TRIUMF

Canada's National Laboratory for Particle and Nuclear Physics
Laboratoire national canadien pour la recherche en physique nucléaire
et en physique des particules

# Towards ab-initio calculations of electromagnetic reactions in exotic nuclei

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In collaboration with:
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**INT workshop on**
**“Reactions and Structure of Exotic Nuclei”**

National Research Council Canada

NSERC
CRSNG

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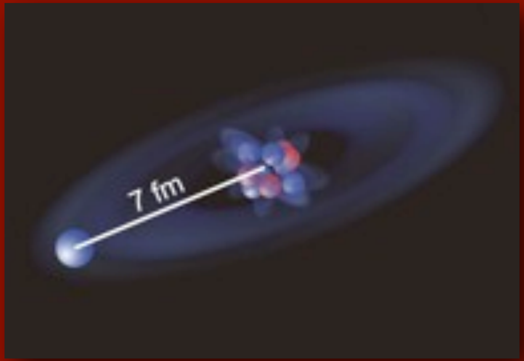
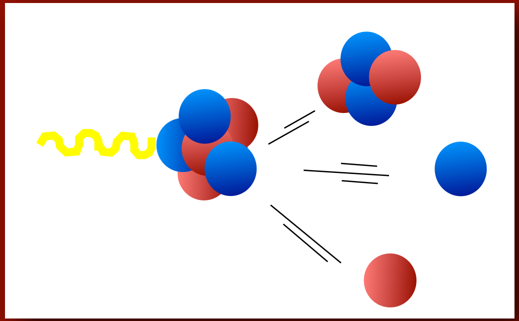
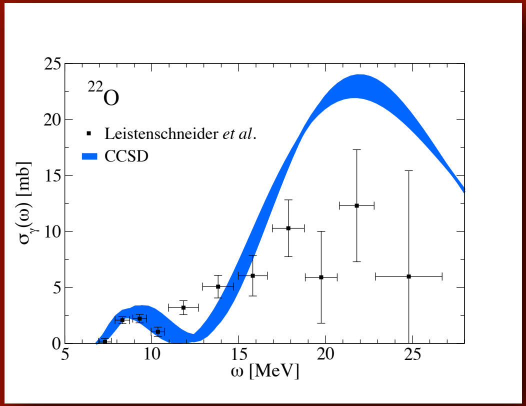


**Nuclear Halo**



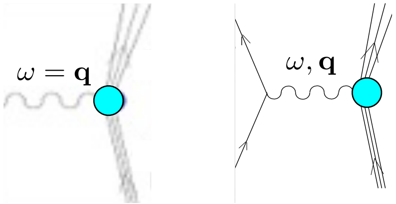
**Electromagnetic Reactions**



**Pigmy Resonance**

# Motivations

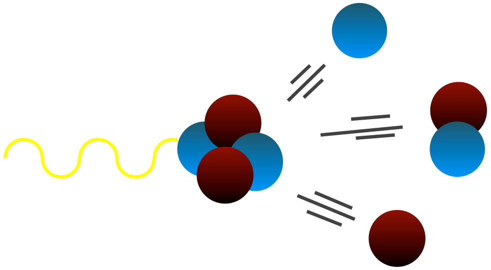
- Electromagnetic probes (coupling constant <<1)



> "*With the electro-magnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"
>
> [De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

- For few-nucleons one can perform exact calculations both for bound and scattering states ➡ test the nuclear theory on light nuclei and **extend it to heavier mass number**



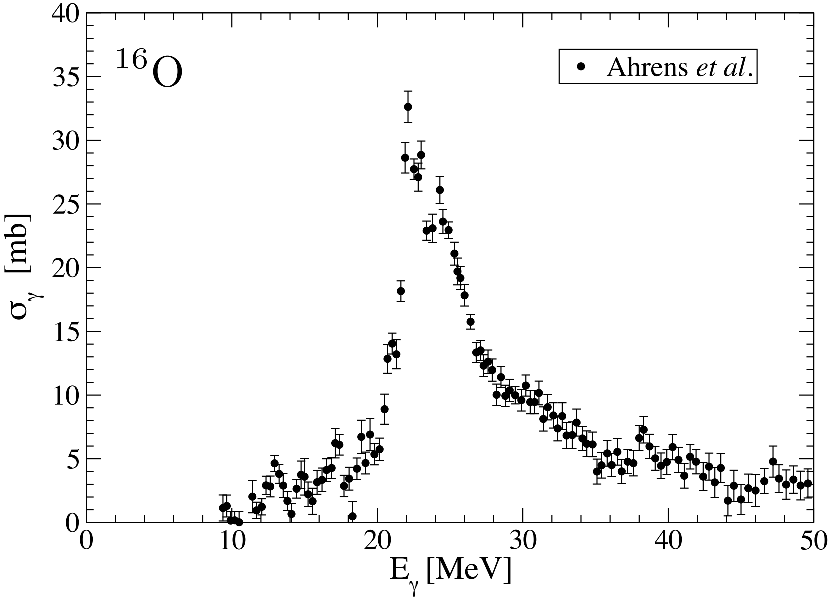
- Provide important informations in other fields of physics, where nuclear physics plays a crucial role:
  - Astrophysics: $\gamma$ interactions with nucleonic matter, radiative capture reactions, $\nu$ interactions with nucleonic matter (vector current as em)
  - Atomic physics (nuclear corrections to atomic levels, etc.)
  - Particle physics (neutrino-less double beta decay, neutrino-nucleus interactions)

# Electromagnetic Reactions

## Photo-nuclear Reactions

Reactions resulting from the interaction of a photon with the nucleus

For photon energy 15-25 MeV stable nuclei across the periodic table show a wide and large peak



# Electromagnetic Reactions

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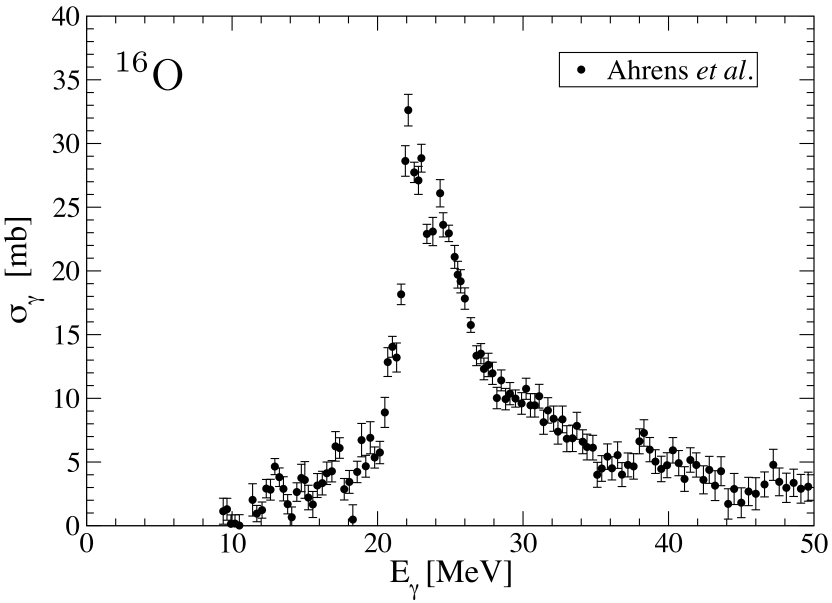
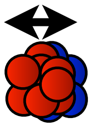


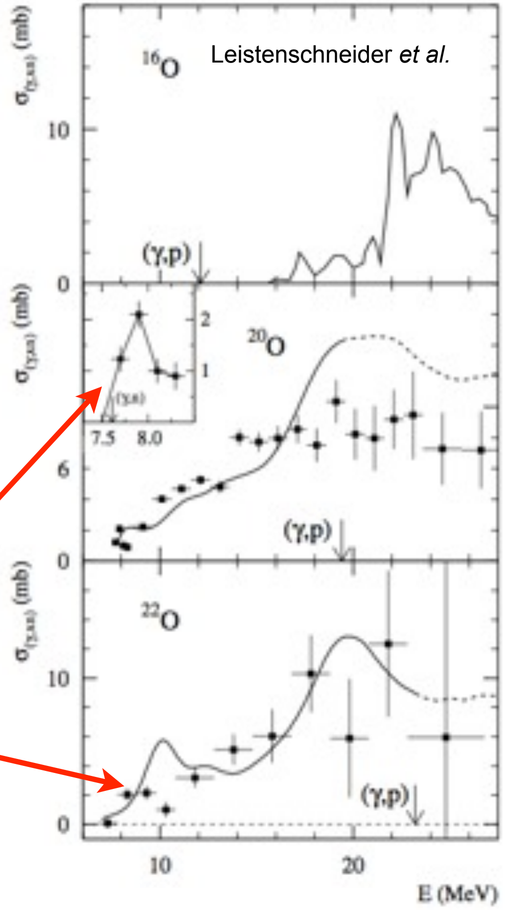
Giant Dipole Resonance



## Coulomb excitations

Inelastic scattering between two charged particles. Can use unstable nuclei as projectiles.

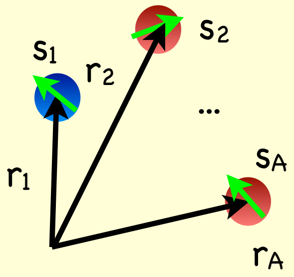
Neutron-rich nuclei show fragmented low-lying strength



Soft Dipole Mode

- Can we give a microscopic explanation of these observations?

# *Ab-initio* Theory Tools



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \dots$$

High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

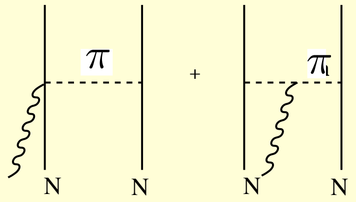
Traditional Nuclear Physics

AV18+UIX, ..., $J_2$

Effective Field Theory

$N^2LO$, $N^3LO$ ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$



two-body currents (or MEC)

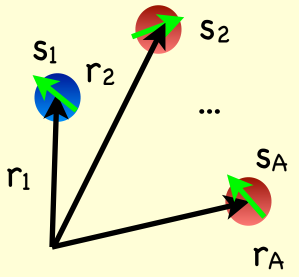
subnuclear d.o.f.

$J^\mu$ consistent with $V$

$$\nabla \cdot J = -i[V, \rho]$$

Review Paper:
*Electromagnetic Reactions on Light Nuclei*
S. Bacca and S. Pastore
J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

# *Ab-initio* Theory Tools



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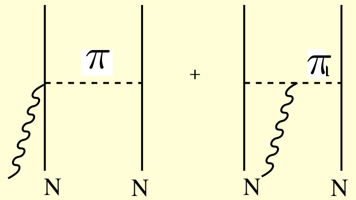

two-body currents (or MEC)

subnuclear d.o.f.

$J^\mu$ consistent with $V$

$$\nabla \cdot J = -i[V, \rho]$$

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Exact Initial state &
Final state in the continuum at different energies and for different A

Review Paper:
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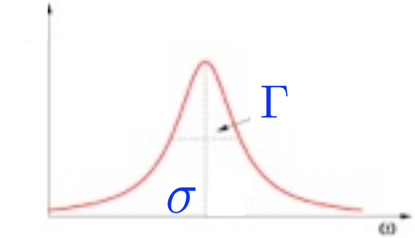
# Lorentz Integral Transform Method

Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

Reduce the continuum problem to a bound-state problem

$$R(\omega) = \sum_f \left| \left\langle \psi_f \middle| J^\mu \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$

$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2 + \Gamma^2} = \left\langle \tilde\psi | \tilde\psi \right\rangle < \infty$$



where $\left|\tilde\psi\right\rangle$ is obtained solving

$$\boxed{(H - E_0 - \sigma + i\Gamma)|\tilde\Psi\rangle = J^\mu|\Psi_0\rangle}$$

- Due to imaginary part $\Gamma$ the solution $|\tilde\psi\rangle$ is unique
- Since $\langle\tilde\psi|\tilde\psi\rangle$ is finite, $|\tilde\psi\rangle$ has bound state asymptotic behaviour

$$L(\sigma,\Gamma) \xrightarrow{\text{inversion}} R(\omega)$$

The exact final state interaction (FSI) is included in the continuum rigorously!

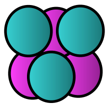
Solved for A=3,4,6,7 with hyper-spherical harmonics expansions and for A=4 with NCSM

# Giant Dipole Resonance in A=6

with Hyperspherical Harmonics

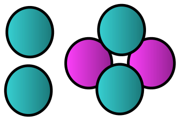
$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

$$E1 = \sum_i^Z (z_i - Z_{cm})$$



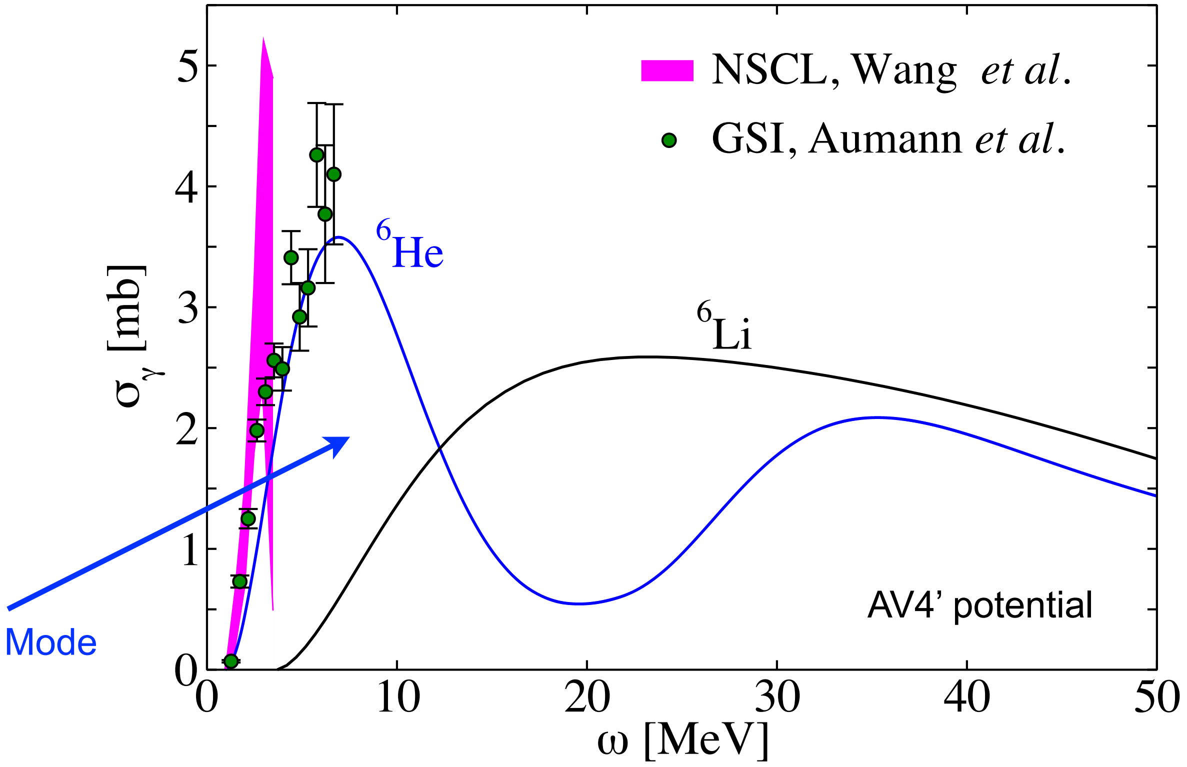
$^6$Li

stable



$^6$He

unstable $T_{1/2} = 806$ ms

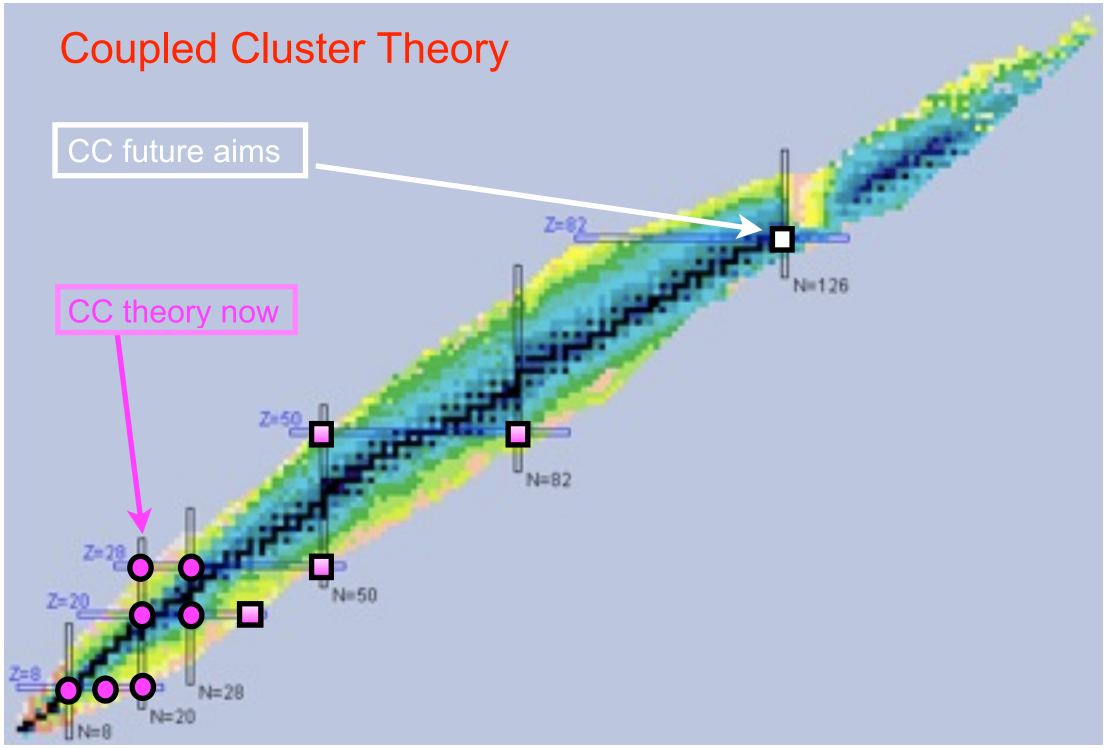


Soft Dipole Mode

S.B. *et al.* PRL **89** 052502/PRC **69** 052502 (2004)

# Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



CC future aims

CC theory now

- CC is optimal for closed shell nuclei (±1, ±2)

Uses particle coordinates

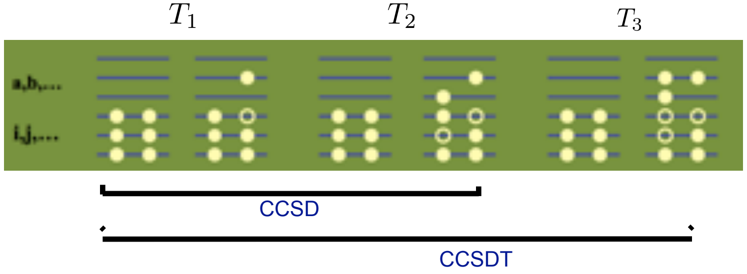
$$|\psi_0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = e^T|\phi_0(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle$$

reference SD with any sp states

$$T = \sum T_{(A)}$$ cluster expansion

$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \frac{1}{4}\sum_{ij,ab} t_{ij}^{ab}\ a_a^\dagger a_b^\dagger a_j a_i \qquad \cdots$$



For the ground state energy

$$E_0 = \langle\phi_0|e^{-T}He^{T}|\phi_0\rangle \qquad \bar{H} = e^{-T}He^{T}$$

similarity transformed Hamiltonian

$$0 = \langle\phi_i^a|e^{-T}He^{T}|\phi_0\rangle$$

$$0 = \langle\phi_{ij}^{ab}|e^{-T}He^{T}|\phi_0\rangle$$

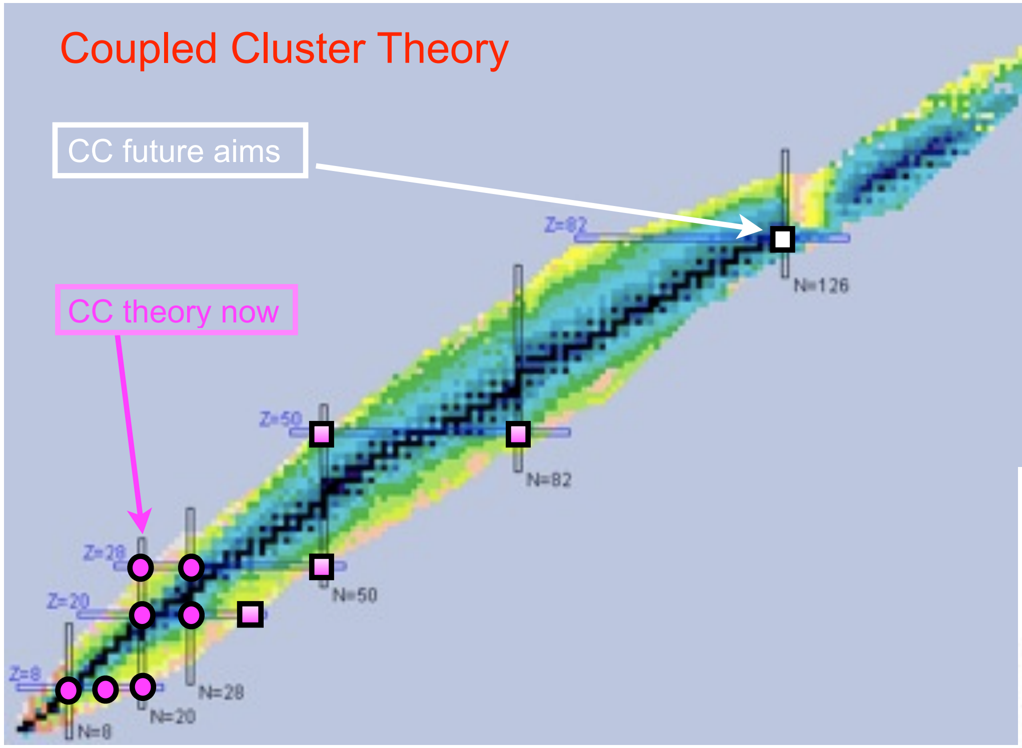
Leads to CCSD equations for the t-amplitudes

Model space truncation $N \le N_{max}$

Computational load $n_o^2 n_u^4$

# Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



Coupled Cluster Theory

CC future aims

CC theory now

- CC is optimal for closed shell nuclei (±1, ±2)

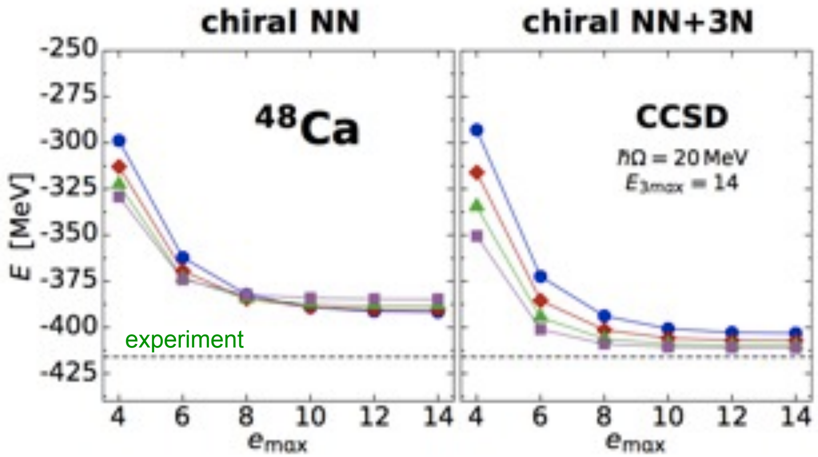
Uses particle coordinates

$$|\psi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

↳ reference SD with any sp states

CC is a very mature theory for g.s., see e.g.

Hagen *et al.* PRL **101**, 092502 (2008),PRC **82**, 03433 (2010)
PRL **108**, 242501 (2012), PRL **109,** 032502 (2012)



R. Roth *et al.*, Phys. Rev. Lett. **109**, 052501 (2012)

What about electro-weak reactions?

LIT+CC can possibly extend calculations of inelastic reactions into medium-mass nuclei!

# LIT with Coupled Cluster Theory

New theoretical method aimed at extending *ab-initio* calculations towards medium-mass

S.B. *et al.*, PRL **111**, 122502 (2013)

$$\boxed{(H - z^*)|\tilde{\Psi}\rangle = J^\mu|\psi_0\rangle} \longrightarrow \boxed{(\bar{H} - z^*)|\tilde{\Psi}_R(z^*)\rangle = \bar{\Theta}|\Phi_0\rangle}$$

with $z = E_0 + \sigma + i\Gamma$

$$\bar{H} = e^{-T} H e^{T}$$

$$\bar{\Theta} = e^{-T} \Theta e^{T}$$

$$L(\sigma,\Gamma) = \left\langle \tilde{\Psi} \middle| \tilde{\Psi} \right\rangle \longrightarrow L(\sigma,\Gamma) = \left\langle \tilde{\Psi}_L \middle| \tilde{\Psi}_R \right\rangle =$$

$$-\frac{1}{2\pi}\Im\left\{\langle \bar{0}_L|\bar{\Theta}^\dagger \left[|\tilde{\Psi}_R(z^*)\rangle - |\tilde{\Psi}_R(z)\rangle\right]\right\}$$

with $|\tilde{\Psi}_R(z^*)\rangle = \hat{R}(z^*)|\Phi_0\rangle$

Formulation based on the solution of an Equation of Motion with source No approximations done so far!

Present implementation in the CCSD scheme

$$T = T_1 + T_2$$

$$\hat{R} = \hat{R}_0 + \hat{R}_1 + \hat{R}_2$$

# Chiral Effective Field Theory

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Meissner, Nogga, Machleidt,…

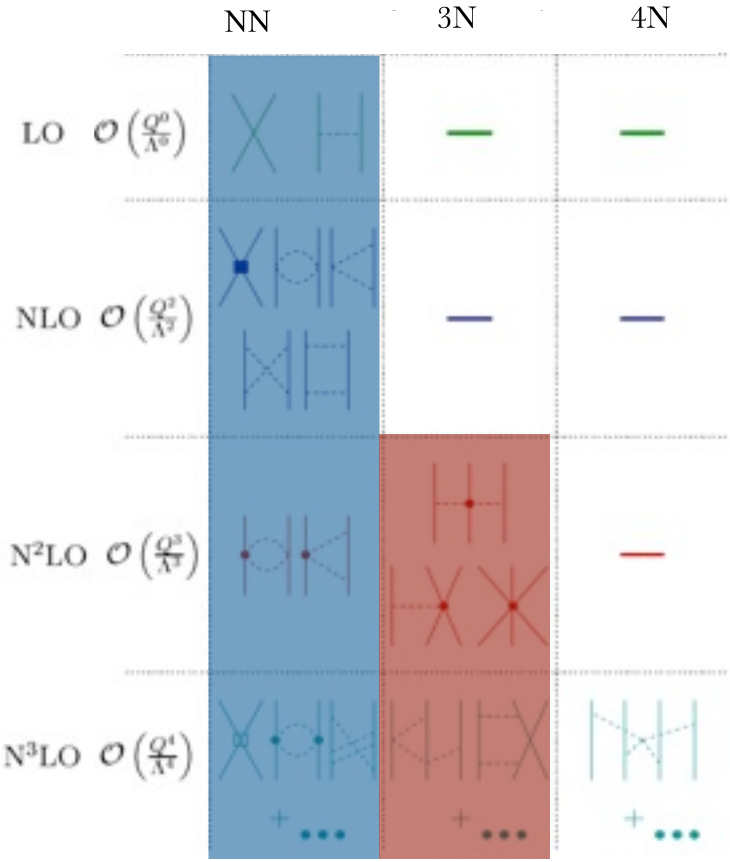
$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \ldots$$

Construct an effective Lagrangian which respects the chiral symmetry and is written as an expansion in powers of $\frac{Q}{\Lambda_b}$

$$\mathcal{L} = \sum_k c_k \left(\frac{Q}{\Lambda_b}\right)^k$$

Details of short distance physics not resolved, but captured in low energy coupling constants, fit to experiment once

Systematic expansion of the potential in many-body terms



**Traditional Paradigm:**
calibrate NN on NN scattering data
calibrate 3N on 3N data

Tests and Results from NN only

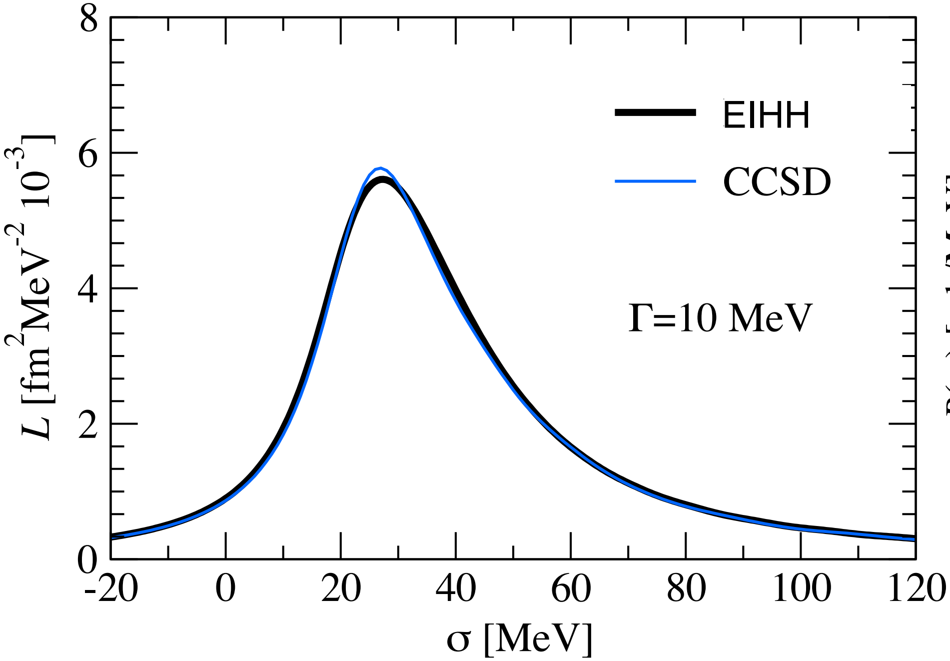
$$V_{NN} > V_{3N} > V_{4N}$$

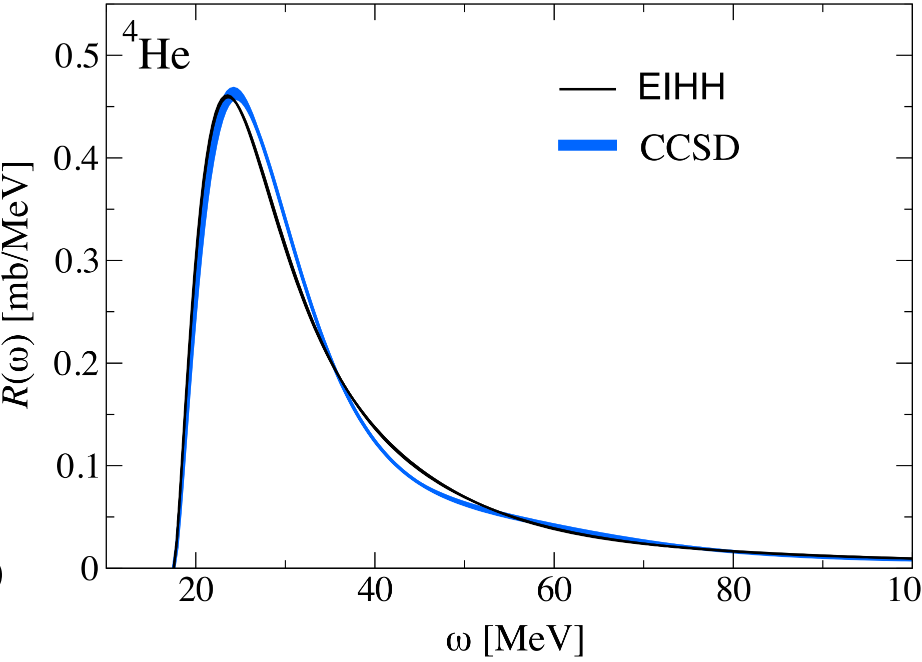
# LIT with Coupled Cluster Theory

New theoretical method aimed at extending *ab-initio* calculations towards medium mass

## Validation for $^4$He

⇨ Comparison of CCSD with exact hyperspherical harmonics (EIHH) with NN forces at N$^3$LO





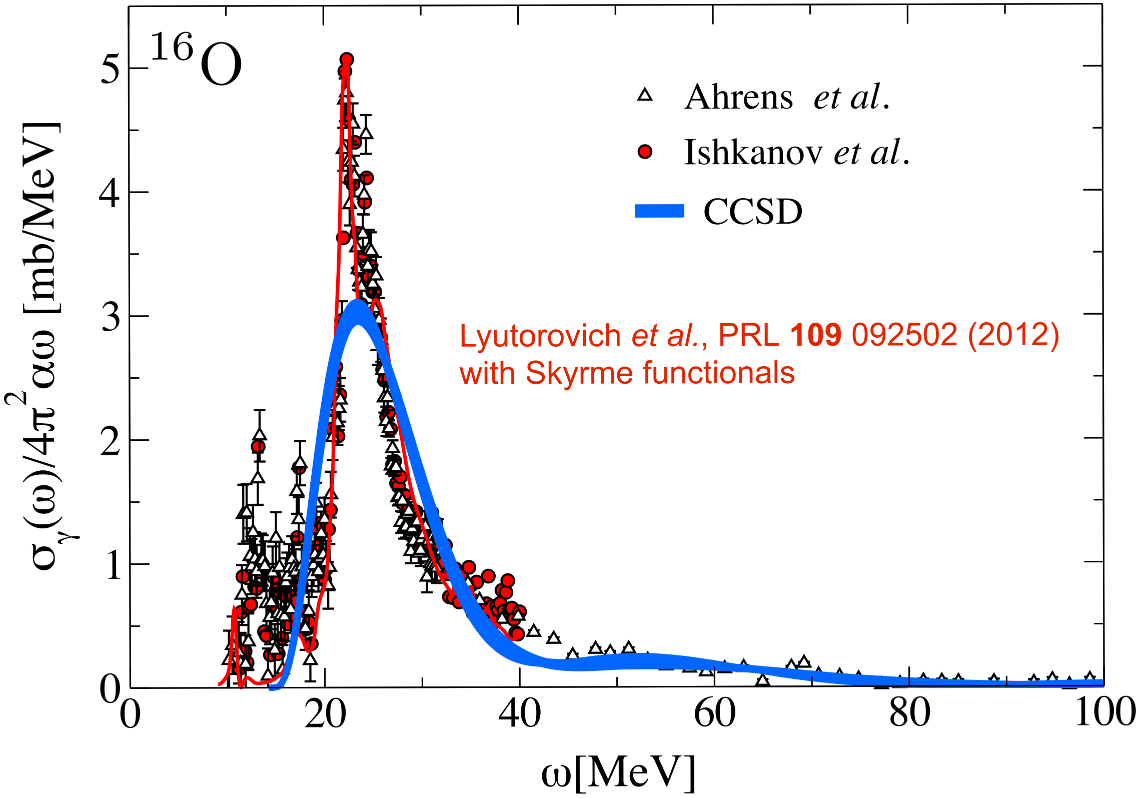
The comparison with exact theory is very good!

# LIT with Coupled Cluster Theory

New theoretical method aimed at extending *ab-initio* calculations towards medium mass

S.B. *et al.*, PRL **111**, 122502 (2013)

Extension to Dipole Response Function in $^{16}$O with NN forces derived from $\chi$EFT (N$^3$LO)



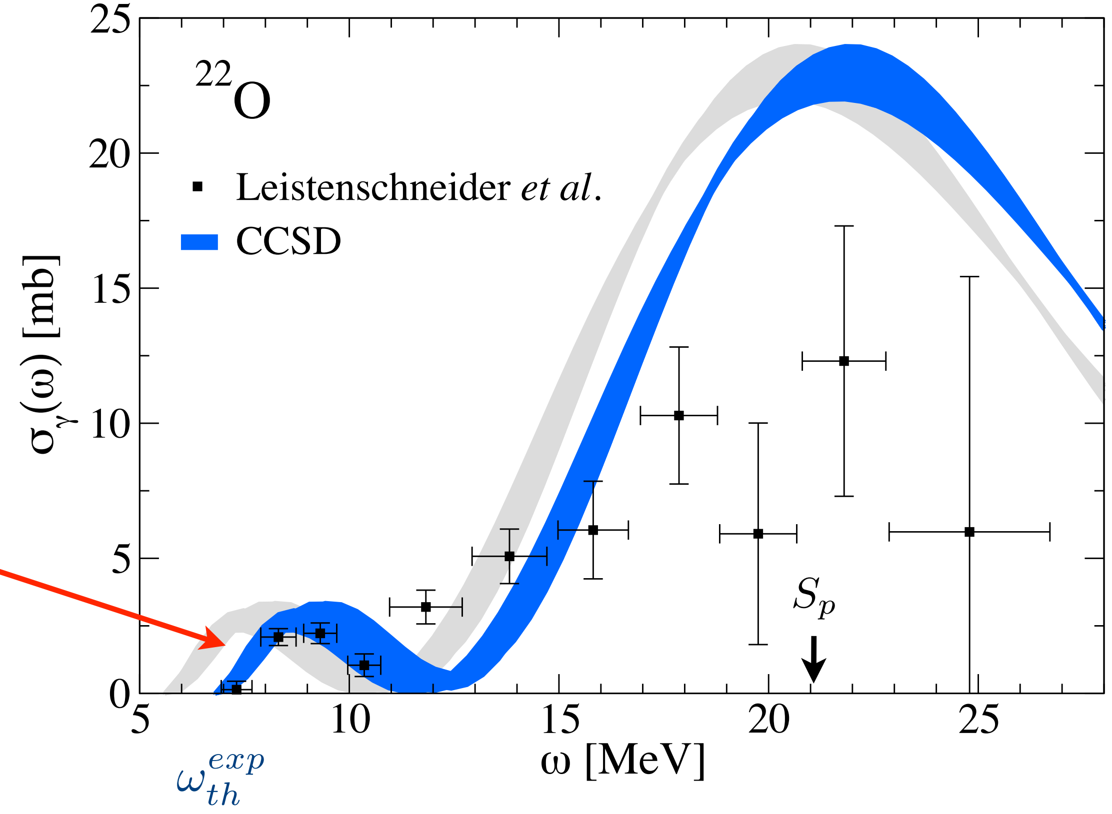
Lyutorovich *et al.*, PRL **109** 092502 (2012)
with Skyrme functionals

The position of the GDR in $^{16}$O is described from first principles for the first time!

# Addressing neutron-rich nuclei

$^{22}$O with NN forces derived from $\chi$EFT (N$^3$LO)

S.B. *et al.*, PRC **90**, 064619 (2014)



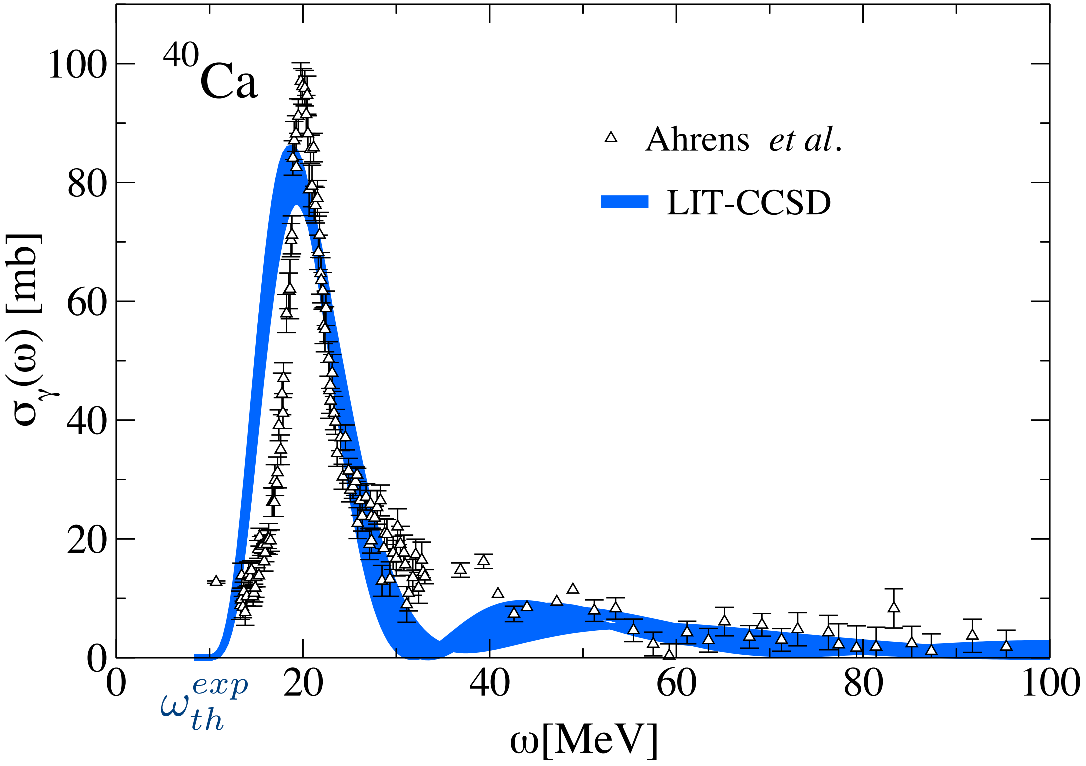
Soft Dipole Mode

⇨ Soft dipole mode emerges from a first principle calculation

# Addressing heavier nuclei

$^{40}$Ca with NN forces derived from $\chi$EFT (N$^3$LO)

S.B. *et al.*, PRC **90**, 064619 (2014)



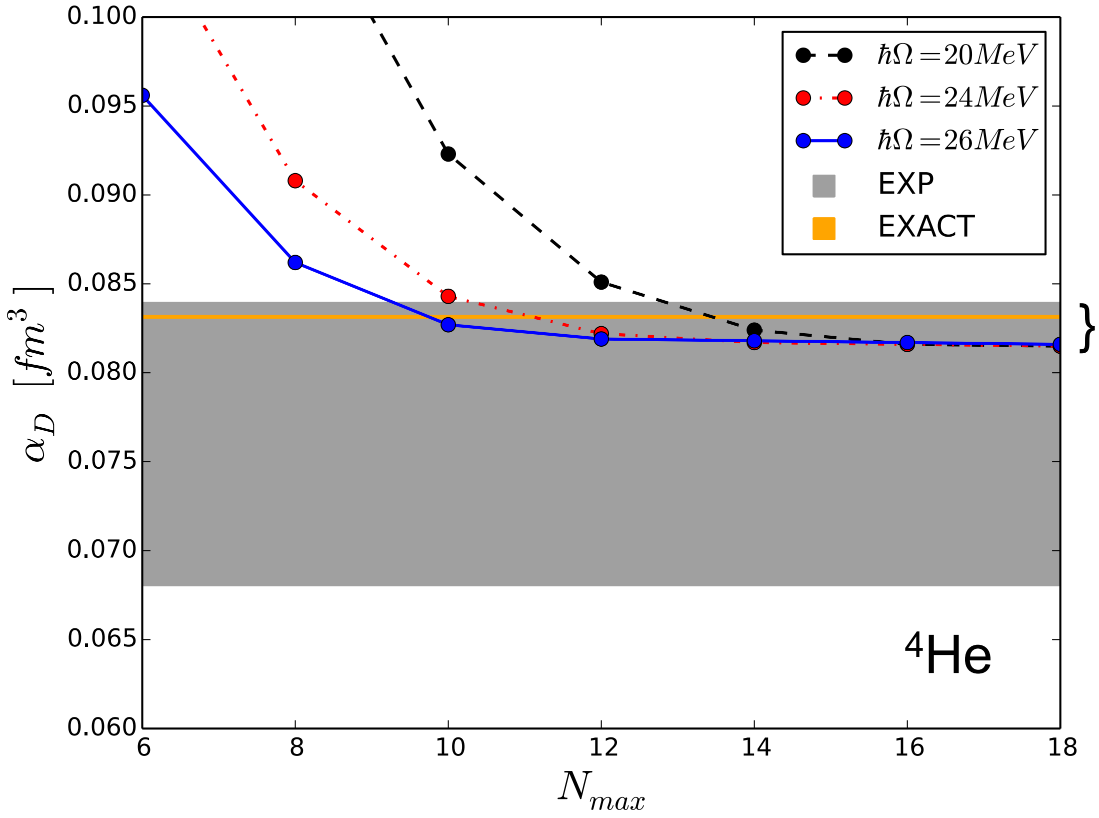
Electric Dipole Polarizability

$$\alpha_D = 2\alpha \int_{\omega_{th}}^{\infty} d\omega \frac{R(\omega)}{\omega}$$

# Electric Dipole Polarizability

M. Miorelli *et al.*, in preparation (2015)

Validation for $^4$He with NN(N$^3$LO)



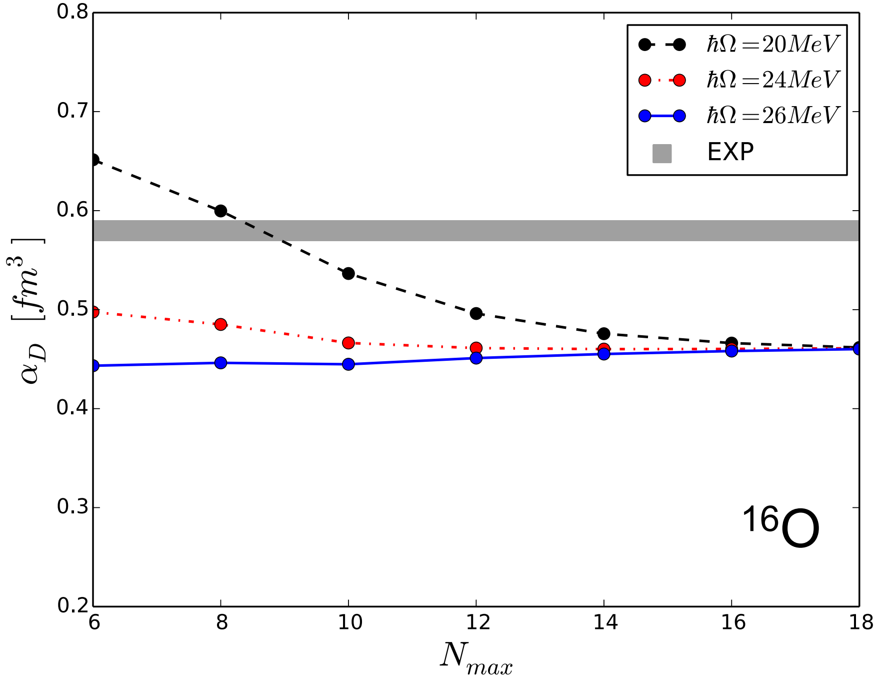
} Small difference due to missing triples and quadrupoles

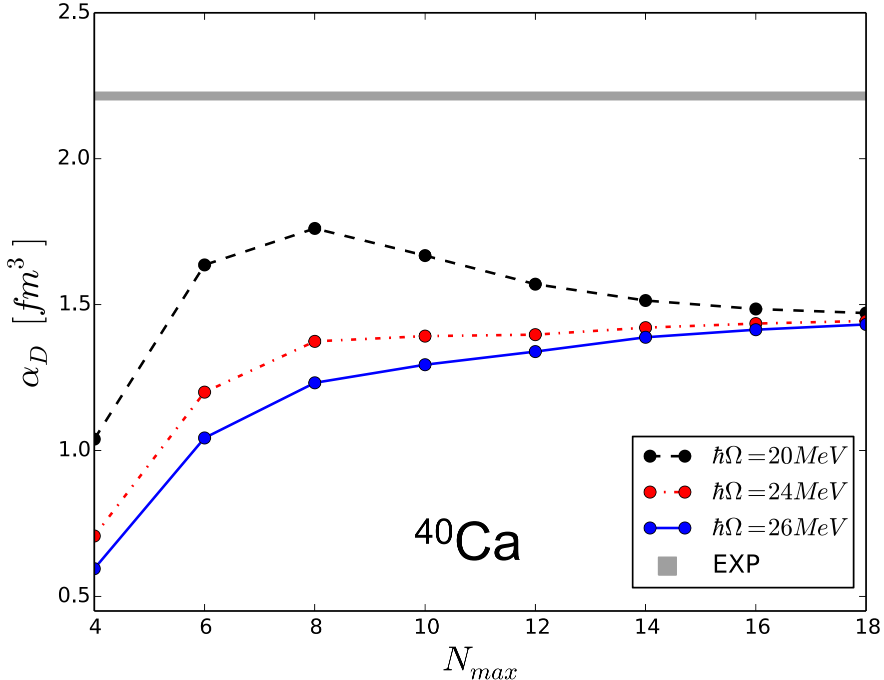
The comparison with experiment is very good

# Electric Dipole Polarizability

M. Miorelli *et al.*, in preparation (2015)

Medium-mass nuclei with NN(N$^3$LO)





$$\alpha_D = 0.46\ \text{fm}^3$$
$$\alpha_D^{exp} = 0.585(9)\ \text{fm}^3$$

$$R_{ch} = 2.3\ \text{fm}$$
$$R_{ch}^{exp} = 2.6991(52)\ \text{fm}$$

$$\alpha_D = 1.47\ \text{fm}^3$$
$$\alpha_D^{exp} = 2.23(3)\ \text{fm}^3$$

$$R_{ch} = 3.05\ \text{fm}$$
$$R_{ch}^{exp} = 3.4776(19)\ \text{fm}$$

The present Hamiltonian underestimates both radii and electric dipole polarizabilities

# Conclusions and Outlook

- Electromagnetic break up reactions are very rich observables to test our understanding of nuclear forces
- Extending first principles calculations to medium mass nuclei is possible and very exciting: more applications/impact on experiments in the future

## Perspectives

- Other multipole excitation: electric quadrupole or monopole, magnetic dipole in medium mass nuclei
- Add triples excitations: $\hat{R}_3, T_3$
- Extend first principle calculations to the weak sector: Gamow-Teller transitions

See talk by Gaute Hagen on Wed

**Thank you!**