Neutron-Antineutron Perturbative Operator Renormalization

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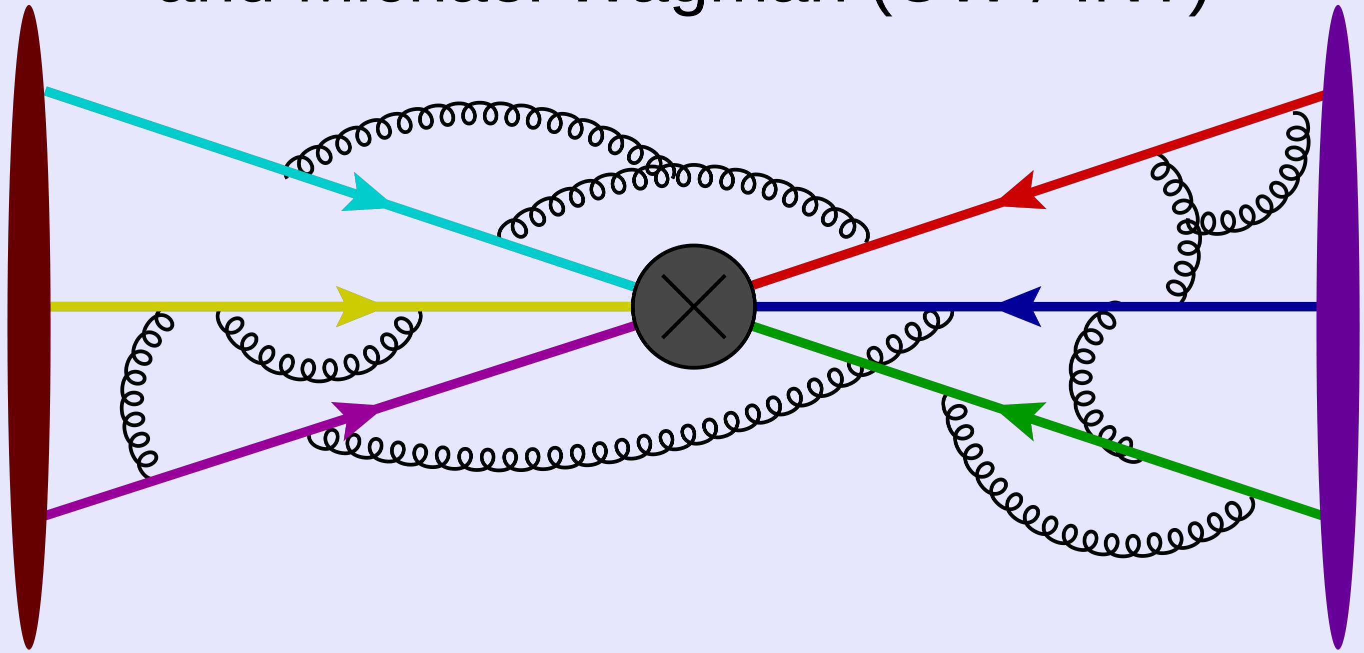
arxiv:1506.00647
arxiv:1502.00044



Michael Wagman, 9/24/2015 Seminar for INT Program:

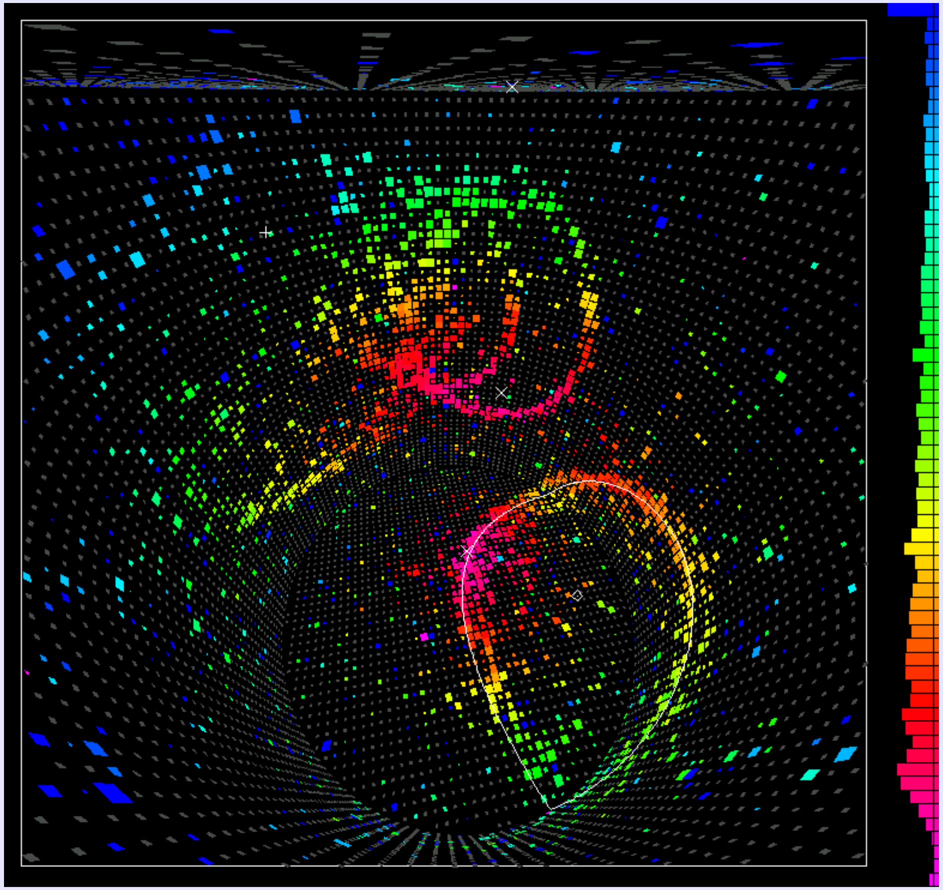
Intersections of BSM Phenomenology and QCD for New Physics Searches

Neutron-Antineutron Transitions

Predicted by some BSM theories relevant for baryogenesis

Experimentally testable predictions require QCD matrix element calculations

Calculations with quantified uncertainty require lattice QCD and perturbative operator renormalization



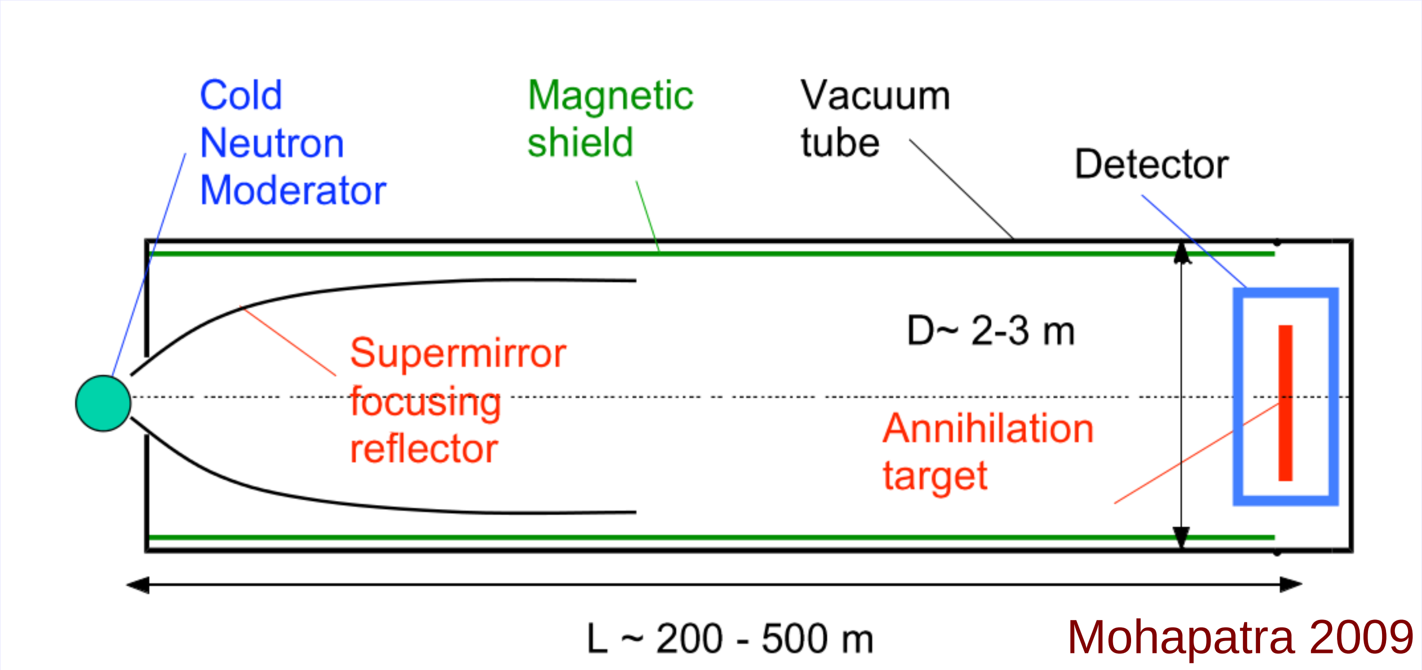
Super K, proton decay candidate event

Neutron-Antineturon Phenomenology

Vacuum transition rate described by two-state Hamiltonian

$$\frac{1}{\tau_{n\overline{n}}} = <\bar{n}|H_{n\bar{n}}|n>$$

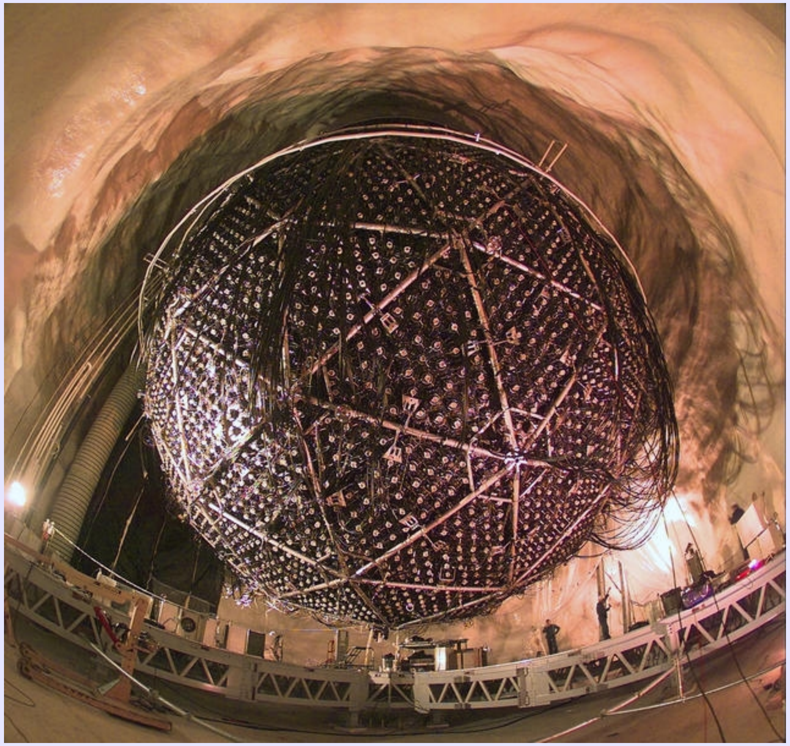
Magnetic fields, nuclear interactions modify transition rate



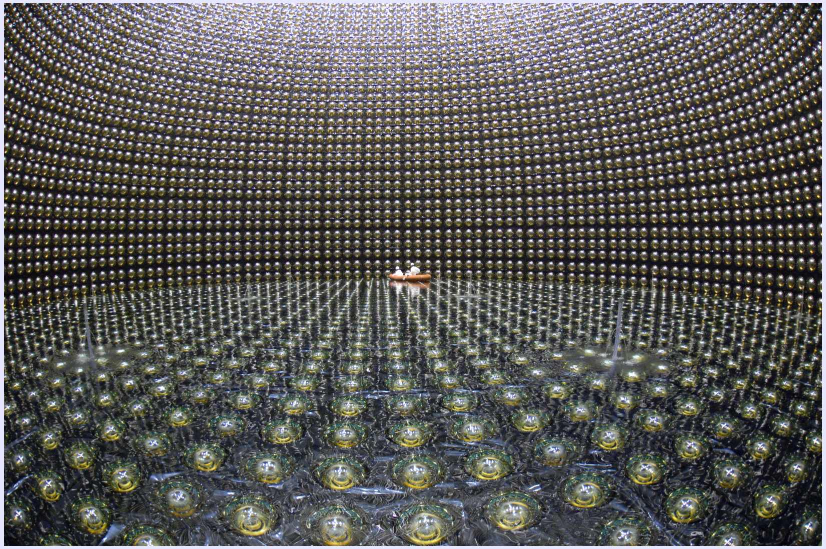
Mohapatra 2009

Experimental Constraints

ILL: $\tau_{n\bar{n}} > 2.7$ years



SNO: $\tau_{n\bar{n}} > 5.7$ years (preliminary)



Super K: $\tau_{n\bar{n}} > 11$ years

Standard Model Results

Complete basis of gauge singlet six-quark operators, bag model estimates for QCD matrix elements

Rao and Shrock (1984)

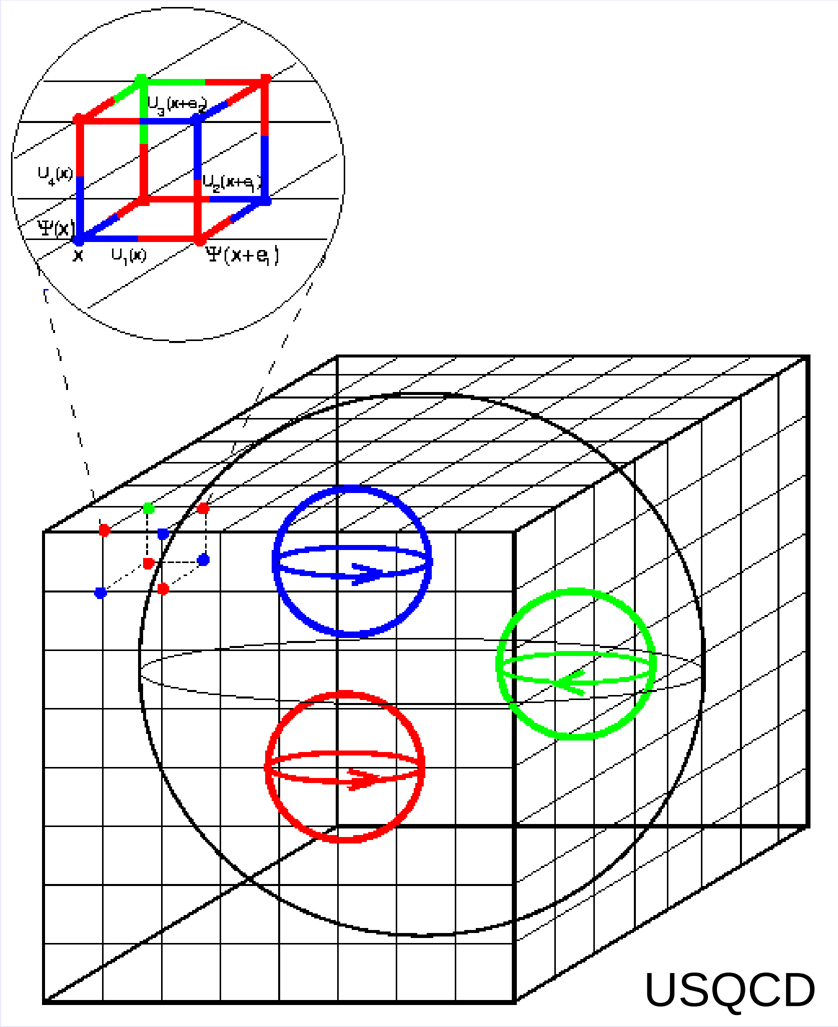
$$\mathcal{H}_{n\overline{n}} = \sum_{I=1}^{14} C_I(\mu) Q_I(\mu)$$

1-Loop QCD (and QED) anomalous dimensions

Caswell, Milutinovic, and Senjanovic (1983)

Preliminary lattice QCD matrix elements

Buchoff, Schroeder, and Wasem (2012)



The Operators

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$Q_1 = (\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \tau^+ \psi) T^{AAS} = \mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{AAS}$$

Chiral Basis	Fixed-Flavor Basis	Chiral Tensor Structure	Chiral Irrep
Q_1	$\mathcal{O}^3_{RRR}$	$\mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_2	$\mathcal{O}^3_{LRR}$	$\mathcal{D}_L \mathcal{D}_R \mathcal{D}_R^+ T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_3	$\mathcal{O}^3_{LLR}$	$\mathcal{D}_L \mathcal{D}_L \mathcal{D}_R^+ T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_4	$4/5\ \mathcal{O}^2_{RRR} + 1/5\ \mathcal{O}^1_{RRR}$	$\mathcal{D}_R^{33+} T^{SSS}$	$(\mathbf{1}_L, \mathbf{7}_R)$
Q_5	$\mathcal{O}^1_{RLL}$	$\mathcal{D}_R^- \mathcal{D}_L^{++} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_6	$\mathcal{O}^2_{RLL}$	$\mathcal{D}_R^3 \mathcal{D}_L^{3+} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_7	$2/3\ \mathcal{O}^2_{LLR} + 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}_R^+ \mathcal{D}_L^{33} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
$\widetilde{Q}_1$	$1/3\ \mathcal{O}^2_{RRR} - 1/3\ \mathcal{O}^1_{RRR}$	$\mathcal{D}_R \mathcal{D}_R \mathcal{D}_R^+ T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
$\widetilde{Q}_3$	$1/3\ \mathcal{O}^2_{LLR} - 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}_L \mathcal{D}_L \mathcal{D}_R^+ T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$

Isospin 3 → Q_4

Not SM gauge invariant → Q_5, Q_6, Q_7

Redundant in $D = 4$ → $\widetilde{Q}_1$, $\widetilde{Q}_3$

Chiral symmetry provides a basis with no operator mixing

Perturbative Renormalization

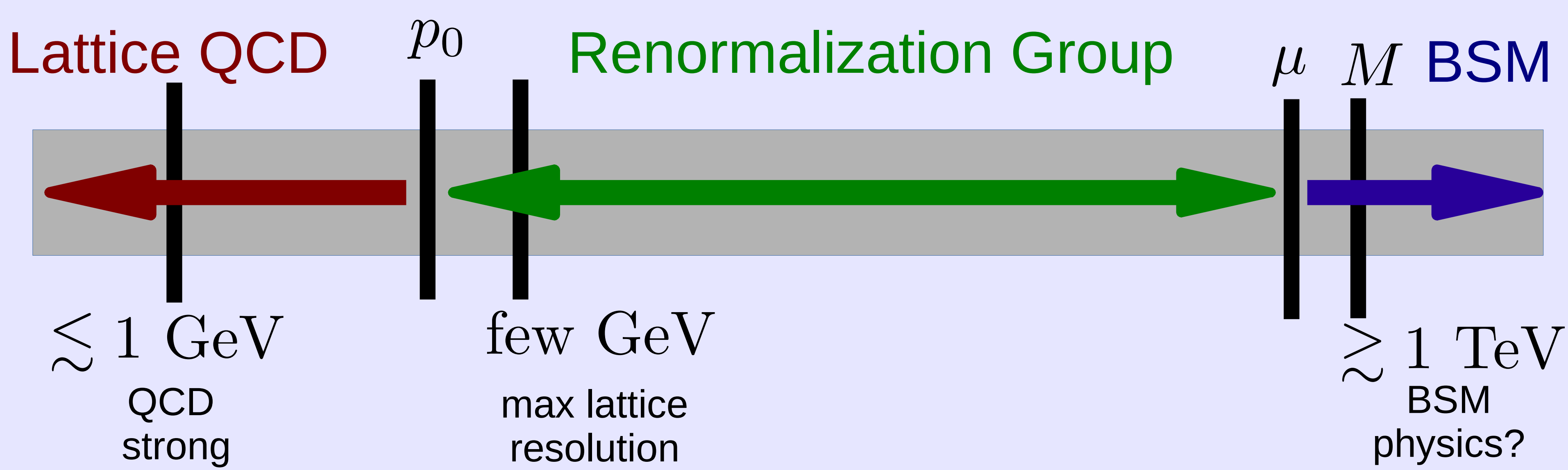
$$\mathcal{H}_{n\bar{n}} = \sum_I C_I^{\overline{MS}}(\mu) Q_I^{\overline{MS}}(\mu) = \sum_I C_I^{\overline{MS}}(\mu) U_I(\mu, p_0) Q_I^{RI}(p_0)$$

$$U_I(\mu, p_0) = \left(\frac{\alpha_s(p_0)}{\alpha_s(\mu)}\right)^{-\gamma_I^{(0)}/2\beta_0} \left[1 - r_I^{(0)} \frac{\alpha_s(p_0)}{4\pi} + \left(\frac{\beta_1 \gamma_I^{(0)}}{2\beta_0^2} - \frac{\gamma_I^{(1)}}{2\beta_0}\right) \frac{\alpha_s(p_0) - \alpha_s(\mu)}{4\pi} + O(\alpha_s^2)\right]$$

One-loop matching

Two-loop running,
needed for $\alpha_s(p_0)$ accuracy

Negligible



Regularization-Independent Renormalization

RI-MOM scheme: apply momentum subtraction to vertex functions Λ_I at lattice matching scale p_0

Martinelli et al (1995)

$$[\Lambda_I]^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn}(p) = \frac{1}{5}\left\langle Q_I(0)\,\bar{u}^\alpha_i(p)\bar{u}^\beta_j(p)\bar{d}^\gamma_k(p)\bar{d}^\delta_l(-p)\bar{d}^\eta_m(-p)\bar{d}^\zeta_n(-p)\right\rangle\Big|_{amp}$$
$$+ \frac{3}{5}\left\langle Q_I(0)\,\bar{u}^\alpha_i(p)\bar{u}^\beta_j(-p)\bar{d}^\gamma_k(p)\bar{d}^\delta_l(p)\bar{d}^\eta_m(-p)\bar{d}^\zeta_n(-p)\right\rangle\Big|_{amp}$$
$$+ \frac{1}{5}\left\langle Q_I(0)\,\bar{u}^\alpha_i(-p)\bar{u}^\beta_j(-p)\bar{d}^\gamma_k(p)\bar{d}^\delta_l(p)\bar{d}^\eta_m(p)\bar{d}^\zeta_n(-p)\right\rangle\Big|_{amp}$$

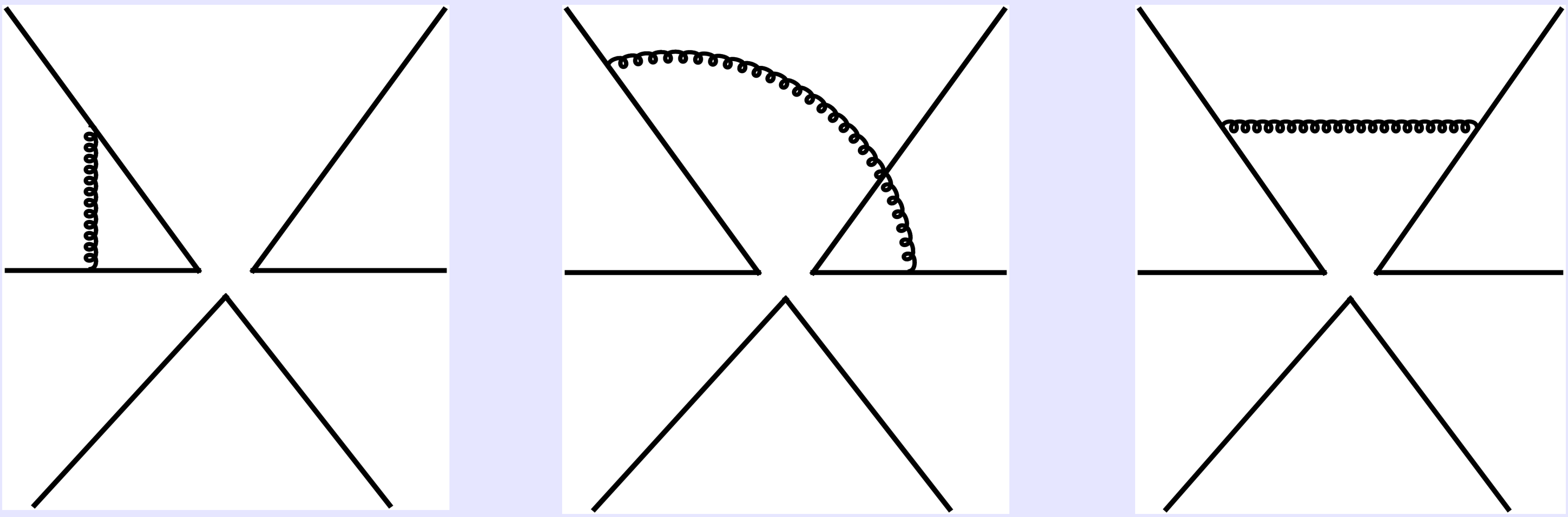
Syritsyn (2015)

RI-MOM condition applicable to lattice and dim reg operators

$\mathcal{P}_I$ projects out contribution from tree-level $\Lambda_I^{(0)}$ → $[\mathcal{P}_I \Lambda_J(p_0, \mu = p_0)] = \delta_{IJ}$

One-Loop Matching

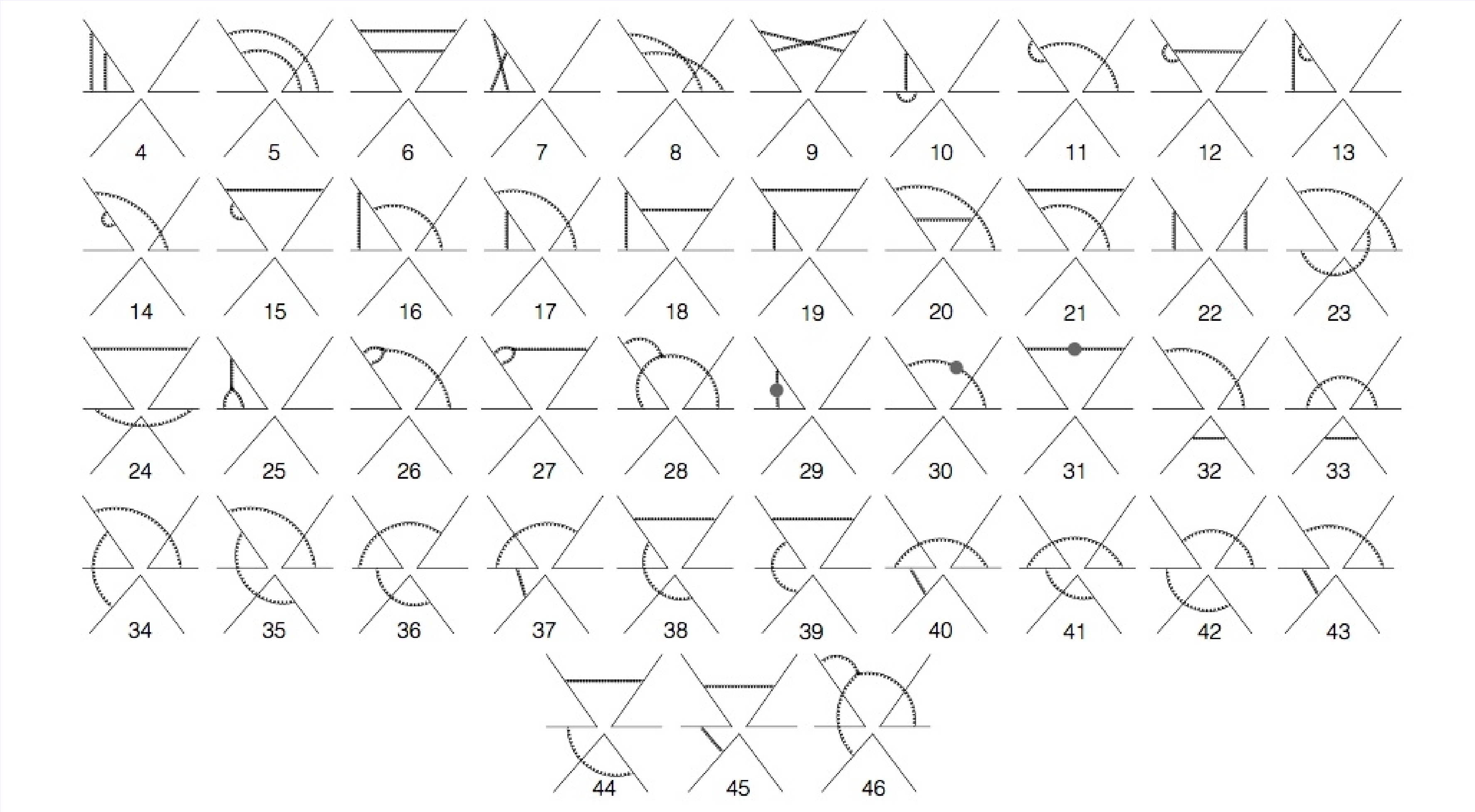
15 one-loop Feynman diagrams contribute



One quark pair is not involved in the interaction. Identical diagrams contribute to four-quark weak matrix elements and proton decay

Two-Loop Running

Need $\ln\mu^2$ coefficients of 320 two-loop Feynman diagrams



Covers all diagram topologies needed for two-loop running of any operator built from spin singlet quark pairs

Evanescent Operators

Operator basis incomplete in dim reg, Fierz identities broken

$$Q_1 = (\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \tau^+ \psi) T^{AAS}$$

Vanishes in *D* = 4

$$E_1^a \equiv (\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R i\tau_2 \tau_+ \psi) T^{SSS} - 12 Q_1$$

Complete basis for one-loop counterterms found by adding evanescent operators

Buras and Weisz (1990), Dugan and Grinstein (1991), Herrlich and Nierste (1995), ...

$$\frac{1}{\bar{\varepsilon}}(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R i\tau_2 \tau_+) T^{SSS} + \left(\overline{\text{MS}} \text{ counterterm}\right)$$

$$= \frac{1}{\bar{\varepsilon}} 12 Q_1 + \frac{1}{\bar{\varepsilon}} E_1^a$$

Evanescent Operators

Complication: evanescent counterterms have different color structures than corresponding two-loop subdiagrams

T^{AAS} color structure

$$(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R i\tau_2 \tau_+ \psi) T^{SSS} \equiv 12 Q_1 + E_1^a$$

Non-local divergences don't cancel diagram-by-diagram

Solution: change of evanescent basis

$$(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R \sigma_{\mu\nu} i\tau_2 \psi)(\psi C P_R i\tau_2 \tau_+ \psi) T^{SSS}$$

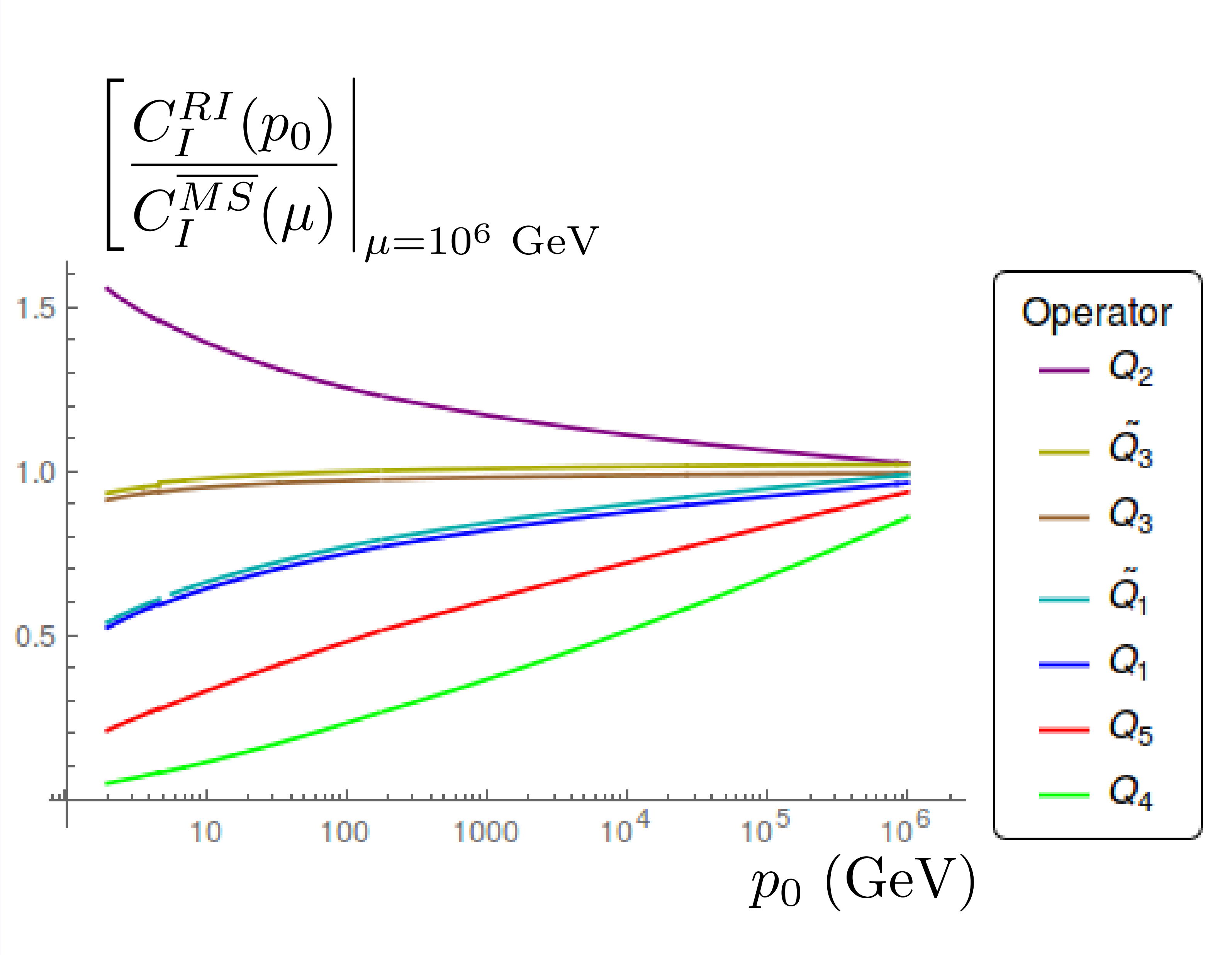
$$\equiv 8(\psi^\alpha [C P_R]^{\alpha\delta} i\tau_2 \psi^\beta)(\psi^\gamma [C P_R]^{\gamma\beta} i\tau_2 \psi^\delta)(\psi^\eta [C P_R]^{\eta\zeta} i\tau_2 \tau_+ \psi^\zeta) T^{SSS} + E_1^{a\prime}$$

$$= 12 Q_1 - 6(Q_1 - \tilde{Q}_1) + E_1^{a\prime}$$

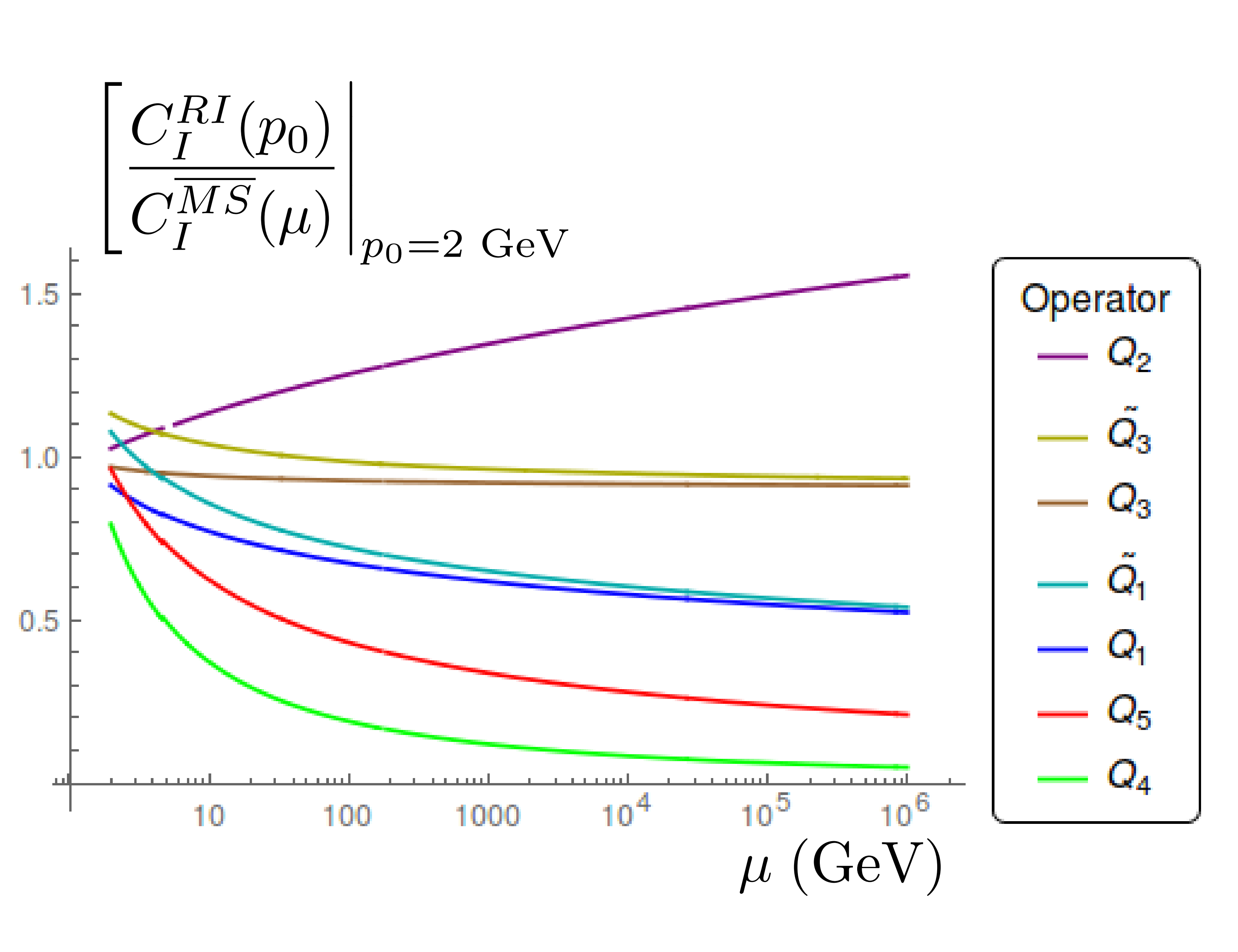
Results

Operator	$\gamma^{(0)}$	$\gamma^{(1)}$	$r^{(0)}$
Q_1	4	$335/3 - 34N_f/9$	$101/30 + 8/15 \ln 2$
Q_2	-4	$91/3 - 26N_f/9$	$-31/6 + 88/15 \ln 2$
Q_3	0	$64 - 10N_f/3$	$-9/10 + 16/5 \ln 2$
Q_4	24	$229 - 46N_f/3$	$177/10 - 64/5 \ln 2$
Q_5	12	$238 - 14N_f$	$49/10 - 24/5 \ln 2$
$\widetilde{Q}_1$	4	$797/3 - 118N_f/9$	$-109/30 + 8/15 \ln 2$
$\widetilde{Q}_3$	0	$218 - 38N_f/3$	$-79/10 + 16/5 \ln 2$

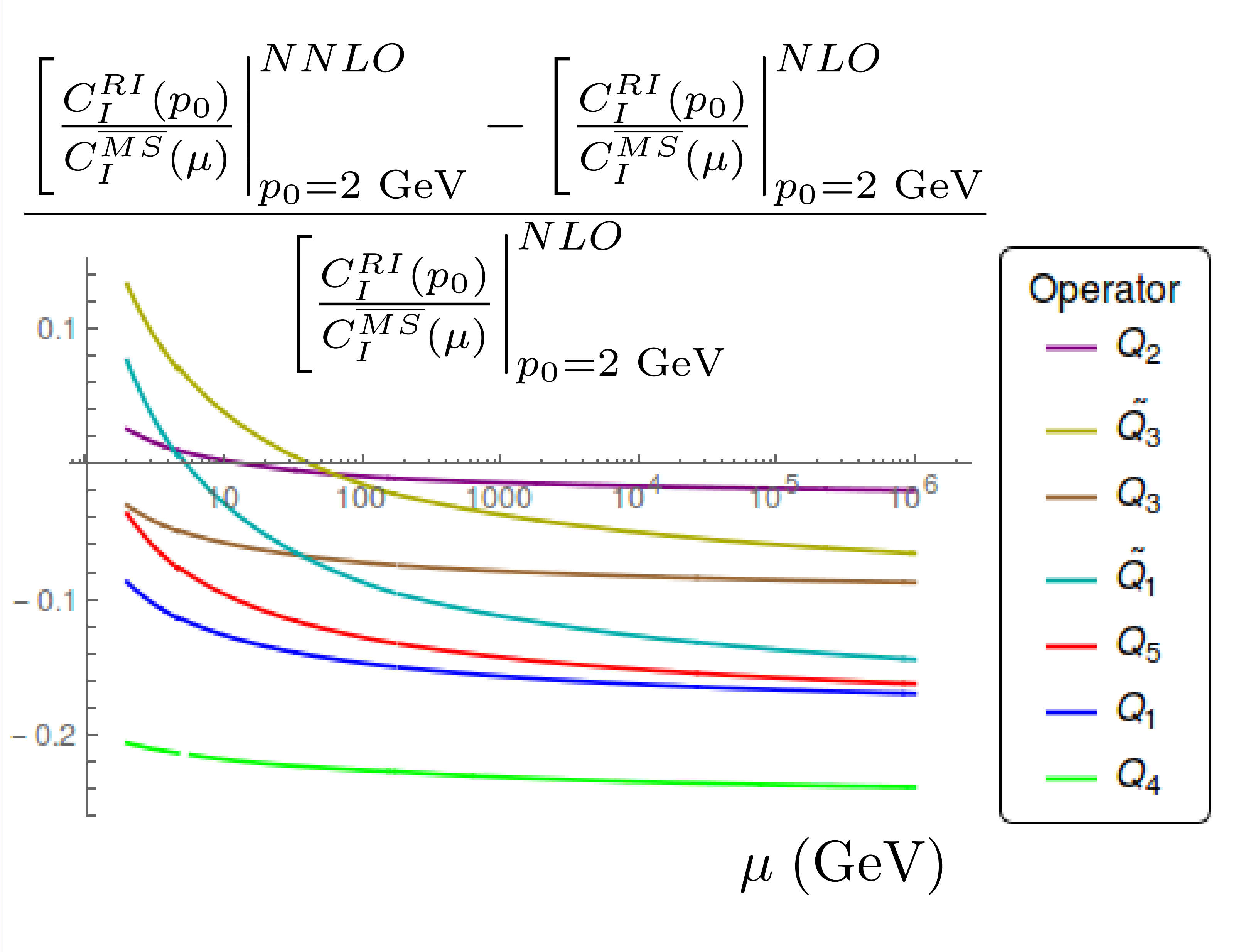
Perturbative Renormalization Results – Theory View



Perturbative Renormalization Results – Experimental View



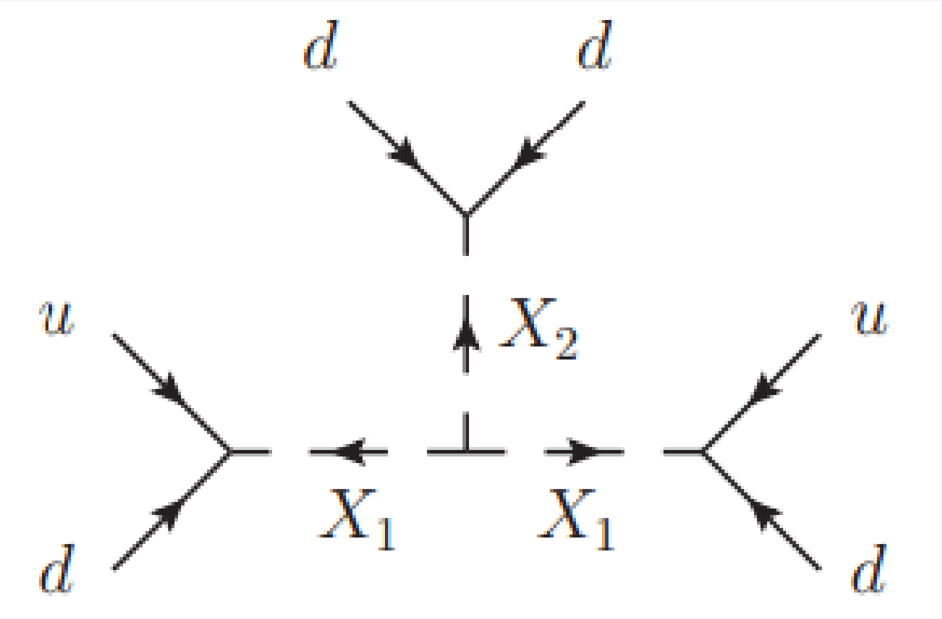
Relative Size of NNLO Corrections



Example BSM Application

Tree-level BSM matching for a simplified model

Arnold, Fornal, and Wise (2013)

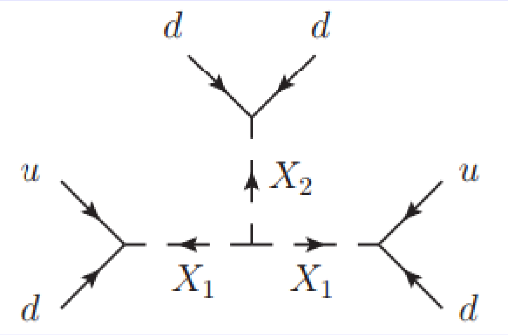


Assuming all dimensionless couplings =1 at $\mu = M$

$$\mathcal{H}_{\rm eff} = \frac{1}{16M^5}\left[Q_4^{\overline{MS}}(M) + \frac{3}{5}\widetilde{Q}_1^{\overline{MS}}(M)\right]$$

$$= \frac{1}{16M^5}\left[U_4(M,p_0)Q_4^{RI}(p_0) + \frac{3}{5}\widetilde{U}_1(M,p_0)\widetilde{Q}_1^{RI}(p_0)\right]$$

Example BSM Application



Arnold, Fornal, and Wise (2013)

LO error: $O\left(\frac{\alpha_s(p_0)}{\pi}\ln(M^2/p_0^2)\right) \sim 2.46$

Uncontrolled perturbative errors
without NLO 1-loop running

NLO error: $O\left(\frac{\alpha_s(p_0)}{\pi}\right) \sim 0.09$ $\quad O\left(\frac{\alpha_s(p_0)^2}{\pi^2}\ln(M^2/p_0^2)\right) \sim 0.23$

Assuming
$M = 500 \text{ TeV}$

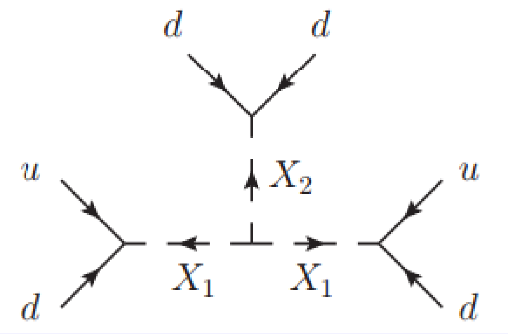
$$1/\tau_{n\bar{n}} = (\underbrace{6.74}_{\text{LO}} \quad \underbrace{-2.44}_{\text{NLO Running}} \quad \underbrace{\pm 0.16}_{\text{Lattice Statistical}}) \times 10^{-34} \text{ GeV}$$

Super K
$1/\tau_{n\bar{n}} < 2 \times 10^{-33} \text{ GeV}$

$$M > (\underbrace{402}_{\text{LO}} \quad \underbrace{-34}_{\text{NLO}} \quad \underbrace{\pm 3}_{\text{Lattice Statistical}}) \text{ TeV}$$

Preliminary lattice QCD matrix elements: Buchoff, Schroeder, and Wasem (2012)
See Sergey Syritsyn Lattice 2015 talk and
Michael Buchoff INT Workshop talk next week
for updated lattice results by Buchoff, Schroeder, Syritsyn, and Wasem

Example BSM Application



NNLO error: $O\left(\frac{\alpha_s(p_0)^3}{\pi^3}\ln(M^2/p_0^2)\right) \sim 0.02$

$O\left(\frac{\alpha_s(p_0)^2}{\pi^2}\right) \sim 0.01$

NLO Convergence Rate
$\sim 0.(14)^2 = 0.02$

Arnold, Fornal, and Wise (2013)

$$1/\tau_{n\bar{n}} = (\underbrace{6.74}_{\text{LO}} \underbrace{-2.44}_{\text{NLO Running}} \underbrace{+0.33}_{\text{NNLO Matching}} \underbrace{-0.94}_{\text{NNLO Running}} \underbrace{\pm 0.16}_{\text{Lattice Statistical}}) \times 10^{-34}\ \text{GeV}$$

$$= (3.68 \pm 0.16) \times 10^{-34}\ \text{GeV}$$

$$M > (\underbrace{402}_{\text{LO}} \underbrace{-34}_{\text{NLO}} \underbrace{+5}_{\text{NNLO matching}} \underbrace{-16}_{\text{NNLO running}} \underbrace{\pm 3}_{\text{Lattice Statistical}})\ \text{TeV}$$

$$M > 357 \pm 3\ \text{TeV}$$

Bound reaches $M > 500\ \text{TeV}$ with factor of 5.4 tighter lifetime bound

Bounds with fully quantified uncertainties can be made once lattice systematics are included Buchoff, Schroeder, Syritsyn and Wasem

Summary

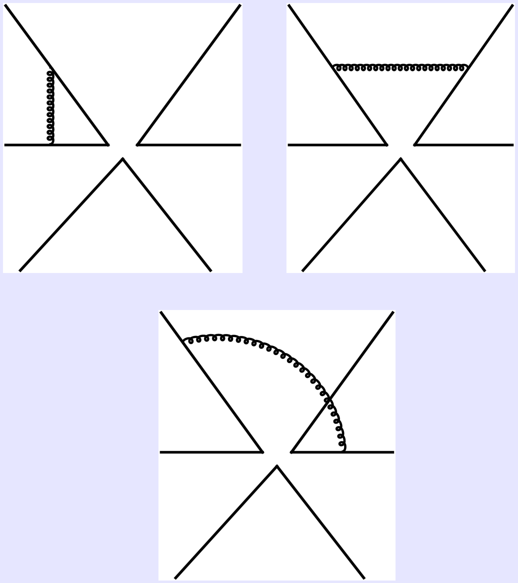
Color and flavor structure matter when determining bounds for particular BSM models:

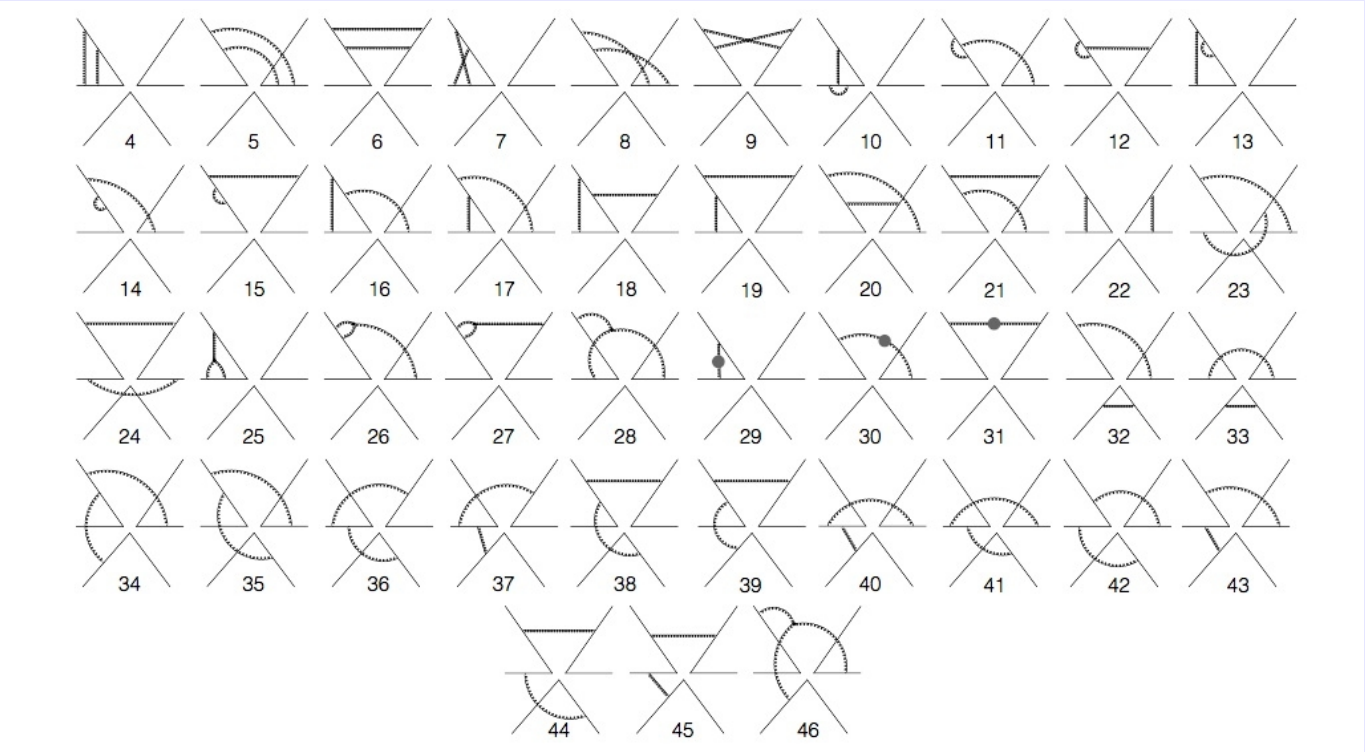
At $M = 500\ \mathrm{TeV}$, perturbative renormalization multiplies lattice results by (1.54) for Q_2 vs (0.05) for Q_4

2-loop running and 1-loop matching correct 1-loop running result of Caswell *et al.* by $\lesssim 26\%$

Assuming same rate of convergence, remaining perturbative uncertainty $\lesssim 7\%$

Summary

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14 15 16 17 18 19 20 21 22 23
24 25 26 27 28 29 30 31 32 33
34 35 36 37 38 39 40 41 42 43
44 45 46


Lattice QCD
p_0
Renormalization Group
μ
M
BSM
$\lesssim 1$ GeV
QCD strong
few GeV
max lattice resolution
$\gtrsim 1$ TeV
BSM physics?