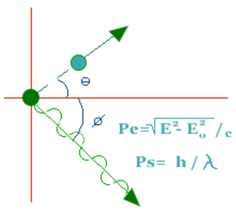
Reactions and Structure employing the Dispersive Optical Model in a broader context

INT 15-58W Seattle, 3/2/2015

Washington University in St.Louis



Wim Dickhoff

Bob Charity

Lee Sobotka

Helber Dussan

Seth Waldecker

Hossein Mahzoon

Carlo Barbieri, Surrey

Arnau Rios, Surrey

Arturo Polls, Barcelona

Dimitri Van Neck, Ghent

Herbert Müther, Tübingen

- Why Green's functions?
- Ab initio and
- as a framework to analyze experimental data (and extrapolate and predict properties of exotic nuclei)

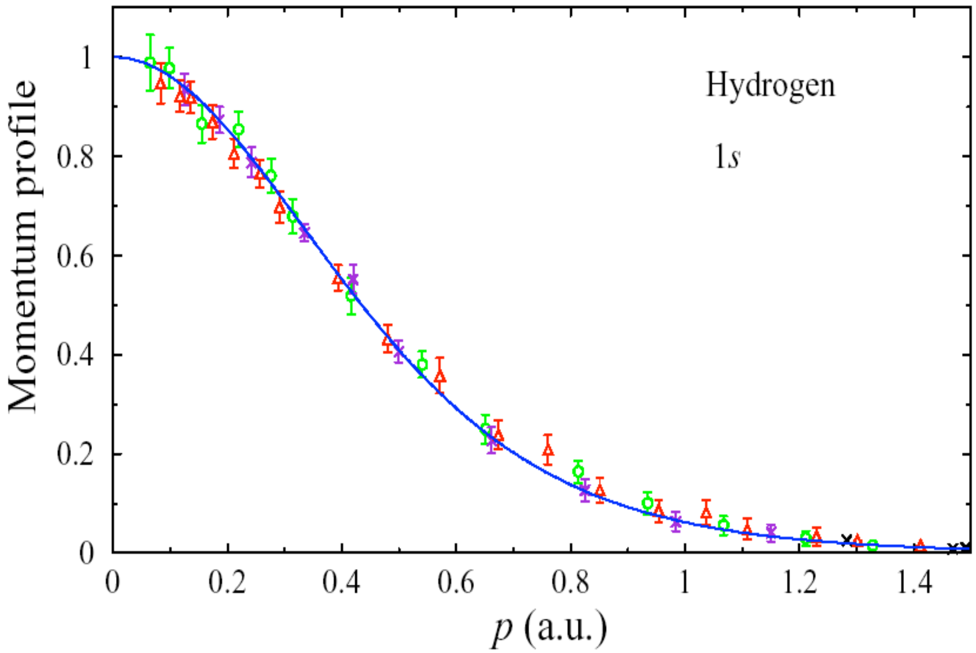
  --> dispersive optical model (DOM)
- Focus on recent DOM —> DSM developments
- Some surprises!
- Conclusions

NSCL

N.B. Nguyen & F. Nuñes

What do nucleons do in the nucleus?

- Shell model from 1949 with residual interaction? Not enough!
- Why do nuclei have the central density they have? **Unanswered**
- Do they sit in independent-particle model orbits all the time?
- Even electrons do other things some of the time!



Hydrogen 1s wave function "seen" experimentally in (e,2e) reaction

Phys. Lett. 86A, 139 (1981)

$$\phi_{1s}(\boldsymbol{p}) = \frac{2^{3/2}}{\pi} \frac{1}{(1+p^2)^2}$$

But in other atoms slight deviations!

- Properly executed --> Green's function answers a question from Sir Denys Wilkinson: "What does a nucleon do in the nucleus?"

Remarks

- Given a Hamiltonian, a perturbation expansion can be generated for the single-particle propagator
- Dyson equation determines propagator in terms of nucleon self-energy
- Self-energy is causal and obeys dispersion relations relating its real and imaginary part
- Data constrained self-energy acts as ideal interface between ab initio theory and experiment

Propagator / Green's function

- Lehmann representation

$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle\Psi_0^A|\, a_{k\ell j}\, |\Psi_m^{A+1}\rangle \langle\Psi_m^{A+1}|\, a^\dagger_{k'\ell j}\, |\Psi_0^A\rangle}{E-(E_m^{A+1}-E_0^A)+i\eta} + \sum_n \frac{\langle\Psi_0^A|\, a^\dagger_{k'\ell j}\, |\Psi_n^{A-1}\rangle \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle}{E-(E_0^A-E_n^{A-1})-i\eta}$$

- Any other single-particle basis can be used
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function

$$S_{\ell j}(k;E) = \frac{1}{\pi}\,\mathrm{Im}\; G_{\ell j}(k,k;E) \qquad E \le \varepsilon_F^-$$

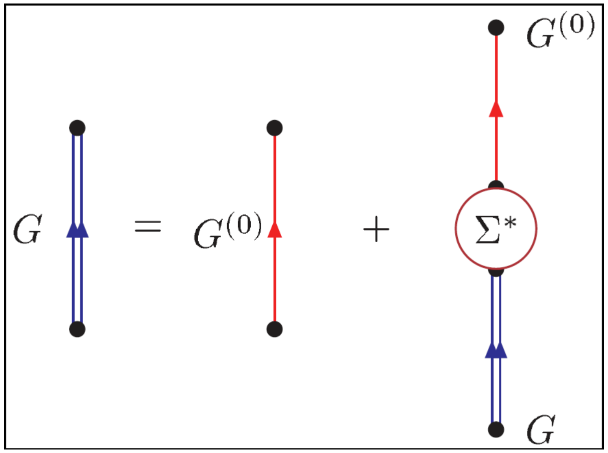
$$= \sum_n \left| \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle \right|^2 \delta(E-(E_0^A-E_n^{A-1}))$$

- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk\; k^2\; S_{\ell j}(k;E)$$

- Discrete transitions $\sqrt{S_{\ell j}^n}\;\phi_{\ell j}^n(k) = \langle\Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle$
- Positive energy —> see later

Propagator from Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

$\Rightarrow$ as extracted from (e,e′p) reaction

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\, q^2\, \Sigma_{\ell j}^*(k,q;E_n^-)\ \phi_{\ell j}^n(q) = E_n^-\ \phi_{\ell j}^n(k)$$

Spectroscopic factor $\quad S_{\ell j}^n = \int dk\, k^2\, \left|\langle \Psi_n^{A-1}|\, a_{k\ell j}\, |\Psi_0^A\rangle\right|^2 < 1$

Dyson equation also yields $\left[\chi_{\ell j}^{elE}(r)\right]^* = \langle \Psi_{elE}^{A+1}|\, a_{r\ell j}^\dagger\, |\Psi_0^A\rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

reactions and structure

Propagator in principle generates

- Elastic scattering cross sections for p and n
- Including all polarization observables
- Total cross sections for n
- Reaction cross sections for p and n
- Overlap functions for adding p or n to bound states in Z+1 or N+1
- Plus normalization --> spectroscopic factor
- Overlap function for removing p or n with normalization
- Hole spectral function including high-momentum description
- One-body density matrix; occupation numbers; natural orbits
- Charge density
- Neutron distribution
- p and n distorted waves
- Contribution to the energy of the ground state from V_{NN}

reactions and structure

Dispersive Optical Model

- Claude Mahaux 1980s
  - connect traditional optical potential to bound-state potential
  - crucial idea: use the dispersion relation for the nucleon self-energy
  - smart implementation: use it in its subtracted form
  - applied successfully to ^{40}Ca and ^{208}Pb in a limited energy window
  - employed traditional volume and surface absorption potentials and a local energy-dependent Hartree-Fock-like potential
  - Reviewed in Adv. Nucl. Phys. **20**, 1 (1991)
- Radiochemistry group at Washington University in St. Louis: Charity and Sobotka propose to use it for a sequence of Ca isotopes —> data-driven extrapolations to the drip line
  - First results 2006 PRL
  - Subsequently —> attention to data below the Fermi energy related to ground-state properties —> Dispersive Self-energy Method (**DSM**)

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\mathrm{Re}\ \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{E-E'}$$

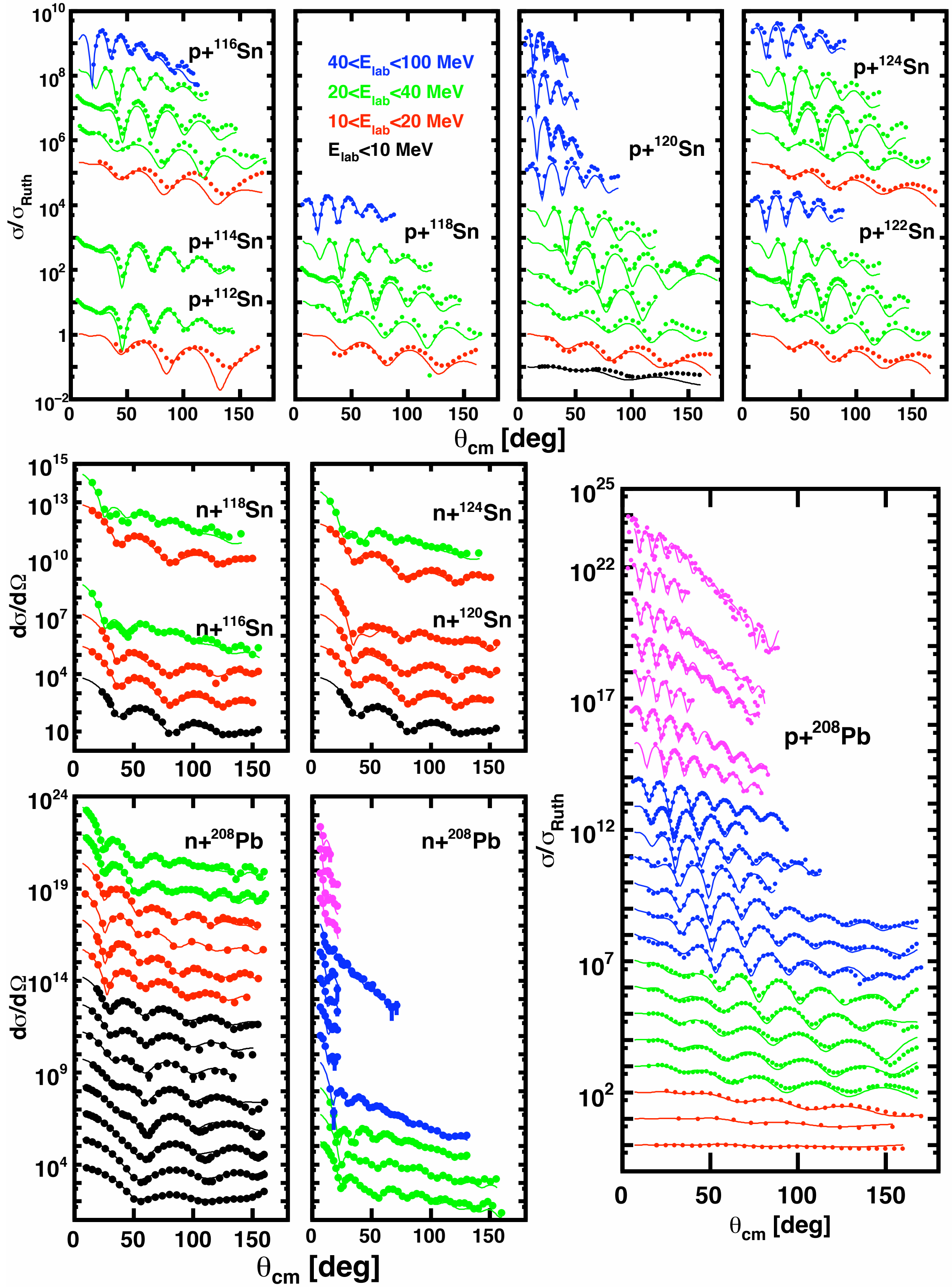
Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1})\right\}$

$$\mathrm{Re}\ \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\mathrm{Re}\ \Sigma(E) = \mathrm{Re}\ \Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\mathrm{Im}\ \Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

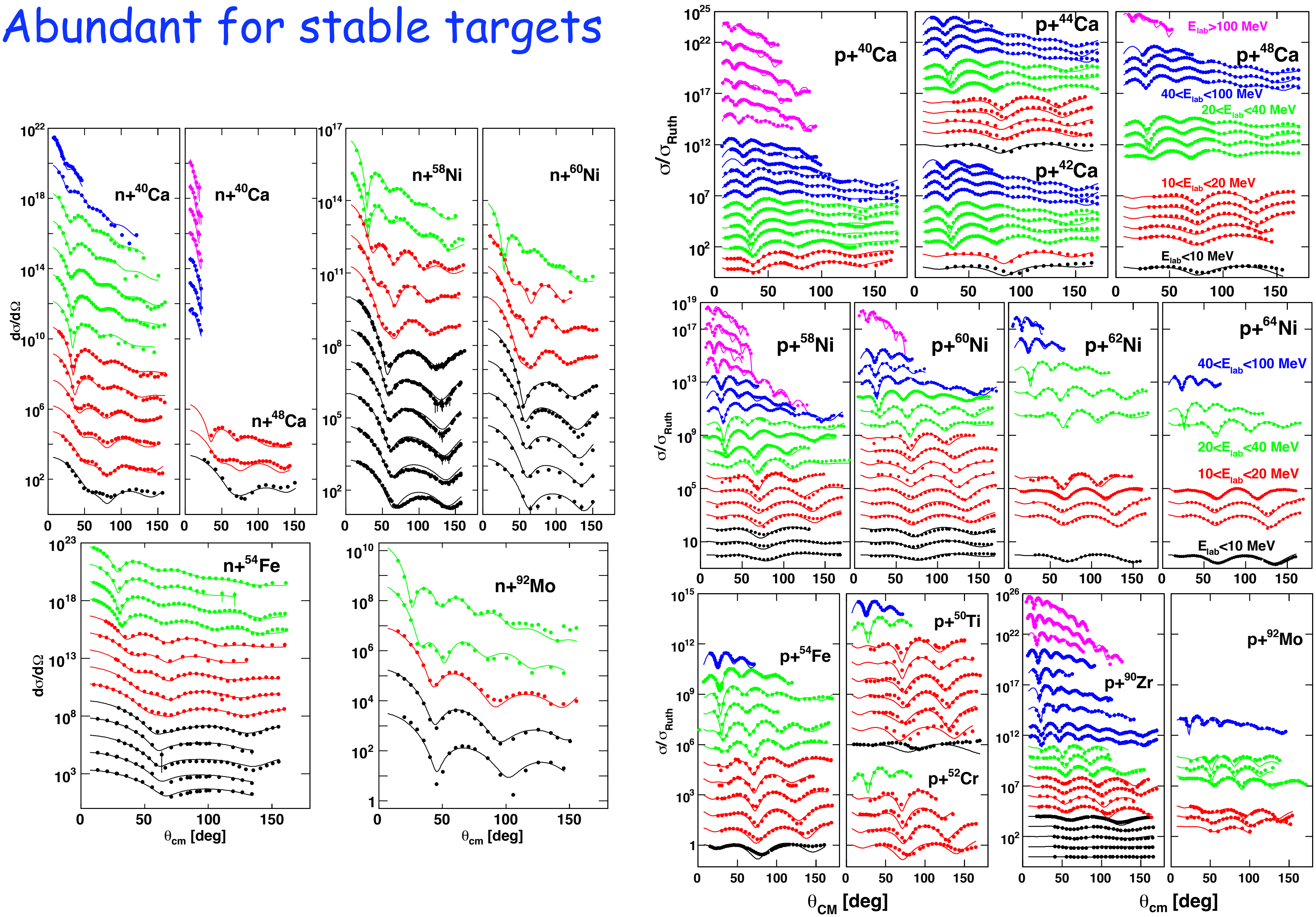


Recent local DOM analysis --> towards global

J. Mueller et al.
PRC83,064605 (2011), 1-32

Elastic scattering data for protons and neutrons

- Abundant for stable targets



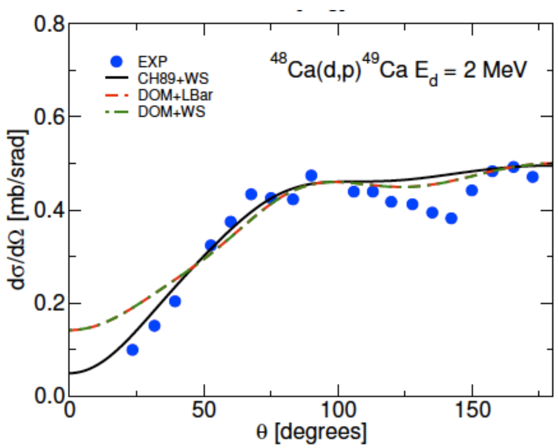
Local DOM ingredients and transfer reactions

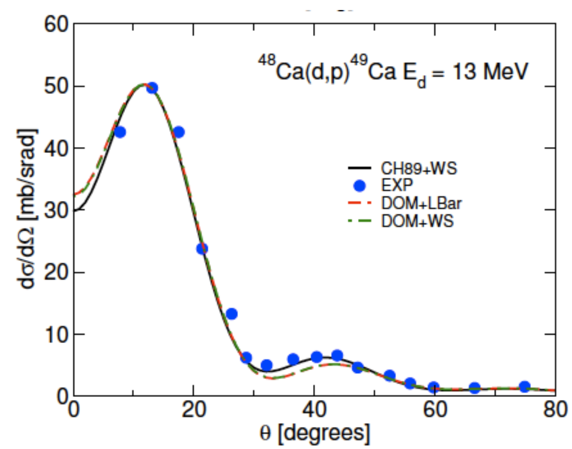
- Overlap function
- p and n optical potential
- ADWA (developed by Ron Johnson)
- MSU-WashU:-->

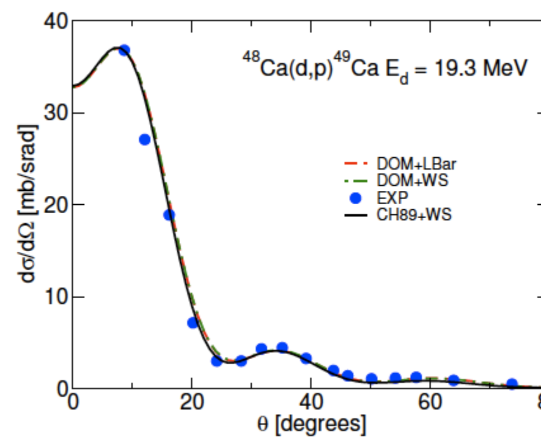
N. B. Nguyen, S. J. Waldecker, F. M. Nuñes, R. J. Charity, and W. H. Dickhoff

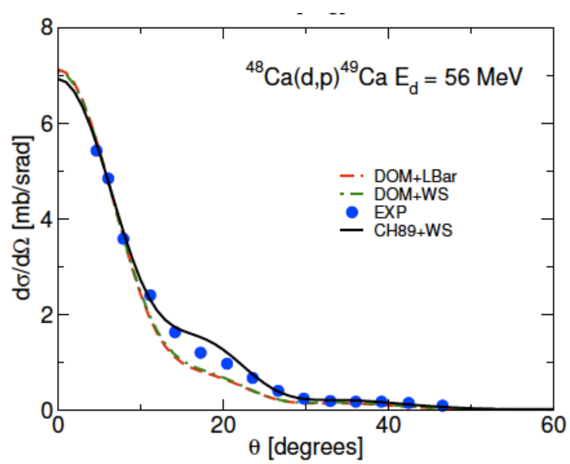
- $^{40,48}Ca$, ^{132}Sn, $^{208}Pb(d,p)$

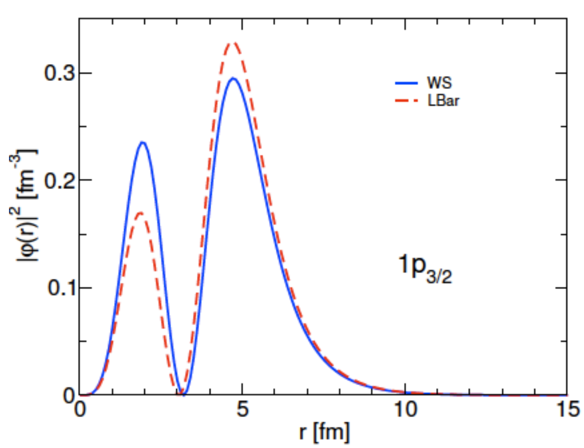
Phys. Rev. C84, 044611 (2011), 1-9









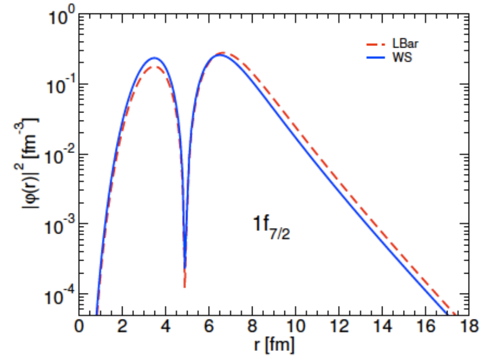


E	CH+ws	DOM
2	0.94	0.72
13	0.82	0.67
19.3	0.77	0.68
56	1.1	0.70

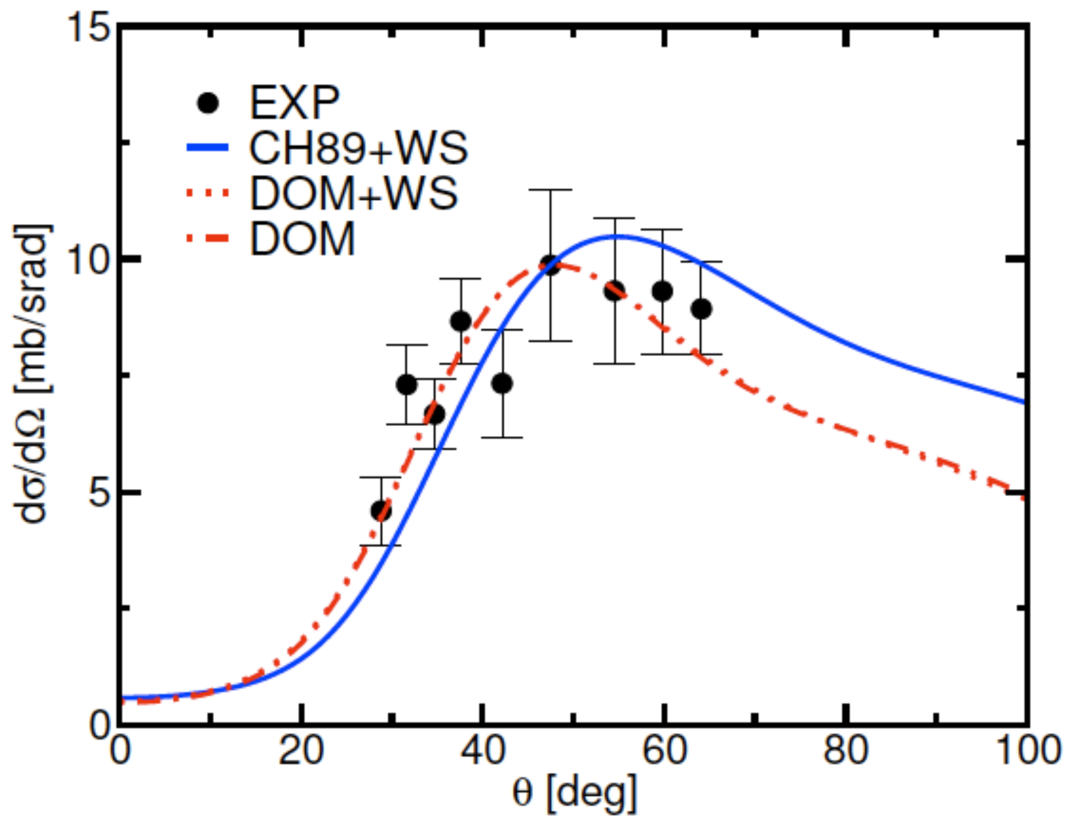
reactions and structure

$^{132}Sn(d,p)$

- Does it work when the potentials are extrapolated?



- Data: K.L. Jones et al., Nature 465, 454 (2010)
- E_d = 9.46 MeV $^{132}Sn(d,p)^{133}Sn$
- CH89+ws --> $S_{1f7/2}$ =1.1
- DOM --> $S_{1f7/2}$ =0.72



Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Microscopic FRPA & SRC --> different nonlocal properties above and below the Fermi energy
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e′p) data from JLab

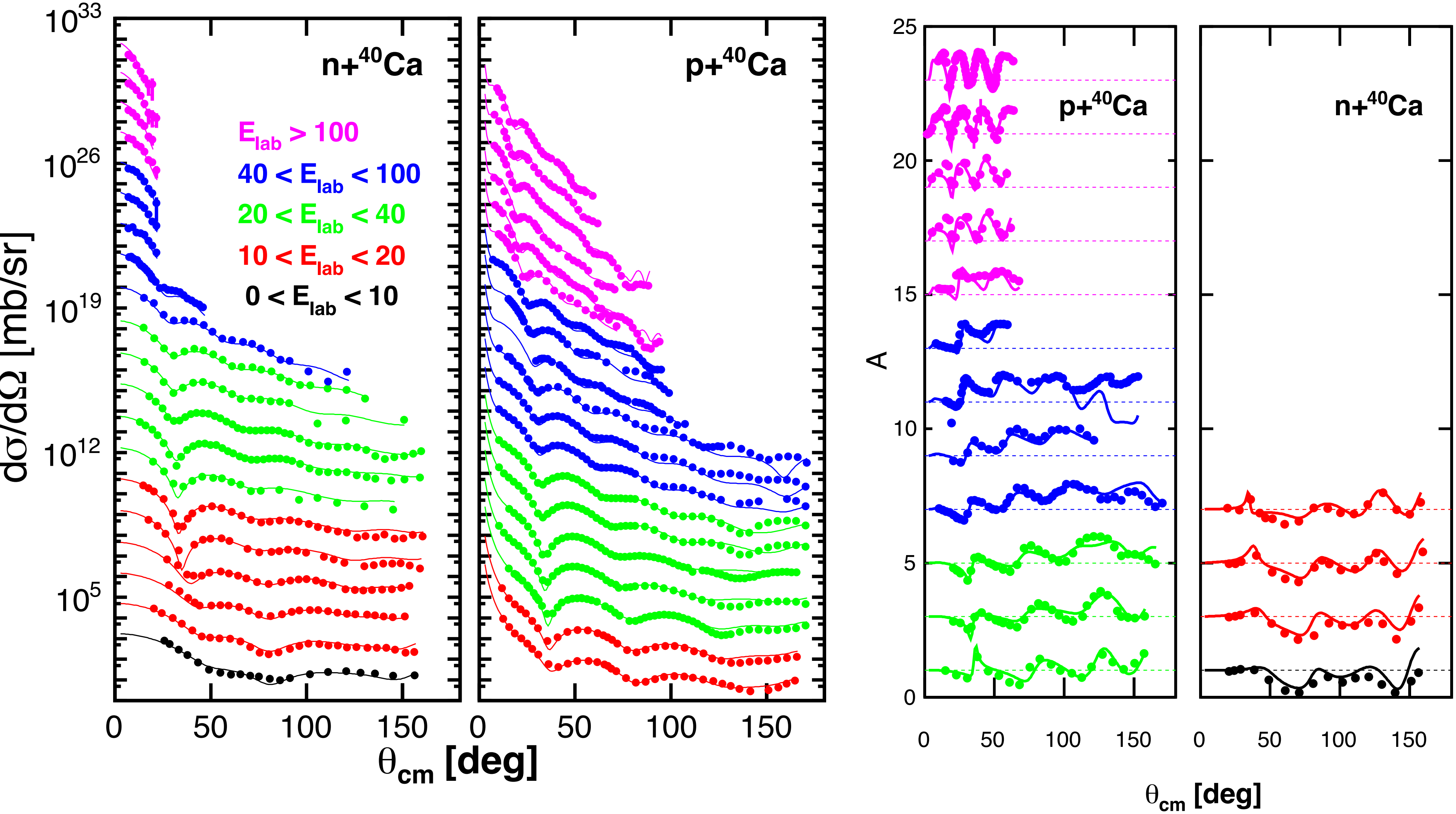
Implications

- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of the interpretation of (e,e′p) possible
- Independent "experimental" statement on size of three-body contribution to the energy of the ground state--> two-body only:

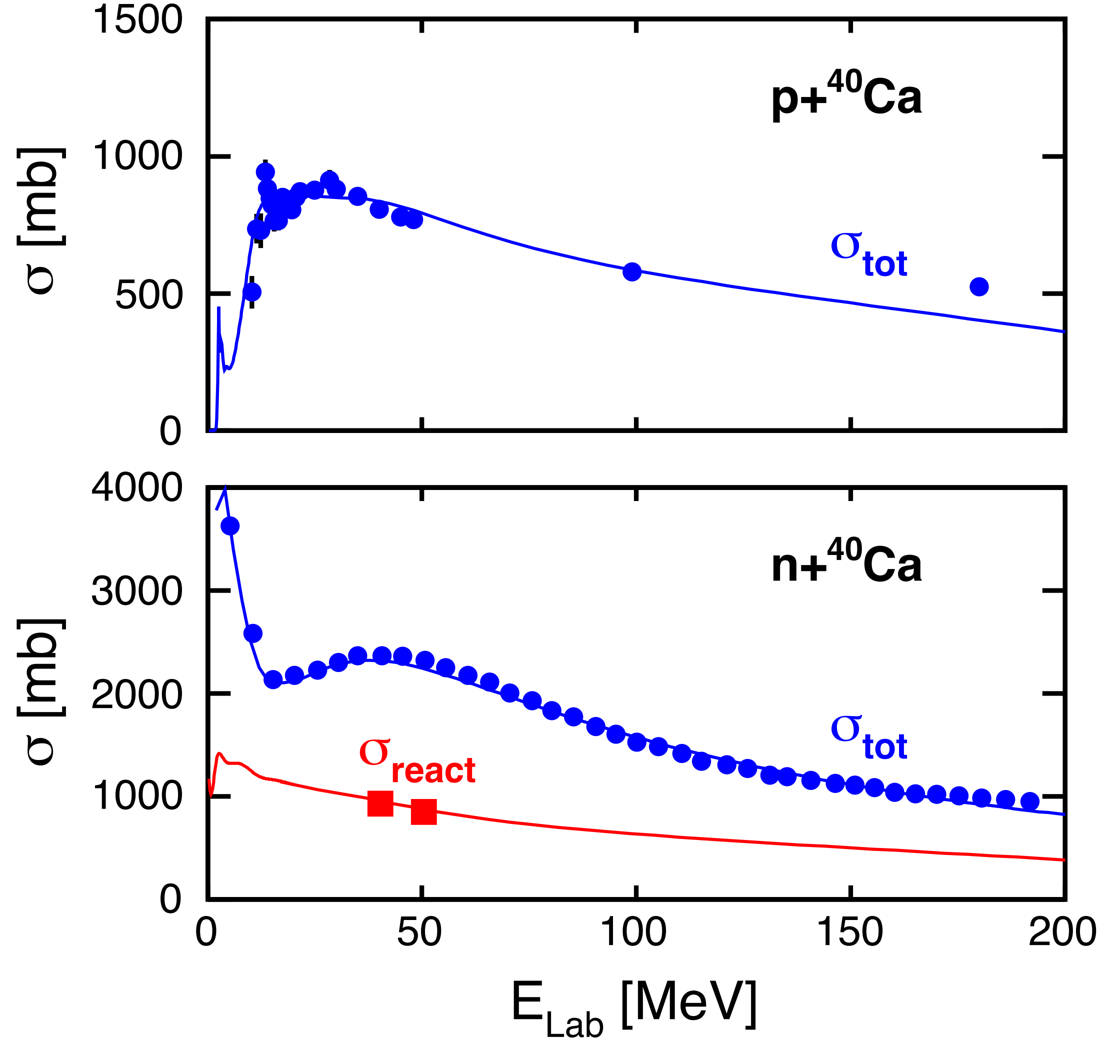
$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

reactions and structure

Differential cross sections and analyzing powers

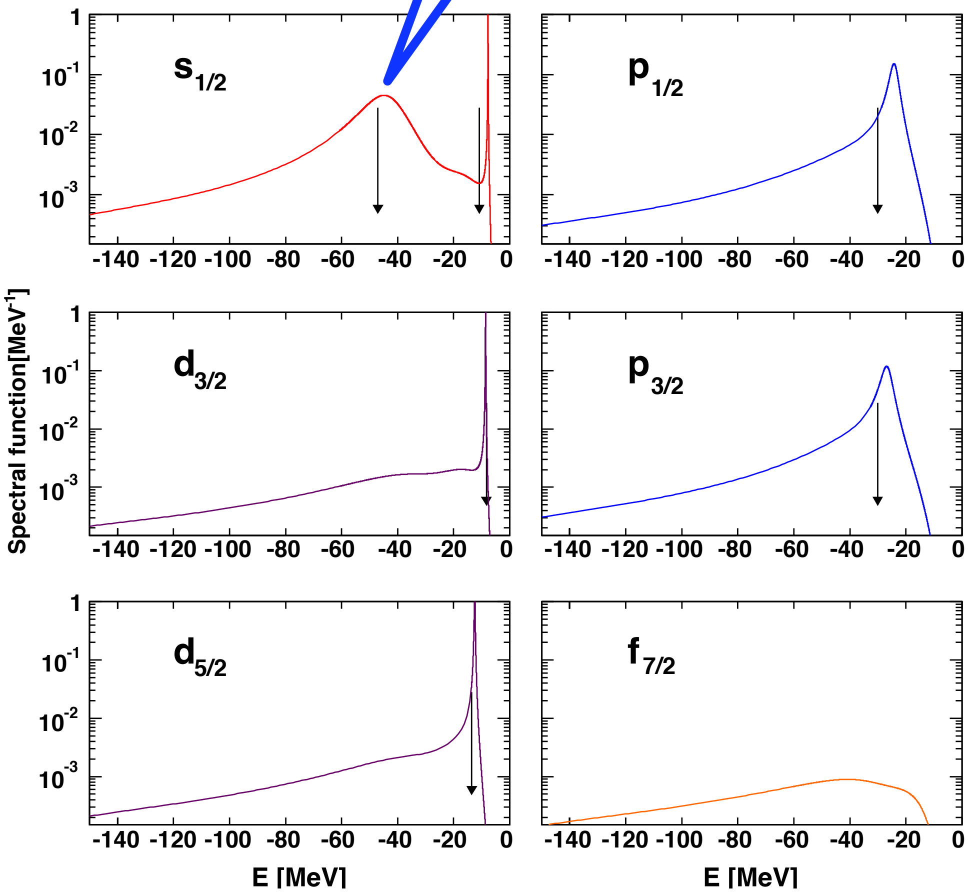


Reaction (p&n) and total (n) cross sections



Below ε_F

^{40}Ca spectral function



Nonlocal imaginary self-energy:

proton number --> 19.88

neutron number -> 19.79

$\ell \leq 5$

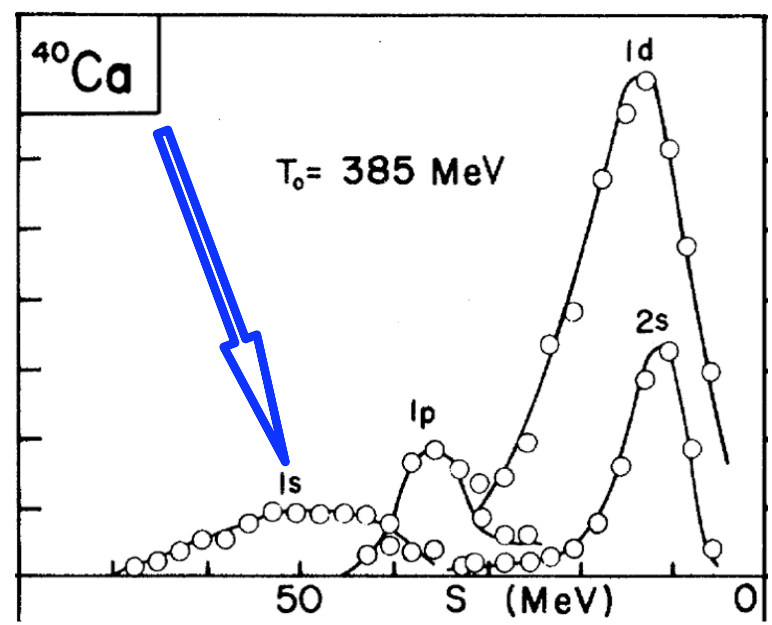
$S_{0d3/2}$= 0.76

$S_{1s1/2}$ = 0.78

Not part of fit!!

0.15 larger than NIKHEF analysis!

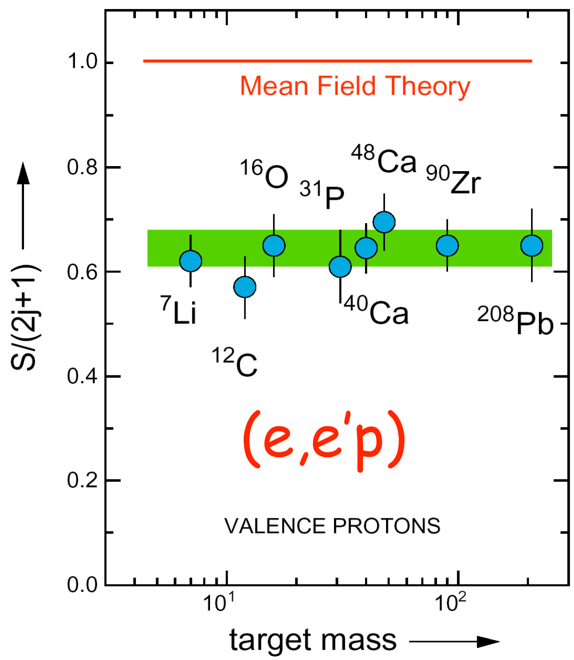
Old (p,2p) data from Liverpool or (e,e'p) from Saclay

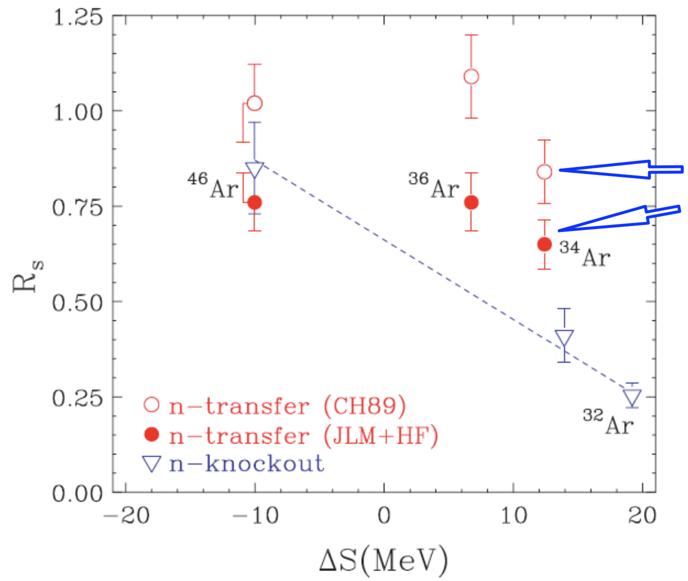


reactions and structure

Linking nuclear reactions and nuclear structure —> DOM

Correlations from nuclear reactions





Different optical potentials --> different reduction factors for transfer reactions

Spectroscopic factors > 1 ???

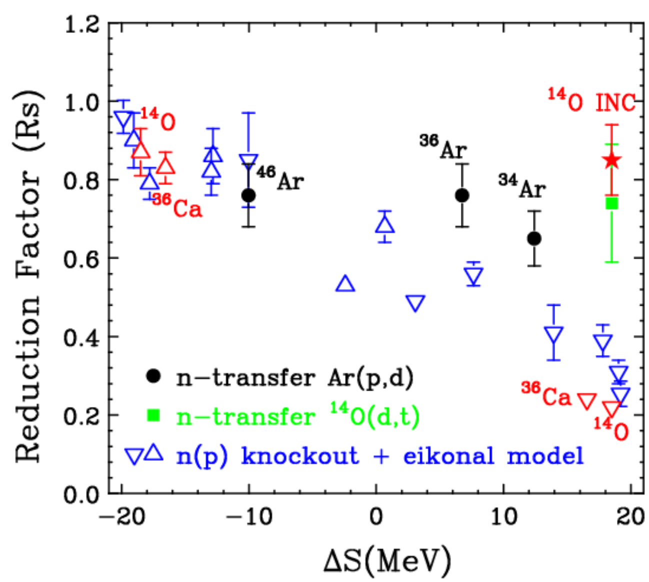
PRL 93, 042501 (2004) HI

PRL 104, 112701 (2010) Transfer

In (e,e'p) proton still has to get out of the nucleus —> optical potential

Nucl. Phys. A553,297c (1993)

Consistency study in progress

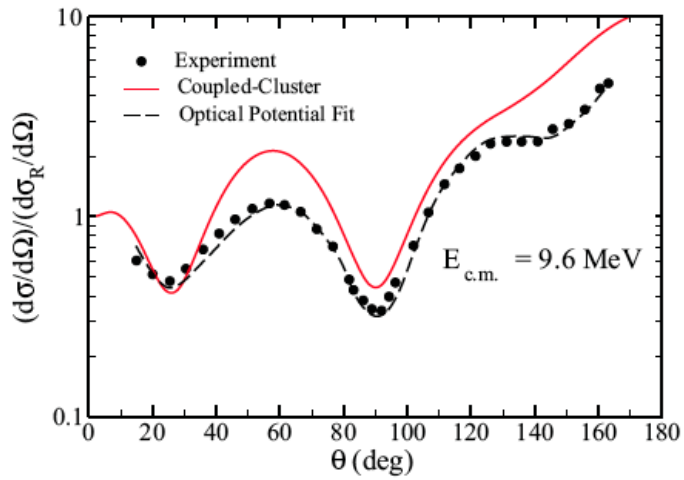


Recent summary —> Jenny Lee

Different reactions different results???

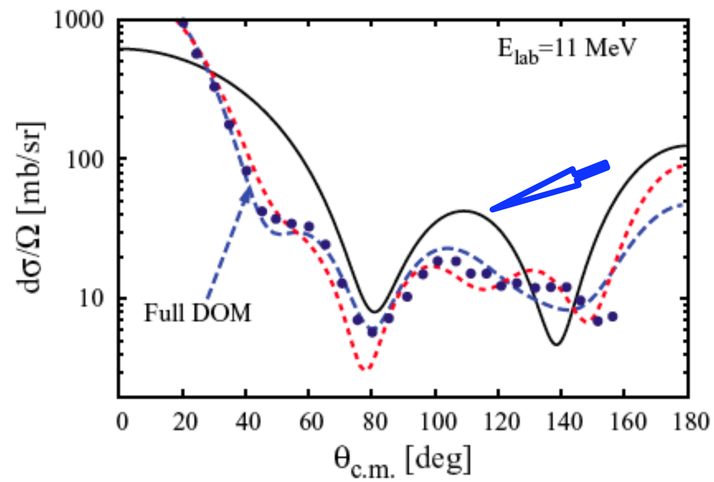
Linking nuclear reactions and nuclear structure

- Extracting information on correlations beyond the independent particle model requires optical potentials in (e,e'p), (d,p),(p,d),(p,pN), etc.
- Quality of ab initio to describe elastic scattering or optical potentials should be improved substantially and urgently



Coupled cluster calculation using overlap functions
PRC86,021602(R)(2012)
Probably limited to low energy

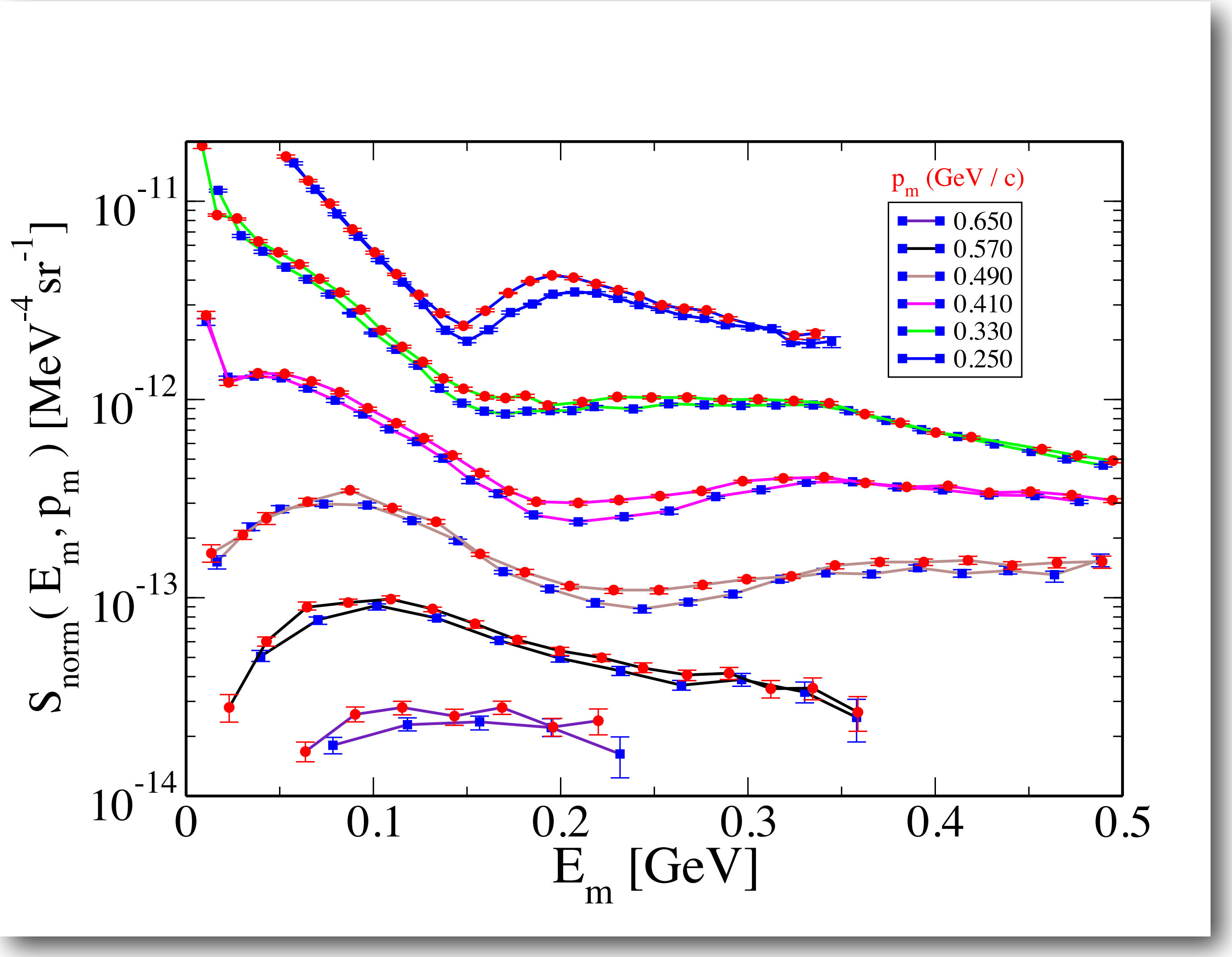
^{40}Ca



Green's function result —> optical potential with emphasis on SRC only
PRC84,044319(2011)

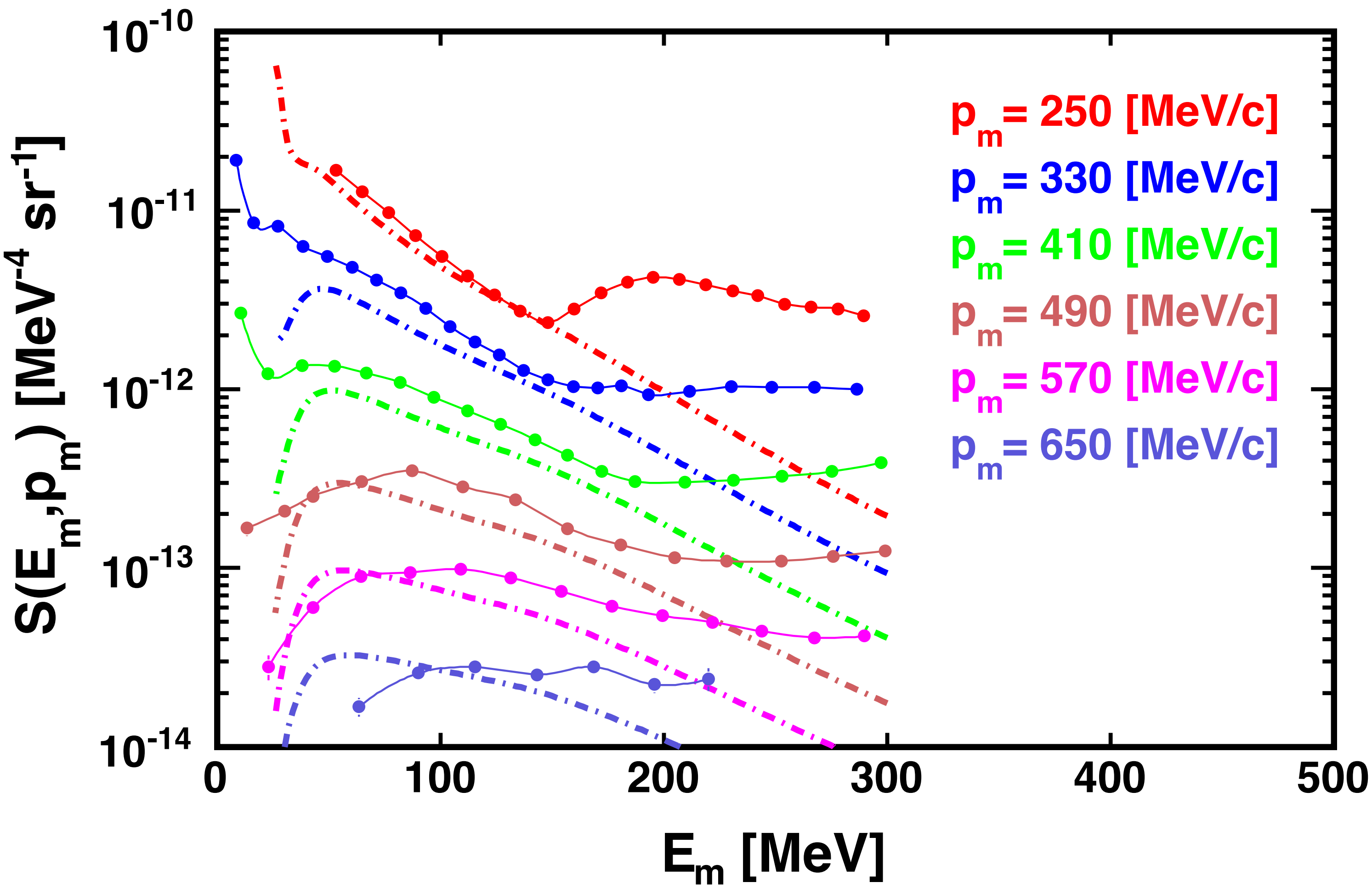
High-momentum components

Rohe, Sick et al. JLab data for Al and Fe (e,e'p) per proton



Jefferson Lab data per proton

- Pion/isobar contributions cannot be described
- Rescattering contributes some cross section (Barbieri, Lapikas)

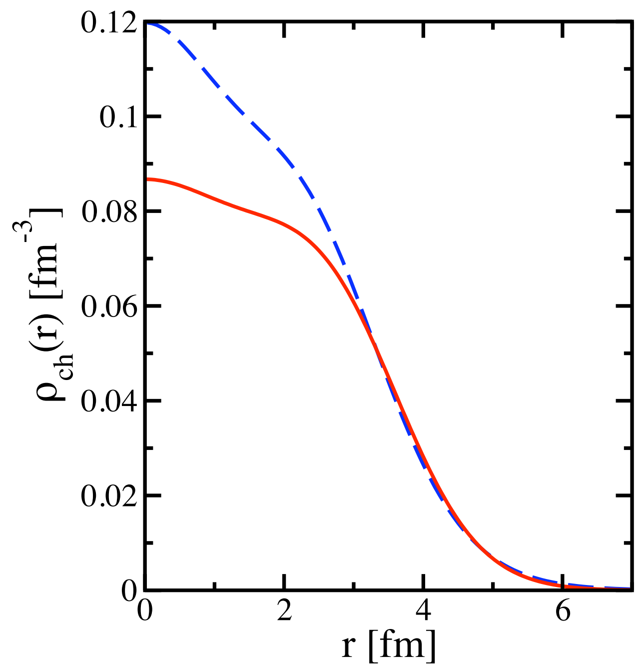


Critical experimental data

Local version

radius correct…

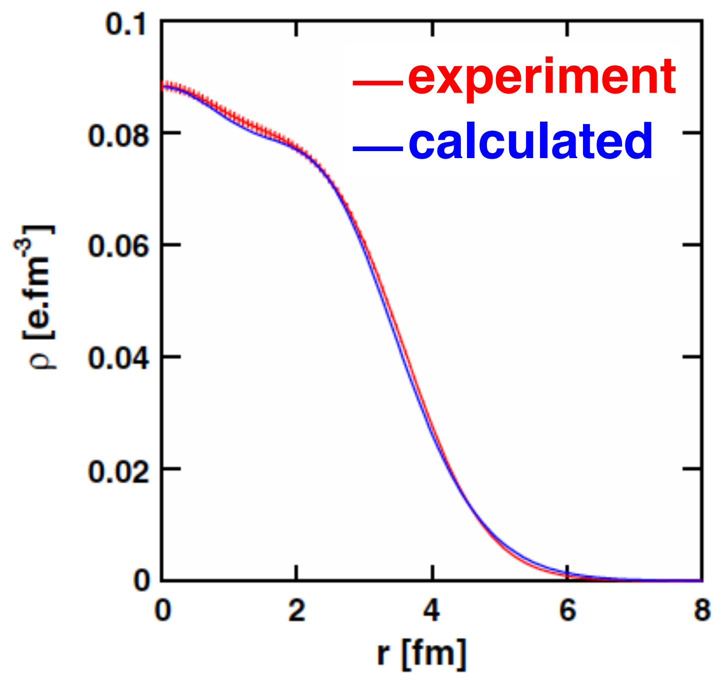
PRC82,054306(2010)



Charge density ^{40}Ca

Non-locality essential

PRL112,162503(2014)



High-momentum nucleons —> JLab can also be described —> E/A

Historical perspective...

- The following authors identify the single-particle propagator (or self-energy) as central quantities in many-body systems

Abrikosov, Gorkov, Dzyaloshinski
(Methods of Quantum Field Theory in Statistical Physics, 1963 Dover Revised edition 1975),

Pines
(The Many-body Problem, 1961 Addison Wesley reissued 1997),

Nozieres
(Theory of Interacting Fermi Systems, 1964 Addison-Wesley reissued 1997),

Thouless
(The Quantum Mechanics of Many-body Systems, 1972 Dover reissue of second edition, 2014),

Anderson
(Concepts in Solids, Benjamin 1963; World Scientific reissued 1998),

Schrieffer
(Theory of Superconductivity, 1964 Benjamin revised 1983),

Migdal
(Theory of Finite Fermi Systems and Applications to Atomic Nuclei (Interscience, 1967),

Fetter and Walecka
(Quantum Theory of Many-particle Systems, 1971 Dover reissued 2003)

- but apart from qualitative features, they don't answer what it looks like for a real system like a nucleus!

reactions and structure

Energy of the ground state

- Energy sum rule (Migdal, Galitski & Koltun)

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

- **Not** part of fit because it can only make a statement about the two-body contribution
- Result:
  - DOM ---> 7.91 MeV/A   T/A ---> 22.64 MeV/A
  - 10% of the particles (those with momenta above 1.4 fm-1) provide ~⅔ of the binding energy!
  - Exp.   8.55 MeV/A
  - Three-body ---> 0.64 MeV/A attraction
  - Argonne GFMC ~ 1.5 MeV/A attraction for three-body <--> Av18

Do elastic scattering data tell us about correlations?

- Scattering T-matrix

$$\Sigma_{\ell j}(k,k';E) = \Sigma^*_{\ell j}(k,k';E) + \int dq q^2 \Sigma^*_{\ell j}(k,q;E) G^{(0)}(q;E) \Sigma_{\ell j}(q,k';E)$$

- Free propagator $\qquad G^{(0)}(q;E) = \dfrac{1}{E - \hbar^2 q^2/2m + i\eta}$

- Propagator

$$G_{\ell j}(k,k';E) = \frac{\delta(k-k')}{k^2} G^{(0)}(k;E) + G^{(0)}(k;E)\Sigma_{\ell j}(k,k';E)G^{(0)}(k;E)$$

- Spectral representation

$$G^p_{\ell j}(k,k';E) = \sum_n \frac{\phi^{n+}_{\ell j}(k)\left[\phi^{n+}_{\ell j}(k')\right]^*}{E - E^{*A+1}_n + i\eta} + \sum_c \int_{T_c}^{\infty} dE' \, \frac{\chi^{cE'}_{\ell j}(k)\left[\chi^{cE'}_{\ell j}(k')\right]^*}{E - E' + i\eta}$$

- Spectral density

$$S^p_{\ell j}(k,k';E) = \frac{i}{2\pi}\left[G^p_{\ell j}(k,k';E^+) - G^p_{\ell j}(k,k';E^-)\right] = \sum_c \chi^{cE}_{\ell j}(k)\left[\chi^{cE}_{\ell j}(k')\right]^*$$

- Coordinate space $\qquad S^p_{\ell j}(r,r';E) = \sum_c \chi^{cE}_{\ell j}(r)\left[\chi^{cE}_{\ell j}(r')\right]^*$

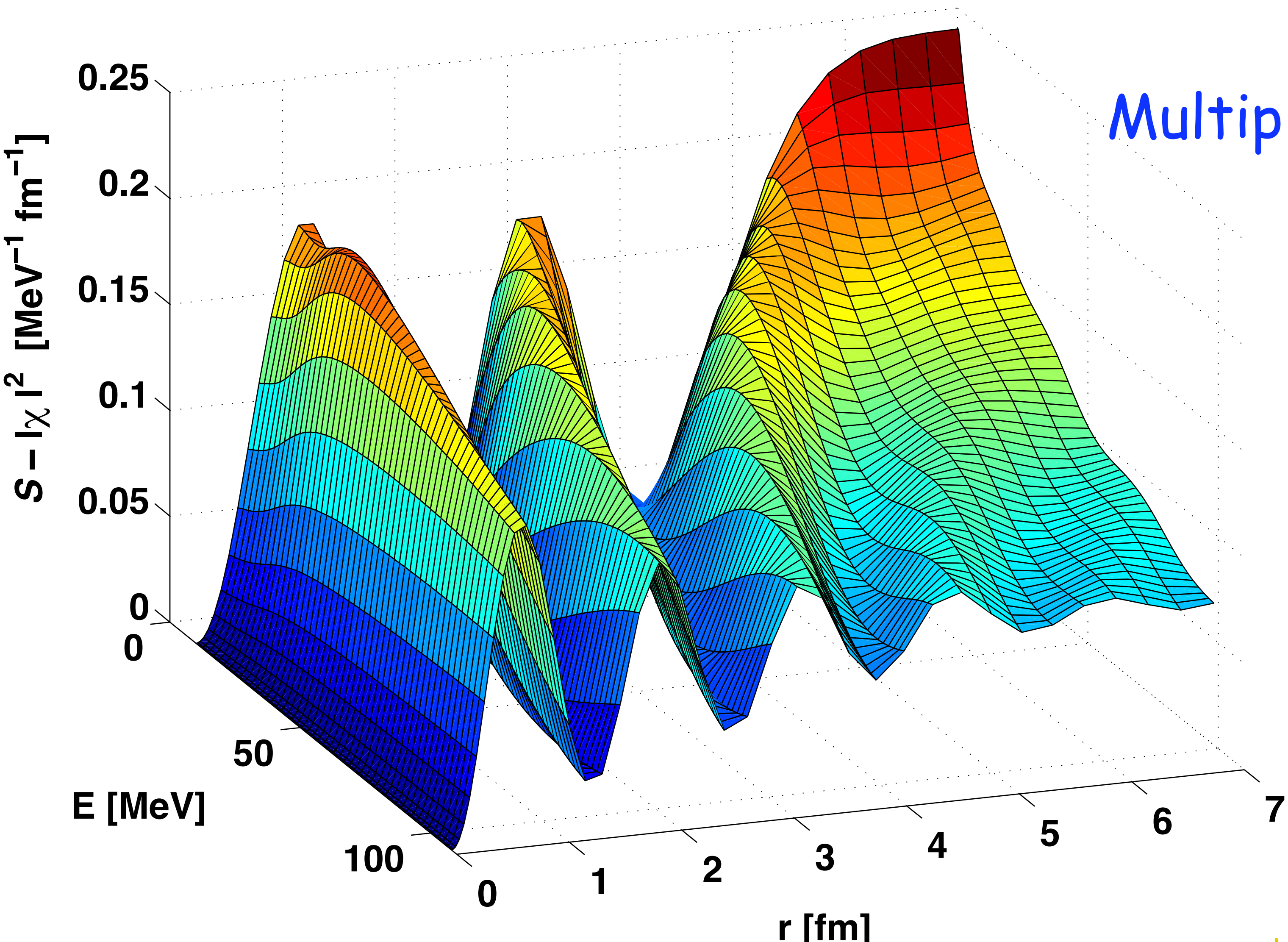
- Elastic scattering explicit

$$\chi^{elE}_{\ell j}(r) = \left[\frac{2mk_0}{\pi\hbar^2}\right]^{1/2}\left\{ j_\ell(k_0 r) + \int dk k^2 j_\ell(kr) G^{(0)}(k;E)\Sigma_{\ell j}(k,k_0;E)\right\}$$

reactions and structure

Adding an $s_{1/2}$ neutron to ^{40}Ca

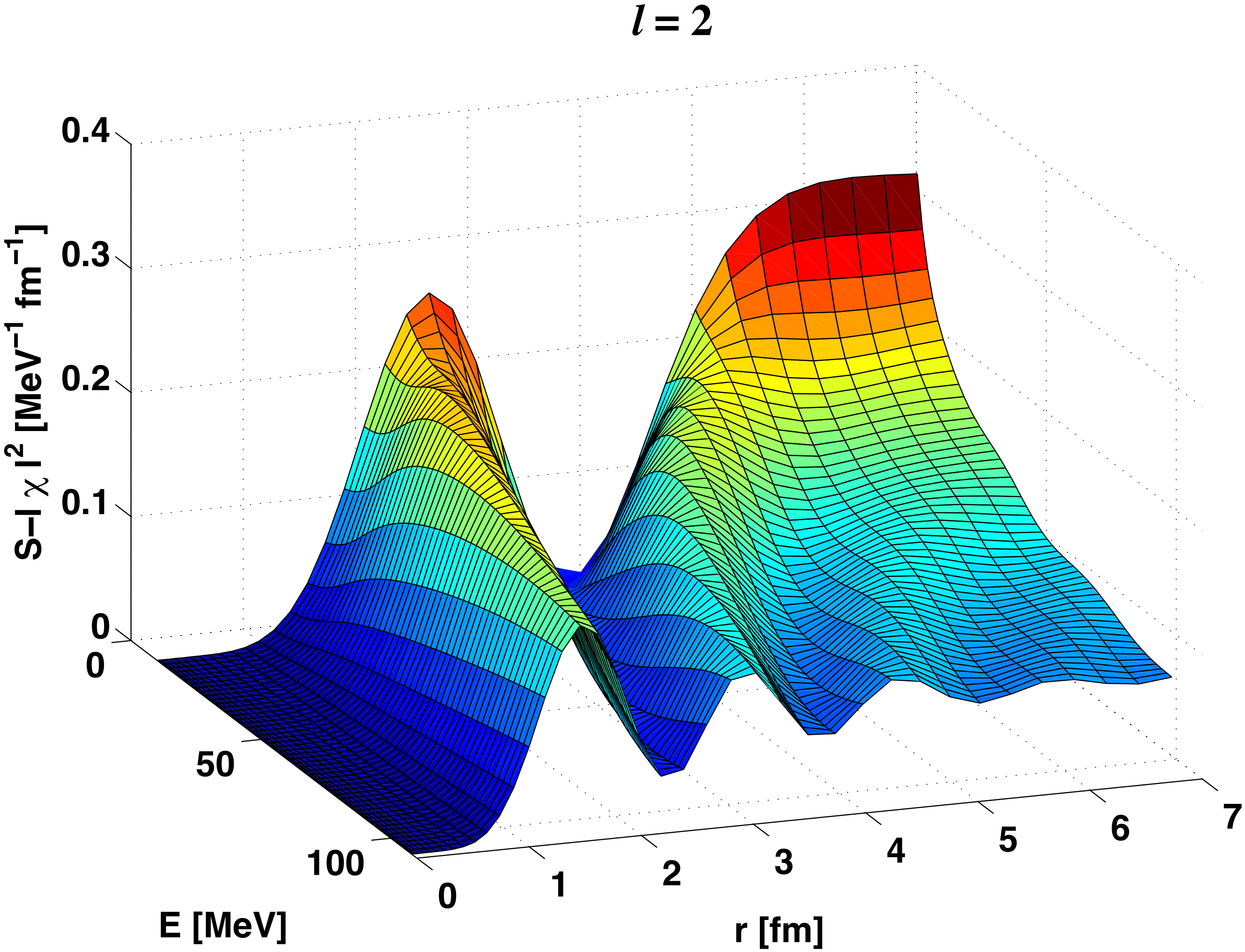
- Inelastically!
- Zero when there is no absorption!



Multiplied by r^2

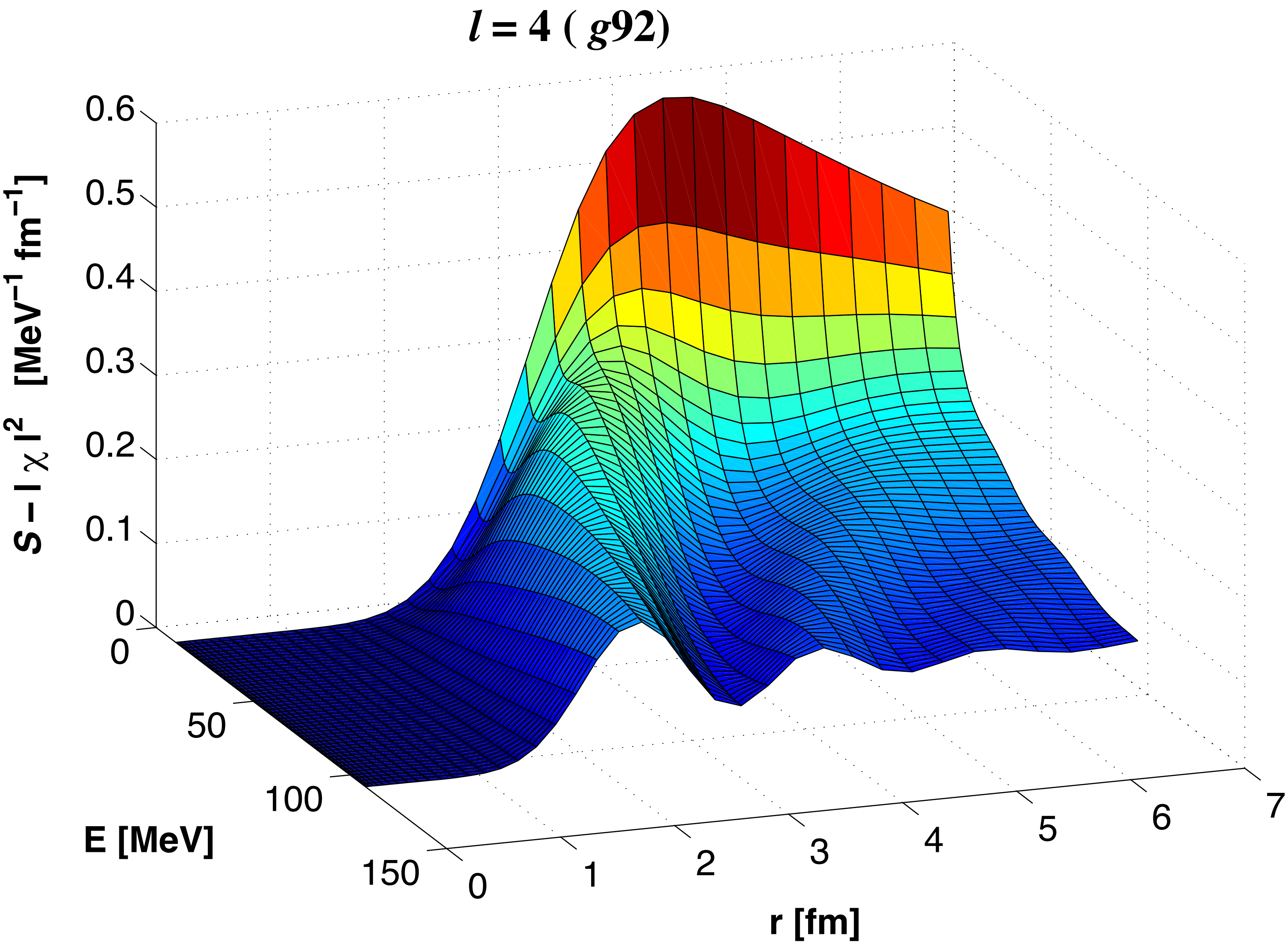
$d_{3/2}$

- One node now



No nodes

- Asymptotically determined by inelasticity



reactions and structure

Determine location of bound-state strength

- Fold spectral function with bound state wave function

$$S_{\ell j}^{n+}(E) = \int dr\ r^2 \int dr'\ r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{p}(r,r';E) \phi_{\ell j}^{n-}(r')$$

- —> Addition probability of bound orbit
- Also removal probability

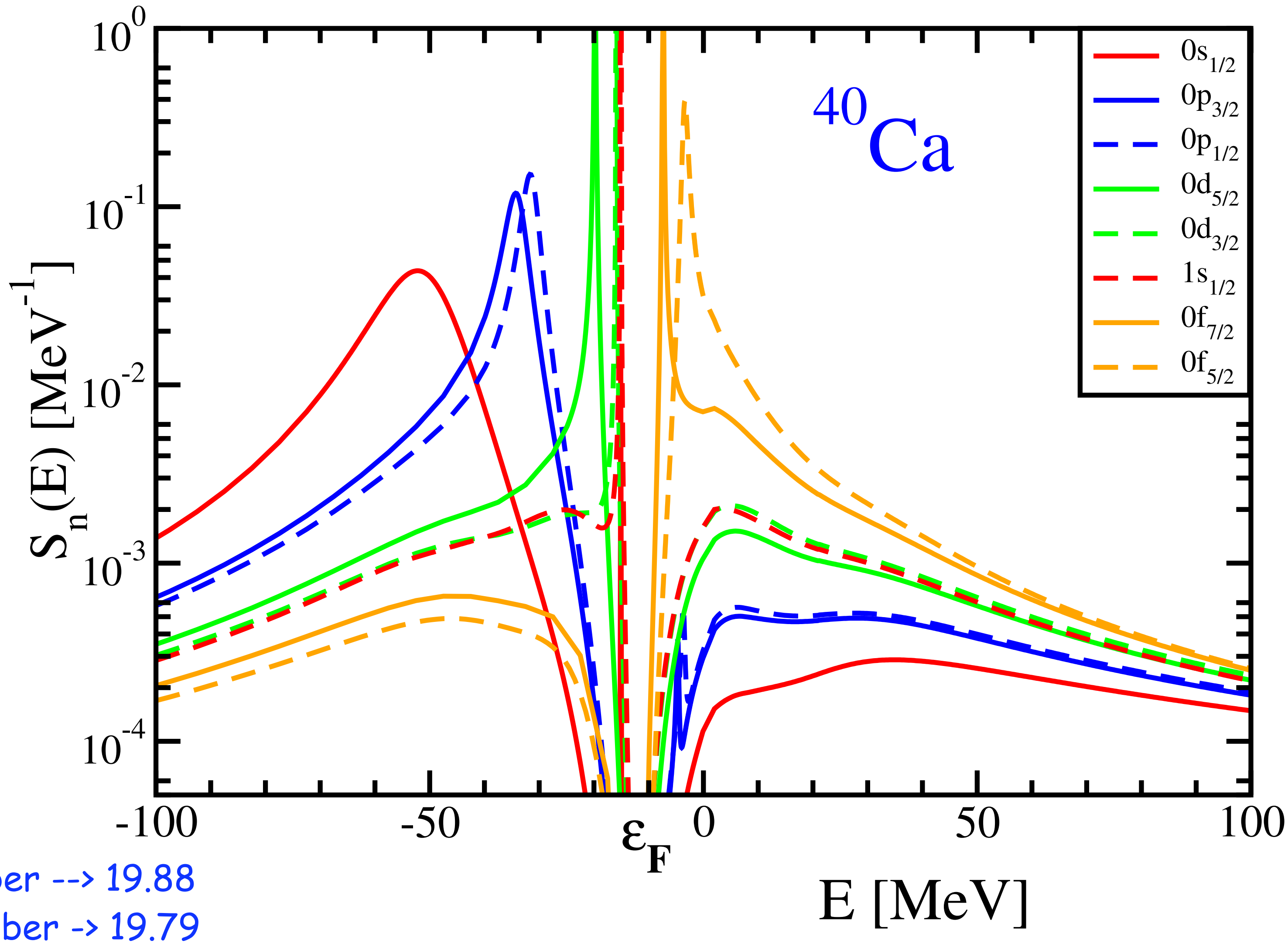
$$S_{\ell j}^{n-}(E) = \int dr r^2 \int dr' r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^{h}(r,r';E) \phi_{\ell j}^{n-}(r')$$

- Overlap function $\sqrt{S_{\ell j}^{n}}\phi_{\ell j}^{n-}(r) = \langle \Psi_n^{A-1} | \, a_{r\ell j} \, | \Psi_0^A \rangle$

- Sum rule $1 = n_{n\ell j} + d_{n\ell j} = \int_{-\infty}^{\varepsilon_F} dE\ S_{\ell j}^{n-}(E) + \int_{\varepsilon_F}^{\infty} dE\ S_{\ell j}^{n-}(E)$

Spectral function for bound states

- [0,200] MeV —> constrained by elastic scattering data



proton number --> 19.88
neutron number -> 19.79
$S_{0d3/2}$= 0.76
$S_{1s1/2}$ = 0.78
0.15 larger than NIKHEF analysis!

reactions and structure

Quantitatively

- Orbit closer to the continuum —> more strength in the continuum
- Note "particle" orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca. $d_{nlj}[0,200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$. Last column $d_{nlj}[0,200]$ depletion numbers for the CDBonn calculation.

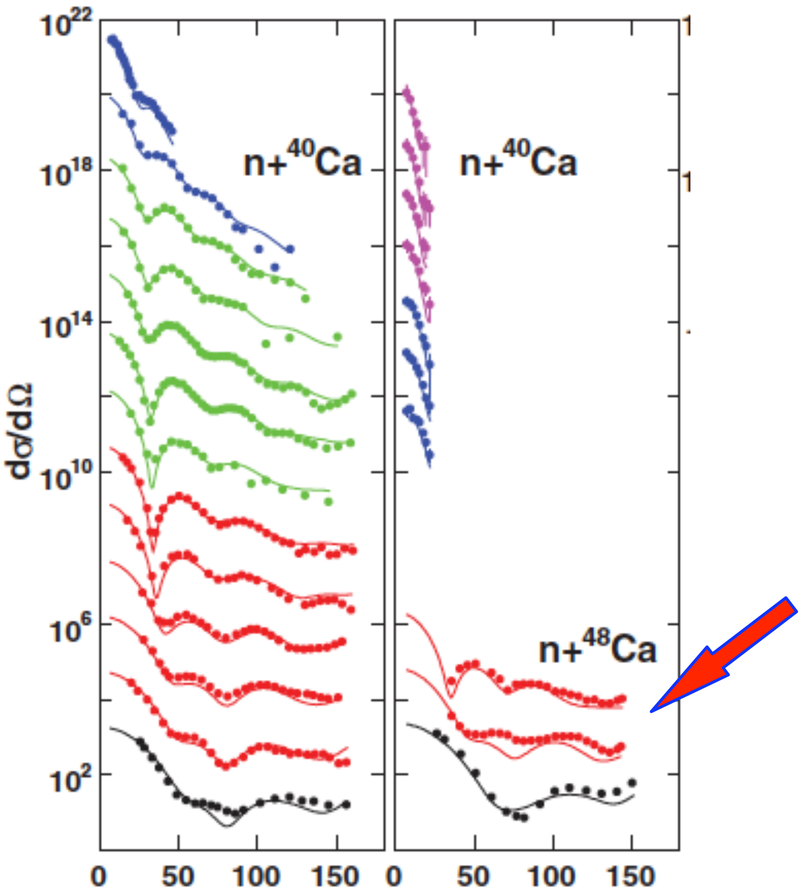
orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0,200]$ DOM	$n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$ DOM	$d_{n\ell j}[0,200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

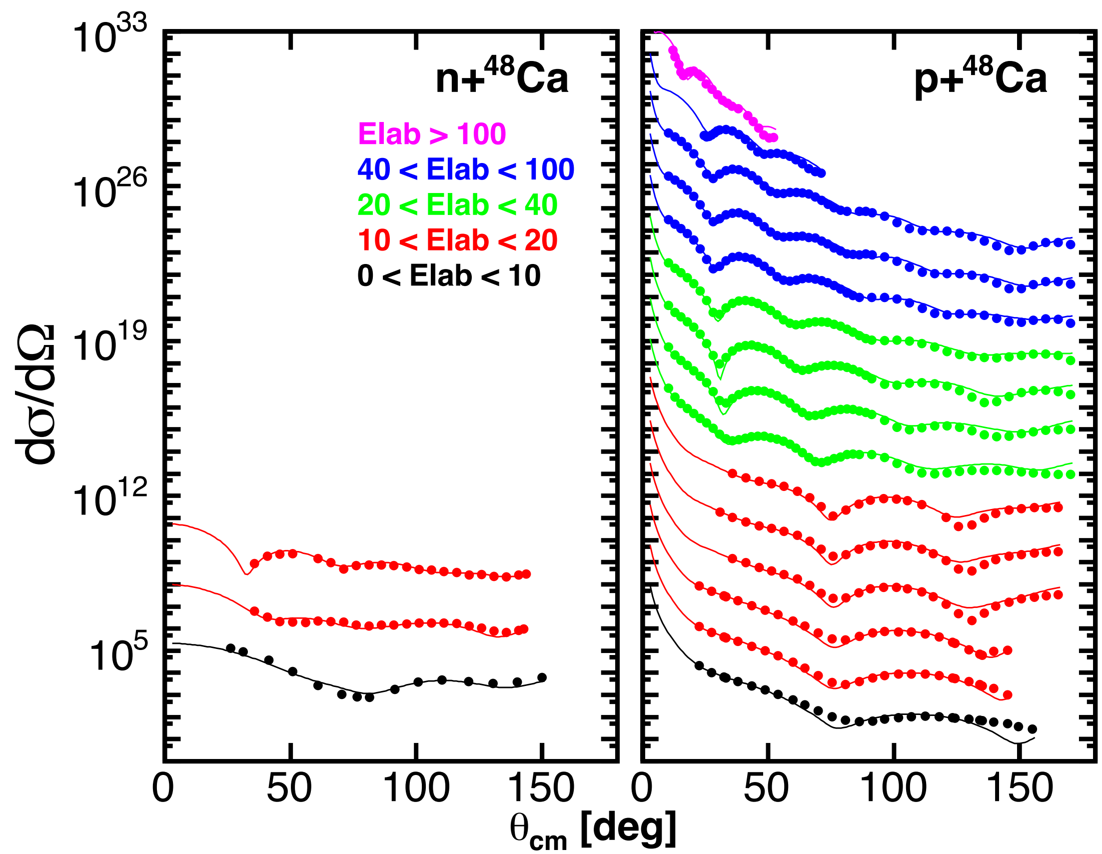
PRC90, 061603(R) (2014)

- ^{48}Ca in progress —> more later this week

In progress

- ^{48}Ca —> charge density has been measured
- Recent neutron elastic scattering **data** —> PRC83,064605(2011)
- Local DOM **OLD** Nonlocal DOM **NEW**

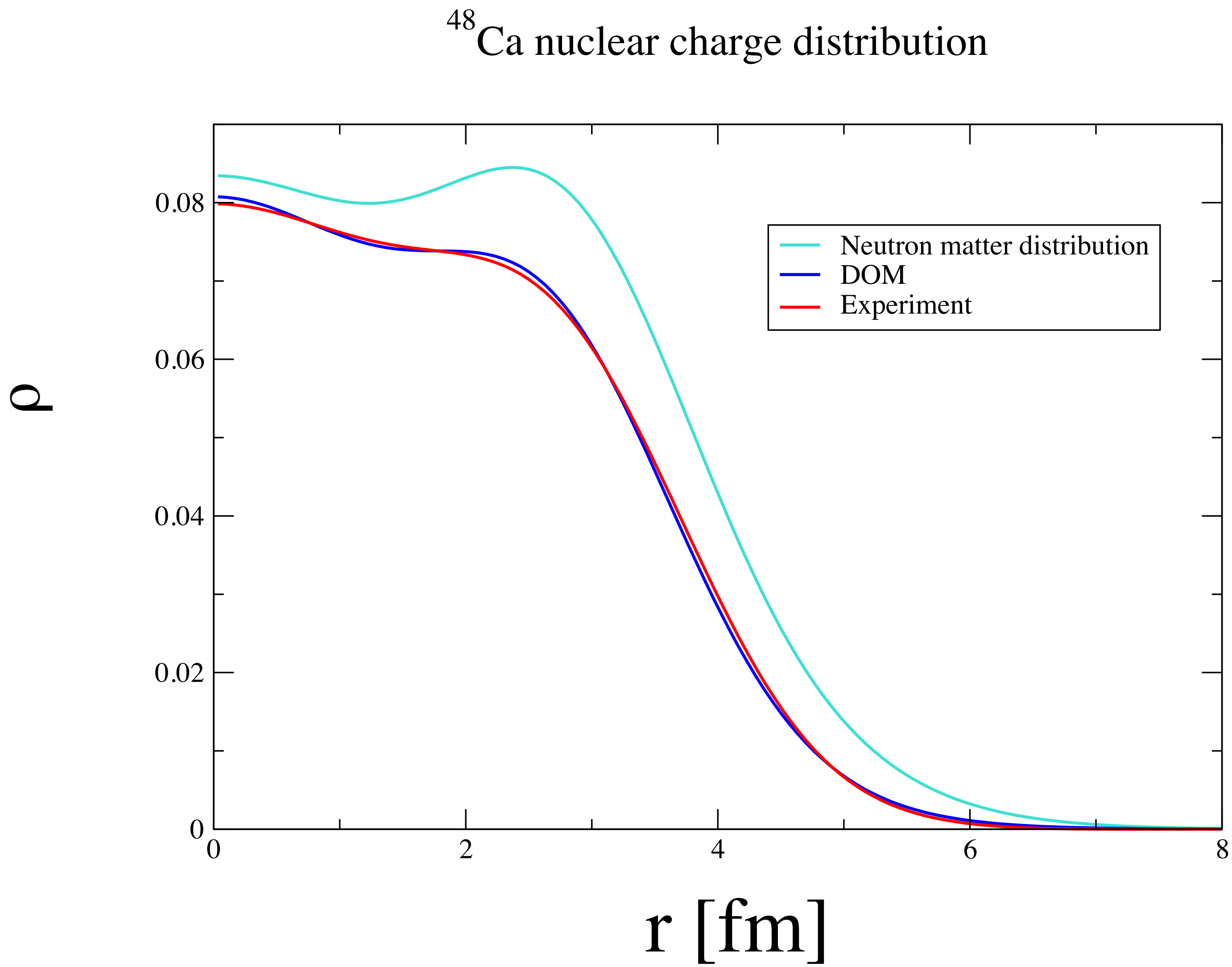




reactions and structure

Preliminary results ^{48}Ca

- Density distributions
- DOM —> neutron distribution —> R_n-R_p



ınd structure

Conclusions

- It is possible to link nuclear reactions and nuclear structure
- Vehicle: nonlocal version of **Dispersive Optical Model** (Green's function method) pioneered by Mahaux —> DSM
- Can be used as input for analyzing nuclear reactions
- Can predict properties of exotic nuclei
- "Benchmark" for ab initio calculations: e.g. V_{NNN} —> binding
- Can describe ground-state properties
  - charge density & momentum distribution
  - spectral properties including high-momentum Jefferson Lab data
- **Elastic scattering determines depletion of bound orbitals**
- Outlook: reanalyze many reactions with nonlocal potentials...
- For N ≳ Z exhibits sensitivity to properties of neutrons —> weak charge in progress