Microscopic approach of nucleus-nucleus bremsstrahlung and radiative captures

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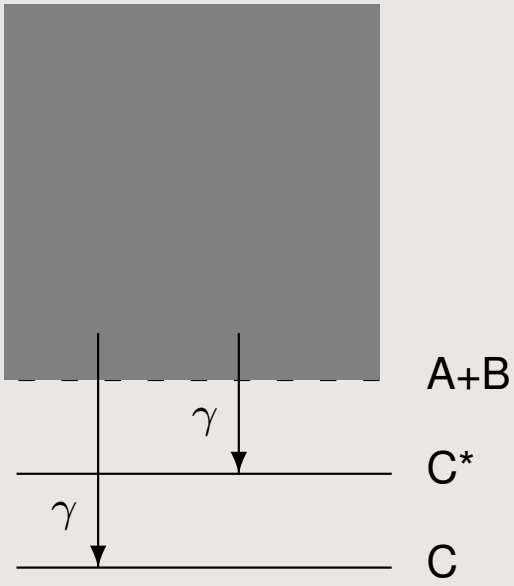
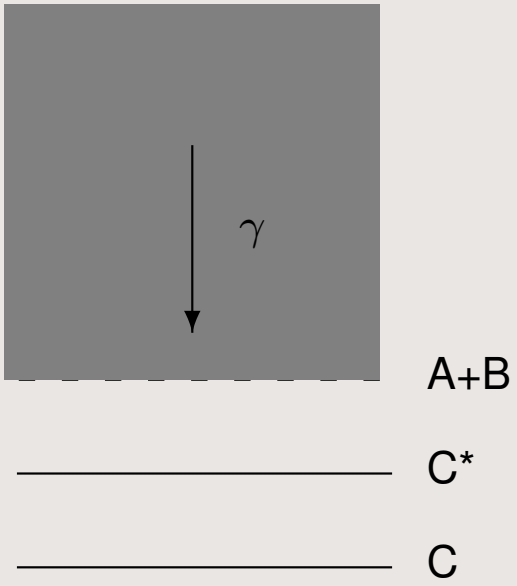
Electromagnetic transitions



Radiative captures



Nucleus-nucleus bremsstrahlung

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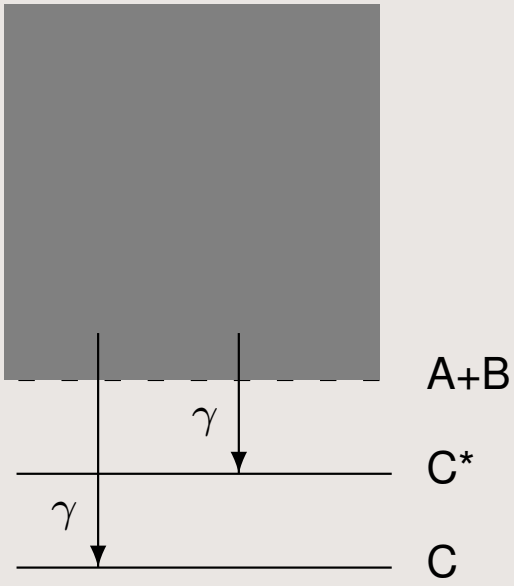

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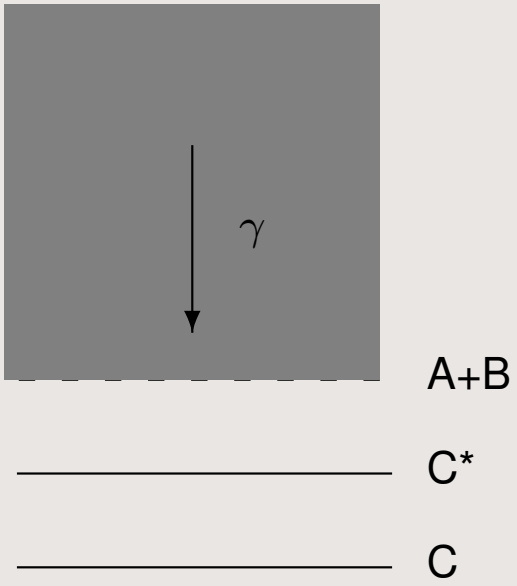

Nucleus-nucleus bremsstrahlung

- Radiative captures have a strong astrophysical interest (cf. Barry Davids's talk)

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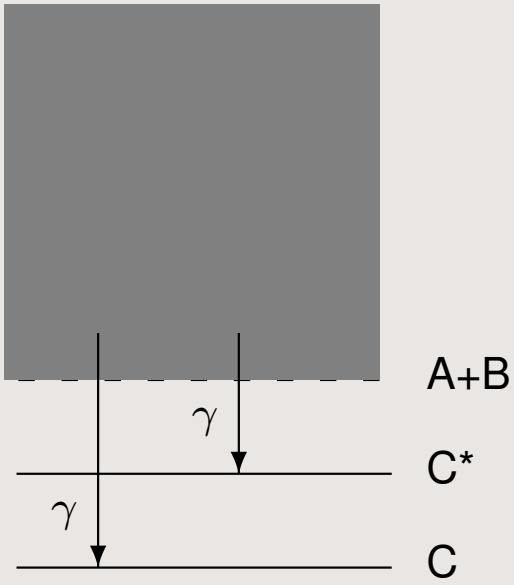

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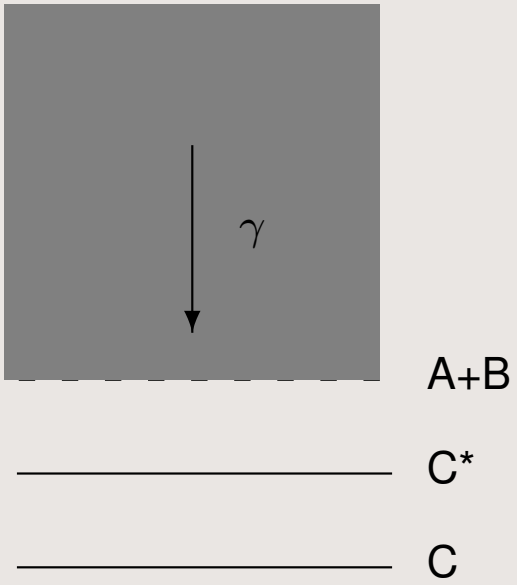

Nucleus-nucleus bremsstrahlung

- Radiative captures have a strong astrophysical interest (cf. Barry Davids's talk)
- Bremsstrahlung
  -to describe the radiative transitions between unstable states

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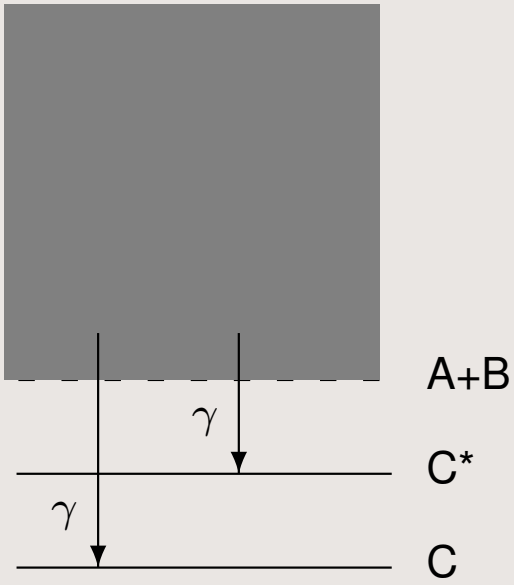

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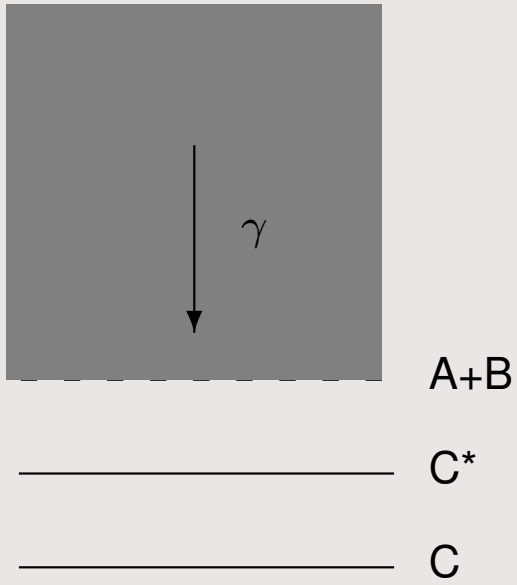


Nucleus-nucleus bremsstrahlung

- Radiative captures have a strong astrophysical interest (cf. Barry Davids's talk)
- Bremsstrahlung
  -to describe the radiative transitions between unstable states
  -to describe the $t(d, n\gamma)\alpha$ radiative transfer reaction (perspective to diagnose plasmas in fusion experiments from this reaction and recent experiment at Ohio university)

Outline

Bremsstrahlung

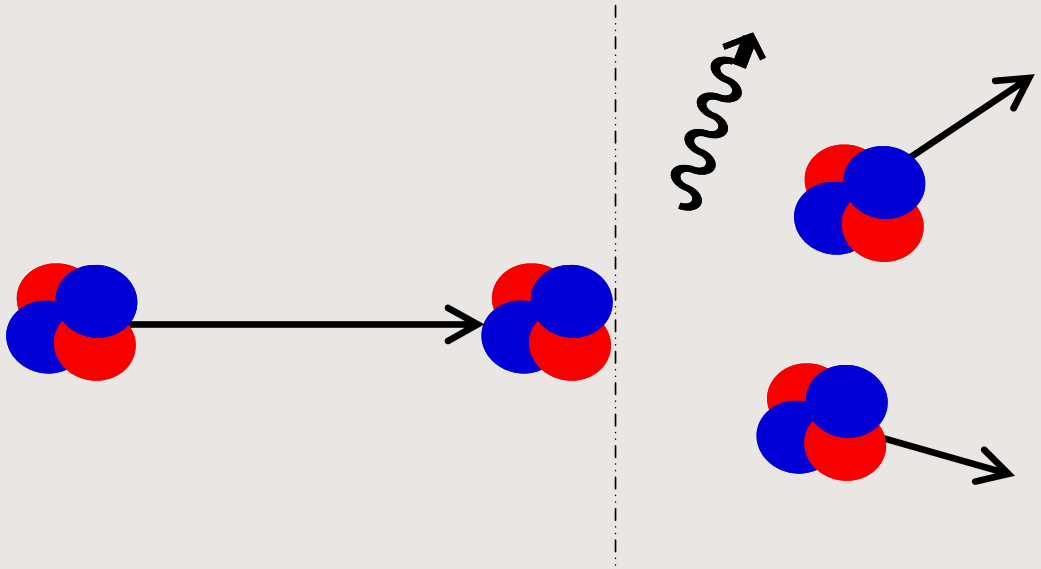
- microscopic cluster approach: application to $\alpha\alpha$ and αN systems
- No-Core Shell Model/Resonating-Group Method approach: application to αp (preliminary)

Radiative capture

- No-Core Shell Model/Resonating-Group Method approach: application to $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ (preliminary)

Nucleus-nucleus bremsstrahlung

- =photon emission induced by a collision between two nuclei
  =radiative transitions between continuum states



- A part of the kinetic energy between the nuclei is converted to a photon during this process.

Why studying the $\alpha + \alpha$ system?

- At low energy, the $\alpha + \alpha$ scattering is well described by (relatively) simple models, as cluster models.

  Since the electromagnetic forces are much weaker than the nuclear ones, the electromagnetic emission process can be seen as a small perturbation of the elastic scattering $\Rightarrow$ necessity to have a fair description of the elastic scattering

- Recent measurements of "4^+-to-2^+" gamma transitions in $^8\mathrm{Be}$ from the $\alpha(\alpha, \alpha\alpha\gamma)$ performed at Mumbai (India).

  [V. M. Datar *et al.*, PRL 94 (2005) 122502] [V. M. Datar *et al.*, PRL 111 (2013) 062502]

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- (Older) experimental differential bremsstrahlung cross sections also available for the $\alpha + \alpha$ system.
- Why studying the $\alpha + \alpha$ system?
  1. We trust in our models for describing the $\alpha + \alpha$ collision
  2. Comparison between theory and experiment is possible

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- Microscopic cluster calculation is a very useful preliminary step to an *ab initio* calculation because
  - some *ab initio* techniques are extensions of the (old) cluster techniques:
    - No-Core Shell Model/Resonating-Group Method (NCSM/RGM) and No-Core Shell Model with Continuum (NCSMC)
    - Fermionic Molecular Dynamics (FMD)/RGM
    - Correlated Gaussian (CG)/RGM ($\Rightarrow$ good way to learn)

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  - provides some guidance (what partial wave is important? what type of configuration/clustering is important?

Method

- Obtaining the elastic wave functions
  - Microscopic cluster approach with an effective nucleon-nucleon (NN) interaction
  - RGM or Generator coordinate method (GCM) and the microscopic *R*-matrix method (MRM)
  - $\alpha + \alpha$ phase shifts
- From the elastic wave functions to the bremsstrahlung cross sections
  - Divergence problem in the continuum-to-continuum transitions
  - Electric transition multipole operators in the Siegert and non-Siegert approaches
  - Application to the $\alpha + \alpha$ system and comparison with experiments

Starting point: microscopic approach

- A pointlike nucleons interacting via inter-nucleon potentials
- Pauli-antisymmetrization between nucleons taken into account
- all physical quantities are derived from the internal many-body Schrödinger equation

$$H\Psi = \left(\sum_{i=1}^{A} \frac{p_i^2}{2m_N} + \sum_{i>j=1}^{A} v_{ij} - T_{\mathrm{c.m.}} \right) \Psi = E_T \Psi,$$

where

- $p_i^2/2m_N$ is the kinetic energy of nucleon i
- v_{ij} is a two-body interaction between nucleons i and j
- $T_{\mathrm{c.m.}}$ is the kinetic energy of the center of mass

Nucleon-nucleon interaction

- The NN interaction is divided in two parts:

$$v_{12} = v_{12}^N + v_{12}^C.$$

- nuclear part v_{12}^N=Minnesota potential
  - well adapted to cluster wave function
  - central potential with spin-isospin dependence (one parameter)
  - (+spin-orbit potential with isospin dependence) (one parameter)

  [D. R. Thompson and M. LeMere and Y. C. Tang, Nucl. Phys. A 286 (1977) 53]
- Coulomb part (v_{12}^C)

Resonating-Group Method

- The α cluster is described by a Slater determinant in the harmonic-oscillator shell model

$$\Phi_\alpha = \mathcal{A}|(0s)^4 p\uparrow p\downarrow n\uparrow n\downarrow\rangle = \phi_\alpha \varphi_{cm}$$

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- Inserting the RGM wave function in Schrödinger equation $\Rightarrow$

$$\int_0^\infty r^2[\mathcal{H}_\ell(r,r') - E\mathcal{N}_\ell(r,r')]\frac{u_\ell(r)}{r}dr$$

where

$$\mathcal{H}_\ell(r,r') = \langle\phi_\alpha\phi_\alpha Y_\ell(\Omega_{12})\frac{\delta(r-r_{12})}{rr_{12}}|H_\ell - E_\alpha - E_\alpha|\mathcal{A}\phi_\alpha\phi_\alpha Y_\ell(\Omega_{12})\frac{\delta(r'-r_{12})}{r'r_{12}}\rangle$$

$$\mathcal{N}_\ell(r,r') = \langle\phi_\alpha\phi_\alpha Y_\ell(\Omega_{12})\frac{\delta(r-r_{12})}{rr_{12}}|\mathcal{A}\phi_\alpha\phi_\alpha Y_\ell(\Omega_{12})\frac{\delta(r'-r_{12})}{r'r_{12}}\rangle$$

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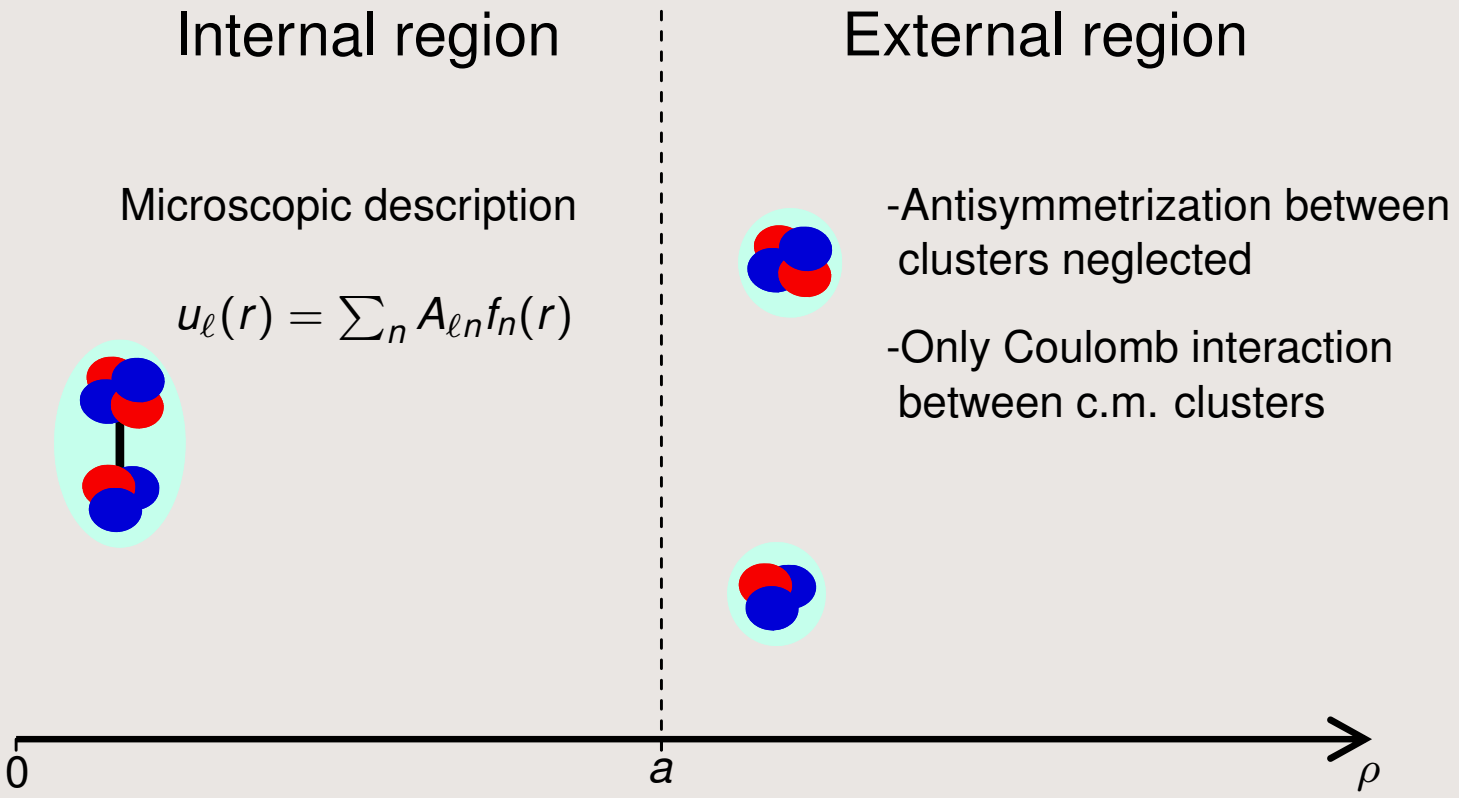
- Solving the RGM equation by expanding u_ℓ with the microscopic R-matrix.

Microscopic *R*-matrix



[D. Baye, P.-H. Heenen, and M. Libert-Heinemann, Nucl. Phys. A 291 (1977) 230]

[M. Hesse, J.-M. Sparenberg, F. Van Raemdonck, and D. Baye, Nucl. Phys. A 640, 37 (1998)]

[P. Descouvemont and D. Baye, Rep. Prog. Phys. 73 (2010) 036301]

Microscopic R-matrix in equations

- RGM equation (formally)

$$H_\ell u_\ell(r) = E u_\ell(r)$$

  where the potential is energy-dependent and non-local!

- replaced by Bloch-Schrödinger equation on internal region

$$(H_\ell + \mathcal{L} - E) u_\ell^{int}(r) = \mathcal{L} u_\ell^{ext}(r)$$

  and

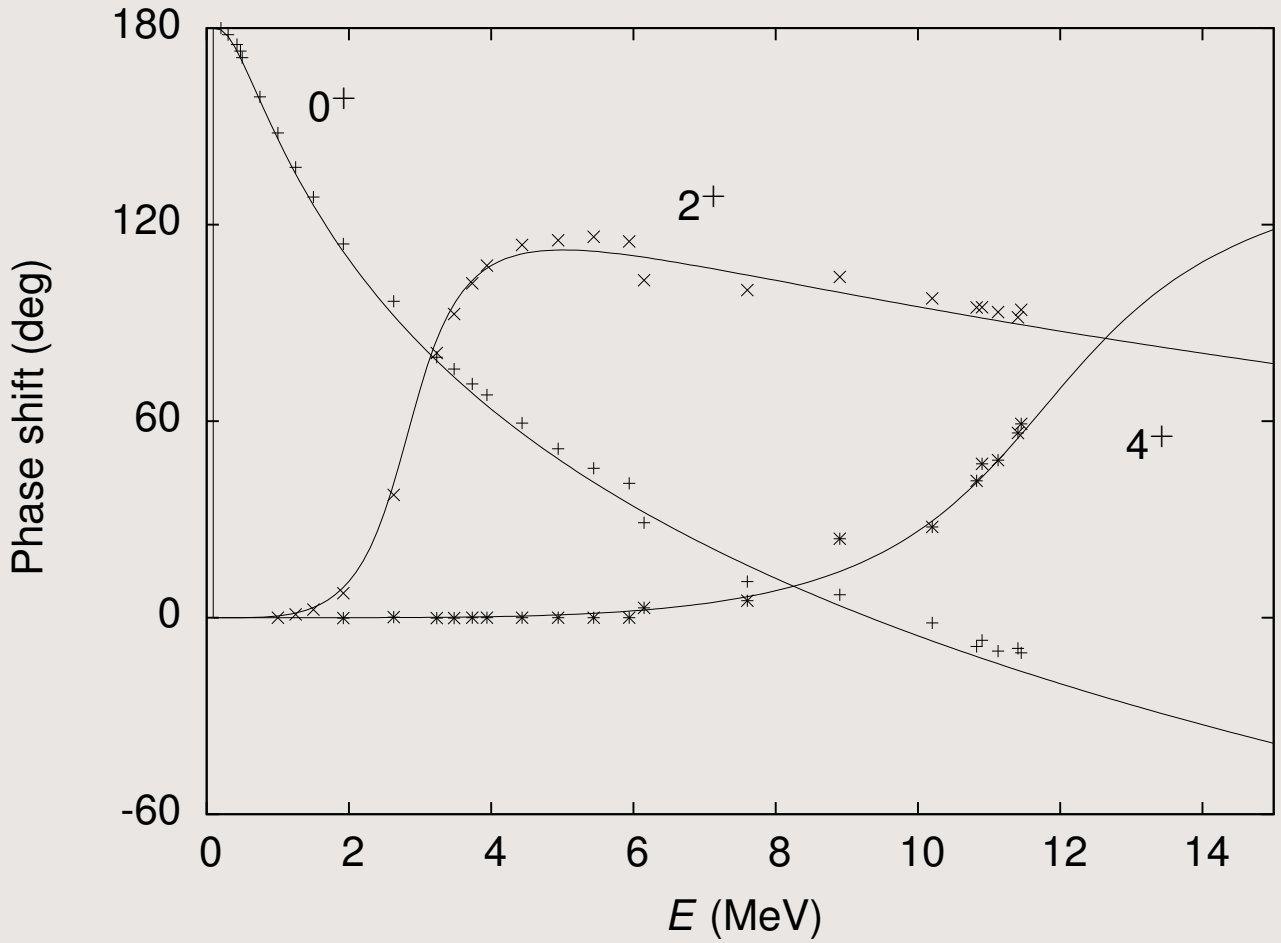
$$u_\ell^{int}(a) = u_\ell^{ext}(a)$$

  where Bloch operator makes $H_\ell + \mathcal{L}$ Hermitian over the internal region and enforces the continuity of the derivative of u_ℓ at $r = a$

$$\mathcal{L} \propto \delta(r-a)\left(\frac{d}{dr} - \frac{B}{r}\right)$$

- Convenient as well for scattering state (E input, B=0), bound state as resonances (E unknown, B energy-dependent $\Rightarrow$ iterations ($\approx$10))
- Enable accurate determination of effective range parameters and zero-energy S-factor. [D. Baye and E. Brainis, PRC 61 (2000) 025801] [O. L. Ramírez Suárez and J-M. Sparenberg, PRC 88 (2013) 014601]

$\alpha + \alpha$ phase shifts



Experimental data from [Afzal, Ahmad and Ali, Rev. Mod. Phys. 41 (1969) 247]

Bremsstrahlung cross sections

- The bremsstrahlung cross sections are obtained from the reduced matrix elements of the electromagnetic transition multipole operators between the collision wave functions at initial colliding energy E_i and final energy E_f

$$\langle \Psi_{l_f}(E_f) || \mathcal{M}^{\sigma}_{\lambda} || \Psi_{l_i}(E_i) \rangle$$

  where $\sigma = E$ or M and λ is the order of the transition
- The photon energy is approximately given by

$$E_\gamma \approx E_i - E_f$$

- For the $\alpha + \alpha$ bremsstrahlung, the E2 transitions are dominant at low photon energy (E1 forbidden and M1 forbidden at the long-wavelength approximation).

Divergence problem

- In most microscopic applications, the electric operators are defined by

$$\mathcal{M}^{E}_{\lambda\mu} = e\sum_{j=1}^{A}\left(\frac{1}{2} - t_{j3}\right)|\boldsymbol{r}_j - \boldsymbol{R}_{\mathrm{c.m.}}|^{\lambda}\, Y^{\mu}_{\lambda}(\Omega_{\boldsymbol{r}_j - \boldsymbol{R}_{\mathrm{c.m.}}})$$

  which corresponds to the Siegert form of the electric multipole transition operator at the long-wavelength approximation.

- However, in the study of the continuum-continuum transitions, this form leads to divergent matrix elements. Indeed,

$$\mathcal{M}^{E}_{\lambda\mu} \underset{\rho\to\infty}{\longrightarrow} e\left[Z_1(A_2/A)^{\lambda} + Z_2(-A_1/A)^{\lambda}\right]\rho^{\lambda}\, Y^{\mu}_{\lambda}(\Omega_{\rho})$$

  and on the external region,

$$\langle \Psi_{I_f}(E_f)||\mathcal{M}^{E}_{\lambda}||\Psi_{I_i}(E_i)\rangle_{\mathrm{ext}} \propto$$

$$\int_a^{\infty} \underbrace{[F_{I_f}(\rho)\cos\delta_{I_f} + G_{I_f}(\rho)\sin\delta_{I_f}]}_{\text{non}-\text{decreasing oscillating function}}\,\rho^{\lambda}\,\underbrace{[F_{I_i}(\rho)\cos\delta_{I_i} + G_{I_i}(\rho)\sin\delta_{I_i}]}_{\text{non}-\text{decreasing oscillating function}}\,d\rho$$

- $\Rightarrow$ divergence!

Electric transition multipole operators

- The electric transition multipole operators are defined by

$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} = (-i)^{\sigma}\sqrt{\frac{\lambda}{\lambda+1}}\frac{(2\lambda+1)!!}{k_{\gamma}^{\lambda}c}\int \boldsymbol{J}(\boldsymbol{r})\cdot\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r},$$

  where $\boldsymbol{J}$ is the intrinsic nuclear current density and

$$\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r}) = \frac{i}{k_{\gamma}\sqrt{\lambda(\lambda+1)}}\left(k_{\gamma}^{2}\boldsymbol{r}+\boldsymbol{\nabla}\frac{\partial}{\partial r}r\right)j_{\lambda}(k_{\gamma}r)\,Y_{\lambda\mu}(\Omega)$$

- The nuclear current is caused by the motion of the nucleons and also by the motion of the mesons which are responsible for the internucleon interaction
- Two difficulties: $\boldsymbol{J}$ depends on the considered NN potential (More complex is the NN potential, more complex is the current) and it is not defined unequivocally
- These difficulties can be bypassed at low-photon energies by using an extended Siegert theorem, which enables us to reduce the nuclear current dependence.

  [K.-M. Schmitt, P. Wilhelm, H. Arenhovel, A. Cambi, B. Mosconi, and P. Ricci, Phys. Rev. C 41, 841 (1990)] [JDE, D. Baye, Phys. Rev. C 88 (2013) 024602] [JDE, Phys. Rev. C 89 (2014) 024617] [JDE, D. Baye, Phys. Rev. C 90, (2014) 034611]

Extended Siegert theorem

Ingredients

- separation of $\boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}$ in a gradient term (dominant at low photon energy) and a rest
- Continuity equation which links current and charge densities

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}(\boldsymbol{r}) + \frac{i}{\hbar}[H, \rho(\boldsymbol{r})] = 0$$

Derivation

$$\begin{aligned}
\mathcal{M}^{\mathrm{E}}_{\lambda\mu} &\propto \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^{\mathrm{E}}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} \\
&\propto \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{\nabla}\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} + \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^{\mathrm{E}\prime}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} \\
&\propto -\int \boldsymbol{\nabla}\boldsymbol{J}(\boldsymbol{r})\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} + \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^{\mathrm{E}\prime}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} \\
&\propto \frac{i}{\hbar}\int [H, \rho(\boldsymbol{r})]\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} + \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^{\mathrm{E}\prime}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}
\end{aligned}$$

- Operator evaluated between eigenstates of the Hamiltonian $\Rightarrow$

$$\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu} \propto -i\frac{E_\gamma}{\hbar}\int \rho(\boldsymbol{r})\Phi_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r} + \int \boldsymbol{J}(\boldsymbol{r}) \cdot \boldsymbol{A}^{\mathrm{E}\prime}_{\lambda\mu}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$$

Extended Siegert theorem

- $\mathcal{M}^{\mathrm{E}}_{\lambda\mu}$ and $\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu}$ lead to the same results if consistent current and charge densities and exact eigenstates of the Hamiltonian are used.
- The dominant part of $\widetilde{\mathcal{M}}^{\mathrm{E(S)}}_{\lambda\mu}$ at low photon energy depends on the charge density.
- The operator $\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu}$ should be preferred in microscopic calculations because
  1. it leads to easier calculations than $\mathcal{M}^{\mathrm{E}}_{\lambda\mu}$ (if some reasonable approximation is done)
  2. derivatives of the wave functions, which are known less accurately than the wave function itself, are avoided (if some reasonable approximation is done)
  3. the charge density is less sensitive to the meson-exchange currents than the current density

Back to the divergence problem

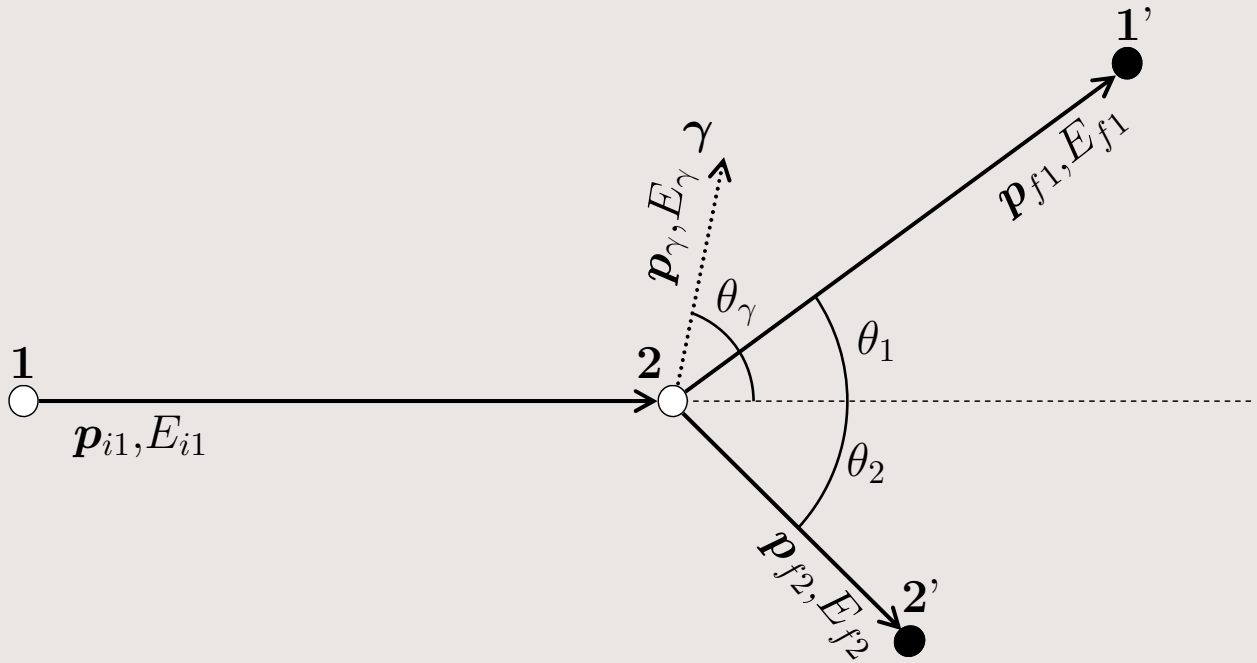
$$\mathcal{M}^{\mathrm{E}}_{\lambda\mu} \underset{\rho\to\infty}{\longrightarrow} \text{oscillating function divided by } \rho^2$$

$$\mathcal{M}^{\mathrm{E(S)}}_{\lambda\mu} \underset{\rho\to\infty}{\longrightarrow} \text{oscillating function divided by } \rho$$

- $\Rightarrow$ no divergence problem even for continuum-continuum transition!

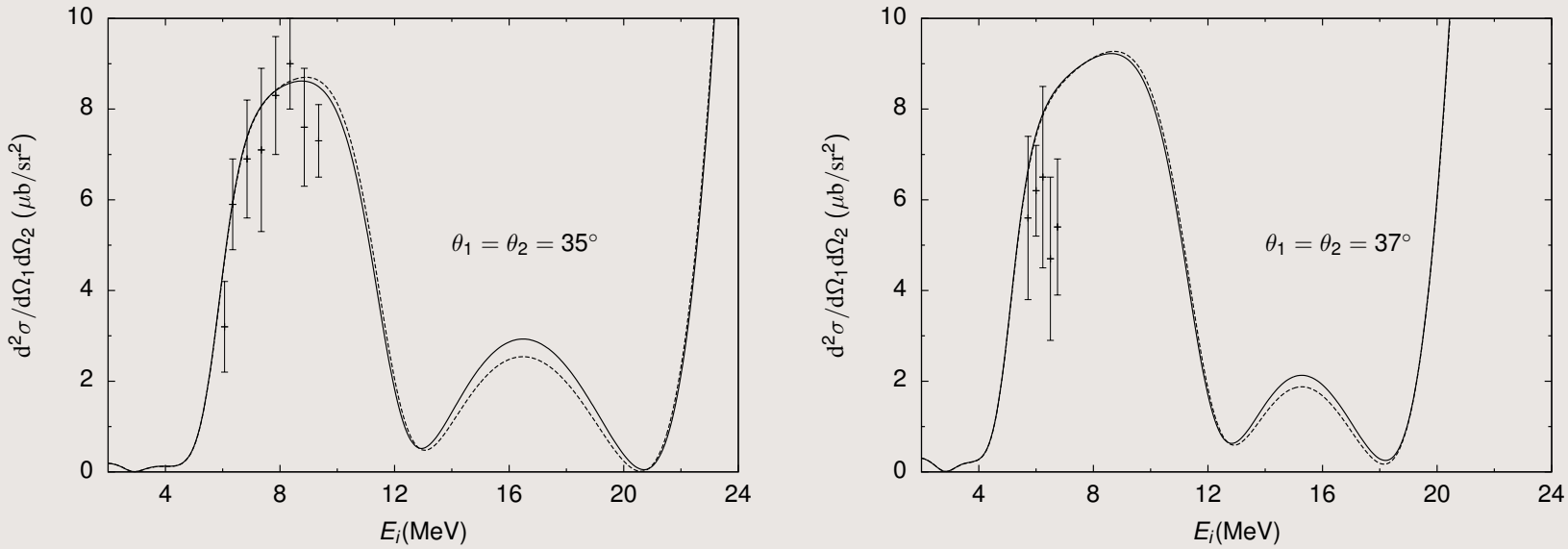
Main reason: the long-wavelength approximation is not done!

Comparison with experiments



In the so-called Harvard geometry, the photon is undetected.

Harvard geometry



- $\alpha + \alpha$ bremsstrahlung cross sections in the Siegert (full lines) and non-Siegert (dashed lines) approaches
- Experimental data from [B. Frois, J. Birchall, C. R. Lamontagne, U. von Moellendorff, R. Roy, and R. J. Slobodrian Phys. Rev. C 8 (1973) 2132] [U. Peyer, J. Hall, R. Muller, M. Suter, and W. Wolfli. Phys. Lett. 41B (1972) 151]
- Figure from [JDE, D Baye, Phys. Rev. C 88 (2013) 024602]

Integrated cross sections

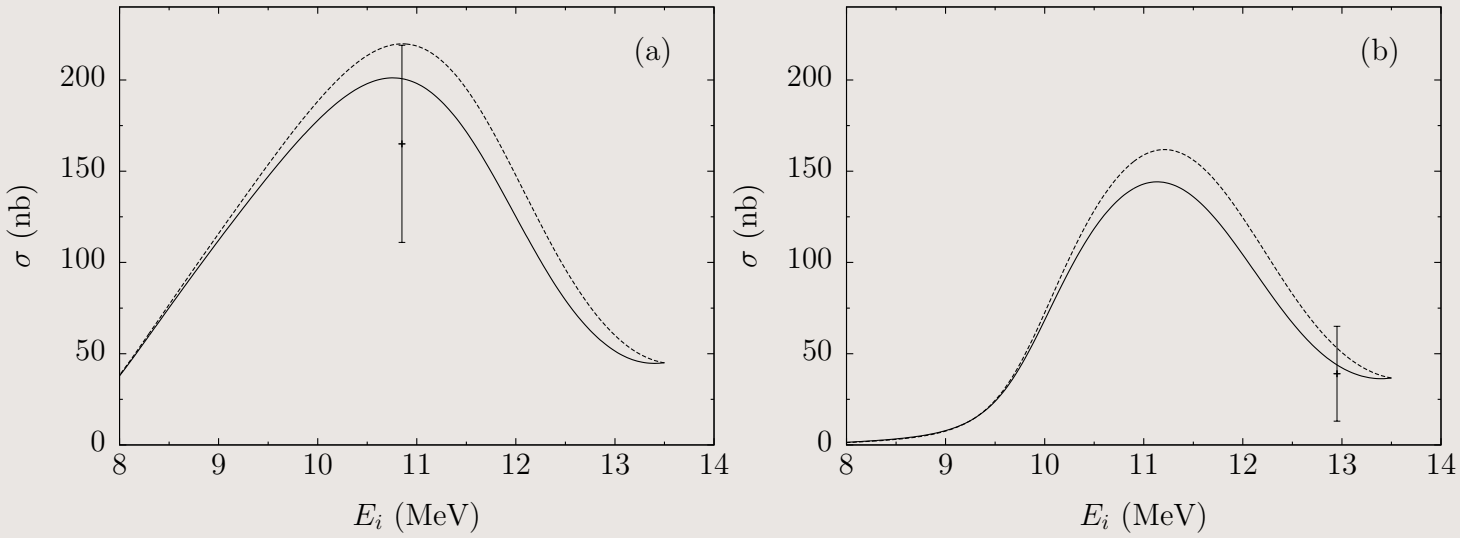
- The bremsstrahlung cross sections integrated over the angles and the photon energy have been measured for two colliding energies E_i

$$\sigma(E_{\min}, E_{\max}) = \int_{E_{\min}}^{E_{\max}} \frac{\mathrm{d}\sigma}{\mathrm{d}E_\gamma} \mathrm{d}E_\gamma .$$

E_i (MeV)	$E_{\min}$ (MeV)	$E_{\max}$ (MeV)	Siegert	non-Siegert	Exp.
10.85	5.0	12.5	200 (178)	219 (199)	165±54
12.95	7.0	14.5	44 (12)	53 (24)	39±26

- Theoretical and experimental cross sections $\sigma(E_{\min}, E_{\max})$ in nb for two initial energies. In parentheses, cross sections considering only 4^+ to 2^+ transitions are given.
- Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 94, (2005) 122502]
- Table from [JDE, D Baye, Phys. Rev. C 88 (2013) 024602]

Integrated cross sections

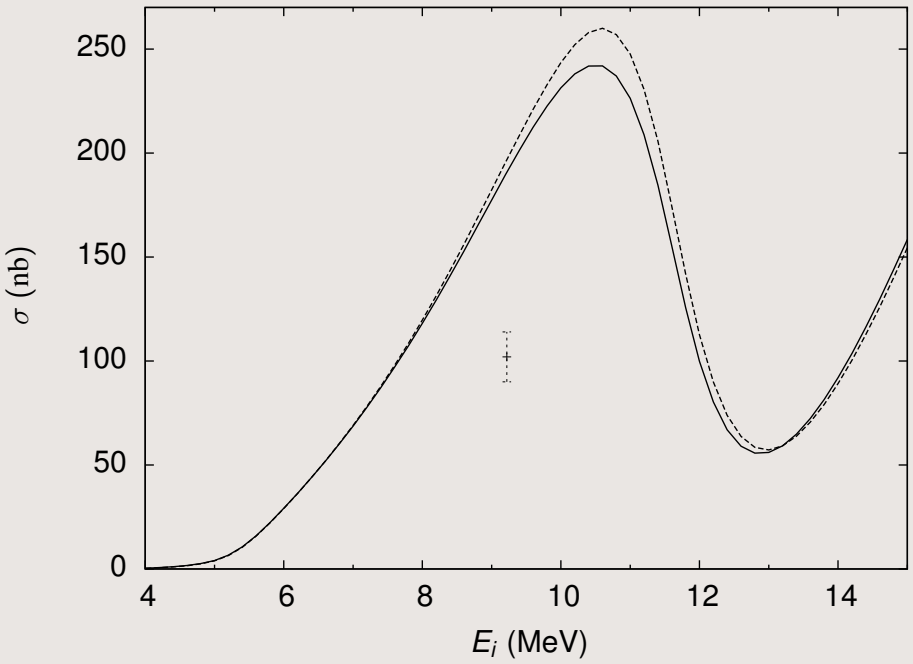


Cross sections $\sigma(E_{\min}, E_{\max})$ as a function of the initial energy E_i for (a) $[E_{\min}, E_{\max}] = [5.0, 12.5]$ MeV and (b) $[E_{\min}, E_{\max}] = [7.0, 14.5]$ MeV in the Siegert (full lines) and non-Siegert (dashed lines) approaches.

Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 94, (2005) 122502]

Figure from [JDE, D Baye, Phys. Rev. C 88 (2013) 024602]

Integrated cross sections

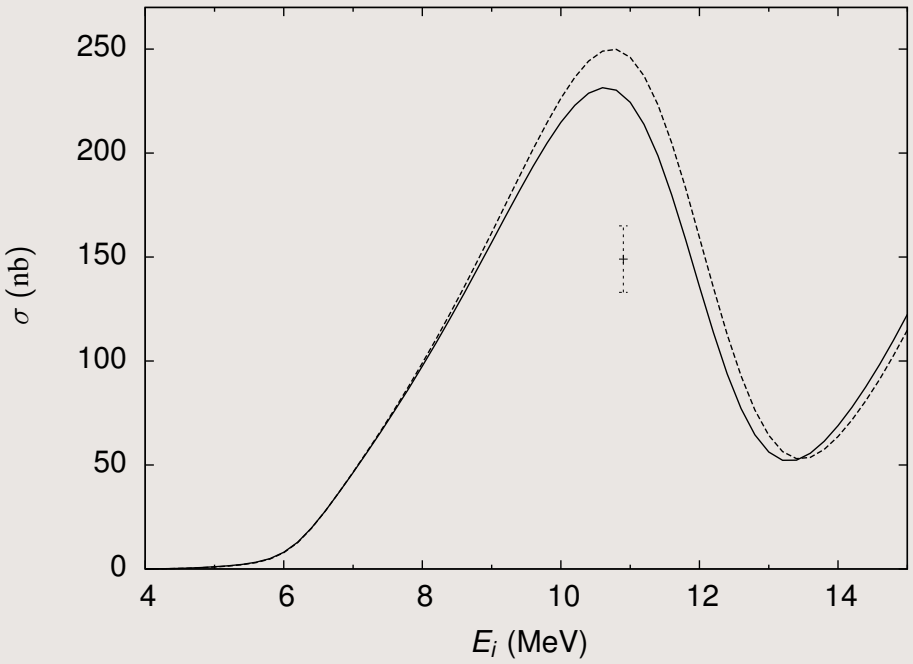


Cross sections $\sigma(E_{\min}, E_{\max})$ as a function of the initial energy E_i for $[E_{\min}, E_{\max}] = [2.5, 8.8]$ MeV in the Siegert (full lines) and non-Siegert (dashed lines) approaches.

Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 111 (2013) 062502]

Figure from [JDE, D Baye, Journal of Physics: Conf. Ser. 569 (2014) 012074]

Integrated cross sections

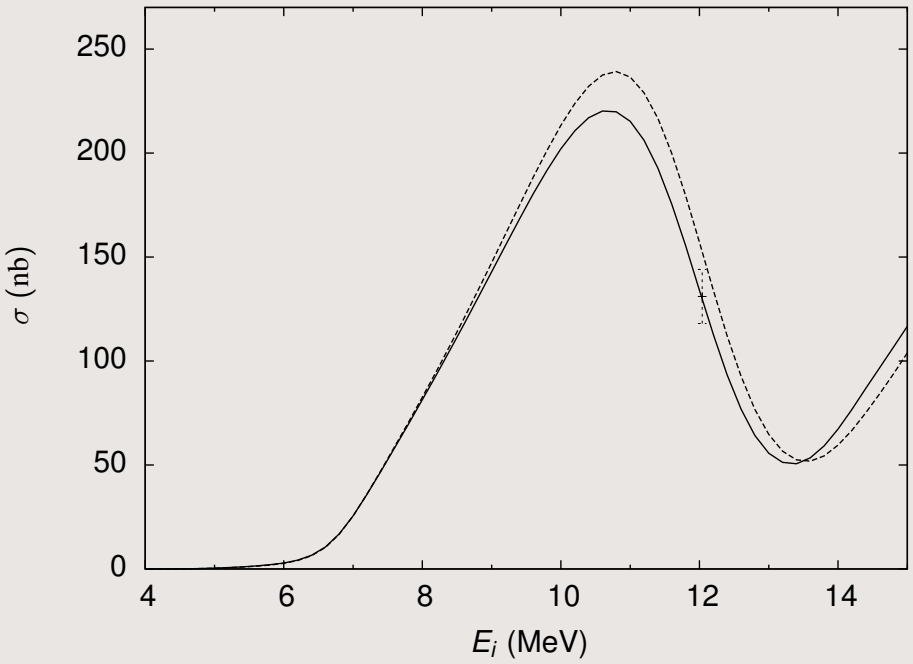


Cross sections $\sigma(E_{\min}, E_{\max})$ as a function of the initial energy E_i for $[E_{\min}, E_{\max}] = [3.4, 10.5]$ MeV in the Siegert (full lines) and non-Siegert (dashed lines) approaches.

Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 111 (2013) 062502]

Figure from [JDE, D Baye, Journal of Physics: Conf. Ser. 569 (2014) 012074]

Integrated cross sections

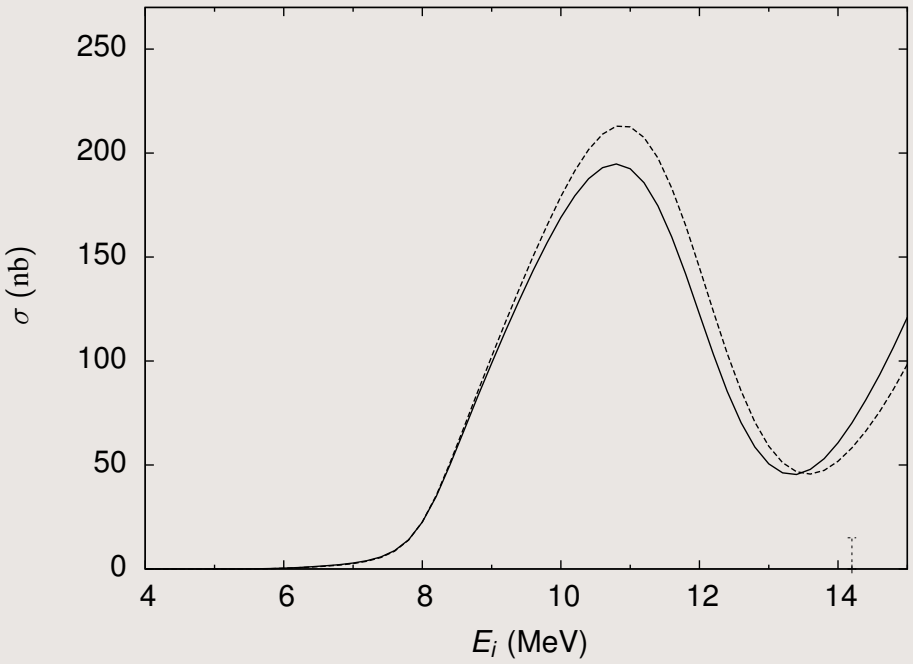


Cross sections $\sigma(E_{\min}, E_{\max})$ as a function of the initial energy E_i for $[E_{\min}, E_{\max}] = [4, 11.6]$ MeV in the Siegert (full lines) and non-Siegert (dashed lines) approaches.

Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 111 (2013) 062502]

Figure from [JDE, D Baye, Journal of Physics: Conf. Ser. 569 (2014) 012074]

Integrated cross sections



Cross sections $\sigma(E_{\min}, E_{\max})$ as a function of the initial energy E_i for (a) $[E_{\min}, E_{\max}] = [5.3, 14]$ MeV in the Siegert (full lines) and non-Siegert (dashed lines) approaches.

Experimental data from [V. M. Datar *et al.*, Phys. Rev. Lett. 111 (2013) 062502]

Figure from [JDE, D Baye, Journal of Physics: Conf. Ser. 569 (2014) 012074]

Comparison with experiments

- Good description of elastic scattering with the microscopic cluster approach
- Good agreement between theory and experiment for the differential $\alpha + \alpha$ bremsstrahlung cross sections in the Harvard geometry (but large error bars).
- Good agreement between theory and experiment for the integrated $\alpha + \alpha$ bremsstrahlung cross sections from [Dat05](but large error bars)
- Bad agreement between theory and experiment for the integrated $\alpha + \alpha$ bremsstrahlung cross sections from [Dat13] (small error bars)
- Differences between Siegert and non-Siegert approaches are rather small for the considered energies and configurations ($\rightarrow \alpha + \alpha$ system is well described by a cluster wave function)
- Importance to compare theoretical and experimental integrated bremsstrahlung cross sections for the same photon-energy range is emphasized
- OPEN QUESTIONS: Are experimental error bars too optimistic? Does the cluster model really fail to reproduce the experimental bremsstrahlung cross sections? Is there some mismatch between the physical quantities which are measured and the ones which are calculated?

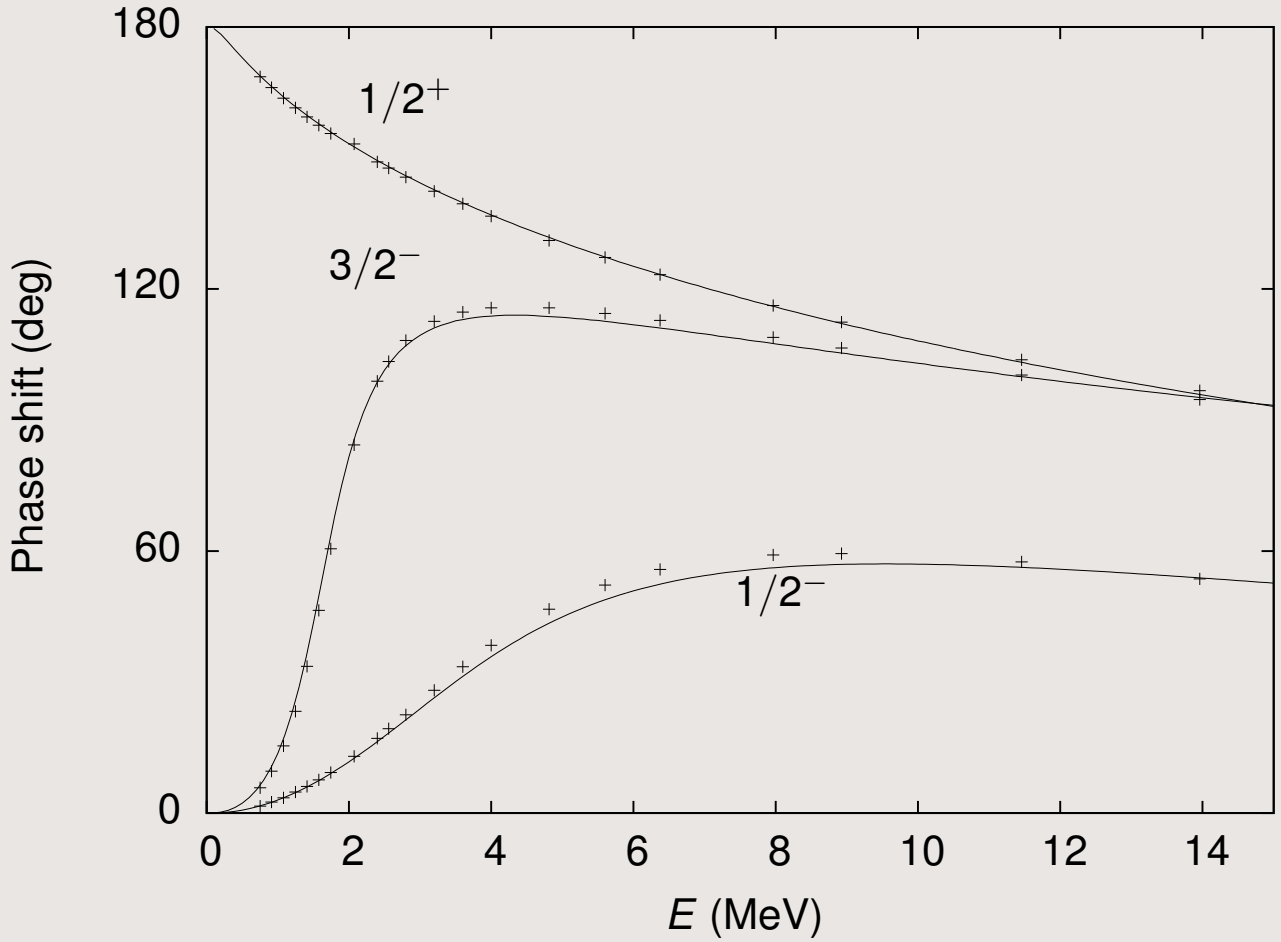
α+N bremsstrahlung

Motivation

- Direct comparison between theory and experiment possible for the $\alpha + p$ bremsstrahlung
- $\alpha + n$ bremsstrahlung describes the final channel of $t(d, \gamma n)\alpha$
- α+N systems well described by microscopic cluster models and ab initio approach*!

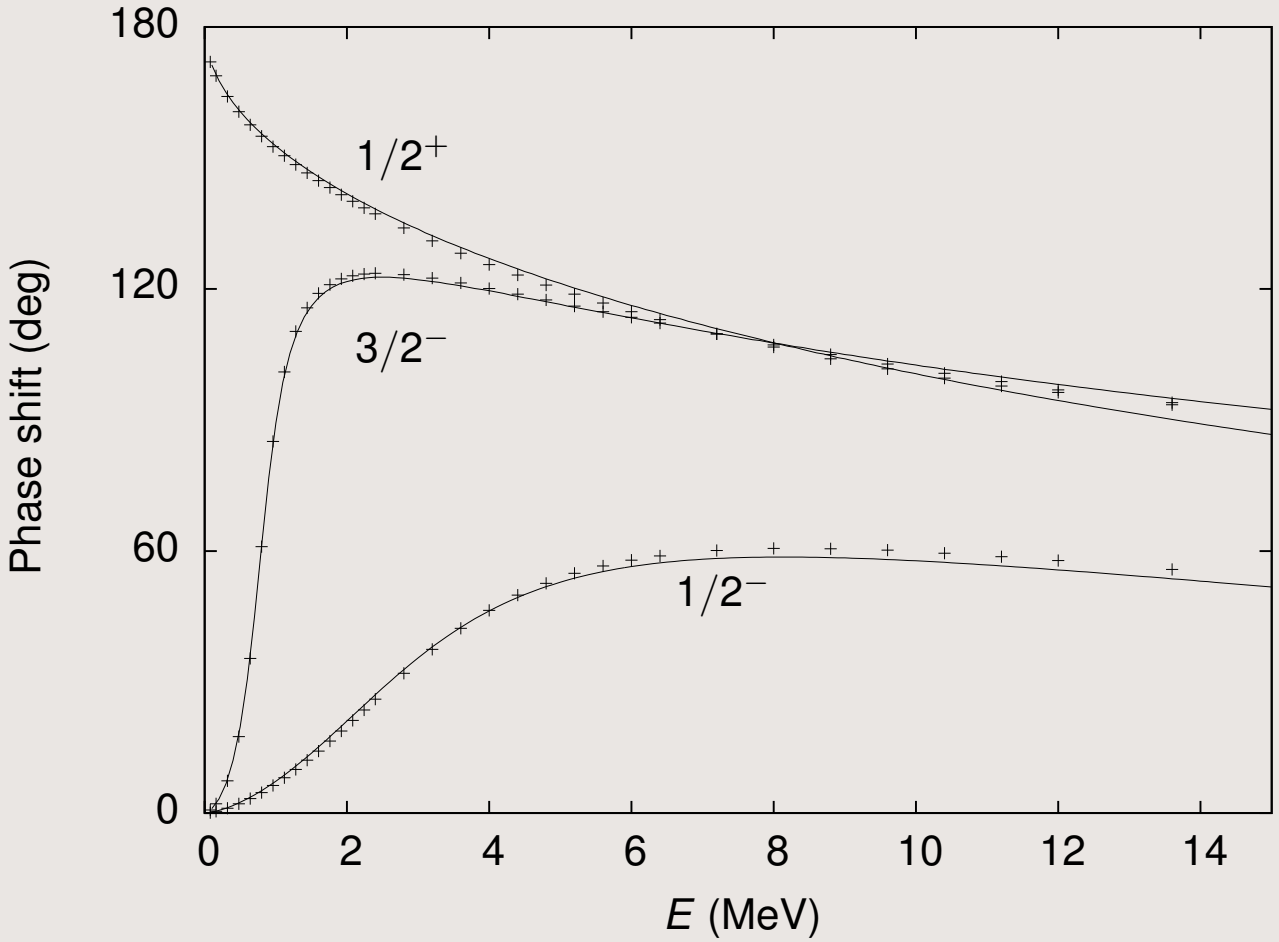
  *G. Hupin, S. Quaglioni, P. Navrátil, PRC 90 (2014) 061601(R)

$\alpha + p$ phase shifts (effective potential)



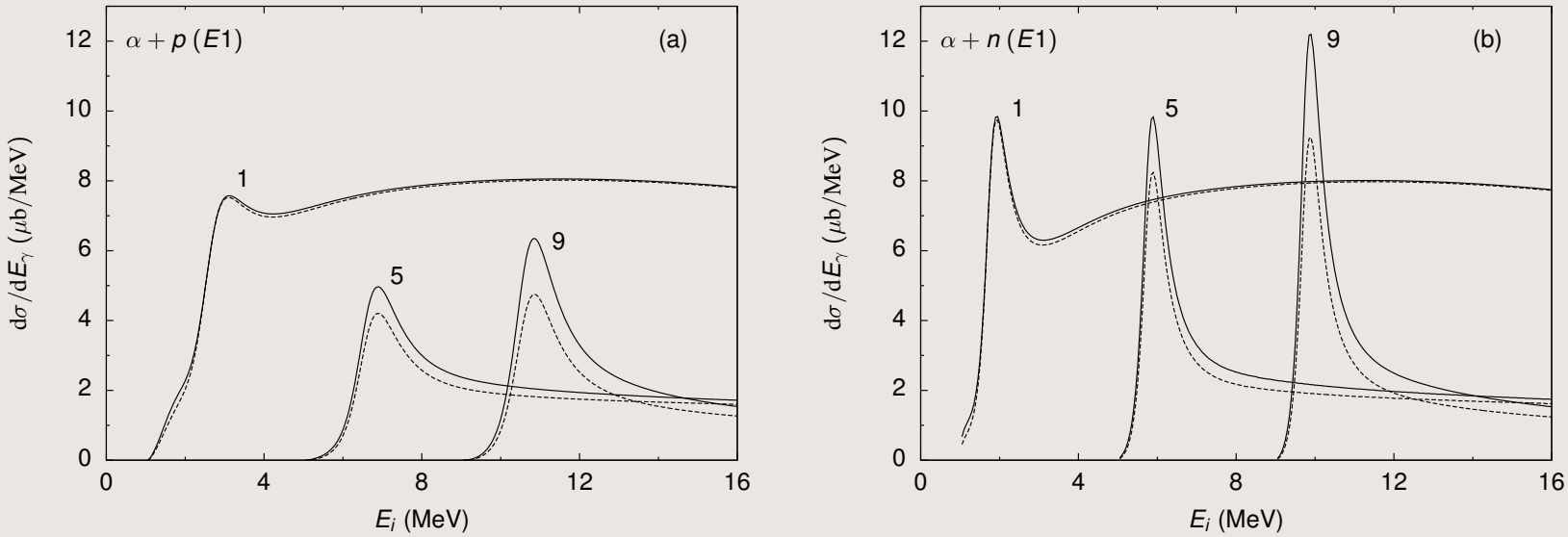
Experimental data from [Satchler, Owen, Elwyn, Morgan and Walter, Nucl. Phys. A 112 (1968) 1]

$\alpha + n$ phase shifts (effective potential)



Experimental data from [Morgan and Walter, Phys. Rev. 168 (1968) 1114]

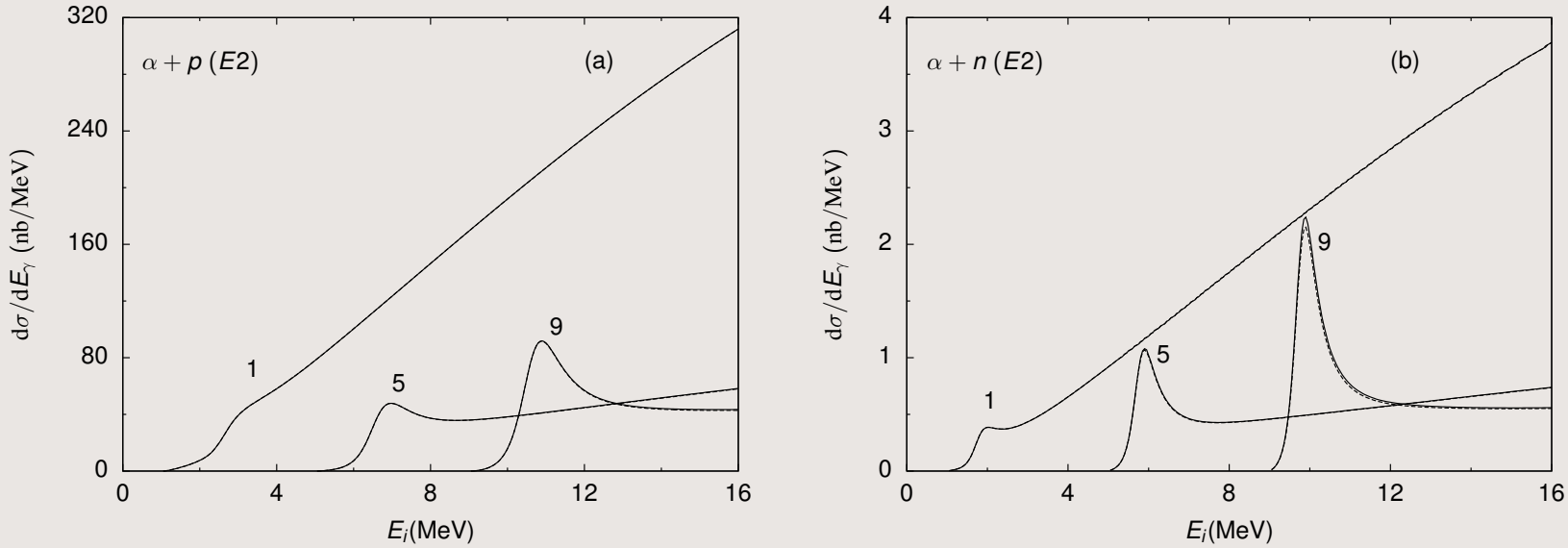
$\alpha + N$ bremsstrahlung



[JDE, Phys. Rev. C 89 (2014) 024617]

- Peak at the final energy corresponding approx. to the $3/2^-$ resonance

$\alpha + N$ bremsstrahlung



[JDE, Phys. Rev. C 89 (2014) 024617]

- Peak at the final energy corresponding approx. to the $3/2^-$ resonance

Effective charge

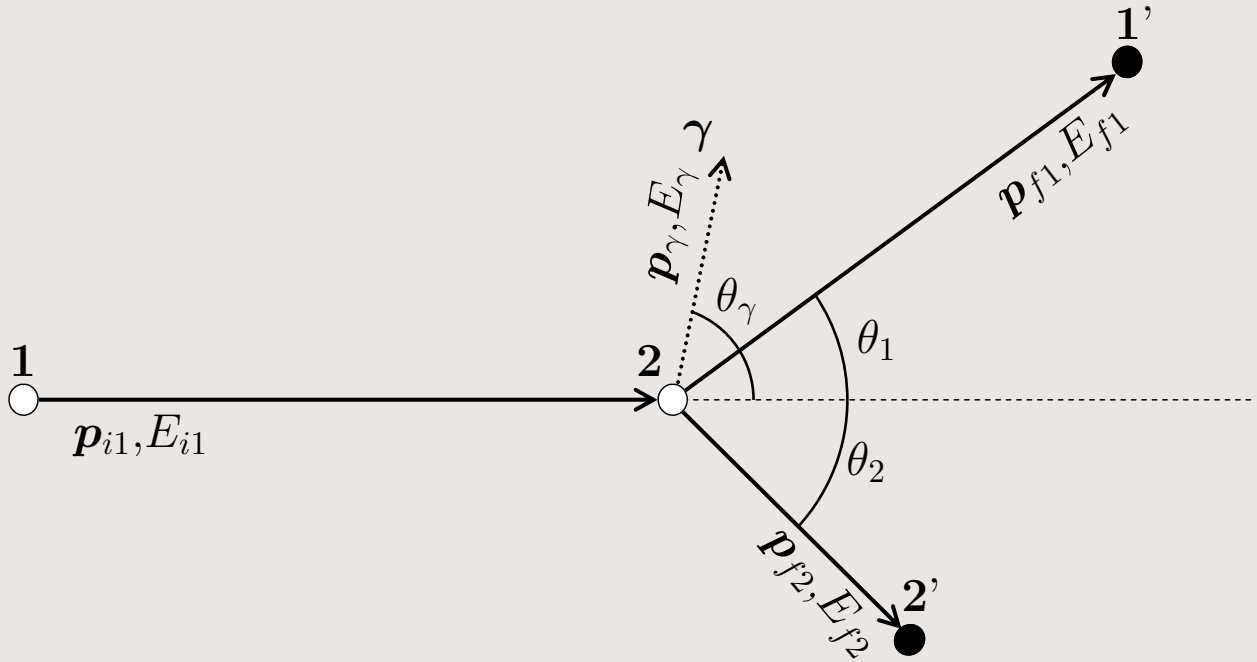
- The ratio of the orders of magnitude of the electric transition contributions can be explained by comparing the effective charges defined by

$$Z_{\mathrm{eff}}^{(\lambda)} = Z_1 \left(\frac{A_2}{A}\right)^{\lambda} + Z_2 \left(\frac{-A_1}{A}\right)^{\lambda}$$

- In first approximation, the ratio between the contributions of a given electric transition for the $\alpha + p$ and $\alpha + n$ bremsstrahlung cross sections is given by the square of the ratio between the effective charges

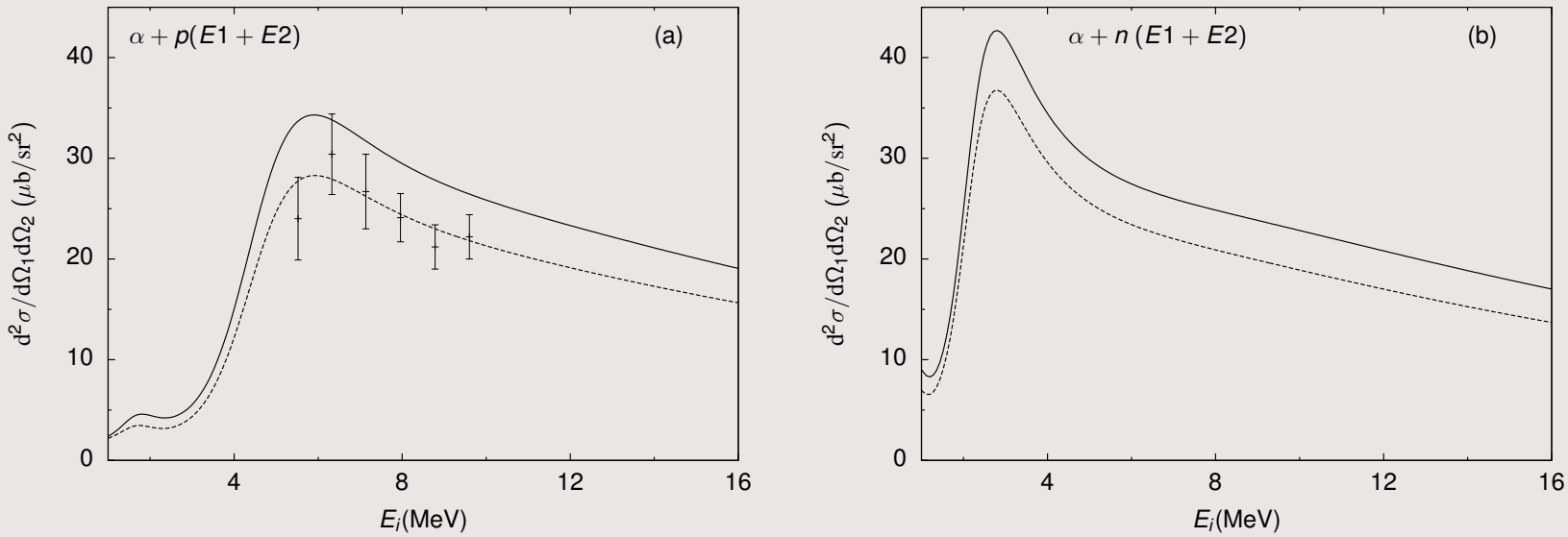
$$\frac{\mathrm{d}\sigma(\alpha p, \mathrm{E}\lambda)}{\mathrm{d}\sigma(\alpha n, \mathrm{E}\lambda)} \approx \left(\frac{Z_{\mathrm{eff},\alpha p}^{(\lambda)}}{Z_{\mathrm{eff},\alpha n}^{(\lambda)}}\right)^2$$

Coplanar configuration



In the so-called Harvard geometry, the photon is undetected.

$\alpha(N, \alpha N)\gamma$ bremsstrahlung



Experimental data from [W. Wölfli, J. Hall, and R. Müller, Phys. Rev. Lett. 27 (1971) 271]

Figure from [JDE, Phys. Rev. C 89 (2014) 024617]

Towards an *ab initio* approach

Motivation

- More fundamental

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- Not restricted to cluster systems (d and t target/projectile can be considered)

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Cluster to *ab initio*

- Effective interaction $\rightarrow$ realistic/chiral EFT interaction

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- $(0s)^4$ cluster $\rightarrow$ NCSM cluster+NCSM A-body state

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- Effective interaction $\rightarrow$ realistic/chiral EFT interaction
- $(0s)^4$ cluster $\rightarrow$ NCSM cluster+NCSM A-body state
- one channel $\rightarrow$ multi-channel (open+closed)
- Microscopic R-matrix on a Lagrange mesh$\rightarrow$ Microscopic R-matrix on a Lagrange mesh

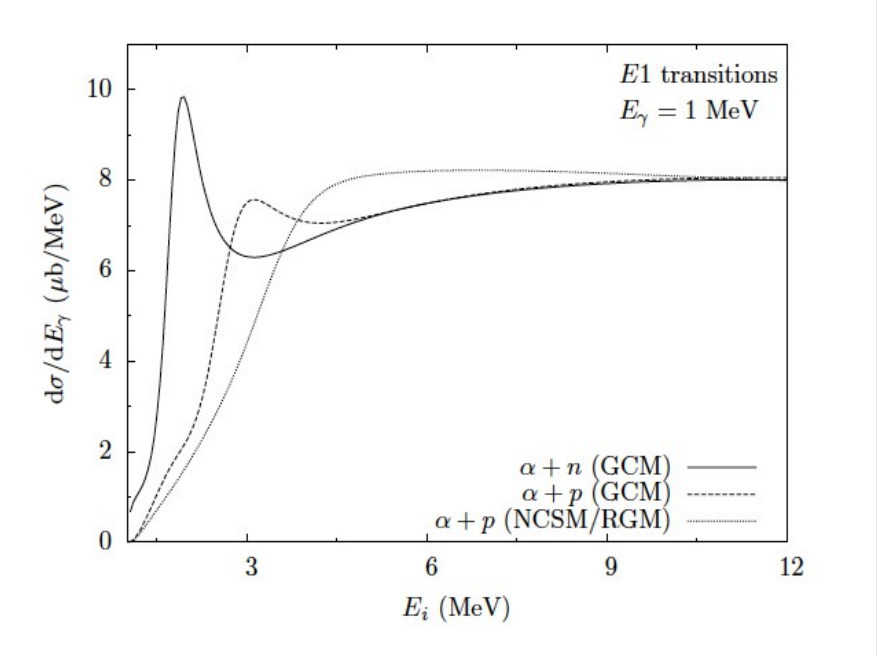
Unitary transformation

- NCSM unadapted to (bare) realistic potential (convergence too slow)
- Solution: modify the model space by a Unitay operator $\Phi \rightarrow U\Phi$
- Calculating the matrix elements by applying the transformations to the operators

$$(\langle\Phi|U^\dagger)O(U|\Phi\rangle) = \langle\Phi|(U^\dagger OU)|\Phi\rangle$$

- In practice, $U^\dagger OU$ is often truncated.
- Implementations: UCOM, SRG,...
- Linked with Couple Cluster, Complex Scaling Method,...

NCSM/RGM (very preliminary!)



J. Dohet-Eraly, S. Quaglioni, P. Navrátil, G. Hupin, arXiv:1501.02744.

Bibliography

- $\alpha + \alpha$ bremsstrahlung
  - JDE, D Baye, Phys. Rev. C 88 (2013) 024602
  - JDE, D Baye, Phys. Rev. C 90 (2014) 034611
  - JDE, D Baye, Journal of Physics: Conf. Ser. 569 (2014) 012074
- α+N bremsstrahlung
  - JDE, Phys. Rev. C 89 (2014) 024617
  - JDE, S Quaglioni, P Navrátil, G Hupin, arXiv:1501.02744

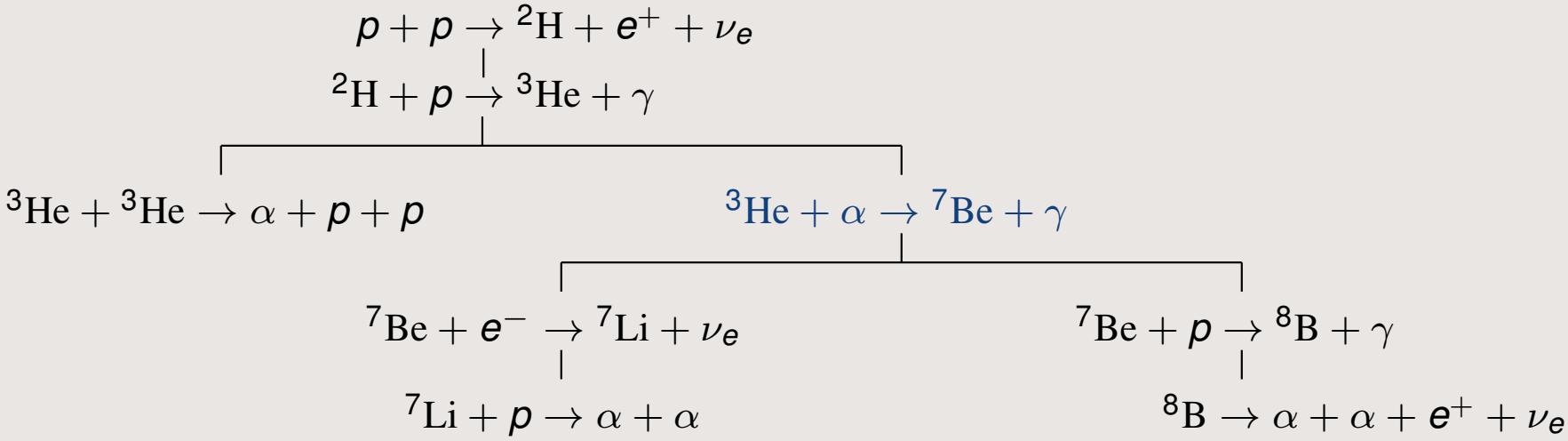
Radiative captures

Motivation: the nuclear reaction in stars

- Radiative captures play an important role in the synthesis of elements in the stars
- Rates of these reactions are essential for describing quantitatively the evolution of the stars
- Radiative capture processes take place at low energies, out of reach of the experiments
- $\Rightarrow$ NUCLEAR MODELS ARE NEEDED

Motivation: *pp*-chains

- Among the nuclear reactions which take place in the stars, the *pp*-chains play a central role. Indeed, they are the first reactions which synthesize nuclear elements since they do not require any catalyst.



Branch I	Branch II	Branch III
$\approx 69\%$	$\approx 30.9\%$	$\approx 0.1\%$

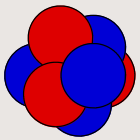
- The relative rates of the ${}^3\mathrm{He}(\alpha,\gamma){}^7\mathrm{Be}$ and ${}^3\mathrm{He}({}^3\mathrm{He},2p){}^4\mathrm{He}$ reactions determines which percentage of the *pp*-chain terminations produces neutrinos.

Bound state with NCSMC

- In the NCSMC, the A-nucleon wave function is expanded as

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda \underbrace{|A\lambda J^\pi T\rangle}_{NCSM} + \sum_\nu \int dr\, r^2 \frac{\gamma_\nu^{J^\pi T}(r)}{r} \mathcal{A}_\nu \underbrace{|\Phi_{\nu r}^{J^\pi T}\rangle}_{NCSM/RGM}$$

$|A\lambda J^\pi T\rangle$ = approximate eigenstates of the A-nucleon Schrödinger equation obtained within the No-Core Shell Model.



$$|\Phi_{\nu r}^{J^\pi T}\rangle = \left[\left(|A_1 \alpha_1 I_1^{\pi_1} T_1\rangle |A_2 \alpha_2 I_2^{\pi_2} T_2\rangle\right)^{IT} Y_\ell(\Omega_{12})\right] \frac{\delta(r - r_{12})}{r r_{12}}$$

= Cluster states where the clusters are approximate eigenstates (ground state and excited states) of the A_1- or A_2- nucleon Schrödinger equation within the No−Core Shell Model

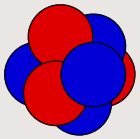
*S. Baroni, P. Navratil, and S. Quaglioni, Phys. Rev. Lett 110, 022505 (2013); Phys. Rev. C 87, 034326 (2013)

Scattering state with NCSMC

- In the NCSMC, the A-nucleon wave function is expanded as

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda \underbrace{|A\lambda J^\pi T\rangle}_{NCSM} + \sum_\nu \int dr\, r^2 \frac{\gamma_\nu^{J^\pi T}(r)}{r} \mathcal{A}_\nu \underbrace{|\Phi_{\nu r}^{J^\pi T}\rangle}_{NCSM/RGM}$$

$|A\lambda J^\pi T\rangle$ = These states are essential to improve the quality of the wave function at short inter-cluster distances.



$$|\Phi_{\nu r}^{J^\pi T}\rangle = \left[\left(|A_1 \alpha_1 I_1^{\pi_1} T_1\rangle |A_2 \alpha_2 I_2^{\pi_2} T_2\rangle\right)^{IT} Y_\ell(\Omega_{12})\right] \frac{\delta(r - r_{12})}{r r_{12}}$$

= Cluster states where the clusters are approximate eigenstates (ground state and excited states) of the A_1- or A_2- nucleon Schrödinger equation within the No−Core Shell Model

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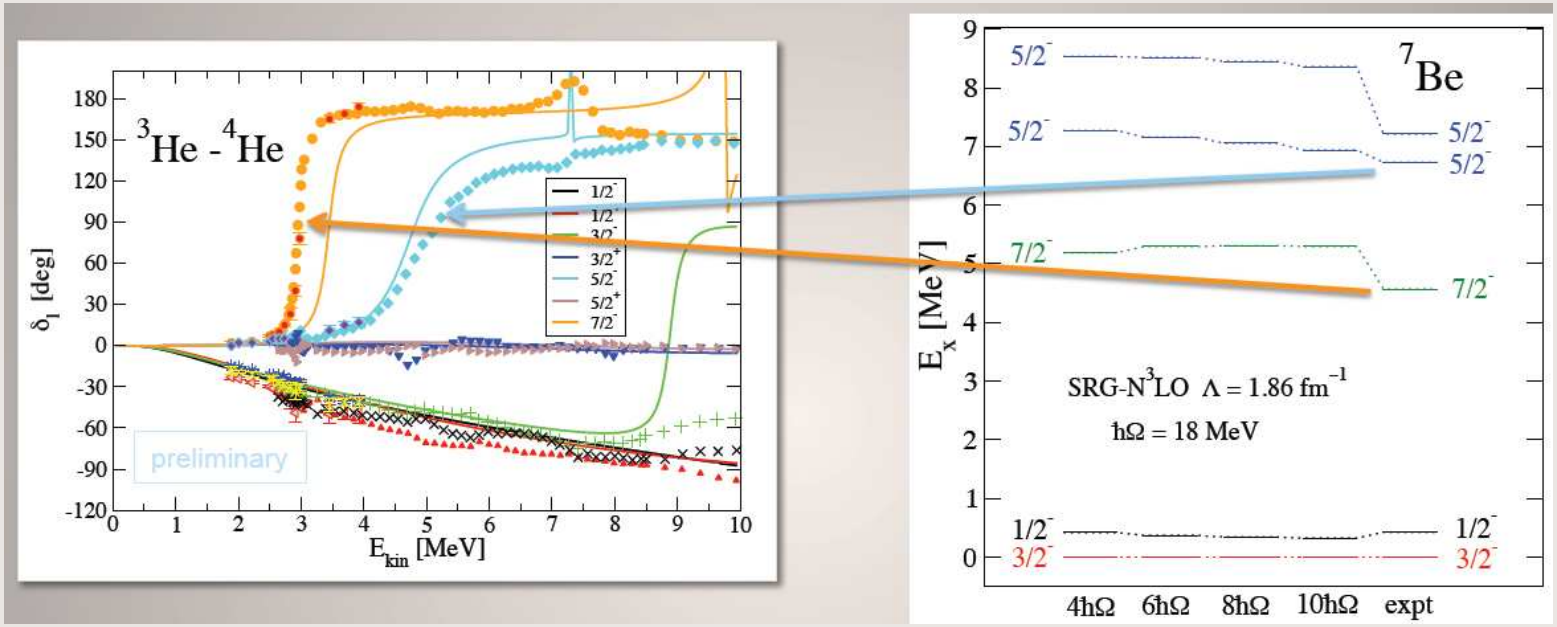
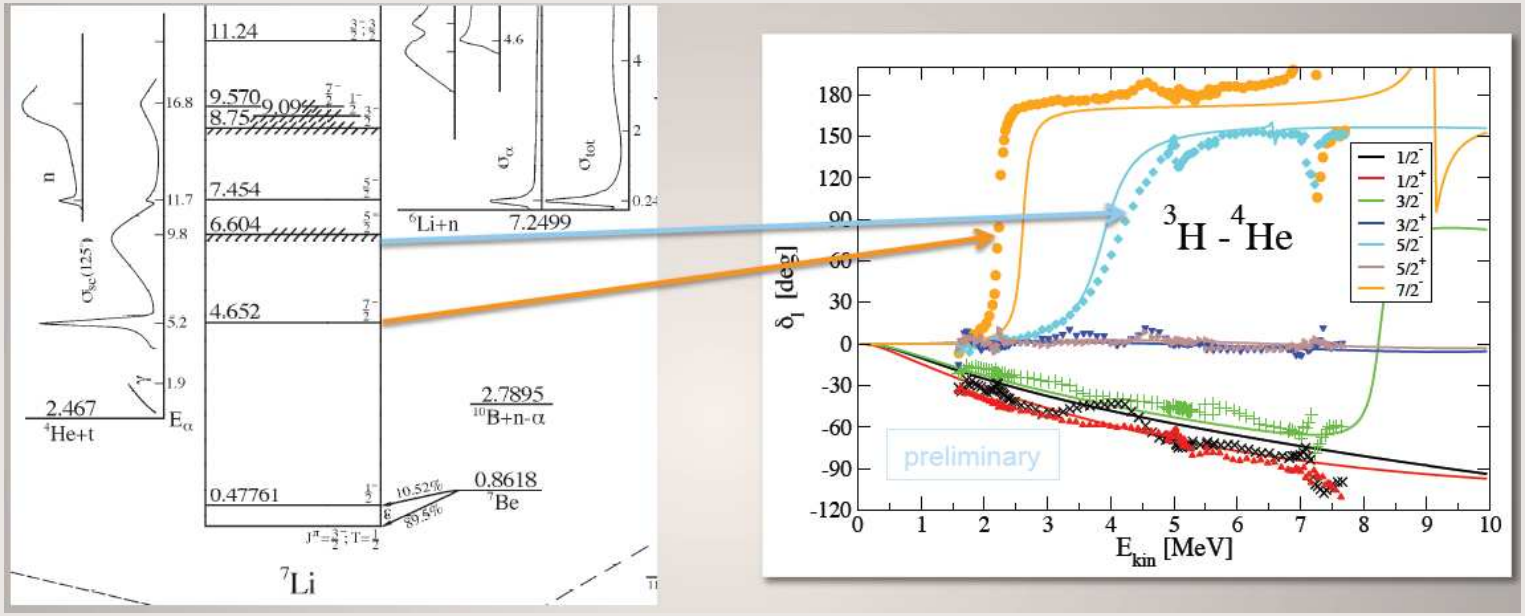
$\alpha + {}^3\mathrm{He}$



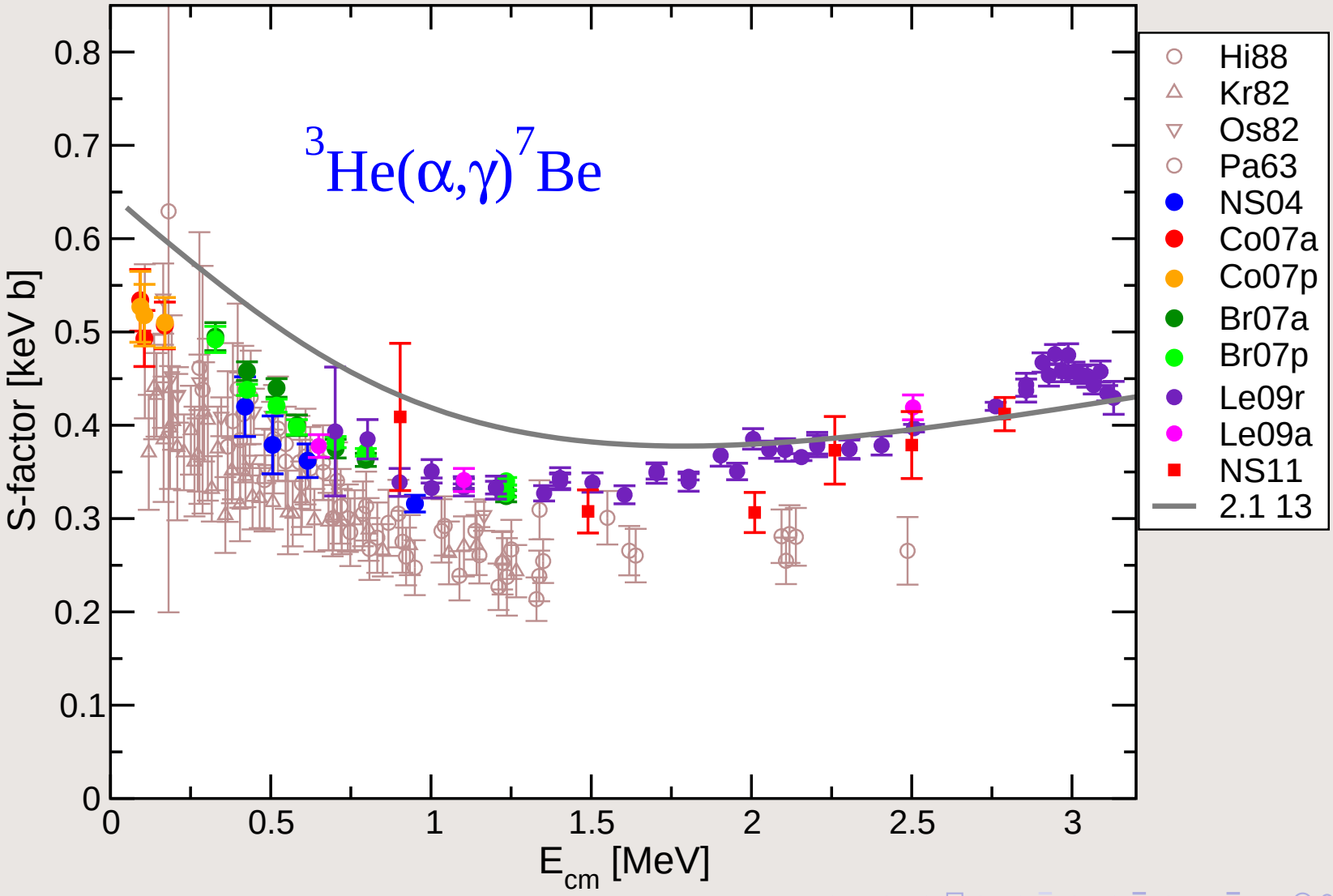
- NCSMC calculations with SRG N^{3}LO *NN* potenital ($\lambda = 2.1$ fm^{-1})
- Preliminary: $N_{max} = 12; \hbar\Omega = 20$ MeV
- $^3\mathrm{He}$, α ground state
- 8 eigenstates with negative parity of $^7\mathrm{Be}$
- 6 eigenstates with positive parity of $^7\mathrm{Be}$
- $E_{th}(^7\mathrm{Be}) = -1.70$ MeV ; $E_{exp}(^7\mathrm{Be}) = -1.59$ MeV

$\alpha + {}^3\mathrm{H}$



- NCSMC calculations with SRG N^{3}LO *NN* potenital ($\lambda = 2.1$ fm^{-1})
- Preliminary: $N_{max} = 12; \hbar\Omega = 20$ MeV
- ${}^3\mathrm{H}$, α ground state
- 8 eigenstates with negative parity of ${}^7\mathrm{Li}$
- 6 eigenstates with positive parity of ${}^7\mathrm{Li}$
- $E_{th}({}^7\mathrm{Li}) = -2.62$ MeV ; $E_{exp}({}^7\mathrm{Li}) = -2.47$ MeV

$^3He(\alpha,\gamma)^7Be$



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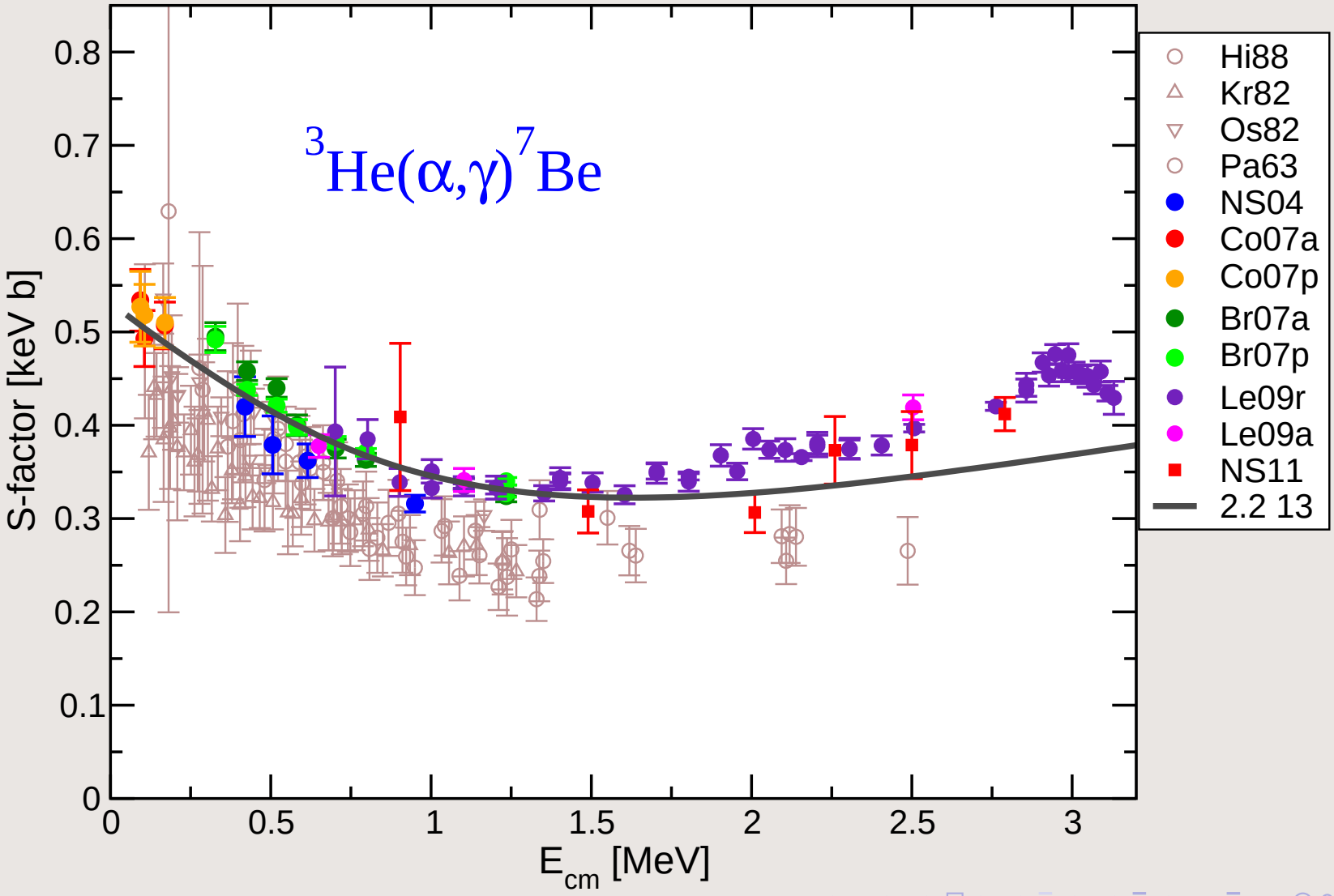

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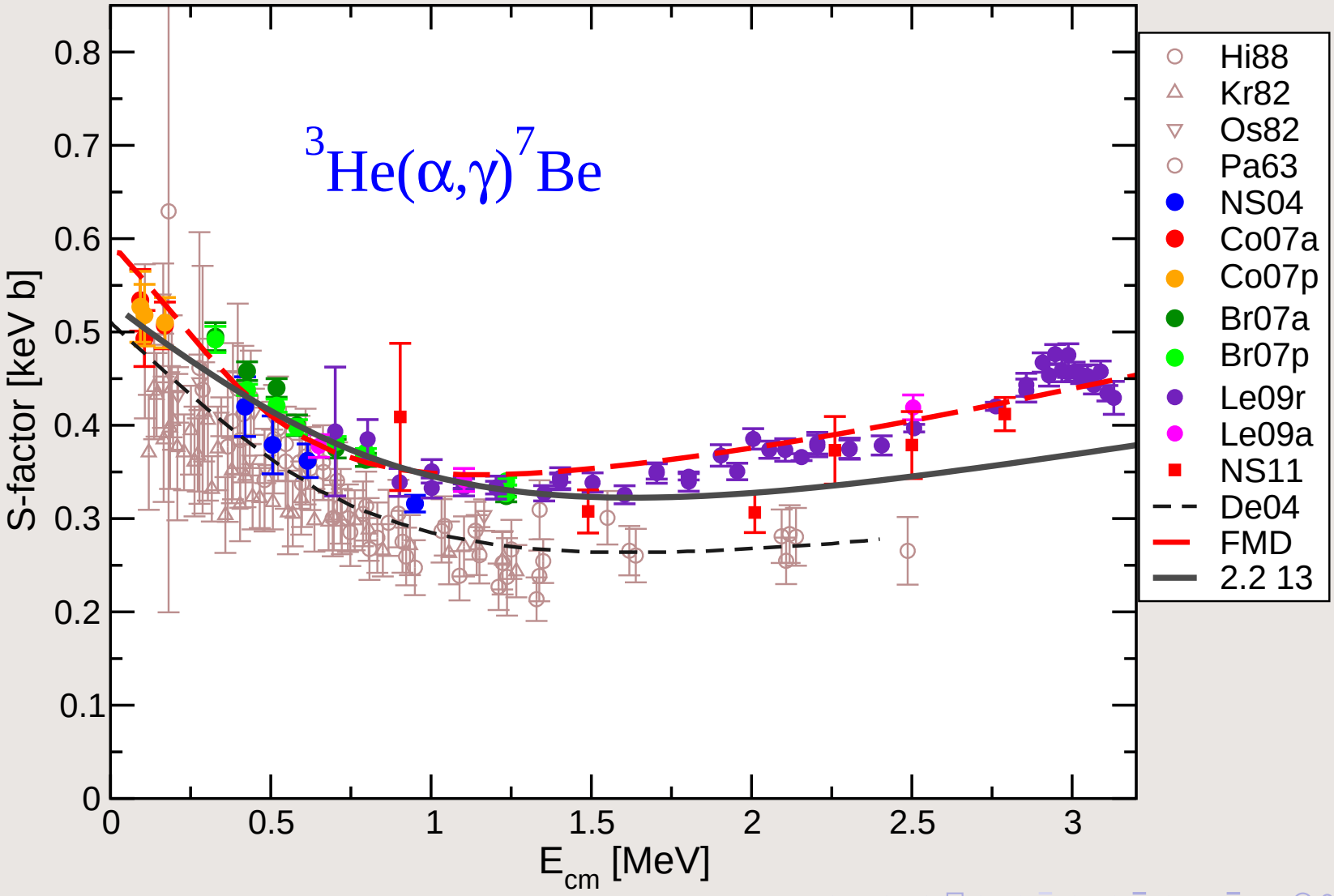
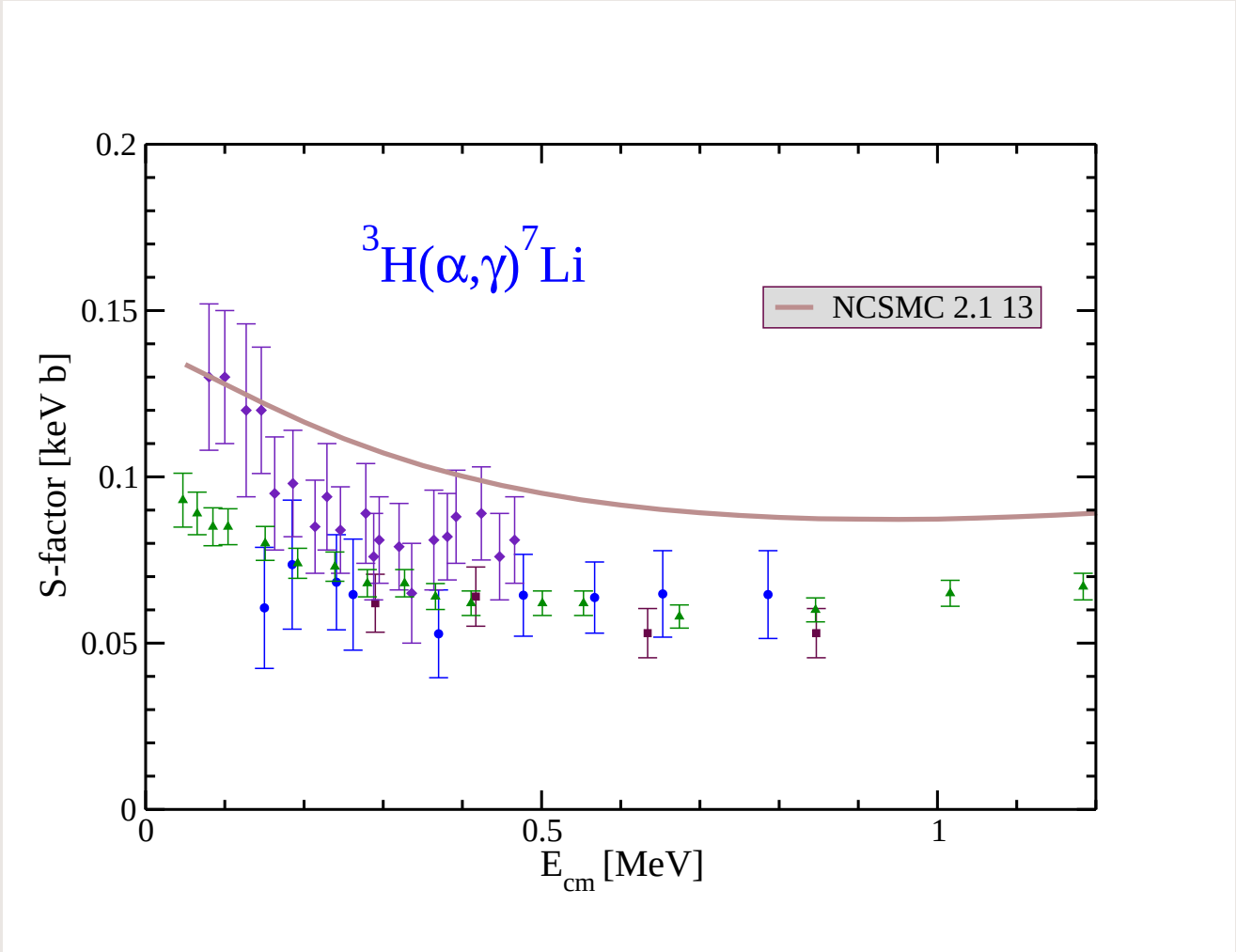


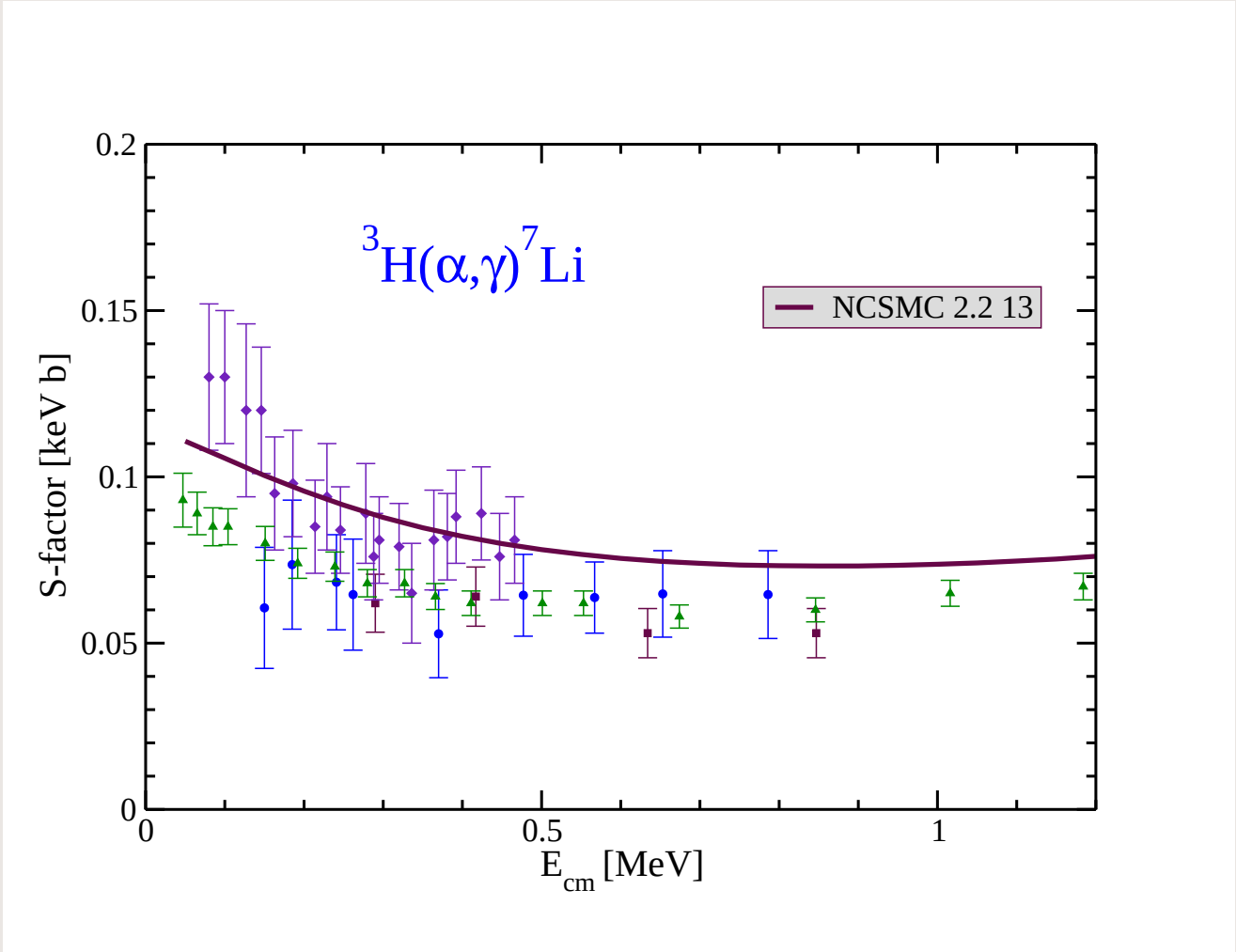
^{7}Be and ^{7}Li properties

		E(MeV)	λ (fm^{-1})
^{7}Be	$3/2^-$	-1.70	2.1
		-1.59	exp
		-1.33	2.2
	$1/2^-$	-1.44	2.1
		-1.16	exp
		-1.08	2.2
^{7}Li	$3/2^-$	-2.62	2.1
		-2.47	exp
		-2.24	2.2
	$1/2^-$	-2.34	2.1
		-1.99	exp
		-1.97	2.2

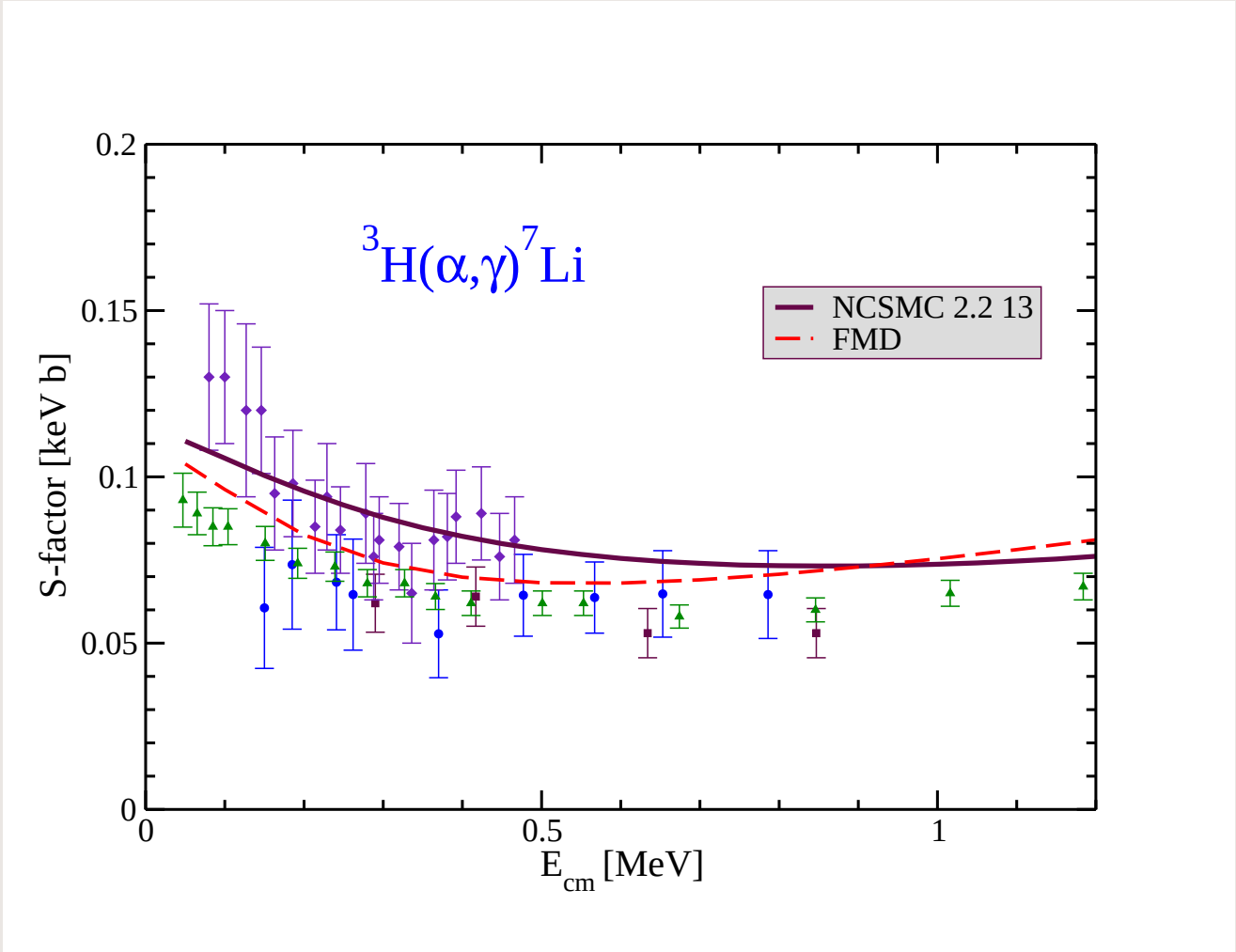
$^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$



$^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$



$^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$



Conclusion

- The NCSMC enables us to describe the bound states and the scattering states within the same framework.
- Hence, the radiative capture processes can be described in a rigorous way.
- The approach is applied to the 7-nucleon system:
  - the ^{7}Be and ^{7}Li ground states
  - the $\alpha + ^3$He and $\alpha + ^3$H elastic scattering
  - and the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ radiative captures

  are studied.
- The results are qualitatively in agreement with the experiments.
- A quantitative comparison requires to increase the size of the NCSMC basis and to include three-nucleon forces.
- The accuracy could be improved by considering the full $E1$ operator (especially for the highest photon energies, which are considered).

Summary

Bremsstrahlung

- Microscopic cluster model of bremsstrahlung was developed
- Divergence problem is solved by an extended Siegert theorem
- Applied to $\alpha\alpha$ bremsstrahlung. Discrepancies with the most recent experimental data
- Applied to αN bremsstrahlung.
- Development of an *ab initio* NCSMC approach of bremsstralung in progress
- Prospect:apply to $N(\alpha, N\gamma)\alpha$ and $t(d, n\gamma)\alpha$

Radiative captures

- *ab initio* NCSMC approach is developed
- Restricted currently to 2-nucleon forces
- Applied to $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$

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et en physique des particules

Thank you!
Merci

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