Proton Decay and other BSM in the lattice

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OUTLINE

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3. Updated result
4. Extension to dark matter model
5. Summary

Proton decay

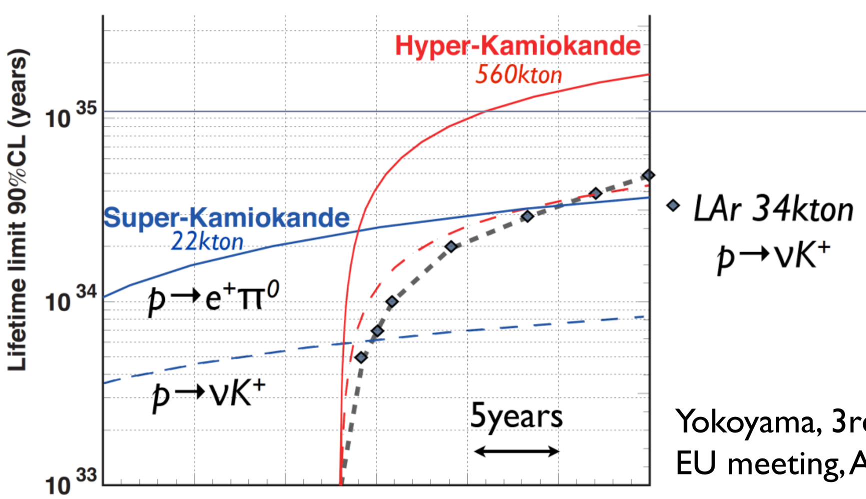
- SK: $\tau(pe^+\pi^0) > 1.3 \times 10^{34}$ years, $\tau(p\nu K) > 5.9 \times 10^{33}$ years SK, PRD90(2014)

⇒ Hyper-K aims to take $\tau(pe^+\pi^0) > 1.3 \times 10^{35}$ years, $\tau(p\nu K) > 3.2 \times 10^{34}$ years

HyperK-proto, PTEP 2015

Selected as one of 27 top priority projects in "Japan Master Plan of Large Research Project", ~$650M+$25M/y for 15y. (also J-PARC, Muon g-2/EDM experiment was)
http://www.scj.go.jp/ja/info/kohyo/pdf/kohyo-22-t188-1.pdf (in Japanese)

SUSY-GUTs: $\tau(pe^+\pi^0) \sim 10^{35}$ year



Yokoyama, 3rd Hyper-Kamiokande EU meeting, April 27, 2015 CERN

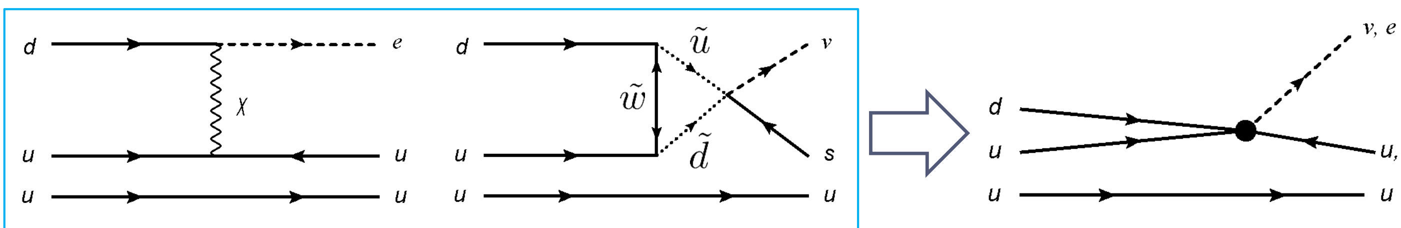
1. Introduction

Decay rate

$$\Gamma_{p\to\pi^0 e^+} = \frac{m_p}{32\pi}\left[1-\left(\frac{m_e}{m_p}\right)^2\right]^2\left|\sum_i C_i(p\to P+l)\right|^2 \sim \frac{\alpha_5^2 m_p^5}{M_X^4}$$

Wilson coefficient, relying on GUTs, + QCD matrix element, summing over quantum number i

$$H_{\rm BV} = \sum_i C_i(\mu)O_i(\mu)/\Lambda_{\rm GUTs}, \quad O_i = (qqql)_i$$



Minimum SU(5), dim-6 SUSY-GUT, dim-5 (qqql) coupling

- Wilson coefficient (p→νK) on SUSY-SU(5)

Matrix element described by ChPT

$$C_\mu(\mu) = \frac{\alpha_2^4}{M_{H_C}^2 m_W^4 \sin^2 2\beta} F(M_2, M_S^2) \frac{\beta_p(\mu)}{f_\pi^2}\left\{1+(D+F)\frac{m_p}{m_B}\right\}\bar{m}_s V_{us}^* \sum_{i=2,3} m_{u_i} V_{u_i d} V_{u_i s} e^{i\phi_i} A_R^{(i,2)}(\mu),$$

$$F(M, M_S) = M\left[\frac{1}{M_S^2 - M^2} - \frac{M^2}{(M_S^2-M^2)^2}\ln\left(\frac{M_S^2}{M^2}\right)\right]$$

M_2 : wino, M_S: sfermion,
M_{HC} : color-triplet Higgs

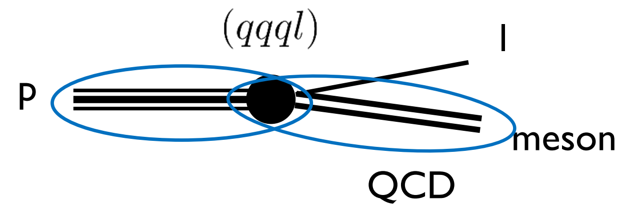
Hisano et al., JHEP07(2013)038

Matrix element with BV operator

- Transition amplitude

$$\langle \pi^0 e^+ | p \rangle_{\rm GUT} = \sum_{i=\Gamma,\Gamma'} C_i \langle \pi^0 e^+ | (ud)_\Gamma (ul)_{\Gamma'} | p \rangle_{\rm SM}$$
$$= \sum_{i=\Gamma,\Gamma'} C_i \langle \pi^0 | (ud)_\Gamma u_{\Gamma'} | p \rangle \bar{v}^c_{e+}$$

Γ, Γ'=L, R



- QCD matrix element appears in p→P(π, K, η)

$$\langle \pi^0(\vec{p}) | (ud)_\Gamma (u)_{\Gamma'} | p(\vec{k}, s) \rangle = P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_p} W_1^{\Gamma\Gamma'}(q^2) \right] u_p(\vec{k}, s)$$
$$= P_{\Gamma'} u_p(\vec{k}, s) W_0^{\Gamma\Gamma'}(0) + \mathcal{O}(m_l/m_N)$$

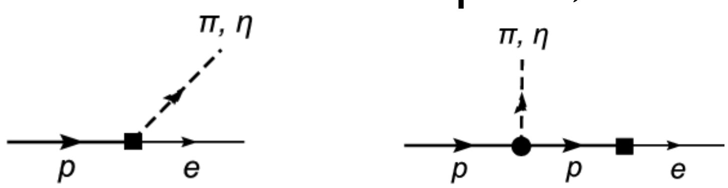
Aoki et al. (JLQCD), PRD62 (2000); Aoki et al.(RBC), PRD75 (2007)

- W_0 contains the matrix element with effective vertex of BSM operator.
- 3-quark operator (qqq) forms L, R combination.
- $m_l/m_N \ll 1 \Rightarrow q^2 = m_l^2 \simeq 0$ is physical kinematics. W_0 is only relevant form factor.

1. Introduction

Matrix element in BChPT

- Leading order interaction in p→π, K transition



Claudson, et al. NPB195(1982), JLQCD, PRD62(2000)

$$
\begin{aligned}
\langle \pi^0 | (ud_R)u_L | p \rangle &= \alpha_p P_L u_p \Big[1 \frac{\sqrt{2} f_\pi}{-} \frac{D+F}{\sqrt{2} f_\pi} \frac{-q^2 + m_N^2}{-q^2 - m_N^2} - \frac{4b_1}{\sqrt{2} f_\pi} \frac{m_u m_N}{-q^2 - m_N^2} \Big] \\
&- \alpha_p P_L \frac{i\not{q}}{m_N} u_p \Big[\frac{D+F}{\sqrt{2} f_\pi} \frac{2m_N^2}{-q^2 - m_N^2} + \frac{4b_1}{\sqrt{2} f_\pi} \frac{m_u m_N}{-q^2 - m_N^2} \Big]
\end{aligned}
$$

α_p : LECs

D+F = $g_A^{(np)}$ = 1.27

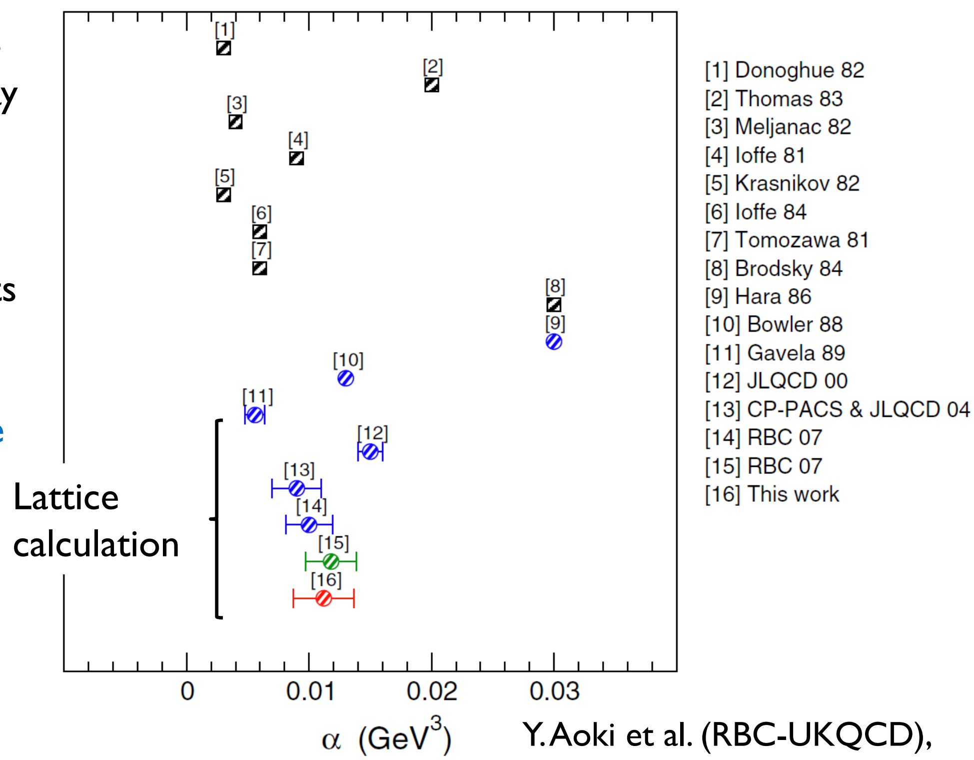
D-F = $g_A^{(n\Sigma)}$ = 0.33

$$
\begin{aligned}
\langle K^+ | (us_R)d_L | p \rangle &= \alpha_p P_L u_N \Big[- \frac{D-F}{2f_\pi} \frac{-q^2 + m_N m_\Sigma}{-q^2 - m_\Sigma^2} - \frac{D+3F}{6f_\pi} \frac{-q^2 + m_N m_\Lambda}{-q^2 - m_\Lambda} \\
&- \frac{b_2}{f_\pi} \frac{(m_u + m_s) m_\Sigma}{-q^2 - m_\Sigma^2} + \frac{b_2 - 2b_1}{3f_\pi} \frac{(m_u + m_s) m_\Lambda}{-q^2 - m_\Lambda^2} \Big] \\
&- \alpha_p P_L \frac{i\not{q}}{m_N} \Big[\frac{D-F}{2f_\pi} \frac{m_N (m_N + m_\Sigma)}{-q^2 - m_\Sigma^2} + \frac{D+3F}{6f_\pi} \frac{m_N (m_N + m_\Lambda)}{-q^2 - m_\Lambda^2} \\
&+ \frac{b_2}{f_\pi} \frac{m_N (m_u + m_s)}{-q^2 - m_\Sigma^2} - \frac{b_2 - 2b_1}{3f_\pi} \frac{m_N (m_u + m_s)}{-q^2 - m_\Lambda^2} \Big]
\end{aligned}
$$

Model calculations

- Estimate of LECs from effective models and lattice

  - Strong model dependence
  - Systematic uncertainty may have O(10) for α
    $\Rightarrow$ O(100) in τ_p
  - Lattice calculation presents LEC is around 0.01 GeV^3.
  - More importantly, lattice QCD is able to determine matrix element itself.



Y. Aoki et al. (RBC-UKQCD), PRD78 (2008)

2. Matrix element of proton decay on the lattice

Method in lattice QCD

- There are two ways on the lattice.
- Called as "indirect" method
  - Measurements of LECs in BChPT.
  - Like: Computation is less cost, and obtain precise value.
  - Dislike: Neglecting higher order mass correction → other systematic error
- Called as "direct" method
  - Measurement extracted from 3-pt function.
  - Like: There is no uncertainty depending on models.
  - Like: Provides each decay mode.
  - Dislike: Expensive calculation, and large statistical noise appears.

S. Aoki et al. (JLQCD), PRD62 (2000), Y. Aoki et al.(RBC), PRD75,, Y. Aoki et al. (RBC-UKQCD), PRD78 (2008)

Indirect method

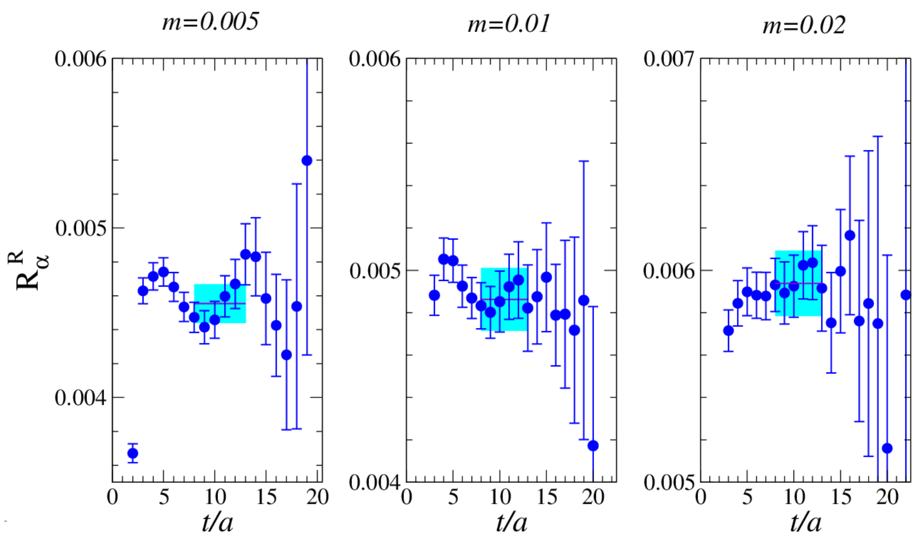
- LECs are defined by amplitude of p→vac with BV op.

$$\langle 0|(ud_R)u_L|p\rangle = \alpha_p P_L u_N, \quad \langle 0|(ud_L)u_L|p\rangle = \beta_p P_L u_N$$

- Two-point function of BV operator and N operator

$$\begin{aligned} C_{NN}(t) &= \langle 0|\eta_N(t)\bar{\eta}_N(0)|0\rangle \simeq |Z_N|^2 e^{-m_N t} \\ C_N^R(t) &= \langle 0|(ud_R)u_L(t)\bar{\eta}_N(0)|0\rangle \simeq \langle 0|(ud_R)u_L|N\rangle Z_N^* e^{-m_N t} \end{aligned}$$

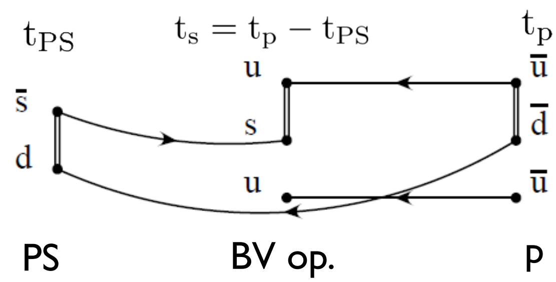
$$\begin{aligned} R_\alpha^R &= Z_N \frac{C_N^R(t)}{C_{NN}(t)} \\ &\simeq \alpha_p \end{aligned}$$



Direct method

- **3-pt function**

  (PS meson)-(BV operator)-(Nucleon)



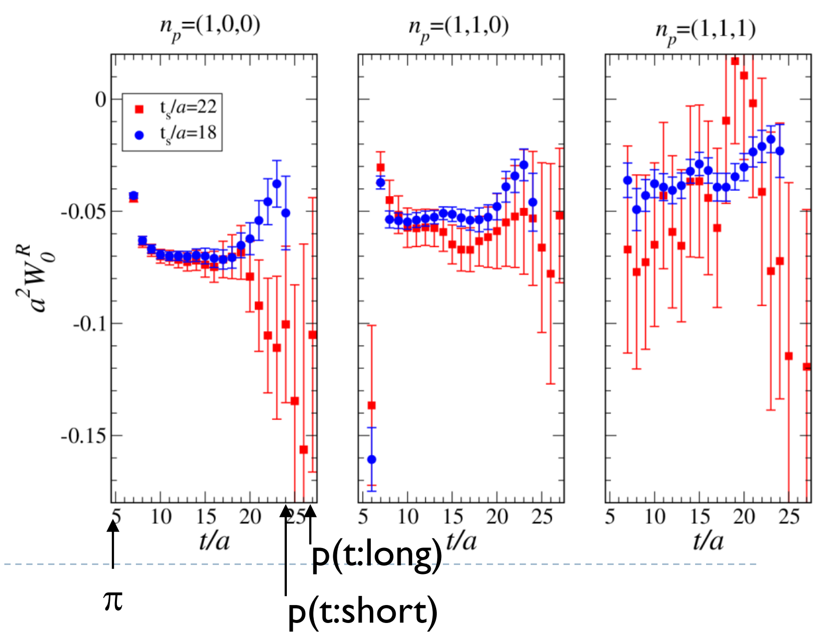
- **Ratio of 3-pt and 2-pt**

$$R_3(t,t_s,p) = \frac{\langle 0|\{\eta_\pi(t_{\rm PS},p)[(ud_R)u_L](t_s,-p)\bar{\eta}_N(0)\}_T|0\rangle}{C_\pi(t_{\rm PS}-t_s,p)C_{NN}(t_s,0)}$$

$$\mathrm{Tr}\Big[R_3(t,t_s,p)P_LP_4\Big] \simeq W_0^L - \frac{iq_4}{m_N}W_1^L,$$

$$\mathrm{Tr}\Big[R_3(t,t_s,p)P_LiP_4\gamma_j\Big] \simeq \frac{q_j}{m_N}W_1^L$$

- Pion and BV op have momentum, however N does not.
- To suppress the excited state, t_s and $t_s - t_{PS}$ should be large.
- Because excited state effect is unknown, we investigate by varying t_s with several values.



2. Matrix element of proton decay on the lattice

Renormalization

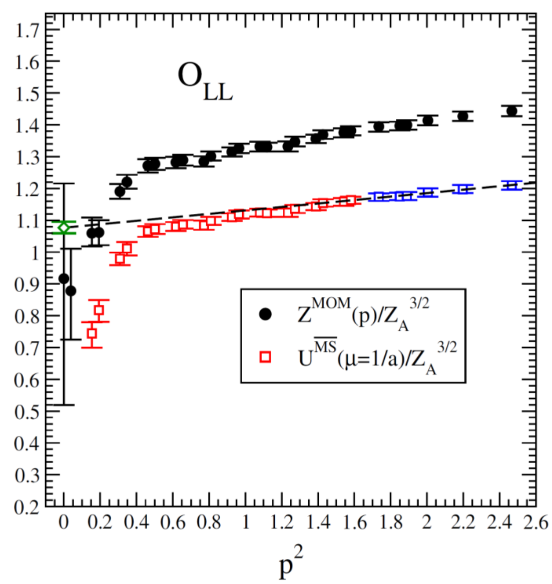
- Renormalization for (qqq) operator (also other kind of high D op.) is also given from matching lattice scheme with MSbar scheme.

See Buchoff's talk

- RI-(s)MOM is easy way to evaluate on present gauge configurations.
  - Non-perturbatively compute three-quark vertex function amputated by quark propagator in Landau gauge fixing.
  - Setting window of p^2 where is small $O((ap)^2)$ at $p \gg \Lambda$, and fitting with function A + B $(ap)^2$
  - One-loop matching

$$U^{\overline{MS}}(\mu) = U^{\overline{MS}}(\mu;p) \underbrace{\frac{Z^{\overline{MS}}(p)}{Z^{MOM}(p)}}_{\text{one loop perturbation}} \underbrace{Z_{ND}(p)}_{\text{Lattice}}$$

$$U^{\overline{MS}}(\mu;p) = \left[\frac{\alpha_s(\mu)}{\alpha_s(p)}\right]^{\gamma_0/2\beta_0} \left[1 + \left(\frac{\gamma_1}{2\beta_0} - \frac{\beta_1\gamma_0}{2\beta_0^2}\right)\frac{\alpha_s(\mu)-\alpha_s(p)}{4\pi}\right]$$



Aoki, et al. (RBC-UKQCD), PRD78(2008)

3. Updated results

Lattice setting

- Domain-wall fermion (DWFs) Nf=2+1 — See also, Blum's talk
  - $24^3 \times 64$ size at a^{-1} = 1.73 GeV (2.5 fm^3 box size)
  - Light quark mass m=0.005, 0.01, 0.02, 0.03 (m_π = 0.3 -- 0.6 GeV)
  - Strange quark mass m_s = 0.04 (m_K = 0.5 GeV)
  - 5^{th} dimension, L_s = 16 in which am_{res} = 0.003
  - AMA with low-mode deflation — Blum, Izubuchi, ES (2013--2014)
- Renormalization constant
  - The mixing with different chirality due to $am_{res} \neq 0$ is negligible.

$$U_L^{\overline{MS}}(\mu = 2\mathrm{GeV}) = 0.705(10), \quad U_R^{\overline{MS}}(\mu = 2\mathrm{GeV}) = 0.706(11)$$

Error is statistical one. In addition, truncation error in one-loop matching is around 8% (roughly), so this will be not negligible.

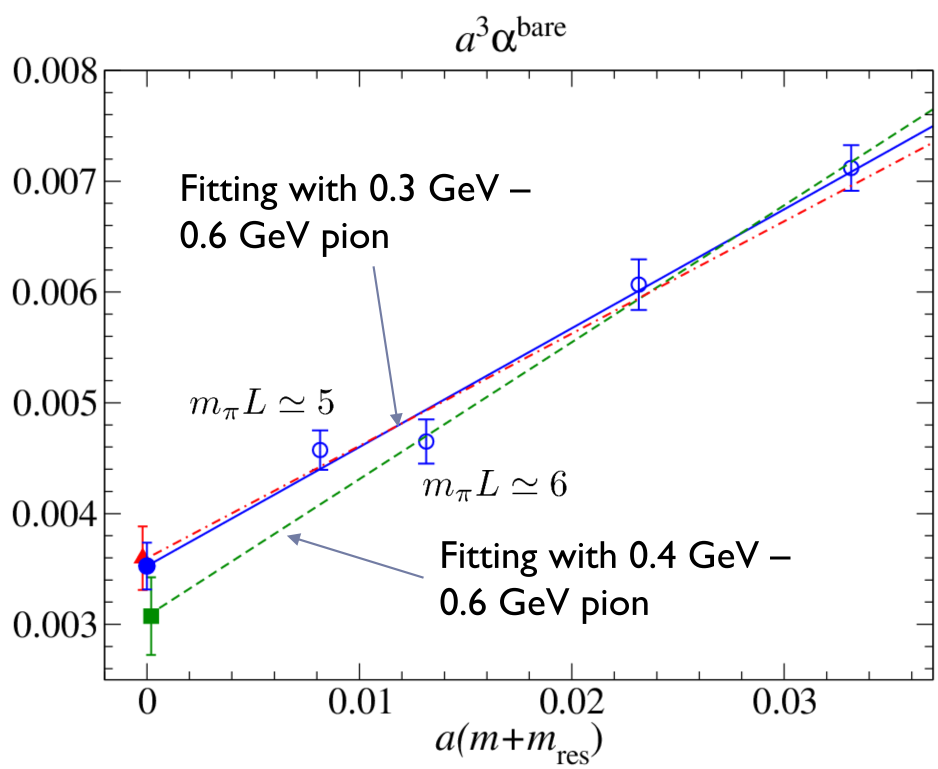
3. Updated results
LECs in BChPT

- Finite volume correction ~ exp[-m_πL]
- Higher order of O(m_π^2), ⇒discrepancy from linearity.

Comparing two fitting, excluding smallest pion or excluding largest pion mass, seemingly finite size is large.

Preliminary:

$$\alpha_p^{\mu=2\mathrm{GeV}} = -0.0142(9)(19)\mathrm{GeV}^3,$$
$$\beta_p^{\mu=2\mathrm{GeV}} = 0.0126(8)(13)\mathrm{GeV}^3$$



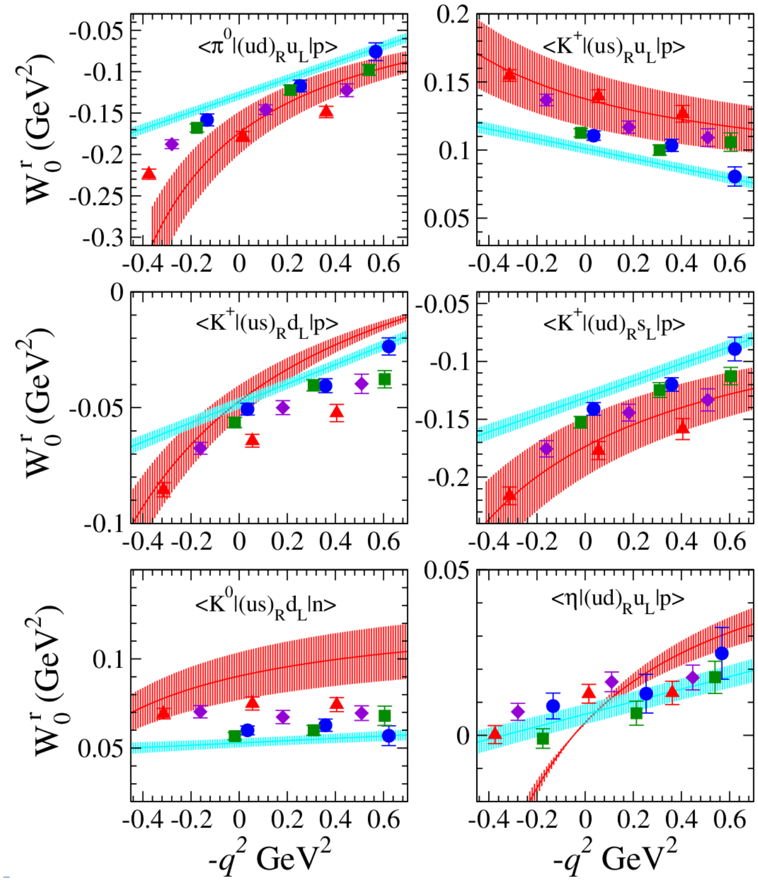
Statistical precision is good, but systematic error of finite size is predominant.
⇒ need to use larger volume than L=3.5fm.

3. Updated results

Behavior on and off physical kinematics

- Straight line: linear extrapolation function in physical pion
- Red line: BChPT using lattice value of α_p .

- Lattice result is not much different from BChPT lines, roughly 10 -- 20%.



3. Updated results

W_0 in physical point

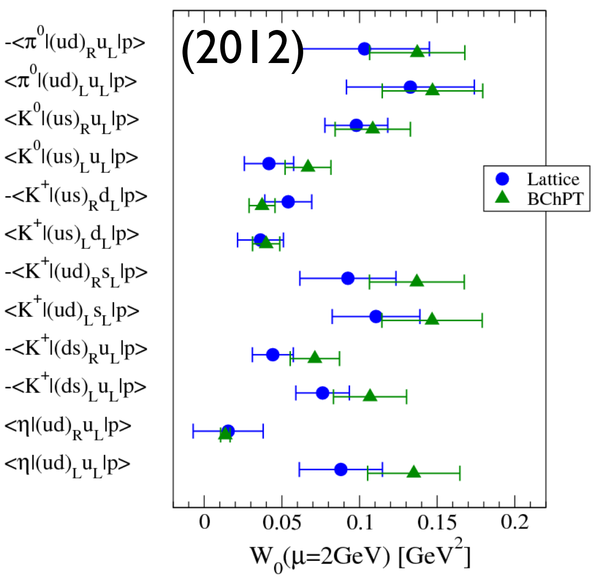
Matrix element	W_0^r GeV^2	fit	artifact	Δ_Z	Δ_a
$-\langle\pi^0\|(ud)_R u_L\|p\rangle$	0.124(6)(14)	0.008	0.006	0.010	0.001
$\langle\pi^0\|(ud)_L u_L\|p\rangle$	0.127(7)(19)	0.016	0.006	0.010	0.001
$\langle K^0\|(us)_R u_L\|p\rangle$	0.099(4)(13)	0.010	0.005	0.008	0.001
$\langle K^0\|(us)_L u_L\|p\rangle$	0.057(3)(6)	0.003	0.003	0.004	0.000
$-\langle K^+\|(us)_R d_L\|p\rangle$	0.046(2)(5)	0.003	0.002	0.004	0.000
$\langle K^+\|(us)_L d_L\|p\rangle$	0.038(2)(8)	0.008	0.002	0.003	0.000
$-\langle K^+\|(ud)_R s_L\|p\rangle$	0.128(6)(14)	0.009	0.006	0.010	0.001
$\langle K^+\|(ud)_L s_L\|p\rangle$	0.136(7)(21)	0.018	0.007	0.010	0.001
$-\langle K^+\|(ds)_R u_L\|p\rangle$	0.052(2)(7)	0.006	0.003	0.004	0.000
$-\langle K^+\|(ds)_L u_L\|p\rangle$	0.097(4)(13)	0.010	0.005	0.007	0.001
$\langle\eta\|(ud)_R u_L\|p\rangle$	0.006(3)(5)	0.005	0.000	0.000	0.000
$\langle\eta\|(ud)_L u_L\|p\rangle$	0.109(6)(14)	0.011	0.005	0.008	0.001

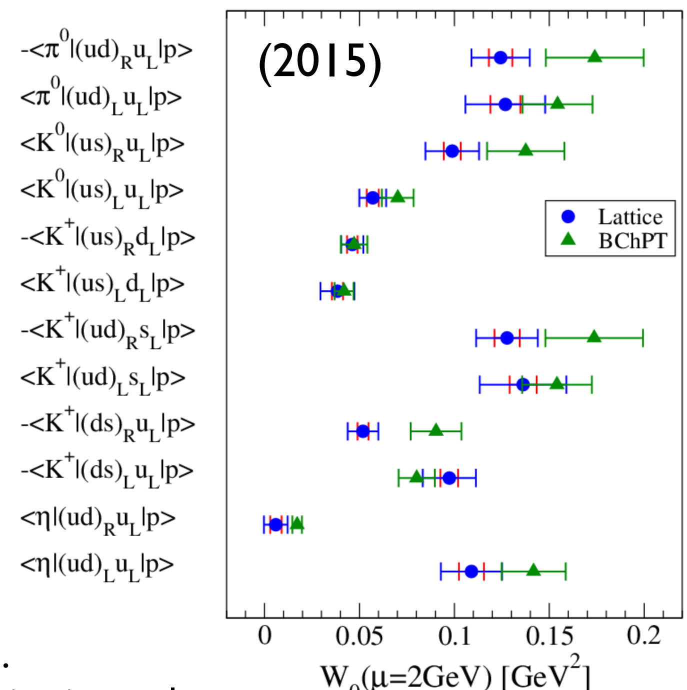
Extrapolation error, finite size correction is rather large.

Truncation error of renormalization constant.

3. Updated results

Comparison with BChPT





- Precision of W_0 is much improved.
- Systematic error are now dominating in total error.
- Decay width ($p \rightarrow \pi$) : factor 1.3 difference,
  $\Rightarrow$ impact to p lifetime is about factor 2.

4. Extension to dark matter model

Induced n decay model

Davoudiasl et al., PRL105(2010), PRD84(2011), Davoudiasl PRL114(2015)

- Assume the dark matter (Φ,Ψ) annihilate nucleon, producing energetic meson,

$$\Phi N \to \bar{\Psi} P(\pi, K, \eta), \quad \Psi N \to \Phi^\dagger P(\pi, K, \eta)$$

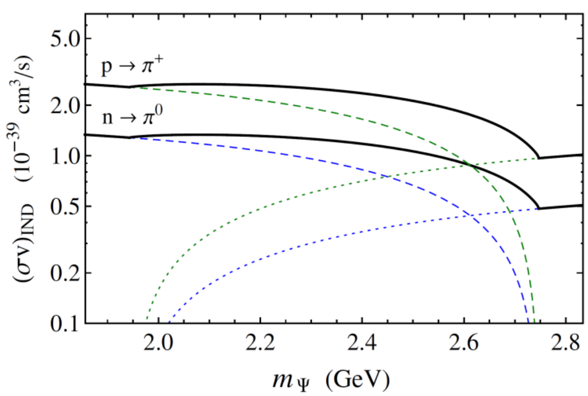
- Assume DM has B number, total $B_\Phi + B_\Psi = -1$, then its coupling is

$$\mathcal{L}_{\text{eff}} = M^{-2} u_R u_R d_R \Psi_R \Phi + h.c.$$

- DM mass is estimated from $\Omega_{DM}/\Omega_N \simeq 5$, $m_{\Phi,\Psi}$ = 1.7 – 2.9 GeV.
- Proton can decay into meson and DM (induced nucleon decay)
  - Proton decay cross section is evaluated as a function of DM mass based on chiral Lagrangian,

$$\mathcal{L}_{\text{IND}} = \beta_p \text{Tr}[c\xi^\dagger (B_R \Psi_R) \Phi \xi]$$

In the IND model, the QCD matrix element is same as ordinal p decay.



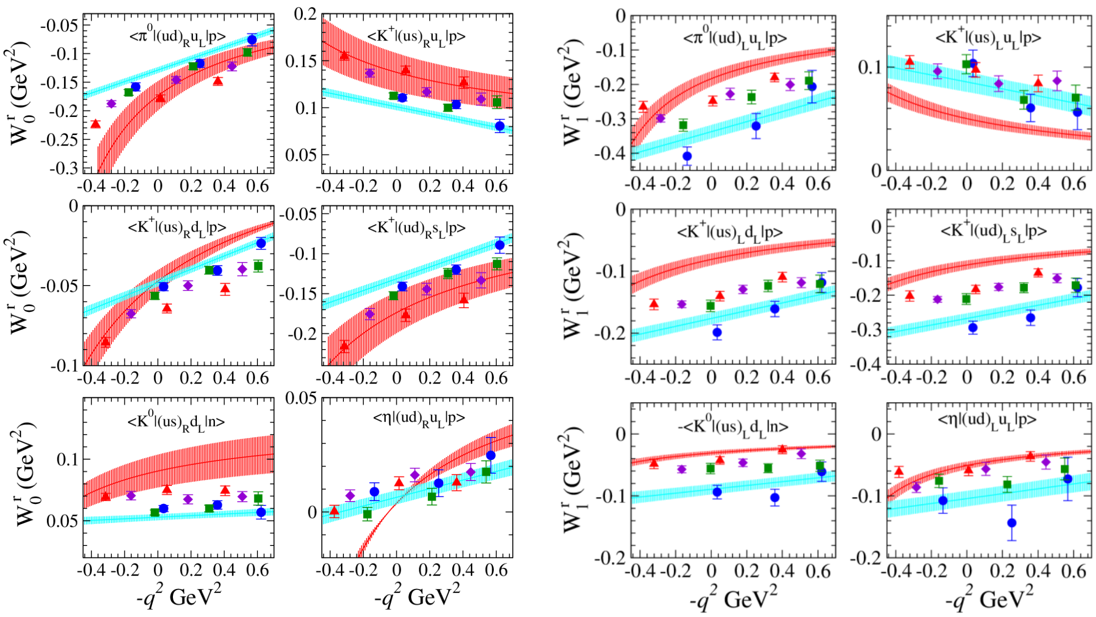
4. Extension to dark matter model

Matrix element in IND model

- Matrix element involves two relevant form factor W_0 and W_1

Standard: $$\langle \pi^0(\vec{p})|(ud)_\Gamma(u)_{\Gamma'}|p(\vec{k},s)\rangle = P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) + \boxed{\frac{m_l}{m_p}W_1^{\Gamma\Gamma'}(q^2)}\right]u_p(\vec{k},s)$$

IND: $$\langle \pi^0(\vec{p})|(ud)_\Gamma(u)_{\Gamma'}|p(\vec{k},s)\rangle_{\mathrm{IND}} = P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) + \boxed{\frac{m_{\Phi,\Psi}}{m_p}W_1^{\Gamma\Gamma'}(q^2)}\right]u_p(\vec{k},s)$$



Lattice W_1 is largely distinct from BChPT.

Large uncertainty may appear into IND model estimate.

Summary

- Improved the precision of matrix element for p decay.
- Limitation of the BChPT became apparent.
- It may increase p lifetime to factor 3.
- Lattice QCD allows to provide more reliable estimate in other kinematics, e.g. induced DM model
- Approaching to physical pion is more important for p decay amplitude.
  Martin and Stavenga, PRD85(2012)
  - There may appear significant effect of pion mass dependence.
    An order of magnitude may be different.
  - Computation close to physical pion (~170 MeV) is underway.

Backup

BV effective operators at low-energy

- 4-fermi BV operators in the SM

$$\begin{aligned}
\mathcal{O}^1_{abcd} &= (D^i_a, U^j_b)_R(q^{k\,\alpha}_c, l^\beta_d)_L\varepsilon^{ijk}\varepsilon^{\alpha\beta}, && : (q,q)_R\ (q,l)_L \\
\mathcal{O}^2_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L(U^k_c, l_d)_R\varepsilon^{ijk}\varepsilon^{\alpha\beta}, && : (q,q)_L\ (q,l)_R \\
\widetilde{\mathcal{O}}^4_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L(q^{k\,\gamma}_c, l^\delta_d)_L\varepsilon^{ijk}\varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma}, && : (q,q)_L\ (q,l)_L \\
\mathcal{O}^5_{abcd} &= (D^i_a, U^j_b)_R(U^k_c, l_d)_R\varepsilon^{ijk} && : (q,q)_R\ (q,l)_R
\end{aligned}$$

a,b,c,d : generation,
$\alpha,\beta,\gamma,\delta$: SU(2) indices,
i,j,k: color indices

Weinberg, PRL43, 1566(1979), Wilczek and Zee, PRL43, 1571(1979)

- 3-quark operator

$$\mathcal{O}^{\Gamma\Gamma'}_{abc} = (q_a q_b)_\Gamma q_{c\Gamma'} = (q^{Ti}_a C P_\Gamma q^j_b) P_{\Gamma'} q^k_c \varepsilon^{ijk}$$

- Reduced number of matrix element with Parity and flavor SU(2) symmetry

$$\langle PS; \vec{p}|\mathcal{O}^{LL}|N; \vec{k}, s\rangle = \gamma_4\langle PS; -\vec{p}|\mathcal{O}^{RR}|N; -\vec{k}, s\rangle,$$
$$\langle PS; \vec{p}|\mathcal{O}^{LR}|N; \vec{k}, s\rangle = \gamma_4\langle PS; -\vec{p}|\mathcal{O}^{RL}|N; -\vec{k}, s\rangle$$
$$\langle \pi^+|\mathcal{O}^{L/R}_{udu}|p\rangle = \sqrt{2}\langle \pi^0|\mathcal{O}^{L/R}_{udu}|p\rangle$$

⇒ Total matrix element is 12.