

Electric Dipole Moment of the Neutron from Lattice QCD

G. Schierholz

Deutsches Elektronen-Synchrotron DESY



With

F.-K. Guo, R. Horsley, U.-G. Meißner, Y. Nakamura, H. Perlt,

D. Pleiter, P.E.L. Rakow, A. Schiller, J. Zanotti

QCDSF Collaboration

Outline

Objective

Dipole Moment

Sources of \mathcal{Q}/P

Lattice Particulars & Results

Conclusions

Objective

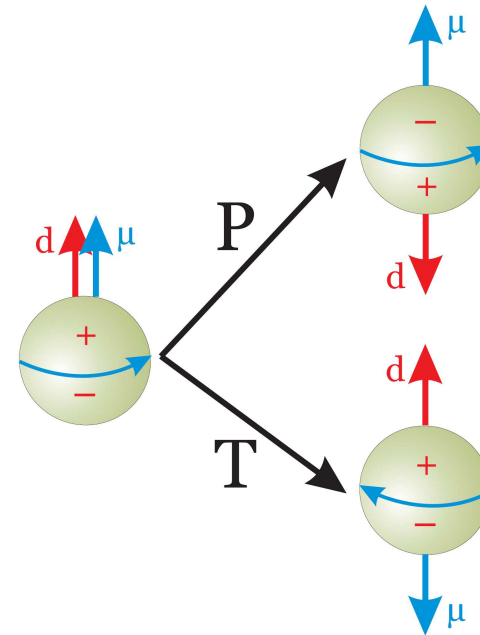
- The electric dipole moment of the neutron provides a unique and sensitive probe to physics beyond the Standard Model. It has played an important part over many decades in shaping and constraining numerous models of CP violation
- While the CP violation observed in K and B meson decays can be accounted for by the phase of the CKM matrix, the baryon asymmetry of the universe cannot be described by this phase alone, suggesting that there are additional sources of CP violation awaiting discovery
- In a wide class of GUTs the diagrams that generate a high baryon/photon asymmetry also contribute to the renormalization of the CP-violating vacuum angle θ , and hence to the electric dipole moment of the neutron
- With the increasingly precise experimental efforts to observe the electric dipole moment, it is now important to have a rigorous calculation directly from QCD

Dipole Moment

$$\vec{d} = \int d^3x \vec{x} \rho(x)$$

$$\vec{d} \vec{\mu} \rightarrow -\vec{d} \vec{\mu}$$

CP



\Rightarrow Lattice

Nucleon electromagnetic current

$$\langle p', s' | J_\mu | p, s \rangle = \bar{u}(\vec{p}', s') \mathcal{J}_\mu u(\vec{p}, s)$$

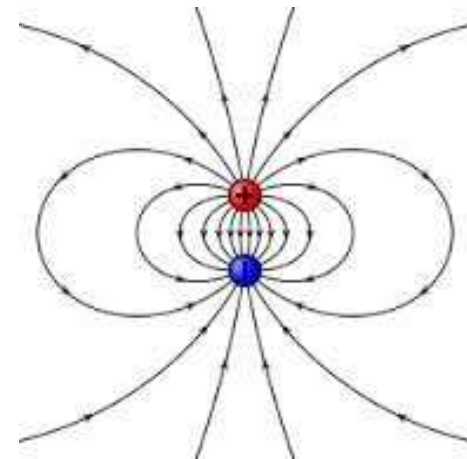
$$\mathcal{J}_\mu = \gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_N} + (\gamma q q_\mu - \gamma_\mu q^2) \gamma_5 F_A(q^2) + \sigma_{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N}$$

anapole

dipole

Dipole moment

$$d_N = \frac{e F_3(0)}{2m_N} \propto e_q \ell$$



Sources of \mathcal{CP}

QCD

$$S = S_{\text{QCD}} + S_\theta$$

$$S_\theta = i\theta Q$$

$$Q = -\frac{1}{64\pi^2} \sum_x F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a \in \mathbb{Z}$$

AWI

\implies

$$S_\theta = -\frac{i}{3} \theta \hat{m} \sum_x (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s)$$
$$\hat{m}^{-1} = \frac{1}{3} (m_u^{-1} + m_d^{-1} + m_s^{-1})$$

GUTs

Quark EDMs (e.g.)

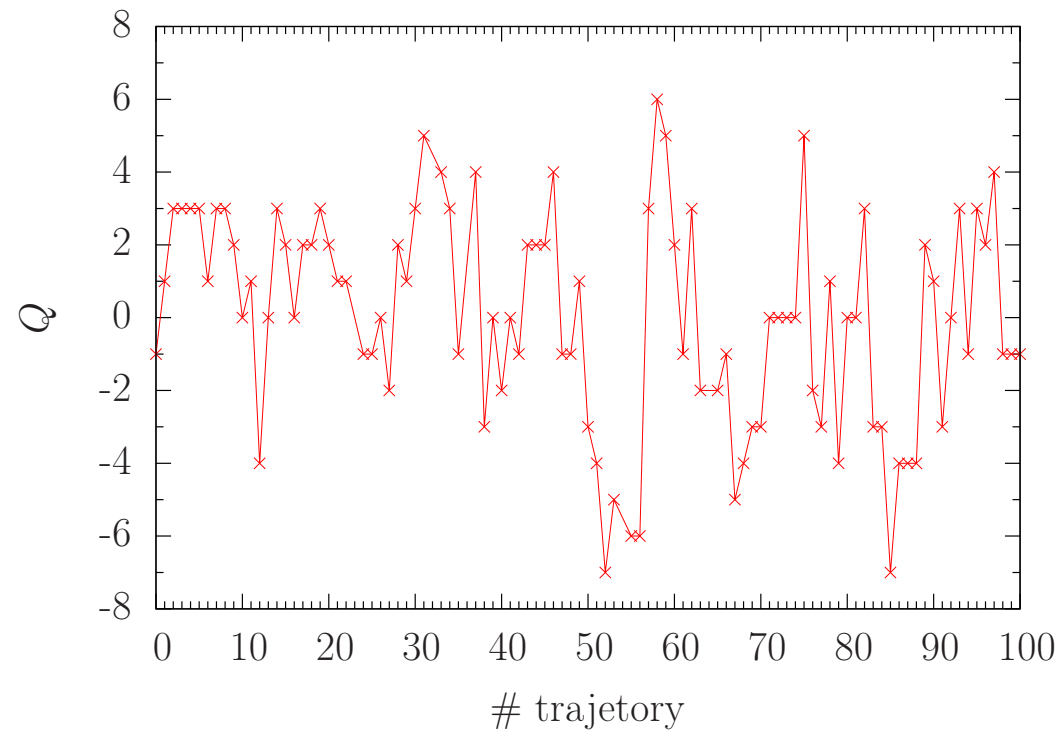
Renormalize θ : $\theta \rightarrow \theta + \theta_q$

Bhattacharya et al.

SUSY

Standard Model

Topological charge?

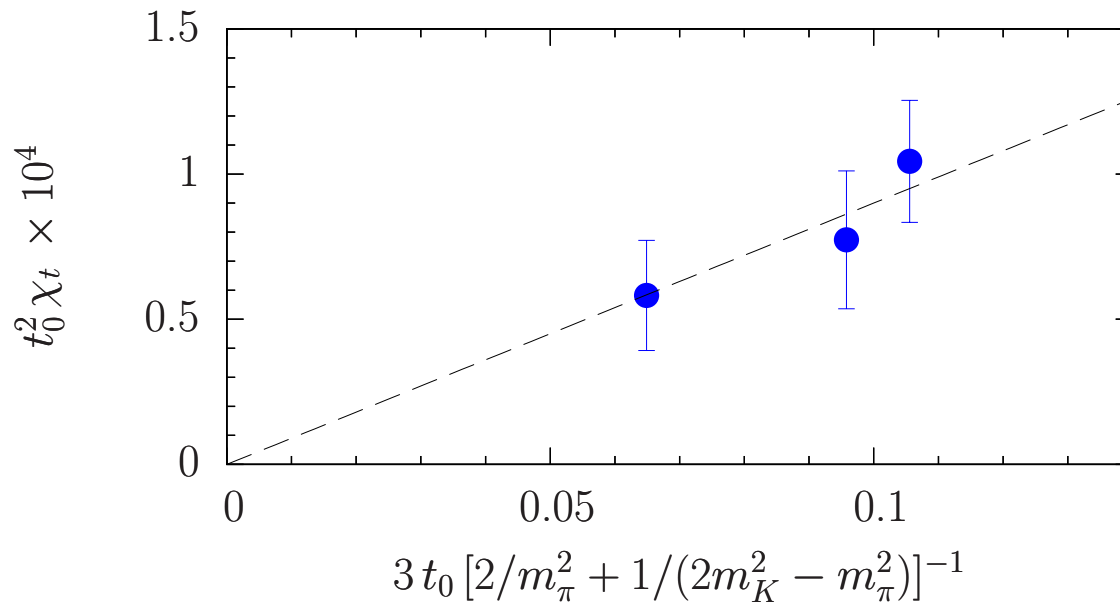


U(1) problem

$$\underbrace{m_{\eta'}^2 + m_{\eta}^2 - 2m_K^2}_{\text{CL}} = \frac{12}{f^2} C \quad C = \chi_t \left[1 - \chi_t \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle} \right]^{-1}, \quad \chi_t = \frac{Q^2}{V}$$

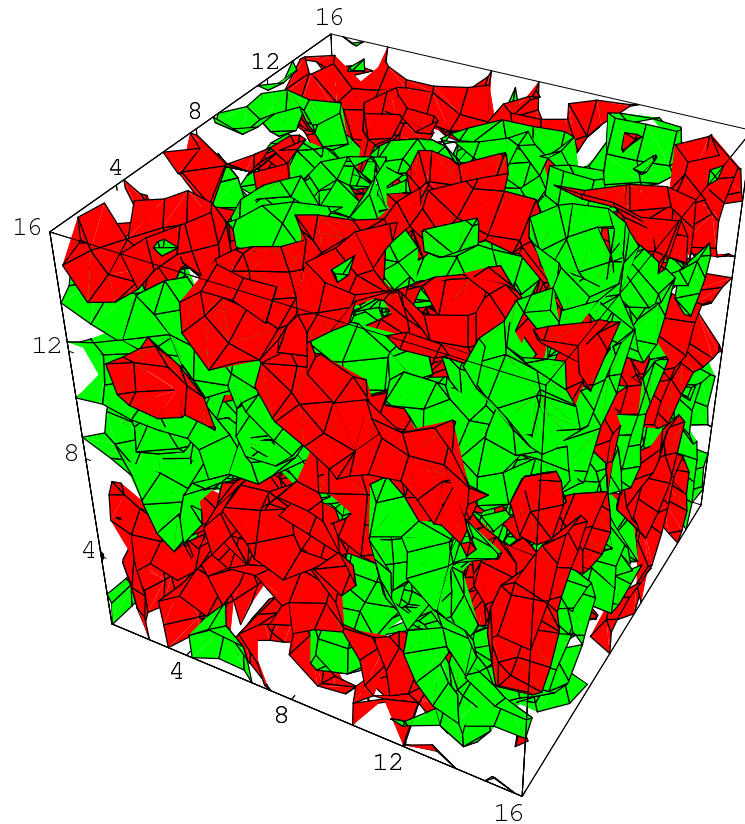
$$\Downarrow \text{CL}$$
$$m_{\eta'}^2$$

Shore & Veneziano



QCDSF

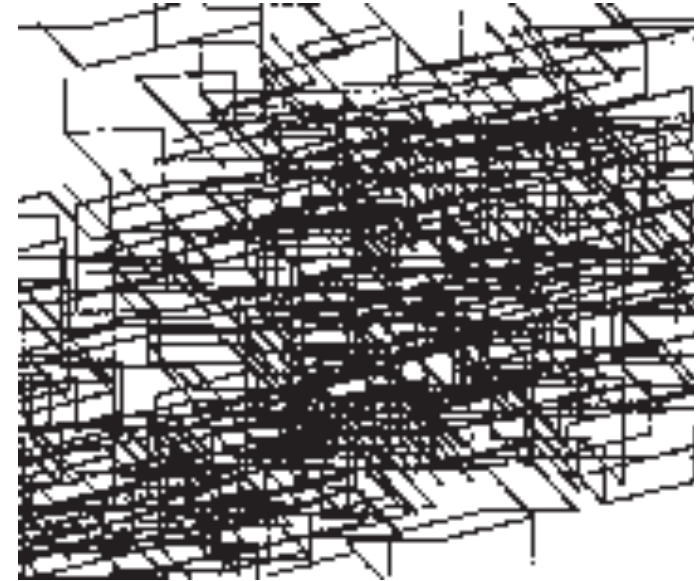
Vacuum



Isosurfaces of positive (red) and negative (green) topological charge density

Weinberg (Thesis)

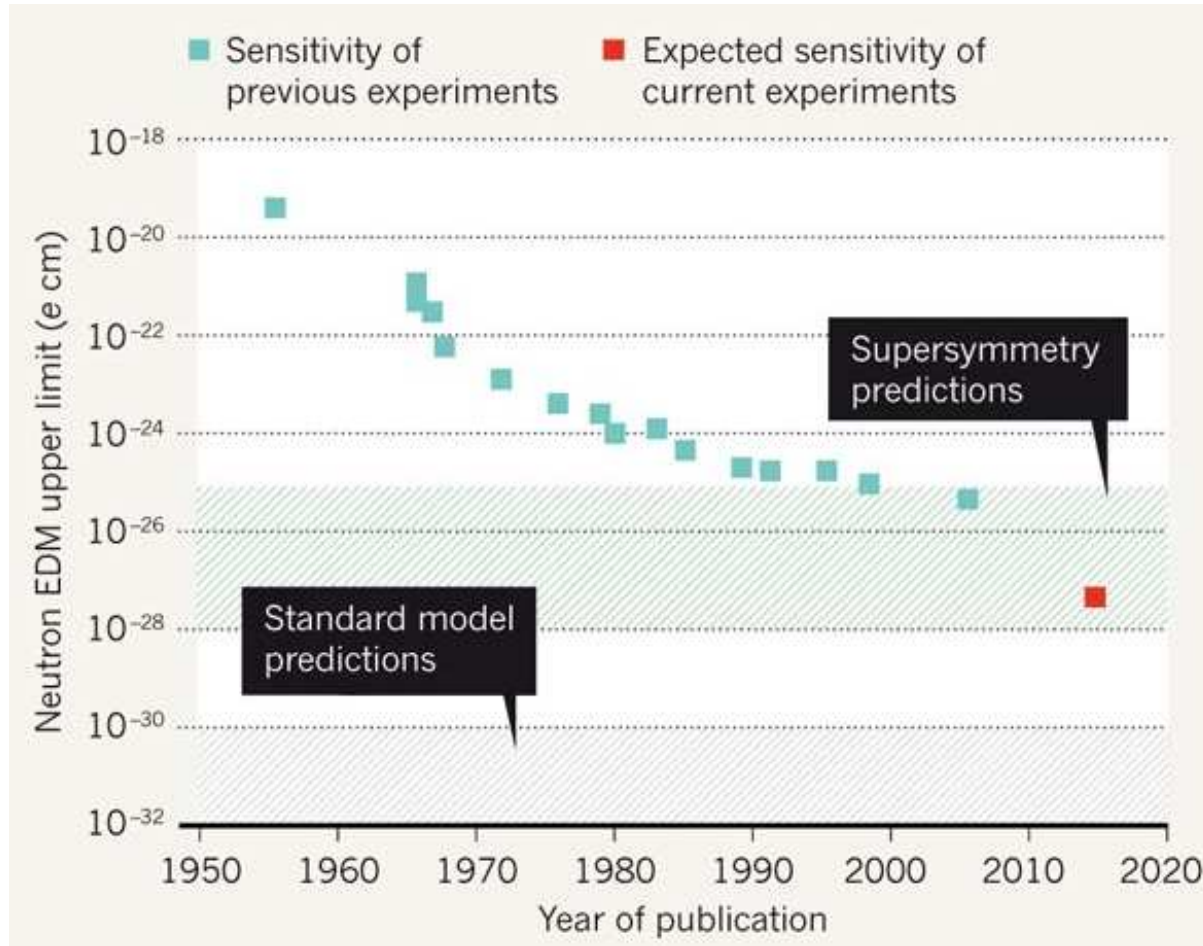
→
IR



Monopole condensation

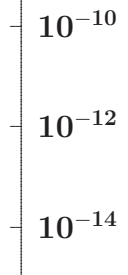
Kronfeld, GS & Wiese

Experiment



QCD

θ

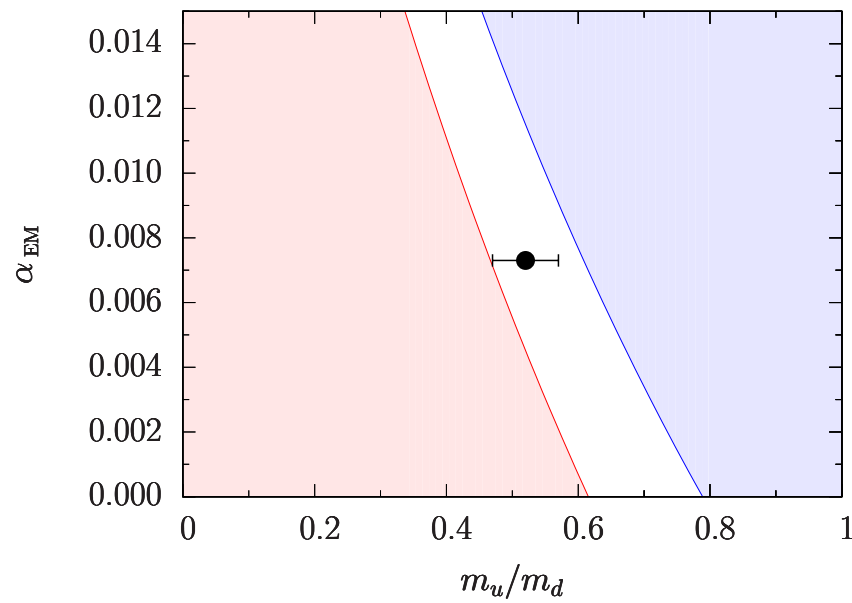


Strong CP problem

PQ, what else? Solution within QCD?

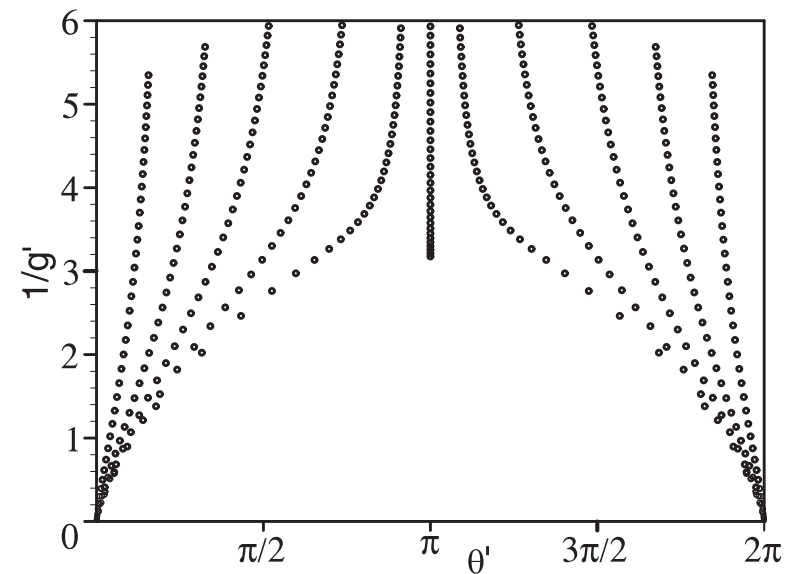
Largely question of QCD vacuum

$m_u = 0?$



QCDSF

NP renormalization?



Knizhnik, Morozov, Levine, Libby, Apenko

Lattice Particulars & Results

Assuming that the theory is analytic for small angles, we rotate θ to imaginary $\bar{\theta} = -i\theta$. This leads to the action

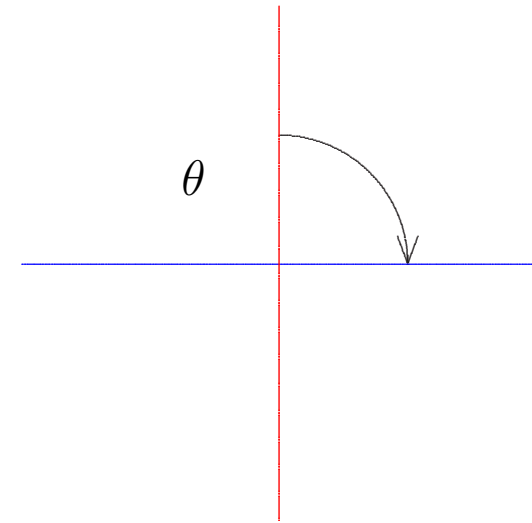
$$S_\theta = \frac{1}{3} \bar{\theta} \hat{m} \sum_x (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s)$$

which is amenable to numerical simulations. At the end of the calculation the results are rotated back to real θ .

We take the up and down quarks to be mass degenerate, $m_u = m_d \equiv m_\ell$, leading to

$$\hat{m} = 3 \frac{m_\ell m_s}{m_\ell + 2m_s}$$

which makes S_θ vanish at $m_\ell = 0$ (i.e. $m_\pi^2 = 0$) and $m_s = 0$ (i.e. $2m_K^2 - m_\pi^2 = 0$)



Power of simulation

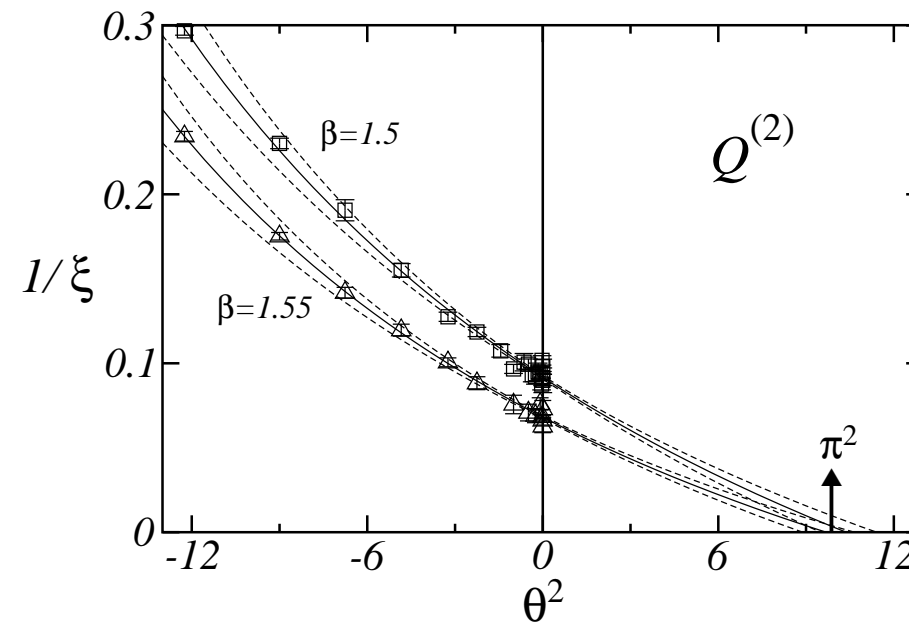
$O(3)$ Sigma Model

Haldane's conjecture: Second order phase transition at $\theta = \pi$

i.e. $\xi \rightarrow \infty$

↑

correlation length



Alles & Papa

Lattices

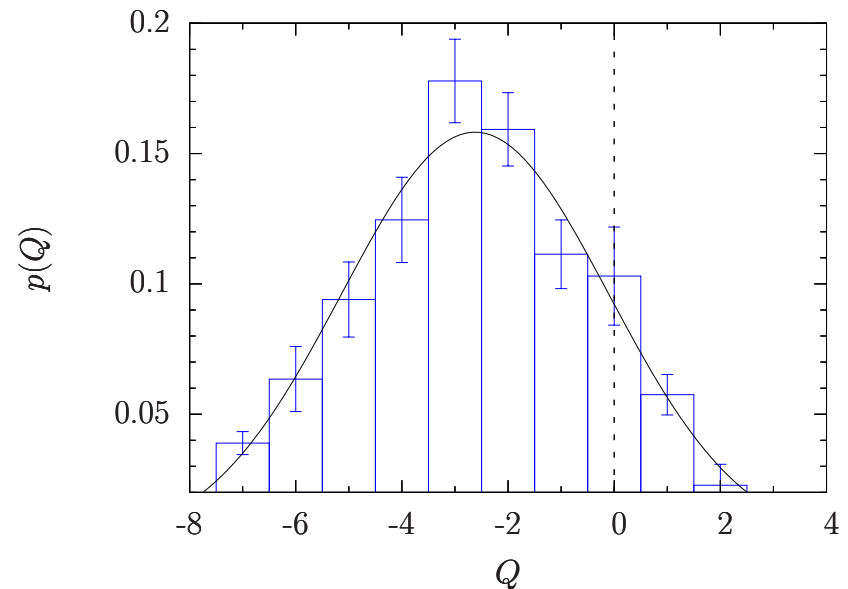
SLiNC action: $\beta = 5.50$, $a = 0.074(2)$ fm

#	κ_ℓ	κ_s	V	m_π [MeV]	m_K [MeV]	λ
1	0.12090	0.12090	$24^3 \times 48$	465	465	0.003
2	0.12090	0.12090	$24^3 \times 48$	465	465	0.005
3	0.12104	0.12062	$24^3 \times 48$	360	505	0.003
4	0.12104	0.12062	$24^3 \times 48$	360	505	0.005
5	0.12104	0.12062	$32^3 \times 64$	360	505	0.003
6	0.121095	0.120512	$32^3 \times 64$	310	520	0.003

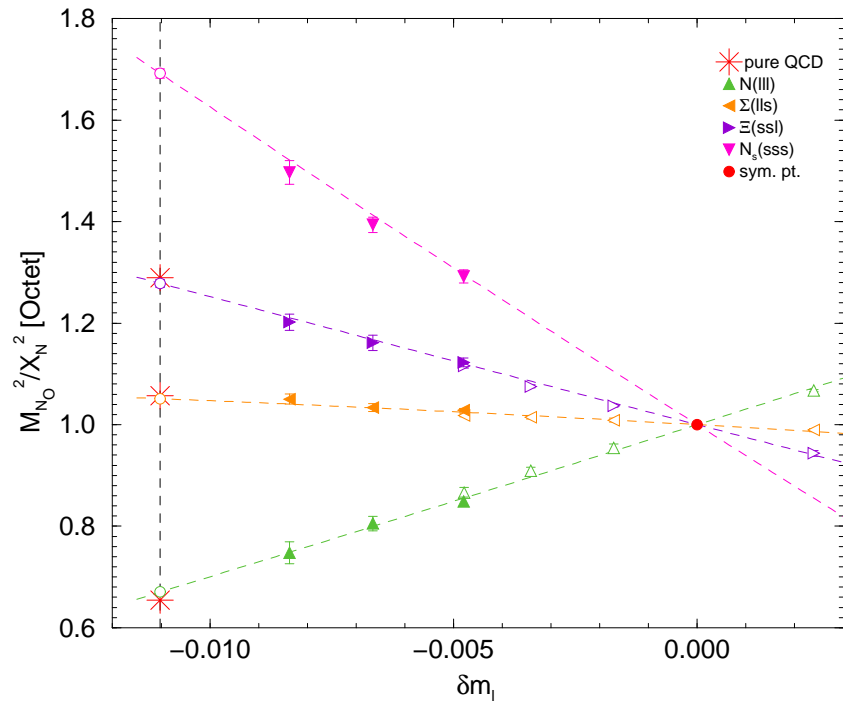
We keep the singlet quark mass $\bar{m} = (m_u + m_d + m_s)/3$ fixed at its physical value, while $\delta m_q = m_q - \bar{m}$ is varied

$$\lambda = \bar{\theta} 2a \frac{m_\ell m_s}{m_\ell + 2m_s}$$

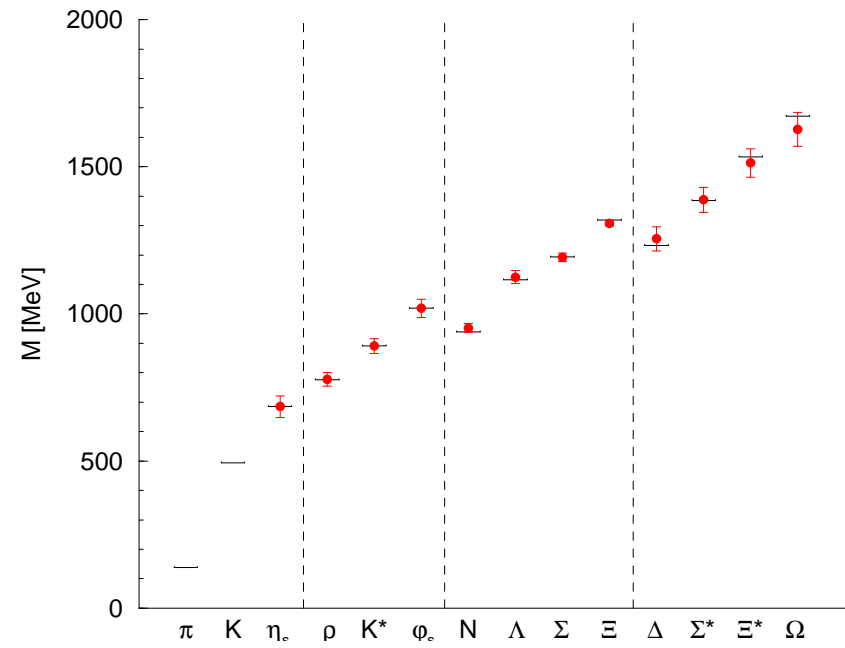
$$am_q = \frac{1}{2\kappa_q} - \frac{1}{2\kappa_{0,c}}$$



QCDSF approach



↑
SU(3) symmetric point



$F_3(0)$

$$\frac{G_{N\mathcal{J}_\mu N}}{G_{NN}} \rightarrow \frac{1}{4} \text{Tr} \Gamma \left[e^{i\alpha(\theta)\gamma_5} \frac{\not{p}' + m_N}{E'} e^{i\alpha(\theta)\gamma_5} \right] \mathcal{J}_\mu \left[e^{i\alpha(\theta)\gamma_5} \frac{\not{p} + m_N}{E} e^{i\alpha(\theta)\gamma_5} \right]$$

\mathcal{CP} phase

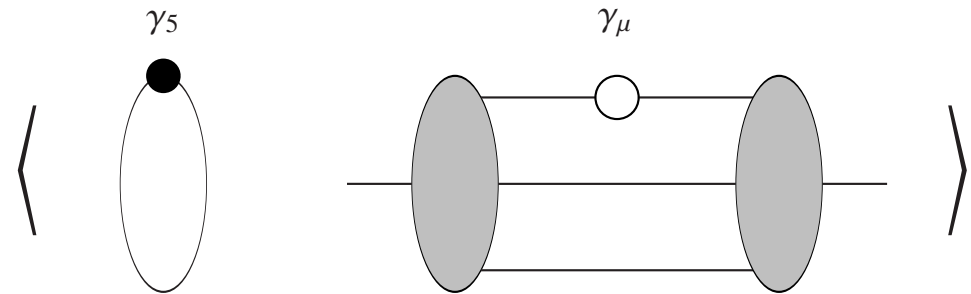
$$u(\vec{p}, s) \rightarrow e^{i\alpha(\theta)\gamma_5} u(\vec{p}, s)$$

$$\bar{u}(\vec{p}, s) \rightarrow \bar{u}(\vec{p}, s) e^{i\alpha(\theta)\gamma_5}$$

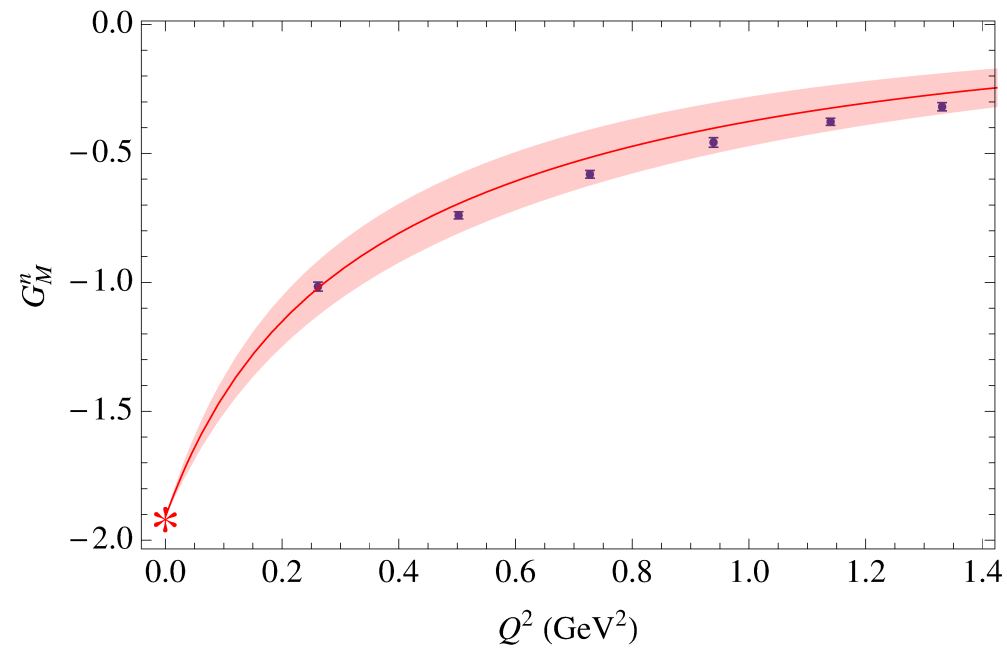
$$\frac{\text{Tr} G_{NN} \Gamma_4 \gamma_5}{\text{Tr} G_{NN} \Gamma_4} = i \frac{\sin 2\alpha(\theta)}{1 + \cos 2\alpha(\theta)}$$

Shintani et al.

The topological θ term couples to internal gluon lines only. Quark propagators are computed with the action S_{QCD} , neglecting the S_θ term

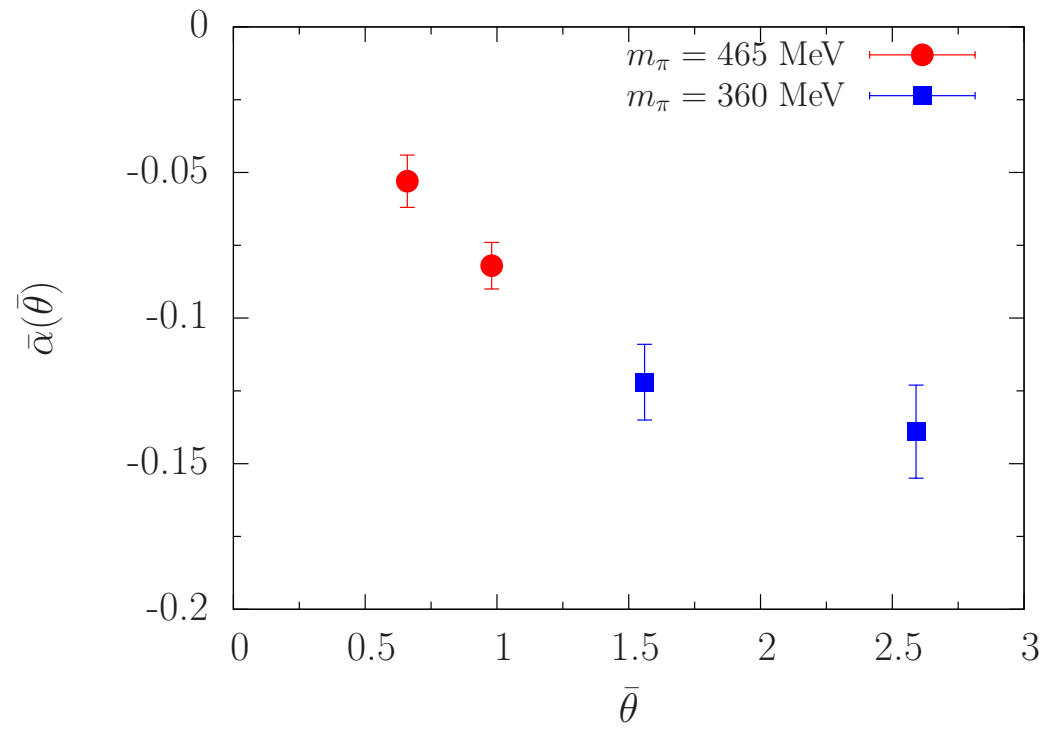


$$G_M^n = F_1^n + F_2^n$$



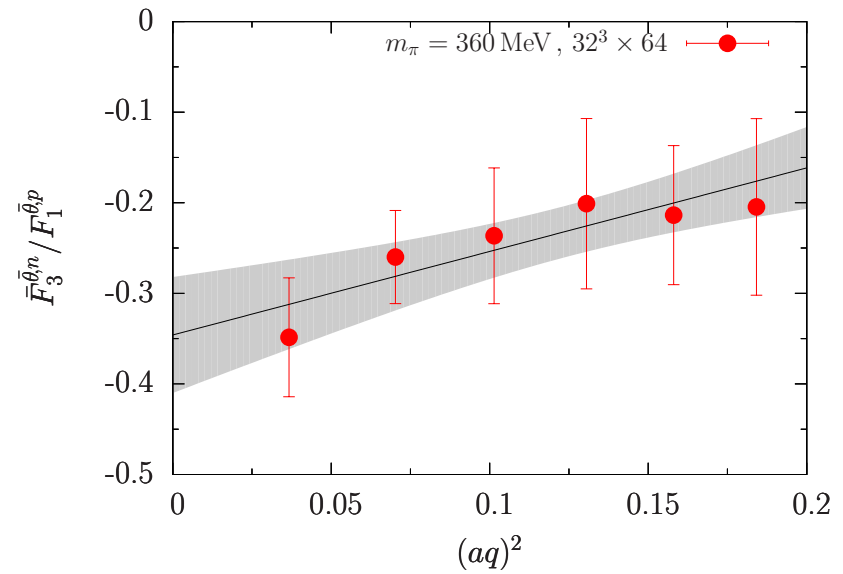
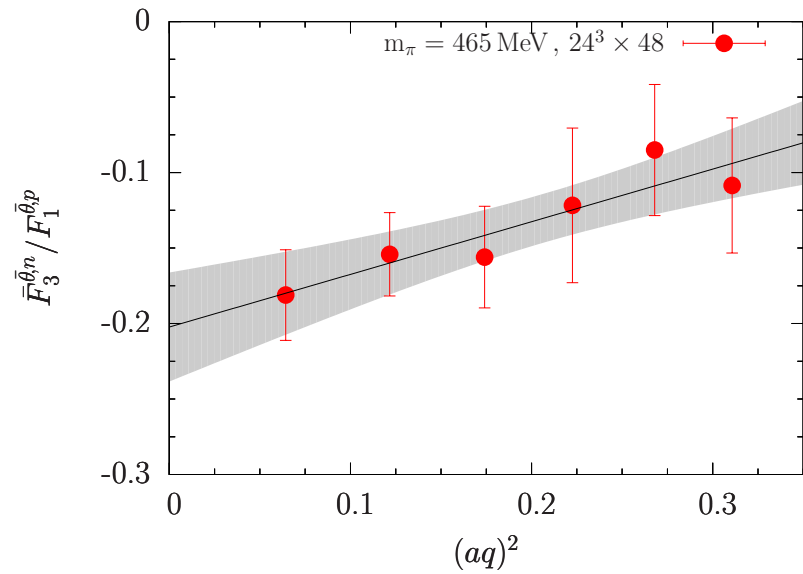
Shanahan et al. (QCDSF)

Phase



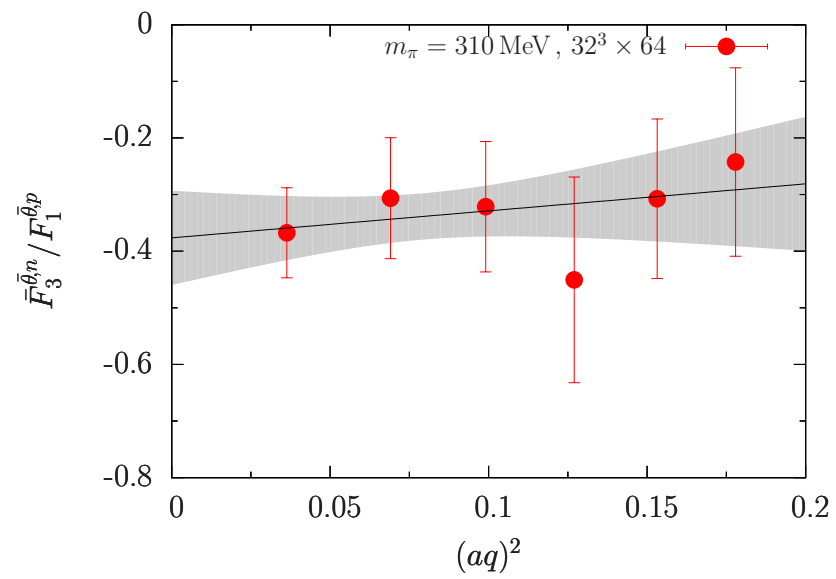
Renormalized $F_3^n(0)$

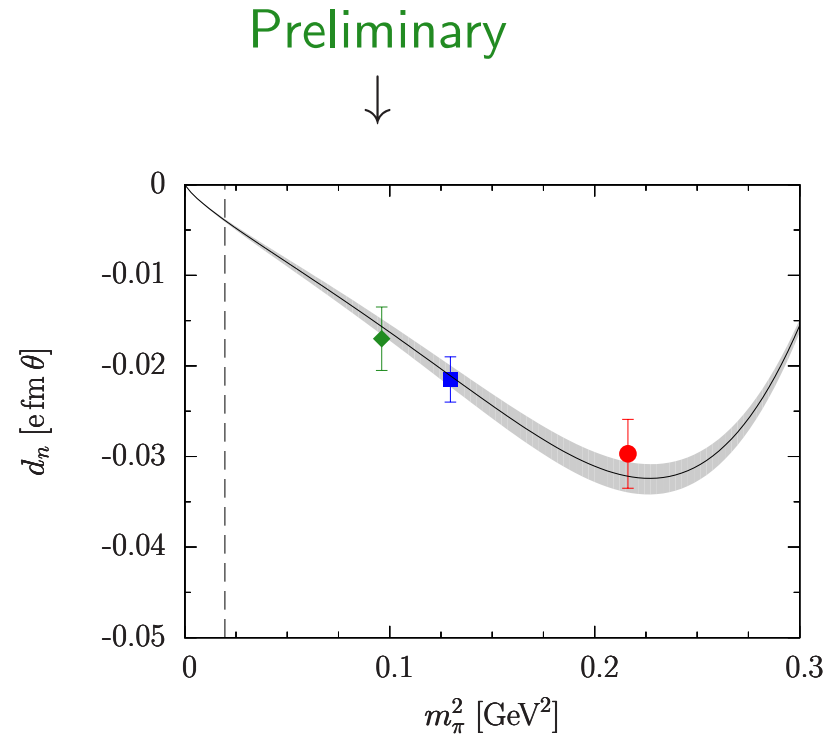
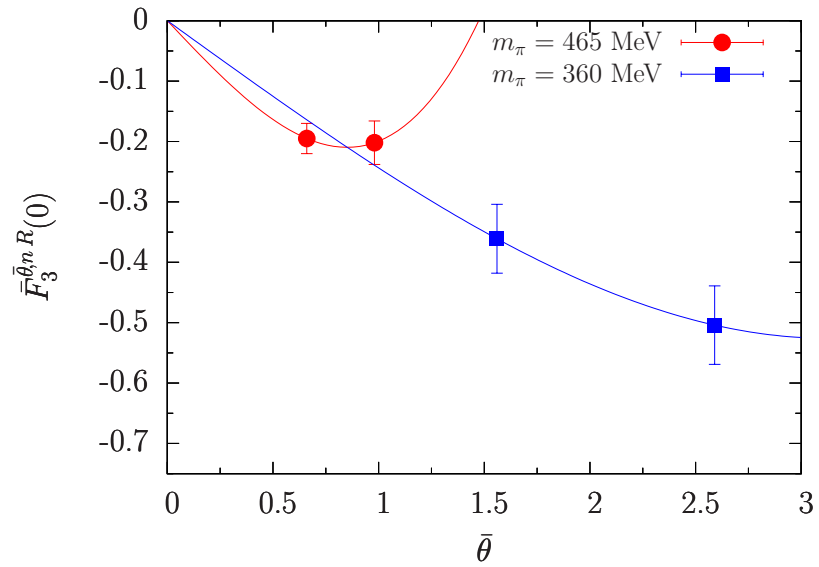
$$F_3^n(0) \propto \bar{\theta} m_\ell m_s$$



Proton case more difficult
due to mixing with $F_1^p(0)$

Preliminary (160 configs only)





m_π	m_K	$d_n [\text{efm } \theta]$
465(13)	465(13)	-0.0297(38)
360(10)	505(14)	-0.0215(25)

Fit to ChPT keeping the flavor singlet mass $2m_K^2 + m_\pi^2$ constant

Guo et al.

At the physical u , d and s quark masses this gives

PRL 115 (2015) 062001

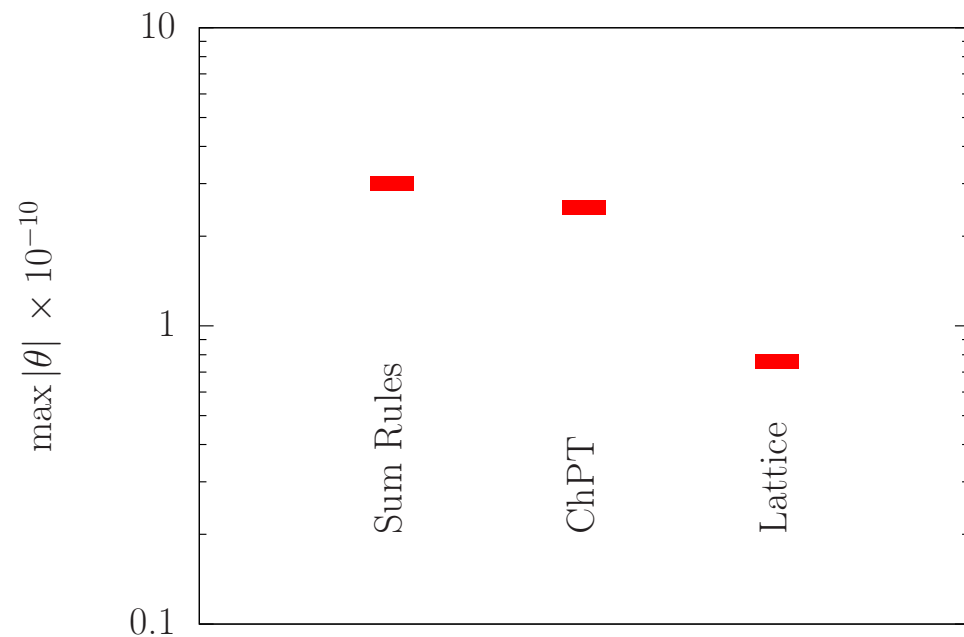
$$d_n = -0.0039(2)(9) [e \text{ fm } \theta]$$

Combining this result with the experimental bound $|d_n| \leq 2.9 \times 10^{-13} [e \text{ fm}]$, we arrive at

$$|\theta| \lesssim 7.4 \times 10^{-11}$$

Comparison with some other results:

Pospelov & Ritz, Guo et al.



Conclusions

- Electric dipole moment can be computed on the lattice with precision for small values of θ
- At the physical light and strange quark masses $d_n = -0.0039(11) [e \text{ fm } \theta]$, $|\theta| \lesssim 7.4 \times 10^{-11}$
- Simulations on larger lattices and smaller quark masses are in progress
- To obtain a full understanding of CP violation within QCD, further observables need to be addressed

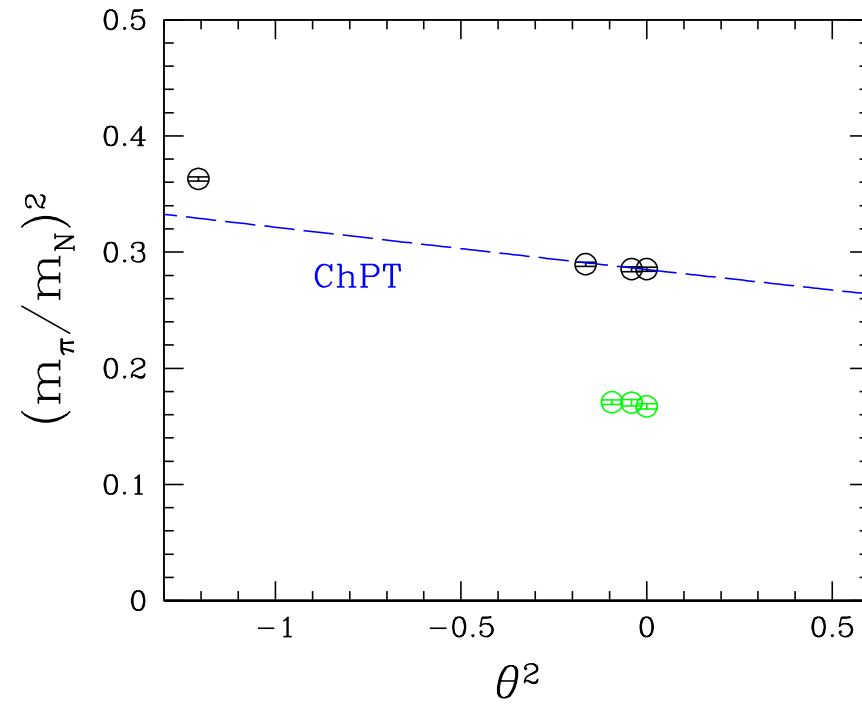
By means of simulations at imaginary vacuum angle θ

Based on chiral extrapolation and upper experimental bound

Possibly also including effects of QED

Hadron masses, hadron decays, vacuum structure, oblique confinement

Masses (first look)



ChPT: Brower et al.