INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Lepton Flavour Violation in Hadronic Tau decays

Emilie Passemar

Indiana University/Jefferson Laboratory

QCD for New Physics at the Precision Frontier
INT, Seattle, September 30, 2015

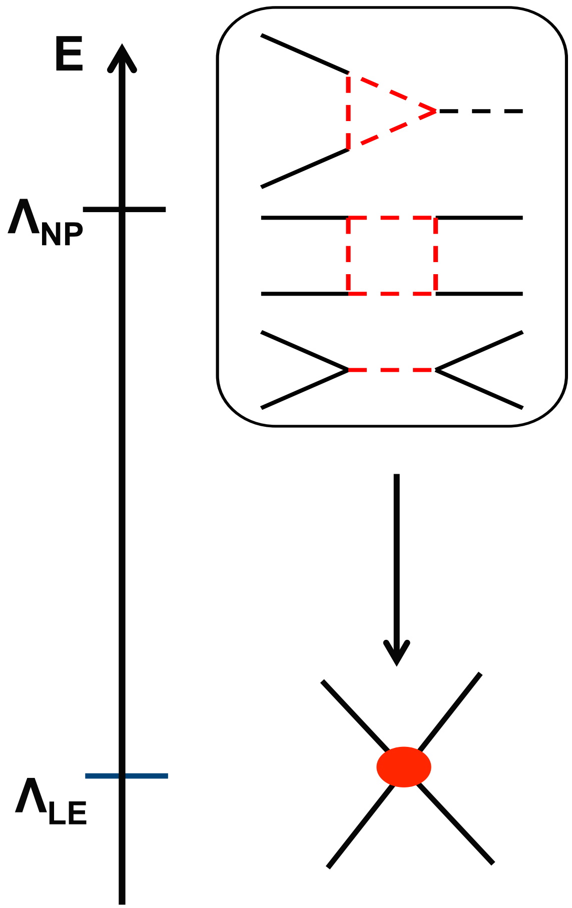
In collaboration with A. Celis (LMU, Munich), and V. Cirigliano (LANL)
PRD 89 (2014) 013008, 095014

Outline

1. Introduction and Motivation
2. Charged Lepton-Flavour Violation from tau decays
3. Special Role of $\tau \to \mu\pi\pi$: hadronic form factors
4. Results
5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why study charged leptons?

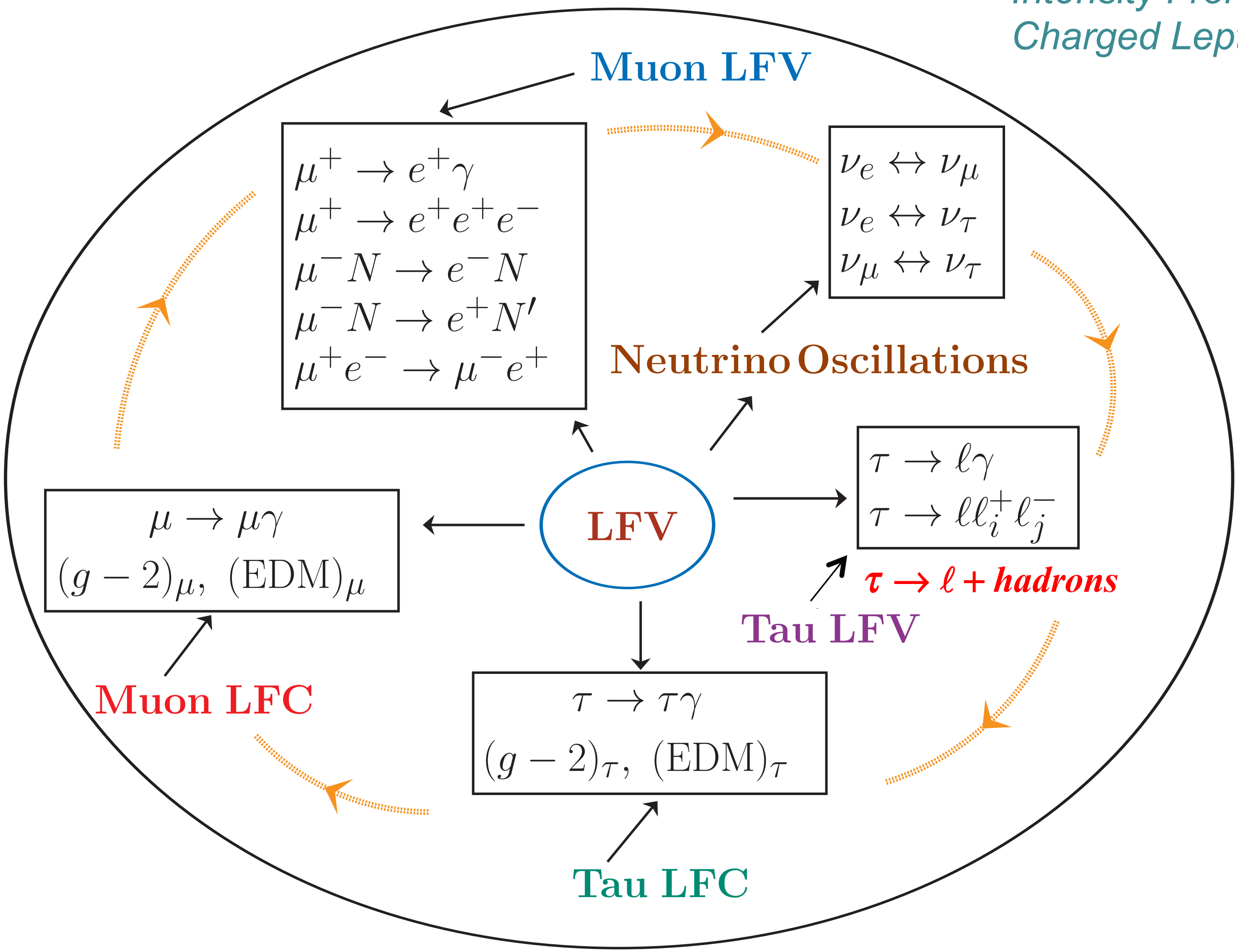


- In the quest of New Physics, can be sensitive to very high scale:
  - Kaon physics: $\frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5$ TeV
    [ε_K]
  - Charged Leptons: $\frac{\mu\bar{e}f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^4$ TeV
    [$\mu \to e\gamma$]
- At low energy: lots of experiments e.g., *MEG*, *COMET*, *Mu2e*, *E-969, BaBar*, *BelleI-II*, *BESIII, LHCb* ⇨ huge improvements on measurements and bounds obtained and more expected
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential

⇨ Charged leptons very important to look for *New Physics*!

1.2 The Program

Intensity Frontier Charged Lepton WG'13



2. Charged Lepton-Flavour Violation

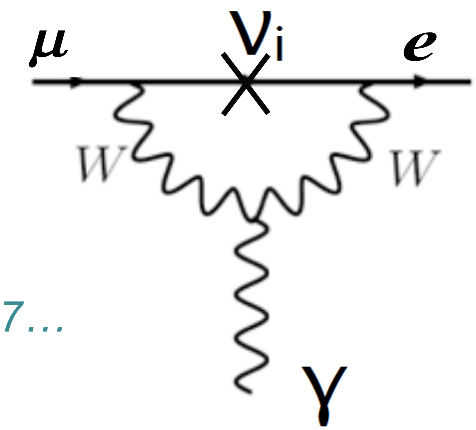
2.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

E.g.: $\mu \to e\gamma$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013

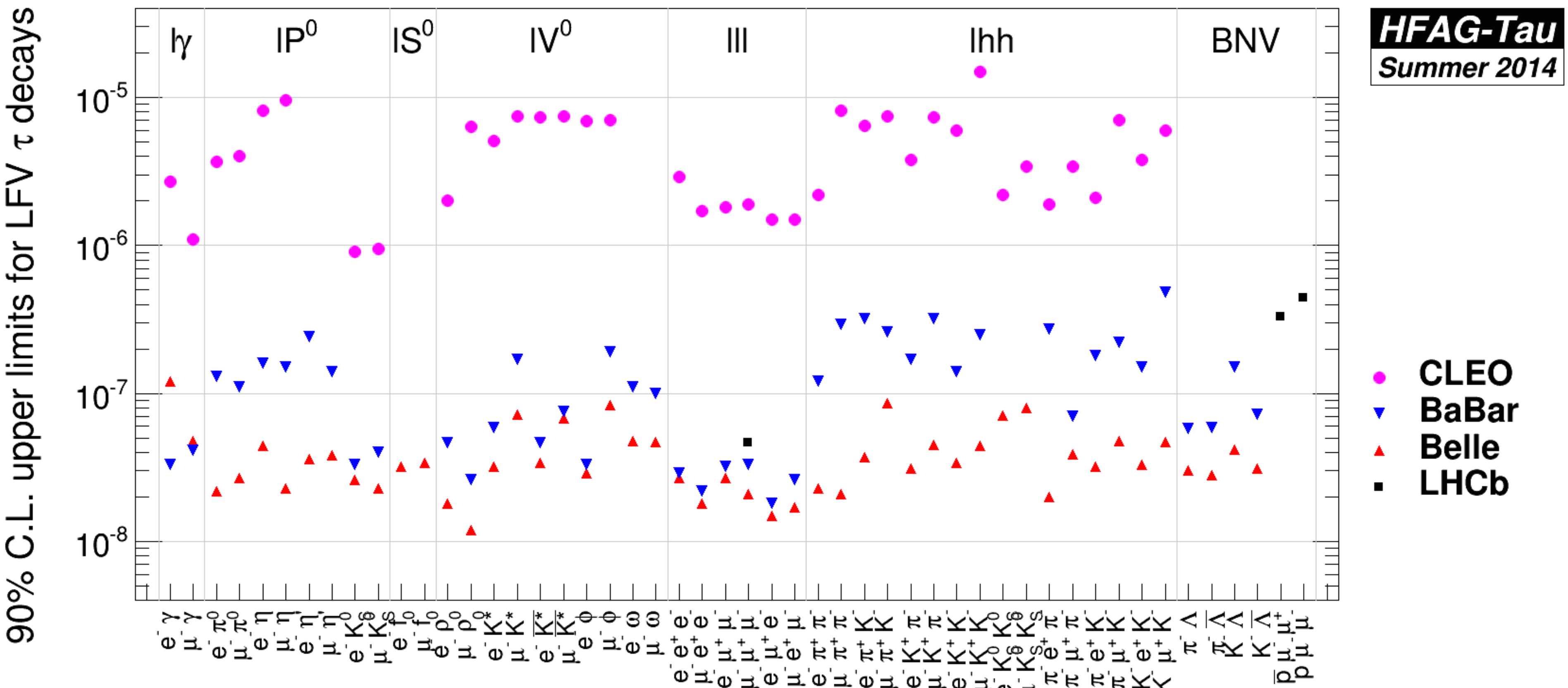
		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent

- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

2.2 CLFV processes: tau decays

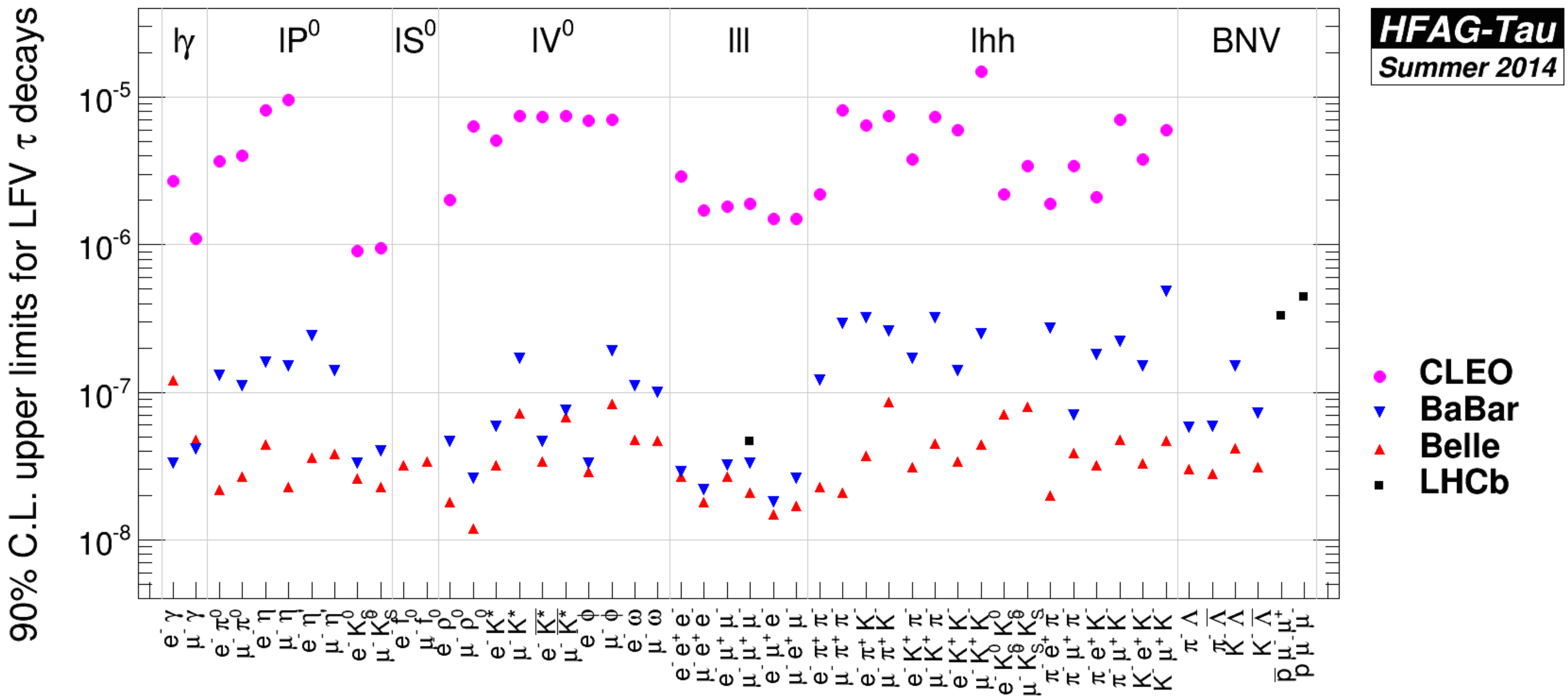
- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P}, \ldots$



- 48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ↖ $P,\ S,\ V,\ P\bar{P}, \ldots$



- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

2.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^a G_a^{\mu\nu}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu$

$\Gamma \equiv 1, \gamma^\mu$

- Each UV model generates a *specific pattern* of them

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇒ key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O_V^q	–	–	✓ (I=1)	✓(I=0,1)	–	–
O_S^q	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O_A^q	–	–	–	–	✓ (I=1)	✓ (I=0)
O_P^q	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

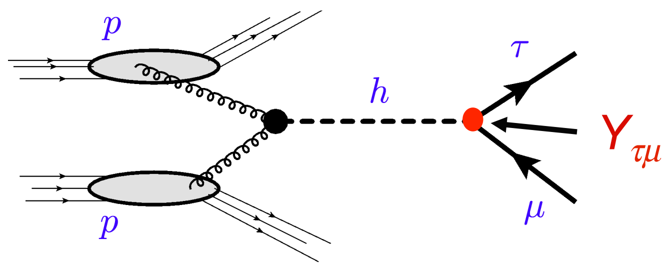
2.5 Ex: Non standard LFV Higgs coupling

- $\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H$ $\Rightarrow$ $-Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

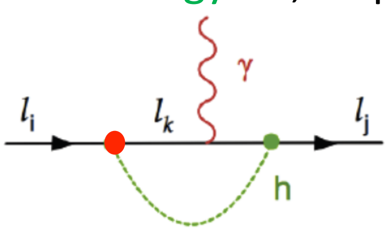
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

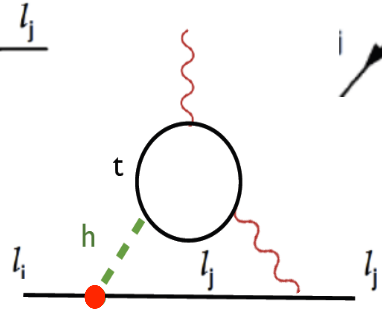
- High energy : LHC

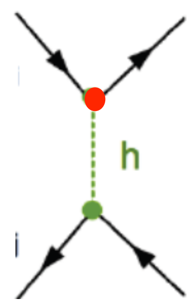


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







2.5 Ex: Non standard LFV Higgs coupling

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Goudelis, Lebedev, Park'11
Davidson, Grenier'10
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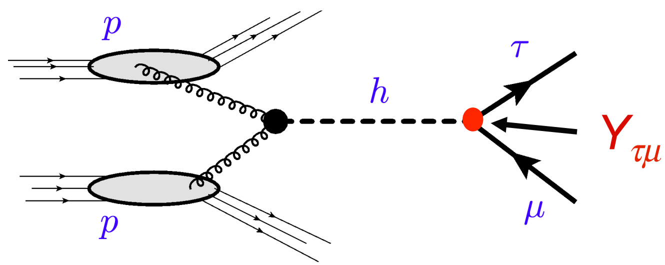
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

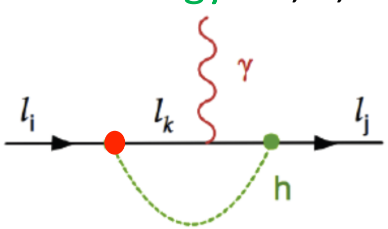
- High energy : LHC

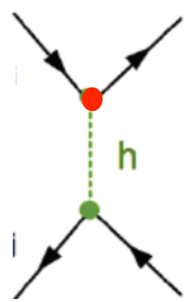


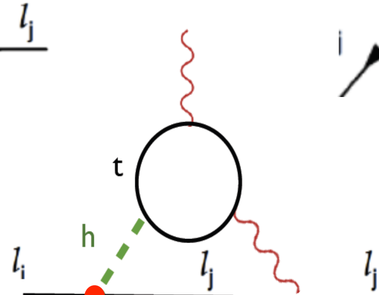
Hadronic part treated with perturbative QCD

Reverse the process

- Low energy : D, S, G operators







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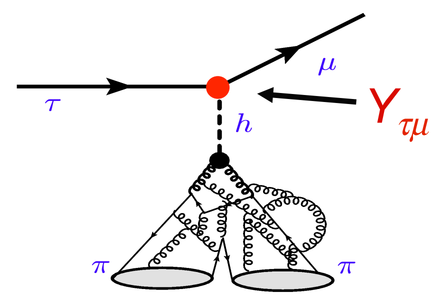
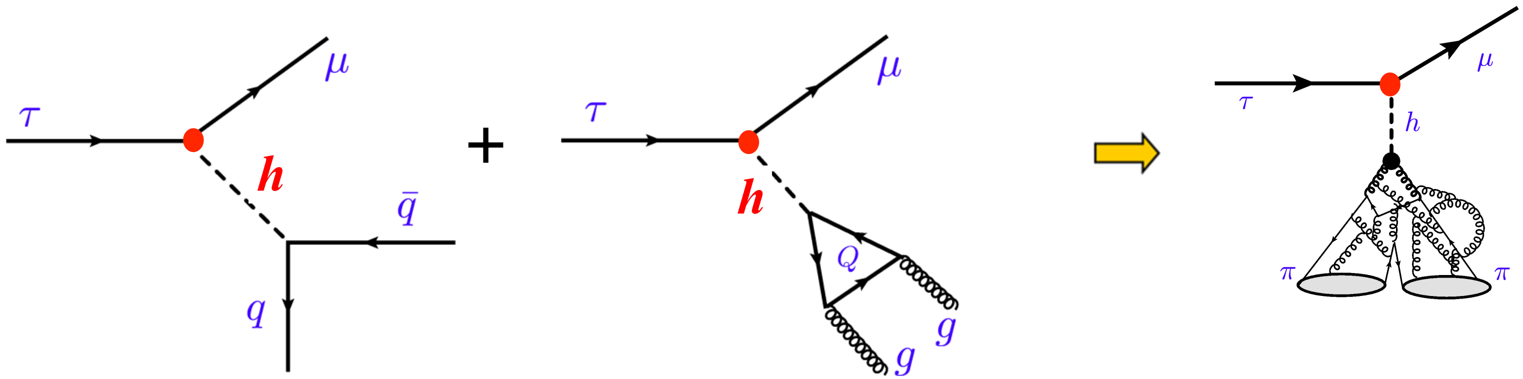


Hadronic part treated with non-perturbative QCD

2.6 Constraints from τ→μππ

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

  ⇒ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

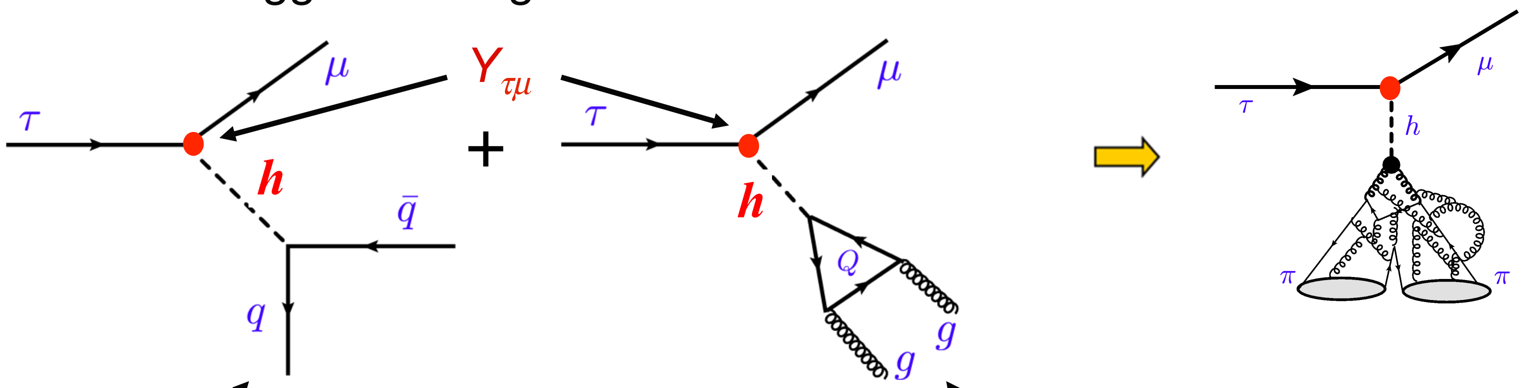
Daub, Dreiner, Hanart, Kubis, Meissner'13

Celis, Cirigliano, E.P.'14

3. Description of the hadronic form factors

3.1 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



$$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

$$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

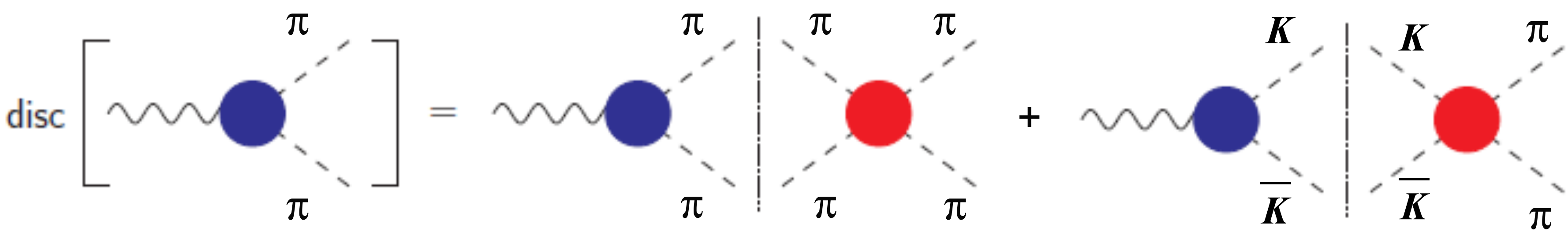
$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\Gamma_{\tau\to\mu\pi\pi} \propto \int \left| \Gamma_\pi(s) + \Delta_\pi(s) + \theta_\pi(s) \right|^2 Y^2_{\tau\mu}$$

with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

3.2 Unitarity

- Unitarity ⇨ the discontinuity of the form factor is known



⇨
$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$
$n = \pi\pi, K\bar{K}$

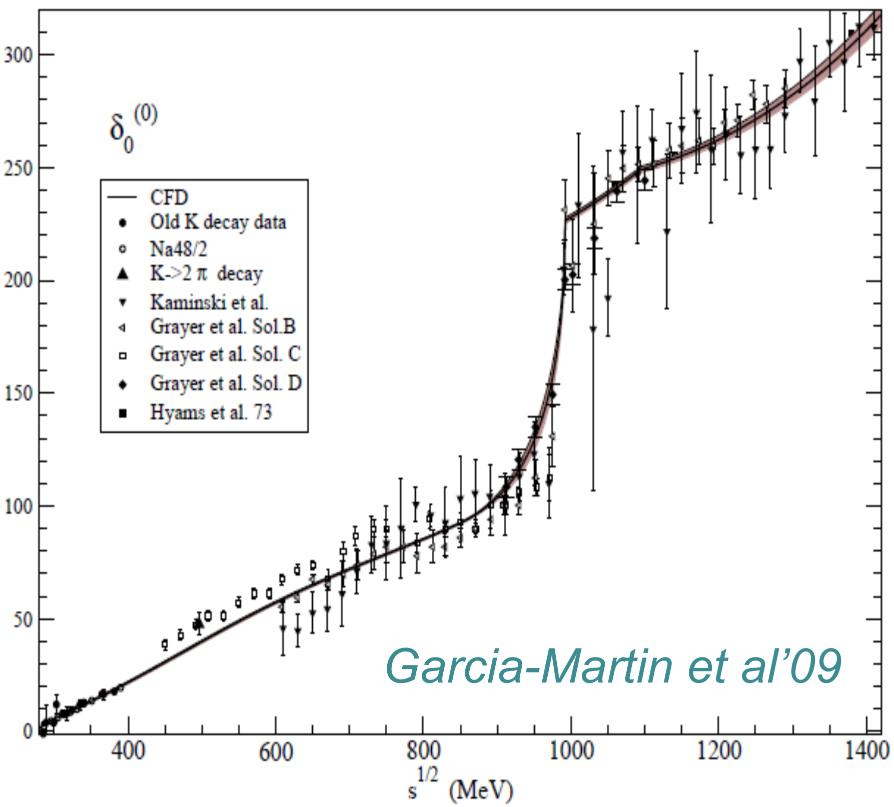
- *Coupled channel analysis* up to $\sqrt{s}\sim 1.4$ GeV
  Inputs: I=0, S-wave ππ and KK data

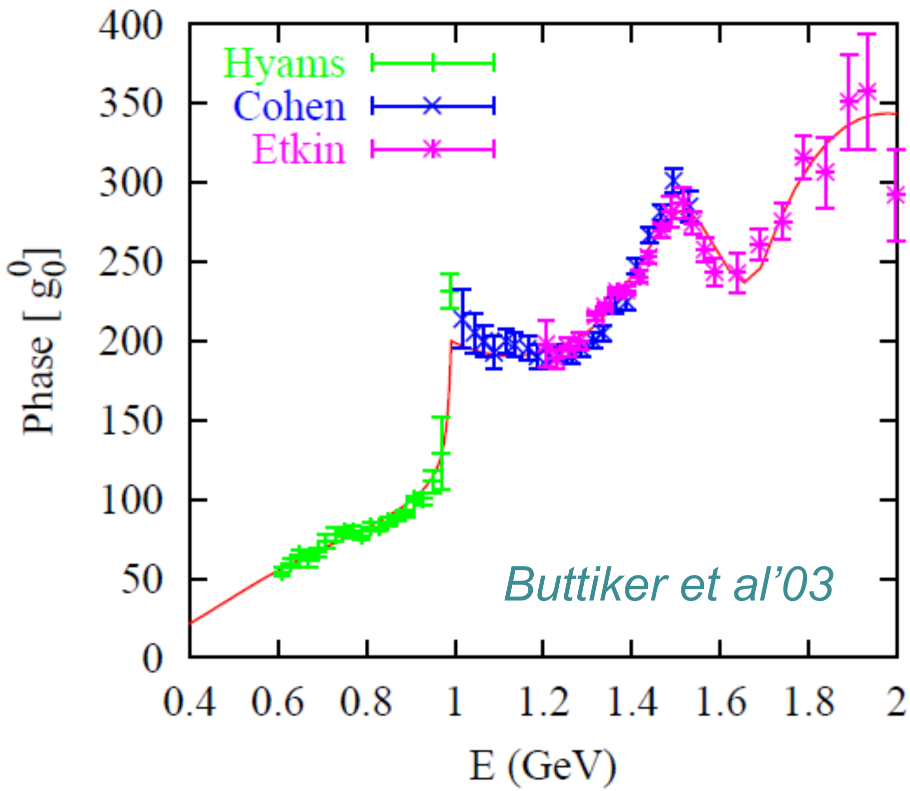
Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub et al'13
Celis, Cirigliano, E.P.'14

3.3 Inputs for the coupled channel analysis

Celis, Cirigliano, E.P.'14

- Inputs : $\pi\pi \to \pi\pi, KK$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree

- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

3.4 Dispersion relations

Celis, Cirigliano, E.P.'14

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

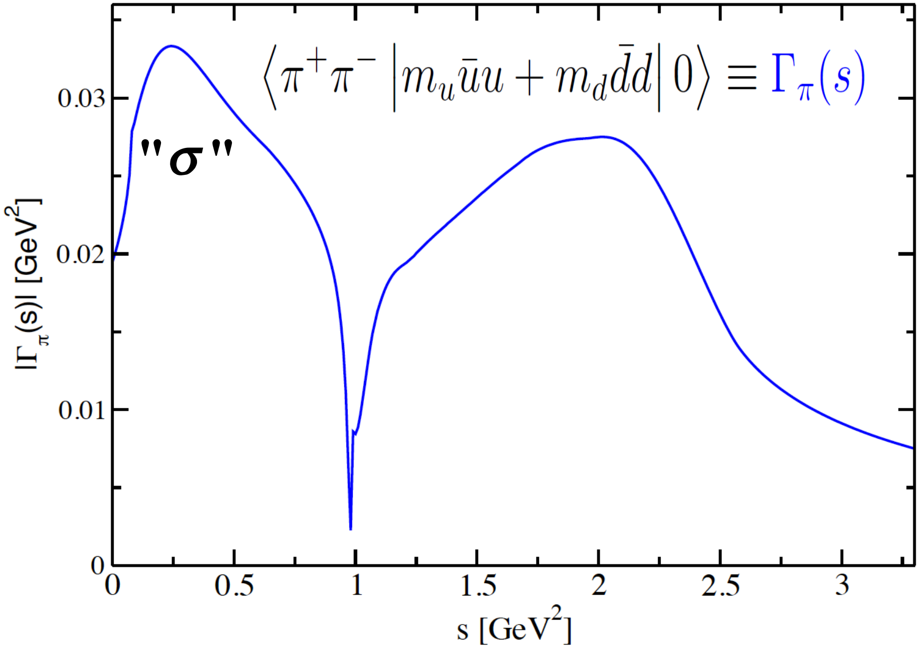
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

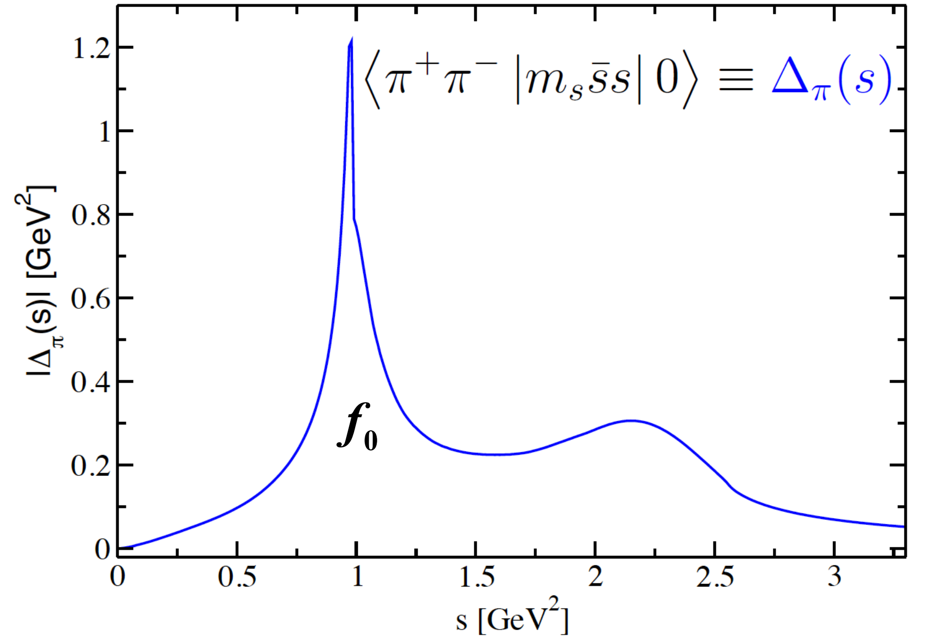
Polynomial determined from a matching to ChPT + lattice

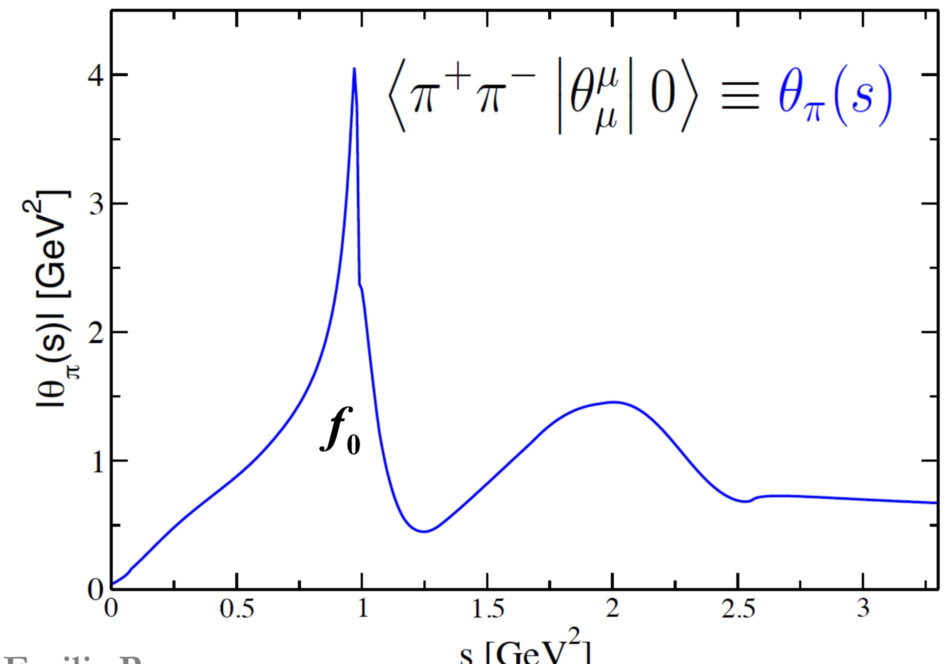
- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

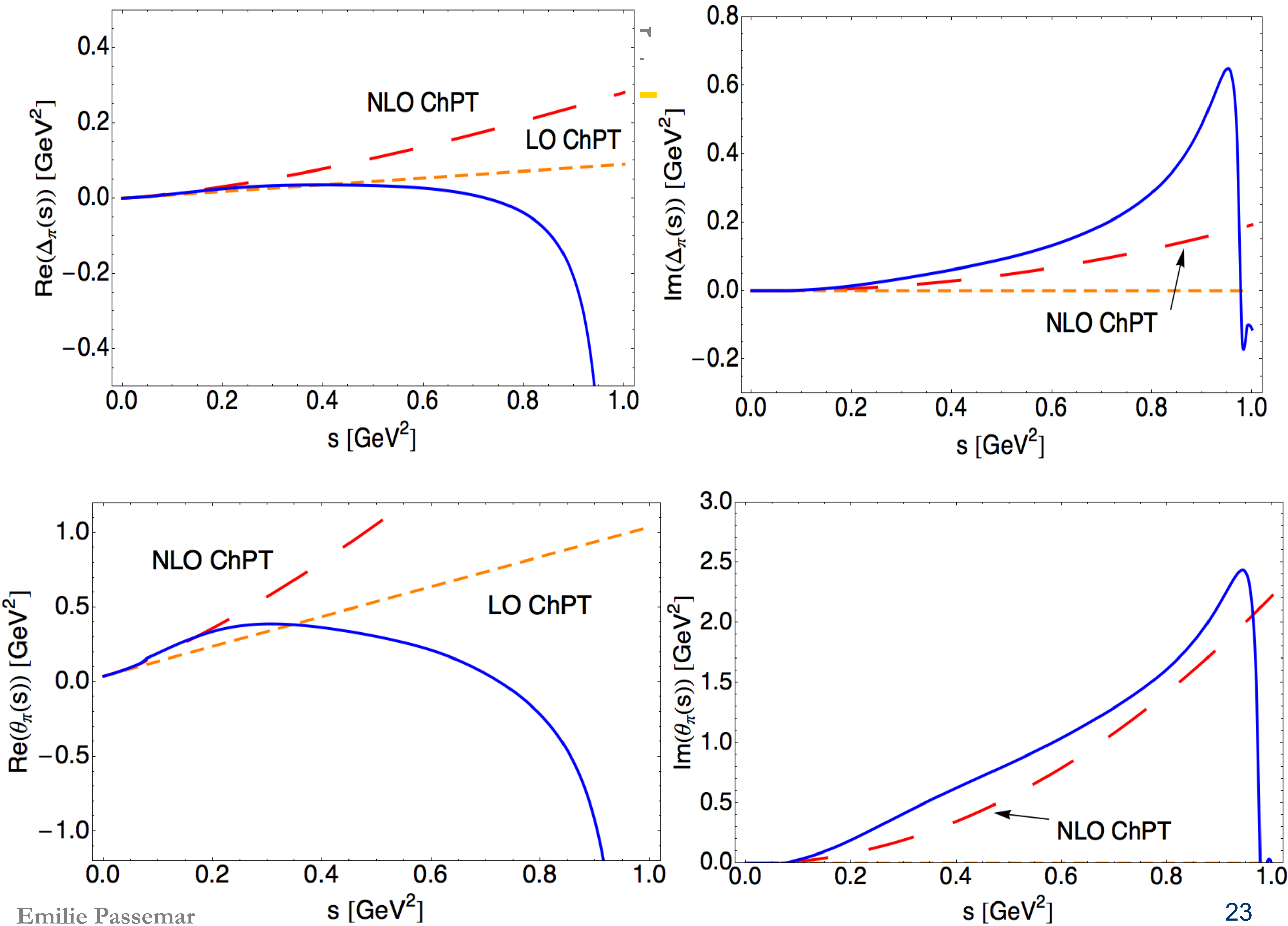
$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} T^*_{mn} \sigma_m(s) X_m^{(N)}(s) \longrightarrow X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} X_n^{(N+1)}(s')}{s'-s}$$







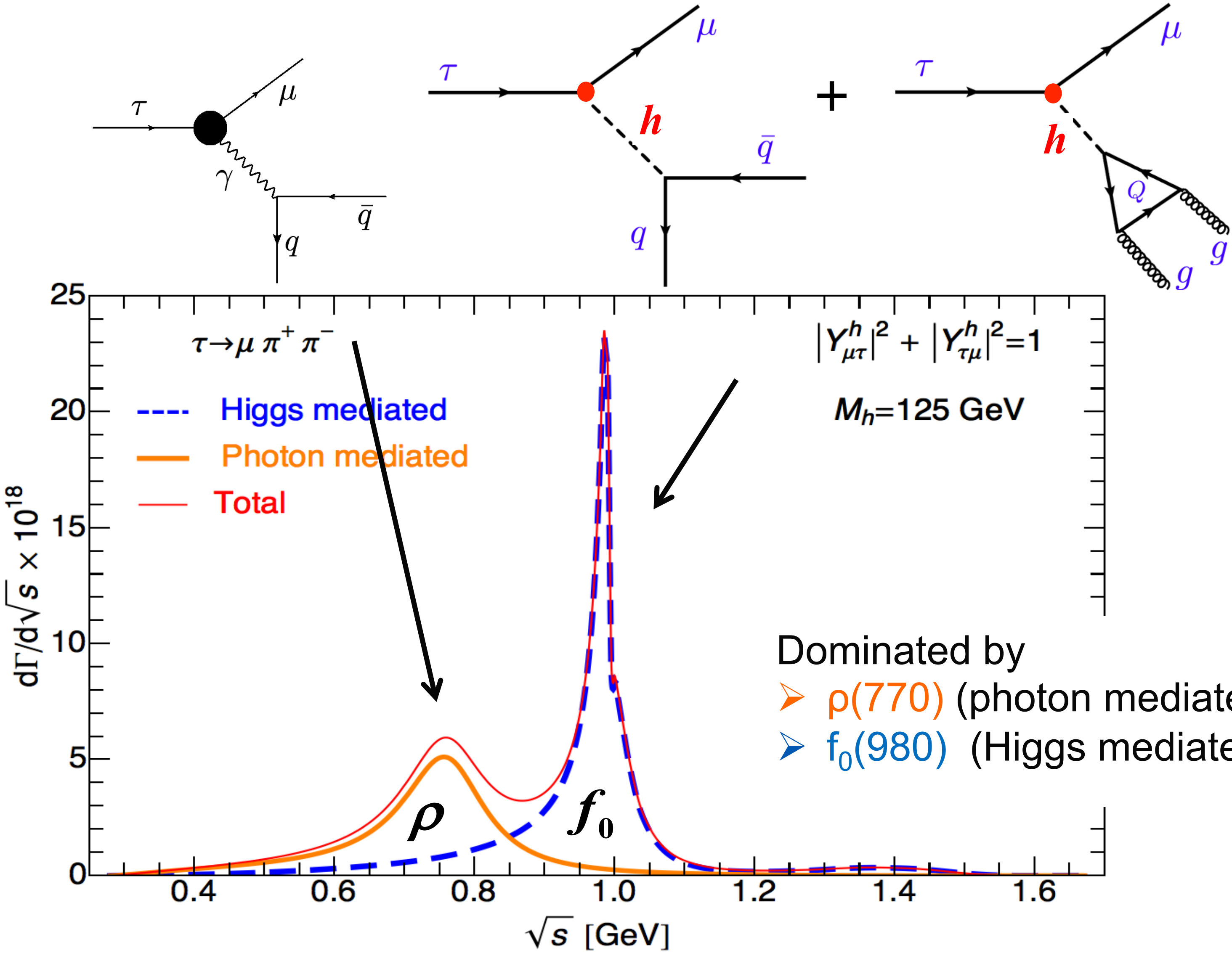
Dispersion relations:
Model-independent method, based on first principles that extrapolates ChPT based on data



NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV²]
s [GeV²]
Im(Δ_π(s)) [GeV²]
NLO ChPT
s [GeV²]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV²]
s [GeV²]
Im(θ_π(s)) [GeV²]
NLO ChPT
s [GeV²]

4. Results

Celis, Cirigliano, E.P.'14

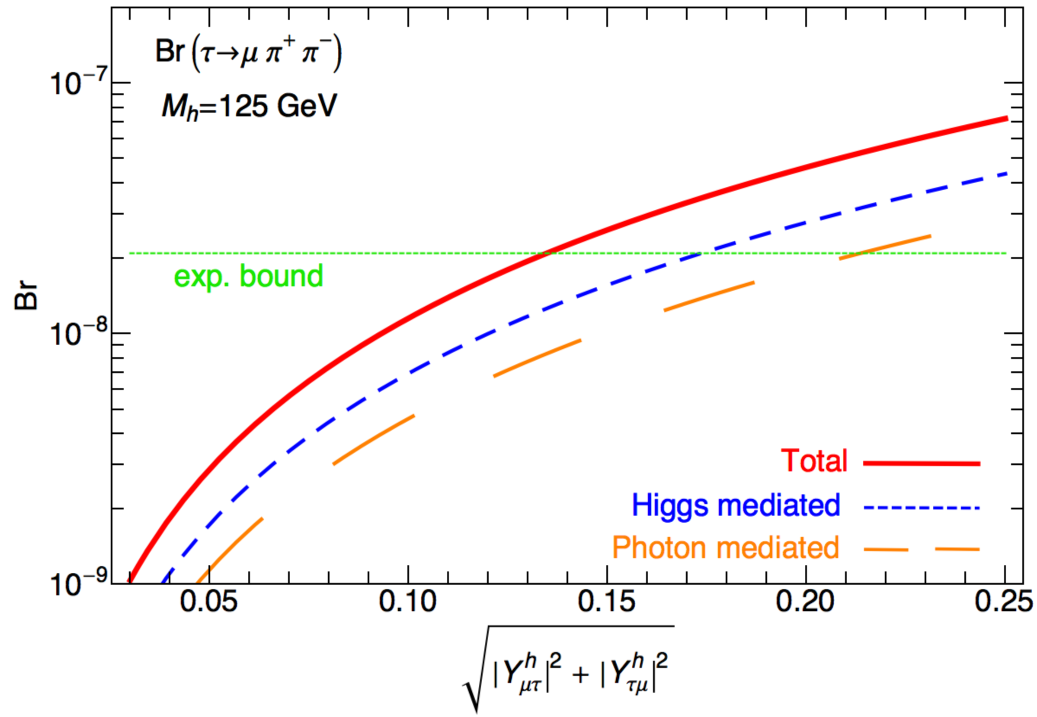


Dominated by

- ρ(770) (photon mediated)
- $f_0(980)$ (Higgs mediated)

4.2 Bounds

Celis, Cirigliano, E.P.'14



Bound:

$$\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13$$

Process	(BR × 10^8) 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	< 1.4×10^3 [87]	< 6.3	Scalar, Gluon

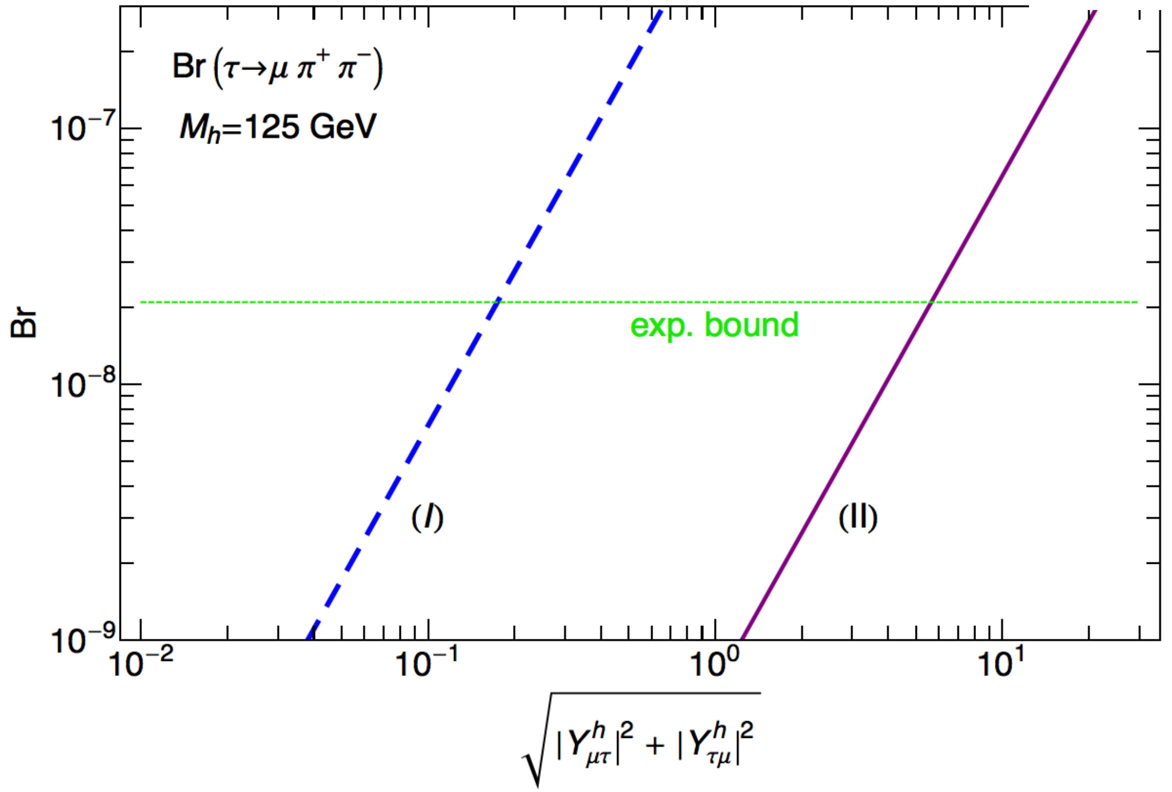
?

Less stringent but more robust handle on LFV Higgs couplings

BaBar'10, Belle'10'11'13 except last from *CLEO'97*

4.3 Impact of our results

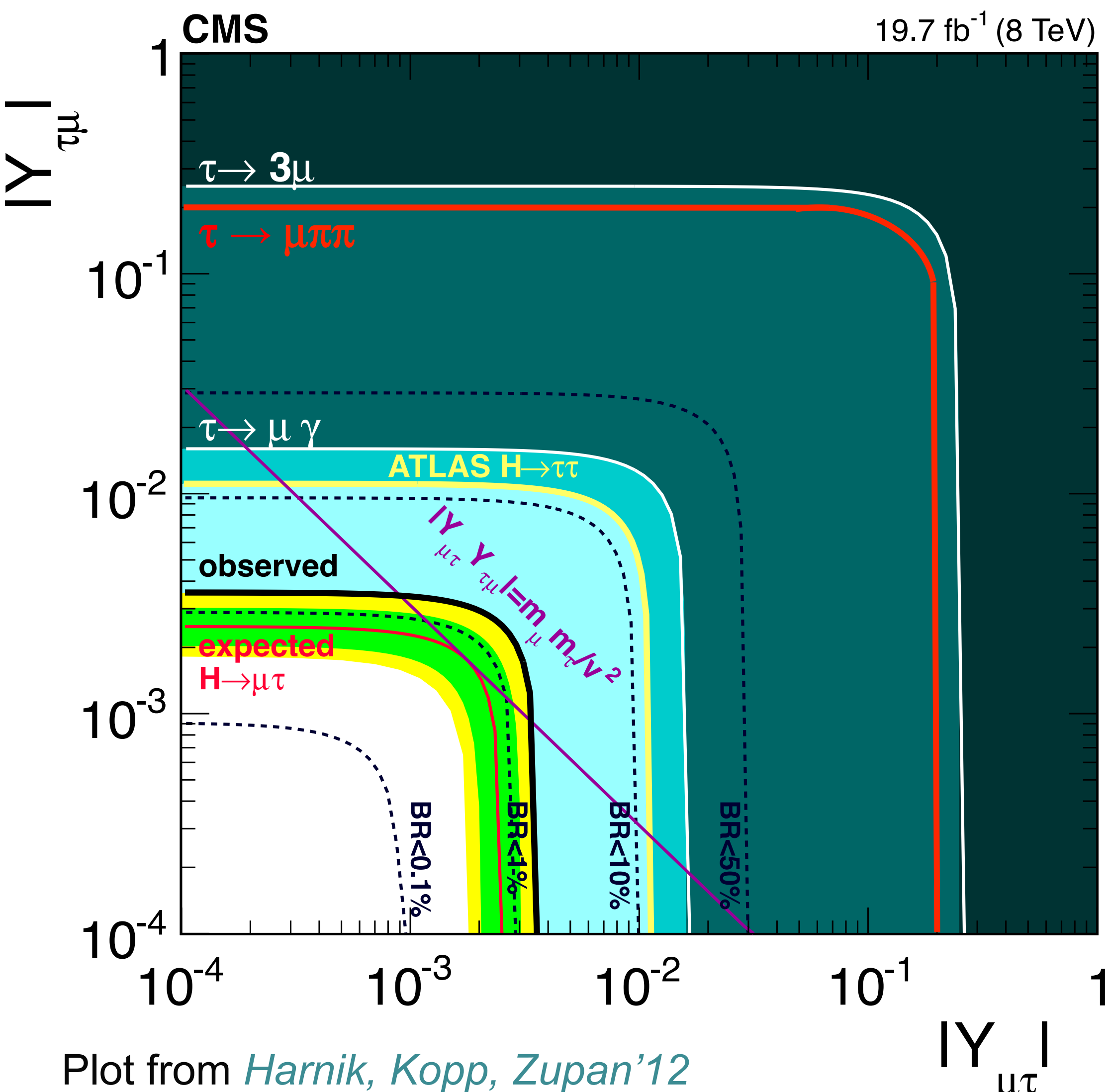
Celis, Cirigliano, E.P.'14



- Rigorous treatment of hadronic part ⇒ bound reduced by one order of magnitude! ⇒ *Robust bounds!*

- ChPT, EFT only valid at low energy for $\mathbf{p << \Lambda = 4\pi f_\pi \sim 1\ GeV}$
  ⇒ *not valid up to $E = (m_\tau - m_\mu)$* !

4.4 Comparison Low Energy & High Energy



Plot from *Harnik, Kopp, Zupan'12* updated *by CMS'15*

"Signal" not confirmed by *ATLAS'15*

- Constraints from LE:
  - $\tau \to \mu\gamma$: best constraints but loop level → sensitive to UV completion of the theory
  - $\tau \to \mu\pi\pi$: tree level diagrams → robust handle on LFV
- Constraints from HE: *LHC* wins for τμ!
- Opposite situation for μe!
- For LFV Higgs and nothing else: LHC bound

→

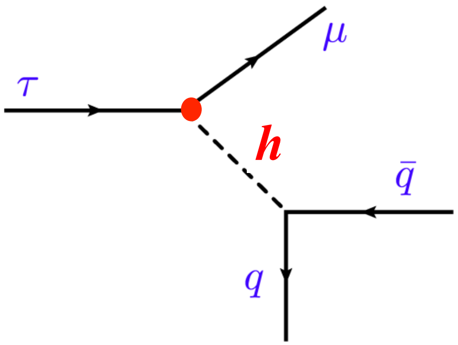
$$BR(\tau \to \mu\gamma) < 2.2 \times 10^{-9}$$

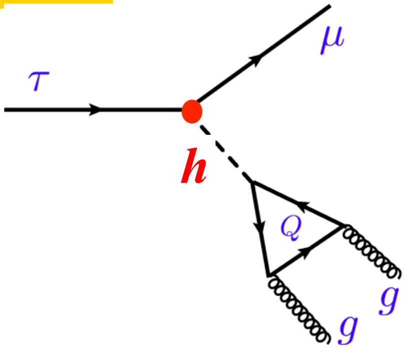
$$BR(\tau \to \mu\pi\pi) < 1.6 \times 10^{-11}$$

4.4 What if $\tau \to \mu(e)\pi\pi$ is observed?

Talk by J. Zupan
@ KEK-FF2014FALL

- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$

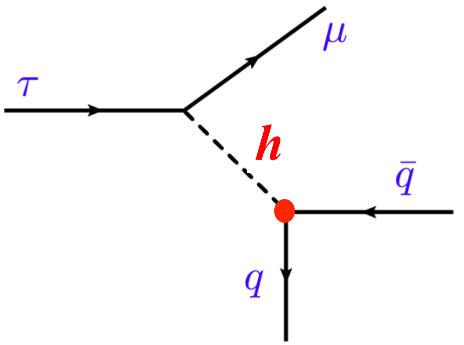


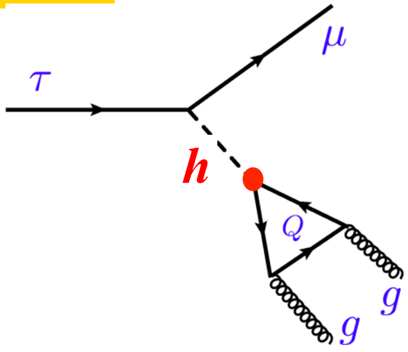


4.4 What if $\tau \to \mu(e)\pi\pi$ is observed?

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \to \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!

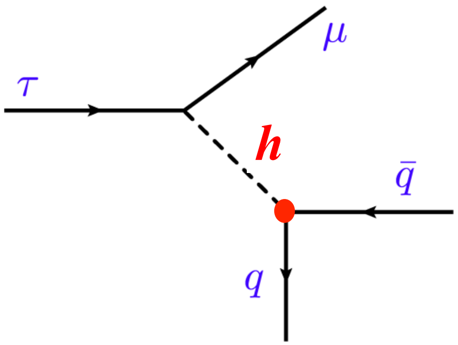


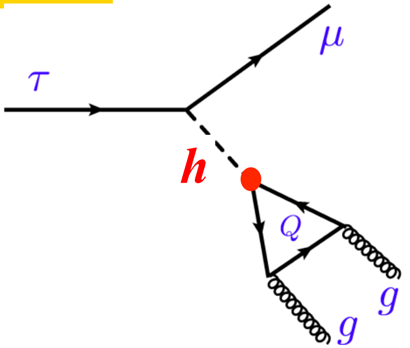


4.4 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}$!





- $Y_{u,d,s}$ poorly bounded

- For $Y_{u,d,s}$ at their SM values :

$$Br(\tau \to \mu\pi^+\pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu\pi^0\pi^0) < 4.6 \times 10^{-12}$$
$$Br(\tau \to e\pi^+\pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e\pi^0\pi^0) < 6.9 \times 10^{-11}$$

- But for $Y_{u,d,s}$ at their upper bound:

$$Br(\tau \to \mu\pi^+\pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu\pi^0\pi^0) < 1.5 \times 10^{-8}$$
$$Br(\tau \to e\pi^+\pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e\pi^0\pi^0) < 2.1 \times 10^{-7}$$

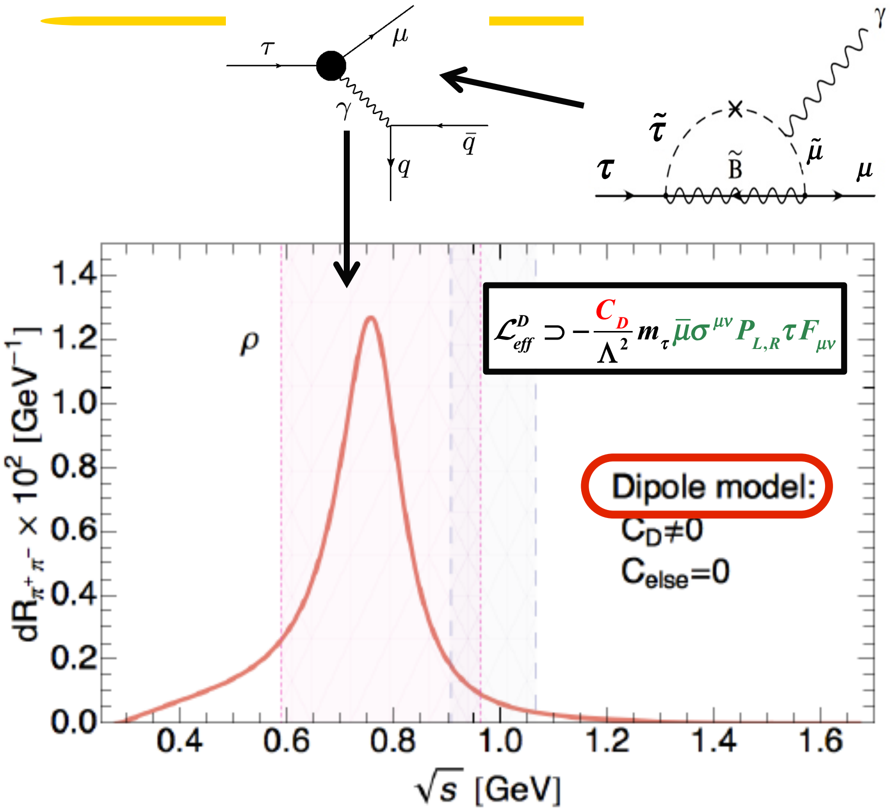
below present experimental limits!

- If discovered ⇨ *upper limit* on $Y_{u,d,s}$!

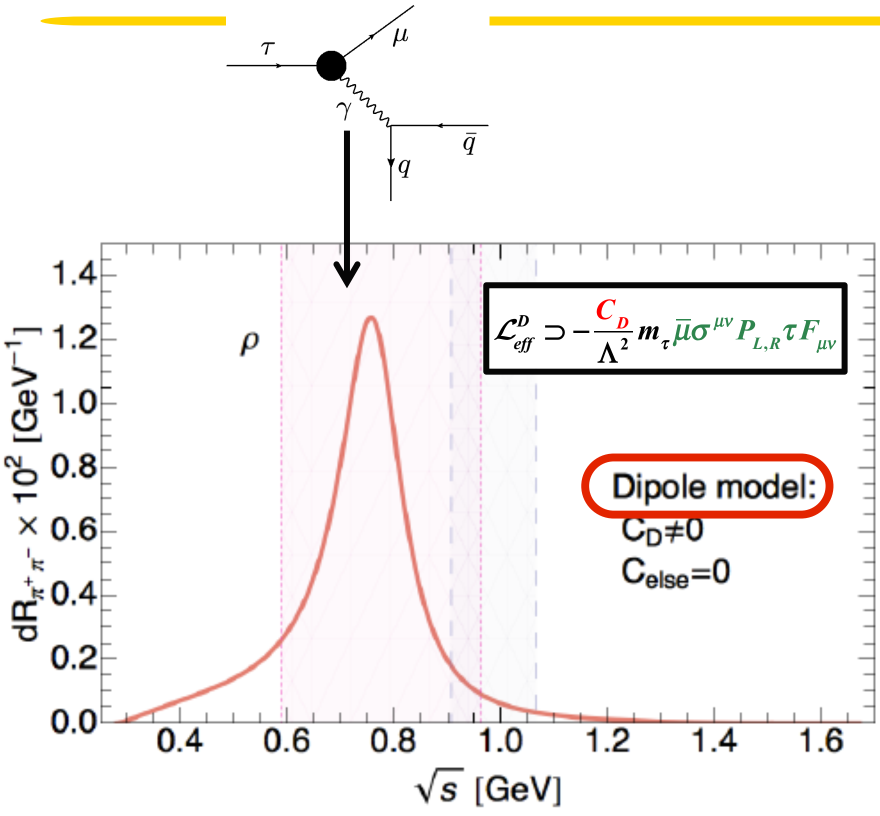
⇨ Interplay between high-energy and low-energy constraints!

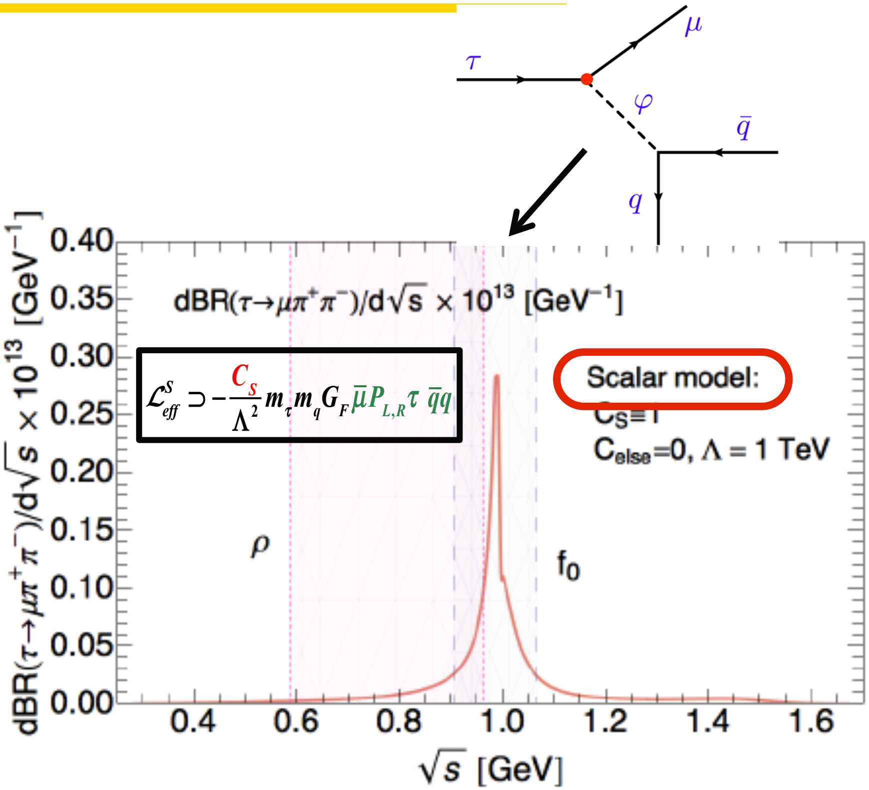
4.5 Discriminating power of $\tau \to \mu(e)\pi\pi$ decays

Celis, Cirigliano, E.P.'14

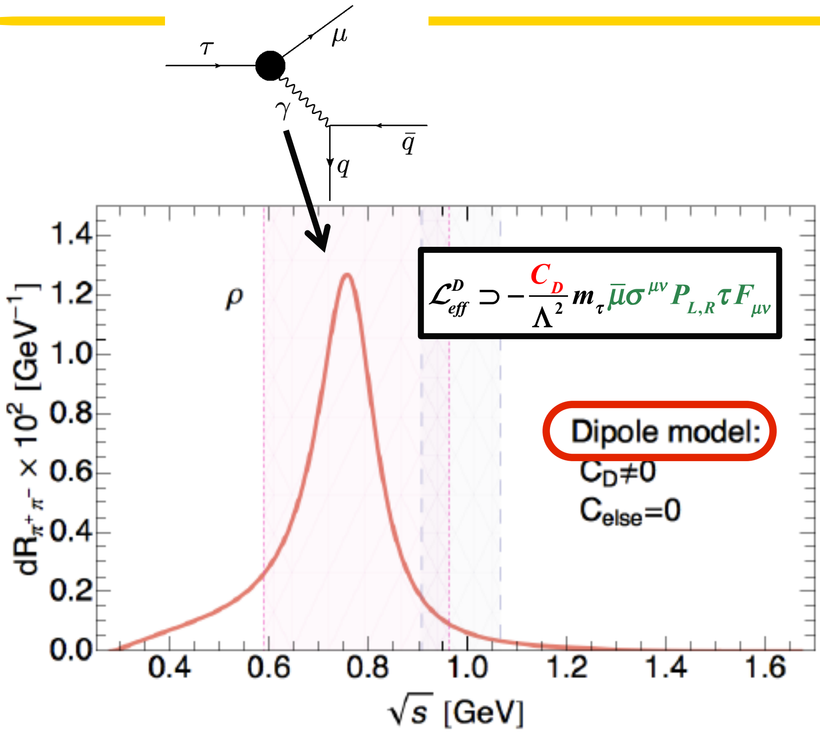


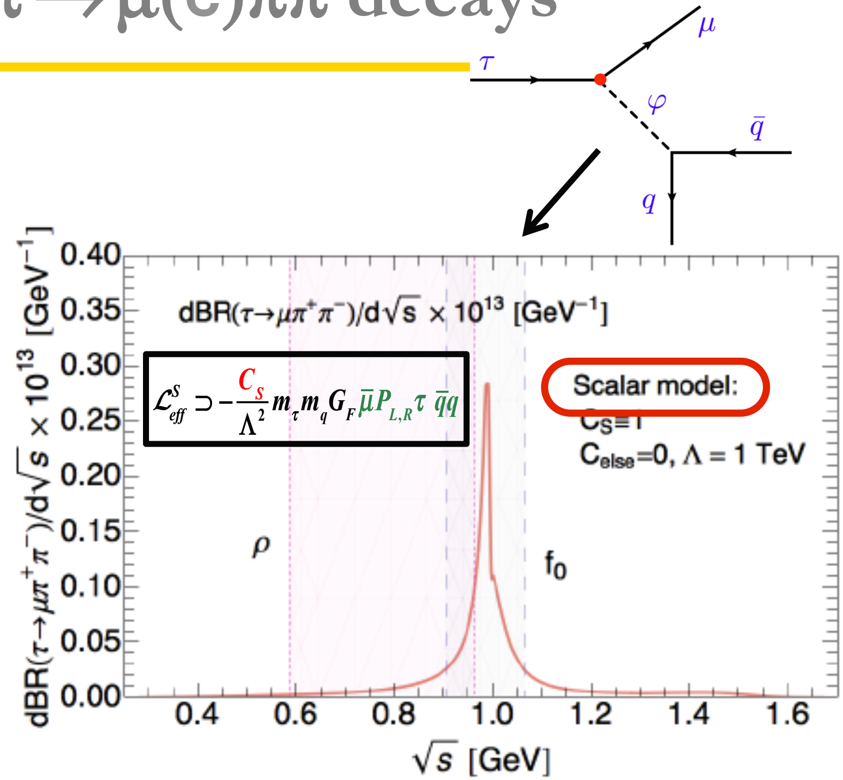
4.5 Discriminating power of τ→μ(e)ππ decays

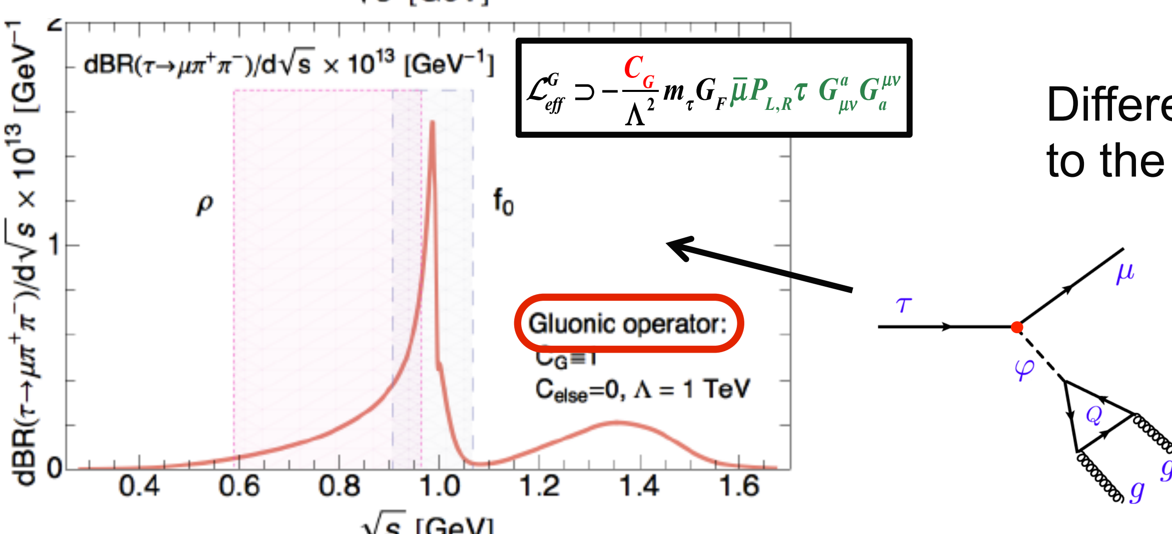




4.5 Discriminating power of τ→μ(e)ππ decays







Different distributions according to the *operator!*

5. Conclusion and Outlook

Summary

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS ⇨ energy frontier

- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the *intensity* and *precision frontiers*

- Charged LFV are a very important probe of new physics
  - ➢ Extremely small SM rates
  - ➢ Experimental results at low energy are very precise
    ⇨ very high scale sensitivity

- CLFV decays excellent model discriminating tools especially τ decays
  ⇨ *Hadronic decays* such as $\tau \to \mu(e)\pi\pi$ important!

Summary

- To consider hadronic decays, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.

- For $\tau \to \mu(e)\pi\pi$: need to know the ππ form factors

  ⇨ Use dispersion relations

- Dispersion relations rely on analyticity, unitarity and crossing symmetry

  ⇨ Rigorous treatment of two and three hadronic final state

- $\tau \to \mu(e)\pi\pi$ gives interesting constraints on LFV new physics operators involving quarks

- Interplay low energy and collider physics: LFV of the Higgs boson

- Complementarity with LFC sector: EDMs, g-2 and colliders:

  ⇨ New physics models usually strongly correlate these sectors

5. Back-up

3.4.4 Determination of the form factors : $\Gamma_\pi(s)$, $\Delta_\pi(s)$, $\theta_\pi(s)$

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$X(s) = C(s), D(s)$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{ T_{nm}^* \sigma_m(s) X_m^{(N)} \right\} \longrightarrow \mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- Fix the polynomial with requiring $F_P(s) \to 1/s$ (*Brodsky & Lepage*) + ChPT:

  Feynman-Hellmann theorem: ⇒

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$

$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

⇒

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

⇒

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  ⇒ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2 M_\pi^2 \right)$$

Dreiner, Hanart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

Determination of the polynomial

- General solution

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

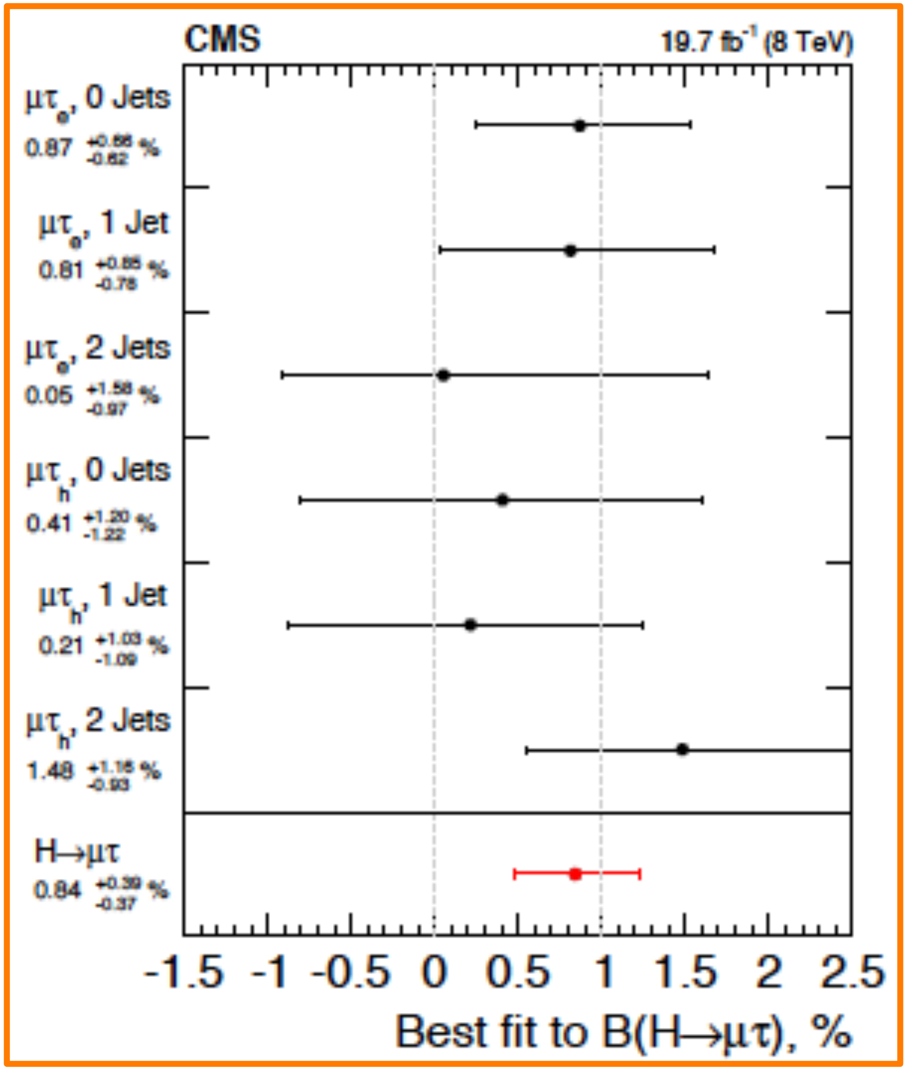
- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

⇨ Relax the constraints and match to ChPT

$$\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot{\theta}_\pi - 2M_\pi^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3} M_\pi^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{aligned}$$

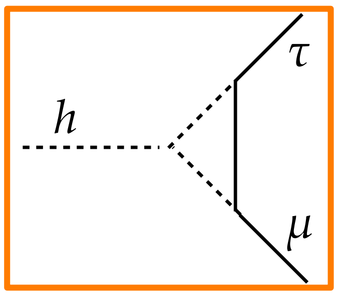
4.4 Hint of New Physics in $h \to \tau\mu$?

CMS'15



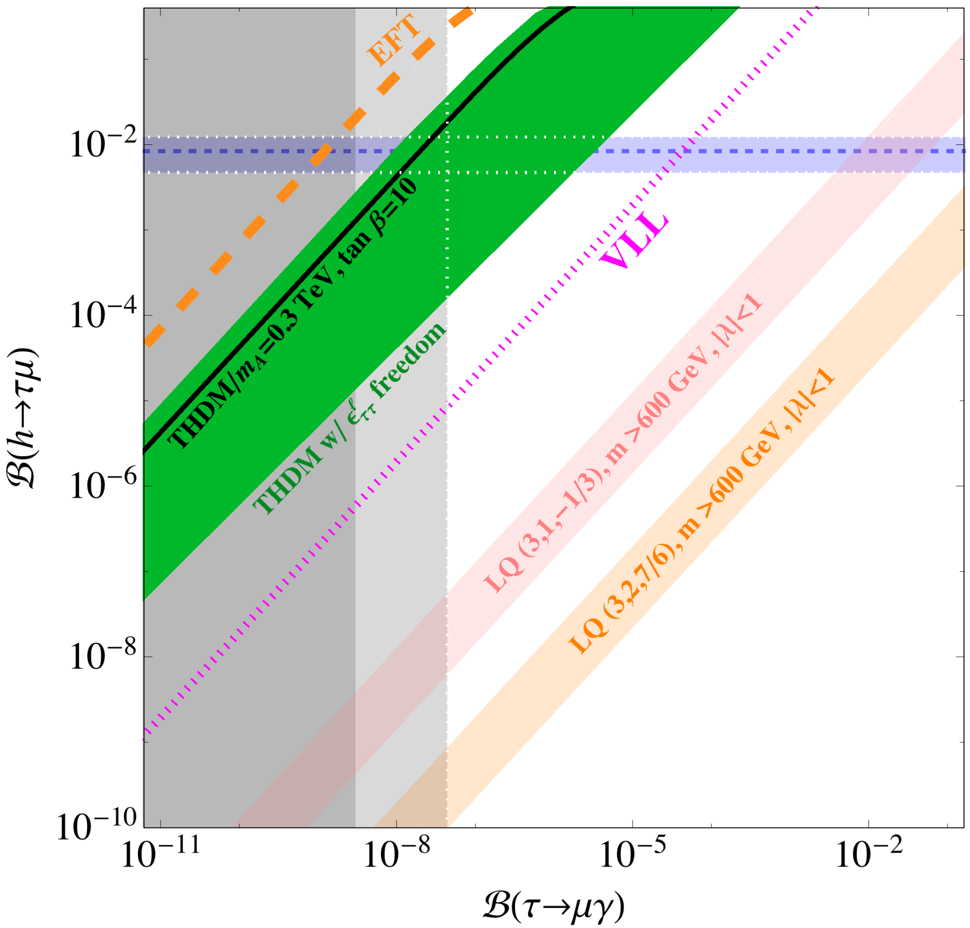
4.5 Interplay between LHC & Low Energy

- If real what type of NP?
- If h → τ μ due to loop corrections:
  - extra charged particles necessary
  - τ → μγ too large



- h → τ μ possible to explain if extra scalar doublet: ⇨ *2HDM of type III*

Dorsner et al.'15

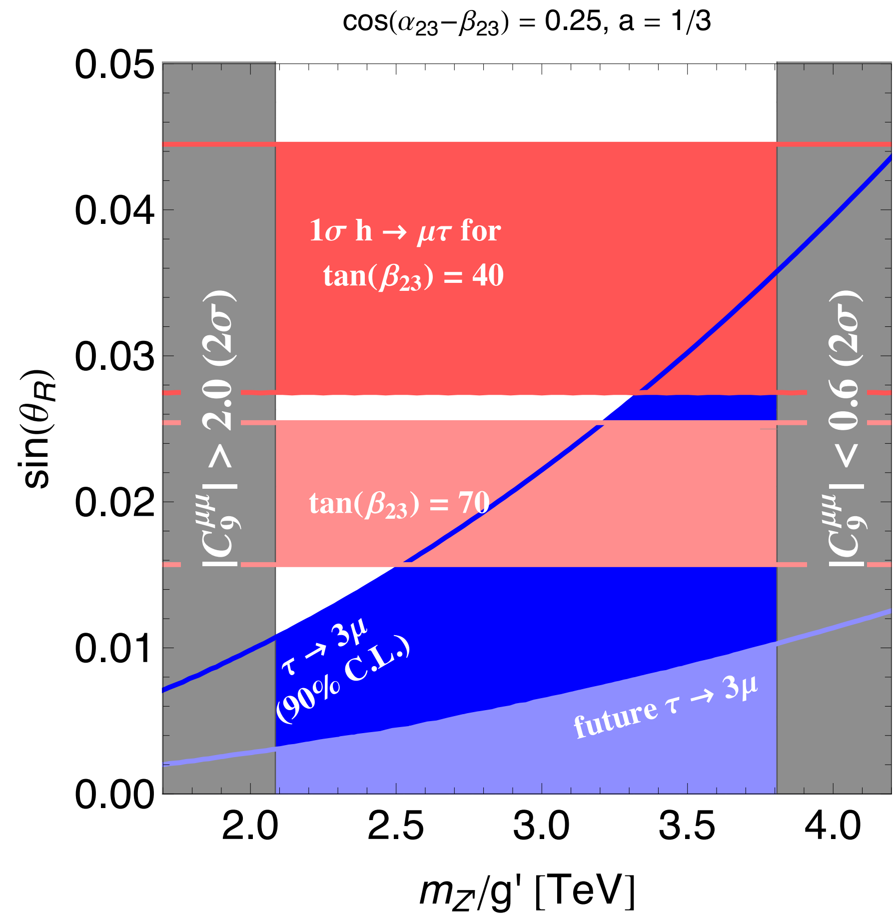


- Constraints from $\tau \to \mu\gamma$ important! ⇨ *Belle II*

4.5 Interplay between LHC & Low Energy

- *2HDMs* with gauged $L_\mu - L_\tau$
  ⇨ Z', explain anomalies for
  - $h \to \tau\mu$
  - $B \to K^*\mu\mu$
  - $R_K = B \to K\mu\mu / B \to Kee$

Altmannshofer & Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15



- Constraints from $\tau \to 3\mu$ *crucial* ⇨ *Belle II, LHCb*

- See also:
  Aristizabal-Sierra & Vicente'14,
  Lima et al'15,
  Omhura, Senaha, Tobe '15

Determination of $F_V(s)$

- Vector form factor
  - Precisely known from experimental measurements

    $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^0\pi^-\nu_\tau$ (isospin rotation)

  - Theoretically: Dispersive parametrization for $F_V(s)$

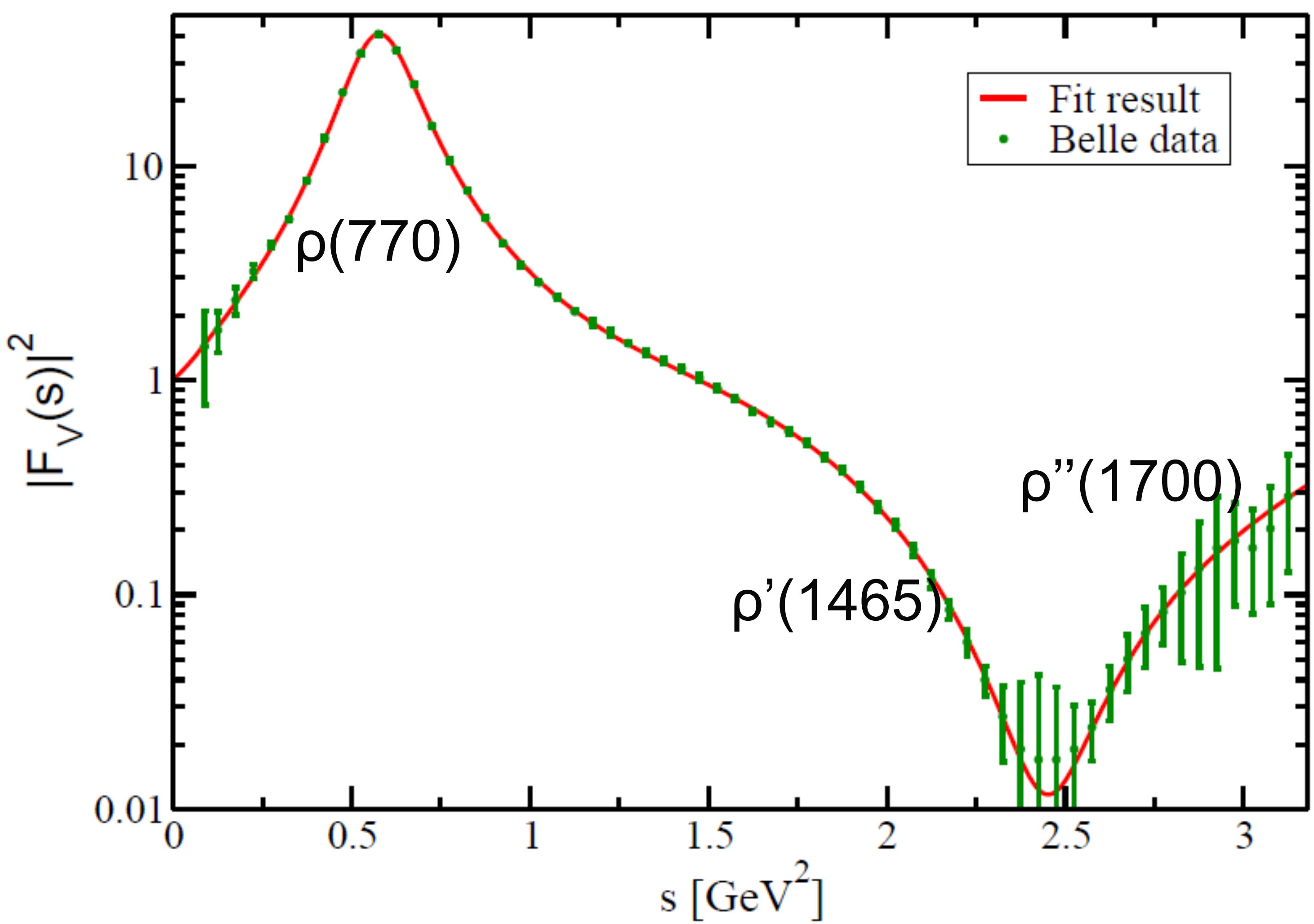
Guerrero, Pich'98, Pich, Portolés'08
Gomez, Roig'13

$$F_V(s) = \exp\left[\lambda_V' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_V'' - \lambda_V'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_V(s')}{(s' - s - i\varepsilon)}\right]$$

Extracted from a model including
3 resonances ρ(770), ρ'(1465)
and ρ''(1700) fitted to the data

  - Subtraction polynomial + phase determined from a *fit* to the *Belle data* $\tau^- \to \pi^0\pi^-\nu_\tau$

Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!