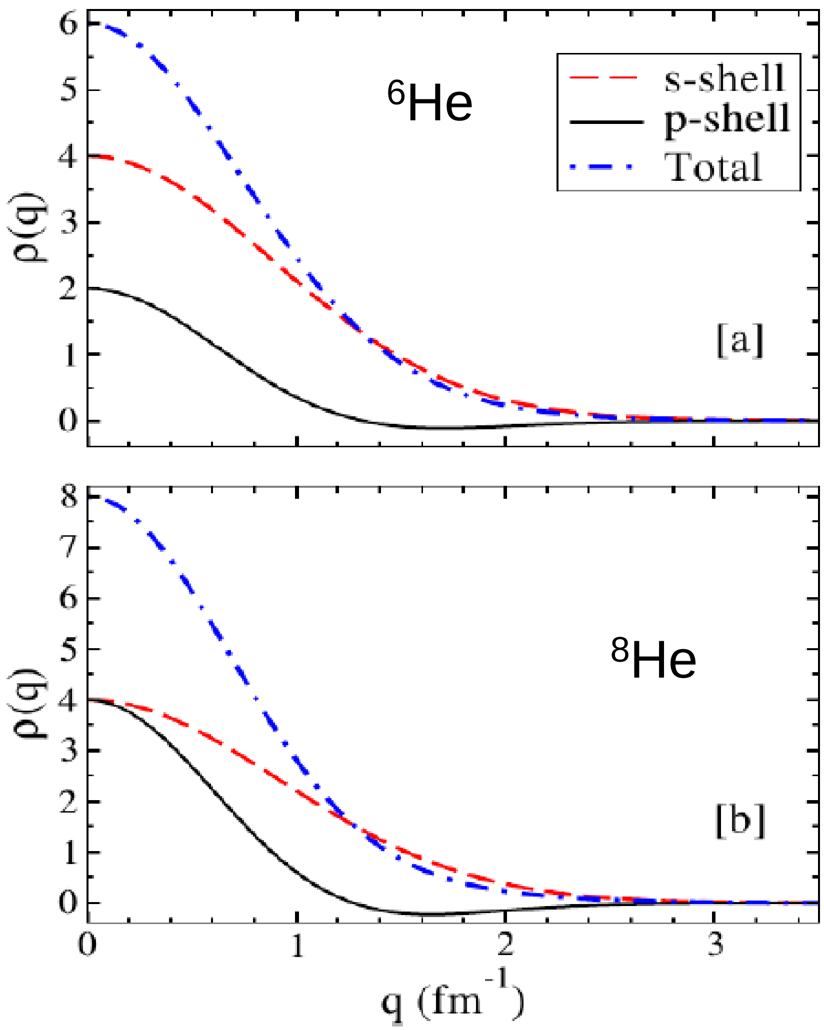
$$\rho_p(\mathbf{q},\mathbf{P}) = \frac{2}{3}\frac{1}{\pi^{3/2}\nu_p^{5/2}}\left(P^2 - \left(\frac{A-1}{2A}\right)^2 q^2\right) exp\left(-\frac{P^2}{\nu_p} - \left(\frac{A-1}{2A}\right)^2\frac{q^2}{\nu_p}\right)$$

$k_s=1$

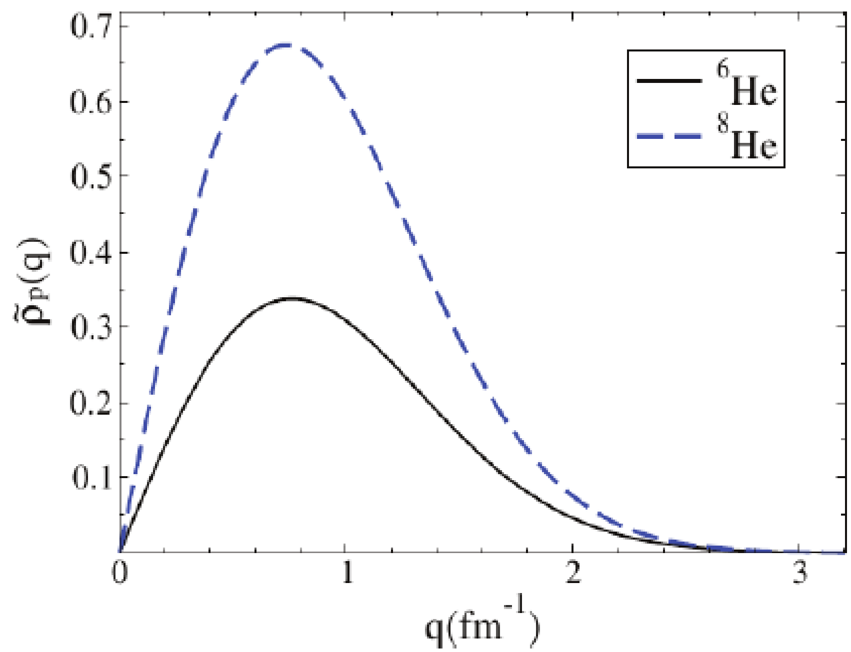
$$\tilde{\rho}_p(\mathbf{q},\mathbf{P}) = N_p\,\frac{2}{9}\frac{(-i)}{\pi^{3/2}\nu_p^{5/2}}\left(\frac{A-1}{2A}\right)|\mathbf{q}\times\mathbf{P}|\, exp\left(-\frac{P^2}{\nu_p} - \left(\frac{A-1}{2A}\right)^2\frac{q^2}{\nu_p}\right)$$

Diagonal Densities

k_s=0



k_s=1



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Comments:

Ansatz for ^{6}He as s-shell core (closed shell) plus two valence neutrons is cluster picture, not shell model.

Two valence neutrons only in $p_{3/2}$ shell is unrealistic from a NCSM point of view

→

Contributions from open shells in this picture are too small to Influence the scattering calculation.

Flow of calculation:

Expectation Values for struck target nucleon in target (A) intrinsic frame

$$I_1 = \frac{1}{8\pi^2}\int d\hat{\zeta}\, d\hat{\zeta}'\, \Phi_0(\hat{\zeta}')\, \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{n}}\, \Phi_0(\hat{\zeta})\, \delta(\hat{\zeta}\cdot\hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

$$I_2 = \frac{1}{8\pi^2}\int d\hat{\zeta}\, d\hat{\zeta}'\, \Phi_0(\hat{\zeta}')\, \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{q}}\, \Phi_0(\hat{\zeta})\, \delta(\hat{\zeta}\cdot\hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

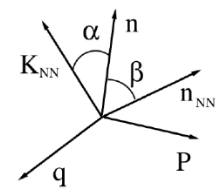
$$I_3 = \frac{1}{8\pi^2}\int d\hat{\zeta}\, d\hat{\zeta}'\, \Phi_0(\hat{\zeta}')\, \boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{P}}\, \Phi_0(\hat{\zeta})\, \delta(\hat{\zeta}\cdot\hat{\zeta}' - \cos\alpha_{\zeta\zeta'})$$

Projection to the NN frame for combining with NN Wolfenstein functions:

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{n}}_{NN}\Psi_p(\hat{\zeta}) = -i\frac{2}{9\pi^{3/2}\nu_p^{5/2}}\,|\zeta\times\zeta'|\, exp\left(-\frac{\zeta^2}{2\nu_p} - \frac{\zeta'^2}{2\nu_p}\right)\ \cos\beta$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{q}}\Psi_p(\hat{\zeta}) = 0$$

$$\Psi_p^*(\hat{\zeta}')\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathbf{K}}_{NN}\Psi_p(\hat{\zeta}) = -i\frac{2}{9\pi^{3/2}\nu_p^{5/2}}\,|\zeta\times\zeta'|\, exp\left(-\frac{\zeta^2}{2\nu_p} - \frac{\zeta'^2}{2\nu_p}\right)\ \cos\alpha$$



Final optical potential as integral over Wolfenstein functions and single particle density matrices calculated in the A+1 c.m. frame as input to the p+A scattering code

Optical Potential for ^{6}He with the terms from the valence neutrons ($p_{3/2}$ shell)

$$U_{^6He}(\mathbf{q},\mathbf{K}) = \sum_{i=N,P} U_{core}(\mathbf{q},\mathbf{K}) + U_{val}(\mathbf{q},\mathbf{K})$$

$$\begin{aligned} U_{val_{central}} &= U_A(\mathbf{q},\mathbf{K}) + U_A^C(\mathbf{q},\mathbf{K}) \\ U_{val_{spin-orbit}} &= U_C(\mathbf{q},\mathbf{K}) + U^M(\mathbf{q},\mathbf{K}) \end{aligned}$$

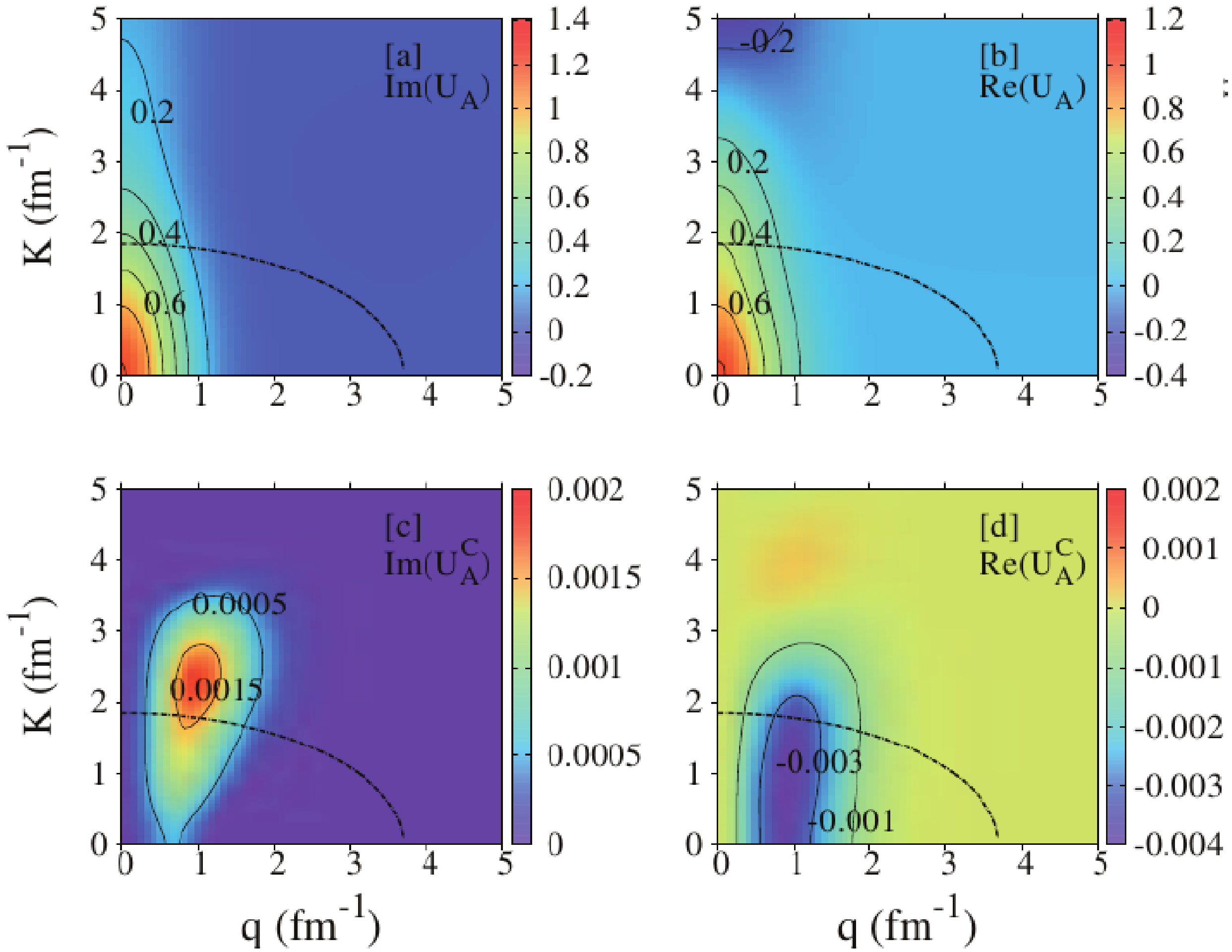
Remark:

- Transition $p_{3/2}$-$d_{3/2}$ → Wolfenstein will G+H and D contribute

Central Terms

[MeV fm^3]

- $k_s=0$
- $U_A(q,K)$



- $k_s=1$
- $U_A^C(q,K)$

$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$

On-shell condition

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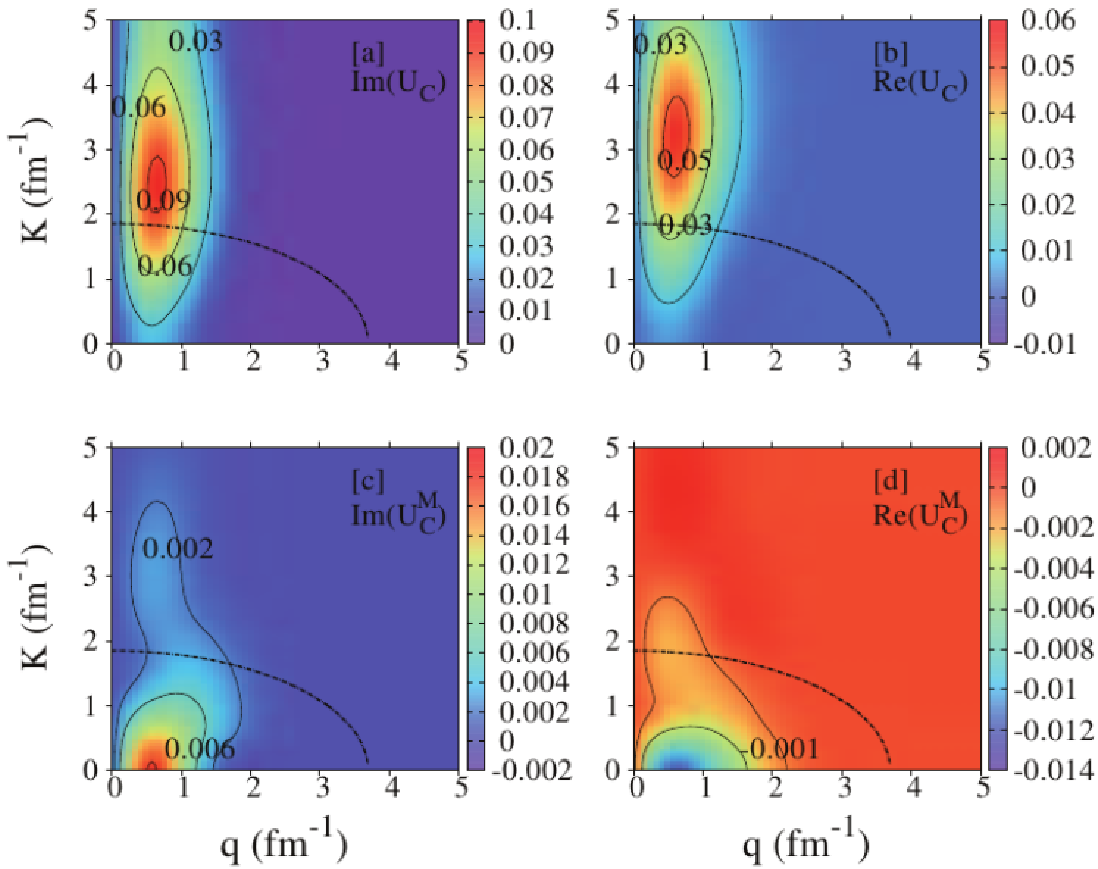
Spin-Orbit Terms

[MeV fm^3]

- $k_s=0$
- $U_c(q,K)$



- $k_s=1$
- $U_C^M(q,K)$

$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$

On-shell condition

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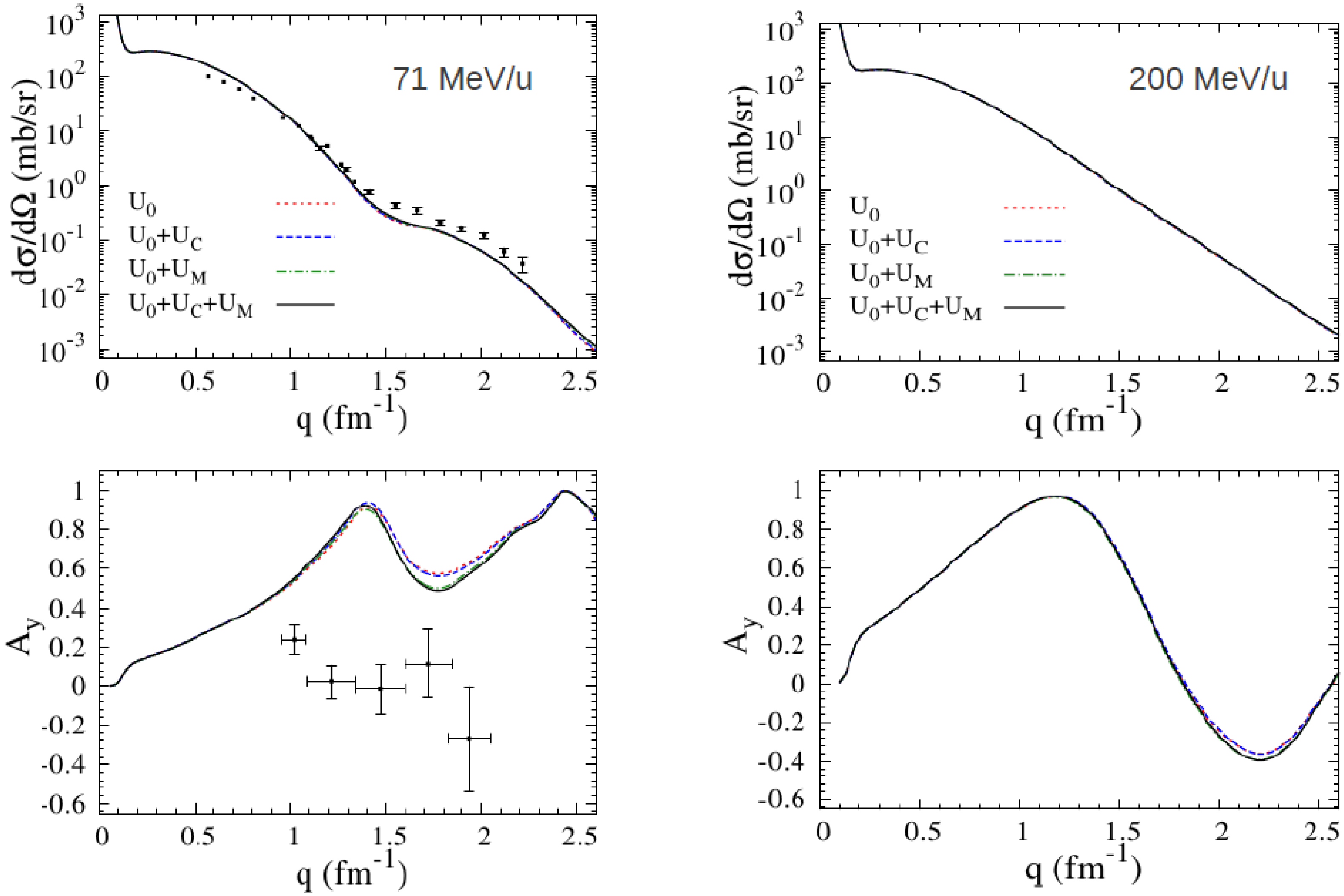
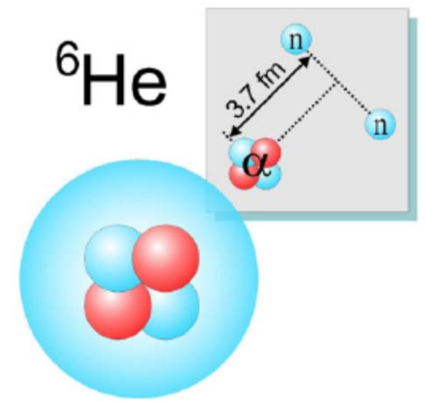
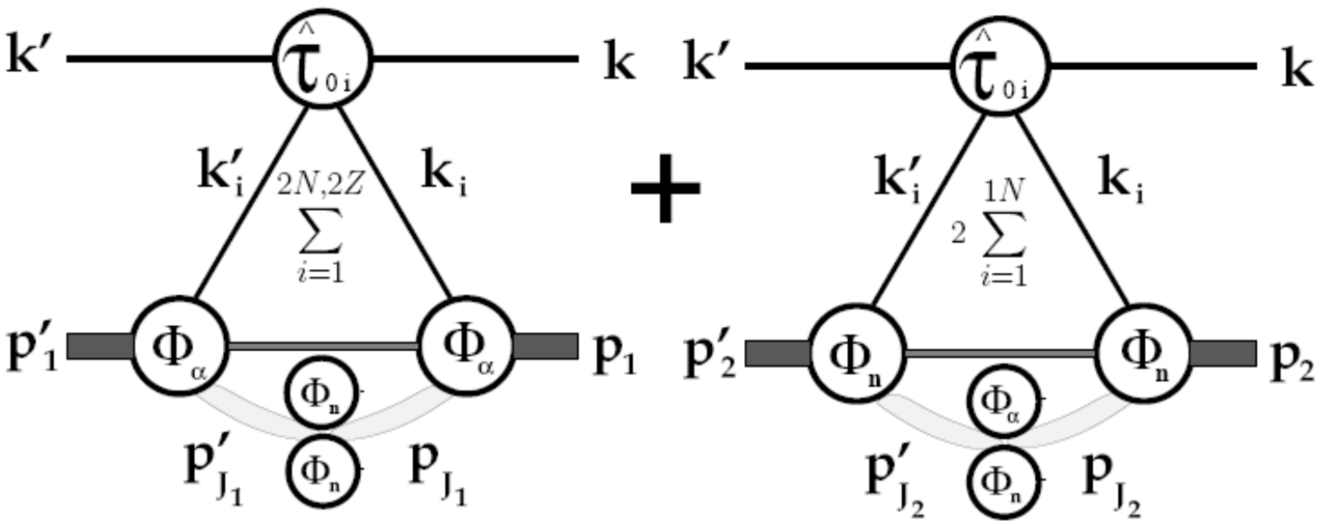
Scattering Observables for ^{6}He



Optical Potential for ^{6}He as cluster α+n+n





Weppner, Elster, PRC 85, 044617 (2012)

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Cluster Folding Optical Potential (n+n+α)

Jacobi momenta

$$\mathbf{p}_{j_i} = \frac{1}{A}(A_{s_i}\mathbf{p}_i - A_i\mathbf{p}_{\mathbf{s}_i})$$

Correlation Density

$$\rho_{corr}(\mathbf{p}_{j_1}, \mathbf{p}'_{j_1}) \equiv \int \prod_{l=2}^{N_c} d\mathbf{p}'_{j_l} \int \prod_{m=2}^{N_c} d\mathbf{p}_{j_m} \langle \phi_A | \mathbf{p}'_{\mathbf{j}_1} \mathbf{p}'_{\mathbf{j}_2} \cdots \mathbf{p}'_{\mathbf{j}_{N_c}} \rangle \langle \mathbf{p}_{\mathbf{j}_1} \mathbf{p}_{\mathbf{j}_2} \cdots \mathbf{p}_{\mathbf{j}_{N_c}} | \phi_A \rangle$$

$p_{3/2}$ HO state

Cluster optical potential

$$U_{el}(\mathbf{q}, \mathbf{K}) = \sum_{c=1,N_c} \sum_{i=n_c,p_c} \int d\mathbf{P}\, d\mathcal{P}_{j_c}\, \rho_{corr}(\mathcal{P}_{j_c})$$

$$\hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right) \rho_{ci}\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right)$$

Cluster folding potential for ^{6}He+p

$$
{}^{^6\mathrm{He}}U_{el}(\mathbf{q},\mathbf{K}) = U_\alpha + 2U_n =
$$

$$
\sum_{i=n,p} \int d\mathbf{P}\, d\mathcal{P}_{j_\alpha}\, \rho_{corr}(\mathcal{P}_{j_\alpha})\, \hat{\tau}_{0i} \left(\mathbf{q}, \frac{1}{2}\left(\frac{A+1}{A}\mathbf{K} - \mathbf{P}\right), \mathcal{E}\right)\, \rho_{\alpha i}\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right)
$$

$$
+2 \int d\mathbf{P}\, d\mathcal{P}_{j_n}\, \rho_{corr}(\mathcal{P}_{j_n})\, \hat{\tau}_{0n} \left(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K} - \mathbf{P}), \mathcal{E}\right)\, \rho_n\left(\mathbf{P} - \frac{A-1}{2A}\mathbf{q}, \mathbf{P} + \frac{A-1}{2A}\mathbf{q}\right).
$$

For calculation:

NN t-matrix: Nijmegen II potential

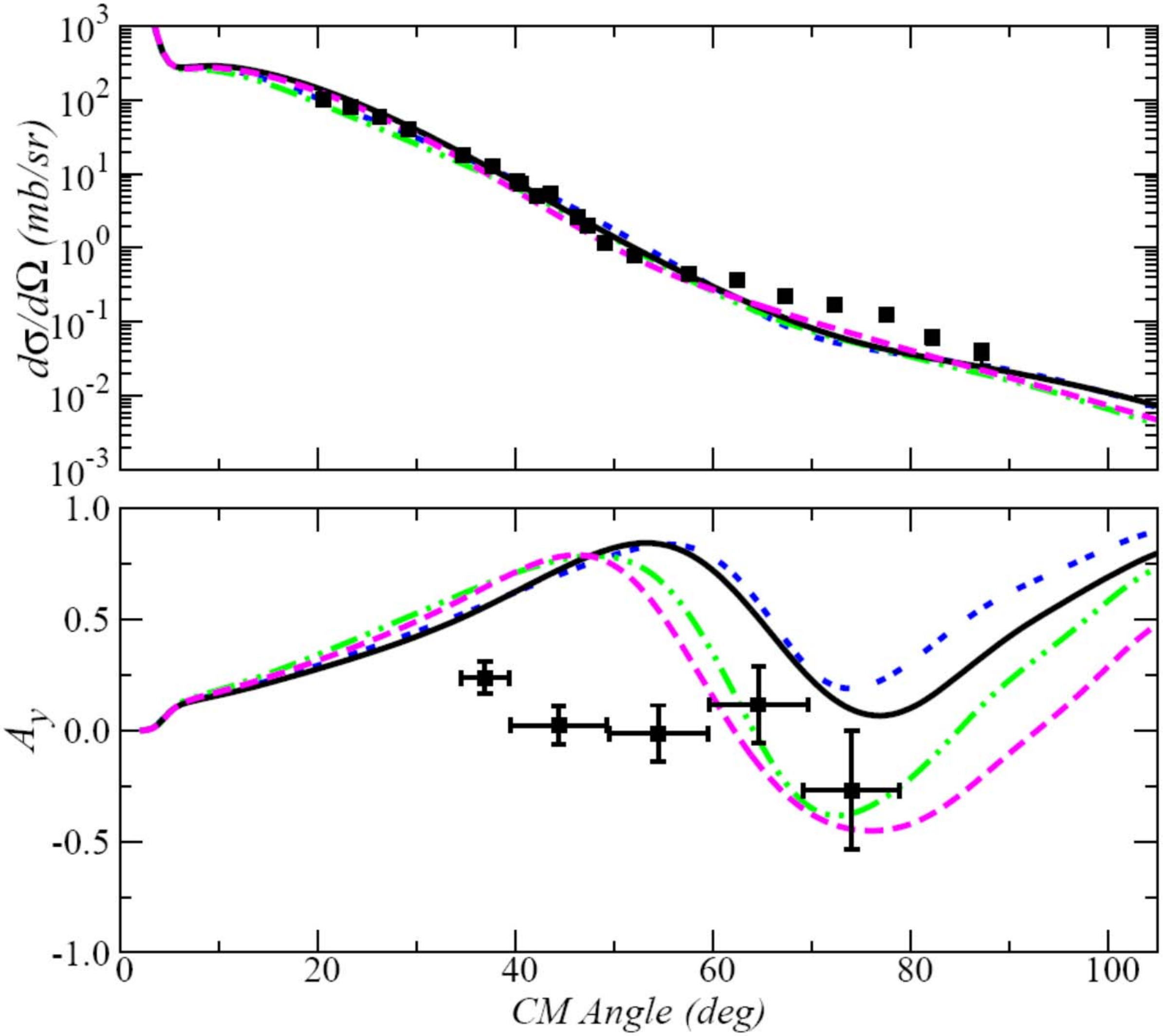
Densities:

COSMA density == s & p- shell harmonic oscillator wave functions

Fitted to give rms radius of ^{6}He (older value)

and for ^{4}He: Gogny density with coupling to medium

^{6}He (p,p) ^{6}He @ 71 MeV



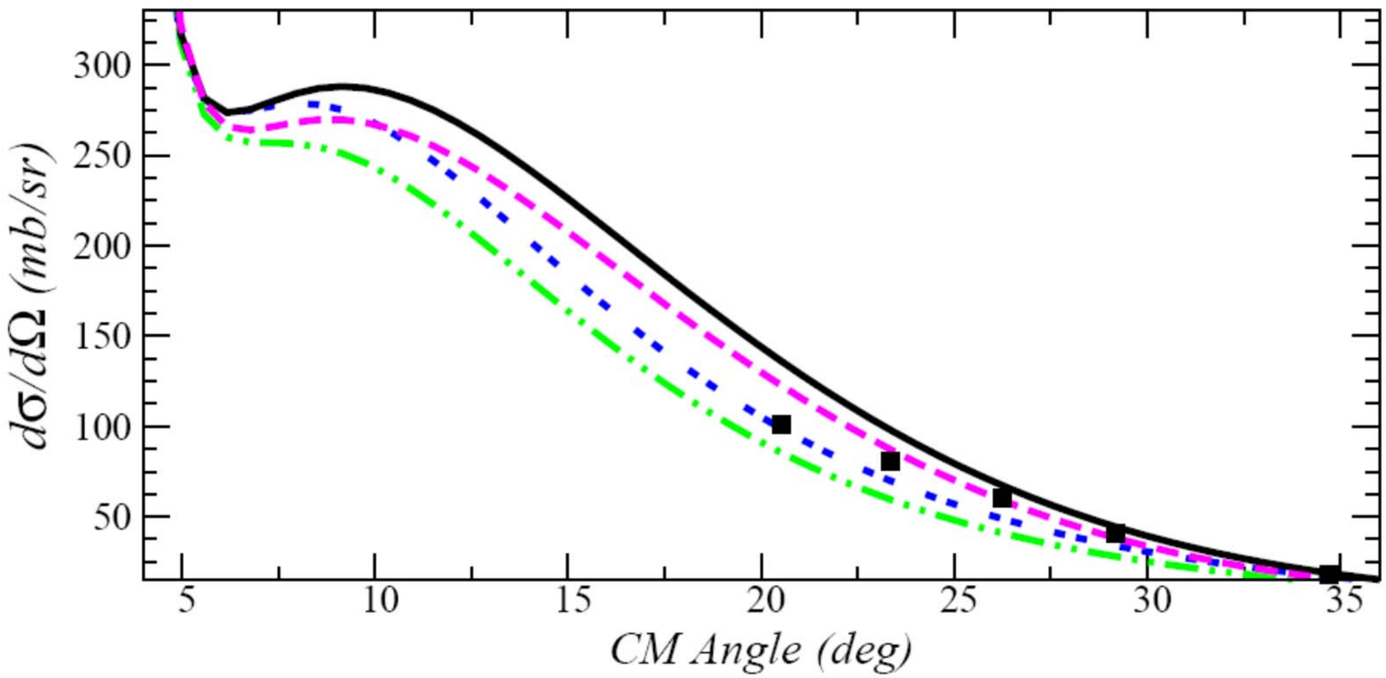
COSMA single particle OP

COSMA cluster OP

α- HFB
n - COSMA

α - HFB
n – COSMA
no correlations

^{6}He (p,p) ^{6}He @ 71 MeV



COSMA
single
particle OP

COSMA
cluster OP

α- HFB

n - COSMA

α - HFB
n – COSMA
no
correlations

Summarizing and Reflecting

- **Microscopic folding potentials: extensive work for closed shell nuclei**
- **Open shell nuclei: single particle density matrix has spin independent and spin dependent parts.**
- **In a microscopic first order optical potential with a shell model density all amplitudes of the NN t-matrix will contribute.**
- **Single scattering: for energies ~ 70 to 500 MeV (relativistic kinematics @ higher energies)**
- **Going lower: Double scattering needs to be considered.**
- **First calculation:**
  - HO ansatz with filled s-shell (alpha core) and valence neutrons in $p_{3/2}$ shell (COSM)
  - Needed: calculation with NCSM density.