Nucleon EDM computation from lattice QCD

Eigo Shintani (RIKEN-AICS)
for RBC/UKQCD collaboration

Program INT-15-3, Intersections of BSM Phenomenology and QCD for New Physics Searches, September 14 - October 23, 2015

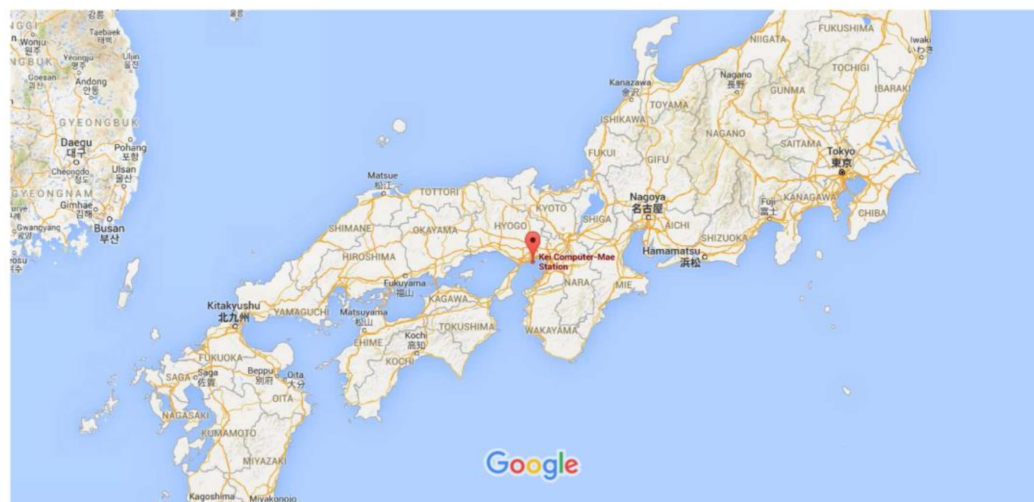

SOAMES/DU
LATTE
2015 09 15

2015 09 21

RIKEN Advanced Institute for Computational Science (RIKEN-AICS)

- Approved K project in 2005
- ~$1 billion budget (2006—2013)
- Research division (Director: A. Ukawa)
  - 19 teams total
  - Field theory research term (leader: Y. Kuramashi)
  - 5 local member (Y. Nakamura, H. Suno, ES, Y. Shimizu, Y. Yoshimura), and 3 visiting member
  - Location: Kobe city

    tasty meats, sweets …



Map data ©2015 Google, SK planet, ZENRIN 100 km

K computer

- Advanced supercomputer for computational science
- Kei(京) = 10^{16} = 10 Peta magnitude performance
- 4th ranking in Top500 in 2015 June

Computation node specs	CPU	SPARC64™ VIIIfx 2GHz
	Performance	128 GF (16 GF x 8 cores)
	Memory	16GB
Number of racks		864
Number of nodes		82,944
Network		Tofu Interconnect (6D Mesh/Torus)
Peak performance		10.62 PF
Total memory capacity		1.26 PB
File system		Fujitsu Exabyte File System (FEFS)
Storage		30 PB

Top500 No.1 in June & November 2011
Graph500 No.1 in June 2014 & July 2015
HPCG No.2 in November 2014 & 2015

The K computer strikes a balance between performance in calculation, memory, and communication

Outline

- Introduction
- Strategy and method in lattice QCD
- Recent update of RBC/UKQCD work
- Summary and future work

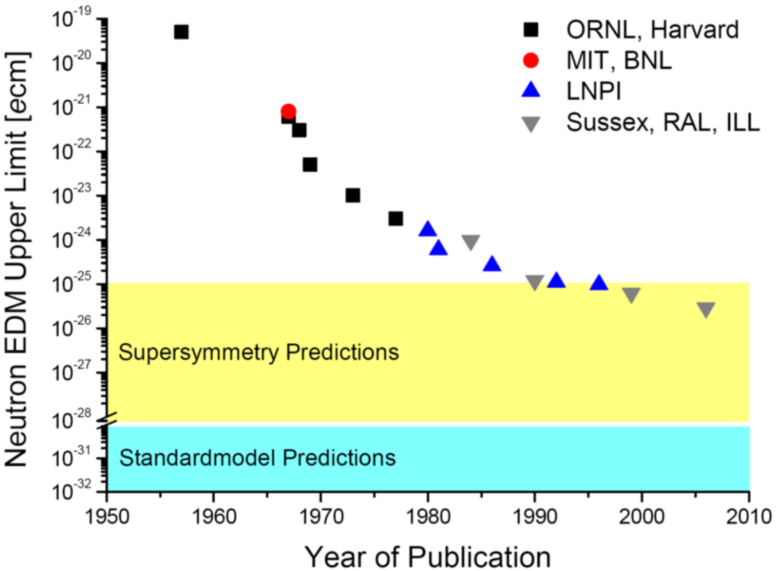
Neutron EDM

- Discovery of CPV in QCD and the new physics (NP)
- Since 1960's, sensitivity of experiment has been developed.
  - Current nEDM upper limit is $|d_N^{\rm exp}| < 2.9 \times 10^{-26}\,{\rm e\cdot cm}$
- Theoretical interests
  - Unnaturally small θ term in QCD. (strong CP problem)
  - CPV from NP

    Severe BSM (SUSY, etc) constraints
  - Intensity frontier physics

    Alternative direction from high energy frontier.

    Precision of the SM calculation is necessary.



/wiki/Neutron_electric_dipole_moment

Also see talk by Andrea Shindler

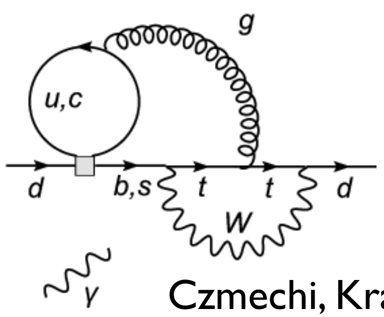
Theoretical estimate of nEDM in EW

- EW contribution to EDM is very small
  - Proportional to weak phase, $\Delta = \mathrm{Im} V_{cs}^* V_{us} V_{cd} V_{ud}^*$
  - No CPV phase in 1-loop ($|V_{dq}|^2$) and 2-loop diagram (cancelation).

Theoretical estimate of nEDM in EW

- EW contribution to EDM is very small
  - Proportional to weak phase, $\Delta = \mathrm{Im} V_{cs}^* V_{us} V_{cd} V_{ud}^*$
  - No CPV phase in 1-loop ($|V_{dq}|^2$) and 2-loop diagram (cancelation).
  - Three-loop order(short) and pion loop correction (long):

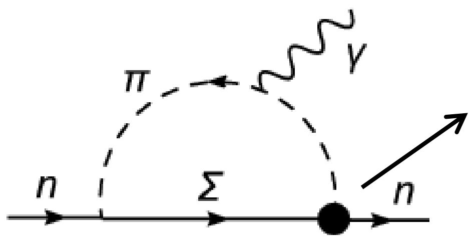
Short distance

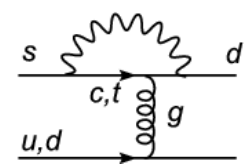


Czmechi, Krause (1997)

$d_N^{\mathrm{KM\,short}} \sim \mathcal{O}(\alpha_s G_F^2) \sim -10^{-34}\,\mathrm{e}\cdot\mathrm{cm}$

Long distance





Khriplovich, Zhitnitsky (1982), Seng (2015)

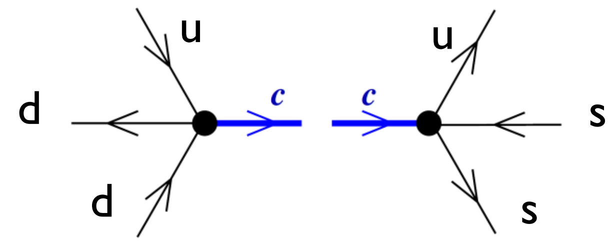
$d_N^{\mathrm{KM\,long}} \sim 10^{-32}\,\mathrm{e}\cdot\mathrm{cm}$

$$\Rightarrow d_N^{\mathrm{KM}} = d_N^{\mathrm{KM\,short}} + d_N^{\mathrm{KM\,long}} \simeq 10^{-30} - 10^{-32}\,\mathrm{e}\cdot\mathrm{cm}$$

which is the 6-order magnitude below the experimental upper limit.
(to confirm, non-pertubative estimate is also needed)

Recent update in EW sector

- Charm contribution Mannel, Uraltsev, PRD85 (2012)
  - Loop-less EDM contribution
  - Integrating out the virtual charm,



$$\tilde{O}_{uds} = (\bar{u}\Gamma_\mu d)(\bar{d}\Gamma^\mu i\not{D}\Gamma^\nu s)(\bar{s}\Gamma_\nu u)$$
$$= \underbrace{(\bar{u}\Gamma_\mu d)(\bar{d}\Gamma^\mu i\gamma^\alpha\Gamma^\nu s)i\partial_\alpha(\bar{s}\Gamma_\nu u)}_{O^\alpha_{uds}} + \underbrace{(\bar{u}\Gamma_\mu d)(\bar{d}\Gamma^\mu i(\not{D}\Gamma^\nu s))(\bar{s}\Gamma_\nu u)}_{O_{uds}}$$

$$\mathcal{L}^\alpha \sim \frac{2}{3}\mathcal{O}^\alpha_{uds} + i\int d^3x J^\alpha_{\rm em} O_{uds}, \quad \langle N(q+p)|\mathcal{L}^\alpha|N(p)\rangle = d_N^{uds} q_\nu \bar{u}_N i\sigma^{\alpha\nu}\gamma_5 u_N$$

Using dimensional analysis, $d_N^{uds} \sim \kappa\mu^5_{\rm had},\ \kappa \simeq 1/3$

$$|d_N| = \frac{32}{3}e\frac{G_F^2\Delta}{m_c^2}|d_N^{uds}| = 3.3\times 10^{-31} e\cdot{\rm cm}\times\kappa\left(\frac{\mu_q}{0.25{\rm GeV}}\right)^6\left(\frac{0.5{\rm GeV}}{\mu_{\rm had}}\right)^6$$
$$\sim 10^{-31} e\cdot{\rm cm}$$

Indicate that higher order of loop-less charm will be same order as chiral loop.
Lattice calculation is helpful, but seems practically hard…

Nucleon EDM in QCD

- θ term in QCD Lagrangian

$$\mathcal{L} = \bar{q}_L M q_R + \bar{q}_R M^\dagger q_L + \theta/(64\pi^2) G\widetilde{G}$$

$$\Rightarrow \mathcal{L}_\theta = \bar{\theta}\frac{1}{64\pi^2}G\widetilde{G}, \quad \bar{\theta} = \theta + \arg\det M, \quad \arg\det M \sim \eta\frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}$$

  - Renormalizable and CPV.
  - $d_N/\theta \sim 10^{-16}$ e・cm (quark model, current algebra, etc)

    θ and arg det M is *unnaturally* canceled. Crewther, et al. (1979), Ellis, Gaillard (1979)

- Possible solution

  1. Massless quark (m_u = 0) Blum, et al. (2010)

  from lattice QCD+QED, m_u = 2.24(35) MeV, m_d = 4.65(35) MeV.

  It is hard to explain $\bar{\theta} = 0$.

  2. *Axion model (assumption of PQ symm.),* invisible axion model

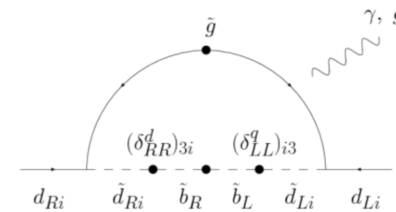
  3. *Spontaneous CP breaking.* θ is calculable in loop order.

Nucleon EDM in BSM

- Higher dimension operators of CPV

$$H_{CP\!\!\!/} = \sum_k C_k(\mu)\, \mathcal{O}_k \qquad \begin{array}{rcll} \mathcal{O}_{\rm qEDM} &=& d_q \bar{q}(\sigma\cdot F)\gamma_5 q & \text{: Quark-photon (5-dim)} \\ \mathcal{O}_{\rm cEDM} &=& d^c_q \bar{q}(\sigma\cdot G)\gamma_5 q & \text{: Quark-gluon (5-dim)} \\ \mathcal{O}_{\rm Weinberg} &=& d^G G G \widetilde{G} & \text{: Pure gluonic (6-dim)} \end{array}$$

- In BSM, q(c)EDM corresponds to CPV phase of heavy particle.
- Need to know low-energy scale matrix element.
- d_q d_q^c are determined by BSM renormalization

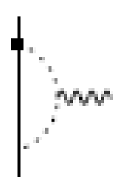


Hisano, et al. (2009)

- QCD effect in nucleon

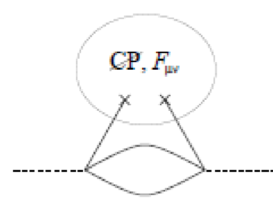
Using baryon CHPT or QCD sum rule, there are several evaluations,

$$\mathcal{L}_{\rm eff} = d_q \bar{q} F_{\mu\nu}\sigma_{\mu\nu}\gamma_5 q, \quad \langle N|L_{\rm eff}|N\rangle = d_N q_\nu \bar{u}_N(p+q) i\sigma_{\mu\nu}\gamma_5 u_N(p)\Big|_{q=0}$$



⋯

Mereghetti, Vries, Hockings, Maekawa, Kolck, Timmermans, ...



⋯

Pospelov, Ritz, Hisano, Shimizu, Nagata, Lee, Yang, …

Constraint on nEDM

- ~10 new proposals of EDM experiment

  nEDM experiment @ ORNL, ILL, FRM-2, FNAL, PSI/KEK/TRIUMF, …
  (p,d)EDM experiment @ BNL,
  Charged particle (d, p)EDM @ COSY
  Lepton EDM @ J-PARC, FNAL

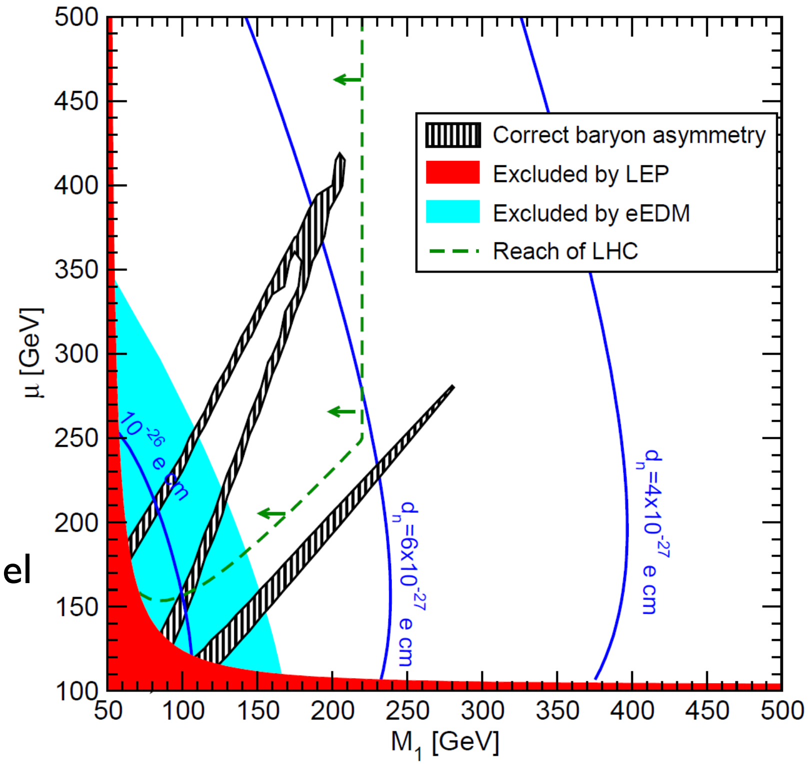
  aiming for a sensitivity to 10^{-29} e･cm !

- SUSY models favor 10^{-25} -- 10^{-27} e cm

$$\begin{aligned} d_N &\sim 10^{-17}[\mathrm{e\cdot cm}]\bar{\theta} \\ &+ (1.4\text{--}0.47)d_d - (0.12\text{--}0.35)d_u \\ &+ O(10^{-2})d_q^c \\ &\sim O(10^{-25}\text{--}10^{-27})\,\mathrm{e\cdot cm} \end{aligned}$$

Effective model estimate

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)



Ramsey, 0907.2716

Non-perturbative test in lattice QCD is important.

What lattice QCD can do ?

- In principle
  - Direct estimate of matrix element of neutron and proton EDM for θ term, higher dim. CPV operators
  - LECs (or condensate) including higher dimension operators

    → check validity of QCD sum rule, ChPT, …

Bhattacharya et al, Lattice 2012--2015

What lattice QCD can do ?

- In principle
  - Direct estimate of matrix element of neutron and proton EDM for θ term, higher dim. CPV operators
  - LECs (or condensate) including higher dimension operators

    → check validity of QCD sum rule, ChPT, …

    Bhattacharya et al, Lattice 2012--2015
- In practice there are some difficulties
  - Statistical noise

    gauge background (topological charge, sea quark) and *disconnected diagram* (flavor singlet contraction) are intrinsically noisy.
  - Systematic study

    Volume effect may be significant. (e.g. BChPT discussion)

    O'Connell, Savage, PLB633(2006), Guo, Meissner, 1210.5887

    Chiral behavior is also important, $d_N \sim O(m)$.

Strategy and method in lattice QCD

Lattice methods of nEDM

- Spectrum method
  - Aoki and Gocksch, PRL63, 1125 (1989).

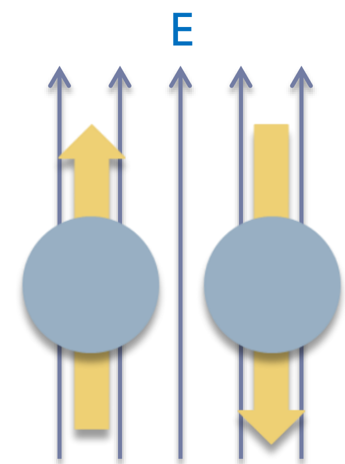
    Aoki, Gocksch, Manohar, Sharpe, PRL65, 1092 (1990), in which they discussed about the possible lattice artifact in Aoki's paper
  - ES, et al., for CP-PACS collaboration, PRD75, 034507 (2007)
  - ES, Aoki, Kuramashi, PRD78, 014503 (2008)
- Form factor
  - ES, et al., for CP-PACS collaboration, PRD72, 014504 (2005), Berruto, et al. for RBC collaboration, PRD73, 05409 (2006).
  - ES et al., RBC Lattice 2008, 2013, Shindler 2015, Izubuchi Lattice 2015.
- Imaginary θ
  - Izubuchi, Lattice 2007.
  - Horsley et al., arXiv:0808.1428 [hep-lat], Guo et al., PRL115(2015)

Spectrum method

Aoki, Gocksch, PRL63(1989); ES, et al., PRD75(2007)

- Measurement of energy shift depending on E

$$\Delta H(\theta, E) = \theta d_N \vec{s} \cdot \vec{E}$$



- given 2-pt function in constant E field

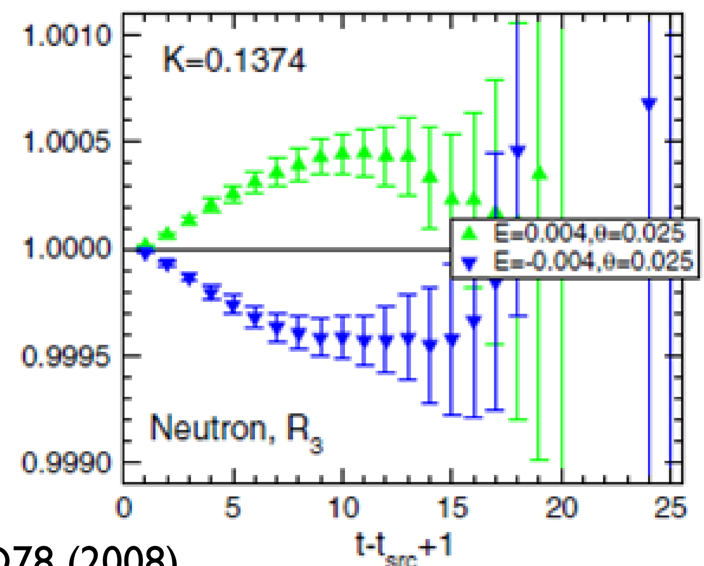
$$m_{\uparrow \text{spin}} - m_{\downarrow \text{spin}} = 2d_N\theta E$$

→ Linear response, gradient is a signal of EDM.

- Ratio of spin up and down

$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\text{up}}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\text{down}}_{\theta,E}} \quad \simeq \quad 1 + d_N E\theta t$$

- Making the spin-projection to up, down, and ratio in θ term ensemble (reweighting).
- Direct extraction without momentum insertion, but error of boundary broken by E field appears.



ES, Aoki, Kuramashi, PRD78 (2008)

EDM Form factor

ES, et al., Phys. Rev. D72, 014504 (2005),
Berruto, et al., Phys. Rev. D73, 05409 (2006).

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle = \bar{u}_N\Big[\underbrace{\frac{F_3^\theta(q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}q_\nu}_{\text{P,T-odd}} + \underbrace{F_A(q^2)(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)}_{\text{P-odd}}$$
$$+ \underbrace{F_1(q^2)\gamma_\mu + \frac{F_2(q^2)}{2m_N}\sigma_{\mu\nu}q_\nu}_{\text{P,T-even}}\Big]u_N$$

- CP-odd phase including N amplitude should be subtracted from 3pt function.
- CP-odd phase is able to be evaluated from 2pt function in θ term with γ_5 projection.

$$\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N} \quad \Rightarrow \text{obtain CPV phase } \alpha_N$$

- Subtraction formula

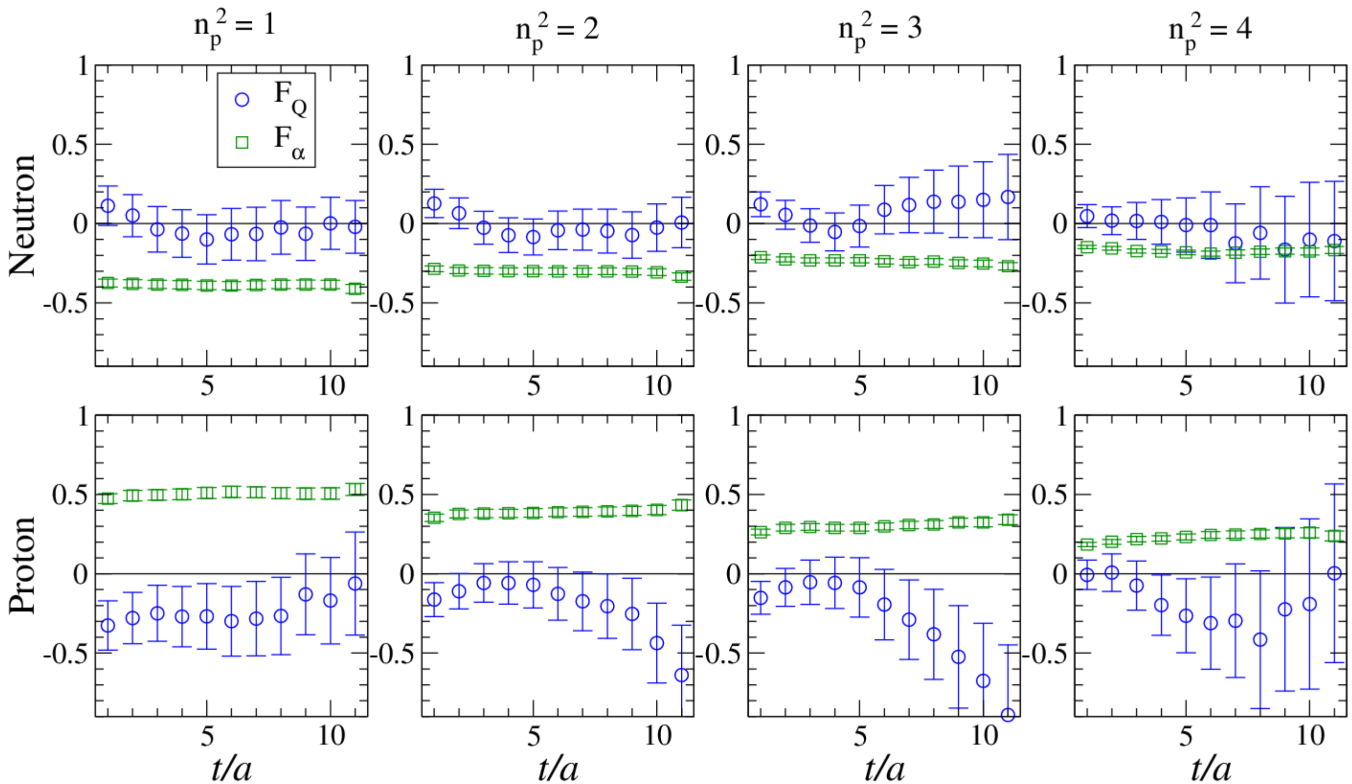
$$\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)\rangle$$
$$= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma+m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5 \Big\}\ \text{Subtraction}$$
$$+\frac{1+\gamma_4}{2}F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N}\frac{ip\cdot\gamma+m_N}{2E_N} \Big\} \Rightarrow \text{EDM form factor}$$

Subtraction term and 3pt function

- Splitting EDM form factor into two parts:

$$F_3 = F_Q + F_\alpha,\ F_Q = C(m_N)\langle N J_t^{\mathrm{EM}} \bar{N} Q\rangle,\ F_\alpha = F(\alpha_N, F_{1,2})$$

- F_α is good precision, but statistical fluctuation of F_Q is still large.



Imaginary θ

Izubuchi(2007), Horsley et al. (2008), Guo et al., (2015)

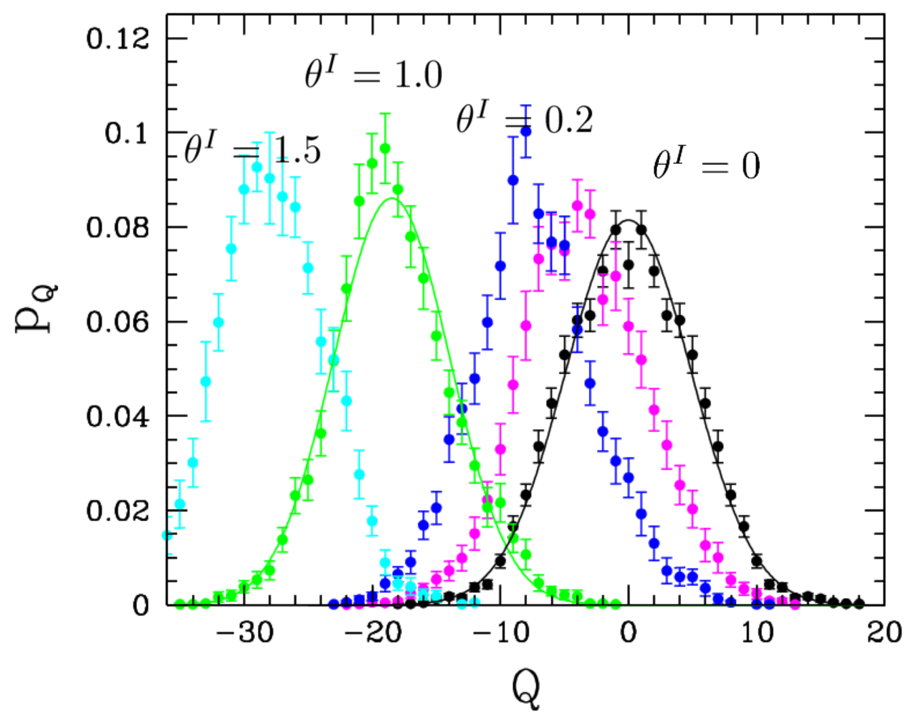
- Analytical continuation to pure imaginary θ

$$\langle O\rangle_\theta \to Z_{\theta^I}^{-1}\int dU O e^{-S} e^{-\theta^I Q} = \sum_i p_i[\theta^I] O_i, \quad p[\theta^I] = e^{-S_{QCD}-\theta^I Q}$$

using $\theta \to \theta^I$ and p is distribution function with imaginary theta term.

- There is no sign problem, then expect better signal.
- Need to generate the new QCD ensemble with θ^I: distribution of Q is shifted by θ^I

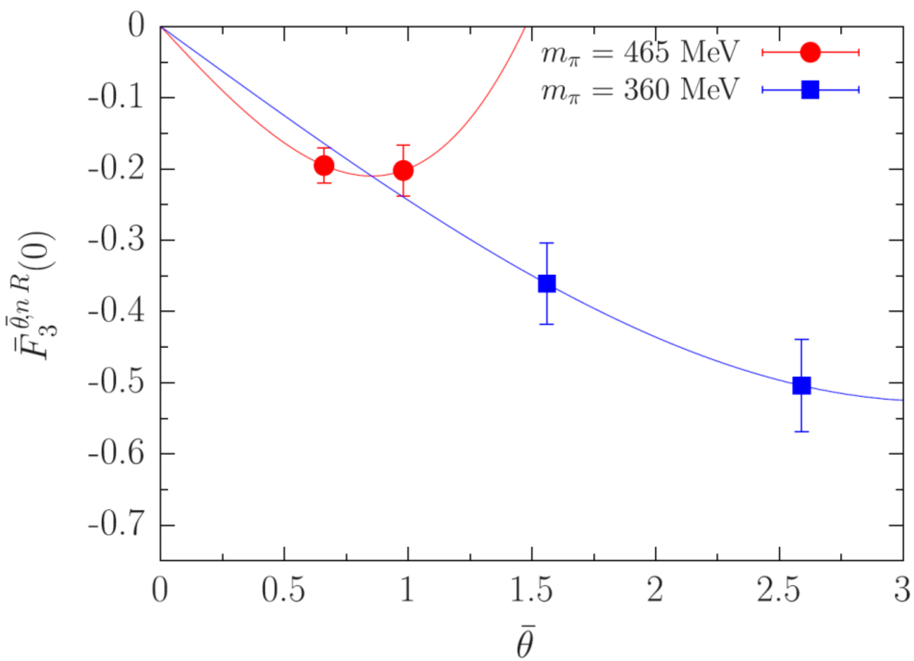
⇒ it will be challenging work when going to realistic lattice (larger lattice and physical quark mass)

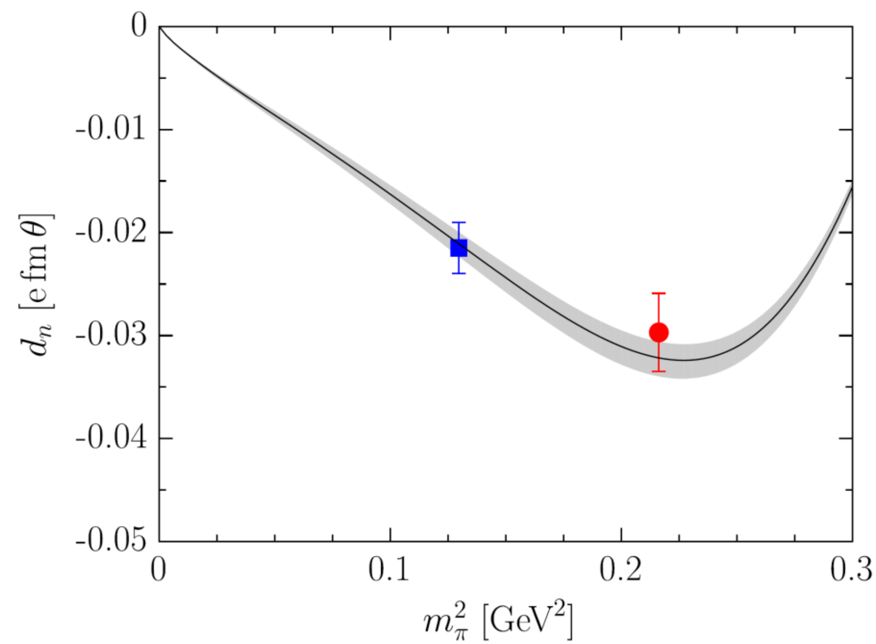


Imaginary θ

- Recent calculation of EDM form factor

Guo et al., PRL115 (2015)





- The slope of imaginary θ dependence for $F_3(\theta^I)$ is regarded as EDM form factor.
- Statistical signal is much better.

Talk by Schierholz in 3rd week.

Recent update of RBC/UKQCD work

Lattice setup of RBC/UKQCD group

- Domain-wall fermion
  - Chiral symmetry (χ) is well conserving in lattice QCD.
  - χ breaking is represented as m_{res} : chiral limit $m+m_{res} \rightarrow 0$
  - Precise result is achieved in Kaon physics (e.g. $K\rightarrow\pi\pi$)

Blum et al., PRD91(2015)

  - Development of algorithm to make speed-up computation of correlation function, (e.g. all-mode-averaging)

Blum, et al., PRD88(2013), ES et al. PRD91(2015)

  - m_π = 170, 330, 420 MeV, lattice size $\simeq$ 4.6 fm^3, 2.7 fm^3
  - Computational cost is much larger than Wilson-type fermion.
  - O(10,000) measurements of EM and EDM form factor

3. Recent update
q^2 dependence

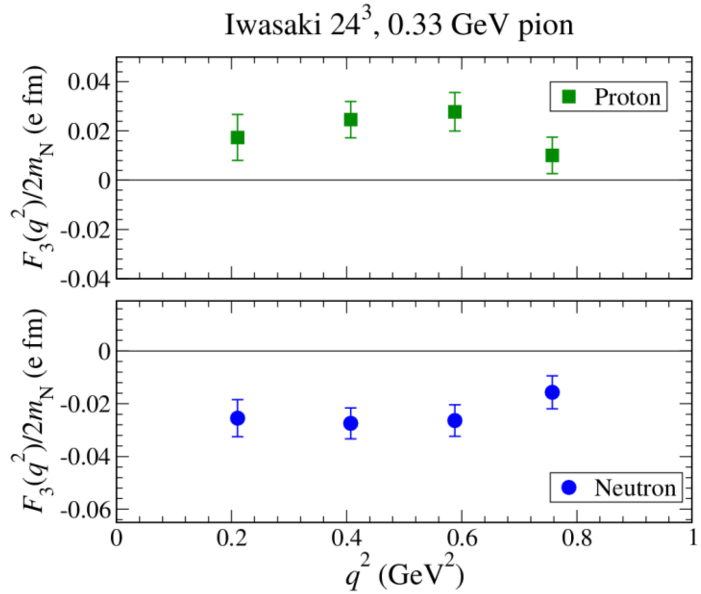
- $d_N = F_3(q^2)/2m_N$ ($q^2 \rightarrow 0$)
- BChPT describe $F_3(q^2)$ with LECs

$$F_3^i(q^2) = d_N^i - S_i' q^2 + H_i(q^2),\ i = \{p+n, p-n\}$$

$$S'_{p+n} = -\frac{e g_A \bar{g}_0 \pi}{12(2\pi F_\pi)^2 m_\pi^2} \frac{\delta m_N}{m_\pi}$$

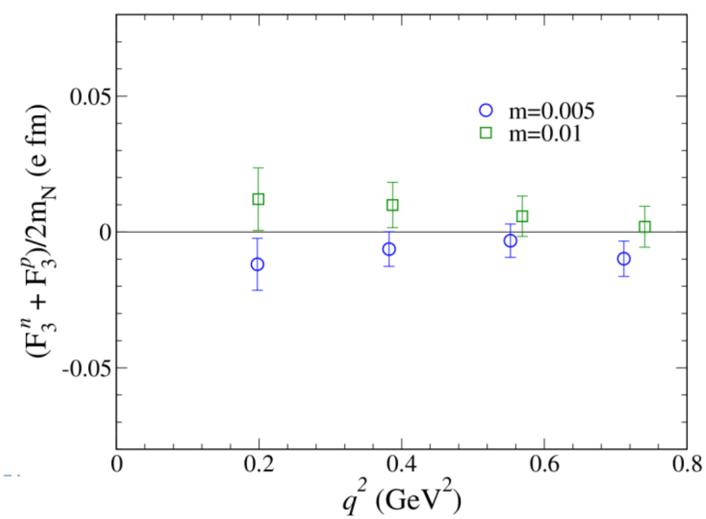
$$S'_{p-n} = \frac{e g_A \bar{g}_0}{6(2\pi F_\pi)^2 m_\pi^2}\left[1 - \frac{5\pi m_\pi}{4 m_N} - \frac{\breve{\delta} m_\pi^2}{m_\pi^2}\right]$$

Mereghetti et al., PLB696 (2011)



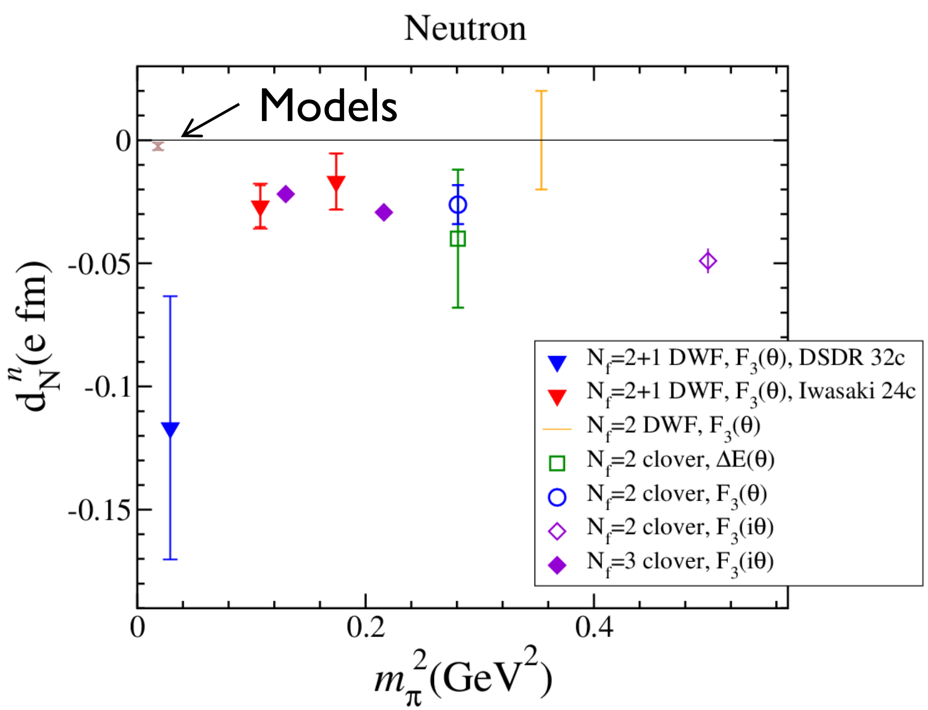
Isoscalar S'_{p+n} << isovector S'_{p-n} at $m_\pi \rightarrow 0$

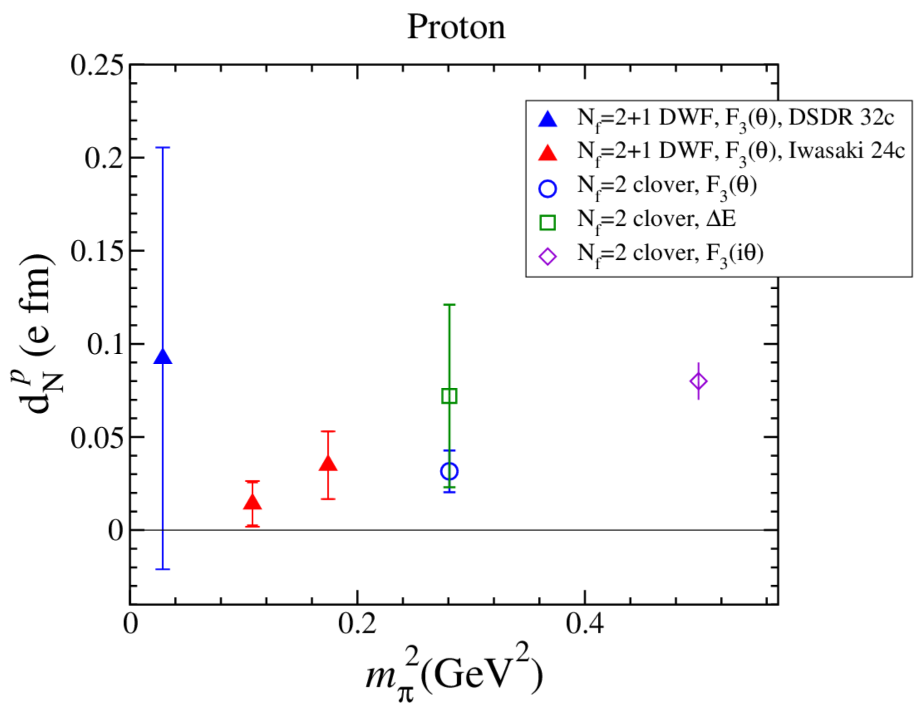
- EDM form factor has opposite sign between p and n, then it tends to be BChPT prediction.
- Lattice QCD (330 MeV pion) gives
  $S'_p = -2(2) \times 10^{-4}$ e fm^3,
  $S'_n = 0.1(1.0) \times 10^{-4}$ e fm^3
- S'_{p+n} may be constant, and it is below 10^{-4} e fm^3



3. Recent update

Mass dependence





- Vanishing at chiral limit (CP conserving), but not clear in RBC result.
- BChPT at NLO Mereghetti et al., PLB696 (2011)

$$|d_n| \simeq |d_p| > \frac{e g_A \delta m_N}{2(2\pi F_\pi)^2 \varepsilon}\left[\ln\frac{m_N^2}{m_\pi^2} + \frac{\pi m_\pi}{2m_N} - \frac{\breve{\delta} m_\pi^2}{m_\pi^2} + \pi\frac{\delta m_N}{m_\pi}\right] \simeq 2\times 10^{-3} e\cdot\mathrm{fm}$$

Lattice QCD is O(10) magnitude large at m_π ~ 300 MeV, strong mass dependence ?

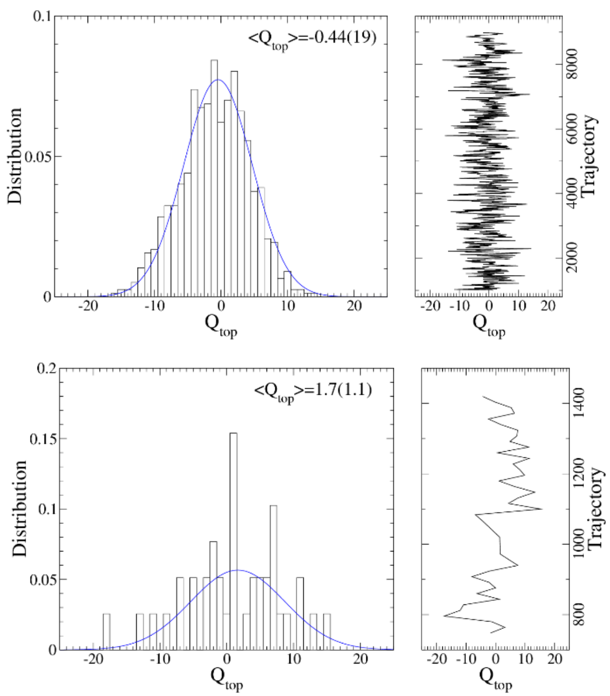
Dependence on topological charge

- According to ChPT argument Abada et al., PLB256(1991), Aoki, Hatsuda PRD45(1992)

$$d_N \simeq \frac{2}{F_\pi^2}\chi_Q^2\mu_N\frac{\bar{g}_{\pi NN}}{2m_N}, \quad \chi_Q = m_\pi^2 F_\pi^2 \frac{m_{\eta'}^2 - m_\pi^2}{2N_f m_{\eta'}^2}$$

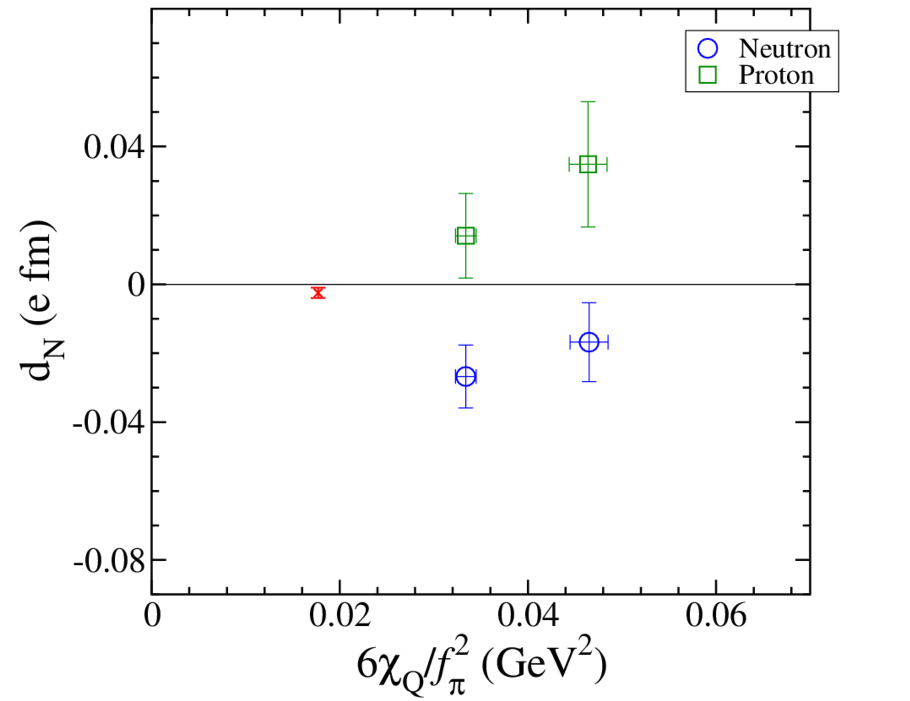
⇒ proportional to magnetic moment and topological charge distribution.

- ➢ Q distribution should be close to Gaussian. See also Shinlder's talk
- ➢ χ_Q has to be mass suppression.



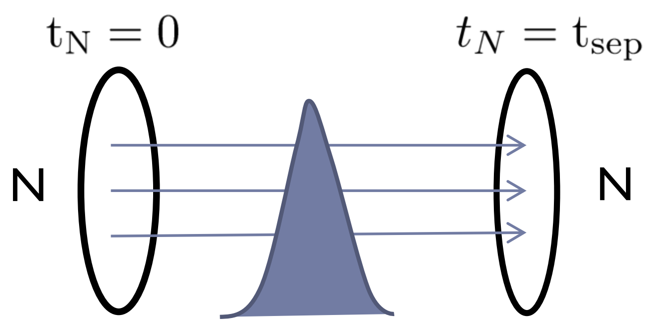
DWF
330 MeV

DWF
170 MeV



Local topology contribution

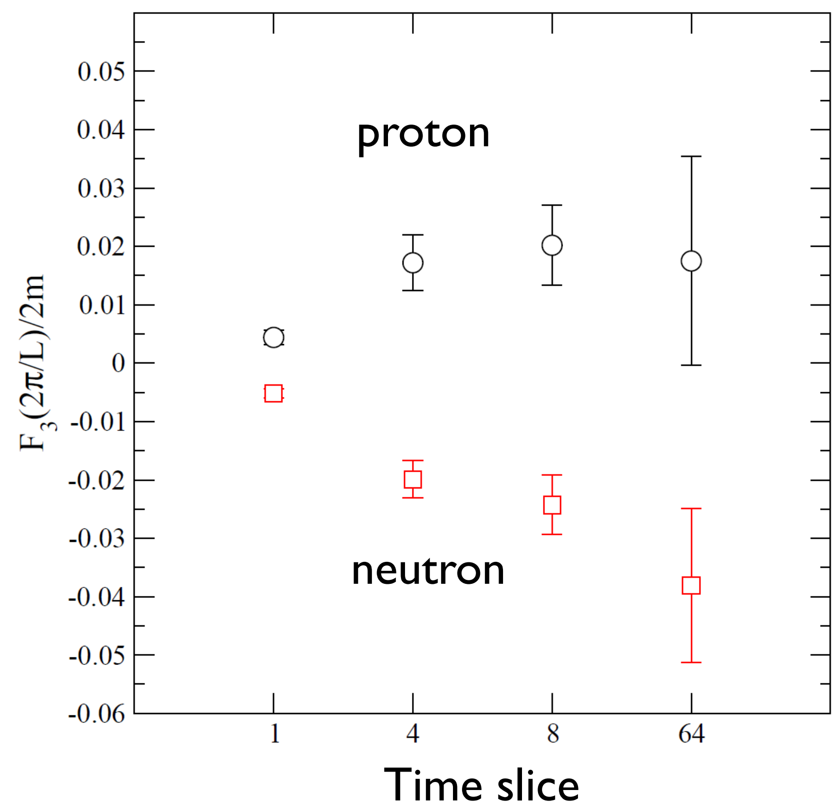
- Investigation of local topology effect near the correlation function
  - Exploring local topological charge fluctuation in correlation function.



$$q(\tau) = \int_0^{\tau} dt \int d^3x q(x)$$

$$\langle C_{J_\mu} \rangle_q = Z^{-1} \int dq dA e^{-S_{QCD}} q(t) C_{J_\mu}$$

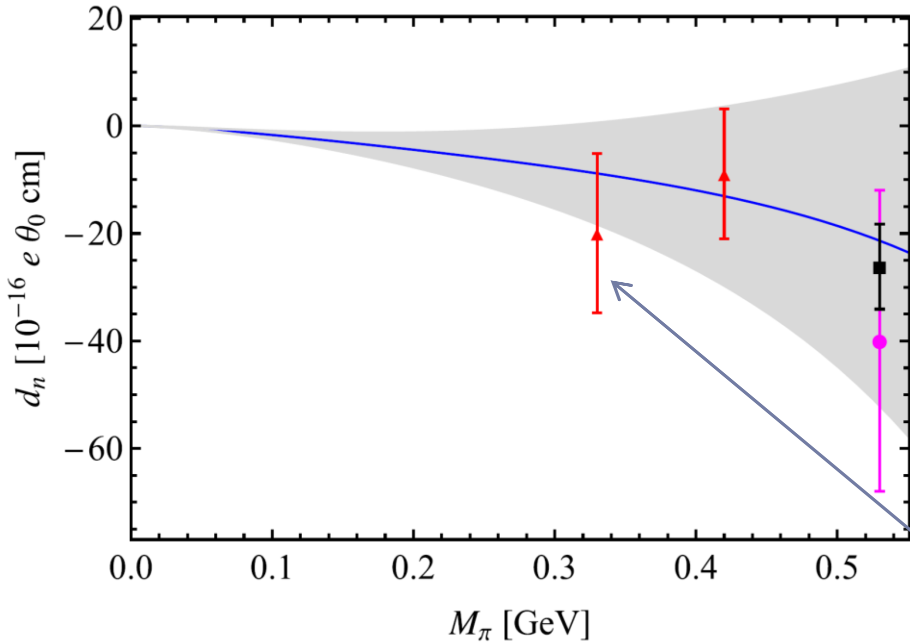
- Statistical fluctuation depends on the time-slice of t integration for q(x).
- Statistical error due to global topological fluctuation is predominant.

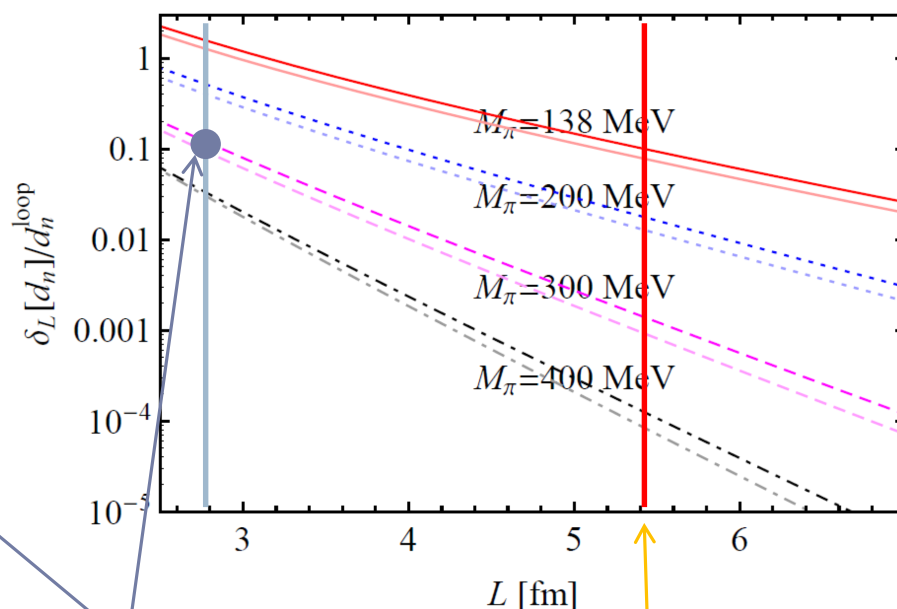


Volume effect

- BChPT analysis

Guo, Meissner, 1210.5887, Akan et al., PLB736(2014)





Simulation point

Physical pion configs. in RBC

- In LO, NLO BChPT analysis, there may be 10--20% finite size effect.
- EDM is related to axial charge, which may have also visible finite size effect.

3. Recent update

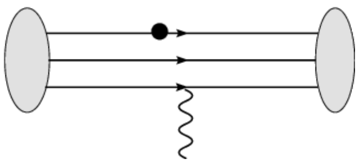
Lattice artifact

Aoki, Gocksh, Manohar, Sharpe (1990)

- Fermionic insertion of CPV term (e.g. θ term) into correlation function should be mixed with χ symmetry breaking on the lattice,
  ⇒ lattice artifact correction, e.g. Wilson-type fermion

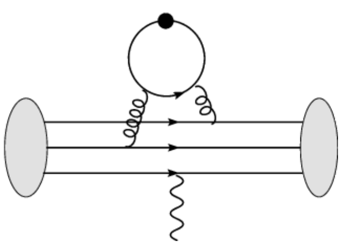
$$\mathcal{L}_\theta = \bar{m}\theta\bar{q}\gamma_5 q/2 \to \mathcal{L}_\theta^W = \bar{m}(1+\kappa_P)\theta\bar{q}\gamma_5 q,\ \kappa_P \sim \mathcal{O}(a) : \text{renom. const.}$$

- χ rotation for valence quark renders valence θ quark unobservable.
  ⇒ χ breaking affects valence quark with θ term.
  ⇒ Leading lattice artifact contribution



$\sim \mathcal{O}(\kappa_P m_{\text{val}})$

- In topological insertion, lattice artifact appears in seq quark contribution.
  ⇒ rather small, but may be visible near chiral limit.



$\sim d_N + \mathcal{O}(\kappa_P m_{\text{sea}} a^2)$

DWF for both valence and sea quarks is able to suppress as to be O(a^2) correction.

Summary and future plan

- Lattice QCD calculation (theta term) is still difficult.
  - Nucleon EDM within 40% statistical error is achievable.
  - 3-pt function is noisy, which seems caused of global Q fluctuation.
  - Systematic error of finite size may be visible, but DWF can suppress lattice artifact.
- (Near) physical point of DWF configurations
  - Ensembles near physical points and large volume. Hantao, Lattice2013
  - Developed algorithm (AMA, Mobius DWF, etc) is helpful.
  - aim to be less than 10% precision

Lattice size	Physical size	a	L_s	Gauge action	Pion mass
$32^3 \times 64$	4.6 fm^3	0.135 fm	32	DSDR	171 -- 241 MeV
$48^3 \times 96$	5.5 fm^3	0.115 fm	16	Iwasaki	135 MeV

Thank you for your attention !

Backup

Correlation function in θ term

- Expansion at O(θ)

Källen Lehmann representation of matrix element

$$
\begin{aligned}
&\langle C^{\theta}_{J_\mu}\rangle(t_1,t,t_0|p_1,p_0)\\
&=\sum_{s_0,s_1}\frac{\bar{Z}_N(p_0)Z_N(p_1)}{4E_N(p_1)E_N(p_0)}u^{\theta}_N(\vec{p}_1,s_1)\langle N(p_1,s_1)|J^{\mathrm{EM}}_\mu|N(\vec{p}_0,s_0)\rangle_\theta\bar{u}_N(\vec{p}_0,t_0)\\
&\times^{-E_N(p_1)(t_1-t)-E_N(p_0)(t-t_0)}\theta(t_1-t)\theta(t-t_0)+(\text{time permutation})+(\text{excited state})
\end{aligned}
$$

EDM matrix element is able to be represented by correlation function at O(θ):

$$
\langle N|J_\mu|N\rangle_\theta=\frac{d_N}{2m_N}\bar{u}_N q_\nu\sigma_{\mu\nu}u_N
$$

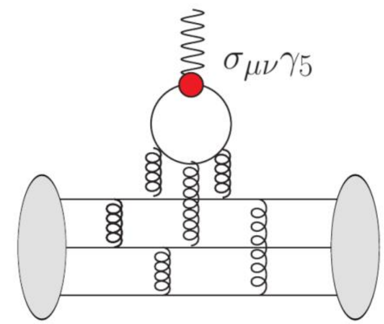
$$
\begin{aligned}
&\langle C^{\theta}_{J_\mu}\rangle(t_1,t,t_0|p_1,p_0)\\
&=Z_\theta^{-1}\int dqDAe^{-S_{\mathrm{QCD}}+i\theta Q}\int d^3xd^3ye^{i(\vec{p}_1\vec{x}+\vec{q}\vec{y})}T\{\eta_N(t_1,\vec{x})J^{EM}_\mu(t,\vec{y})\bar{\eta}_N(t_0,0)\}\\
&=Z^{-1}\int dqDAe^{-S_{\mathrm{QCD}}}\int d^3xd^3ye^{i(\vec{p}_1\vec{x}+\vec{q}\vec{y})}\Big[T\{\eta_N(t_1,\vec{x})J^{\mathrm{EM}}_\mu(t,\vec{y})\bar{\eta}_N(t_0,0)\}\\
&+\underbrace{i\theta QT\{\eta_N(t_1,\vec{x})J^{\mathrm{EM}}_\mu(t,\vec{y})\bar{\eta}_N(t_0,0)\}}_{\text{EDM term}}\Big]+\mathcal{O}(\theta^2)
\end{aligned}
$$

q(c)EDM term into QCD action

Bhattacharya et al, Lattice 2012--2015

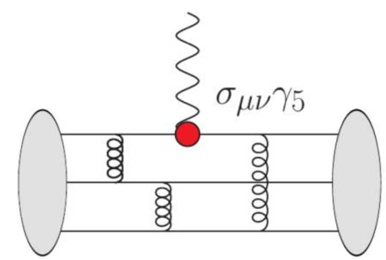
- Plan to do extension toward BSM action
  - Matrix element including BSM operator, quark EDM and chromo EDM (PQ symmetry is assumed)
  - The q(c)EDM term is CP-violating tensor charge of nucleon, connected diagram should be leading contribution→ statistical signal will be clear.
  - External E filed method may be easy way.
  - qEDM
    - Tensor charge matrix element + matrix element with qEDM:

$$\partial_{A_\mu} \langle N | d_q (\bar{q}\gamma_5 \sigma \cdot F q) | N \rangle_E$$
$$= \langle N | d_q (\bar{q}\gamma_5 q^\nu \sigma_{\mu\nu} q) | N \rangle + \langle N | J_\mu d_q (\bar{q}\gamma_5 \sigma \cdot F q) | N \rangle$$



  - Quark chromo EDM
    - Matrix element with chromo EDM term:

$$\partial_{A_\mu} \langle N | d_{cq} (\bar{q}\gamma_5 \sigma \cdot G q) | N \rangle_E = \langle N | J_\mu d_{cq} (\bar{q}\gamma_5 \sigma \cdot G q) | N \rangle$$



Imaginary θ

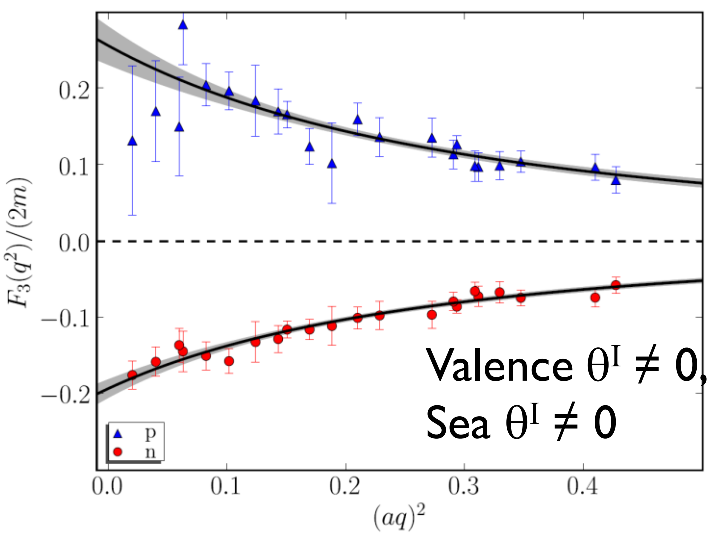
Izubuchi(07), Horsley et al. (08)

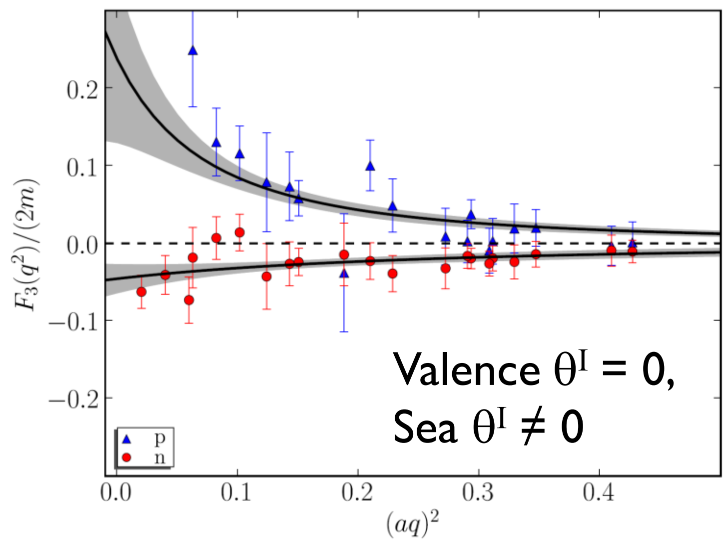
- Valence θ term in Wilson-type fermion

$$Z(\bar\eta,\eta) = \int dA \det(\not{D}+m+\theta^I\gamma_5 m)\exp\left[\bar\eta(\not{D}+m+\theta^I\gamma_5 m)^{-1}\eta\right]$$

$$= \int dA \det(\not{D}+m+\theta^I\gamma_5 m)\exp\left[\bar\eta(\not{D}+m)^{-1}\eta\right] \quad \eta \to e^{-\gamma_5\theta^I/2}\eta$$

$$\Rightarrow Z^W(\eta,\bar\eta) = \int dA \det(\not{D}+m+\theta^I\gamma_5 m)\exp\left[\bar\eta(\not{D}+m+\theta^I\kappa_P\gamma_5 m)\eta\right]$$





N_f=2 Wilson

Izubuchi, 08, Blum, 2010

Seems to be clear signal.

Signal disappears in valence $\theta^I = 0$

Error reduction techniques

Blum, Izubuchi, ES, 1208.4349,
ES (lattice 2012),
Chulwoo, plenary on Fri

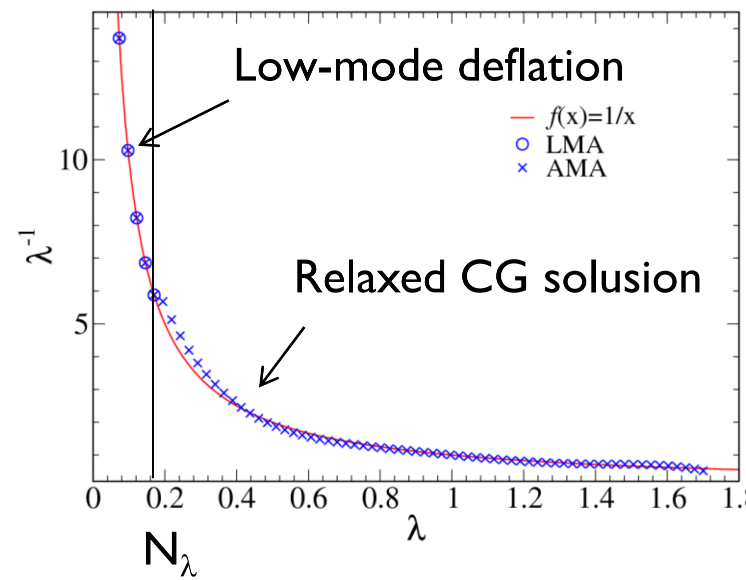
- Covariant approximation averaging (CAA)
  - For original correlator O, (unbiased) improved estimator is defined as

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

  - $\langle O\rangle = \langle O^{(\mathrm{imp})}\rangle$ if approximation has covariance under lattice symmetry g
  - Improved error $\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}$
  - Computational cost of $O^{(\mathrm{imp})}$ is cheap.
- All-mode-averaging (AMA)
  - Relaxed CG solution for approximation

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],\ S_l = \sum_{\lambda=1}^{N_\lambda} v_\lambda v_\lambda^\dagger \frac{1}{\lambda} + P_n(\lambda)|_{|\lambda|>N_\lambda}$$

  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.

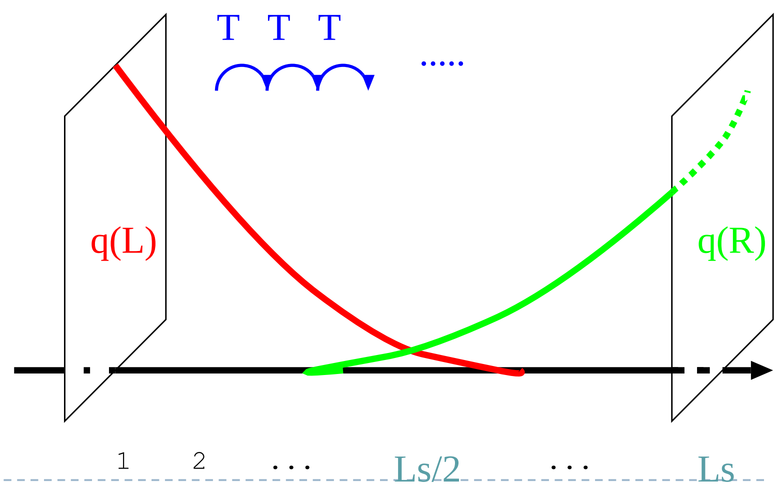


Choice of lattice fermion

- There are several kinds of fermion definition on the lattice.

  Wilson(-clover), staggered, domain-wall, overlap, …
- Require "realistic" fermion for the precise calculation
  - good approximated chiral symmetry on the lattice.
  - suppression of O(a) effect, but trading computational time
  - Domain-wall fermion is appropriate selection.
- Domain-Wall fermion (DWF) [Blum Soni, (97), RBC/UKQCD. (05 --)]

  - L, R fermion are localized on boundaries
    Exact chiral symmetry is realized if $L_s \to \infty$.
  - In finite L_s
    Violation of chiral symm. is suppressed as
    $am_{res} \sim \exp(-L_s) \ll 1$.

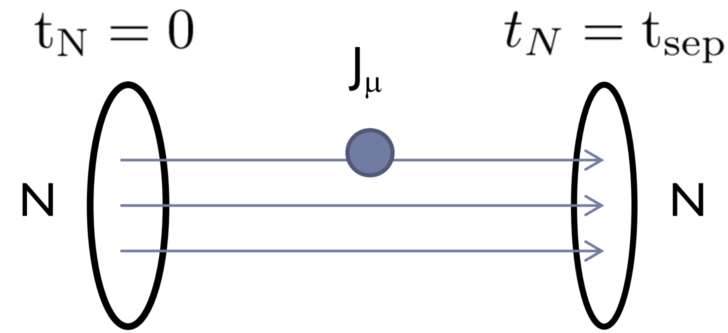


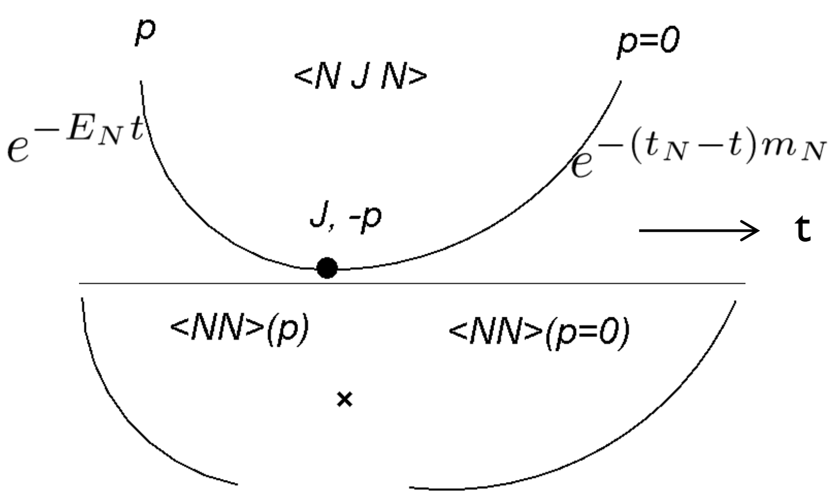
3-pt function

- (Nucleon)-(EM current)-(Nucleon)
- Location of operators

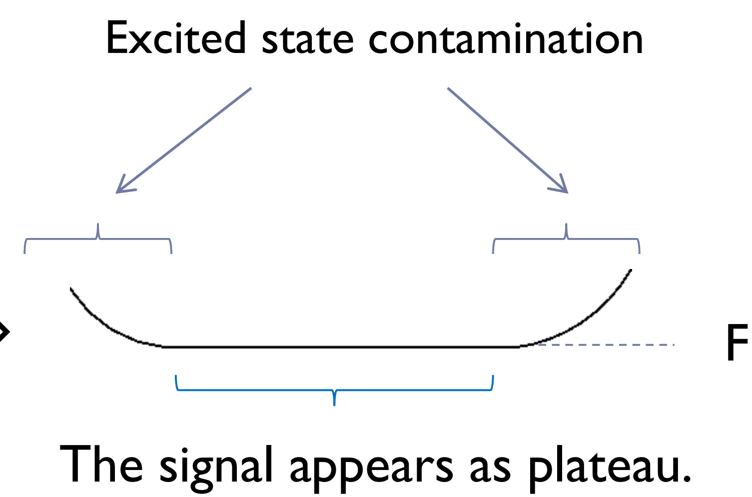
  t = 0 and t = t_N : nucleon op.

  EM current inserts between nucleon ops.
- Comparison of different t_{sep} is good check of excited state contamination.
- Ratio of 3-pt and 2-pt





⇒



Parameters

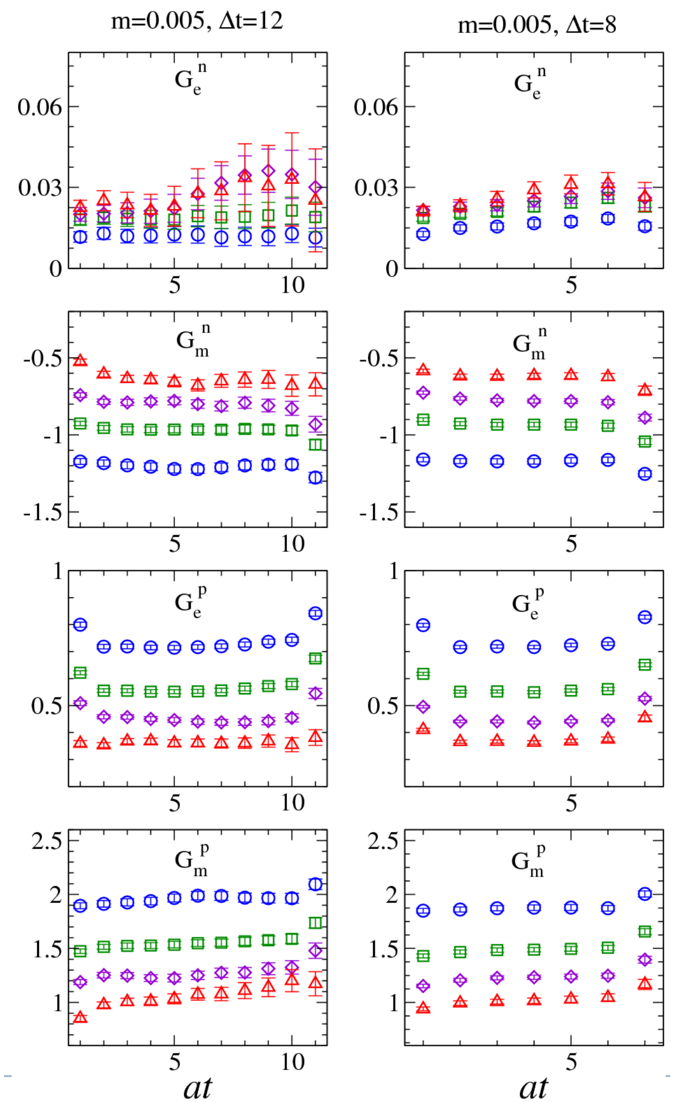
- DWF
  - $24^3 \times 64$ lattice, a^{-1} = 1.73 GeV (~2.7 fm^3 lattice)
  - L_s = 16 and am_{res} = 0.003
  - m = 0.005, 0.01 corresponding to m_π = 0.33, 0.42 GeV
  - Two temporal separation of N sink and source in 3 pt. function

    t_{sep} = 12 (t_{source} = 0, t_{sink} = 12), t_{sep} = 8 (t_{source} = 0, t_{sink} = 8)
  - # configs = 751 (m=0.005), 700 (m=0.01) [t_{sep} = 12]

    # configs = 180 (m=0.005), 132 (m=0.01) [t_{sep} = 8]

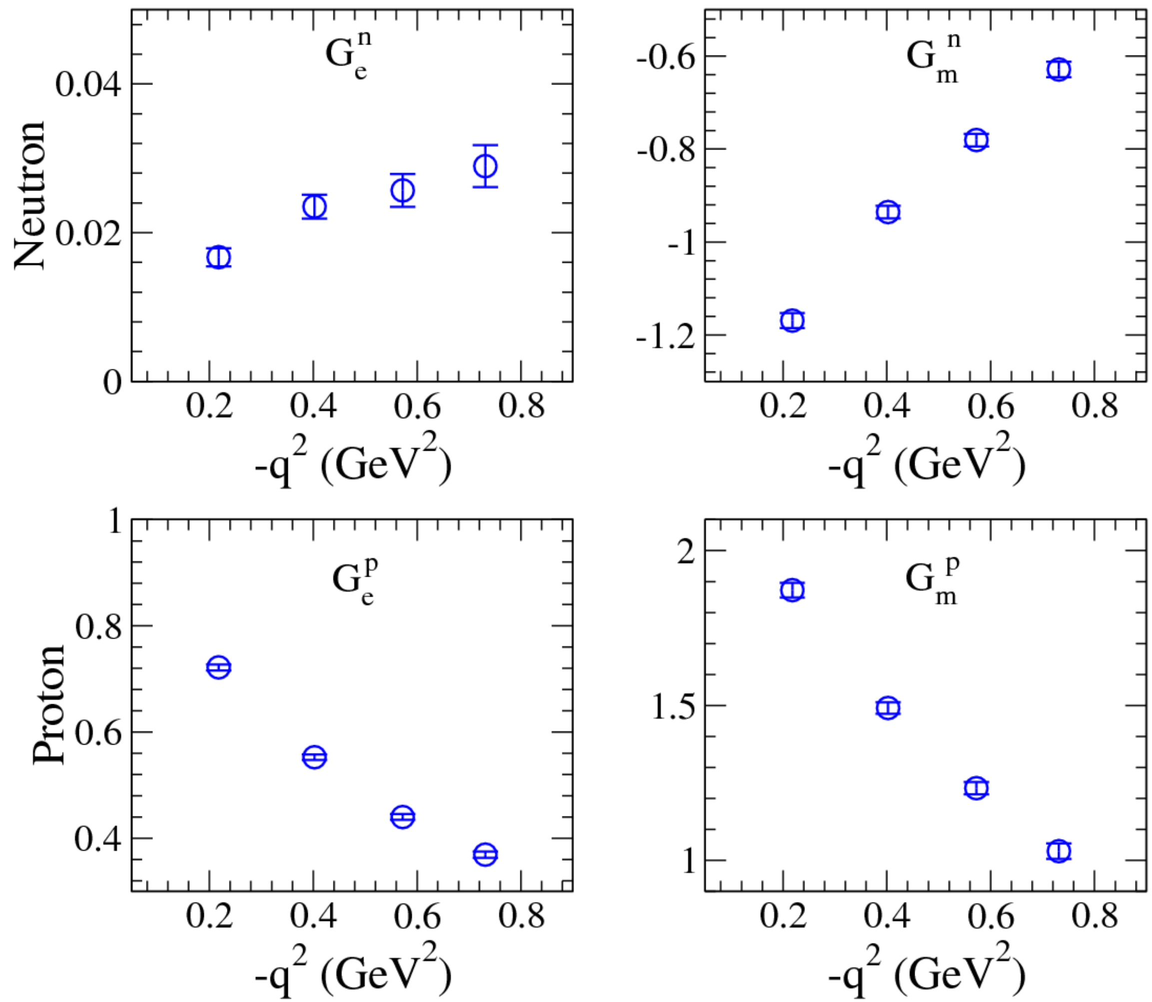
    } Comparison to check the higher excited state contamination
- AMA
  - # of low-mode : N_λ = 400 (m=0.005), 180 (m=0.01)
  - Stopping condition, $|r| < 0.003$
  - N_G = 32 (2 separation for spatial, 4 separation for temporal direction of source localtion) → effectively O(10^4) statistics

EM form factor



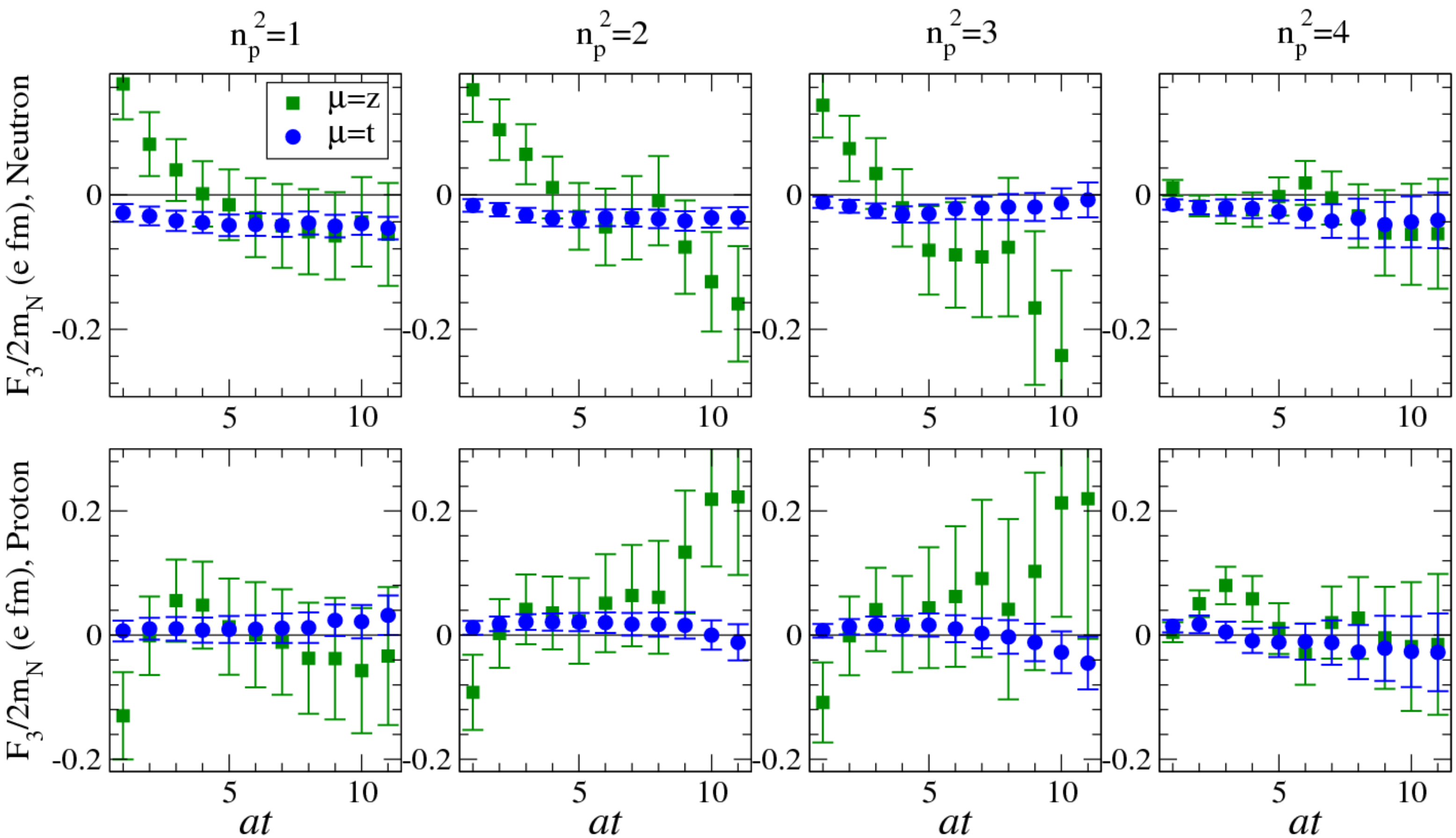
EM form factor

- By using AMA algorithm, statistical error of these observables achieve below 5% level.
- Compared with previous works (RBC PhysRevD79(2009)), computational time can be reduced by factor 5 and more.
  ⇒ higher precision



Comparison with μ = t, z

- EDM form factor is given from two directions of EM current
- Two signals are consistent, and data in t direction is much stable.

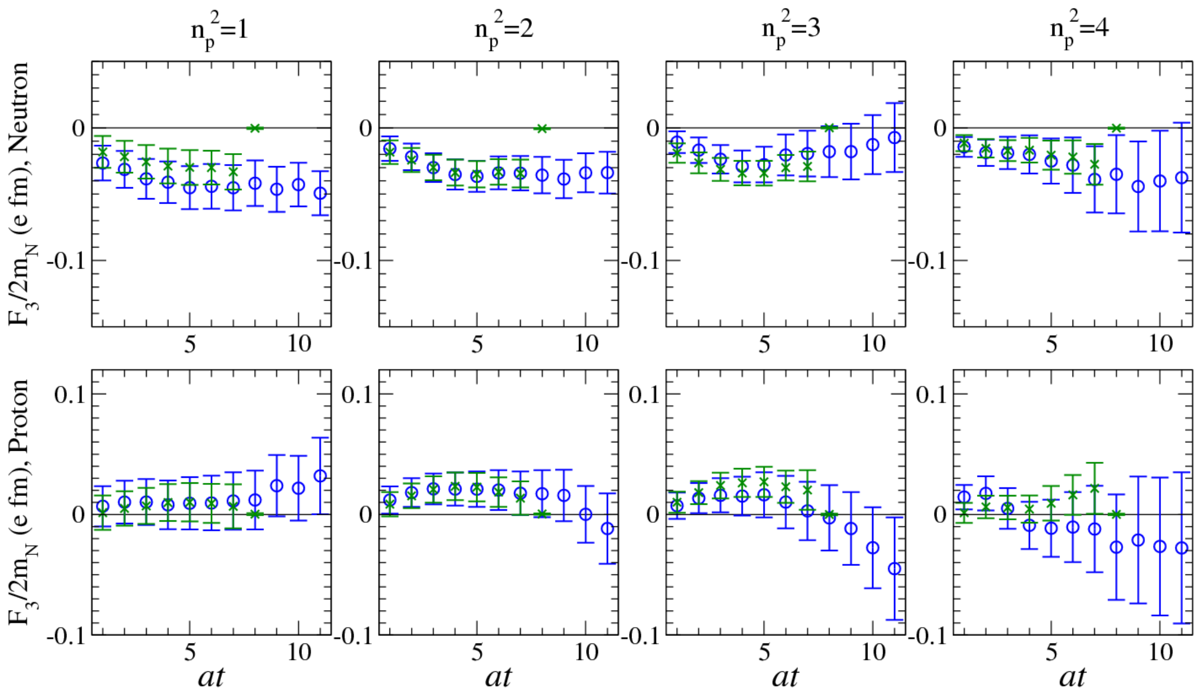


Comparison with different t_{sep}

- The sink and source separation in 3pt function enables us to control the statistical noise and excited state contamination
  - Short: statistical fluctuation < excited state contamination
  - Long: statistical fluctuation > excited state contamination
- Comparison

t_{sep} = 12 (blue),
[N_{conf} = 751]
t_{sep} = 8 (green)
[N_{conf} = 180]

- Good consistency between them.
- Precision in t_{sep}=8 is much better.

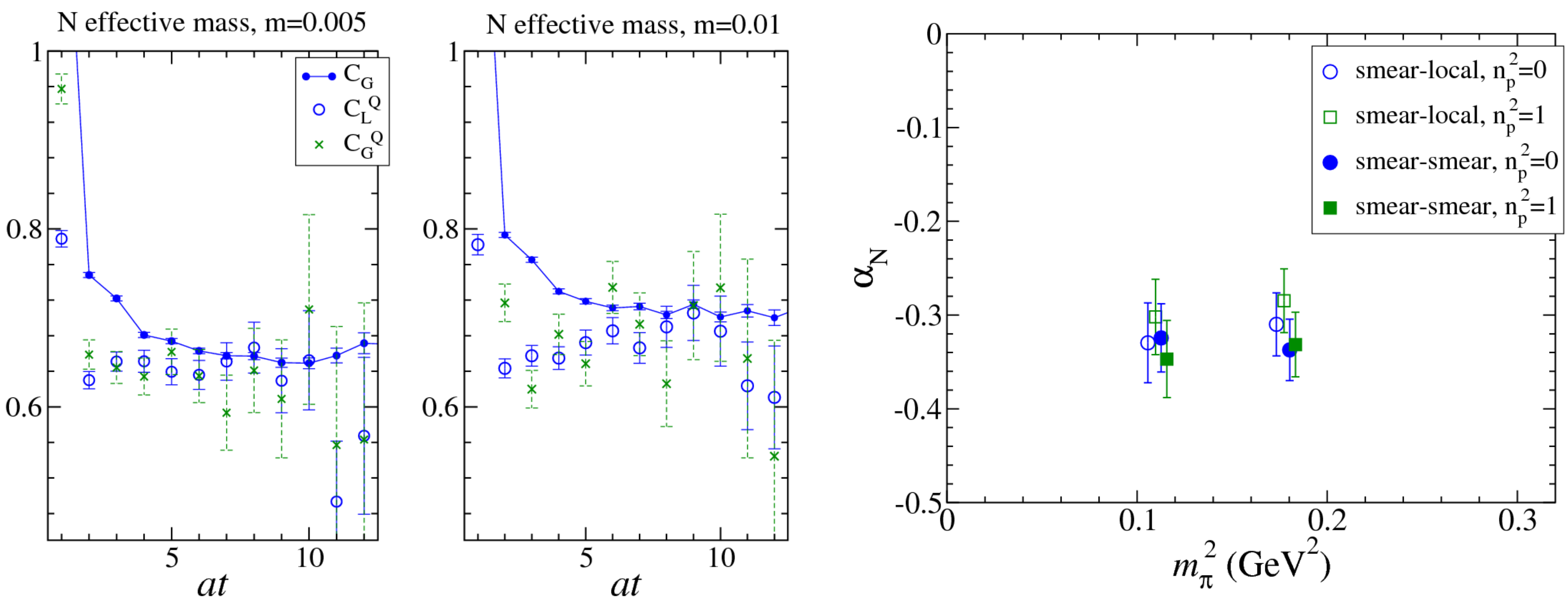


α_N : CP-odd phase of wave function

- Projection with γ_5 for 2 pt in θ term, perform global fitting

$$\mathrm{tr}\left[\gamma_5 \langle N(t)\bar{N}(0)Q\rangle\right] = Z_N \frac{2m_N}{E_N}\alpha_N e^{-E_N t} + O(e^{-E_{N^*}})$$

- By using AMA, this factor is determined within15 % error.
- It does not depend on smearing and momentum, but mass dependence is not so clear.



Statistical error

- Comparison between AMA error reduction and number of configurations.
- Number of configurations : reduce stat. error and relating to Q distribution

  AMA error reduction : reduce stat. error
- Error rate

  = Error(full)/Error(N)
- AMA works well
- Reduction rate when

  increase of confgs.

  is slightly better.

w/o correlation

↓

36%

52%

73%

35%

50%

Full statistics →