The $T(t, 2n)\alpha$, $T(^3\text{He}, np)\alpha$, and $^3\text{He}(^3\text{He}, 2p)\alpha$ Reactions at Low Energies

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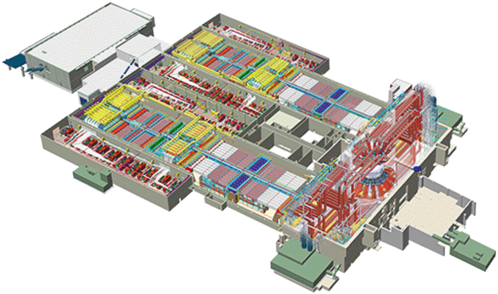
Overview of Presentation

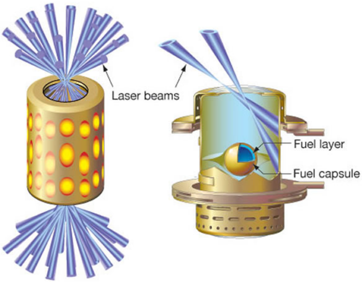
- $\mathrm{T}(t,2n)\alpha$ Inertial Confinement Fusion Experiment
- Results and Analysis of Neutron Energy Spectrum
- Measurement of Thermonuclear Reaction Rate
- $\mathrm{T}(^3\mathrm{He},np)\alpha$ and $^3\mathrm{He}(^3\mathrm{He},2p)\alpha$ Experiments
- Conclusions and Outlook

Motivation and Background

- Study reaction mechanism: ^{5}He and di-neutron correlations
- R-Matrix description of 3-particle final states
- Study mirror symmetry
- Demonstrate measurement of charged-particle reaction rate in plasma
- The cross section and neutron spectrum are important for inertial confinement fusion

National Ignition Facility





images courtesy LLNL

Similar capabilities exist at the Laboratory for Laser Energetics (LLE) at Rochester (Omega Laser), but $\approx 50\times$ less powerful

Unique Features of ICF Environment for Nuclear Physics

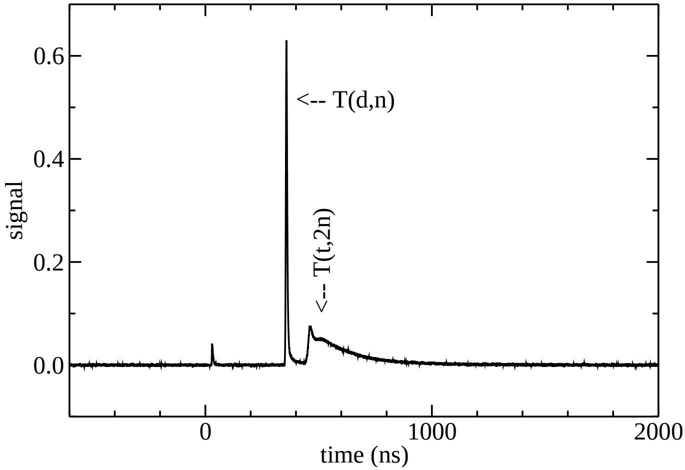
as compared to accelerator-based approaches

- Low mass near target
- Sharp time structure
- Possibility of high neutron fluxes
- Willingness to work with tritium

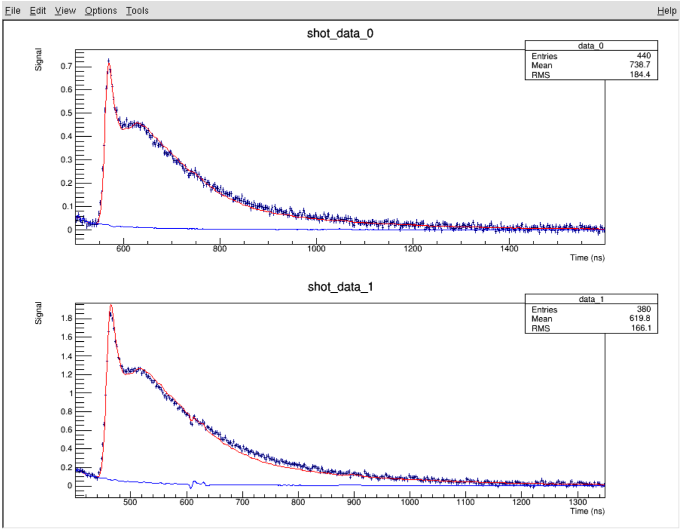
Measurement of the $T(t, 2n)\alpha$ at the National Ignition Facility

- Nearly pure tritium gas (0.1% D), low areal density "symcap" (gas-filled plastic capsule)
- $\approx$ 200 ps thermonuclear burn time
- $kT = 3.3(3)$ keV $\quad \rightarrow \quad E_{\text{Gamow}}(\text{T}+\text{T}) = 16$ keV
- 2 organic liquid scintillators (xylene) @ 20 and 22 meters, respectively
- Modeling includes:
  - Instrument Response Function (time response)
  - Scintillator response (efficiency)
  - Attenuation and scattering
  - Thermal broadening
  - Background from $\text{T}(d, n)$ (small)

Raw Data from Equator Detector @ 20.1 m



Fits to Time Spectra



General Comments on the Phenomenological R-Matrix Method (2-Body Case)

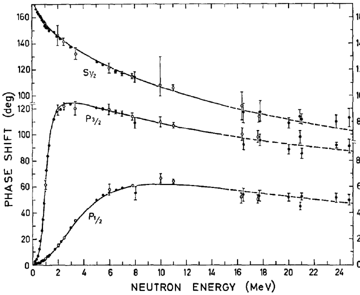
- Exact implementation of quantum-mechanical symmetries and conservation laws (Unitarity)
- Treats long-ranged Coulomb potential explicitly
- Wavefunctions are expanded in terms of unknown basis functions
- Energy eigenvalues and the matrix elements of basis functions are adjustable parameters, which are typically optimized via χ^2 minimization
- A wide range of physical observables can be fitted (e.g. cross sections, E_x, Γ_x,...)
- The fit can then be used to determine unmeasured observables
- Better than the alternatives (effective range, K-matrix,..)
- Major Approximations: truncation (levels / channels), channel radius

$T(t, 2n)\alpha$ R-Matrix Modeling (3-Body Case)

Carl Brune, Dan Sayre, Jac Caggiano, Andy Bacher, Gerry Hale, Mark Paris

- Three-body final state treated in Faddeev-inspired approach
- Kinematics (recoil) is more complicated
- Angular correlation effects on spectrum
- Identical particles / antisymmetrization
- F.C. Barker formalism + angular momentum coupling + antisymmetrization
  - D.P. Balamuth, R.W. Zurmühle, and S.L. Tabor, Phys. Rev. C **10**, 975 (1974).
  - D.F. Geesaman *et al.*, Phys. Rev. C **15**, 1835 (1977).
  - H.O.U. Fynbo *et al.*, Phys. Rev. Lett **91**, 082502 (2003).

Two-Body Interactions are Modeled in an R-matrix Approach



$n - \alpha$ R-matrix parameters: Stammbach and Walter (1972).

The singlet nn is modeled with a one-level R-matrix that reproduces the scattering length and effective range of the Argonne V18 potential.

Some Formulas

- Our form for the matrix element:

$$\mathcal{M}_{\nu_1\nu_2} = \sum_c u_c(12) f^{lJ}_{\nu_1\nu_2}(\Omega_1, \Omega_{23}) - u_c(21) f^{lJ}_{\nu_2\nu_1}(\Omega_2, \Omega_{13})$$

- u_c is given by an R-matrix expression:

$$u_c(12) = \left[\frac{P_1 P_{23}}{p_1 p_{23}}\right]^{1/2} e^{i(\omega_1 - \Phi_1)} e^{i(\omega_{23} - \Phi_{23})} \frac{\sum_\lambda \frac{A_{c\lambda}\gamma_{c\lambda}}{E_{c\lambda} - E_{23}}}{1 - [S_{23} - B_c + iP_{23}]R_c}$$

- $f^{lJ}_{\nu_1\nu_2}$ contains the spin and angular information:

$$f^{lJ}_{\nu_1\nu_2}(\Omega_1, \Omega_{23}) = \sum_{m, m_l, m_l'} \frac{(-1)^{J+m}}{\sqrt{2J+1}} \langle l m_l \tfrac{1}{2} \nu_1 | J m \rangle \langle l m_l' \tfrac{1}{2} \nu_2 | J {-m} \rangle Y_{l m_l}(\hat{\boldsymbol{p}}_1) Y_{l m_l'}(\hat{\boldsymbol{p}}_{23})$$

- The particle distribution is given by

$$\frac{d^3 N}{dE_i\, \Omega_i\, d\Omega_j} = \sum_{\nu_1, \nu_2} |\mathcal{M}_{\nu_1\nu_2}|^2\, p_i p_{jk} \mathcal{J}_{ijk}$$

- A 0^+ $(l = 0)$ initial $t + t$ state is assumed, and $c = 1/2^+$, $1/2^-$, $3/2-$ $n + \alpha$ or an $l = 0$ spin-singlet di-neutron state.

The resulting formula for the particle spectra...

is not so simple, and I will not repeat it here. The key step is the application of an obscure addition theorem for spherical harmonics that was first given by M.E. Rose [Journal of Mathematics and Physics **37**, 215 (1958)]:

$$Y_{lm}(\hat{\boldsymbol{c}}) = \sum_{\substack{\lambda_1+\lambda_2=l \\ \nu_1+\nu_2=m}} a^{\lambda_1} b^{\lambda_2} \langle \lambda_1 \nu_1 \lambda_2 \nu_2 | lm \rangle \sqrt{\frac{4\pi(2l+1)!}{(2\lambda_1+1)!(2\lambda_2+1)!}} Y_{\lambda_1\nu_1}(\hat{\boldsymbol{a}}) Y_{\lambda_2\nu_2}(\hat{\boldsymbol{b}}),$$

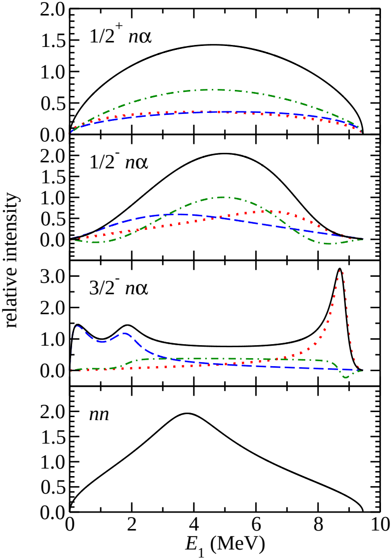
where $\hat{\boldsymbol{c}} = \vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$ with $\vec{\boldsymbol{a}} = a\hat{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}} = b\hat{\boldsymbol{b}}$.

Findings:

- Antisymmetrization is very important
- Angular correlations are important for the $3/2^-$ $n+\alpha$ channel:

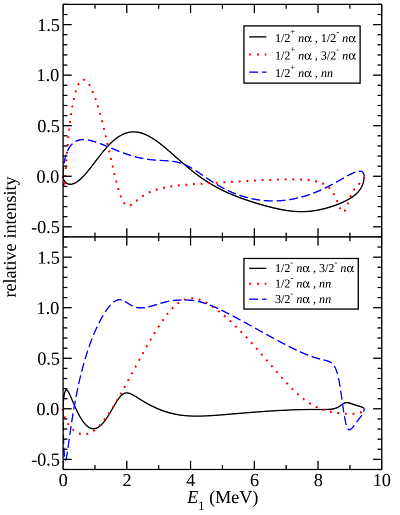
  $W(\theta) = 1 + P_2(\cos\theta)$
- There *is* coherent interference between different partial waves

Neutron Energy Distributions



Neutron energy distributions for each channel considered separately. The primary, secondary, exchange, and total are given by the dotted, dashed, dot-dashed, and solid curves, respectively. Only the total is shown for the nn case.

Coherent Interference Effects



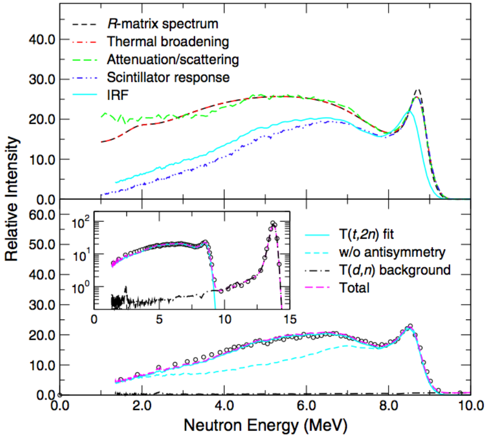
- Interference contributions to the neutron energy distributions for partial wave combinations indicated.
- There is minimal coherent interference between the 3/2− and 1/2− contributions.

R-Matrix Fitting

- Assume $3/2^-$, $1/2^-$, $1/2^+$ n-α and singlet nn channels.
- Explore all combinations of channels.
- Fit both detectors simultaneously.
- Best fit yields $\chi^2_{\text{min}} = 632$ when all A_λ included (812 data points):

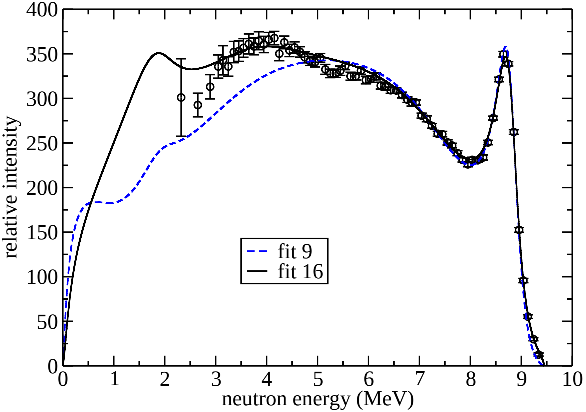
channel	λ	$E_{c\lambda}$ (MeV)	$\gamma^2_{c\lambda}$ (MeV)	$A_{c\lambda}$
$1/2^+$ $n\alpha$	1	50.00	12.00	-18(3)
$1/2^+$ $n\alpha$	2	1000	-40	0
$1/2^-$ $n\alpha$	1	6.43	12.30	-18.2(3)
$1/2^-$ $n\alpha$	2	1000	300	-306(16)
$3/2^-$ $n\alpha$	1	0.97	7.55	9.86(6)
$3/2^-$ $n\alpha$	2	1000	300	155(9)
nn	1	3.119	31.95	12.5(5)

$T(t,2n)\alpha$ Neutron Spectrum $E_{c.m.} = 16$ keV



Sayre, Caggiano *et al.*, Rev. Lett. **111**, 052501 (2013). Di-neutron not included.

$T(t,2n)\alpha$ Neutron Spectrum $E_{c.m.} = 16$ keV



Deconvoluted and re-binned spectrum.

Fit 9: no di-neutron.

Fit 16: with di-neutron (best fit).

Jarmie and Brown, NIM B10/11 405 (1985)
Measured alphas – preliminary results...

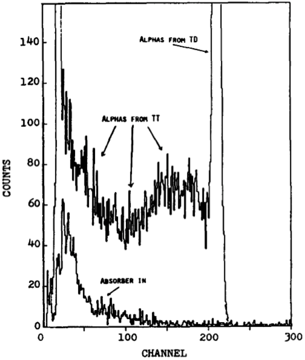


Fig. 8. T(t, α)nn reaction raw data for 45° lab angle and 115 keV bombarding energy. Note the large peak of alpha-particles from the 0.5% deuterium contaminant in the target gas.

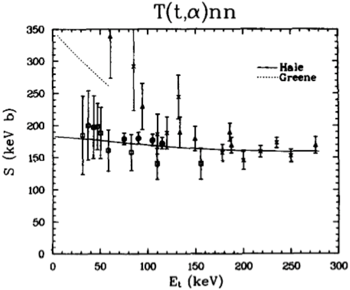
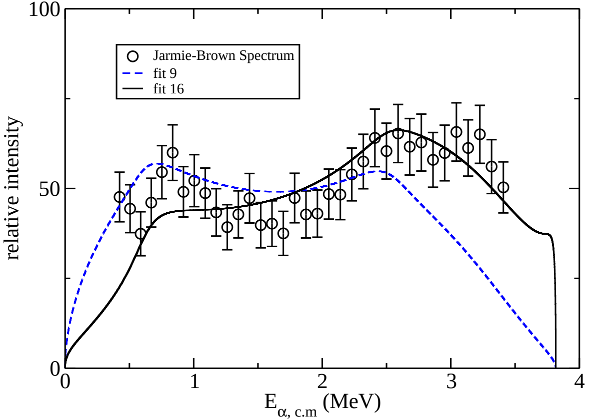


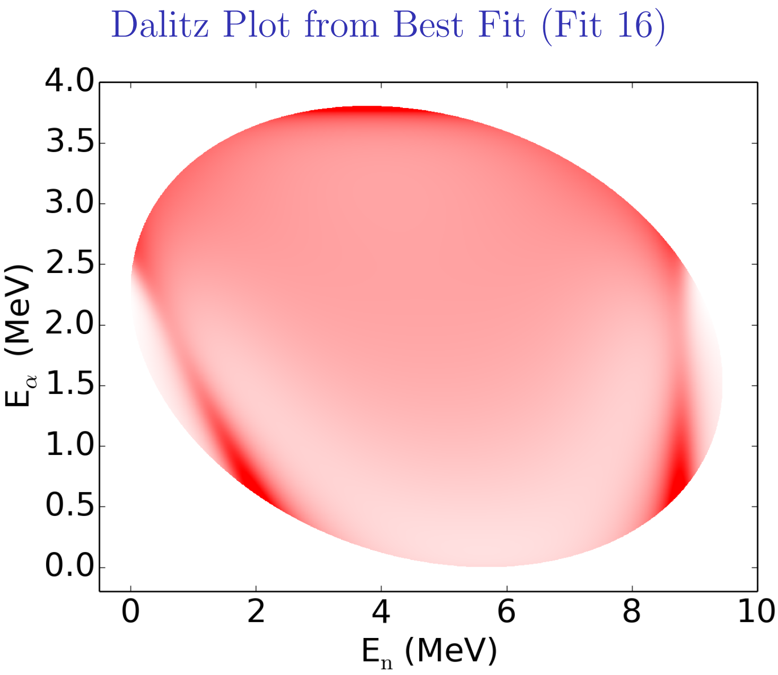
Fig. 9. Integrated *S* functions for the T(t, α)nn reaction. Our preliminary data are the black circles with 5% absolute errors. Also shown are the data of Govorov et al. (triangles) ref. [10]; Agnew et al. (crosses) [11]; and Serov et al. (squares) [12]. The solid curve is an *R*-matrix prediction of Hale [13], and the dashed curve is from the compilation of Greene [14].

α-Particle Spectrum Extracted from Jarmie and Brown (1985)



The prediction from the fit with the di-neutron (Fit 16) is much better:

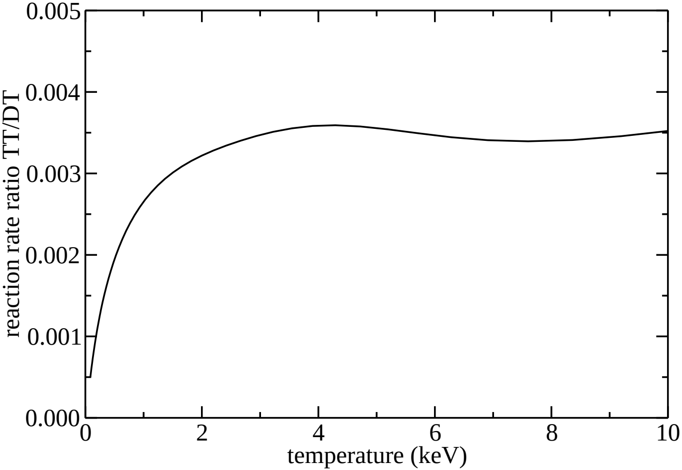
$\chi^2 = 46$ versus 140 for 35 data points.


Dalitz Plot from Best Fit (Fit 16)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
E$_\alpha$ (MeV)
0
2
4
6
8
10
E$_n$ (MeV)

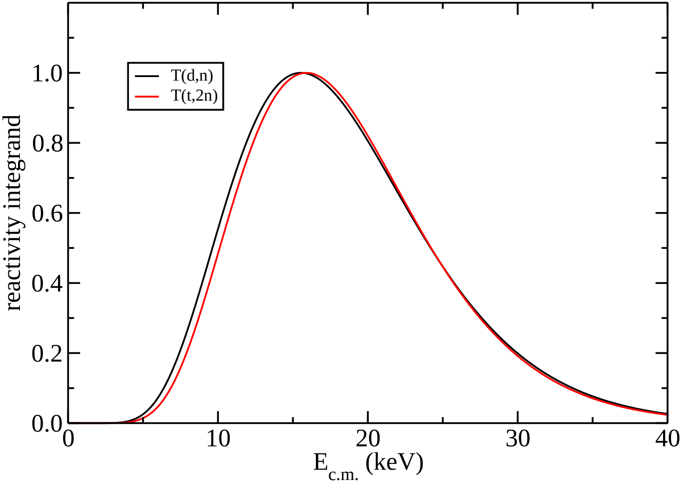
Determination of Thermonuclear Reaction Rate

- Definition:
  $$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi\mu(kT)^3}} \int_0^\infty E\sigma(E) \exp[-E/(kT)]\, dE$$
- Principle of measurement:
  - Measure ratio to $\mathrm{T}(d,n)$ reaction rate (known to $\approx 1\%$)
  - H.-S. Bosch and G.M. Hale, Nucl. Fusion **32** 611 (1992)
  - Assume constant S factor for $\mathrm{T}(t,2n)\alpha$
- Mass spectrometry of capsule fill gas:
  - tritium: 99.598(4) %
  - deuterium: 0.082(1) %
  - remainder: protium and ^{3}He
- Yield-weighted ion temperature determination:
  - use width of "14 MeV" neutron peak from $\mathrm{T}(d,n)$
  - Brysk Formula: $\sigma[E_n] \approx \sqrt{\frac{2M_n\langle E_n\rangle}{M_\alpha + M_n}(kT)}$
  - H. Brysk, Plasma Physics **15**, 611 (1973)
  - Actual analysis uses a more sophisticated approach, including, e.g., relativistic kinematics

Reaction Rate Ratio is Insensitive to Temperature



T(d,n) and T(t,2n) Reactivity Integrands
for $kT = 3.3$ keV



Systematic Errors Considered:

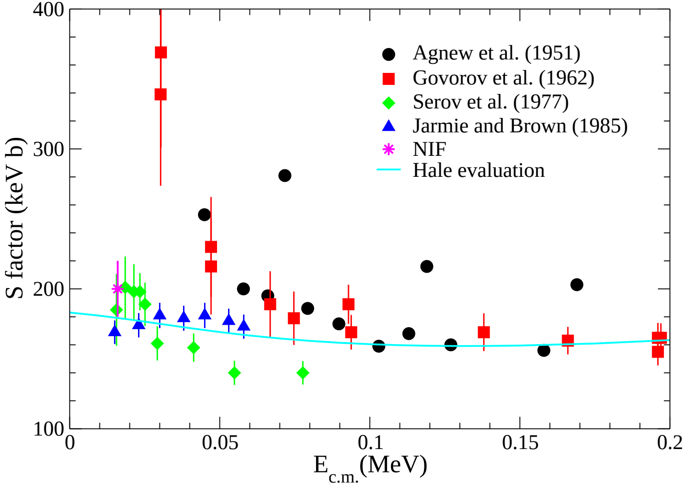
- Fuel mixture uncertainty
- Spectrum fitting
- Ion temperature determination (small)
- Total systematic error is estimated to be 10%

Analysis and Results

- Numbers of neutrons:
  - $N_{DT} \propto n_D n_T \langle \sigma v \rangle_{DT}$
  - $N_{TT} \propto \frac{n_T^2}{2} \langle \sigma v \rangle_{TT} \times 2$
  - $\frac{N_{TT}}{N_{DT}} = \frac{n_T}{n_D} \frac{\langle \sigma v \rangle_{TT}}{\langle \sigma v \rangle_{DT}}$
  - watch factors of two!
- Spectral fitting:
  - $N_{DT} = 3.9(3) \times 10^{12}$
  - $N_{TT}/N_{DT} = 4.5(4)$
  - $kT = 3.3(3)$ keV (burn-weighted)
- $S(16 \text{ keV}) = 200(20)$ keV-b

Comparison to other Data

Note the energy averaging in the plasma is not that different than many of the accelerator measurements, e.g., if a “stopping” target is used.



Summary, Open Questions, and Outlook for $\mathrm{T}(t, 2n)\alpha$

- Only the $3/2^-$ (^{5}He g.s.) provides a distinct feature in the neutron spectrum.
- Interpretation of the continuum remains somewhat ambiguous.
- It would be interesting to measure the neutron spectrum below 4 MeV.
- It would also be nice to measure the α-particle spectrum, particularly near the endpoint.
- The reaction rate for $\mathrm{T}(t, 2n)\alpha$ has been measured in plasma conditions.
- Improved neutron neutron detectors are now online at NIF:
  - solid bibenzyl crystals
  - better sensitivity and less time response tail
  - $\rightarrow$ improved neutron spectrum measurements
- A study of the temperature dependence of the neutron spectrum and reaction rate is underway at Omega (LLE facility at Rochester) and NIF.

$T(^3He, np)\alpha$ and $^3He(^3He, 2p)\alpha$

- Measurements are underway at LLE by a group from MIT: Johan Frenje, Alex Zylstra, Maria Gatu-Johnson, *et al.*
- Requires proton detection, for example by a Magnet Recoil Spectrometer (MRS).
- Allows tests of isospin and mirror symmetry.

Outlook

- Measurements of particle spectra and cross sections for the T$(t, 2n)\alpha$, T$(^3\mathrm{He}, np)\alpha$, and ^{3}He$(^3\mathrm{He}, 2p)\alpha$ reactions have been recently completed or are in progress.
- Further work on R-matrix approaches to three-body states is underway.
- This approach could also be applied to other three-body final state problems where particle correlations can be measured, such as
  $^{16}\mathrm{Be} \rightarrow 2n + {}^{14}\mathrm{Be}$,
  where evidence for the di-neutron has been reported:
  A. Spyrou *et al.*, Phys. Rev. Lett. **108**, 102501 (2012).

Thanks to collaborators:

- D.T. Casey, J.A. Caggiano, R. Hatarik, D.P. McNabb, D.B. Sayre, V.A. Smalyuk (Lawrence Livermore National Lab)
- G.M. Hale, M.W. Paris (Los Alamos National Lab)
- J.A. Frenje, M. Gatu-Johnson, A.B. Zylstra (MIT)
- A.D. Bacher (Indiana), M. Couder (Notre Dame)