proton mass decomposition

from Lattice QCD

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Outline

- ***Motivation and scheme* of the proton mass decomposition based on energy-momentum tensor**

  The source of the proton mass, the simulation scheme and setup.

- *The quark mass term* in proton

  πNσ term, strangeness and also the QCD anomaly contribution.

- *The quark/glue energy in proton.*

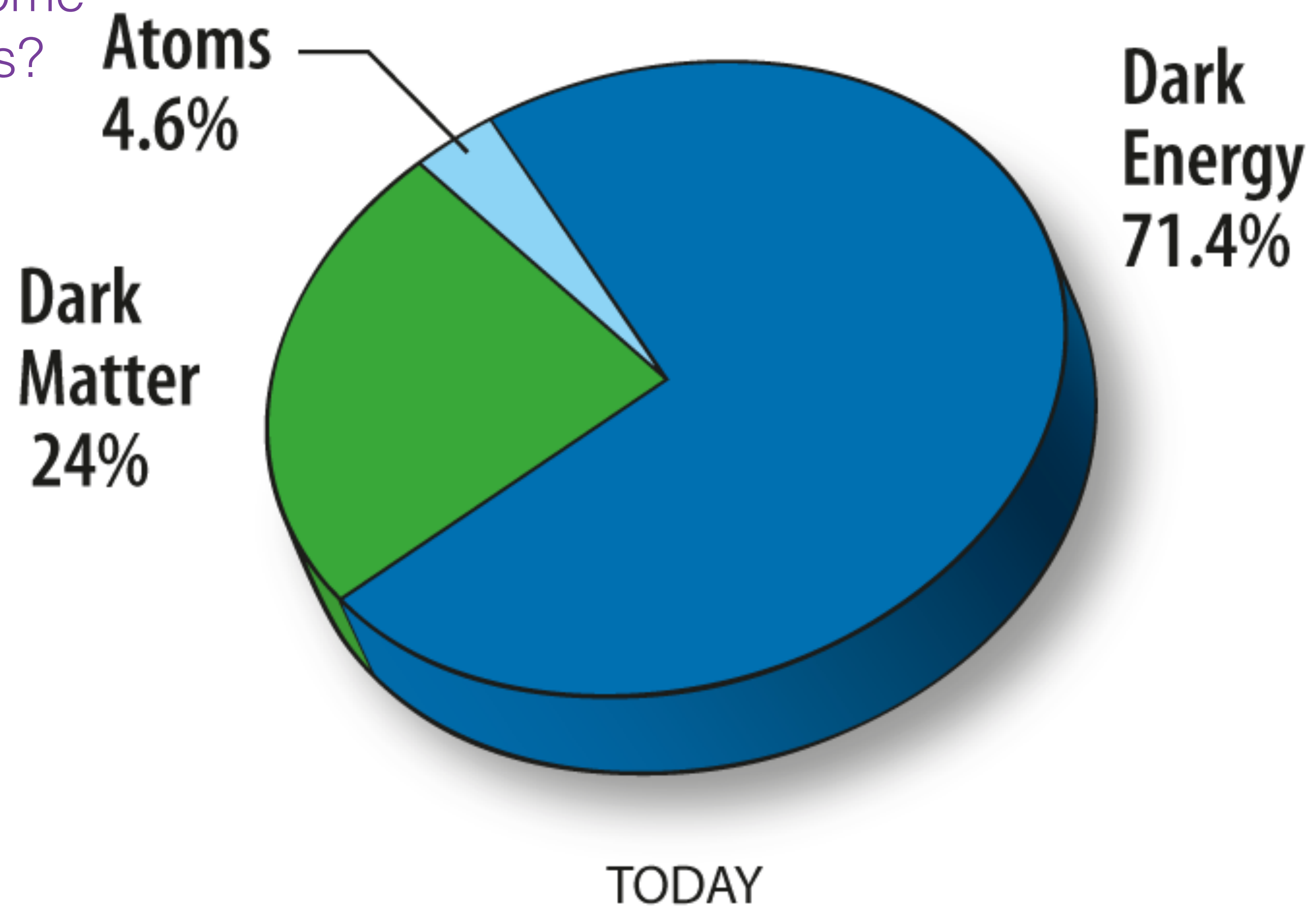
  Based on the direct calculation of the quark/gluon components of the second moment of proton.

- *The simulation results and Open questions needed to be answered by further research*

  The renormalization/mixing issues and the challenge of the simulation on the physical point

Motivation

Where does this observable 4.6% come from, due to Higgs?



Motivation

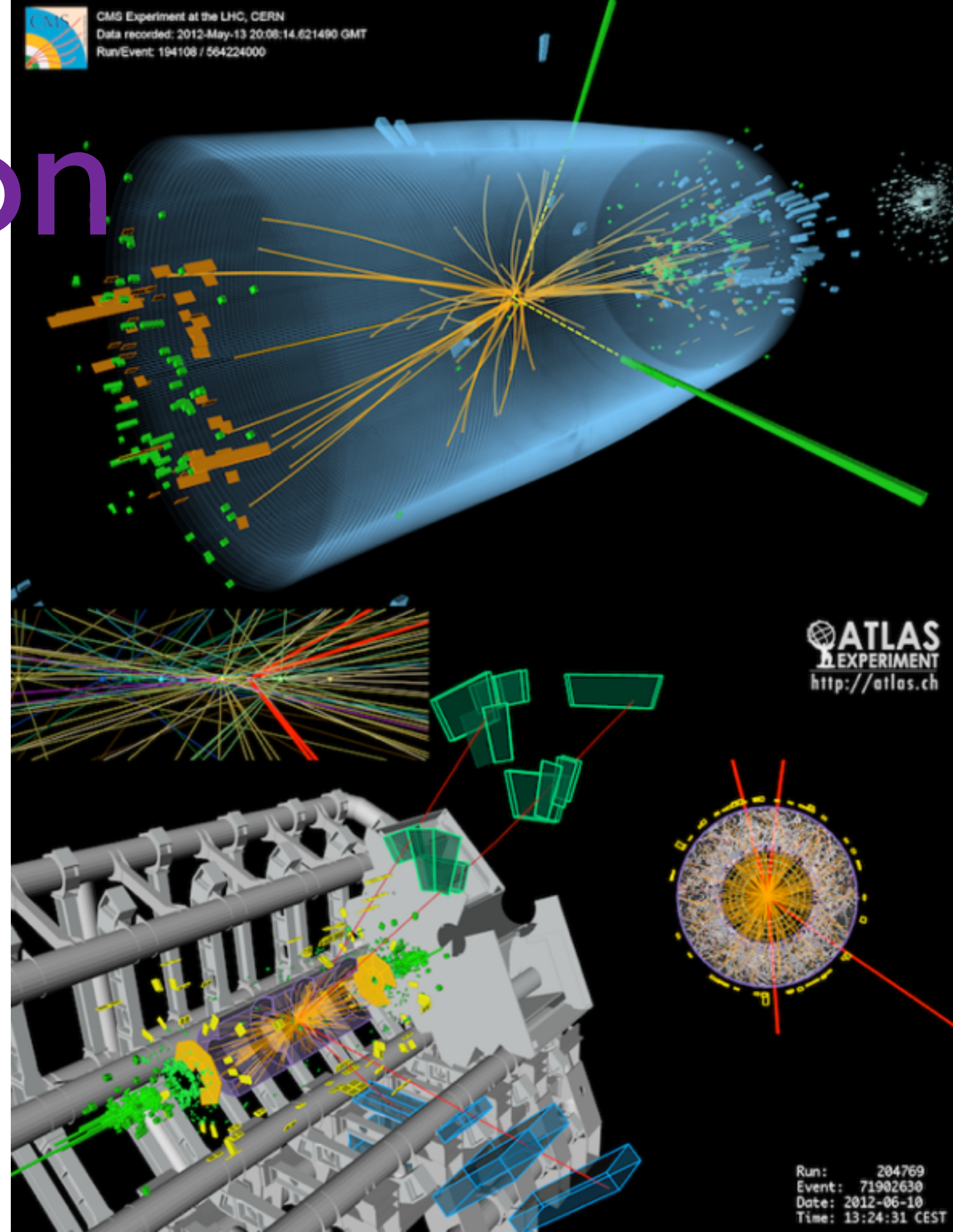
The Higgs boson makes the u/d quark having masses:

$m_u = 2.08(9)$ MeV

$m_d = 4.73(12)$ MeV

But the mass of the proton is *938.272046(21) MeV*. ~100 times of the sum of the quark masses.

Where does the mass of proton come from, and ***how*** ?



Formalism

$$T_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2,$$

The energy momentum tensor
in the classic level

$$\overline{T}_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu}\psi - \frac{1}{16}g_{\mu\nu}\overline{\psi}\gamma_{(\rho}\overleftrightarrow{D}_{\rho)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$

The traceless part of the energy momentum tensor

$$T_{\mu\mu} = -m\bar{\psi}\psi - \gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2$$

The trace part of the energy momentum tensor with equation of motion (EOM) applied, add the quantum trace anomalies.

Xiangdong Ji, PRL 74 (1995), 1071-1074

Xiangdong Ji, PRL 74 (1995), 1071-1074

Formalism

With the relation $$\langle T_{\mu\nu}\rangle \equiv \frac{\langle P|\int d^3x T_{\mu\nu}(x)|P\rangle}{\langle P|P\rangle} = P_\mu P_\nu/M$$

$$\langle \hat{T}_{44}\rangle_{\vec{p}=0} = -1/4M, \qquad \langle \bar{T}_{44}\rangle_{\vec{p}=0} = -3/4M.$$

Even more, the traceless part can be decomposed into its quark/gluon part,

$$\langle \bar{T}^{q,g}_{\mu\nu}\rangle_{\vec{p}=0} = \langle x\rangle_{q,g}(\mu^2)(P_\mu P_\nu - \frac{1}{4}\delta_{\mu\nu}P^2)/M,$$

$$\langle x\rangle_q(\mu^2) + \langle x\rangle_g(\mu^2) = 1.$$

Xiangdong Ji, PRL 74, 1071-1074 (1995)
Y. Yang, et.al. [χQCD], Phys. Rev. D 91, 074516 (2015)

Formalism

Then we have

$$\begin{aligned} M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\ &= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\ \frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle, \end{aligned}$$

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$\begin{aligned} H_a &= H_g^a + H_m^\gamma, \\ H_g^a &= \int d^3x\, \frac{-\beta(g)}{4g}(E^2 + B^2), \\ H_m^\gamma &= \sum_{u,d,s\cdots} \int d^3x\, \frac{1}{4}\gamma_m m\bar{\psi}\psi. \end{aligned}$$

The glue anomaly

The quark mass anomaly

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

Formalism

Then we have

$$
\begin{aligned}
M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\
&= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\
\frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,
\end{aligned}
$$

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\, \overline{\psi}\psi,$$

The quark mass

- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.
- The term where the Higgs boson contributes.
- **Can be calculated directly in the lattice simulation.**

Formalism

Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**

- The total QCD anomaly is renormalization scheme/scale independent.

- Calculate the glue anomaly and the quark mass one separated is complicated.

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x \, \frac{-\beta(g)}{4g}(E^2 + B^2),$$
$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x \, \frac{1}{4}\gamma_m m \bar{\psi}\psi.$$

The quark mass anomaly

Formalism

Then we have

$$\begin{aligned} M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\ &= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\ \frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle, \end{aligned}$$

- The quark energy and quark total energy suffer from the systematic uncertainty of the EOM breaking and possible mixing of the quark mass term, in the bare quantities level.
- In the renormalization, these two quantities will mix between each other, and mix with the quark mass term.
- The total energy term is renormalization scheme/ scale independent.

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

Formalism

Then we have

$$\begin{aligned} M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\ &= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\ \frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle, \end{aligned}$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\begin{aligned} \langle H_E\rangle &= \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle, \\ \langle H_q\rangle &= \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle. \end{aligned} \qquad \langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

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The scalar couplings

in proton

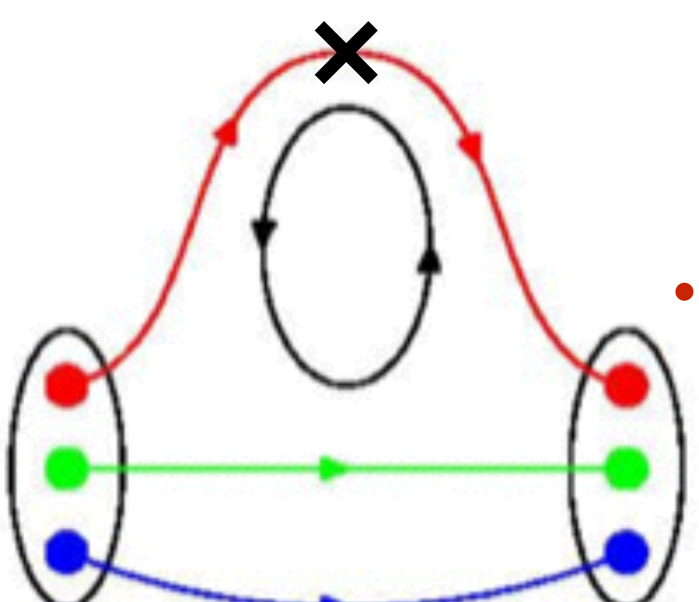
- πNσ term $\sigma_{\pi N} = \hat{m}\langle N|\bar{u}u + \bar{d}d|N\rangle$, as the amount of the light quark contribution in proton.
- Iso-vector coupling $\langle N|\bar{u}u - \bar{d}d|N\rangle$, in constraining posible scalar interactions at the TEV scale.
- Strangeness $f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N}$, also in dark matter searches.

The lattice setup

2 + 1 flavor domain-wall fermion (DWF) configurations from RBC-UKQCD Collaboration.

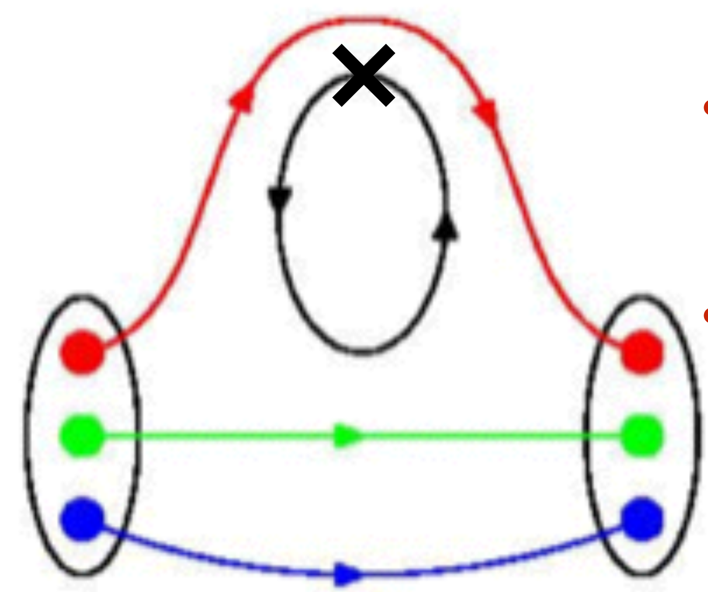
$L^3 \times T$	a (fm)	$m_s^{(s)}$ (MeV)	$m_\pi^{(s)}$ (MeV)	N_{cfg}
$24^3 \times 64$	0.112(3)	120	330	203
$32^3 \times 64$	0.084(2)	110	300	309

Overlap fermion with exact chiral symmetry as the valence quark.



Connected Insertion

- t_f-t_i~1fm



Disconnected Insertion

- Looped over t for better statistics
- Results of all kinds of t_f-t_i can be archived.

The overlap fermion inversion

The overlap operator D_c is defined as

$$D_c = \frac{\rho D_{ov}}{1 - D_{ov}/2}, \; D_{ov}(\rho) = 1 + \gamma_5 \varepsilon(\gamma_5 D_{\rm w}(\rho)),$$

where ε is the matrix sign function and $D_{\rm w}(\rho)$ is the usual Wilson fermion operator with $\rho = 1.5$.

In order to speed up the inversion of D_c, one can deflate the source η with the low lying eigenmodes of D_c,

$$D_c v_i = \lambda_i v_i, \; |\lambda_i| < \epsilon^c$$

where the cutoff ϵ^c as the upper band of the eigenvalue depends on the trade-in of the storage and performance. With the deflation, the propagator of the overlap operator can be rewritten into,

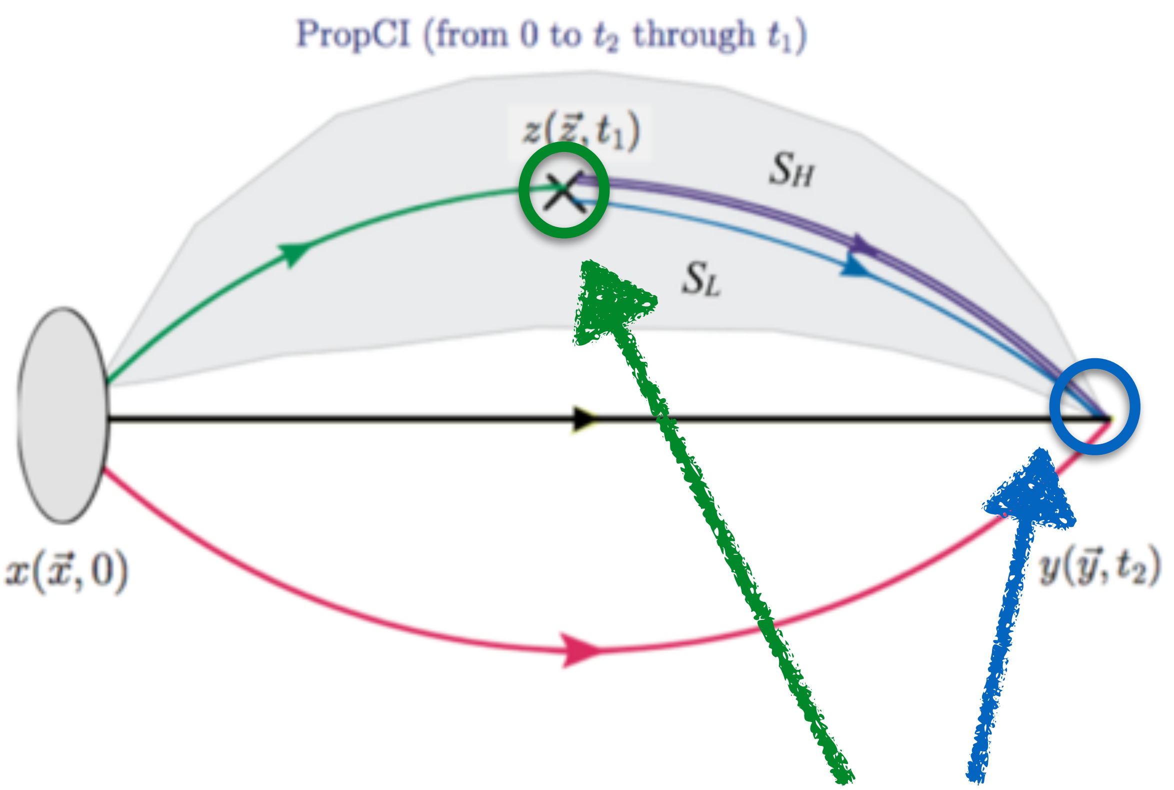
$$S\eta \equiv D_c^{-1}\eta = \sum_i \frac{1}{\lambda_i + m} v_i v_i^\dagger \eta + D_c^{-1}(1 - v_i v_i^\dagger)\eta.$$

Since the first term is enhanced by $\frac{1}{\lambda_i}$ when m is small, that eigensystem $\{\lambda_i, v_i\}$ can be used to construct the long distance part of the all-to-all propagator,

$$S^L = \sum_i \frac{1}{\lambda_i + m} v_i v_i^\dagger.$$

The connected insertion

The sequential method



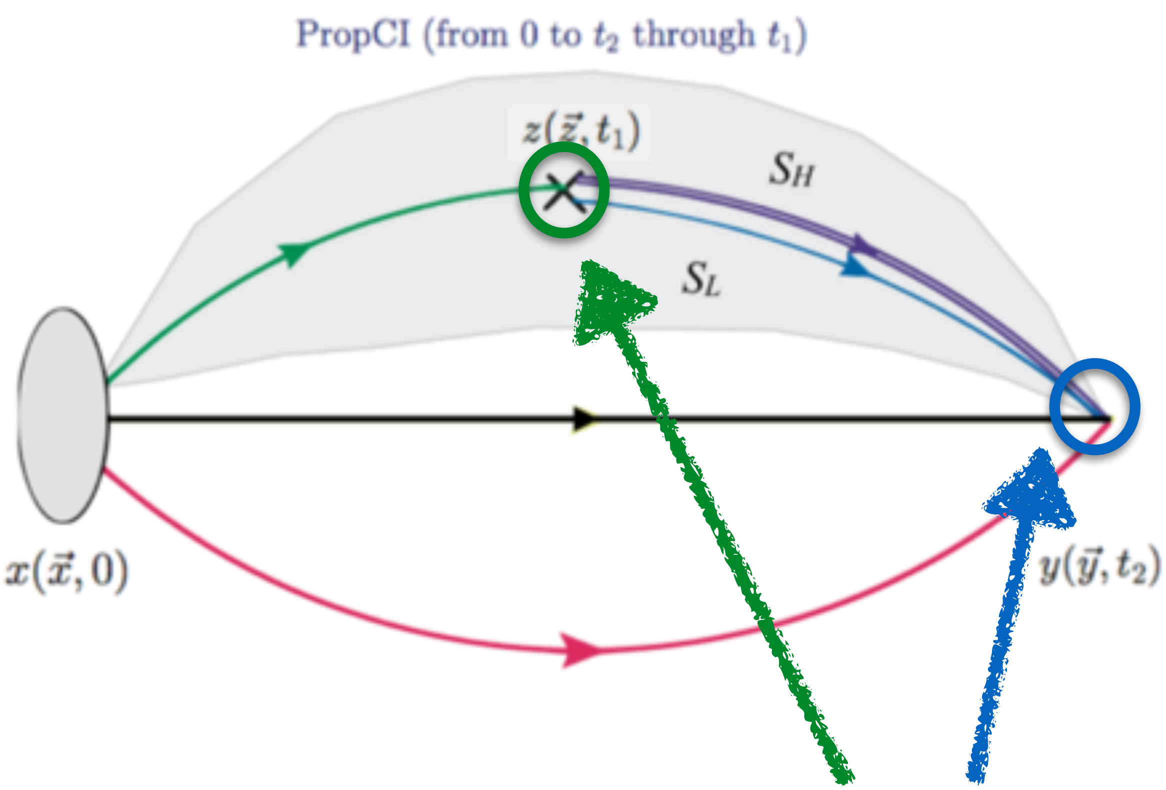
- An all-to-all propagator from z to y is required in the connected insertion calculation.

- The sequential method:
  - Use the product of two propagator at y as a source to construct the propagator from y to z.
  - Need to repeat for u/d, different polarization and momentum.
  - Expensive.
  - Best signal.

T. Draper, Ph. D. thesis, UMI-84-28507 (1984)
C. W. Bernard, T. Draper, G. Hockney, A. M. Rushton and A. Soni, Phys. Rev. Lett. 55, 2770 (1985)

The connected insertion

The stochastic method



- The stochastic estimate:
  - Can be very cheap if just one noise used.
  - Require O(20) noises to reach the sweet point and then still expansive.

$$S(\vec{y}_1, t_2, \vec{z}, t_1) \cong \sum_i \theta^{(i)}_{\vec{y}_1} \gamma_5 (S^{(i)}_{noi}(\vec{z}, t_1, t_2))^\dagger \gamma_5$$

- An all-to-all propagator from z to y is required in the connected insertion calculation.

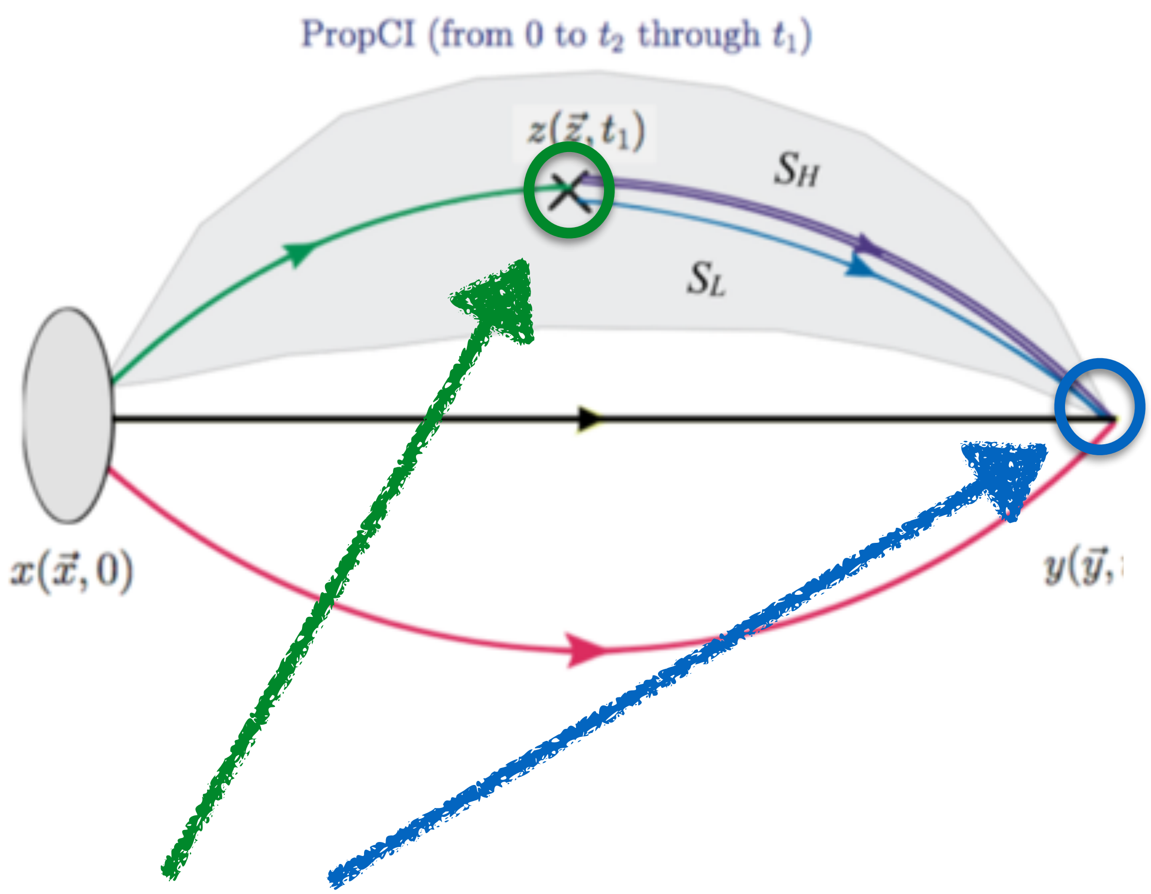
with

$$S^{(i)}_{noi}(\vec{z}, t_1, t_2) = \sum_{\vec{y}_1} S(\vec{z}, t_1, \vec{y}_1, t_2) \theta^{(i)\dagger},$$

R. Evans, G. Bali and S. Collins, Phys. Rev. D 82, 094501 (2010)
G. S. Bali et al., PoS LATTICE 2013, 271
C. Alexandrou et al. [ETM Collaboration], Eur. Phys. J. C 74, no. 1, 2692 (2014)

The connected insertion

The low mode substitution (LMS)



$$S^{LMSs}(\vec{y}_1, t_2, \vec{z}, t_1) = \sum_i \theta^{(i)}_{\vec{y}_1}\gamma_5 (S^{(i),H}_{noi}(\vec{z}, t_1, t_2))^\dagger \gamma_5$$

$$+ \sum_i \frac{1}{\lambda_i + m} v_i(\vec{y}, t_2) v_i^\dagger(\vec{z}, t_1).$$

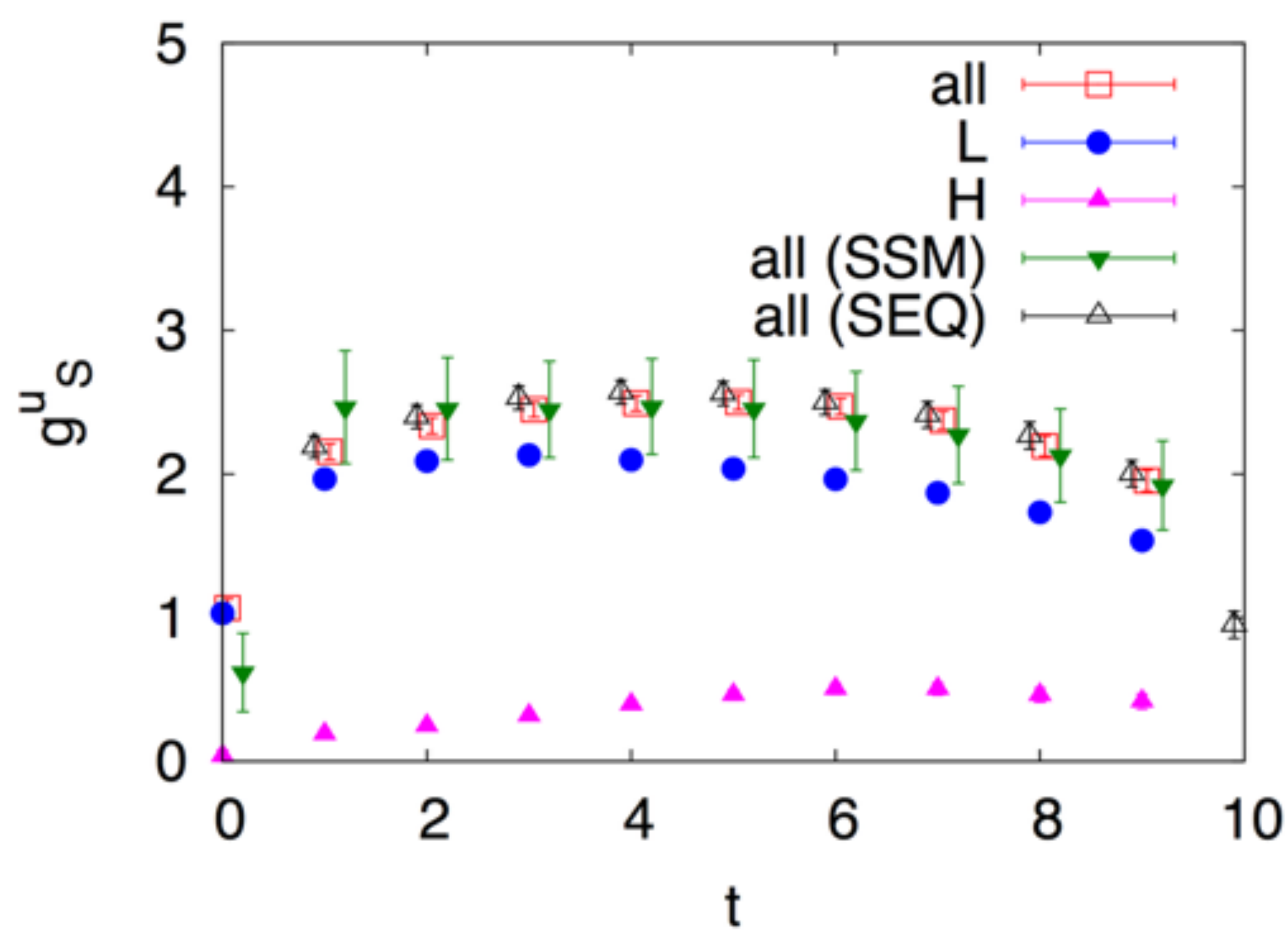
Exact long distance part of the all-to-all propagator from the low lying eigensystem of Dirac operator.

- The stochastic estimate with the low mode substitution:

  Relatively expansive but still cheaper than fully all-to-all.

  Just need O(2) noises in the lattice we used.

Y. Yang, et al. [χQCD], arXiv:1509.04616



The disconnected insertion

The summed method

A standard 3pt/2pt ratio in the forward matrix element case is

$$R(t_2,t_1,0)=\frac{\sum_{i,j} Z_{\mathbf{i}}^{(i)} Z_{\mathbf{f}}^{(i)} e^{-E^{(i)}(t_2-t_1)-E^{(j)}t_1}\langle\chi_{\mathbf{f}}^{(i)}|J|\chi_{\mathbf{i}}^{(j)}\rangle}{\sum_k Z_{\mathbf{i}}^{(k)} Z_{\mathbf{f}}^{(k)} e^{-E^{(k)}t_2}}$$

$$\xrightarrow[t_2\gg 0]{} \langle\chi_{\mathbf{f}}^{(0)}|J|\chi_{\mathbf{i}}^{(0)}\rangle + \frac{Z_f^{(1)}}{Z_f^{(0)}}\langle\chi_{\mathbf{f}}^{(1)}|J|\chi_{\mathbf{i}}^{(0)}\rangle e^{-\Delta E(t_2-t_1)} + \frac{Z_i^{(1)}}{Z_i^{(0)}}\langle\chi_{\mathbf{f}}^{(0)}|J|\chi_{\mathbf{i}}^{(1)}\rangle e^{-\Delta E t_1}$$
$$+\frac{Z_f^{(1)}Z_i^{(1)}}{Z_f^{(0)}Z_i^{(0)}}(\langle\chi_{\mathbf{f}}^{(1)}|J|\chi_{\mathbf{i}}^{(1)}\rangle - \langle\chi_{\mathbf{f}}^{(0)}|J|\chi_{\mathbf{i}}^{(0)}\rangle)e^{-\Delta E t_2}$$

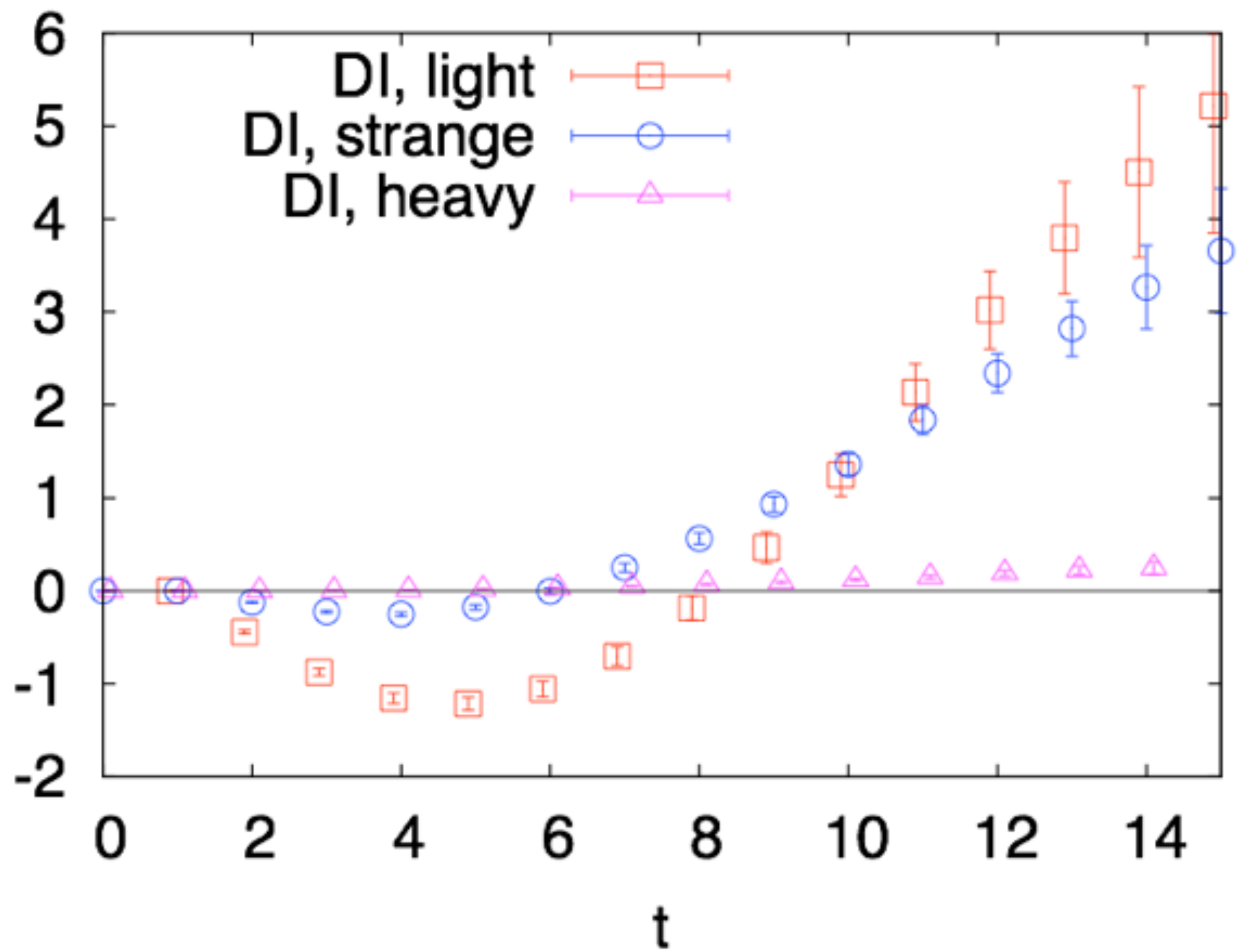
If we sum over different t_1 between t_2 and 0,

$$SR(t_2,t_1,0)=\sum_{0<t_1<t_2} R(t_2,t_1,0)$$
$$=(t_2-1)\langle\chi_{\mathbf{f}}^{(0)}|J|\chi_{\mathbf{i}}^{(0)}\rangle + \frac{e^{-\Delta E}}{1-e^{-\Delta E}}\left(\frac{Z_f^{(1)}}{Z_f^{(0)}}\langle\chi_{\mathbf{f}}^{(1)}|J|\chi_{\mathbf{i}}^{(0)}\rangle + \frac{Z_i^{(1)}}{Z_i^{(0)}}\langle\chi_{\mathbf{f}}^{(0)}|J|\chi_{\mathbf{i}}^{(1)}\rangle\right)$$
$$+O(e^{-\Delta E t_2})+\ldots$$

When t_2 is large, one can fit the summed ratio above as a linear function of t_2, and obtain the slope as the ground state matrix element.

The disconnected insertion

The slope



M. Gong, et al. [χQCD], Phys. Rev. D 88, 014503 (2013)

The quark loop require the calculation of the propagator from all points back to theirselves. Its scholastic estimate with the low mode average is:

$$S^{LMA}(x,x) = \sum_i \theta_x^{(i)} \gamma_5 (S_{noi}^{(i),H}(x))^\dagger \gamma_5 + \sum_i \frac{1}{\lambda_i + m} v_i(x) v_i^\dagger(x)$$

With

$$S_{noi}^{(i)}(y) = \sum_x S(y,x)\theta_x^{(i)\dagger}.$$

- The plot shows the summed ratio. The corresponding matrix element is its slope.
- The low mode contribution dominate the scalar loop.
- The signal to noise ratios are very good for the **light**/**strange** quarks loops.

The chiral extrapolation

$$H_{m,l} = m\frac{\partial}{\partial m}M_N(m) \simeq \frac{1}{2}m_\pi\frac{\partial}{\partial m_\pi}M_N(m_\pi)$$

- The χPT suggest:

$$M_N(m_\pi) = C_0 + C_1 m_\pi^2 + C_2 m_\pi^3, \text{then, } H_{m,l} = C_1 m_\pi^2 + \frac{3}{2}C_2 m_\pi^3.$$

for relatively small m_π.

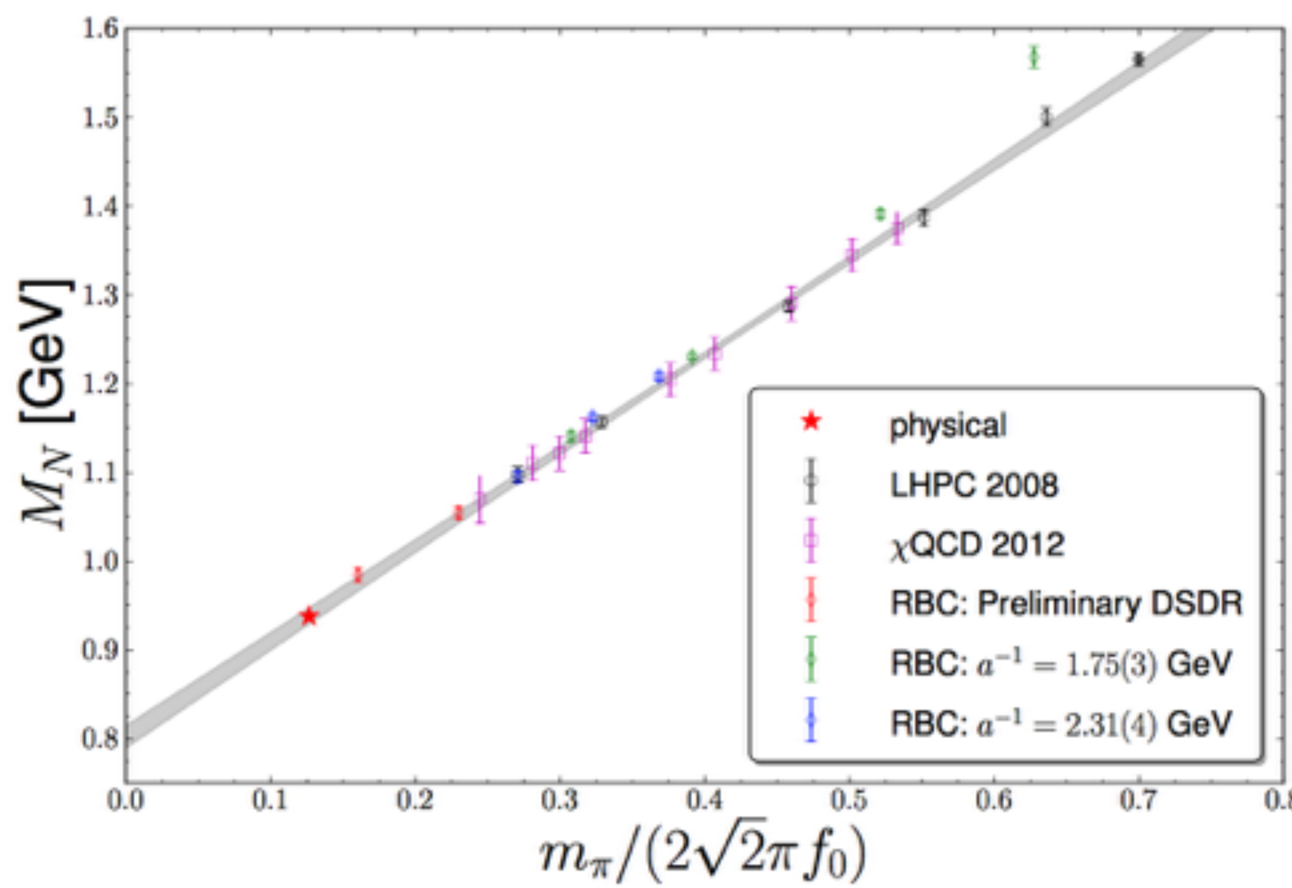
- The lattice simulation observed:

$$M_N(m_\pi) = C_0 + C_1 m_\pi$$

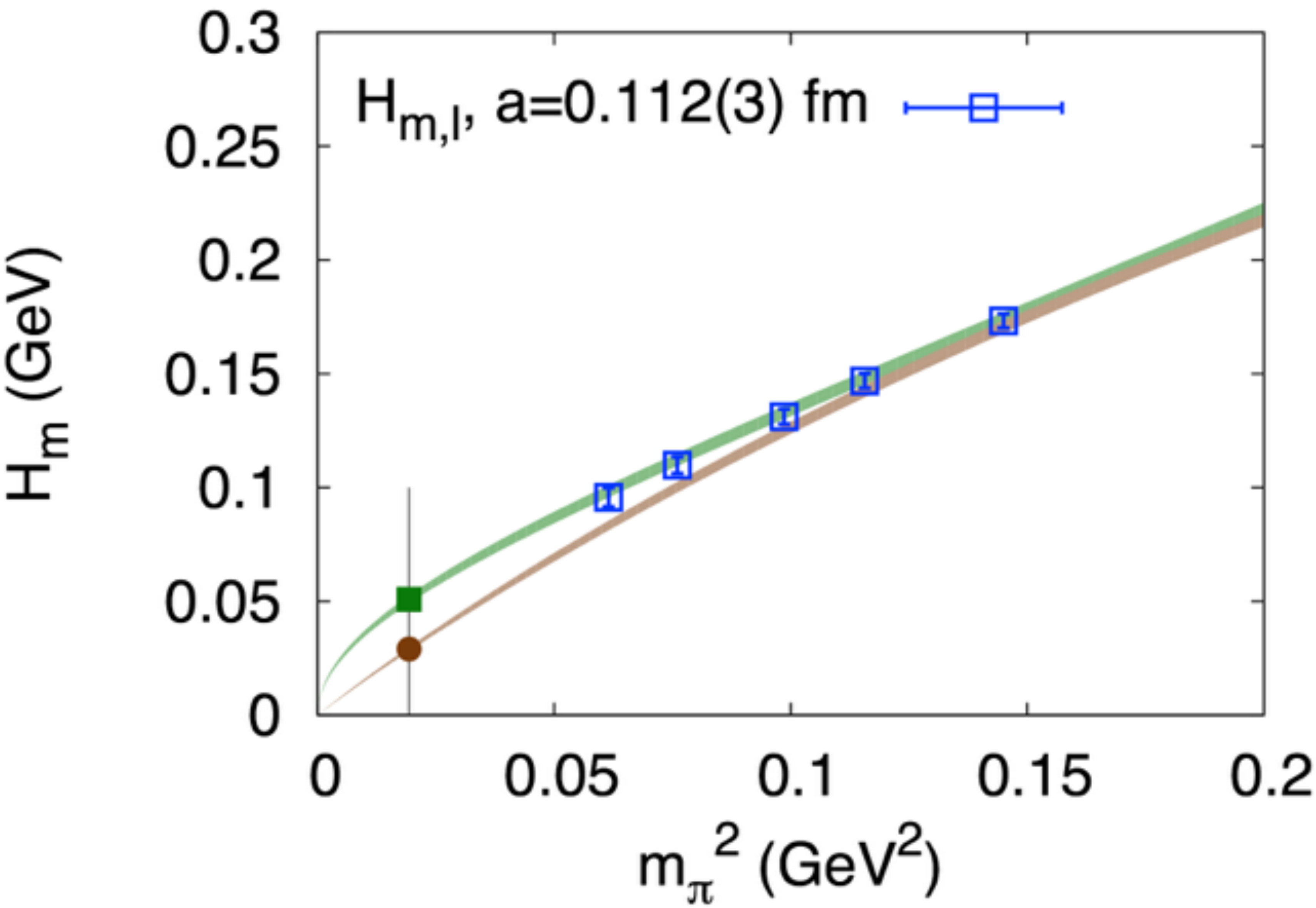
for $m_\pi \lesssim 760$ MeV.

$$H_{m,l} = \frac{1}{2}C_1 m_\pi + O(m_\pi^3).$$

A. Walker-Loud, et al., Phys. Rev. D 79, 054502 (2009)
A. Walker-Loud PoS CD12 (2013) 017



The chiral extrapolation

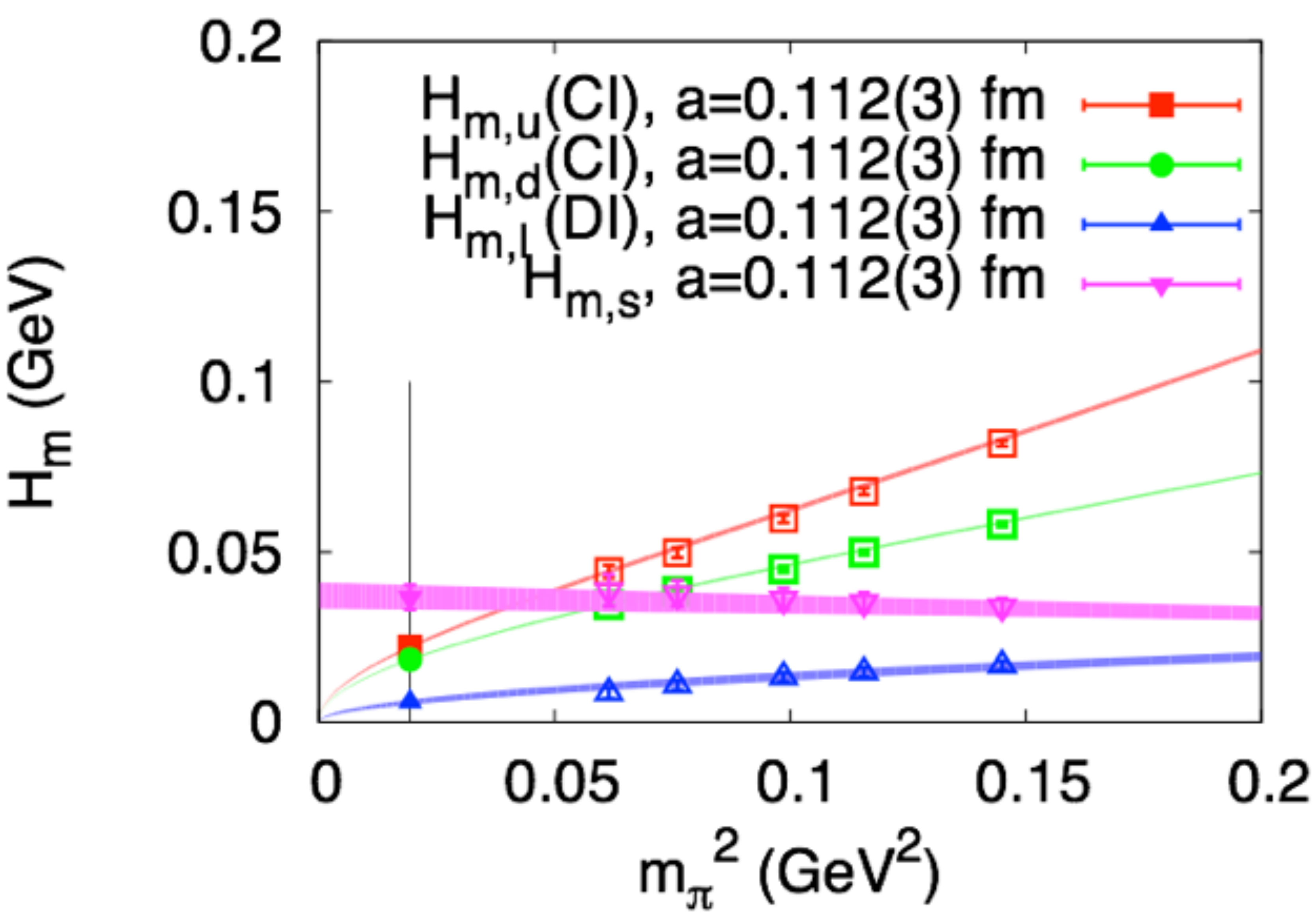


- The data prefers the ruler approximation, compares with the form which the χPT suggest.
- The partially quenching effect would appear here.
- Need the ensemble with lighter sea quark (larger volume) to confirm the exact chiral behavior.

$$\text{green}: \; H_{m,l} = C_0 m_\pi + C_1 m_\pi^3$$

$$\text{red} \; : \; H_{m,l} = C_0 m_\pi^2 + C_1 m_\pi^3$$

The results



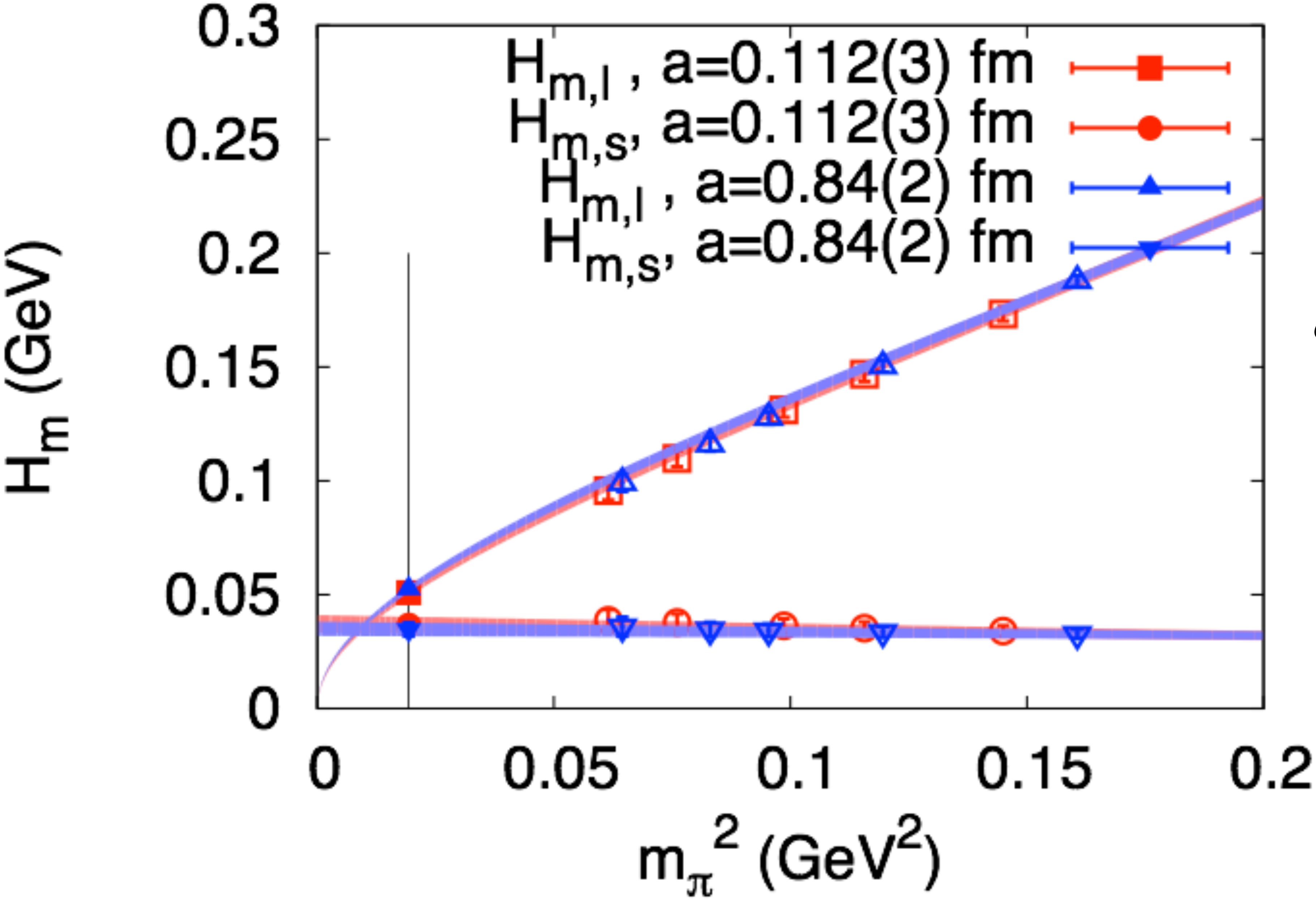
$$H_{m,l}(m_\pi) = C_0 m_\pi + C_1 m_\pi^3,$$
$$H_{m,s}(m_\pi) = C_0 + C_1 m_\pi^2.$$

In the chiral limit of the valance quark:

- The contribution of **u**/**d** are rather close to each other, ~20 MeV;
- That from the disconnected insertion of light quark is small, ~5 MeV for u/d;
- That from the strange quark is ~30 MeV.

Preliminary

The results



The linear extrapolation of a^2:

$$\sigma_{\pi N} = \langle H_{m,l} \rangle = 54(4) MeV,$$
$$\langle H_{m,s} \rangle = 33(7) MeV.$$

$$\langle H_{m,u+d+s} \rangle / M = 9(1)\%.$$

Preliminary

$$H_{m,l}(m_\pi) = C_0 m_\pi + C_1 m_\pi^3,$$
$$H_{m,s}(m_\pi) = C_0 + C_1 m_\pi^2.$$

The QCD anomaly contribution

With the relation $\frac{1}{4}M = -\langle\hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle$

Preliminary

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x \, \frac{-\beta(g)}{4g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x \, \frac{1}{4}\gamma_m m \overline{\psi}\psi.$$

The quark mass anomaly

- **The joint contribution of the QCD anomaly is 23(1)%.**
- The quark mass anomaly contribution is highly suppressed as ~1%.
- Most of the contribution comes from the glue anomaly.

Quick summary

- Benefitted from LMS, we can obtain πNσ term as 54(4) MeV, Strange contribution as 33(7) MeV.

- The combined contribution in the chiral limit of the valence quark and the linear extrapolation of a^2 is 9(1) %.

- The joint contribution of the QCD anomaly is 23(1)%.

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The quark/glue energy

from the momentum fractions

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

The quark/glue energy can be deduced from the momentum fraction and the quark mass term,

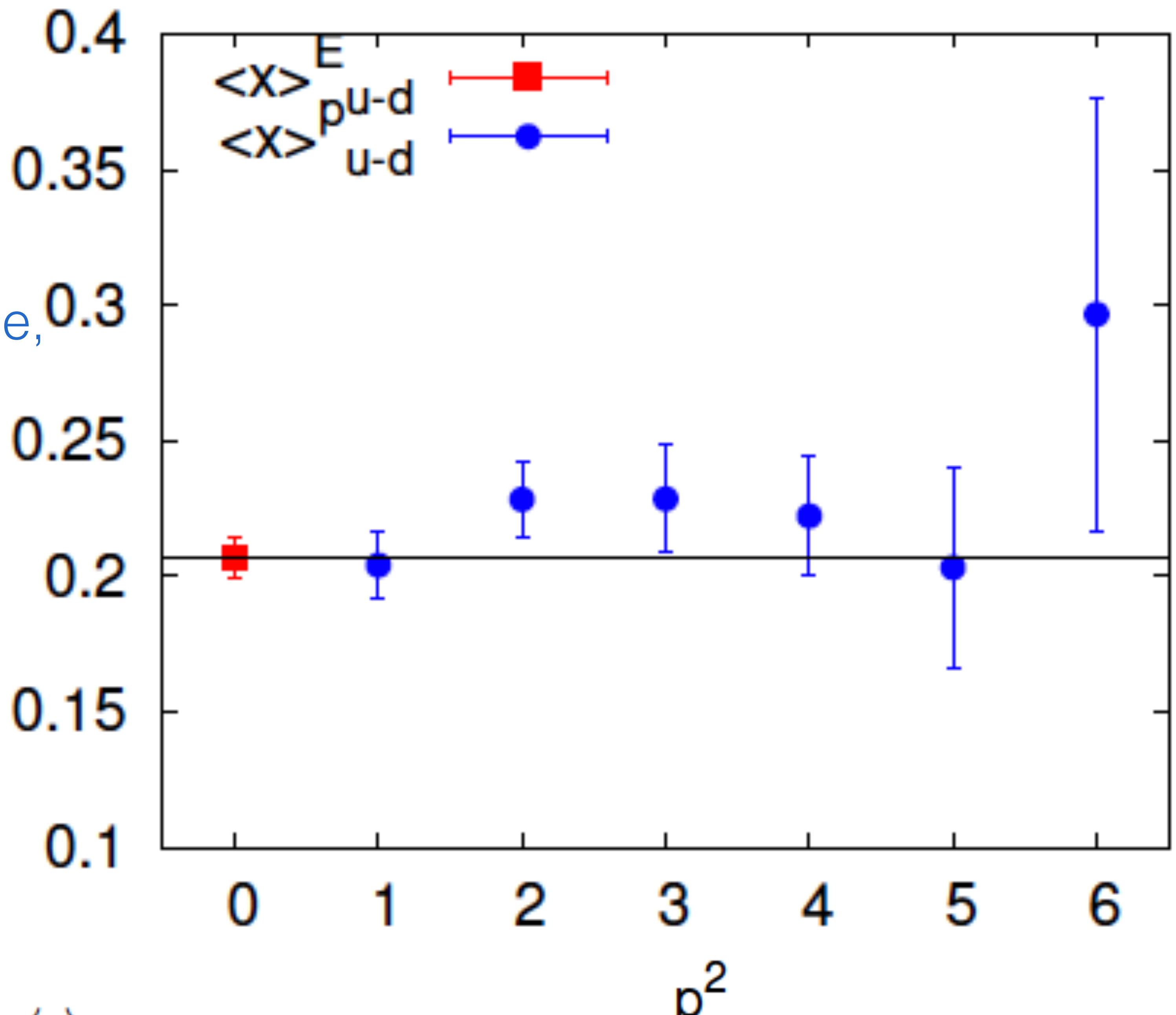
$$\langle H_E \rangle = \frac{3}{4}\langle x \rangle_q M - \frac{3}{4}\langle H_m \rangle,$$

$$\langle H_q \rangle = \frac{3}{4}\langle x \rangle_q M + \frac{1}{4}\langle H_m \rangle. \quad \langle H_g \rangle = \frac{3}{4}\langle x \rangle_g M.$$

The **quark momentum fraction**

obtained from the diagonal/off-diagonal ME

The result obtained with diagonal matrix element in the rest frame is consistent with that with the off-diagonal one in the moving frame, while the former one has better signal to noise ratio.

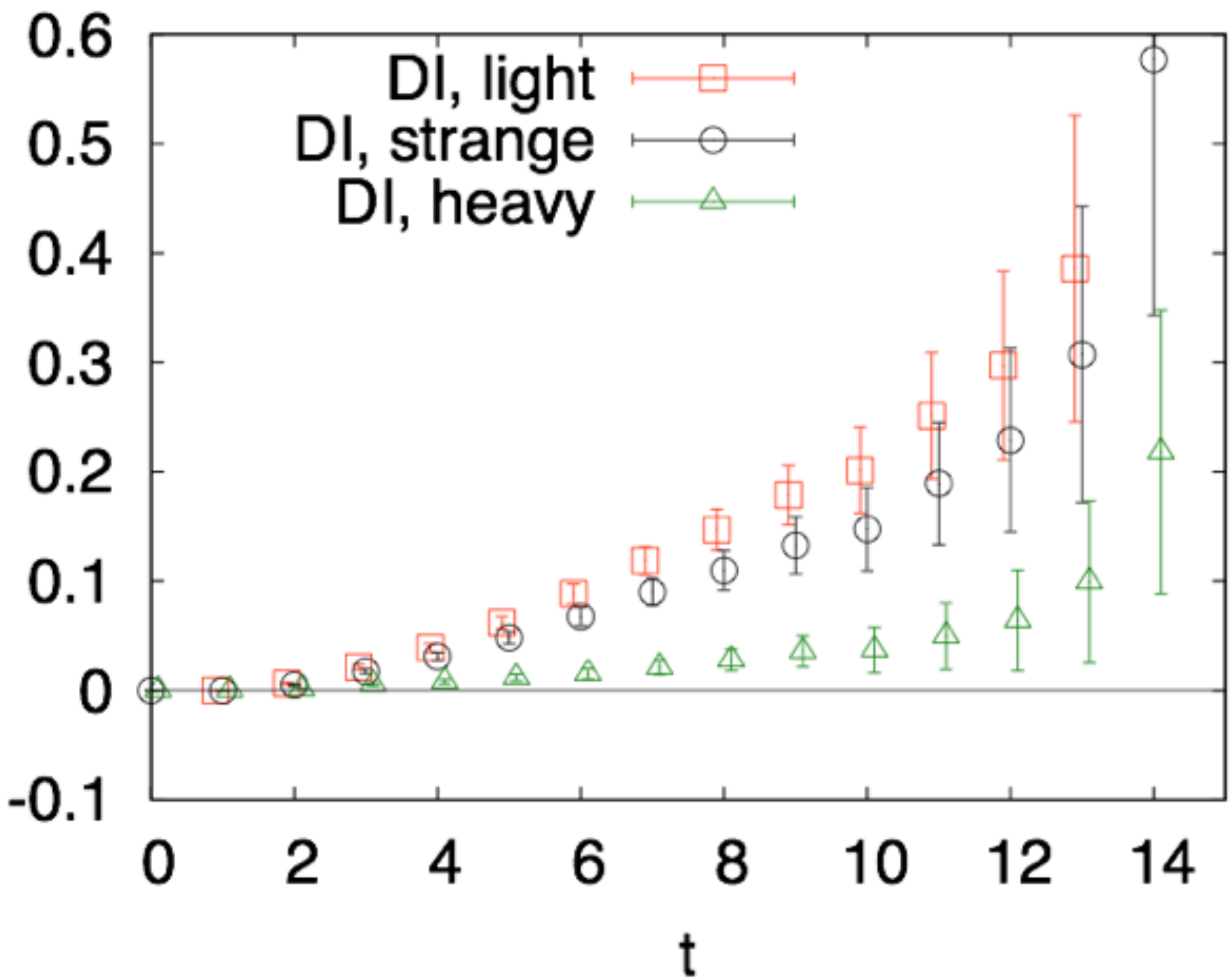


$$\langle x\rangle^{\mathrm{E}} \equiv \frac{\mathrm{Tr}[\Gamma^e\langle P|\int d^3x O^{\mathrm{E}}(x)|P\rangle]}{E\mathrm{Tr}[\Gamma^e\langle P|P\rangle]}.$$

$$O^{\mathrm{E}}(x)=\overline{\psi}(x)\tfrac{1}{2}(\gamma_4\overleftrightarrow{D}_4-\tfrac{1}{3}\textstyle\sum_{i=1,2,3}\gamma_i\overleftrightarrow{D}_i)\psi(x)$$

$$\langle x\rangle^{\mathrm{P}} \equiv \frac{\mathrm{Tr}[\Gamma^e\langle P|\int d^3x\overline{\psi}(x)\frac{1}{4}(\gamma_i\overleftrightarrow{D}_4+\gamma_4\overleftrightarrow{D}_i)\psi(x)|P\rangle]}{p_i\mathrm{Tr}[\Gamma^e\langle P|P\rangle]}$$

The disconnect part of the **quark momentum fraction**



The plot shows the summed ratio.

- The high mode part of the quark loop contribute much on the momentum fraction case, so the results are more noisy than the scalar case.
- The contribution from **the strange quark** and **the DI part of the light quark** is comparable, but that from **the heavy quark** is very small (while preferring a positive value)

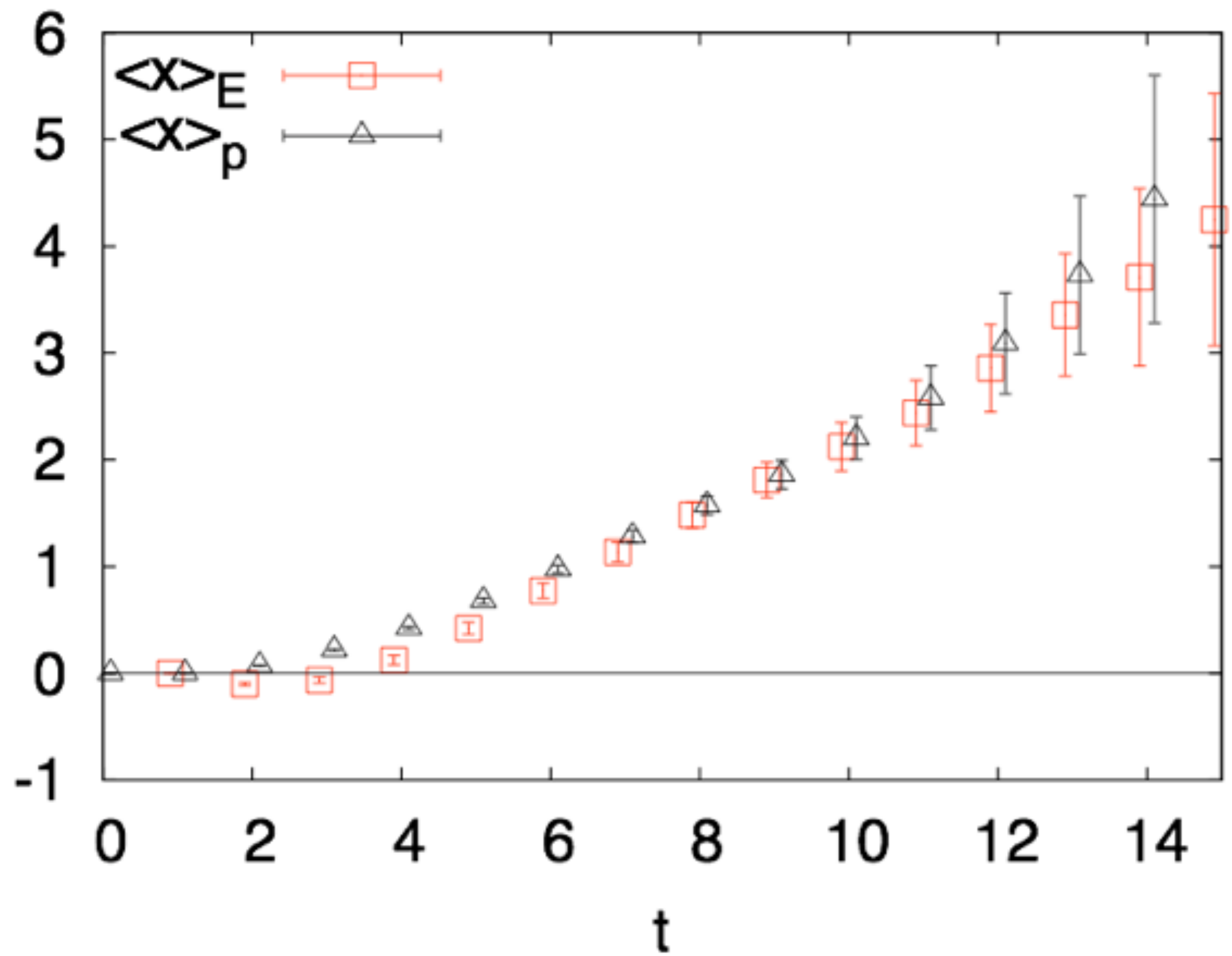
M. Sun, et al. [χQCD], PoS LATTICE 2014, 142

The **glue momentum fraction**

The plot shows the summed ratio.

The glue field operator on HYP smeared configuration.

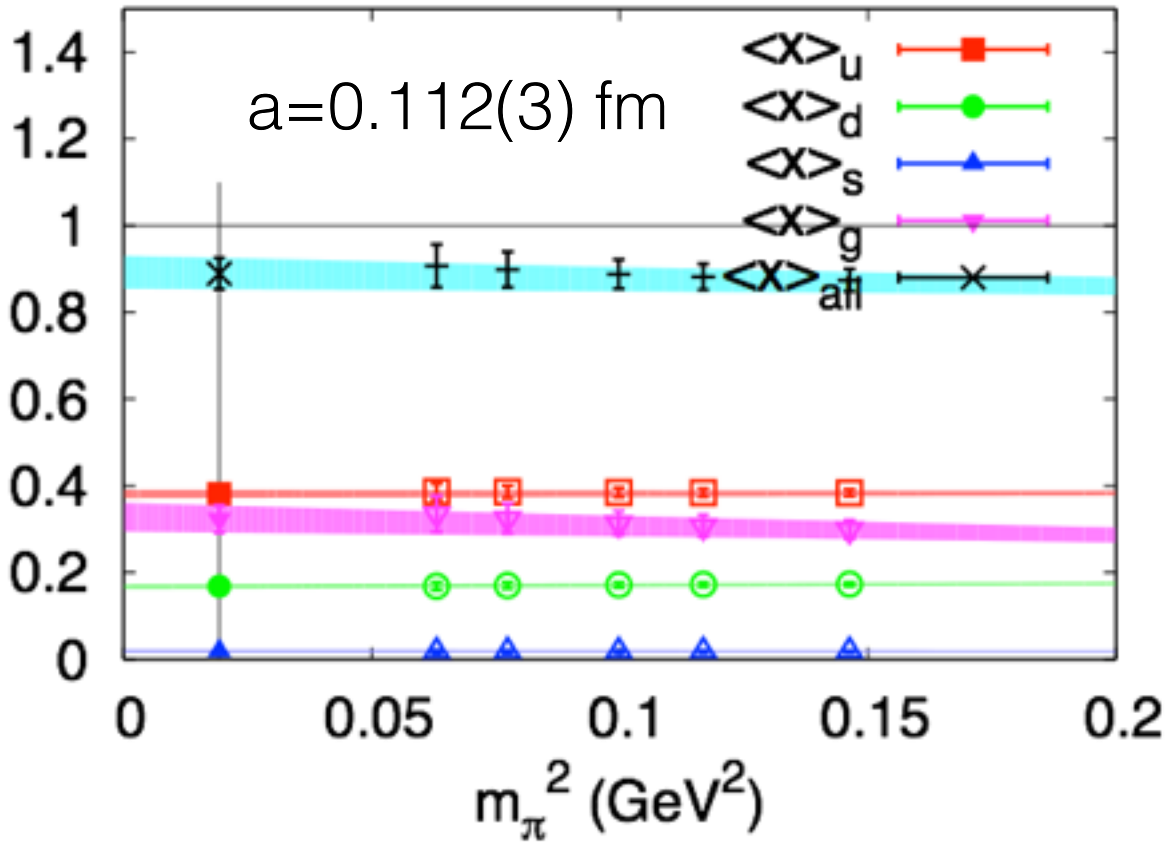
The result obtained with the diagonal matrix element in the rest frame is consistent with that with the off-diagonal one in the moving frame.

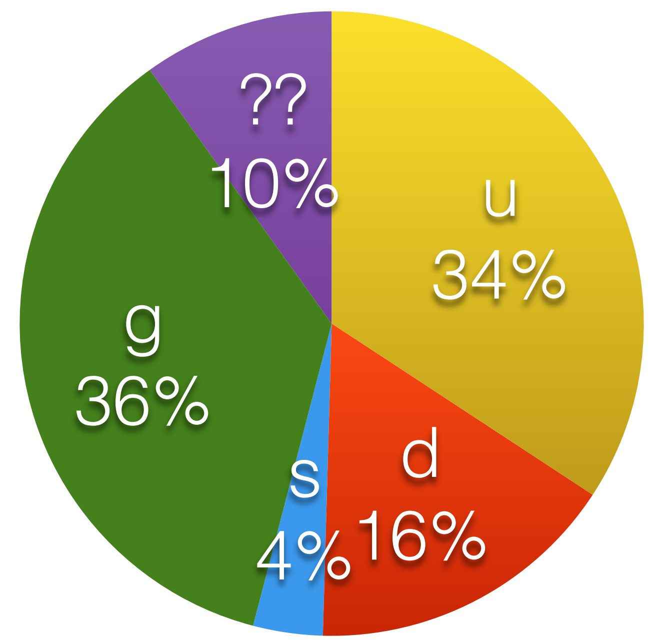


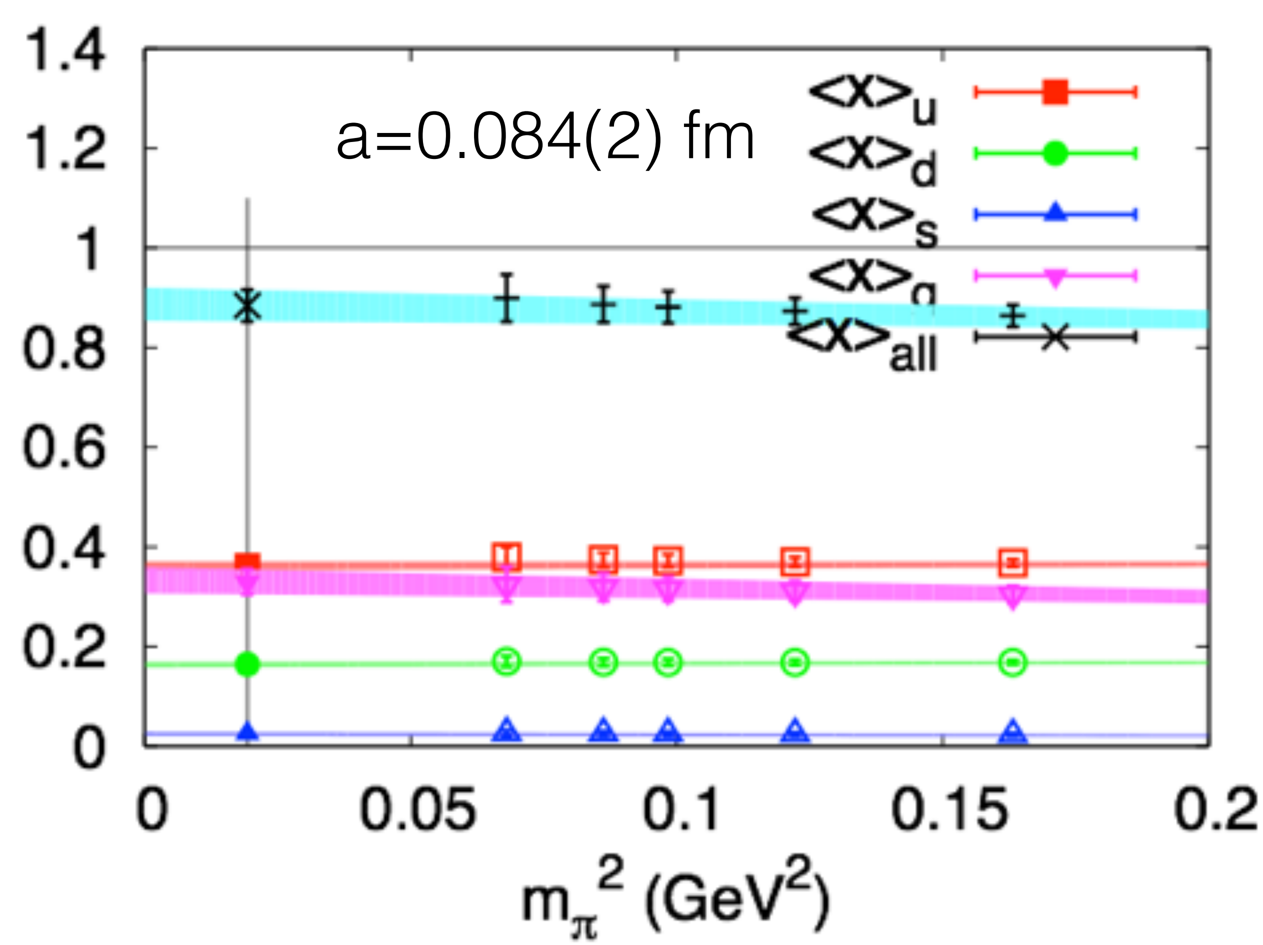
$$\langle x \rangle^E \equiv \frac{\mathrm{Tr}[\Gamma^e \langle P | \int d^3x \frac{1}{2}(B^2 - E^2) | P \rangle]}{\frac{3}{4} M \mathrm{Tr}[\Gamma^e \langle P | P \rangle]}$$

$$\langle x \rangle^p \equiv \frac{\mathrm{Tr}[\Gamma^e \langle P | \int d^3x (E \times B) | P \rangle]}{p \mathrm{Tr}[\Gamma^e \langle P | P \rangle]}.$$

The **sum rule** of the momentum fraction







10% missing. normalization required.

$$\langle x\rangle(m_\pi) = C_0 + C_1 m_\pi^2.$$

The **normalization**

with momentum/angular-momentum sum rules

- 10% of the momentum fraction is missing. A nontrivial normalization is required to satisfy the sum rule.
- The matrix element of T^{0i} between two nucleon states can be written in terms of three form factors,

$$
(p', s'|\mathcal{T}^{\{0i\}q,g}|p, s) = \left(\frac{1}{2}\right) \bar{u}(p', s') \Big[T_1(q^2)(\gamma^0 \bar{p}^i + \gamma^i \bar{p}^0) + \frac{1}{2m} T_2(q^2) \left(\bar{p}^0 (i\sigma^{i\alpha}) + \bar{p}^i (i\sigma^{0\alpha})\right) q_\alpha + \frac{1}{m} T_3(q^2) q^0 q^i \Big]^{q,g} u(p, s),
$$

Then the sum rules are,

$$
\begin{aligned}
Z_q \langle x \rangle^q + Z_g \langle x \rangle^g &= Z_q T_1(0)^q + Z_g T_1(0)^g \\
&= 1, \\
Z_q J^q + Z_g J^g &= \frac{1}{2} \{ Z_q [T_1^q(0) + T_2^q(0)] + Z_g [T_1^g(0) + T_2^g(0)] \} \\
&= \frac{1}{2}
\end{aligned}
$$

- So we can normalize the quark/glue contribution with the above two relations.

M. Deka, et al. [χQCD], Phys. Rev. D 91, 014505 (2015)

The **mixing** between the quark and glue components

- *The bare scale of two ensembles are different.*
- *Running to the same scale before continuum extrapolation is required.*
- *The mixing exists between the quark and glue parts, when the scale changed:*

$$\begin{bmatrix} \langle x\rangle_q(\mu',\mathrm{CI}) \\ \langle x\rangle_q(\mu',\mathrm{DI}) \\ \langle x\rangle_g(\mu') \end{bmatrix} = \begin{bmatrix} Z_q(\frac{\mu'}{\mu},g_0) & 0 & 0 \\ 0 & Z_q(\frac{\mu'}{\mu},g_0) & 1-Z_g(\frac{\mu'}{\mu},g_0) \\ 1-Z_q(\frac{\mu'}{\mu},g_0) & 1-Z_q(\frac{\mu'}{\mu},g_0) & Z_g(\frac{\mu'}{\mu},g_0) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q(\mu,\mathrm{CI}) \\ \langle x\rangle_q(\mu,\mathrm{DI}) \\ \langle x\rangle_g(\mu) \end{bmatrix},$$

To one-loop order, the renormalization/mixing coefficients are:

$$\begin{aligned} Z_q(\frac{\mu'}{\mu},g_0) &= 1+\frac{g_0^2}{16\pi^2}C_F\left(\frac{8}{3}\log(\frac{\mu'^2}{\mu^2})\right), \\ Z_g(\frac{\mu'}{\mu},g_0) &= 1+\frac{g_0^2}{16\pi^2}\left(\frac{2}{3}N_f\log(\frac{\mu'^2}{\mu^2})\right). \end{aligned}$$

- *We use this to run the scale from the lattice spacing of each of two ensembles to 2GeV, for both quark and gluon components.*

M. Glatzmaier, et al. [χQCD], arXiv:1403.7211

The bare **values** at 2 GeV

a	CI(u)	CI(d)	DI(u/d)	DI(s)	Glue	Sum
0.112(3) fm	0.364(7)	0.146(2)	0.023(3)	0.018(3)	0.32(3)	0.89(4)
0.084(2) fm	0.326(5)	0.132(2)	0.031(4)	0.025(4)	0.34(3)	0.89(3)
0	0.30(1)	0.118(4)	0.038(8)	0.031(8)	0.36(6)	0.89(7)
normalized	0.34(3)	0.13(1)	0.043(7)	0.035(7)	0.40(4)	100%

- ***Since the calculation of AM decomposition is unfinished yet, an uniform normalization is applied on both quark and glue part.***
- *The glue fraction is consistent with the experiment.*
- *Iso-vector quark momentum fraction (~0.20(1)) is larger than the experiment, likes the other lattice calculations.*

The quark/glue energy

from the momentum fractions

$$\langle H_E \rangle = \frac{3}{4}\langle x \rangle_q M - \frac{3}{4}\langle H_m \rangle,$$
$$\langle H_q \rangle = \frac{3}{4}\langle x \rangle_q M + \frac{1}{4}\langle H_m \rangle.$$
$$\langle H_g \rangle = \frac{3}{4}\langle x \rangle_g M.$$

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D} \cdot \vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

From the last section, $\langle H_{m,u+d+s} \rangle / M = 9(1)\%.$

Then

$$\langle H_E \rangle / M = 38(4)\%,$$
$$\langle H_q \rangle / M = 47(5)\%,$$
$$\langle H_g \rangle / M = 30(3)\%.$$

Quick summary

- The sum of the bare momentum fraction components explains ~90% of the total momentum, regardless the valance quark masses and lattice spacings.
- The non-perturbative renormalization, or the normalization based on the sum rules of M/AM is required.
- Supposing the normalization factor of the quark/gluon components are similar, and use the fraction of the quark mass term:

$$\langle H_E \rangle / M = 38(4)\%,$$
$$\langle H_q \rangle / M = 47(5)\%,$$
$$\langle H_g \rangle / M = 30(3)\%.$$

Outline

- *Motivation and scheme* of the proton mass decomposition based on energy-momentum tensor

  The source of the proton mass, the simulation scheme and setup.

- *The quark mass term* in proton

  πNσ term, strangeness and also the QCD anomaly contribution.

- *The quark/glue energy in proton.*

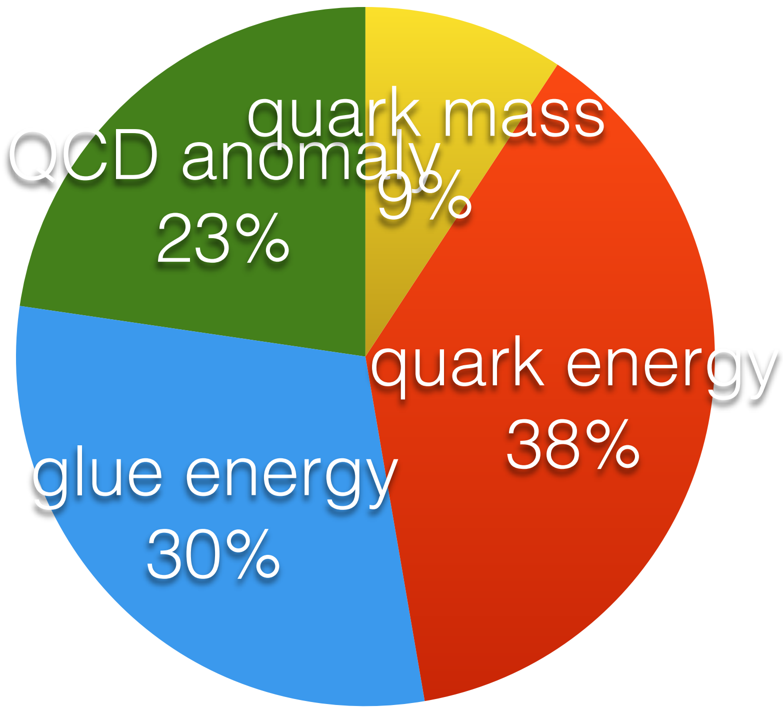
  Based on the direct calculation of the quark/gluon components of the second moment of proton.

- ***The simulation results and Open questions needed to be answered by further research***

  The renormalization/mixing issues and the challenge of the simulation on the physical point.

Proton Mass decomposition

by type



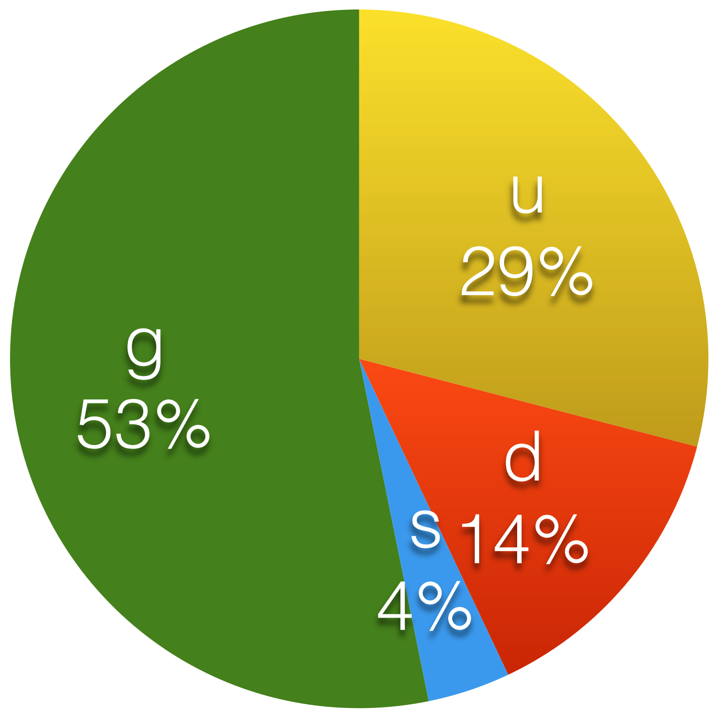
Preliminary

quark mass	quark energy
9(1) %	38(4) %
glue energy	**QCD anomaly**
30(3) %	23(1) %

- *Supposing the total mass of proton as 1 GeV.*
- *Bare momentum fraction at 2GeV.*
- *QCD anomaly deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

Proton Mass decomposition

by u/d/s flavors+glue



- *Quark part: quark mass term+quark energy term.*
- *Glue part: glue energy + QCD anomaly.*

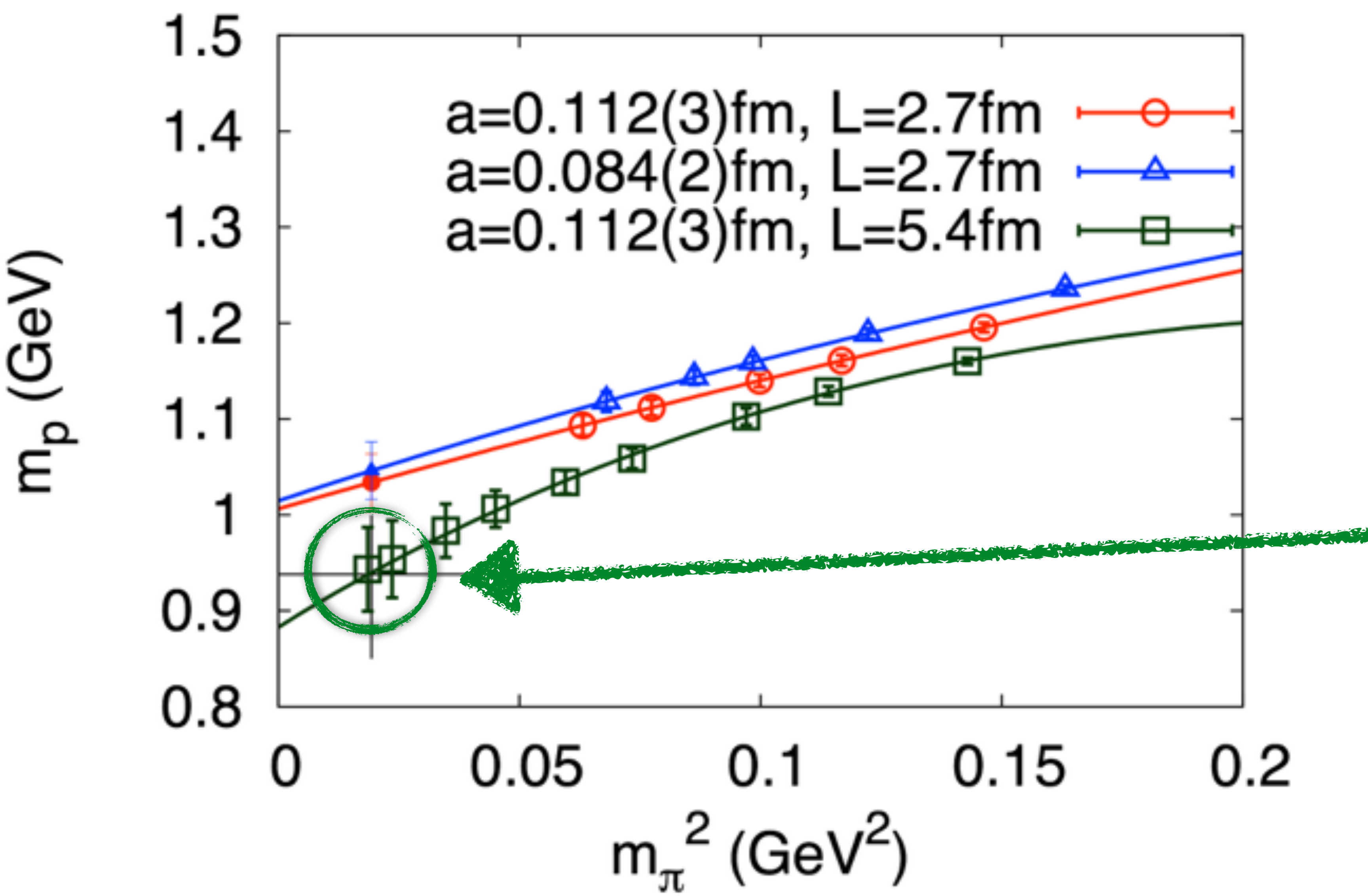
u	d
29(3) MeV	14(1) %
s	**g**
4(1) %	53 (7) %

- *Supposing the total mass of proton as 1 GeV.*
- *Bare momentum fraction at 2GeV.*
- *QCD anomaly deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

Preliminary

Question needed to answer:

The volume and partially quenching effect



$$M = C_0 + C_1 m_{\pi,vv}^2 + C_2 m_{\pi,vs}^3$$

- Two sigma bias at the physical point (based on extrapolation), for two ensembles used here.
- Perfect match with experiment in the ensemble with large volume and physical quark masses.
- We will to repeat the decomposition calculation on 48^3x96 lattice (L=5.4 fm) with physical quark masses.

Summary

- **The mass decomposition based on the energy-momentum tensor is an old story, while Lattice QCD simulation gives it a second life.**

- **We decompose the proton mass into quark and gluon components in lattice simulation.**

1. **The joint u/d/s quark mass term contribute ~9%.**
2. **The glue momentum fraction is ~40%.**
3. **The joint quark/glue energy contribute ~68%.**
4. **The joint glue contribute half of the proton mass.**

- **Further calculations at the physical point are in progress.**