Lattice Axial Form Factors for Upcoming Neutrino Experiments

Aaron Meyer (asmeyer2012@uchicago.edu)
University of Chicago/Fermilab
September 29, 2015

INT 2015
QCD FOR NEW PHYSICS AT THE PRECISION FRONTIER

Outline

- High-precision cross section determinations
  - How is an oscillation measurement done
  - What is needed for a precise cross section prediction
- Where is improvement needed
  - Nucleon form factors
  - Nuclear models
- How are improvements being made
  - Model-independent z-expansion
  - Revisited deuterium bubble chamber data
  - Ab-initio calculations with Lattice QCD
- Lattice QCD contributions
  - Why a Lattice QCD calculation is invaluable to neutrino physics
  - Studying the axial form factor on the lattice
  - Survey of past calculations (and the challenges they faced)
  - Current effort from Fermilab/MILC

High-Precision Cross Section Measurements

Recipe: How to Perform a Neutrino Oscillation Measurement

1. Measure a near-detector event distribution
2. Perform a Monte-Carlo study to reconstruct energy of initial neutrino interaction
3. Use oscillation model to determine far-detector spectrum
4. Invert Monte-Carlo energy reconstruction to get expected event distribution
5. Ratio of predicted to actual events gives oscillation measurement

Monte Carlo is an important tool in a neutrino oscillation measurement
$\Rightarrow$ need robust control of both nuclear- and nucleon-level amplitudes!

Quasi-elastic Scattering

Within a single nucleus, there are still unknowns...

Cross section is parameterized by a series of form factors (see Formaggio, Zeller [arXiv:1305.7513]):

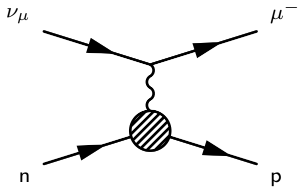
$$\eta \equiv \frac{Q^2}{4M^2} \qquad s-u = E_\nu M - Q^2 - m^2$$

$$\frac{d\sigma}{dQ^2}(E_\nu, Q^2) = \frac{M^2 G_F^2 \cos^2\theta_C}{E_\nu^2}\left[A(Q^2) \pm B(Q^2)\left(\frac{s-u}{M^2}\right) + C(Q^2)\left(\frac{s-u}{M^2}\right)^2\right]$$

$$A(Q^2) = \frac{m^2+Q^2}{M^2}\left[(1+\eta)\,{F_A}^2(Q^2) - (1-\eta)\left(F_1^2(Q^2) - \eta\left(F_2(Q^2)\right)^2\right)\right.$$
$$\left. + 4\eta F_1(Q^2)F_2(Q^2) - \mathcal{O}(m^2)\right]$$

$$B(Q^2) = 4\eta\left[F_A(Q^2)\left(F_1(Q^2)+F_2(Q^2)\right)\right]$$

$$C(Q^2) = \frac{1}{4}\left[{F_A}^2(Q^2) + F_1^2(Q^2) + \eta F_2^2(Q^2)\right]$$



F_P supressed, within $\mathcal{O}(m^2)$ term

F_1 and F_2 constrained by $e - N$ scattering

$\Rightarrow$ Focus on F_A: most potential for improvement!

Where do we need to improve?

Why Do We Still Need Better Theory?

Neutrino physics uses near detector/far detector paradigm, measures number distribution:

$$\frac{N_{\mathrm{CCQE,near}}(E_\nu)}{N_{\mathrm{CCQE,far}}(E_\nu)} = \frac{\phi_{\mathrm{near}}(E_\nu)\,\sigma_{\mathrm{CCQE}}(E_\nu)\,\epsilon_{\mathrm{near}}}{\phi_{\mathrm{far}}(E_\nu)\,\sigma_{\mathrm{CCQE}}(E_\nu)\,\epsilon_{\mathrm{far}}}$$

Problems:

- ϵ depends on near/far detector technology
- σ depends on nuclear models/nuclear target at near/far
- ϕ depends on beam angular distribution
  $\rightarrow$ near/far detector sample different energy distributions

Why Do We Still Need Better Theory?

Neutrino physics uses near detector/far detector paradigm,
measures number distribution:

$$\frac{N_{\text{CCQE,near}}(E_\nu)}{N_{\text{CCQE,far}}(E_\nu)} = \frac{\phi_{\text{near}}(E_\nu)\,\sigma_{\text{CCQE}}(E_\nu)\,\epsilon_{\text{near}}}{\phi_{\text{far}}(E_\nu)\,\sigma_{\text{CCQE}}(E_\nu)\,\epsilon_{\text{far}}}$$

More Problems:

- σ is modified by nuclear and radiative corrections
- Effects of corrections removed by energy reconstruction via Monte Carlo
- Monte Carlo uses σ as input
- σ calculated by measuring N

Degenerate uncertainties $N \to \text{MC} \to \sigma \to N$

Why Do We Still Need Better Theory?

Neutrino physics uses near detector/far detector paradigm,
measures number distribution:

$$\frac{N_{\mathrm{CCQE,near}}(E_\nu)}{N_{\mathrm{CCQE,far}}(E_\nu)} = \frac{\phi_{\mathrm{near}}(E_\nu)\,\sigma_{\mathrm{CCQE}}(E_\nu)\,\epsilon_{\mathrm{near}}}{\phi_{\mathrm{far}}(E_\nu)\,\sigma_{\mathrm{CCQE}}(E_\nu)\,\epsilon_{\mathrm{far}}}$$

Even More Problems:

- Model for σ constructed from single-nucleon cross section
- single-nucleon cross section constrained by assuming a model for σ

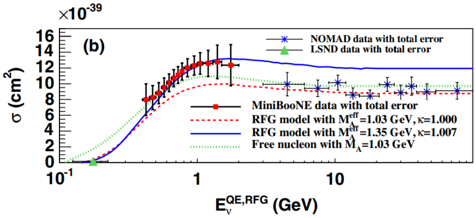
Degenerate uncertainties $\sigma_A \to \sigma_N \to \sigma_A$

Discrepancies with the Axial-Vector Form Factor

Most analyses assume the "Dipole form factor":

$$F_A^{\text{dipole}}(q^2) = g_A \frac{1}{\left(1 - \frac{q^2}{m_A^2}\right)^2}$$

Dipole is an ansatz:
unmotivated in interesting energy range
→ uncontrolled systematics and underestimated uncertainties
Essential to replace ansatz with
model-independent parameterization



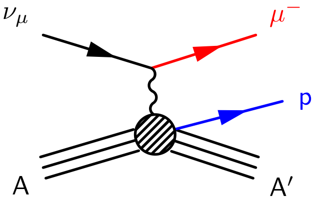
MiniBooNE Collab., PHYS REV D 81, 092005 (2010)

Problematic Nuclear Effects

Nuclear effects not well understood

→ Models which are best for one measurement are worst for another

Need to break F_A/nuclear model entanglement



(assumed $m_A = 0.99$ GeV)

NuWro Model (χ^2/DOF)	RFG [GENIE]	RFG+ TEM	assorted others
leptonic(rate) leptonic(shape) [arXiv:1305.2243]	3.5 4.1	2.4 1.7	2.8-3.7 2.1-3.8
hadronic(rate) hadronic(shape) [arXiv:1409.4497]	1.7[1.2] 3.3[1.8]	3.9 5.8	1.9-3.7 3.6-4.8

How are we improving?

Deuterium Fitting

with M. Betancourt, R. Gran, R. Hill

Fitting done on deuterium bubble chamber data (better control over nuclear effects) using the z-expansion

Three datasets:

- ANL 1982 [Phys.Rev.D 26, 537]: 1737 events, 0.5 GeV peak
- BNL 1981 [Phys.Rev.D 23, 2499]: 1138 events, 1.0 GeV peak
- FNAL 1983 [Phys.Rev.D 28, 436]: 362 events, 20 GeV peak

Flux assumptions from literature removed to recover event energy distribution

PRELIMINARY shape-only fits to QE differential cross section data

Gaussian priors used on z-Expansion coefficients:
if (k $\leq$ 5) $\sigma_k = 5$, else $\sigma_k = 25/k$

Sum rule applied to ensure $F_A \sim 1/Q^4$ as $Q^2 \rightarrow \infty$

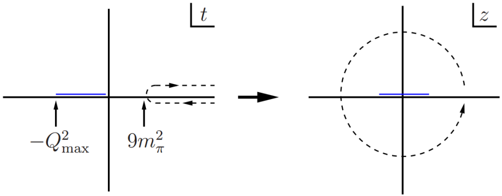
Lightning Introduction of z-Expansion

z-Expansion gives a model-independent description of the axial form factor

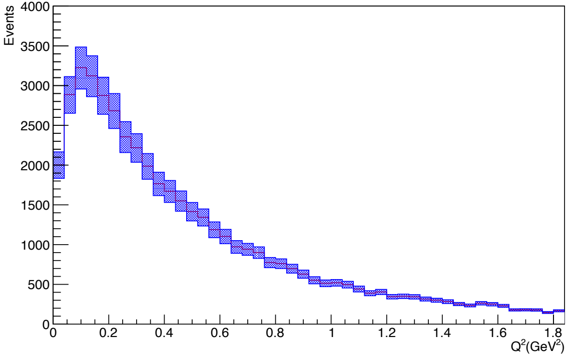
- Conformal mapping to bring $Q^2 \to z$ for $|z| < 1$:

$$F_A(z) = \sum_k^{\infty} a_k z^k$$

- Motivated by analyticity arguments
- Coefficients shown to be bounded, decreasing
- Provides a prescription for introducing more parameters as data improves
- Allows quantification of systematic errors
- z-Expansion in incubator project for GENIE, target release v2.12



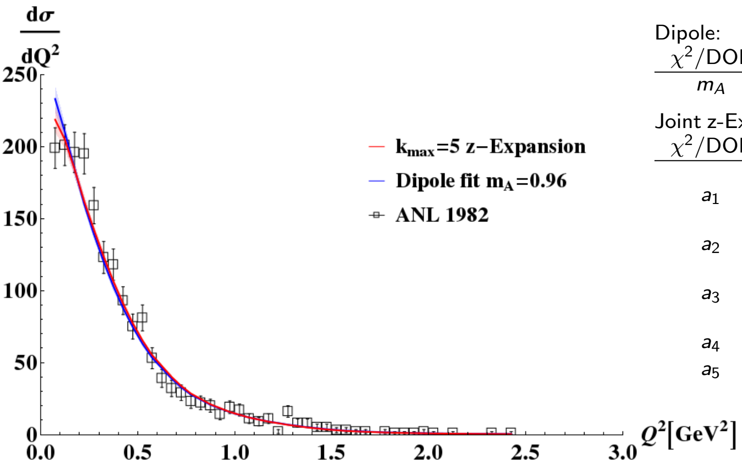
z-Expansion in GENIE



Plot by M. Betancourt

- Working prototype of z-expansion exists in GENIE (target release: v2.12)
- Can replace dipole ansatz with z-expansion
- Even in the absence of Lattice QCD, z-expansion parameterization will be available soon
- In time, replace z-expansion parameterization with suitable average of lattice results

Deuterium Fitting Results (PRELIMINARY)



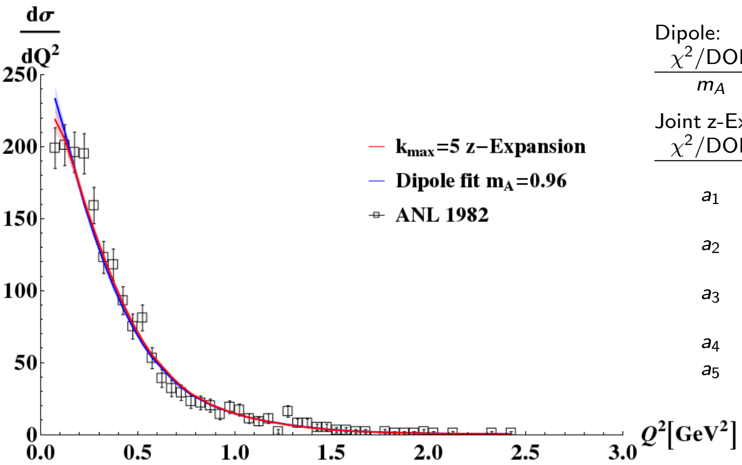
Dipole:

χ^2/DOF	129/100
m_A	0.96(3)

Joint z-Expansion:

χ^2/DOF	125/96
a_1	$2.36^{+0.10}_{-0.10}$
a_2	$1.67^{+1.03}_{-0.98}$
a_3	$-8.06^{+2.58}_{-2.65}$
a_4	$3.40^{+3.64}_{-3.63}$
a_5	3.56

Deuterium Fitting Results (PRELIMINARY)



Dipole:

χ^2/DOF	129/100
m_A	0.96(3)

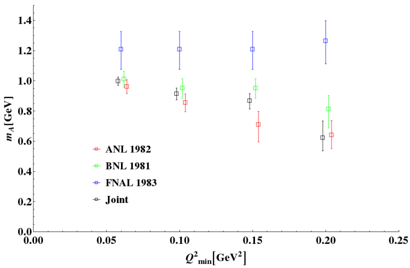
Joint z-Expansion:

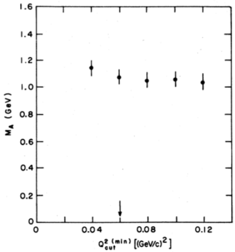
χ^2/DOF	125/96
a_1	$2.36^{+0.10}_{-0.10}$
a_2	$1.67^{+1.03}_{-0.98}$
a_3	$-8.06^{+2.58}_{-2.65}$
a_4	$3.40^{+3.64}_{-3.63}$
a_5	3.56

Why the large a_3?

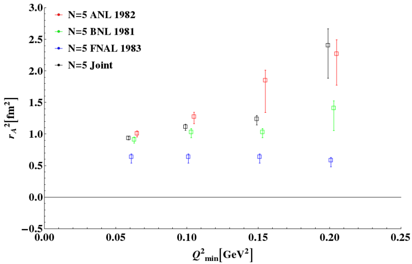
Is there something wrong with low Q^2?

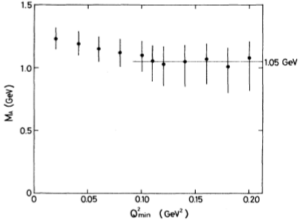
Previous Dipole Analyses (PRELIMINARY)





BNL

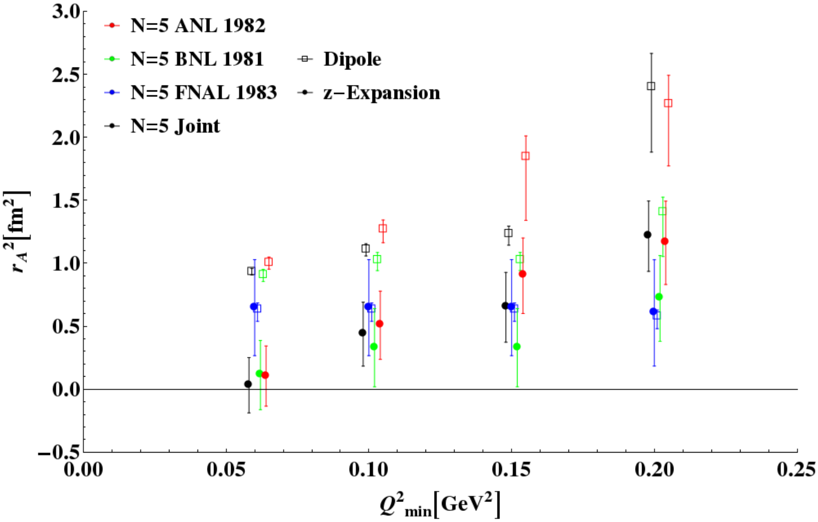




FNAL

Previous evaluations have not shown any Q^2 cut dependence

New Analysis (PRELIMINARY)



Extraction of r_A^2 dependent on Q^2_{min} cut (FNAL excluded)

Possible Explanations

Several possibilities could be cause of low-Q^2 behavior:

- Acceptance corrections
- Real form factor shape
- Deuterium corrections

Possible Explanations

Several possibilities could be cause of low-Q^2 behavior:

- ~~Acceptance corrections~~

  $\Rightarrow$ would have to generate a large factor increase in error bars at low Q^2 (backup slide)

- Real form factor shape

- Deuterium corrections

Possible Explanations

Several possibilities could be cause of low-Q^2 behavior:

- ~~Acceptance corrections~~

  $\Rightarrow$ would have to generate a large factor increase in error bars at low Q^2 (backup slide)

- Real form factor shape (?)

  $\Rightarrow$ Tension relieved by increasing Q^2_{min} (backup slide)
  $\Rightarrow$ Large coefficients disfavored by
  bounds from studying dispersion relations

- Deuterium corrections

Possible Explanations

Several possibilities could be cause of low-Q^2 behavior:

- ~~Acceptance corrections~~

  ⇒ would have to generate a large factor increase in error bars at low Q^2 (backup slide)

- Real form factor shape (?)

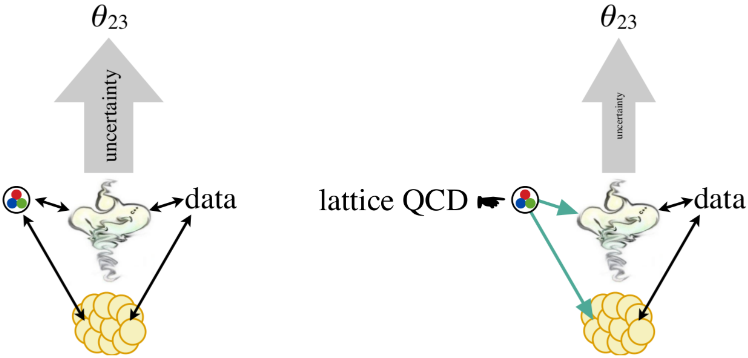
  ⇒ Tension relieved by increasing Q^2_{min} (backup slide)
  ⇒ Large coefficients disfavored by bounds from studying dispersion relations

- Deuterium corrections

  ⇒ Corrections from Singh [Nuclear Physics B36 (1972) 419-435]
  ⇒ Warrants closer scrutiny [under investigation]

Lattice QCD and the Axial Form Factor

Lattice QCD in Neutrino Physics



LQCD acts as disruptive techology to break nucleon-level and nucleus-level degeneracy

LQCD calculations are free from nuclear corrections
$\Rightarrow$ direct access to nucleon-level amplitudes

How to construct F_A on the lattice

All calculations are vacuum-to-vacuum correlation functions:

$$\langle\Omega|\,\mathcal{B}^{\alpha}(t)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle = \sum_{n}\langle\Omega|\,\mathcal{B}^{\alpha}\,|n\rangle\,\langle n|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\,e^{-E_n t}\underset{t\to\infty}{\longrightarrow}\langle\Omega|\,\mathcal{B}^{\alpha}\,|0\rangle\,\langle 0|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\,e^{-E_0 t}$$

How to construct F_A on the lattice

All calculations are vacuum-to-vacuum correlation functions:

$$\langle\Omega|\,\mathcal{B}^{\alpha}(t)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle = \sum_{n} \langle\Omega|\,\mathcal{B}^{\alpha}\,|n\rangle\,\langle n|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_n t} \underset{t\to\infty}{\longrightarrow} \langle\Omega|\,\mathcal{B}^{\alpha}\,|0\rangle\,\langle 0|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_0 t}$$

Three point functions rearrange quantum numbers:

$$\langle\Omega|\,\mathcal{B}^{\beta}(t)\mathcal{A}^{\alpha}_{\beta}(\tau)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle = \sum_{n,n'} \langle\Omega|\,\mathcal{B}^{\beta}\,|n\rangle\,\langle n|\,\mathcal{A}^{\alpha}_{\beta}\,|n'\rangle\,\langle n'|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_n(t-\tau)}e^{-E_{n'}\tau}$$

$$\underset{\tau,t-\tau\to\infty}{\longrightarrow} \langle\Omega|\,\mathcal{B}^{\beta}\,|0\rangle\,\langle 0|\,\mathcal{A}^{\alpha}_{\beta}\,|0\rangle\,\langle 0|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_0 t}$$

How to construct F_A on the lattice

All calculations are vacuum-to-vacuum correlation functions:

$$\langle\Omega|\,\mathcal{B}^{\alpha}(t)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle = \sum_n \langle\Omega|\,\mathcal{B}^{\alpha}\,|n\rangle\,\langle n|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_n t} \underset{t\to\infty}{\rightarrow} \langle\Omega|\,\mathcal{B}^{\alpha}\,|0\rangle\,\langle 0|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_0 t}$$

Three point functions rearrange quantum numbers:

$$\langle\Omega|\,\mathcal{B}^{\beta}(t)\mathcal{A}^{\alpha}_{\beta}(\tau)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle = \sum_{n,n'} \langle\Omega|\,\mathcal{B}^{\beta}\,|n\rangle\,\langle n|\,\mathcal{A}^{\alpha}_{\beta}\,|n'\rangle\,\langle n'|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_n(t-\tau)}e^{-E_{n'}\tau}$$

$$\underset{\tau,t-\tau\to\infty}{\rightarrow} \langle\Omega|\,\mathcal{B}^{\beta}\,|0\rangle\,\langle 0|\,\mathcal{A}^{\alpha}_{\beta}\,|0\rangle\,\langle 0|\,\mathcal{B}'^{\dagger}_{\alpha}\,|\Omega\rangle\, e^{-E_0 t}$$

Further fine-tuning with ratios:

$$\frac{\langle\Omega|\,\mathcal{B}^{\beta}(t)\mathcal{A}^{\alpha}_{\beta}(\tau)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle}{\sqrt{\langle\Omega|\,\mathcal{B}'^{\alpha}(t)\mathcal{B}'^{\dagger}_{\alpha}(0)\,|\Omega\rangle\,\langle\Omega|\,\mathcal{B}^{\alpha}(t)\mathcal{B}^{\dagger}_{\alpha}(0)\,|\Omega\rangle}} \underset{\tau,t-\tau\to\infty}{\rightarrow} \langle 0|\,\mathcal{A}^{\alpha}_{\beta}\,|0\rangle$$

Ratios for cancelling renormalization, prefactors, statistical fluctuations. . .

How to construct F_A on the lattice

$$\partial^\mu A^a_\mu(x) = 2\hat{m}P^a(x)$$

$$A^\mu_\perp(q^2) = A^\mu(q^2) - \frac{q^\mu}{q^2} q \cdot A$$

$$\langle N'(p')| A^a_\mu(x) |N(p)\rangle \big|_{x=0} = \bar{u}_{N'}(p') \left[\gamma_\mu \gamma_5 F_A(q^2) + \frac{q_\mu}{2m_N}\gamma_5 F_P(q^2)\right] \frac{t^a}{2} u_N(p)$$

$$F_A(q^2)\,\overline{u}_p(q)\left(\gamma^\mu - \frac{q^\mu}{q^2}\not{q}\right)\gamma^5 u_n(0) \sim \frac{\langle N'| Z_A A^a_{\perp\,\mu}(q^2)|N\rangle}{\langle 0| Z_A A^a_0(0)|\pi^a\rangle\,\omega^2} \to \frac{F_A(q^2)}{f_\pi}$$

Lattice g_A Calculation

$g_A = F_A(q^2 = 0)$ is a historically difficult calculation

What makes it hard:

- Finite size effects
- Excited state contamination
- Chiral extrapolation
- Explicit Chiral symmetry breaking - for some formalisms
- Baryons(!)

For further discussion, see Martha Constantinou's talk

$F_A(q^2)$ Survey

Recent (past 5 years) F_A calculations with q^2 dependence

Mainz, ETM, RBC/LHP collaborations

Also interested in F_P when available,
though F_P is less important to neutrino physics

F_P related by "pion-pole approximation" to F_A due to chiral symmetry:

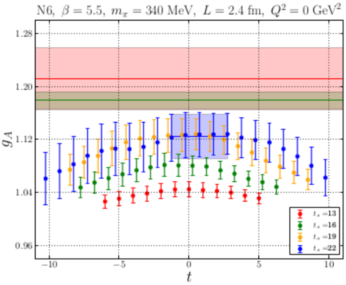
$$F_P(Q^2) = \frac{2M_N^2 F_A(Q^2)}{Q^2 + m_\pi^2}$$

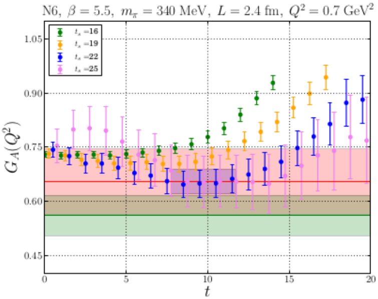
(Apologies if any other relevant works were missed)

$F_A(q^2)$ Survey: Mainz [arXiv:1411.5828]

$N_f = 2$, $a \sim 0.05$ fm, $N_{\text{sep}} = (12 - 28)a$,
$m_\pi = 340$ MeV, $M_\pi L \sim 4.0$

Employ a plateau fit (blue), summation fit (red), simultaneous excited state fit (green)





More advanced handling of excited states

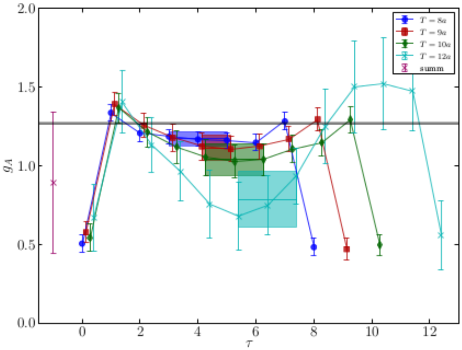
Disagreement between plateau and summation indicates presence of excited states

$F_A(q^2)$ Survey: RBC-LHP [arXiv:1412.3175]

$N_f = 2+1$, $a \sim 0.11$ fm, $N_{sep} = (12-28)a$,
$m_\pi = 139$ MeV, $M_\pi L \sim 3.9$

Low statistics sample (20 configs + noise reduction techniques)

Physical m_π!



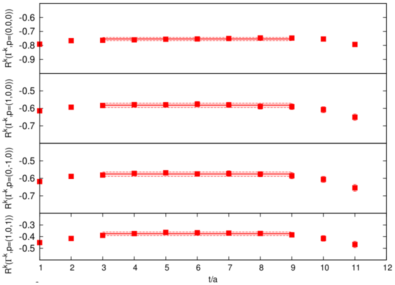
Significant excited state contamination

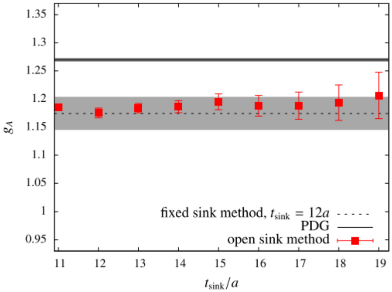
$F_A(q^2)$ Survey: ETMC [arXiv:1303.5979, 1112.2931]

$N_f = 2+1+1$, $a \sim 0.066, 0.082, 0.092$ fm, $N_{\text{sep}} = (12-18)a$,
$m_\pi = 213, 373$ MeV, $M_\pi L \sim 3.4 - 5.0$

Many box sizes, lattice spacings
$\Rightarrow$ good control over continuum/infinite volume extrapolations

Dedicated excited state study, no sign of contamination from excited states



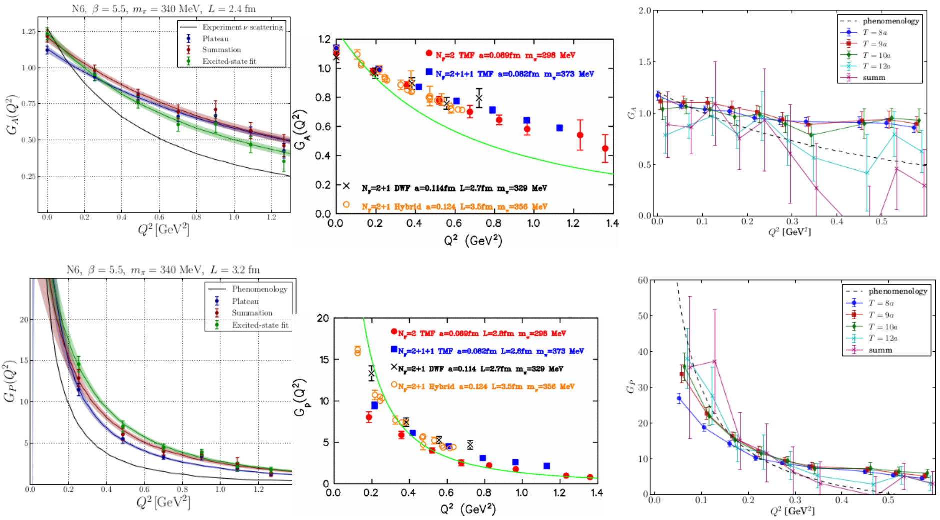


$a \sim 0.078$, $m_\pi \sim 380$

Find summation method reduces g_A

Claim large volume extrapolation could be a problem for form factor calculations

$F_A(q^2)$ Survey: Side by side



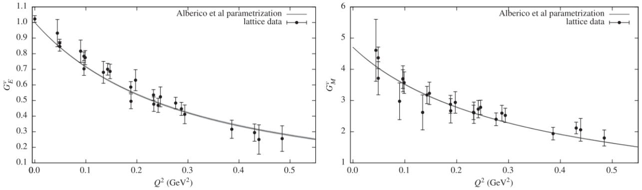
Smaller Q^2 range for RBC-LHP

Rough agreement between calculations

Why do F_A now?

Experience abounds

- Earlier g_A, $F_A(Q^2)$ work
- Systematics for vector form factors are coming under control
- z-Expansion well tested in B-meson physics



Green et. al, Phys.Rev.D 90, 074507

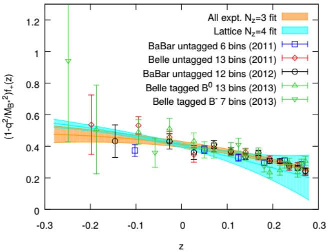
Much to learn from previous attempts to calculate g_A, $F_A(Q^2)$
(as well as other form factor calculations!)

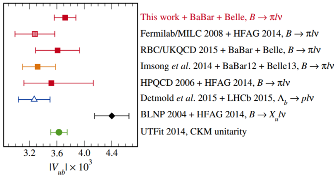
z-Expansion in B-meson Physics

Only a few expansion coefficients necessary to accurately represent data

Coefficients bounded, falloff required by perturbative QCD
(see general analysis Hill [arXiv:hep-ph/0606023])

- For recent $|V_{ub}|$ determination, see Fermilab/MILC [arXiv:1503.07839]
- For recent $|V_{cb}|$ determinations, see Fermilab/MILC [arXiv:1503.07237] and HPQCD [arXiv:1505.03925]
- For recent $|V_{ub}|/|V_{cb}|$ determination, see LHCb [arXiv:1504.01568]





Current Effort

The Fermilab Lattice and MILC Collaborations are calculating the axial form factor using HISQ action

What we bring to the table:

- Large physical volumes ⇒ Control finite size effects
- Variational method ⇒ Reduce excited state contamination
- Physical quark masses ⇒ Avoid chiral extrapolation
- High statistics ⇒ Obtain small statistical errors
- Staggered quarks ⇒ Computationally efficient
- Several lattice spacings ⇒ Take continuum limit
- Blind analysis ⇒ Eliminate bias toward well-known g_A

⇒ We will carry out a complete analysis, with continuum limit and a full error budget

Taste Mixing

Complications with staggered quarks come from extra tastes:

- Messy group theory
  ⇒ Needed to understand which correction functions are necessary
  ⇒ DONE [Bailey, Phys.Rev.D 75, 114505]
- Extra tastes in resulting correlation functions must be fitted away
  ⇒ Encouraging results so far!

$SU(2)_I \times GTS$ irrep	#N	#Δ
$\left(\frac{3}{2}, \mathbf{8}\right)$	3	2
$\left(\frac{3}{2}, \mathbf{8'}\right)$	0	2
$\left(\frac{3}{2}, \mathbf{16}\right)$	1	3
$\left(\frac{1}{2}, \mathbf{8}\right)$	5	1
$\left(\frac{1}{2}, \mathbf{8'}\right)$	0	1
$\left(\frac{1}{2}, \mathbf{16}\right)$	3	4

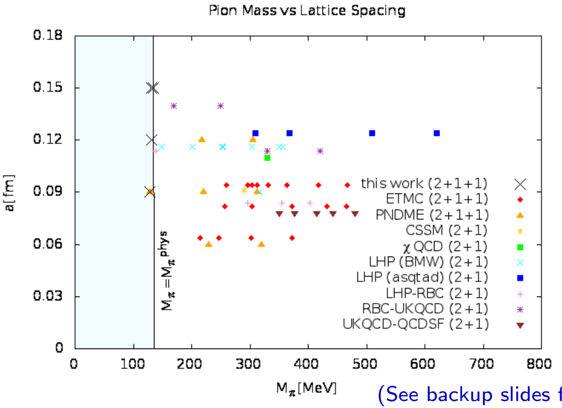
3-point functions use same operator basis as 2-point functions

Can get priors from fitting different taste Δ states
from $\left(\frac{3}{2}, 8'\right)$ and $\left(\frac{1}{2}, 8'\right)$ operators

Current Calculations of g_A

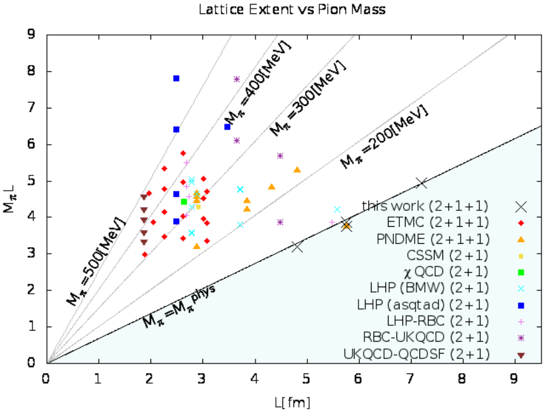
Other collaborations have at most one ensemble for one lattice spacing at physical pion mass

We plan to have full continuum extrapolation at physical m_π



(See backup slides for references)

Finite Size Effects



Doing as well as other calculations at physical masses

MILC $g-2$ proposal to generate $a = 0.15$ fm ensemble at larger L
$\rightarrow$ can use for finite volume study

Estimate finite-size effects with χPT and Lüscher methodology

Summary

Neutrino physics is subject to
underestimated and model-dependent systematics

→ To reduce systematics from modeling,
need to understand nuclear physics

→ To understand nuclear physics, need to understand
nucleon-level amplitudes in an ab initio way

- z-Expansion removes model assumptions and permits better understanding of systematic errors
- With a systematic treatment of the axial form factor, the deuterium bubble chamber data exhibits low-Q^2 behavior in disagreement with our current understanding of form factor shape/nuclear corrections
- LQCD offers a way to access nucleon form factors directly, without nuclear corrections
- Possible now for Lattice QCD calculations of the axial form factor with full, reliable error budgets
- Understanding of axial form factor is crucial for success of next-gen neutrino experiments
- Direct pipeline to disseminate form factor calculation to neutrino community

Thanks!

Backup Slides

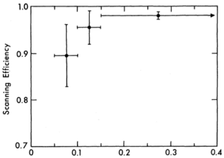
Acceptance Corrections

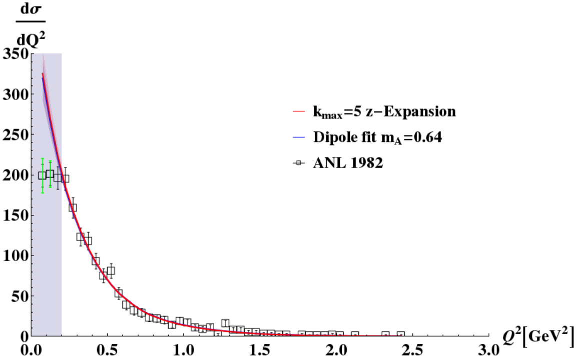
Added uncertainty due to difficult to detect proton tracks with small momentum

Acceptance corrections implemented as an extra error
added in quadrature with statistical error

Correction necessary to put best fit of z-expansion ($Q^2_{\text{min}} = 0.2\ \text{GeV}^2$)
within 1σ bounds is $\mathcal{O}$(few) times larger than statistical error

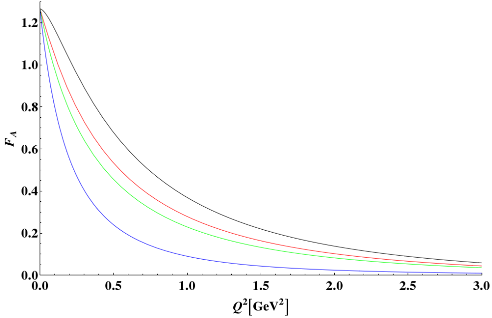
Even with normalization only allowed to float within $\pm 1\sigma$ of nominal,
still see $\mathcal{O}$(few) and penalty in χ^2/DOF





Form Factor Shape

Shape and χ^2/DOF change rapidly with Q^2 cut



Q^2_{cut}	χ^2/DOF
0.06	1.31
0.10	1.27
0.15	1.28
0.20	1.13

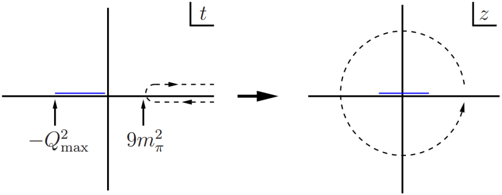
z-Expansion

The z-Expansion (Bhattacharya, Hill, Paz arXiv:1108.0423 [hep-ph]) is a conformal mapping which takes the kinematically allowed region ($t \leq 0$) to within $|z| < 1$

$$t = q^2 = -Q^2 \qquad t_c = 9m_\pi^2$$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}}$$

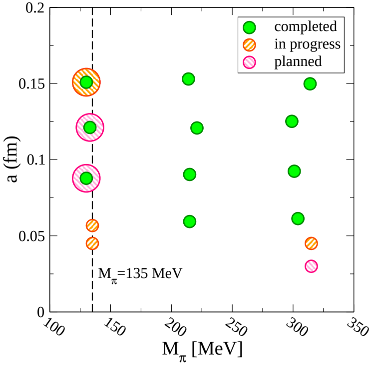
$$F_A(z) = \sum_{n=0}^{\infty} a_n z^n$$



MILC HISQ Lattices

Several lattice spacings and pion masses available for use

Several physical box sizes also exist (not shown)



g_A Calculation references

ETMC
S. Dinter et al. arXiv:1108.1076 [hep-lat]
PNDME
T. Bhattacharya et al. arXiv:1306.5435 [hep-lat]
T. Bhattacharya, R. Gupta, and B. Yoon arXiv:1503.05975 [hep-lat]
R. Gupta, T. Bhattacharya, A. Joseph, H.-W. Lin, and B. Yoon arXiv:1501.07639 [hep-lat]
CSSM
B. J. Owen et al., arXiv:1212.4668 [hep-lat]
χQCD
Y.-B. Yang, M. Gong, K.-F. Liu, and M. Sun arXiv:1504.04052 [hep-ph]
LHP(BMW)
J. Green et al., arXiv:1211.0253 [hep-lat]
S. N. Syritsyn et al., arXiv:0907.4194 [hep-lat]
S. Dürr et al. arXiv:1011.2711 [hep-lat]
LHP(asqtad)
S. N. Syritsyn, Exploration of nucleon structure in lattice QCD with chiral quarks, Ph.D. thesis, Massachusetts Institute of Technology (2010).
LHP-RBC
S. Syritsyn et al. arXiv:1412.3175 [hep-lat]
RBC-UKQCD
S. Ohta arXiv:1309.7942 [hep-lat]
UKQCD-QCDSF
M. Göckeler et al. arXiv:1102.3407 [hep-lat]