Parton Distribution Functions from Large Momentum Effective Theory

Yong Zhao
University of Maryland, College Park

Jianhui Zhang
University of Regensburg

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Outline

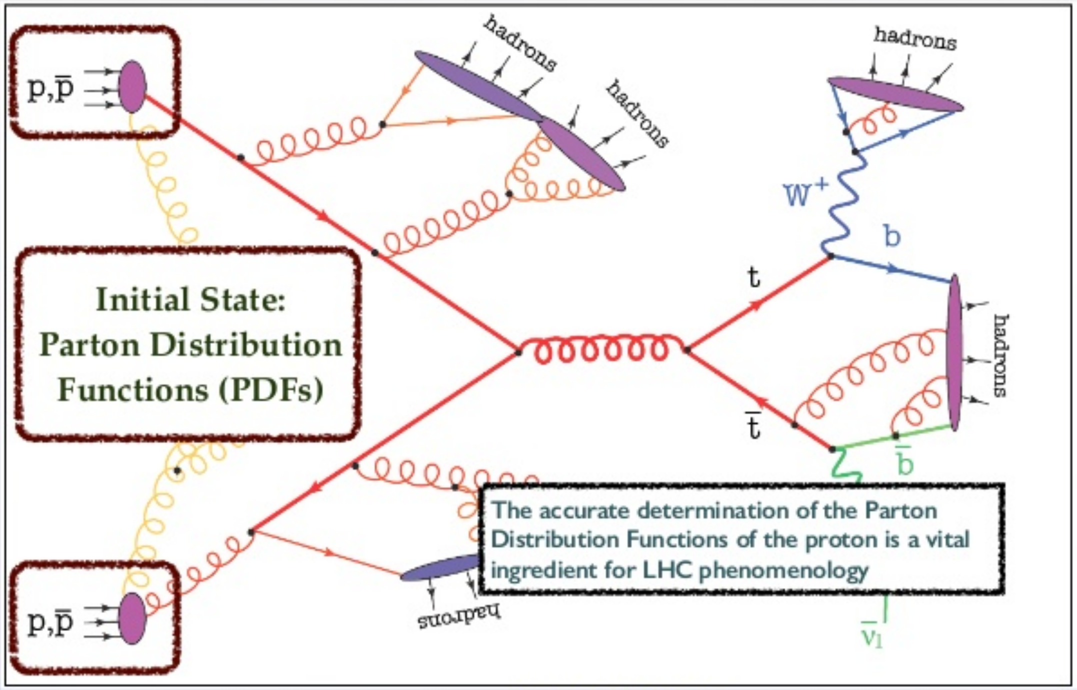
- Review of parton distribution functions (PDF)
  - Parton physics and the infinite momentum frame
  - Why it is difficult to calculate PDFs in lattice QCD?
- A new way of calculating parton physics
  - Lesson from the gluon polarization
  - Large momentum effective theory approach
- PDFs from large momentum effective theory
  - From a quasi-PDF to the normal PDF
  - Requirements for the computation resources

Parton distribution functions

- PDFs characterize the momentum distributions of quarks and gluons inside the nucleon.
- They are important inputs for making predictions in high-energy scattering experiments.

$$\sigma = \sum_{a,b} f_a \otimes f_b \otimes \sigma_{ab}$$

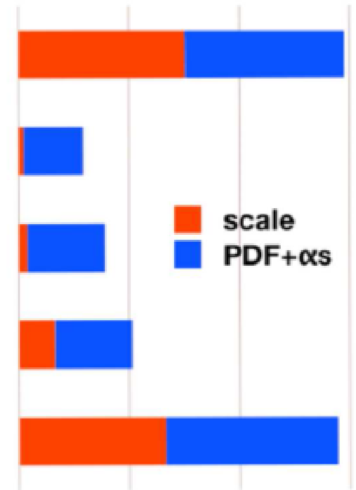
From J. Rojo's talk at SLAC, 2015



Parton distribution functions

Higgs production

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
	VBF	1.56 pb	2.9%
NNLO QCD +NLO EW	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



(J. Campbell, HCP2012)

- PDFs induce large uncertainty in Higgs production at the LHC.
- A better understanding and precise determination of PDFs required to help us understand the standard model and disentangle new physics effects.

Parton distribution functions

- PDFs are the universal properties of nucleons
  - Nucleons are bound states of quarks and gluons described by the fundamental theory of QCD.
  - The quarks and gluons are strongly interacting and relativistic, PDFs are intrinsically non-perturbative.
- Our knowledge of PDFs is mainly driven by the data from state-of-the-art high-energy programs, BCDMS, SLAC, NMC, JLab, HERA, E866, CDF, DØ, COMPASS, RHIC, JPARC, LHC, …, (see talks by Nocera and Schmidt), but a first principle study of PDFs is still behind.

Physics of PDFs

- The nucleon wavefunction |P> is usually defined at equal time and includes lots of complexities: valence quarks, sea quarks, gluons;

- An equal-time wavefunction is not frame independent:
  - Under a Lorentz boost Λ, |P> transforms as

$$\left|P\right\rangle = U(\Lambda(p))\left|P=0\right\rangle$$

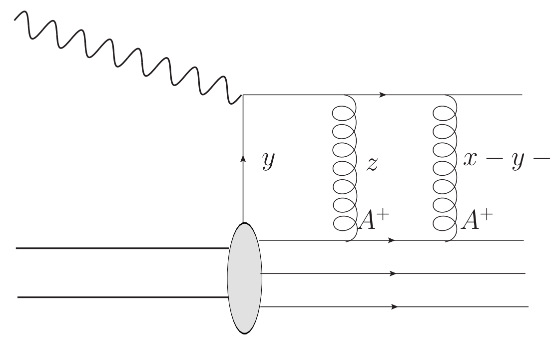
  - U(Λ) depends on the interaction, so the transformation U(Λ(P)) is not kinematic, but dynamical.

Physics of PDFs

- In high energy scattering, the nucleon is travelling at a large momentum relative to the probe;
  - The time scale of the interaction between quarks and gluons is dilated due to relativistic effects;
  - When the nucleon exchanges a hard momentum with the probe within an "impulse", the struck quark or gluon almost does not feel the other constituents;
  - In the infinite momentum frame (IMF), this picture becomes simple as the quarks and gluons appear like free particles, i.e., partons.

Physics of PDFs

- However, the IMF can never be achieved in reality, so this picture is an approximation for high-energy scattering:
  - Leading twist effect
  - Sensitive to the soft scales at higher-twist
  - One has to fix to the light-cone gauge $A^+=0$ to eliminate the longitudinal gluons, and thus have a clear partonic picture.
  - AP (Altarelli, Parisi, 1977) equation was derived in the IMF.



From Ji, Xu, and Zhao, JHEP, 2012

PDFs in light-cone quantization

- The IMF picture of parton physics is replaced by light-cone quantization, in which the probe is moving at the speed of light and the nucleon is at rest:
  - In light-cone quantization, the wavefunction is defined at equal light-cone time $\xi^+=(x^0+x^3)/\sqrt{2}=0$;
  - The Hamiltonian $H_{LC}=P^-$ can be used to formulate "time"-independent perturbation theory as in non-relativistic quantum mechanics;
  - The nucleon wavefunction can have a Fock space expansion

$$|P\rangle = \sum_{n\alpha,\lambda_i} \int \Pi_i \frac{dx_i d^2k_{\perp i}}{\sqrt{2x_i}(2\pi)^3} \psi_{n\alpha}(x_i, k_{\perp i}, \lambda_i) \left| n\alpha : x_i P^+, k_{\perp,i}, \lambda_i \right\rangle \ ,$$

PDFs in light-cone quantization

- The PDFs can be defined as light-cone correlations along the "new spatial direction" $\xi^- = (x^0 - x^3)/\sqrt{2}$:

$$
\begin{aligned}
q(x,\mu^2) &= \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \overline{\psi}(\xi^-) \gamma^+ \\
&\quad \times \exp\left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right) \psi(0) | P \rangle \ ,
\end{aligned}
$$

  - The light-cone gauge link ensures its gauge invariance;
  - Invariant under the boost along the nucleon momentum direction;
  - In the light-cone gauge $A^+=0$, has clear interpretation as parton density:

$$
q(x) \sim \int d^2k_\perp \, dk^+ n(k^+, k_\perp) \delta(x - k^+ / P^+)
$$

First principle calculation of PDFs

- Despite its nice features, calculating PDFs in light-cone quantization is difficult,
  - No systematic approximation has been found in solving QCD in light-cone quantization in 3+1 dimensions;
  - AdS/QCD: no exact correspondence can be made.

- The only practical approach to solve non-perturbative QCD is the lattice theory.
  - Defined in the Euclidean space, with $\tau = it$ being real;
  - $A^4 = iA^0$ is real;
  - Has been successful in calculating hadron spectroscopy, nucleon form factors, finite temperature QCD, etc.

Traditional way of calculating PDFs in Lattice QCD

- Lattice QCD cannot directly access light-cone quantities due to their dependence on real time:
  - Lattice can calculate the moments of PDFs which are local operators;

$$\int x^{n-1} q(x) dx \quad \sim \quad n_{\mu_1} ... n_{\mu_n} \langle P | \overline{\psi}(0) \gamma^{\mu_1} i D^{\mu_2} ... i D^{\mu_n} \psi(0) | P \rangle$$

  - Parametrize the PDFs with a smooth functional form
  - Determine the parameters from the lattice computed moments
  - Problem: number of calculable moments is limited (H.W. Lin's talk)

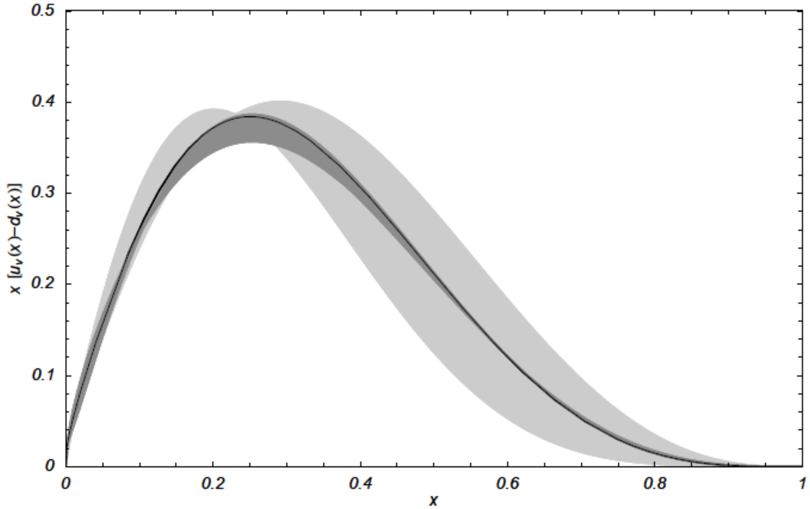


Fig. 5. Reconstructed isovector valence quark distribution $x(u_v - d_v)$ in the proton at $Q^2 = 4$ GeV^2. The central fit curve (solid line) and error band (lightly shaded) are compared with the envelope of the phenomenological distributions[20] (darkly shaded).

Detmold et al., Mod.Phys.Lett.A 03'

$$xf(x) = Ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$

Traditional way of calculating PDFs in Lattice QCD

- The number of calculable moments strongly limits the power of lattice QCD to calculate PDFs;

- Given that lattice QCD can only calculate the matrix elements of time-independent operators at zero or finite nucleon momentum, can we find a way to obtain PDFs directly from lattice QCD?

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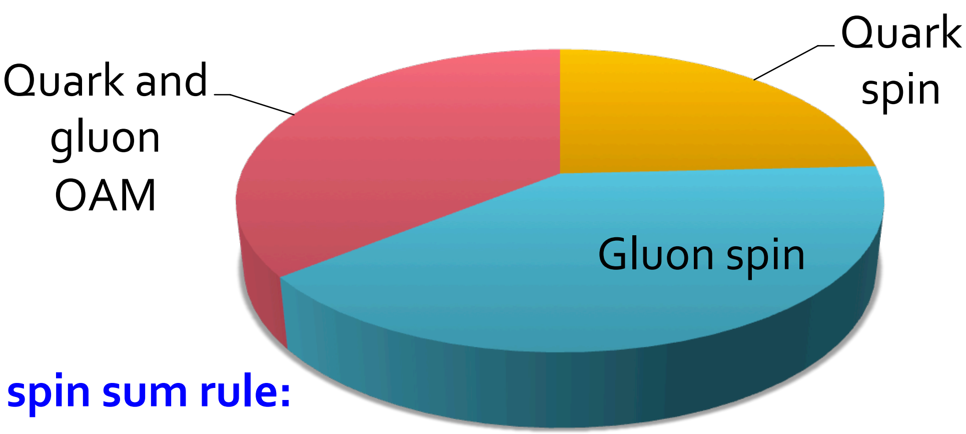
  - Why it is difficult to calculate PDFs in lattice QCD?
- A new way of calculating parton physics
  - Lesson from the gluon polarization
  - Large momentum effective theory approach
- Lattice QCD calculation of PDFs
  - From a quasi-PDF to the normal PDF
  - Requirements for the computation resources

Lesson from the gluon polarization

- The gluon polarization ΔG contributes to the longitudinal nucleon spin, and is measured in polarized deep inelastic scattering (DIS) and proton-proton collisions.

Longitudinal proton spin structure



$\Delta\Sigma(Q^2=10\ \mathrm{GeV}^2) = 0.242$, de Florian et al., 2009

SLAC
HERMES (DESY)
COMPASS (CERN)
JLab
RHIC

Naïve spin sum rule:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + l_q^z + l_q^z$$

$\Delta G(Q^2=10\ \mathrm{GeV}^2) \sim 0.2$, de Florian et al., 2014

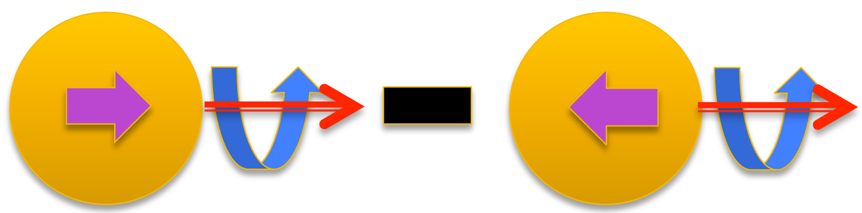
Lesson from the gluon polarization

- ΔG is defined as the first moment of the polarized gluon distribution function (A.V. Manohar, PRL 1990):

$$\Delta g(x) = \frac{i}{2xP^+} \int d\xi^- e^{-ixP^+\xi^-} \langle PS | F_a^{+,i}(\xi^-) L^{ab}(\xi^-,0) \tilde{F}^+_{b,i}(0) | PS \rangle$$

- In light-cone quantization,

$\Delta g(x) =$



- However, the first moment of $\Delta g(x)$ does not correspond to any local gauge-invariant operators;

- As a result, ΔG cannot be directly calculated on the lattice, and has been a problem for a long time.

Lesson from the gluon polarization

- A recent proposal tries to define the gauge-invariant gluon spin operator at equal time in the rest frame of the nucleon (Chen et al., PRL 2008).

$$S_g = \int d^3x\, \vec{E} \times \vec{A}_\perp$$

  - In Abelian gauge theory,

$$\vec{A} = \vec{A}_\perp + \vec{A}_{//}, \quad \vec{\nabla} \cdot \vec{A}_\perp = 0, \quad \vec{\nabla} \times \vec{A}_{//} = 0$$

A_{perp} is gauge invariant under a gauge transformation.

  - However, A_{perp} is not frame independent. In QED, its transformation under a Lorentz boost is dynamical.
  - This makes the physical meaning of $E \times A_{perp}$ as spin not clear.

Lesson from the gluon polarization

- Actually, in the rest frame of the nucleon, due to the existence of unphysical degrees of freedom (d.o.f.), there is no physical meaning of gluon spin at all;

- Nevertheless, if we boost the nucleon to the IMF, longitudinal gluon field strength is suppressed, A_{perp} fully captures the physical d.o.f., and therefore $E \times A_{perp}$ can be regarded as the spin of free gluon fields in this limit.

- This is the famous Weizsäcker-Williams approximation (or equivalent photon method) in classical electrodynamics (Jackson, Classical Electrodynamics).

Lesson from the gluon polarization

- Moreover, we can prove that (Ji, Zhang, Zhao, PRL 2013)

$$\vec{E}\times\vec{A}_{\perp} \xrightarrow{P\to\infty} \int dx \frac{i}{x} \int d\xi^{-} e^{-ixP^{+}\xi^{-}} F_{a}^{+,i}(\xi^{-}) L^{ab}(\xi^{-},0) \tilde{F}_{b,i}^{+}(0)$$

  - The nucleon matrix element of $E\times A_{perp}$ is just the gauge-invariant gluon spin defined from the first momentum of $\Delta g(x)$.
  - $E\times A_{perp}$ is time independent, and therefore can be readily calculated in lattice QCD.
  - One can obtain ΔG by studying the IMF limit of the matrix element of $E\times A_{perp}$.

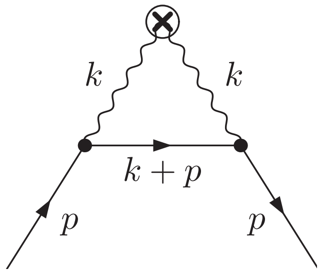
Lesson from the gluon polarization

- However, since $E\times A_{perp}$ is frame dependent, its matrix element will depend on the nucleon momentum P, which becomes a large scale of theory;
- In perturbation theory, if one does the ultraviolet (UV) regularization first, the matrix element of $E\times A_{perp}$ can contain logarithms of P, which is singular in the IMF limit.
- Instead, if one takes the IMF limit of $E\times A_{perp}$ first, and then does the UV regularization, one obtains the correct ΔG.
- The tricky point is that lattice QCD can only calculate finite momentum matrix elements.

Lesson from the gluon polarization

One loop example:

Ji, Zhang, Zhao, PRL 2013



- At large but finite $\vec{p}$, taking the loop momentum $k \to \infty$ first

$$\langle p,s| \left(\vec{E}\times\vec{A}_\perp\right)^3 |p,s\rangle = \frac{\alpha_S C_F}{4\pi}\left[\frac{5}{3\epsilon} + \frac{5}{3}\ln\frac{\mu^2}{m^2} + \frac{4}{3}\ln\frac{4\vec{p}^2}{m^2} - \frac{1}{9}\right] u^\dagger\Sigma^3 u + O(\frac{m^2}{\vec{p}^2}),$$

- Taking $\vec{p}^2 \to \infty$ first,

$$\langle p,s| \left(\vec{E}\times\vec{A}_\perp\right)^3 |p,s\rangle = \frac{\alpha_S C_F}{4\pi}(\frac{3}{\epsilon} + 3\ln\frac{\mu^2}{m^2} + 7)\ u^\dagger\Sigma^3 u$$

- The light-cone gauge result,

$$\langle p,s| \left(\vec{E}\times\vec{A}\right)^3 |p,s\rangle \overset{A^+=0}{=} \frac{\alpha_S C_F}{4\pi}(\frac{3}{\epsilon} + 3\ln\frac{\mu^2}{m^2} + 7)\ u^\dagger\Sigma^3 u$$

Lesson from the gluon polarization

- By observation we find that the IMF limit does not change the collinear or infrared (IR) divergence in the matrix element, and the difference is just in the UV divergent part.
- The UV divergent part is calculable in perturbative QCD, and therefor we can take this advantage to match the matrix element at finite momentum and in the IMF.
- A systematic matching procedure is provided by the large momentum effective theory (LaMET).

Large momentum effective theory

The large momentum effective theory (LaMET) is a theory that expands in powers of $1/P$, where P is the nucleon momentum (Ji, Sci. China Phys. Mech. Astro., 2014):

1. Construct a Euclidean quasi-observable $\tilde{O}$ which can be calculated in lattice QCD;
2. The IMF limit of $\tilde{O}$ is constructed to be a parton observable O;
3. The matrix elements of $\tilde{O}$ is dependent on the lattice cut-off Λ and generally P, i.e., $\tilde{O}(P/\Lambda)$, while the parton observable depends on the renormalization scale μ (if in the MSbar scheme), i.e., $O(\mu)$;

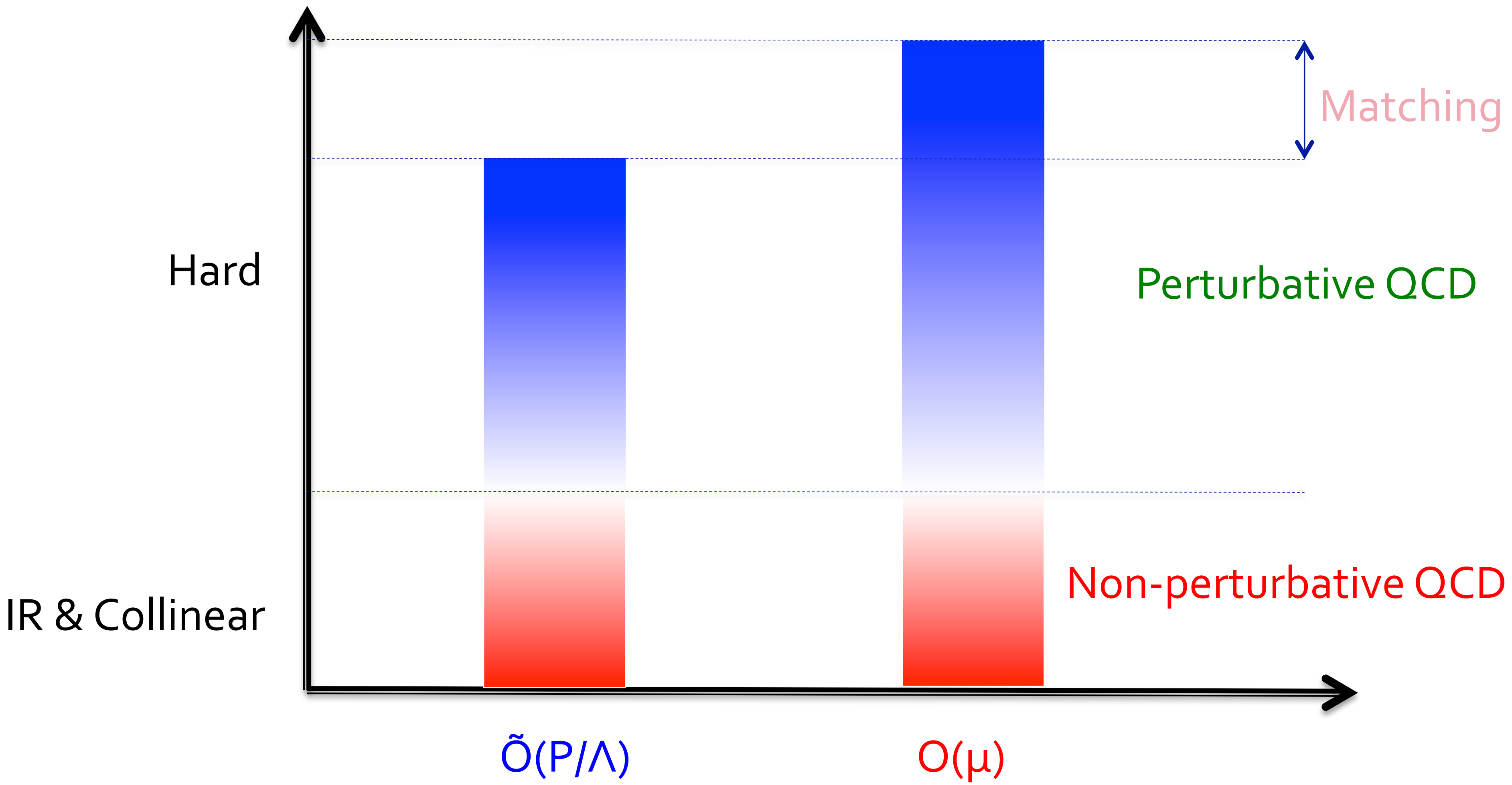
Large momentum effective theory

4. Taking the $P \to \infty$ limit of $\tilde{O}(P/\Lambda)$ is generally ill-defined due to the singularities in quantum field theory, but it can be related to $O(\mu)$ through a factorization formula:

$$\tilde{O}(P/\Lambda) = Z(P/\Lambda, \mu/\Lambda) O(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \ldots$$

- P is much larger than Λ_{QCD} as well as the nucleon mass M;
- $O(\mu)$ captures all the infrared (IR) and collinear divergences in $\tilde{O}(P/\Lambda)$, and thus the leading term can be factorized into $O(\mu)$ and a perturbative coefficient Z.

How matching works?



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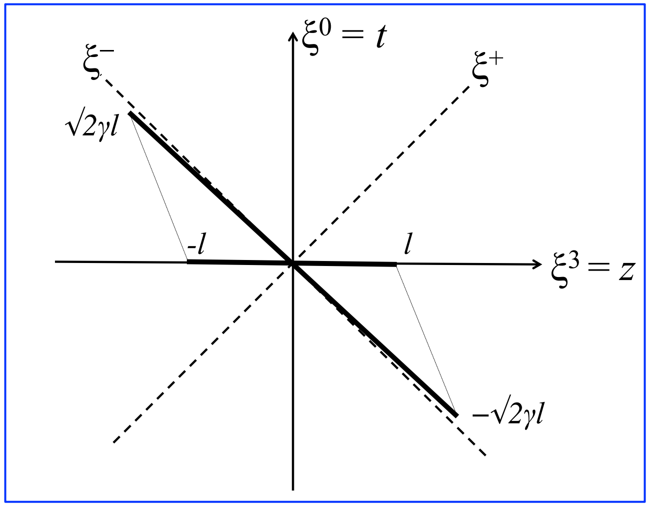
Quasi parton distribution

- Consider the spatial correlation of two quarks in the nucleon with momentum P along the z direction:

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

Ji, PRL 2013

- Equal-time correlation;
- The matrix element of the quasi distribution depends on the nucleon momentum P^z;
- In the IMF limit, $z \to \xi^-$, and $A^z \to A^+$, the quasi PDF approaches normal PDF.



Matching quasi PDF and normal PDF

- According to LaMET, the quasi PDF is related to the normal PDF through a perturbative matching formula for the non-singlet case (Xiong, Ji, Zhang, and Zhao, PRD 2014):

$$\tilde{q}(x,\Lambda,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/(P^z)^2, M^2/(P^z)^2\right)$$

  - Integration range is determined by the support of light-cone parton distribution;
  - Both light-cone and quasi PDFs contain soft divergences, but they cancel in themselves;
  - They also have the same collinear divergences;
  - Z factor captures the difference in their UV behavior, and is thus perturbatively calculable;
  - Higher twist effects suppressed by the large momentum.

Matching quasi PDF and normal PDF

- Take unpolarized quark distribution as an example [Ji, Xiong, Zhang and Zhao PRD, 13]
  - @LO

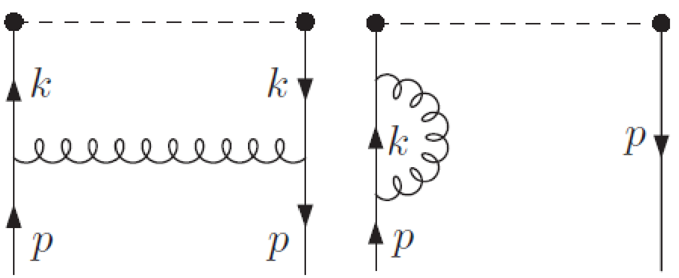
    Both quasi and light-cone quark distribution yield a simple delta function

$$\tilde{q}^{(0)}(x) = q^{(0)}(x) = \delta(1 - x)$$

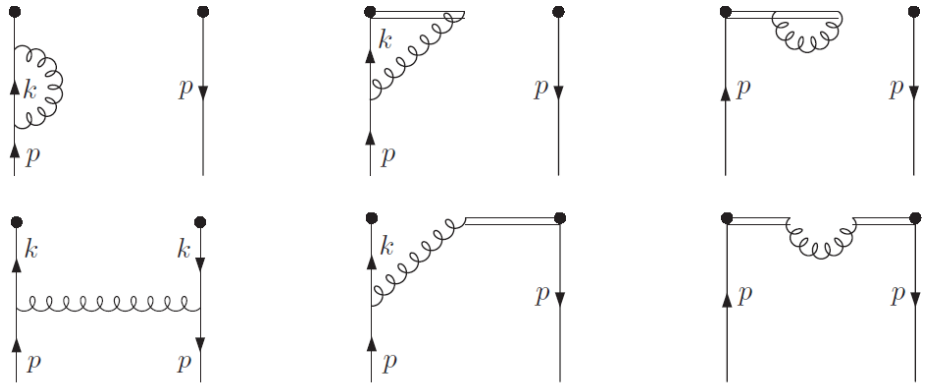
  - @NLO

    The computation can be carried out in any gauge:

- In axial gauge



- In Feynman gauge



Matching quasi PDF and normal PDF

- One-loop matrix element of the normal PDF:

$$q(x,\Lambda) = (1 + Z_F^{(1)}(\Lambda) + \dots)\delta(x-1) + q^{(1)}(x,\Lambda) + \dots$$

$$q^{(1)}(x,\Lambda) = \frac{\alpha_S C_F}{2\pi} \begin{cases} 0\ , & x>1 \text{ or } x<0\ , \\ \frac{1+x^2}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x}\ln(1-x)^2 - \frac{2x}{1-x}\ , & 0<x<1\ , \end{cases}$$

$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} 0\ , & y>1 \text{ or } y<0\ , \\ -\frac{1+y^2}{1-y}\ln\frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y}\ln(1-y)^2 + \frac{2y}{1-y}\ , & 0<y<1\ , \end{cases}$$

Remarks:

- In the IMF or on the light-cone, partons can only have momentum fraction 0<x<1;

Matching quasi PDF and normal PDF

- One-loop matrix element of the normal PDF:

$$q(x,\Lambda) = (1 + Z_F^{(1)}(\Lambda) + \dots)\delta(x-1) + q^{(1)}(x,\Lambda) + \dots$$

$$q^{(1)}(x,\Lambda) = \frac{\alpha_S C_F}{2\pi}\begin{cases} 0\,, & x>1 \text{ or } x<0\,, \\ \frac{1+x^2}{1-x}\ln\frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x}\ln(1-x)^2 - \frac{2x}{1-x}\,, & 0<x<1\,, \end{cases}$$

$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} 0\,, & y>1 \text{ or } y<0\,, \\ -\frac{1+y^2}{1-y}\ln\frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y}\ln(1-y)^2 + \frac{2y}{1-y}\,, & 0<y<1\,, \end{cases}$$

Remarks:

- Light-cone singularity cancels between real and virtual contributions;
- Collinear divergence $\ln(m^2)$ exists in the PDF.

Matching quasi PDF and normal PDF

- One-loop matrix element of the quasi PDF:

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots\ ,$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2P^z}\ , & x>1\ , \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2P^z}\ , & 0<x<1\ , \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2P^z}\ , & x<0\ , \end{cases}$$

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2P^z}\ , \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2P^z}\ , \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2P^z}\ , \end{cases}$$

Remarks:

- In the finite momentum frame, the quark momentum fraction $-\infty < x < +\infty$.

Matching quasi PDF and normal PDF

- One-loop matrix element of the quasi PDF:

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots \,,$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & x>1\,,\\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & 0<x<1\,,\\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & x<0\,,\end{cases}$$

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Remarks:

- No UV divergence (logΛ). Logarithm of P^z only exists in 0<x<1.

Matching quasi PDF and normal PDF

- One-loop matrix element of the quasi PDF:

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots ,$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2 P^z} , & x>1 , \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2 P^z} , & 0<x<1 , \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2 P^z} , & x<0 , \end{cases}$$

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Remarks:

- Axial singularity cancels between real and virtual contributions.

Matching quasi PDF and normal PDF

- One-loop matrix element of the quasi PDF:

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \ldots ,$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2 P^z} , & x>1 , \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2 P^z} , & 0<x<1 , \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2 P^z} , & x<0 , \end{cases}$$

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2 P^z} , \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2 P^z} , \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2 P^z} , \end{cases}$$

Remarks:

- Collinear divergence exists only in $0<x<1$, and is the same as the normal PDF.

Matching quasi PDF and normal PDF

- By comparing their one-loop matrix elements, the matching factor can be read as:

$$Z\left(\xi, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) = \delta(\xi - 1) + \frac{\alpha_s}{2\pi} Z^{(1)}\left(\xi, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) + \ldots$$

$$Z^{(1)}(\xi)/C_F = \frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \quad \xi>1$$

$$Z^{(1)}(\xi)/C_F = \frac{1+\xi^2}{1-\xi}\ln\frac{(P^z)^2}{\mu^2} + \frac{1+\xi^2}{1-\xi}\ln[4\xi(1-\xi)] - \frac{2\xi}{1-\xi} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \quad 0<\xi<1$$

$$Z^{(1)}(\xi)/C_F = \frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi} - 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \quad \xi<0$$

Matching quasi PDF and normal PDF

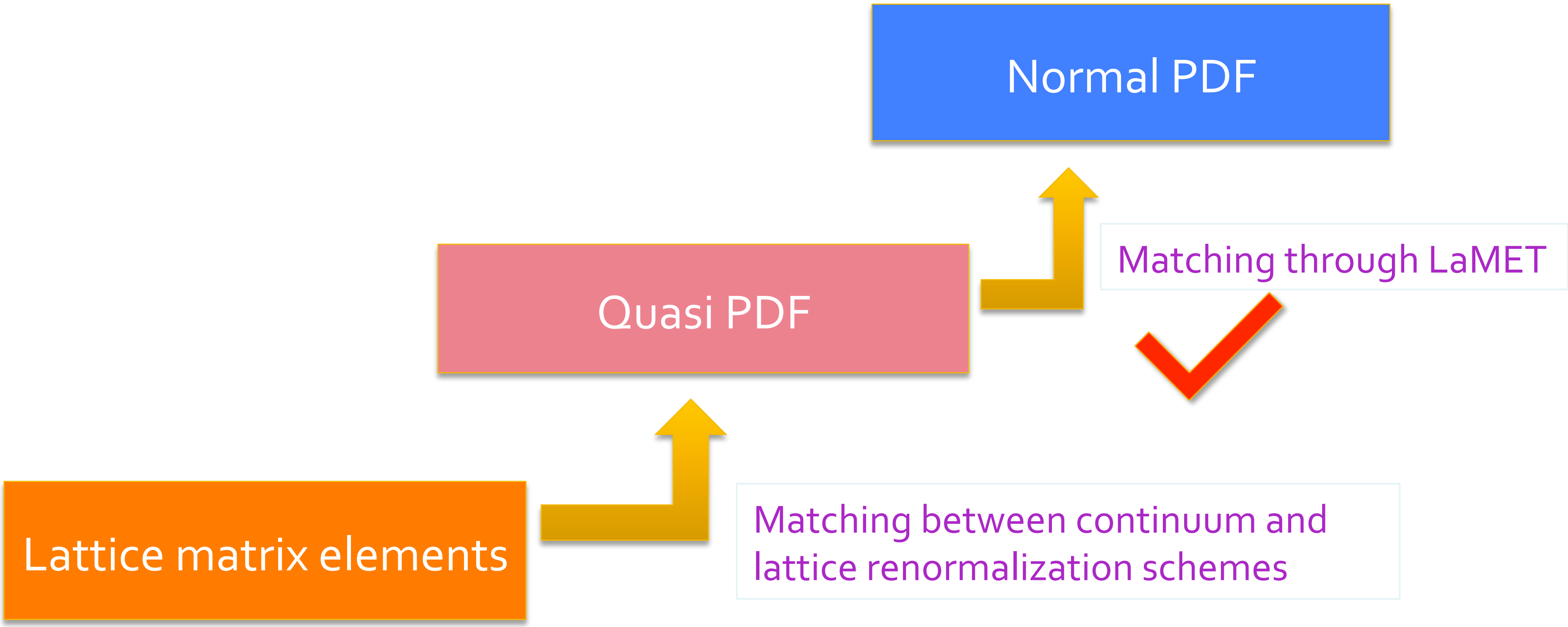
Near $\xi=1$, there is also virtual contribution:

$$Z^{(1)}(\xi) = \delta Z^{(1)}(2\pi/\alpha_s)\delta(\xi - 1) \qquad (18)$$

with

$$\delta Z^{(1)} = \frac{\alpha_S C_F}{2\pi} \int dy \begin{cases} -\frac{1+y^2}{1-y} \ln \frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2 P^z} \ , & y > 1 \ , \\ -\frac{1+y^2}{1-y} \ln \frac{(P^z)^2}{\mu^2} - \frac{1+y^2}{1-y} \ln \left[4y(1-y)\right] + \frac{2y(2y-1)}{1-y} + 1 - \frac{\Lambda}{(1-y)^2 P^z} \ , & 0 < y < 1 \ , \\ -\frac{1+y^2}{1-y} \ln \frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2 P^z} \ , & y < 0 \ , \end{cases}$$

Program of lattice calculation



Renormalizability of quasi PDF

- Quasi PDF is the matrix element of a non-local operator
  - In the renormalization of local operators, all UV divergences can be removed by local counterterms.
  - Renormalization of non-local operator matrix element is rather distinct. Needs non-local counterterms, and to deal with axial singularities.

Renormalizability of quasi PDF

- In the continuum theory, quasi-PDF can be multiplicatively renormalized at two-loop order in dimensional regularization. [Ji and Zhang, PRD, 15']
- Vertex contribution contains sub-divergences only, which can be removed by counterterms from interaction (UV counterterms in the axial gauge).
- Renormalization of quasi PDF reduces to the renormalization of two separate (axial gauge) quark fields, which is equivalent to the renormalization of the heavy-light quark vector current in HQET.
- Such features are expected to hold for all orders, to be checked.

Renormalizability of quasi PDF

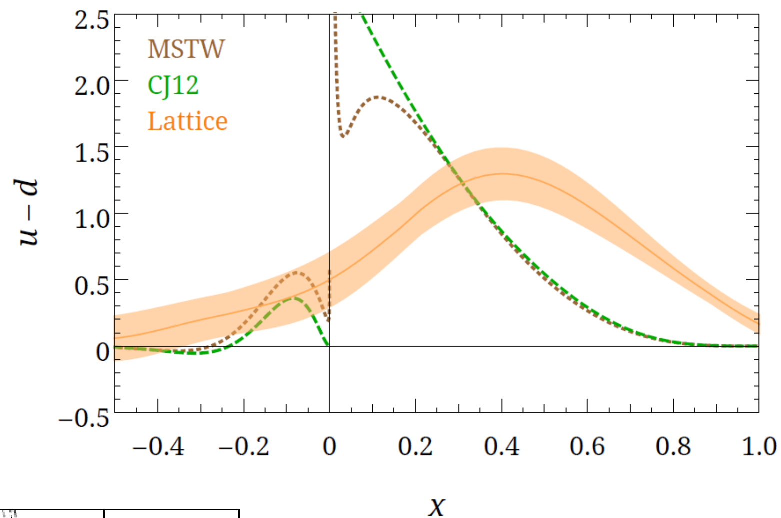
- The renormalization of nonlocal operators in lattice QCD is desirable, but difficult. It is not tested HYP smearing can make the renormalization factor ~1;
- With a proper renormalization scheme in the lattice theory, we will be able to calculate the matching between lattice and continuum theory matrix elements with perturbative QCD.

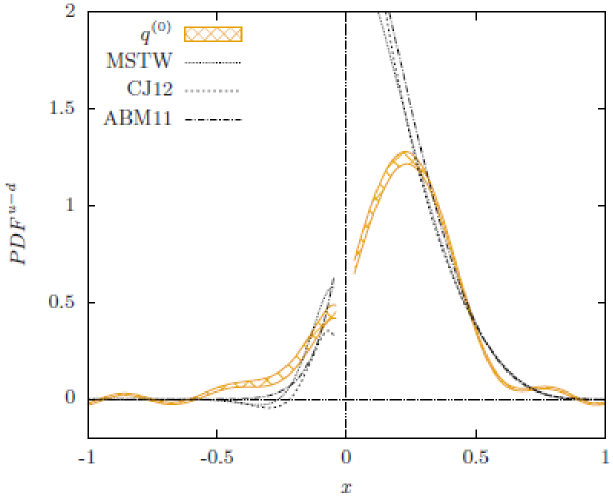
Higher twist corrections

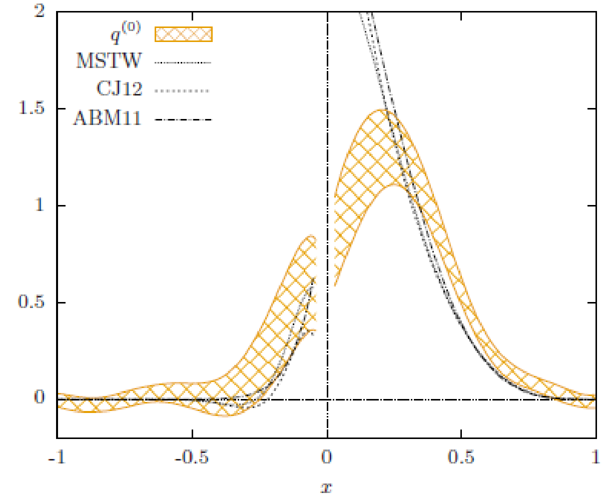
- Similar to target mass corrections from the trace of the nucleon matrix element, which is of order ~$O(M^2/(P^z)^2)$, has been derived (J.W. Chen, in preparation);
- Higher twist correction to the correlation operator itself, which is of order ~$O(\Lambda_{QCD}^2/(P^z)^2)$, has been derived, and its matrix element is to be calculated in lattice QCD (J.W. Chen, in preparation).

Exploratory study of PDFs in lattice QCD

- Isovector (sea) quark distribution calculated with the LaMET approach
- HYP smearing applied to smoothen Wilson line gauge links
- One-loop matching + mass corrections included







Lin et. al. PRD 15'
See H.W. Lin's talk.

Alexandrou et. al.
PRD 15'

Requirements for lattice resources

- Parton with momentum xP^z has correlation length ~$1/xP^z$, so with a boosted nucleon the distribution the valence quark is shrunk by the boost factor γ.

- Needs higher resolution along the z direction, lattice spacing should be $1/\gamma$ to transverse directions, or, with the same box size lattice sites must be γ times that of the transverse directions.

Ji, Sci. China Phys. Mech. Astro., 2014

Small x PDFs

- The smallest x for PDF is ~ Λ_{QCD}/P^z. The correlation length of valence quark does not change under the Lorentz boost, but to achieve extremely small x one needs extremely large P^z.
- For example, for $x \sim 10^{-4}$, $P^z \sim 3$TeV, which means that the lattice sites must be 3000 times of those in the transverse directions.
- This requires a lot of computation resources! Current computation power is good at studying large x PDFs, while small x can be probed in experiments.

Lattice size along the time axis

- Since the energy difference between the lowest excited state and the ground state will be suppressed by a factor of γ, one also requires a long evolution to extract the matrix elements.
- For a typical 24^4 lattice, the ideal choice of lattice size for such calculation will be $24^2 \times (24\gamma)^2$.

Application of the LaMET

- LaMET can be applied to the lattice QCD calculation of all the other parton physics.
  - Light-cone wavefuntions
  - Pion distribution amplitude [Ji, Schäfer, Xiong and Zhang, 15’]
  - Transverse momentum distributions
  - Generalized parton distributions [Ji, Schäfer, Xiong and Zhang, 15']
  - Wigner distributions
  - Fragmentation functions
  - ...

Summary

- LaMET enables us to extract parton observables from the lattice matrix elements of certain time-independent, frame-dependent operators.
- The normal PDF is related to quasi PDF through a perturbative matching condition that has been calculated using LaMET.
- A program for the lattice calculation of PDFs is outlined.

Back up slides

IMF limit of Chen et al.'s sum rule

- For QED,

$$A_\perp^i = A^i - \partial^i \frac{1}{\vec{\partial}^2}\vec{\partial}\cdot\vec{A} \rightarrow A^i - \partial^i \frac{1}{\partial^+} A^+$$

$$\frac{1}{\partial^+} f(\xi'^-) = \frac{1}{2}\int d\xi'^- K(\xi^- - \xi'^-) f(\xi'^-),$$

$$K(\xi^- - \xi'^-) = \mathrm{sgn}(\xi^- - \xi'^-), \text{ or } \pm 2\theta(\xi^- - \xi'^-).$$

- For QCD, the non-Abelian Coulomb condition leads to the IMF limit of A_{phys},

$$A^+_{\text{phys}} = 0, \quad A^+_{\text{pure}} = A^+.$$

Polarized gluon distribution

- Polarized gluon distribution:

$$\Delta g(x) = \frac{i}{2xP^+} \int d\xi^- e^{-ixP^+\xi^-} \langle PS | F_a^{+,i}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}_{b,i}^{+}(0) | PS \rangle$$

Δg(x) can be measured in polarized DIS and proton-proton collisions.

- Total gluon polarization:

$$\begin{aligned}\Delta G &= \int dx\, \Delta g(x) \\ &= \frac{1}{2P^+} \langle PS | \int \frac{dx}{x} \int d\xi^- e^{-ixP^+\xi^-} F_a^{+,i}(\xi^-) L^{ab}(\xi^-, 0) \tilde{F}_{b,i}^{+}(0) | PS \rangle \\ &= -\frac{1}{2P^+} \langle PS | [A^{i,a} - \frac{1}{\partial^+} \partial^i A^{+,b}(\xi'^-) L^{ba}(\xi'^-, \xi^-)] \tilde{F}^+{}_{i,b}(0) | PS \rangle\end{aligned}$$

Back

IMF limit of Chen et al.'s sum rule

Moment of the quark GTMD

Zhao, Liu, Yang, 2015

$$\int dx d^2 k_\perp k_\perp^i f(x, \vec{k}_\perp, \vec{\Delta}_\perp) = \frac{1}{2\bar{P}^+} \left\langle P' S \middle| \bar{\psi}(0) \gamma^+ i \overleftrightarrow{D}^i_{\text{pure}} \psi(0) \middle| PS \right\rangle$$

$$\varepsilon^{ij} \lim_{\Delta \to 0} \frac{\partial}{\partial i \Delta^i} \int dx d^2 k_T \; k_T^j f(x, k_T, \Delta_T) = \frac{1}{2\bar{P}^+} \frac{\left\langle PS \middle| \int d^3\xi \; \bar{\psi}(\xi) \gamma^+ \varepsilon^{ij} \xi^i i \overleftrightarrow{D}^j_{\text{pure}} \psi(\xi) \middle| PS \right\rangle}{(2\pi)^3 \delta^{(3)}(0)},$$

$$\lim_{P^z \to \infty} \left\langle L_q^c \right\rangle = \varepsilon^{ij} \lim_{\Delta \to 0} \frac{\partial}{\partial i \Delta^i} \int dx d^2 k_T \; k_T^j f(x, k_T, \Delta_T)$$

$$= -\int dx d^2 k_T \frac{k_T^2}{M^2} F_{14}^q(x, 0, k_T^2, 0, 0).$$

IMF limit of Chen et al.'s sum rule

Moment of the quark GTMD

- F_{14} is a GTMD whose measurement is unknown right now, but its possibility has been studied (Courtoy et al., 2014);

$$L_q^{\text{can}}(x) = \varepsilon^{ij} \lim_{\Delta \to 0} \frac{\partial}{i\partial\Delta^i} \int \frac{d\xi^-}{2\pi} e^{-ix\bar{P}^+\xi^-} \langle P'S | \bar{\psi}(\xi^-)\gamma^+ i\overleftrightarrow{D}^j_{\text{pure}} \psi(0) | PS \rangle$$

- The canonical quark OAM density is related to a twist-three GPD measureable in hard exclusive processes (Ji et al., 2012; Hatta, 2012).

IMF limit of Chen et al.'s sum rule

- The gluon GTMD

Zhao, Liu, Yang, 2015

$$g(x,\vec{k}_\perp,\vec{\Delta}_\perp) \equiv -\frac{i}{2x\bar{P}^+}\int\frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ix\bar{P}^+ z^-}\langle P'S|F^{+\alpha}(-\frac{z^-}{2},-\frac{z_\perp}{2})\gamma^+$$

$$\times W^-_{-z^-/2,\pm\infty} W^T_{-z_\perp/2,z_\perp/2} W^-_{\pm\infty,z^-/2} F^+{}_\alpha(\frac{z^-}{2},\frac{z_\perp}{2})|PS\rangle$$

- The canonical gluon OAM density is also related to a twist-three GPD (Hatta et al., 2012, 2013).

$$\lim_{P^z\to\infty}\langle L_g^c\rangle = \varepsilon^{ij}\lim_{\Delta\to 0}\frac{\partial}{\partial i\Delta^i}\int dx d^2k_T\ k_T^j g(x,k_T,\Delta_T)$$