Lattice QCD calculation of nucleon charges g_A, g_S and g_T for nEDM and beta decay

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PNDME Collaboration

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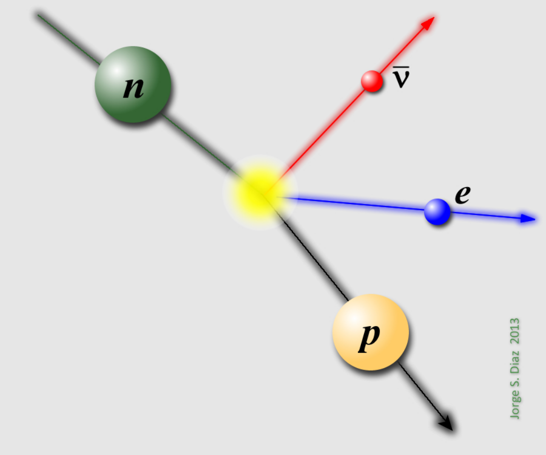
Nucleon Charge g_Γ^q

$$\langle N| \, \bar{q} \, \Gamma \, q \, |N\rangle = g_\Gamma^{q,N} \; \bar{\psi}_N \Gamma \psi_N$$

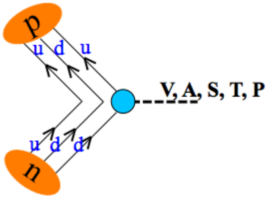
- Can be calculated using lattice QCD
- Plays important role in understanding SM, probing BSM

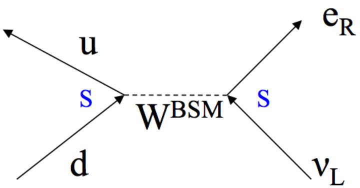
Neutron Decay



- SM : V-A Weak decay (e_L with ν_L)
- BSM : Novel S, T interactions (e_R with ν_L)

Leading BSM Contribution and Nucleon Charges





$$H_{\text{eff}} \supset G_F \left[\varepsilon_S \, \bar{u}d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T \, \bar{u}\sigma_{\mu\nu}d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e\right]$$

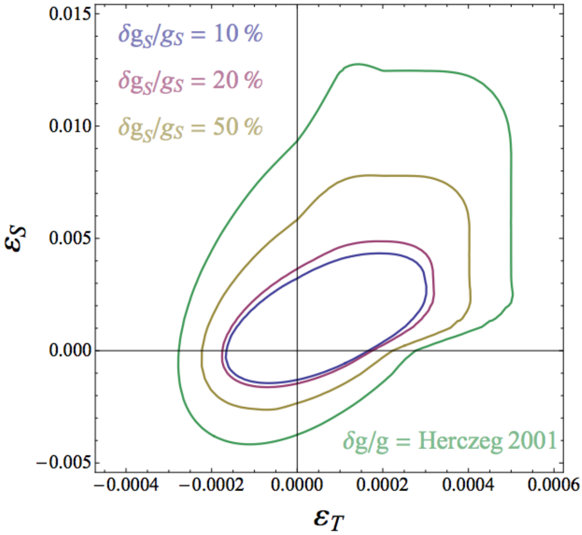
- Low energy hadronic part can be calculated from lattice:

$$g_S^{u-d} \sim \langle p | \, \bar{u}d \, | n \rangle, \qquad g_T^{u-d} \sim \langle p | \, \bar{u}\sigma_{\mu\nu}d \, | n \rangle$$

  Note: $\langle p | \, \bar{u}\Gamma d \, | n \rangle = \langle p | \, \bar{u}\Gamma u \, | p \rangle - \langle p | \, \bar{d}\Gamma d \, | p \rangle \equiv g_\Gamma^{u-d}$ in isospin limit

- Combined with neutron decay experiments, lattice calculation of g_S^{u-d} and g_T^{u-d} can be used to **constrain BSM physics** $(\varepsilon_S, \varepsilon_T)$

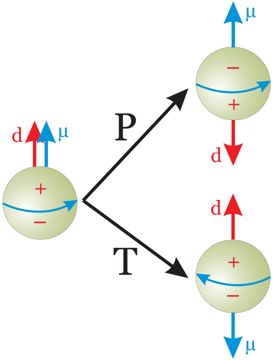
Required Precision of g_S and g_T



Bhattacharya, *et al.*, PRD85 054512 (2012)

- 90% CL contours in $[\varepsilon_S, \varepsilon_T]$ plane
- (Near) future expt. b and B_1 at 10^{-3}
- Impact limited by g_S and g_T
- $\delta g_T = \frac{2}{3}\delta g_S$
- Goal: 10% accuracy in g_S and g_T

Neutron Electric Dipole Moment (nEDM)



- Measure for the distribution of positive and negative charge inside the neutron
- Violates CP
- Current expt. upper limit :
  $|d_N| < 2.9 \times 10^{-26} e \cdot$ cm
  [Baker, *et al.*, PRL, 2006]
- Standard model estimate :
  $|d_N| \sim 10^{-31} e \cdot$ cm
  [Dar, arXiv:hep-ph/0008248]

- CPV in SM is not enough to explain observed baryon asymmetry
- nEDM : good probe of new CP-violating BSM physics

Neutron EDM, Quark EDM and Tensor Charge

- Sources of nEDM in $\mathcal{L}_{\text{eff}}^{\text{CPV}}$:
  θ_{QCD}, quark-EDM, chromo-EDM, Weinberg 3-gluon, ...

- Quark EDMs

$$\mathcal{L} = -\frac{i}{2} \sum_{q=u,d,s} d_q \; \bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$$

- Neutron EDM from qEDMs (assuming qEDM is the major source)

$$d_N = d_u \; g_T^{u,N} + d_d \; g_T^{d,N} + d_s \; g_T^{s,N}$$

- Hadronic part: nucleon tensor charge

$$\langle N| \, \bar{q}\sigma_{\mu\nu}q \, |N\rangle = g_T^{q,N} \; \bar{\psi}_N \sigma_{\mu\nu} \psi_N$$

Neutron EDM, Quark EDM and Tensor Charge

- $d_q \propto m_q$ in many models

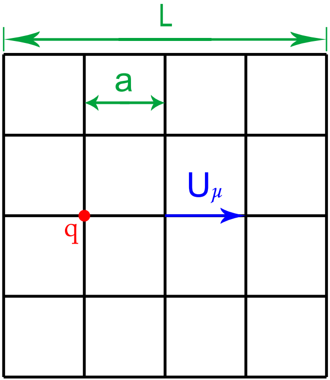
$$d_q = y_q \delta_q; \qquad \frac{y_u}{y_d} \approx \frac{1}{2}, \qquad \frac{y_s}{y_d} \approx 20$$

$$\begin{aligned} d_N &= d_u\ g_T^{u,N} + d_d\ g_T^{d,N} + d_s\ g_T^{s,N} \\ &= d_d \left[g_T^{d,N} + \frac{1}{2}\frac{\delta_u}{\delta_d}\ g_T^{u,N} + 20\frac{\delta_s}{\delta_d}\ g_T^{s,N} \right] \end{aligned}$$

$\Rightarrow$ Precision determination of $g_T^{s,N}$ is important

Lattice QCD

- **Non-perturbative** approach to understand QCD
- Formulated on **discretized Euclidean space-time**
  - Hypercubic lattice
  - Lattice spacing "a"
  - Quark fields placed on sites
  - Gauge fields on the links between sites; U_μ



Systematics for Lattice Calculation

- **Finite Lattice Spacing**
  - Simulations at finite lattice spacings $a \approx 0.06, 0.09$ & 0.12 fm

  $\Rightarrow$ Extrapolate to continuum limit, $a = 0$

- **Heavy Quark Mass**
  - Smaller quark mass $\longrightarrow$ Larger computational cost
  - Simulations at (heavy) pion masses $M_\pi \approx 130, 210$ & 310 MeV

  $\Rightarrow$ Extrapolate to physical pion mass, $M_\pi = M_\pi^{\text{phys}}$

- **Finite Volume**
  - Simulations at finite lattice volume

    $M_\pi L = 3.2 \sim 5.4$ ($L = 2.9 \sim 5.8\,\text{fm}$)

  $\Rightarrow$ Extrapolate to infinite volume, $M_\pi L = \infty$

- **Excited State Contamination**

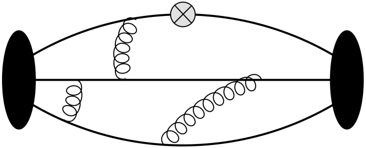
MILC HISQ Lattices, $n_f = 2 + 1 + 1$

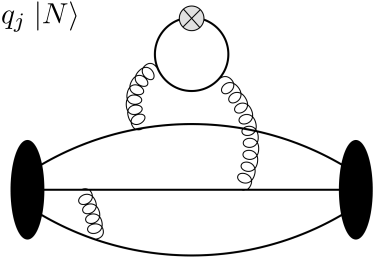
ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$
a12m310	0.1207(11)	305.3(4)	$24^3 \times 64$	4.54
a12m220S	0.1202(12)	218.1(4)	$24^3 \times 64$	3.22
a12m220	0.1184(10)	216.9(2)	$32^3 \times 64$	4.29
a12m220L	0.1189(09)	217.0(2)	$40^3 \times 64$	5.36
a09m310	0.0888(08)	312.7(6)	$32^3 \times 96$	4.50
a09m220	0.0872(07)	220.3(2)	$48^3 \times 96$	4.71
a09m130	0.0871(06)	128.2(1)	$64^3 \times 96$	3.66
a06m310	0.0582(04)	319.3(5)	$48^3 \times 144$	4.51
a06m220	0.0578(04)	229.2(4)	$64^3 \times 144$	4.25

- Fermion discretization : Clover (valence) on HISQ (sea)
- HYP smearing – reduce discretization artifact
- $m_u = m_d$

Three-point Function Diagrams

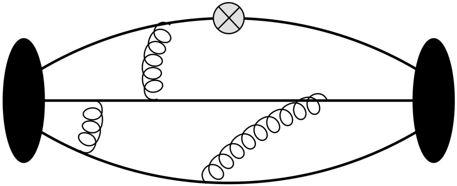
ME $\sim \langle N | \, \overline{q}_i \Gamma q_j \, | N \rangle$





- Quark-line **connected** / **disconnected** diagrams
- Disconnected diagrams : Complicated and expensive on lattice
  Canceled in isovector charges g_Γ^{u-d}
- Neutron decay needs only connected diagrams (g_T^{u-d}, g_S^{u-d}),
  Quark EDM needs both diagrams (g_T^u, g_T^d, g_T^s)

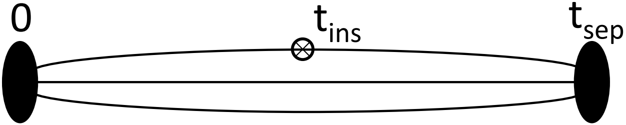
Connected Quark Loop Contribution



Nucleon Charge on Lattice

- Nucleon charges g_Γ^q (Γ = V,A,S,T) defined by

$$\langle N|\ \bar{q}\ \Gamma\ q\ |N\rangle = g_\Gamma^q\ \bar{\psi}_N \Gamma \psi_N$$



- On lattice, g_Γ^q is extracted from ratio of 3-pt and 2-pt function

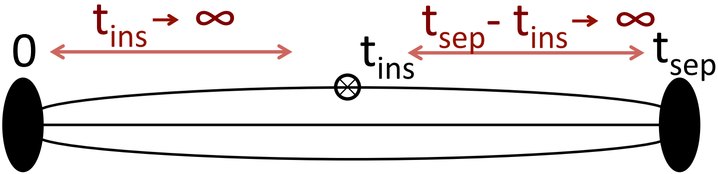
$$C^{\text{3pt}}/C^{\text{2pt}} \longrightarrow g_\Gamma^q$$

  - $C^{\text{2pt}} = \langle 0|\ \chi(t_{\text{s}})\ \overline{\chi}(0)\ |0\rangle, \quad C^{\text{3pt}} = \langle 0|\ \chi(t_{\text{s}})\ \mathcal{O}(t_{\text{i}})\ \overline{\chi}(0)\ |0\rangle$
  - χ : **interpolating operator** of proton

- χ introduces **excited states** of proton

Removing Excited States Contamination

- (1) Gaussian smearing on src/snk, (2) Seperation of src/snk in T

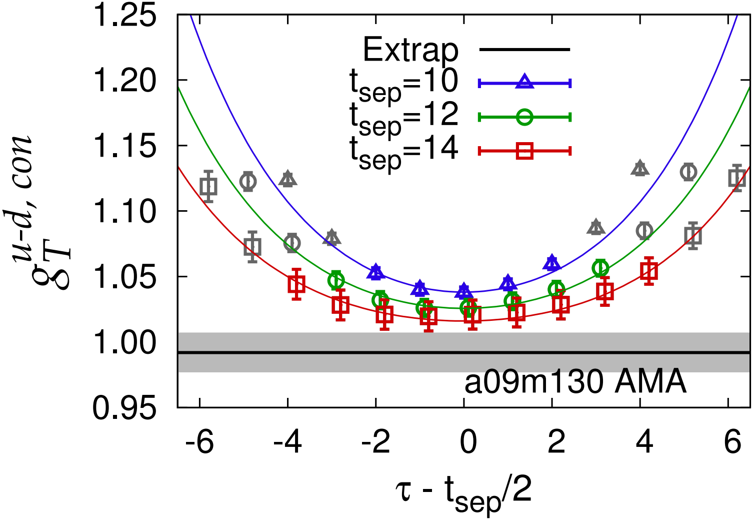


- **Separating** proton sources **far from each other** $\longrightarrow$ small excited state effect, but **weak signal**
- Separating in reasonable range, remove excited state by fitting to

$$C^{\text{2pt}}(t_{\text{sep}}) = A_0^2 e^{-M_0 t_{\text{sep}}} + A_1^2 e^{-M_1 t_{\text{sep}}}$$

$$C^{\text{3pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_0 e^{-M_0 t_{\text{sep}}} + B_1 e^{-M_1 t_{\text{sep}}} + B_{01}\left[e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}} - t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}} - t_{\text{ins}})}\right]$$

Removing Excited States Contamination (a09m130)



All Mode Averaging (AMA) Technique

$$C^{\text{imp}} = \underbrace{\frac{1}{N_{\text{LP}}} \sum_{i=1}^{N_{\text{LP}}} C_{\text{LP}}(\mathbf{x}_i^{\text{LP}})}_{\text{LP estimate}} + \underbrace{\frac{1}{N_{\text{HP}}} \sum_{j=1}^{N_{\text{HP}}} \left[C_{\text{HP}}(\mathbf{x}_j^{\text{HP}}) - C_{\text{LP}}(\mathbf{x}_j^{\text{HP}}) \right]}_{\text{Correction term}}$$

- All-mode averaging (AMA)
  [Blum, Izubuchi and Shintani, PRD 88, 094503 (2013)]
- Exploiting translation symmetry & small fluctuation of low-modes
- "LP" term is cheap low-precision estimate (inverter with $r \sim 10^{-3}$)
- "HP" (high-precision) correction term
  Systematic error $\Rightarrow$ Statistical error
- $N_{\text{LP}} \gg N_{\text{HP}}$ brings computational gain (e.g., N_{LP} = 60, N_{HP} = 4)

MILC HISQ Lattices, $n_f = 2 + 1 + 1$

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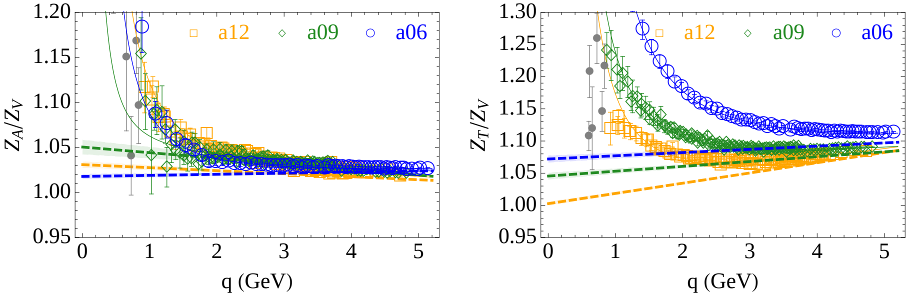
Renormalization of Bilinear Operators $\bar{q}\ \Gamma\ q$

- Lattice results $\Longrightarrow$ $\overline{\text{MS}}$ scheme at 2GeV
- **Non-perturbative** renormalization using RI-sMOM scheme
- Calculate ratio Z_Γ/Z_V : reduce lattice artifact

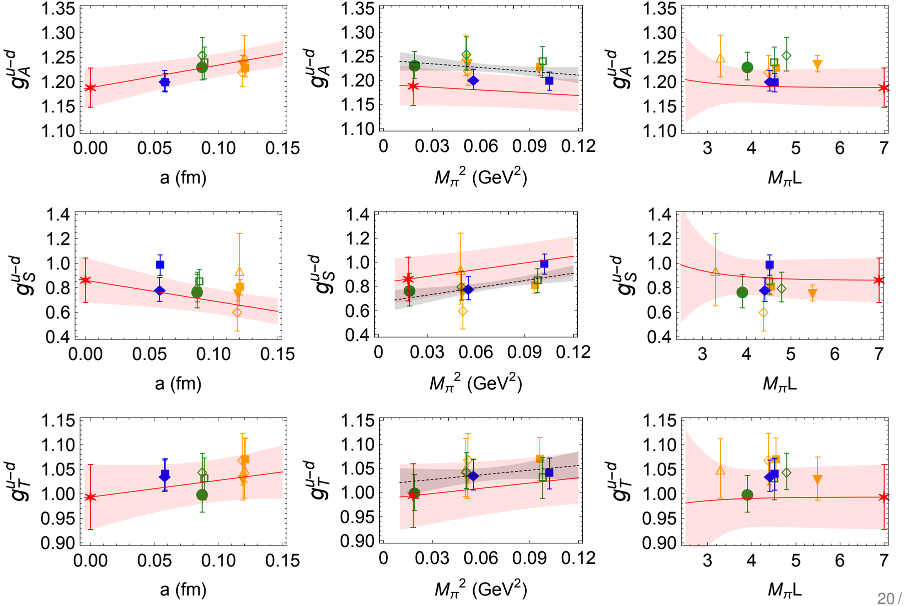
$$g_\Gamma^{\text{renorm}} = \frac{Z_\Gamma}{Z_V} \times \frac{g_\Gamma^{\text{bare}}}{g_V^{\text{bare}}} \qquad \text{(Use } Z_V g_V^{u-d} = 1\text{)}$$



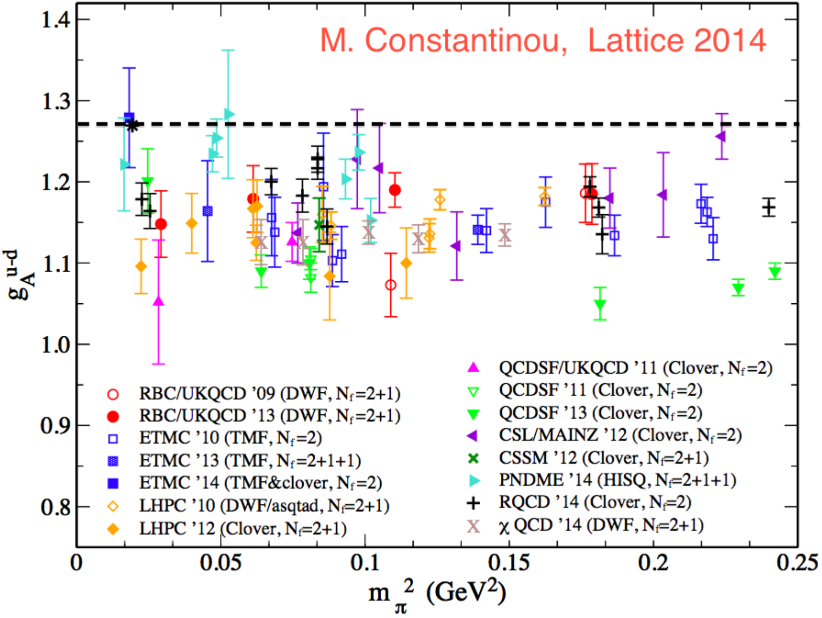
[PRD 89, 094502 (2014)], [arXiv:1506.06411]

Simultaneous Extrapolation of $(a,\ M_\pi,\ M_\pi L)$

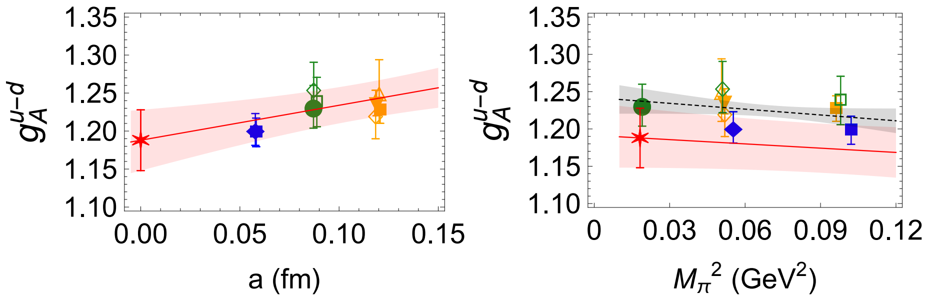
$$g_\Gamma(a, M_\pi, L) = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$



Axial Charge g_A on the Lattice

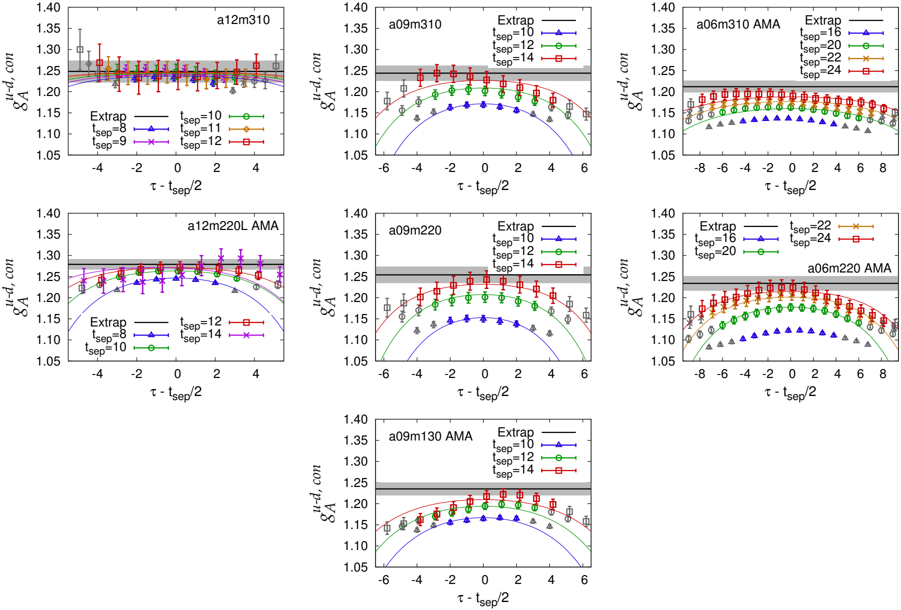


Axial Charge g_A



- Smaller lattice spacing shows smaller value of g_A?
- Excited state contamination!

Excited State Effect of g_A (bare)



Excited State Effect of g_A (bare)

- **Source/Sink gaussian smearing radius**:

  0.66 fm (a12), 0.50 fm (a09) and 0.39 – 0.33 fm (a06)

  – Tuned so that it minimizes computation cost and statistical noise but underestimated excited state effect in smaller a ensembles

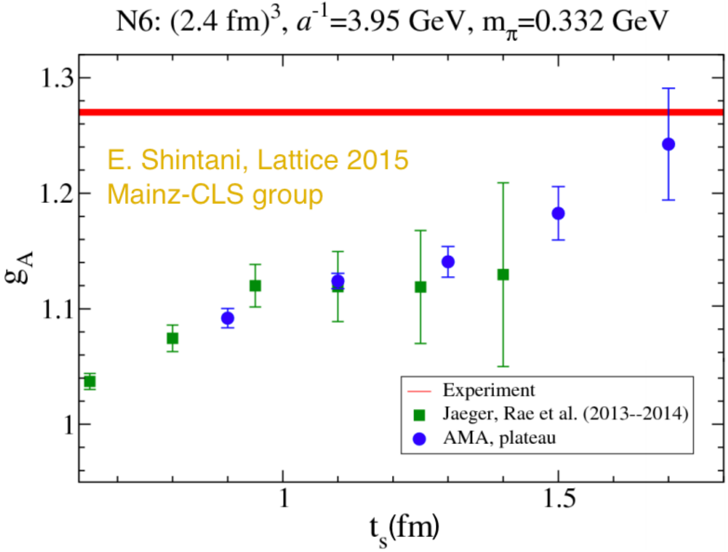
- **Larger excited state effect observed in smaller a ensembles**

$$C^{\text{2pt}}(t_{\text{sep}}) = A_0^2 e^{-M_0 t_{\text{sep}}} + A_1^2 e^{-M_1 t_{\text{sep}}}$$

$A_1^2/A_0^2 \approx$ 0.7 (a12), 1.2 (a09), and 1.7–3.5 (a06)

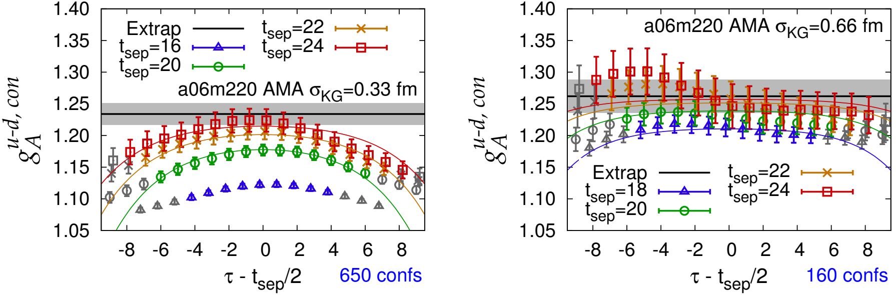
- Excited state contamination makes g_A smaller

Excited State Effect of g_A



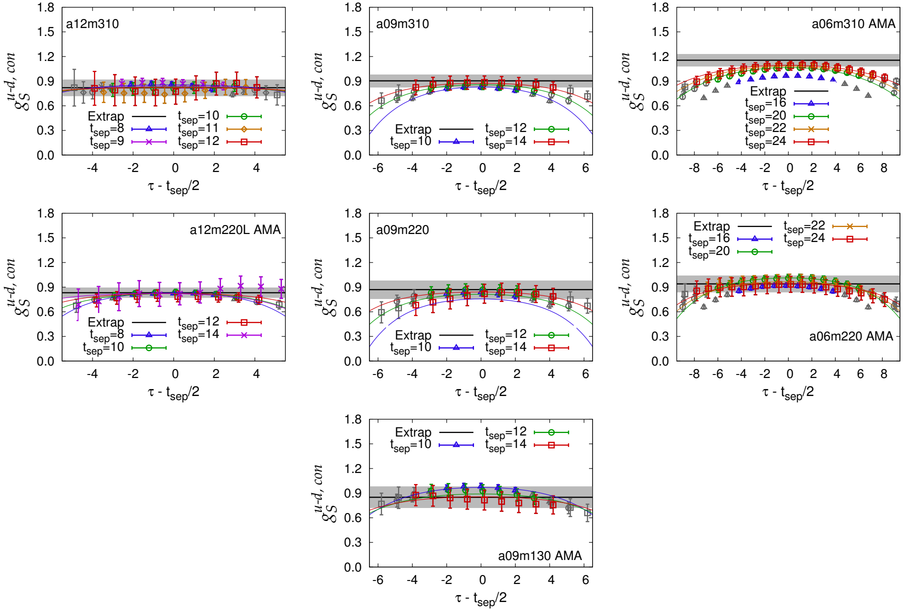
- Fixed source/sink smearing, large src-snk separation

Excited State Effect of g_A with Different Smearing

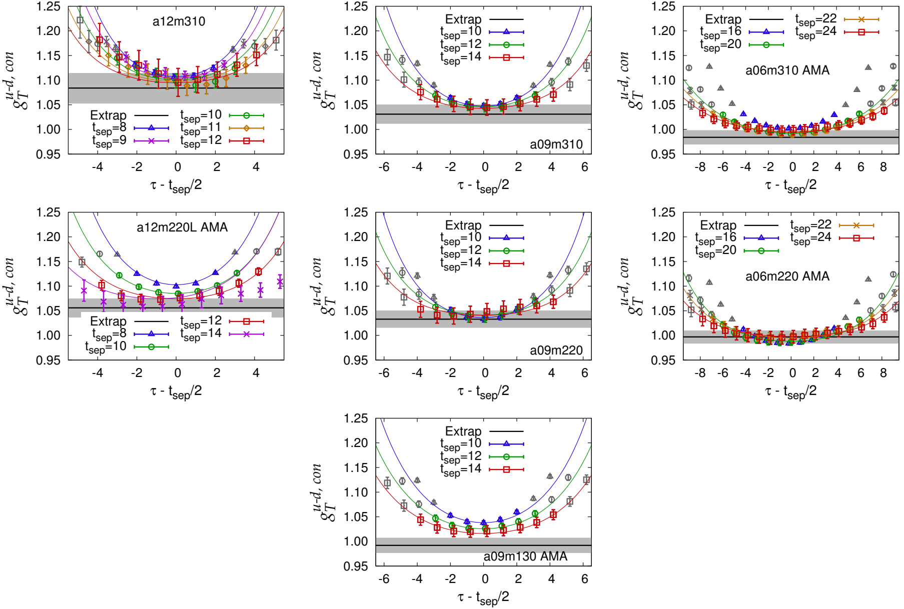


- Aggressive smearing reduces excited state effect, and pushes g_A up closer to experimental estimate
- g_A is under investigation

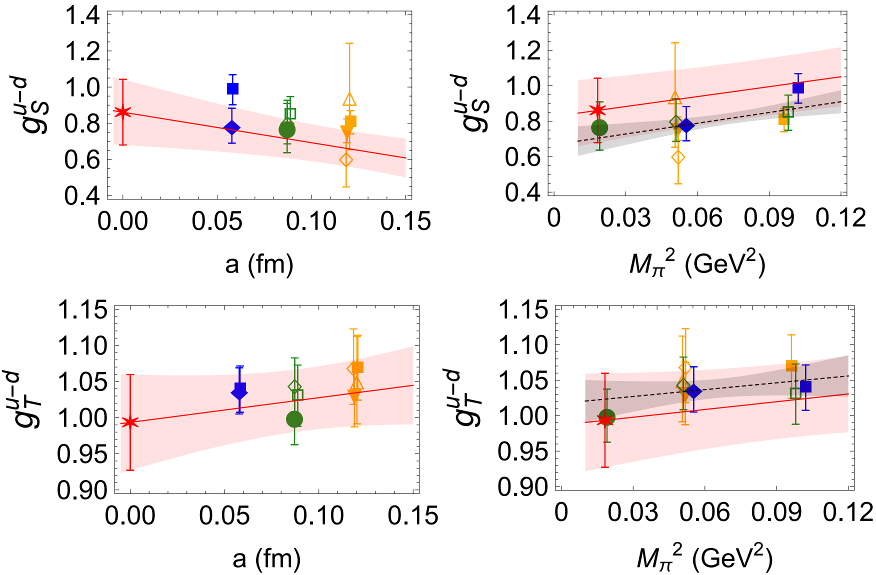
Small Excited State Contamination in g_S and g_T



Small Excited State Contamination in g_S and g_T

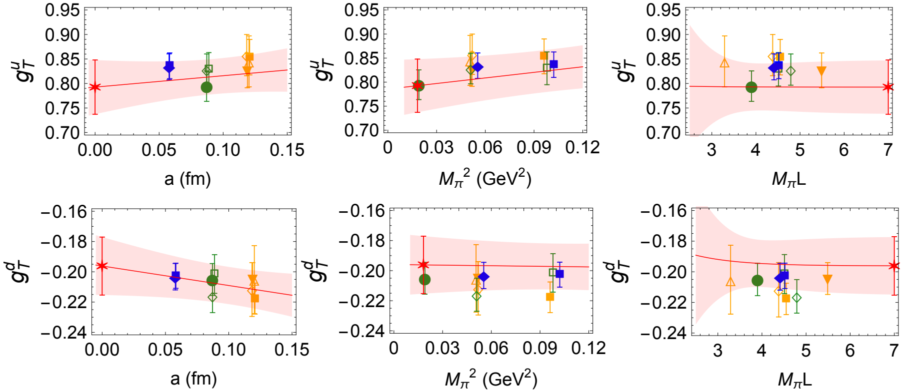


Extrapolation to Physical Limit: g_S^{u-d}, g_T^{u-d}

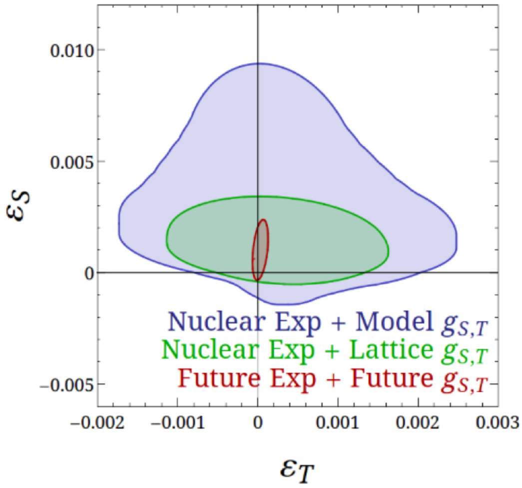


- Volume extrapolation is not displayed, but included

Extrapolation to Physical Limit: g_T^u, g_T^d

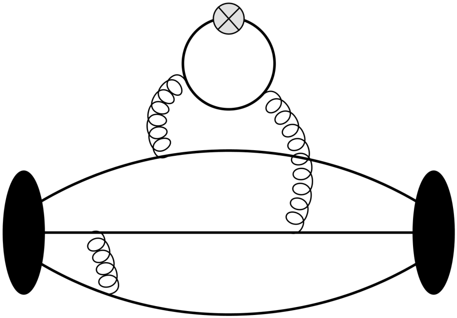


BSM Constraints in β-decay from g_T^{u-d}, g_S^{u-d}



- 90% CL contours in $[\varepsilon_S, \varepsilon_T]$ plane
- Nuclear Experiments : $0^+ \to 0^+$ transitions, Asym in Gamow-Teller β-decay, ...
- Future UCN expt. b and B_1 at 10^{-3}

Disconnected Quark Loop Contribution

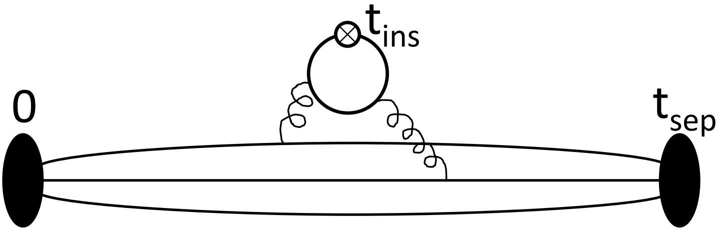


Disconnected Contribution to the Nucleon Charges

Disconnected part of the ratio of 3pt func to 2pt func

$$\left[\frac{C^{\mathsf{3pt}}}{C^{\mathsf{2pt}}}\right]^{\mathsf{disc}} = -\frac{\langle C^{\mathsf{2pt}}(t_{\mathsf{s}}) \sum_{\mathbf{x}} \mathrm{Tr}[M^{-1}(t_{\mathsf{i}},\mathbf{x};t_{\mathsf{i}},\mathbf{x})\ \Gamma\ ]\rangle}{\langle C^{\mathsf{2pt}}(t_{\mathsf{s}})\rangle}$$

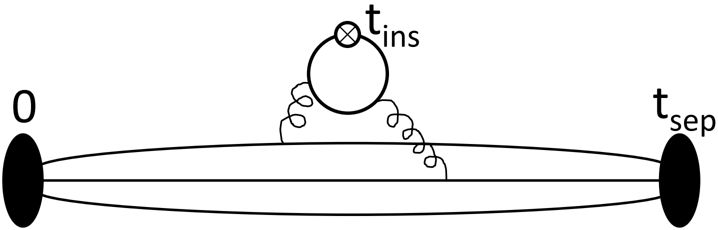
- M : Dirac operator
- $\mathrm{Tr}[M^{-1}(t_{\mathsf{i}},\mathbf{x};t_{\mathsf{i}},\mathbf{x})\ \Gamma]$: disconnected quark loop



Difficulties in Disconnected Diagram Calculation

$$\left[\frac{C^{\text{3pt}}}{C^{\text{2pt}}}\right]^{\text{disc}} = -\frac{\langle C^{\text{2pt}}(t_{\text{s}})\,\sum_{\mathbf{x}}\text{Tr}[M^{-1}(t_{\text{i}},\mathbf{x};t_{\text{i}},\mathbf{x})\sigma_{\mu\nu}]\rangle}{\langle C^{\text{2pt}}(t_{\text{s}})\rangle}$$

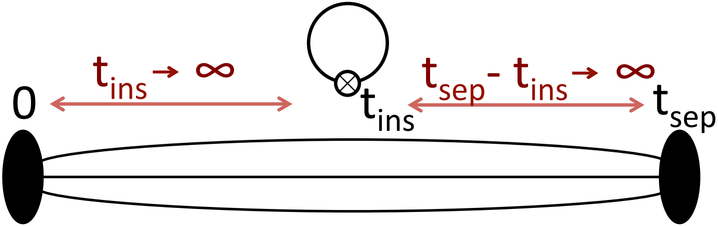
- Connected calculation needs only point–to–all propagators
  Disconnected quark loop needs all–$\mathbf{x}$–to–all propagators
  $\Rightarrow$ Computationally L^3 times more expensive; need new technique
- Noisy signal $\Rightarrow$ Need more statistics



Improvement & Error Reduction Techniques

- Multigrid Solver [Osborn, *et al.*, 2010; Babich, *et al.*, 2010]
- All-Mode Averaging (AMA) for Two-point Correlators [Blum, Izubuchi and Shintani, 2013]
- Hopping Parameter Expansion (HPE) [Thron, *et al.*, 1998; McNeile and Michael , 2001]
- Truncated Solver Method (TSM) [Bali, Collins and Schäfer, 2007]
- Dilution [Bernardson, *et al.*, 1994; Viehoff, *et al.*, 1998]

Removing Excited States Contamination

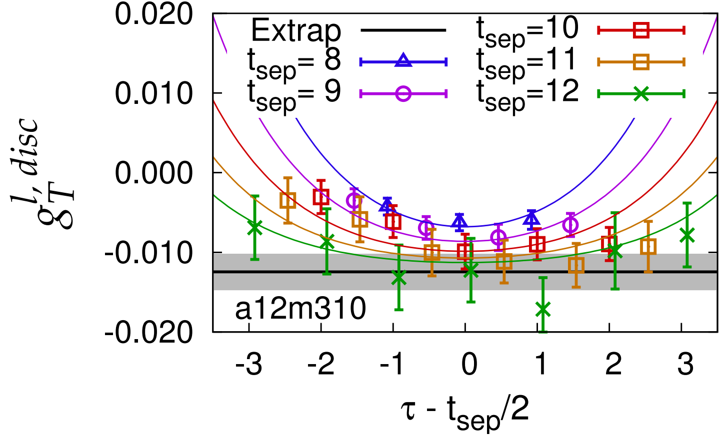


- Interpolating operator introduces excited state contamination
- Remove excited state by fitting to

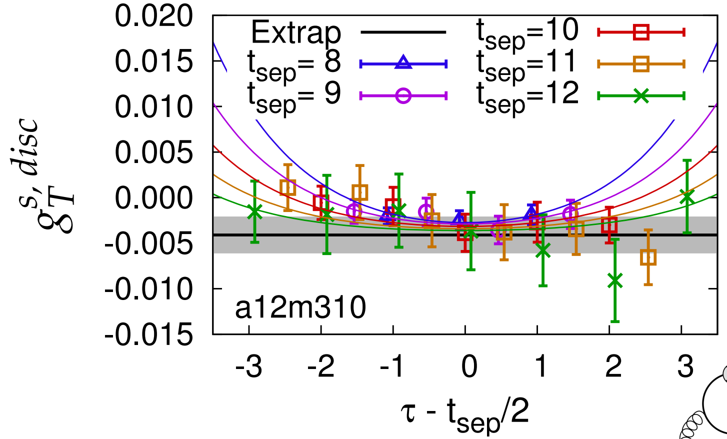
$$C^{\text{2pt}}(t_{\text{sep}}) = A_1 e^{-M_0 t_{\text{sep}}} + A_2 e^{-M_1 t_{\text{sep}}}$$

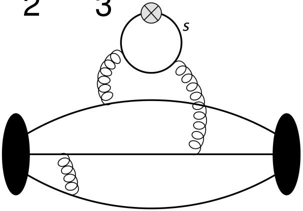
$$C^{\text{3pt}}(t_{\text{sep}}, t_{\text{ins}}) = B_1 e^{-M_0 t_{\text{sep}}} + B_2 e^{-M_1 t_{\text{sep}}} + B_{12}\left[e^{-M_0 t_{\text{ins}}} e^{-M_1 (t_{\text{sep}} - t_{\text{ins}})} + e^{-M_1 t_{\text{ins}}} e^{-M_0 (t_{\text{sep}} - t_{\text{ins}})}\right]$$

Removing Excited States Contamination (a12m310, l)



Removing Excited States Contamination (a12m310, s)





Proton Tensor Charge : Connected / Disconnected

- **Connected Contribution**

$$g_T^u = 0.77(6), \qquad g_T^d = -0.20(2)$$

- **Disconnected Contribution**

Ens	g_T^l	g_T^s
a12m310	$-0.0121(23)$	$-0.0040(19)$
a12m220	$-0.0037(40)$	$-0.0010(27)$
a09m310	$-0.0050(22)$	$-0.0005(21)$
a09m220	—	$-0.0021(54)$
a06m310	$-0.0037(65)$	$-0.0005(55)$

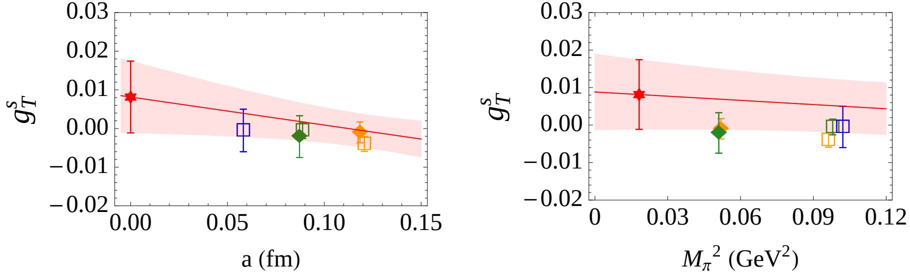
– $g_T^{l,\text{disc}}$ is tiny compared to the connected contributions
$\Rightarrow$ Take maximum value as systematic error

– No connected diagrams for g_T^s $\Rightarrow$ Extrapolate to physical point

– Disconnected contributions are non-trivial in g_S and g_A ($\mathcal{O}(10\%)$)

[arXiv:1506.04196], [arXiv:1506.06411]

Simultaneous extrapolation of g_T^s in (a, M_π)



$$g_T^s = 0.008(9)$$

Results

Lattice Results of Nucleon Tensor Charge

- **Proton Charges** ($\mu^{\overline{\text{MS}}} = 2\,\text{GeV}$)

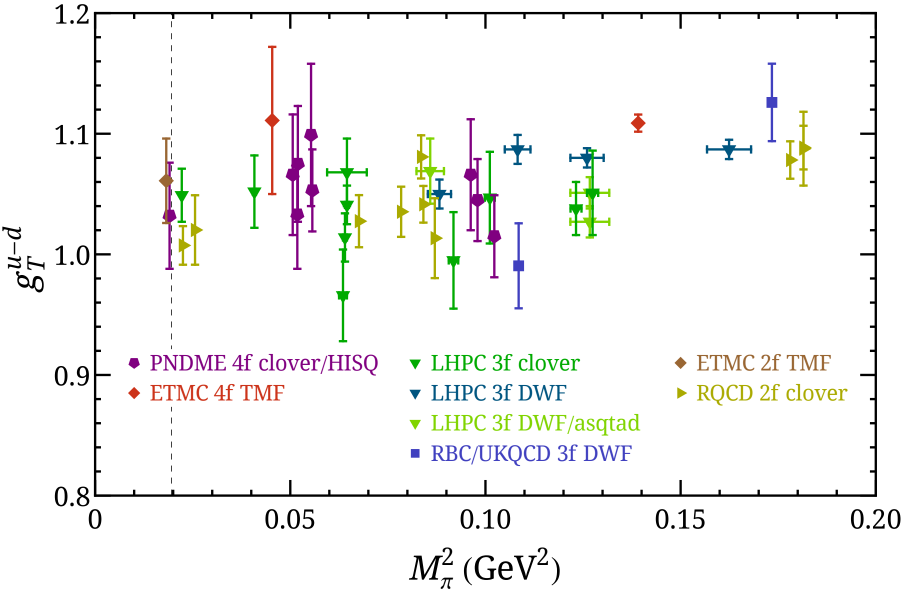
$$
\begin{aligned}
g_S^{u-d} &= 0.86(18) \\
g_T^{u-d} &= 0.99(7)
\end{aligned}
$$

$$
\begin{aligned}
g_T^u &= 0.77(6) \\
g_T^d &= -0.20(2) \\
g_T^s &= -0.008(9)
\end{aligned}
$$

- **Neutron Tensor Charge**

  In isospin limit ($m_u = m_d$), $u \leftrightarrow d$ from proton g_T

Lattice Calculations of Isovector Tensor Charge



Disconnected Contribution to Tensor Charge

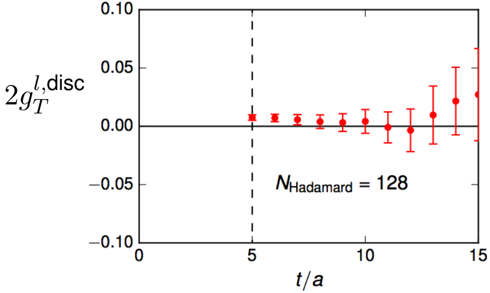
- This study

$$|g_T^{l,\text{disc}}| \leq 0.0121, \qquad g_T^{s,\text{disc}} = 0.008(9)$$

- Lattice, Abdel-Rehim, *et al.*, 2014,
  $a = 0.082\,\text{fm}$, $M_\pi = 370\,\text{MeV}$, Twisted mass

$$g_T^{l,\text{disc}} = 0.0008(7)$$

- Lattice, S. Meinel, *et al.*, 2014,
  $a = 0.11\,\text{fm}$, $M_\pi = 317\,\text{MeV}$, Clover



Tensor Charge

	g_T^d	g_T^u	g_T^s	μ
This study	$-0.20(2)$	$0.77(6)$	$0.008(9)$	2 GeV
Quark model	$-1/3$	$4/3$	–	–
QCD Sum Rules [1]	$-0.35(17)$	$1.4(7)$	–	?
Dyson-Schwinger [2]	$-0.11(2)$	$0.55(8)$	–	2 GeV
Transversity 1 [3]	$-0.18(33)$	$0.57(21)$	–	$\sim$ 1 GeV
Transversity 2 [4]	$-0.25(20)$	$0.39(15)$	–	$\sim$ 1 GeV
Transversity 3 [5]	$-0.22^{+0.14}_{-0.08}$	$0.39^{+0.07}_{-0.11}$	–	3.2 GeV

[1] Pospelov and Ritz, 1999
[2] Pitschmann, *et al.*, 2014
[3] Bacchetta, *et al.*, 2012
[4] Anselmino, *et al.*, 2013
[5] Kang, *et al.*, 2015

qEDM and Tensor Charge

$$d_N = d_u \, g_T^{u,N} + d_d \, g_T^{d,N} + d_s \, g_T^{s,N}$$

- Known parameters

  $|d_N| < 2.9 \times 10^{-26} e$ cm (90% C.L.) [Baker, *et al.*, PRL 2006]

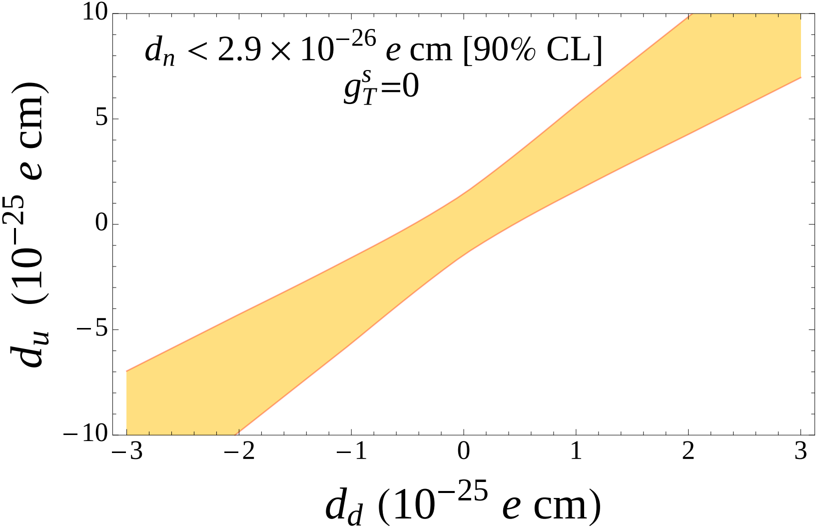
  $g_T^{u,N} = -0.20(2)$

  $g_T^{d,N} = 0.77(6)$

  $g_T^{s,N} = -0.008(9)$

$\Rightarrow$ Place constraints on d_q

qEDM Constraints

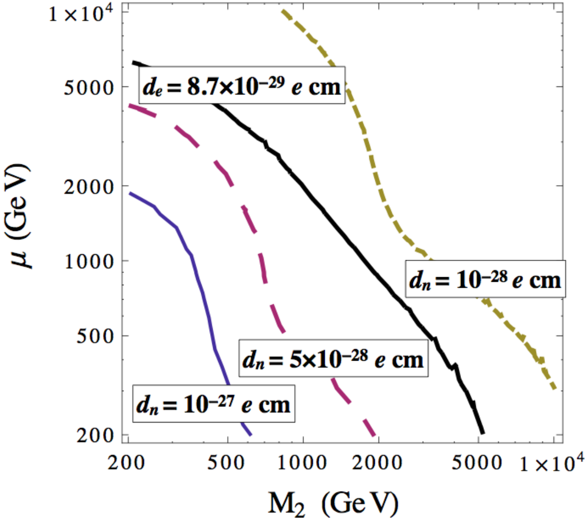


Constraints in Split SUSY Model

Split SUSY: Quark EDM is the dominant BSM source of CPV

[Giudice and Romanino, Nucl.Phys. B699, 65 (2004)]

[Arkani-Hamed and Dimopoulos, JHEP 0506, 073 (2005)]



- Bounds of gaugino (M_2) and Higgsino (μ) mass params placed
- Upper limit of $d_n < 4\times10^{-28}e$ cm in split SUSY placed

[arXiv:1506.04196]

Conclusion

- g_A is under investigation, focusing on the excited state effect
- Isovector g_S and g_T are obtained within 20% and 7% uncertainty
- Presented first lattice QCD calculation of nucleon tensor charge including all systematics (a, M_π, $M_\pi L$, disconnected diagrams)
- Constrained qEDMs and split SUSY model
  by using the lattice results combined with experiment