Constraining gravity with the equation of state in neutron stars

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In collaboration with M. Aparicio, A. de la Cruz Dombriz, V. Zapatero (work in progress) and A. Dobado, A. Oller Phys.Rev. C85 (2012) 012801

Constraining gravity with the equation of state in N stars

Outline

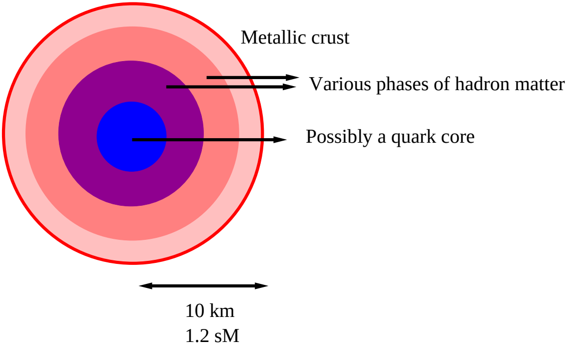
Motivation

Static neutron stars

Constraining Cavendish's constant with heavy stars

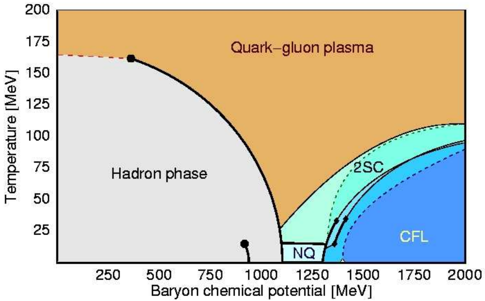
Modified gravity

Often emphasized: Neutron stars test structure of matter



include INT TALENT 2015: Nuclear Physics of Neutron Stars and Supernovae

Only window to a part of the QCD phase diagram



(hep-ph/0503184)

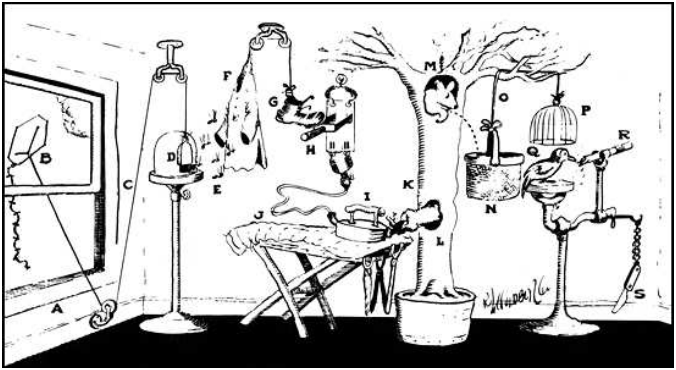
But consider...

Inside the star

- $\rho < 3 - 4\rho_0$
- $g = O(10^{12})\text{m/s}^2$ vs. $O(300)\text{m/s}^2$ outside (where GR tests with pulsars are performed) or $O(10^6)\text{m/s}^2$ at white dwarves.

i.e. we extrapolate General Relativity 6 orders of magnitude to learn nuclear physics only a factor 3-4 away !!??

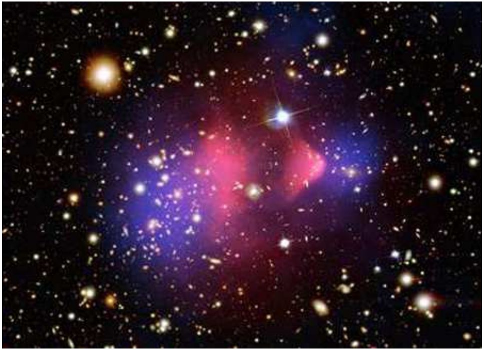
Rube Goldberg contraption



Turn it around

Can nuclear physics constrain General Relativity inside a neutron star?

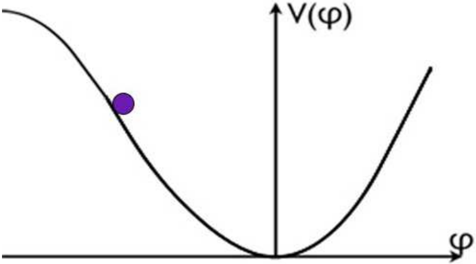
Motivations to keep testing gravity



Dark matter is obscure. Who missed something?

D. Clowe *et al.* Astrophys. J. Lett. **648** L109 (2006)

Motivations to keep testing gravity



What about accelerated expansion, *e.g.* primordial inflation?

Add incompatibility of GR with quantum theory, and prediction of spacetime singularities

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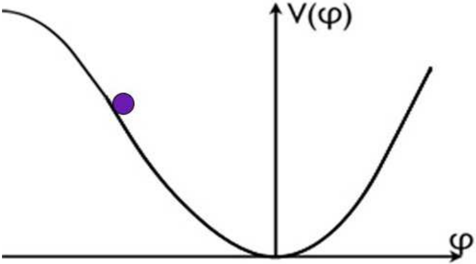

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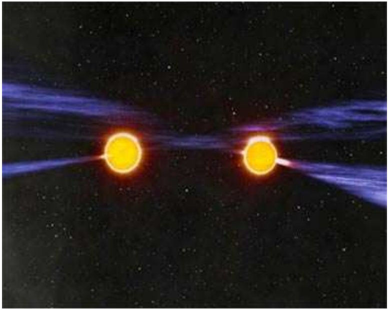
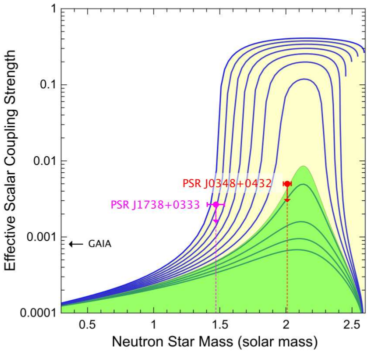
Binary neutron star systems used to constrain gravity



Constraints “external” to star, e.g. Tensor+Scalar theories

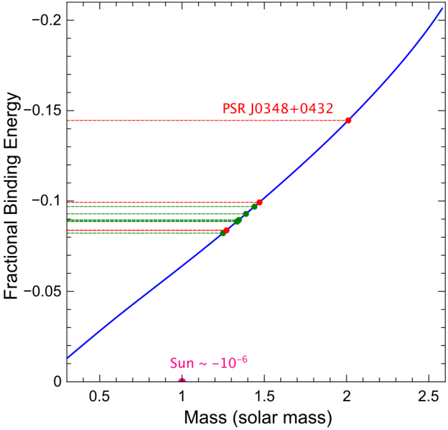


- Two-solar mass pulsar J0348+0432 with white dwarf companion
- Grav. wave emission constrained from dT/dt
- Constraint on α_{PSR} (scalar/tensor coupling ratio),

Science **340** 1233232 (2010)

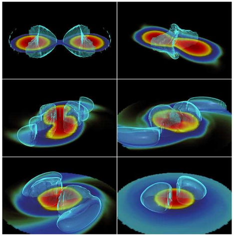
Neutron star interior great for Gen. Rel. tests

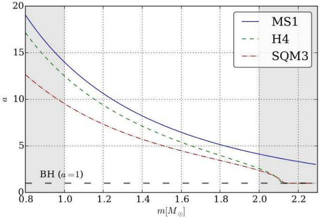
Large fractional binding energy



J.Antoniadis *et al.*, Science **340**, 1233232 (2013)

Future: perhaps gravity waves



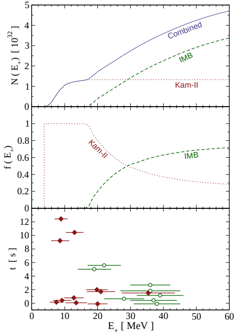


Left: binary merger simulation by Alan Calder (not equilibrium either).
Right: theory prediction of quadrupole moment Q/J^2 for three equations of state

(arXiv:1503.05405)

Only probes of interior that might get to us are neutrinos

(arXiv:0702613)



Analysis of SN87A; not a neutron star in equilibrium

Resort to indirect means, such as bulk properties (M, R, T, L, τ_{cooling} ...) (very much like nuclear physics)

So let's do something about it

Outline

Gravity acceleration in General Relativity ($G = c = 1$)

$$g = \frac{d\Phi}{dr} = \frac{M(r)}{r^2}$$

$$g = \frac{M(r) + 4\pi r^3 P(r)}{r(r - 2M(r))} \ .$$

$g \simeq O(10^{12}) m/s^2$

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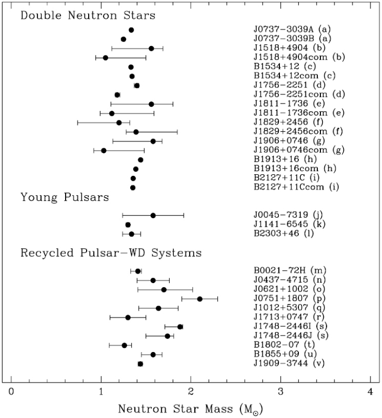
Measured pulsar masses before 2010



PRC77 065803 (2008)

Maximum mass for a given eq. of state

- Hydrostatic equilibrium:
  pressure compensates weight of upper layers
- Causality limits the achievable pressure, $c^2 \geq c_s^2 = \frac{dP}{d\rho}$
- But nothing limits the amount of matter falling on the star
- Only solution: increase density
- When $R_* < R_{\rm Schwarzschild} = 2M_*$, Gen. Rel. predicts collapse
- Finding heavier N-stars tests General Relativity

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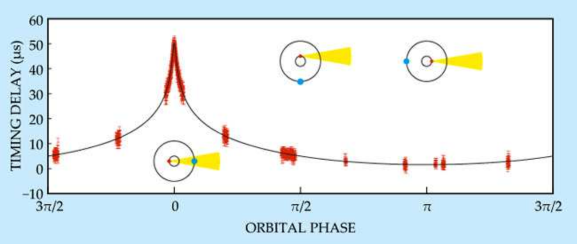
Two-solar mass neutron star

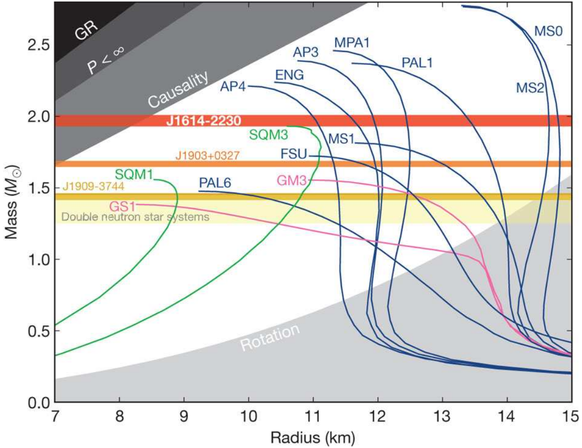
Shapiro delay: allows for absolute measurement of the mass (Demorest et al. Nature 2010)



$$\Delta t = -\log(1 - \hat{n}_1 \cdot \hat{n}_2)\frac{R_s}{c}$$

Confirmed with 2nd example,
Antoniadis *et al.* Science **340** 1233232 (2013)

Used back then to exclude many models



Tolman-Oppenheimer-Volkoff equation

FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

On Massive Neutron Cores

J. R. OPPENHEIMER AND G. M. VOLKOFF
Department of Physics, University of California, Berkeley, California
(Received January 3, 1939)

It has been suggested that, when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. In this paper we study the gravitational equilibrium of masses of neutrons, using the equation of state for a cold Fermi gas, and general relativity. For masses under $\frac{1}{3}\odot$ only one equilibrium solution exists, which is approximately described by the nonrelativistic Fermi equation of state and Newtonian gravitational theory. For masses $\frac{1}{3}\odot < m < \frac{3}{4}\odot$ two solutions exist, one stable and quasi-Newtonian, one more condensed, and unstable. For masses greater than $\frac{3}{4}\odot$ there are no static equilibrium solutions. These results are qualitatively confirmed by comparison with suitably chosen special cases of the analytic solutions recently discovered by Tolman. A discussion of the probable effect of deviations from the Fermi equation of state suggests that actual stellar matter after the exhaustion of thermonuclear sources of energy will, if massive enough, contract indefinitely, although more and more slowly, never reaching true equilibrium.

FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

Static Solutions of Einstein's Field Equations for Spheres of Fluid

RICHARD C. TOLMAN
Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California
(Received January 3, 1939)

A method is developed for treating Einstein's field equations, applied to static spheres of fluid, in such a manner as to provide explicit solutions in terms of known analytic functions. A number of new solutions are thus obtained, and the properties of three of the new solutions are examined in detail. It is hoped that the investigation may be of some help in connection with studies of stellar structure. (See the accompanying article by Professor Oppenheimer and Mr. Volkoff.)

Tolman-Oppenheimer-Volkoff equation

Newtonian Hydrostatic equilibrium

$$dP = -\frac{G_N M(r)}{r^2}\frac{dM}{dA}$$

General Relativistic Hydrostatic equilibrium (static, spherical body)

$$\frac{dP}{dr} = -\frac{G_N}{r^2}\frac{(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{1 - \frac{2G_N M(r)}{r}} .$$

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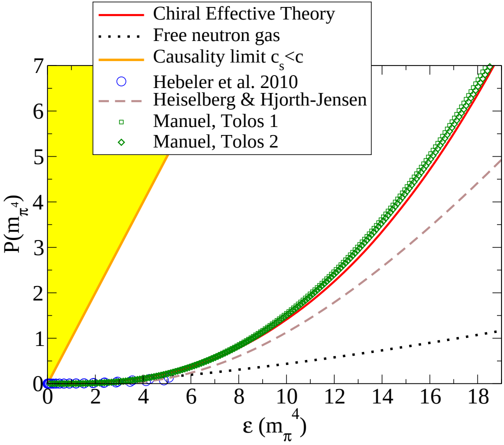
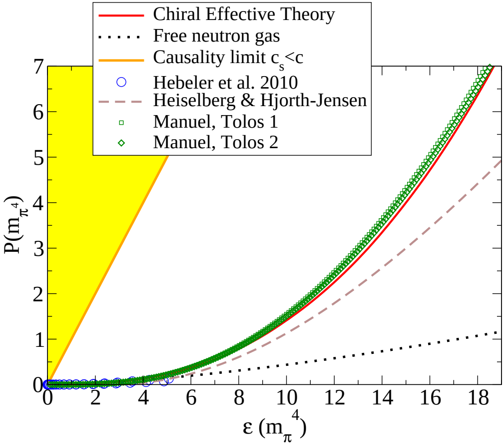
Supplemented by equation of state



So what is your crazy model of the star's inside?

Supplemented by equation of state



So what is your crazy model of the star's inside?

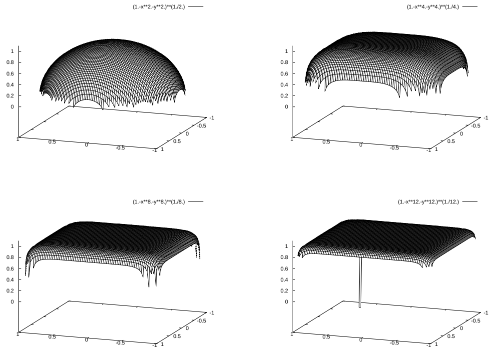
New degrees of freedom→ softer eq. of state

The optimal "random" sphere packing fraction is about 63.5%
The optimal "crystal" packing fraction is about 74%
(Kepler's conjecture)

The neutron is not pointlike

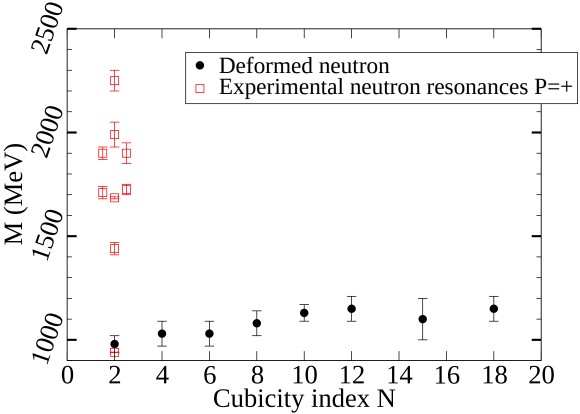
- Nucleon radius (measured in $e^- p$ scattering) $r_N \simeq 0.88\ fm$
- Nuclear radius $r_A \simeq 1.2\ fmA^{1/3}$
- When $\rho \simeq 133\ MeV/fm^3 \simeq 2.8\ m_\pi^4$
  the volume evacuated by the neutrons is important

Wavefunction symmetry



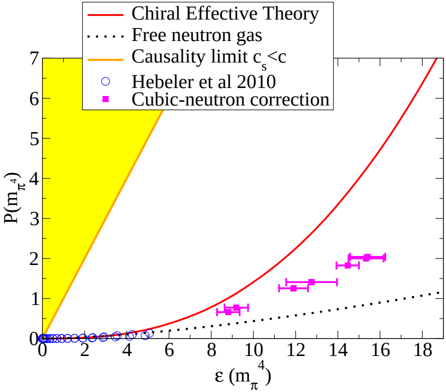
(plotted are N=2,4,8,12)

Cubic neutrons



Cost of "cubing" the neutron: approximately 150 MeV

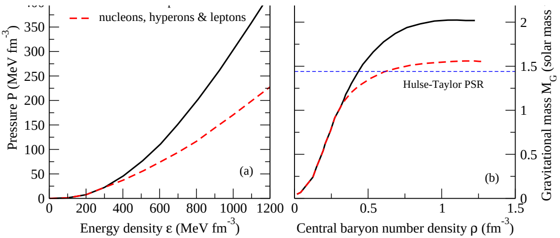
Effect on the equation of state



FLE, Moreno-Navarro Mod.Phys.Lett. A**27** (2012) 1250033

Another example: Hyperon puzzle

Hyperons are expected at high density



I. Vidaña, 1509.03587

But this is characteristic of any new QCD physics

- Most often, just soften
- Exploit it to put bounds by working from the "stiffest" side

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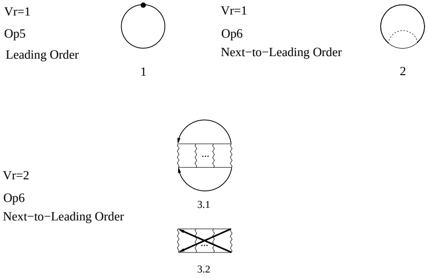
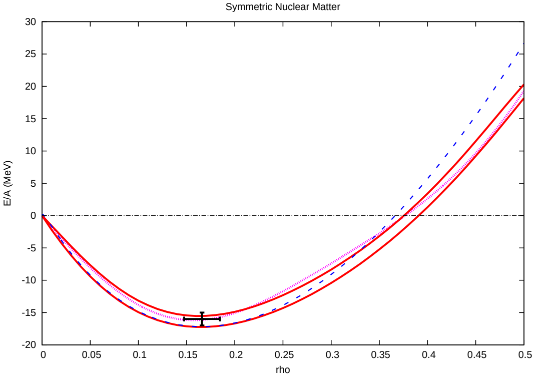
Modern calculations in effective theory



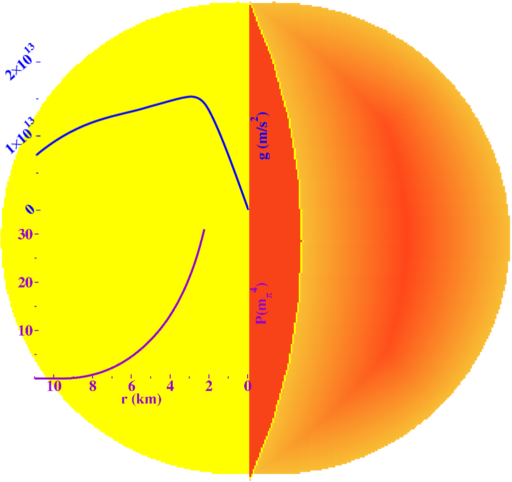
(Lacour, Oller and Meissner Ann. Phys. **326** (2011) 241, consistent power counting in nuclear matter)

Modern calculations in effective theory

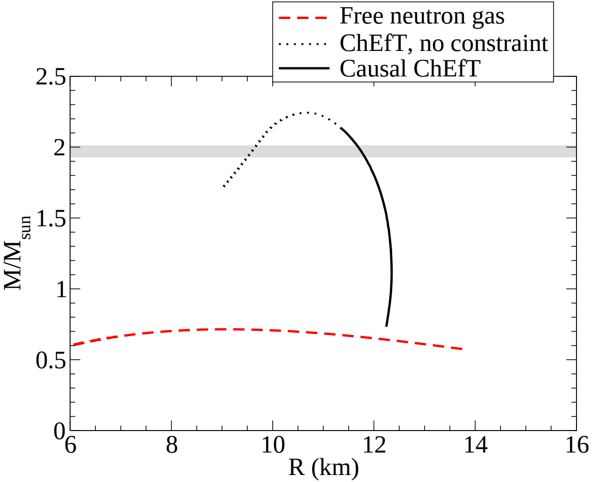


(Lacour, Oller and Meissner 2009, symmetric nuclear matter $N_p = N_n$)

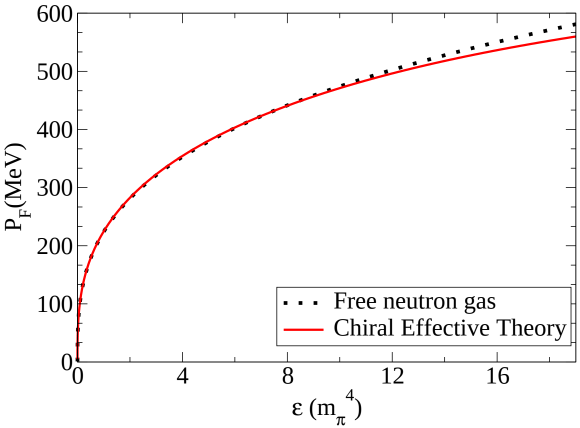
Pressure (and g-acceleration) profile



Mass/radius plot



Closer look at the equation of state



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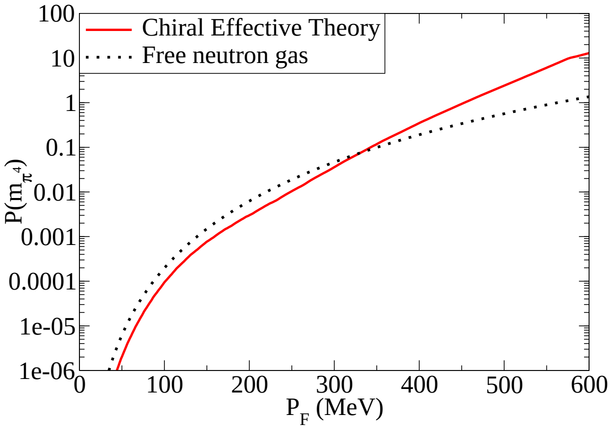
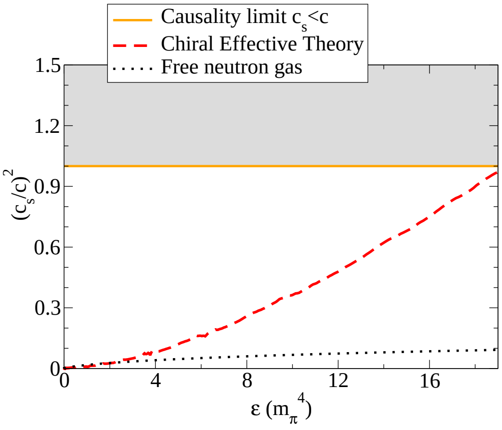


Causality: $c_s \leq c$



Outline

Vary Cavendish's constant

Extensions of General Relativity predict $G_N(g)$

For example, arXiv:0410117

$$G_N \simeq \text{constant} \quad r \to 0$$
$$G_N \propto \frac{1}{k^q} \propto r^q \quad r \to \infty$$
$$q \simeq 10^{-6}$$

Dozens of works 0901.2963, hep-th/9504014, hep-ph/0207282, astro-ph/9501066 ...

Vary Cavendish's constant

$$\frac{dP}{dr} = -\frac{G_N}{r^2}\frac{(\varepsilon(r)+P(r))(M(r)+4\pi r^3 P(r))}{1-\frac{2G_N M(r)}{r}}$$

Where Effective Theory becomes unreliable use the steepmost equation of state

$$P = P_0 + c^2(\rho - \rho_0)$$

(lack of knowledge of dense QCD does not alter conclusions)

Vary Cavendish's constant

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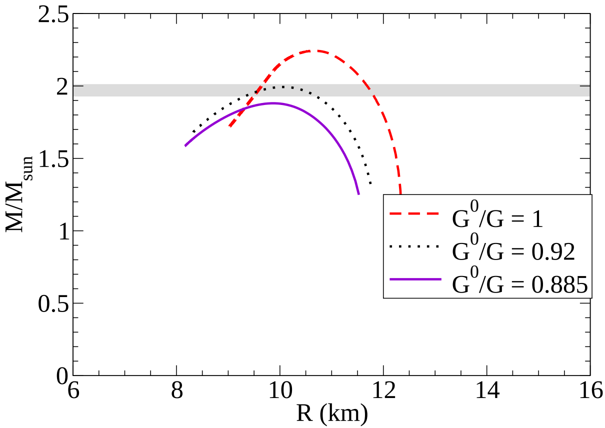
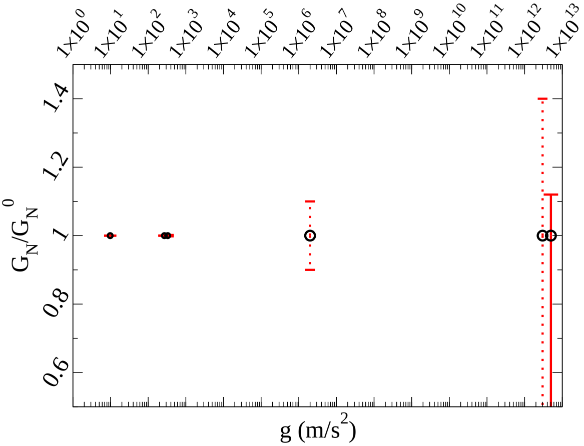
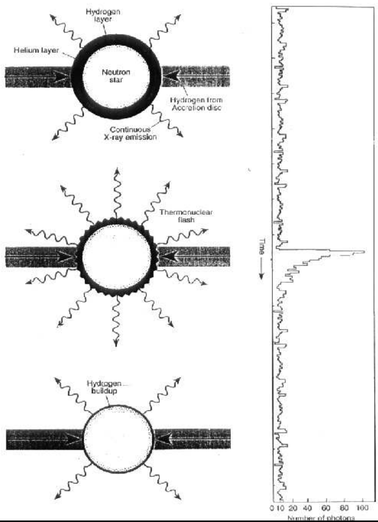


Constraint on the constant at high field intensity



Dobado, LL-E and Oller PRC**85** (2012) 012801

Measurement of radii

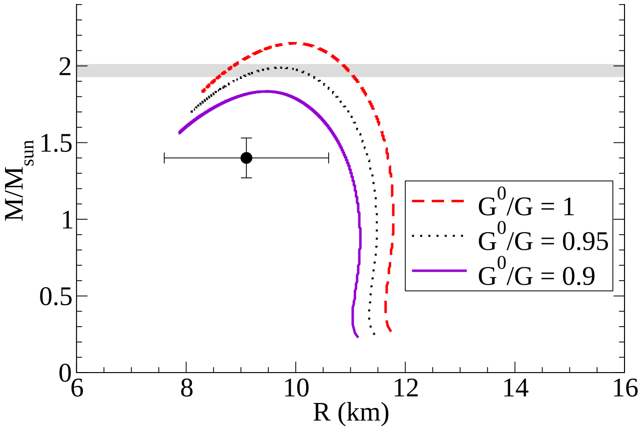


$r_* = 9.4 \pm 1.2$ km
Guillot and Rutledge APJ**796**,1 (2014).

Typical calculated radii with GR: $r_* \simeq 12 - 13 km$

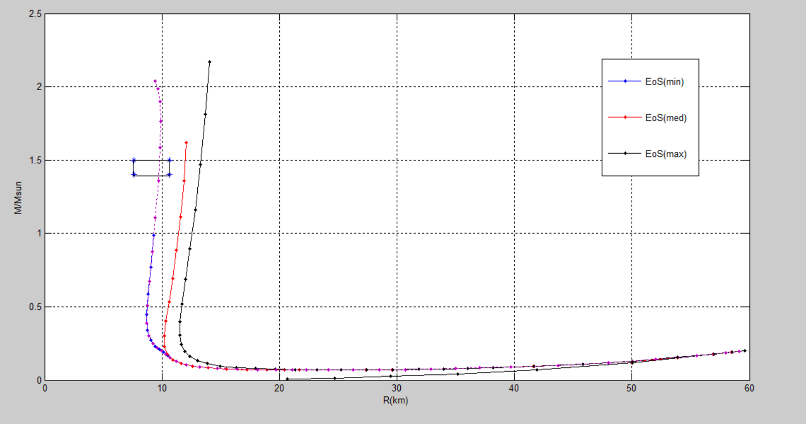
New light *U*-boson exchange?
(1509.02128)

Additional constraint on the mass/radius plot



This measurement brings tension to the field

Additional constraint on the mass/radius plot



The three EOS of Hebeler *et al.*, APJ **773** 11 (2013)

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Modified gravity

Typical models: $R + f(R)$

Hu-Sawicki model that provides accelerated universe's expansion:

$$S = R - \frac{b\frac{R}{cH0^2}}{1 + d\frac{R}{cH0^2}}$$

with $c = 6 * (1 - \Omega_m) * d/b$; $H0 = 0.999456$
$\Omega_m = 0.35831$ $d = 1$, $b = 209.7263765$

But many others too, Tsujikawa: $R - \mu R_T tanh\left(\frac{R}{R_T}\right)$

Starobinsky: $R + \lambda R_S \left(\left(1 + R^2/R_S^2\right)^{-n} - 1 \right)$

Exponential: $R - \beta R_E (1 - e^{-R/R_E})$

K. Bamba *et al.*, 1108.2557

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K. Bamba *et al.*, 1108.2557

Describe inflation

Two possibilities

- $\lim_{R\to\infty} f(R) = -\Lambda$
- $f(R) \sim_{R\to\infty} \alpha R^n$

D.Saez Gomez, 1207.5472

Static, spherically symmetric ansatz

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,[R + f(R)]$$

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = \frac{1}{1+f_R}\left[-8\pi G\,T_{\mu\nu} - \nabla_\mu\nabla_\nu f_R \right.$$
$$\left. + g_{\mu\nu}\nabla^\alpha\nabla_\alpha f_R + \frac{1}{2}(f(R) - R\,f_R)\,g_{\mu\nu}\right]$$

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\phi)$$

Three independent quantities

- In GR's Schwarzschild: $A(r)$, $B(r)$, but $R = 8\pi GT$ is algebraically constrained
- In $f(R)$ theories, a differential equation for R:
  $R = \frac{8\pi G\,T - 2\,f(R) - 3\,\nabla^\alpha\nabla_\alpha f_R}{1 - f_R}$

Equations of hydrostatic equilibrium generalizing TOV

$$
\begin{aligned}
R'' &= R'\left[\frac{A'}{2A} - \frac{B'}{2B} - \frac{2}{r}\right] - \frac{A}{3f_{RR}}\left[8\pi G(\rho - 3p) - (1 - f_R)R - 2f(R)\right] \\
A' &= \frac{2rA}{3(1+f_R)}\left[8\pi GA(\rho + 3p) + \frac{A}{2}R - \frac{3B'}{2rB}\right. \\
&\quad \left. - f_R\left(\frac{A}{2}R + \frac{3B'}{2rB}\right) - \left(\frac{3}{r} + \frac{3B'}{2B}\right) f_{RR}\, R' + Af(R)\right] \\
B'' &= \frac{B'}{2}\left(\frac{A'}{A} + \frac{B'}{B}\right) + \frac{2A'B}{rA} + \frac{2B}{(1+f_R)}\left[-8\pi GAp\right. \\
&\quad \left. - \frac{A}{2}R + \left(\frac{B'}{2B} + \frac{2}{r}\right) f_{RR}\, R' - \frac{A}{2}f(R)\right] \\
p' &= -\frac{\rho + p}{2}\frac{B'}{B}
\end{aligned}
$$

Nonperturbative system

- Unlike earlier studies, e.g. Astashenov *et al.* 1309.1978 we are not using perturbation theory in a
- System solved by 4th order Runge-Kutta
- Initial conditions to obtain regularity, finite pressure, matching

EFT point of view

- Solar system tests: $|f(R_{\rm ss})| < 10^{-6}$ but what is $R_{\rm ss}$?
- $R_{FRW} \sim 3 \times 10^{-46}$ km^{-2}
- $R_{Schwz} = 0$; dimensionally, Kretschmann's scalar $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{12 r_s^2}{r^6} \sim 3 \times 10^{-17}$km^{-2} (at $r = R_\odot$)
- $R + aR^2 + O(R^4)$ a can currently be huge

(Note: not general-most extension, can play with indices $R_{\mu\nu} R^{\mu\nu}$)

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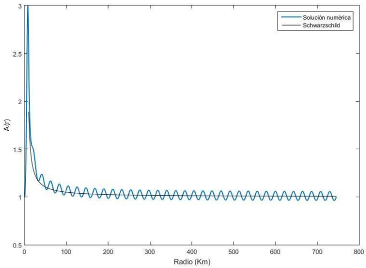
What $R + aR^2$ really means

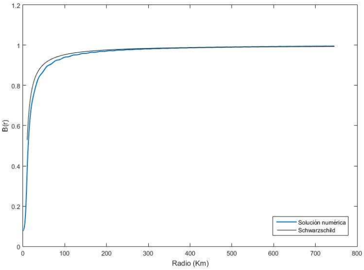
Does the burden grow quadratically with the paperwork?

http://www.freshtracks.co.uk/blog/how-to-lead-a-happy-committee/

Metric functions

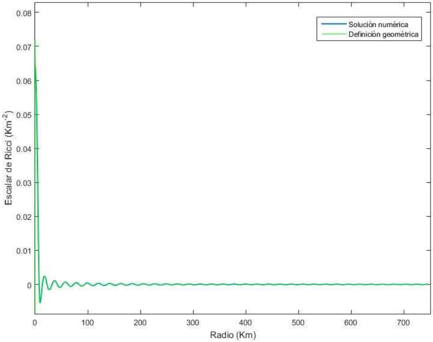
(obtained by forcing A, B, B' to be Schwarzschild-like at R_*)





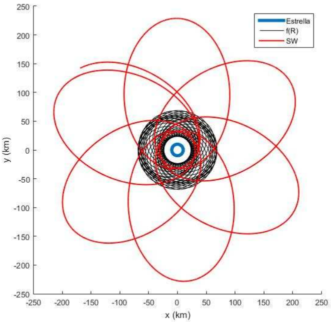
$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\phi)$$

R does not vanish outside the star



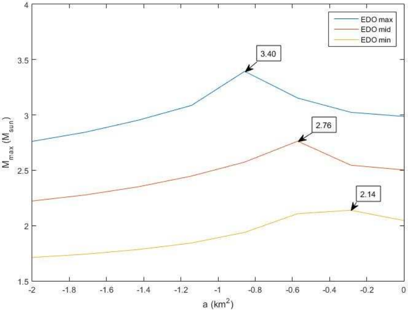
(It actually oscillates indefinitely with a small amplitude $\propto a$)

Typical satellite trajectories around the star



(solution of the geodesic eqs. courtesy of M. Aparicio)

Constraining $R + aR^2$



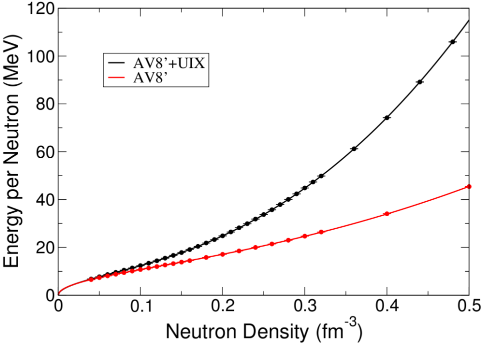
- State equations of Hebeler *et al.* APJ**773**:11 (2013)
- Matching to exterior Schwarzschild (careful)
- Systematic improvement needed: counting in the eq. of state
- Find heavier stars

Wait, what is the meaning of "heavier"?

- In Gen. Rel. "quantity of matter" $M(r) = 4\pi \int_0^r r^2 dr \epsilon(r)$ coincides with Schwarzschild's mass
- Shapiro mass, Newtonian potential at infinity...
- In grav. wave detection, "Chirp mass" $\mathcal{M} = \frac{(m_1+m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ *in general relativity*; so m_1, m_2 are Schwarzschild masses.
- In modified gravity the solutions are not tagged by $M(r)$ in the same way

Equation of state: learning from other groups

Effect of three-body forces from potential models

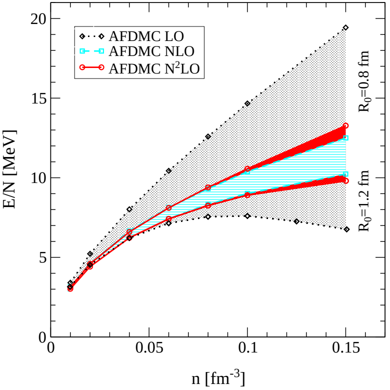


Expect repulsion, higher pressure, when including them.

S. Gandolfi *et al.*, 1307.5815

Equation of state: learning from other groups

NNLO not much better than NLO (large two-pion exchange)



Ongoing work

- Extend dispersive analyses of the EFT one more order (with J. Oller)
- Systematize the matching of solutions to general relativity
- Explore several $f(R)$ alternatives

Summary

- Neutron star properties depends on QCD equation of state
- 2 solar-mass star quite high: several equations of state ruled out *in General Relativity*
- From relatively safe knowledge: constrain Cavendish constant
- $\frac{\Delta G}{G} \leq 12\%$ at neutron star

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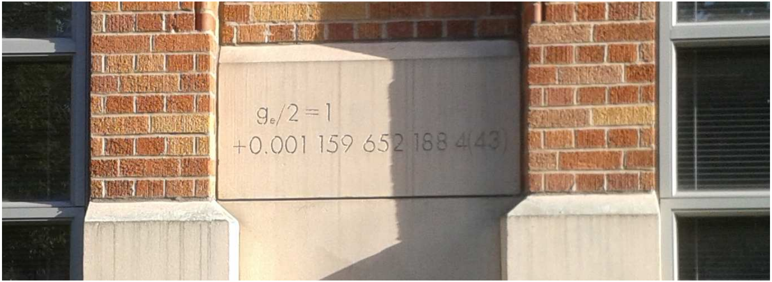
Summary

- aR^2 mod. gravity: a not well constrained elsewhere
- With current nuclear knowledge, $O(1)$ bound
- Outer metric not quite Schwarzschild, how to tag solutions with a mass precisely?
- Safer: provide controlled errors and counting at the expense of some "realism" in the EOS

What a better place than the INT



Constraining gravity with the equation of state in neutron stars

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Institute of Nuclear Theory Seminar
September 16^{th} 2015, U. of Washington at Seattle

In collaboration with M. Aparicio, A. de la Cruz Dombriz, V. Zapatero (work in progress) and A. Dobado, A. Oller Phys.Rev. C85 (2012) 012801