Quark and Glue Components of Proton Spin

- Status of nucleon spin components
- Momentum and angular momentum sum rules
- Lattice results
- Quark spin from anomalous Ward identity
- Glue spin

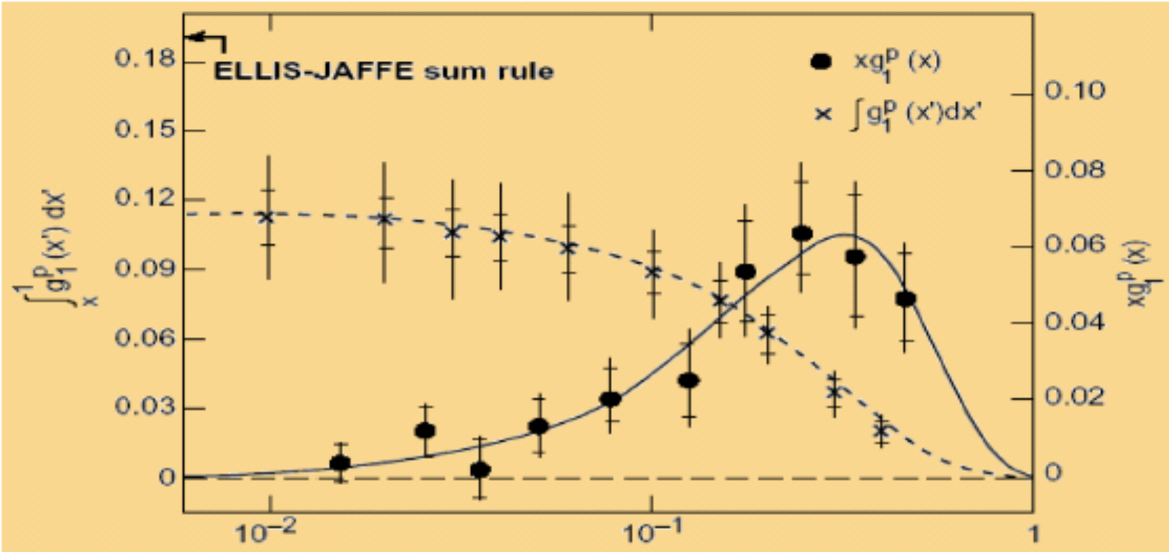
χ QCD Collaboration

INT, Sept. 22, 2015

Where does the spin of the proton come from?

Twenty years since the “spin crisis”

- EMC experiment in 1988/1989 – “the plot”:

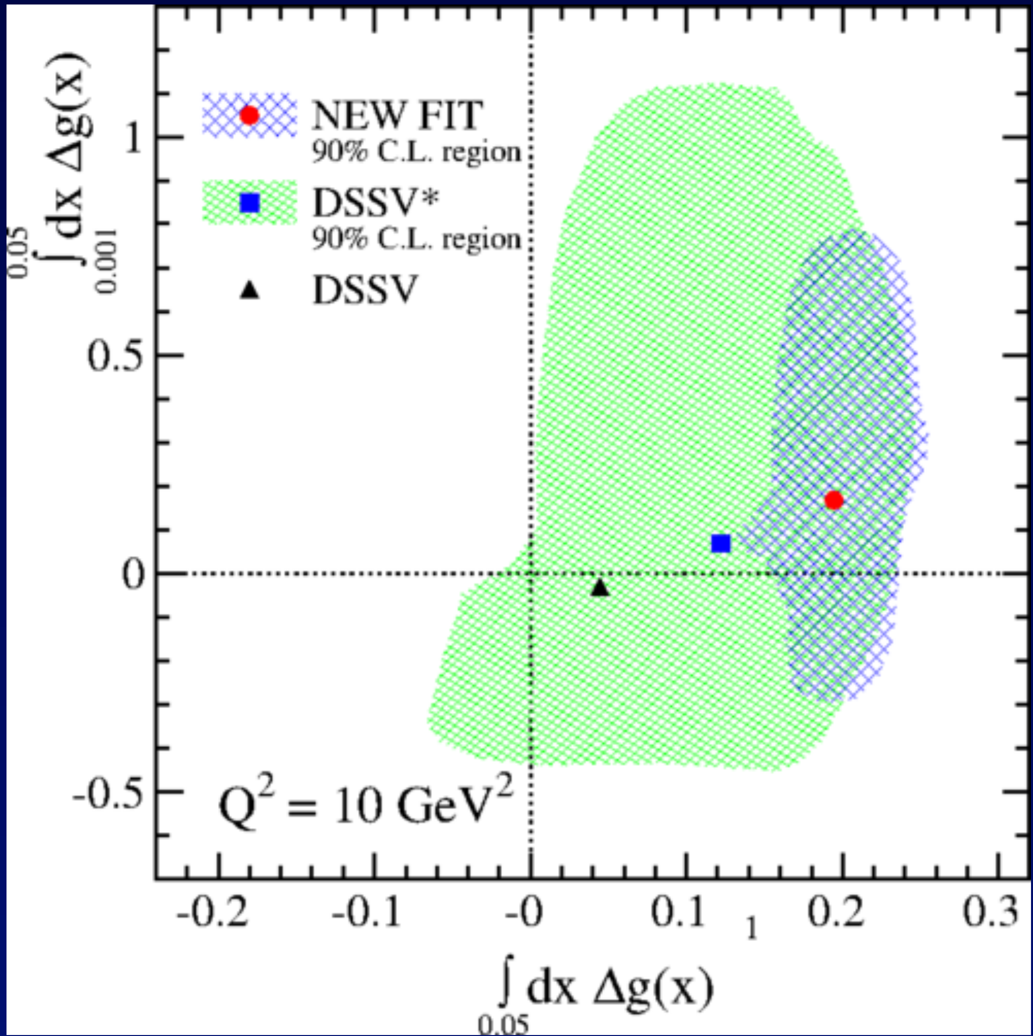


$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_{\parallel}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{\parallel}\rangle$$

- “Spin crisis” or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Glue Helicity ΔG

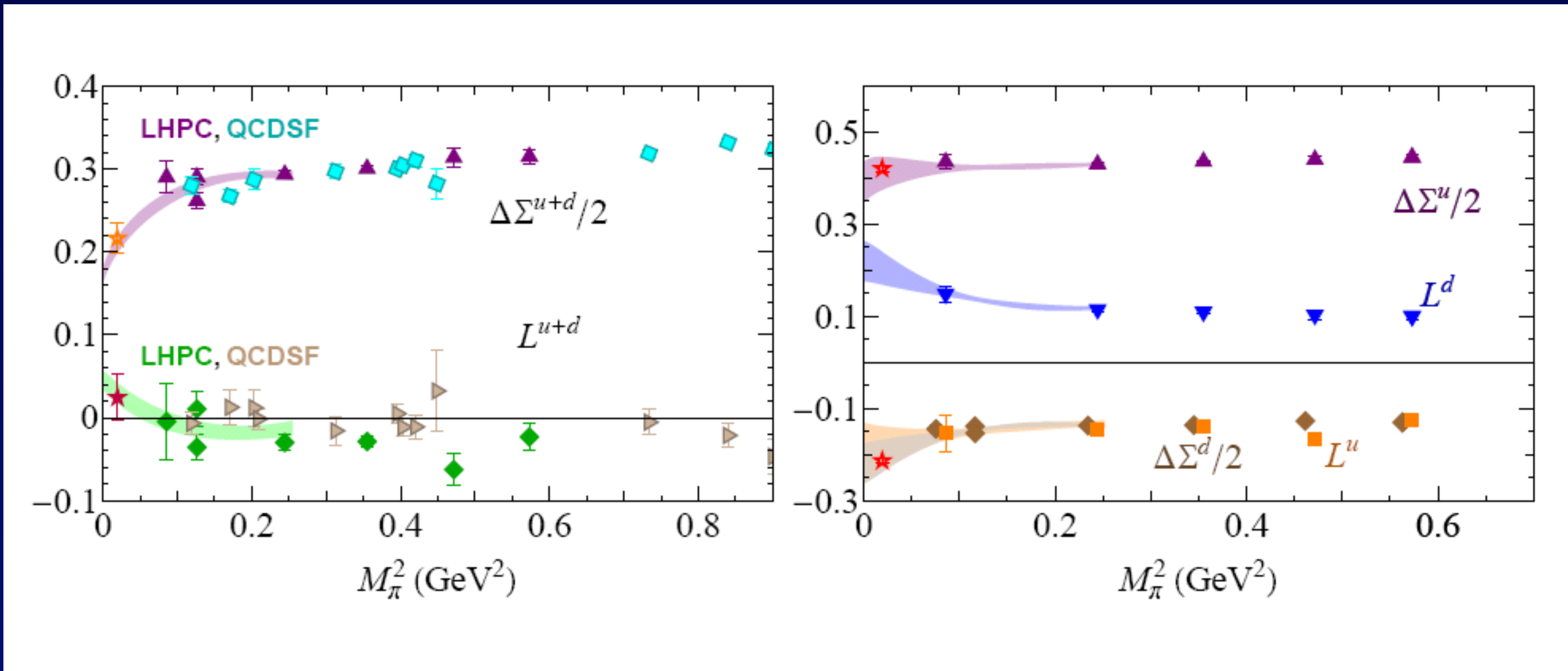


Experimental results from
STAR [1404.5134]
PHENIX [1402.6296]
COMPASS [1001.4654]

ΔG ~ 0.2 with large error

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang,
PRL 113, 012001 (2014)

Quark Orbital Angular Momentum
(connected insertion)



Status of Proton Spin

- Quark spin $\Delta\Sigma$ ~ 20 - 30% of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation of CI (LHPC,QCDSF)→ ~ 0)
- Glue spin ~ 0.4 (STAR, PHENIX, COMPASS)
- Glue orbital angular momentum is zero (Brodsky and Gardner).
- \`proton spin crisis' → \`missing spin puzzle'.
- If the spin sum rules not satisfied experimentally ⟶ Dark Spin

Spin Sum Rules

- Jaffe and Manohar sum rule (1990)

$$J = \frac{\Sigma}{2} + L_q + S_G + L_G$$

$$\vec{J}_{Tot} = \int d^3x\ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x\ \vec{x} \times \psi^\dagger \vec{\nabla}\psi + \int d^3x\ \vec{E}^a \times \vec{A}^a$$
$$+ \int d^3x\ \vec{x} \times E^{aj} (\vec{x} \times \nabla) A^{aj}$$

  - Canonical EM tensor on light-cone with light-cone gauge
  - Not directly accessible on the lattice

- Ji sum rule (1997)

$$J = \frac{\Sigma}{2} + L_q + J_G$$

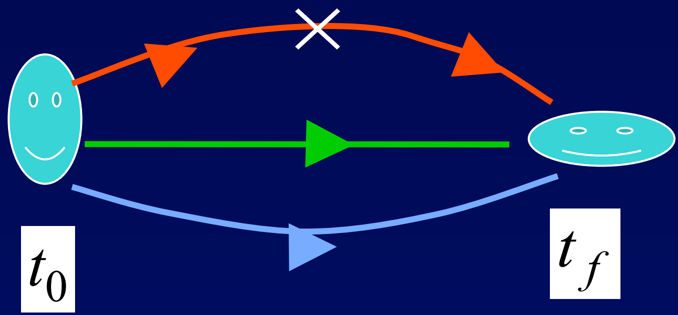
$$\vec{J}_{Tot} = \int d^3x\ \psi^\dagger \frac{1}{2}\Sigma\psi + \int d^3x\ \vec{x} \times \psi^\dagger \vec{D}\psi + \int d^3x\ \vec{x} \times\ (\vec{E}^a \times \vec{B}^a)$$

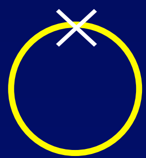
  - Symmetric EM tensor (Belafonte) → gauge invariant and frame independent.

Hadron Structure with Quarks and Glue

- Quark and Glue Momentum and Angular Momentum in the Nucleon

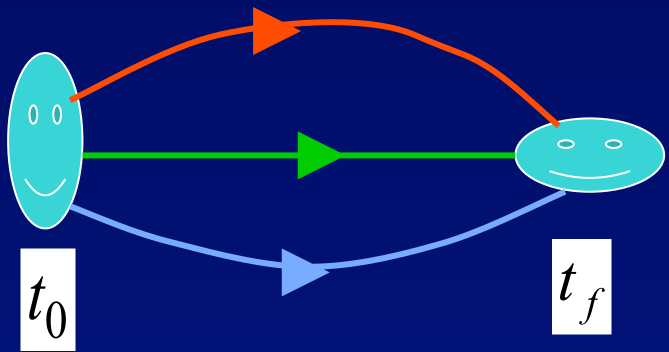
$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$

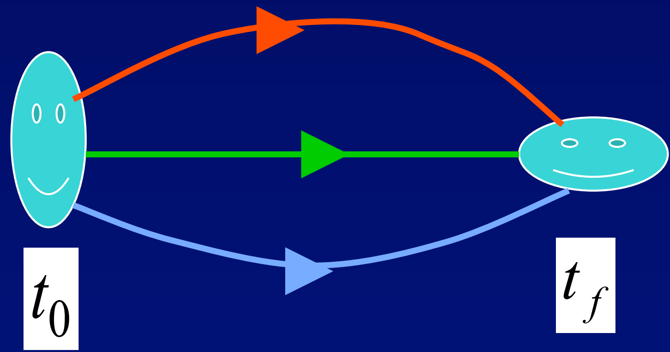




$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$

$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$





Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\left\langle p,s\,|\,T_{\mu\nu}\,|\,p's'\right\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

$$-i\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ \left[\mathrm{OPE}\right] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Lattice Parameters

- Quenched $16^3 \times 24$ lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- κ = 0.154, 0.155, 0.1555 (m_π = 650, 538, 478 MeV)
- 500 configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

$T_1\,(q^2)$ and $T_2\,(q^2)$

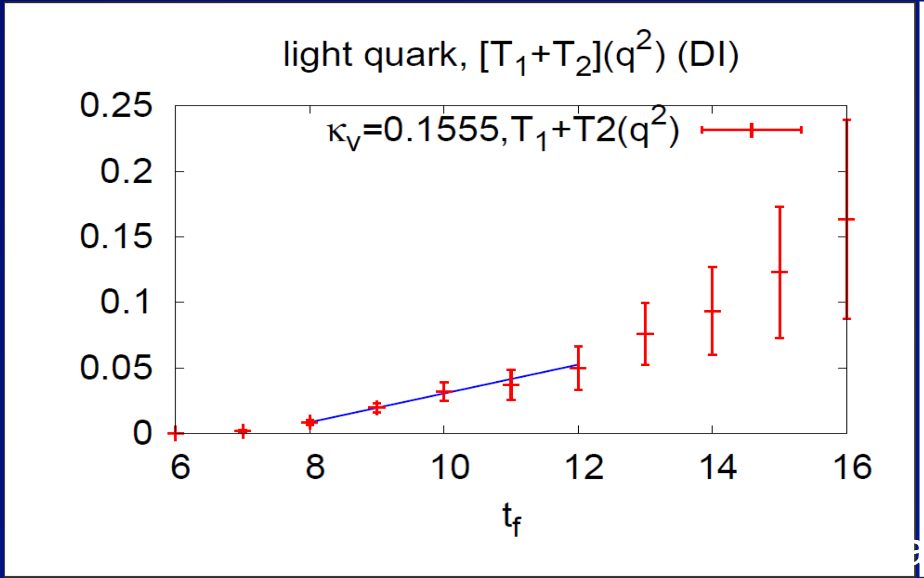
- 3-pt to 2-pt function ratios

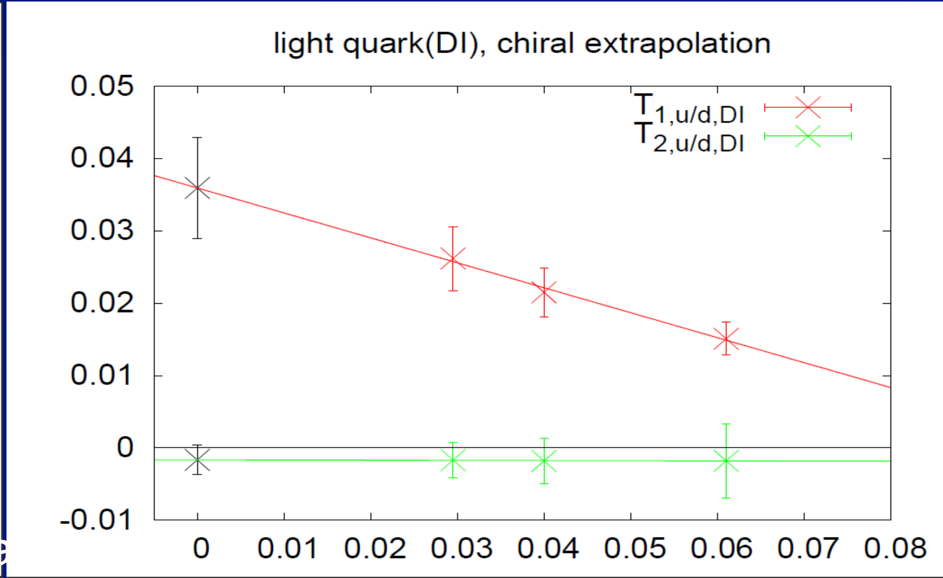
$$G^{3pt}_{\mu\nu}(\vec{p},t_2;\vec{q},t_1)=\sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}\cdot\vec{x}_2+i\vec{q}\cdot\vec{x}_1}\left\langle 0\,|\,T\left[\chi_N(\vec{x}_2,t_2)T_{\mu\nu}(t_1)\bar{\chi}_N(0)\right]\right\rangle;$$

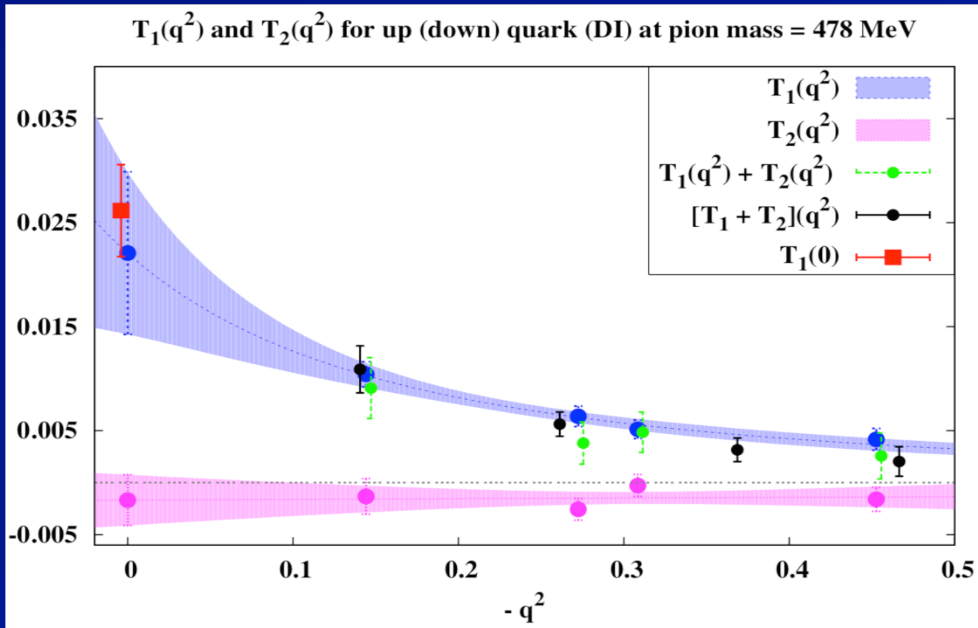
$$\mathrm{Tr}\left[\Gamma_m G^{3pt}_{\mu\nu}(\vec{p}=0,t_2;\vec{q},t_1)\right]=We^{-m(t_2-t_1)}e^{-Et_1}\left[T_1(q^2)+T_2(q^2)\right]$$

- Need both polarized and unpolarized nucleon and different kinematics (p_i, q_j, s) to separate out $T_1\,(q^2)$, $T_2\,(q^2)$ and $T_3\,(q^2)$

Disconnected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks







Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \quad H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5(1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5(1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu}(x) + O(a^3),$$

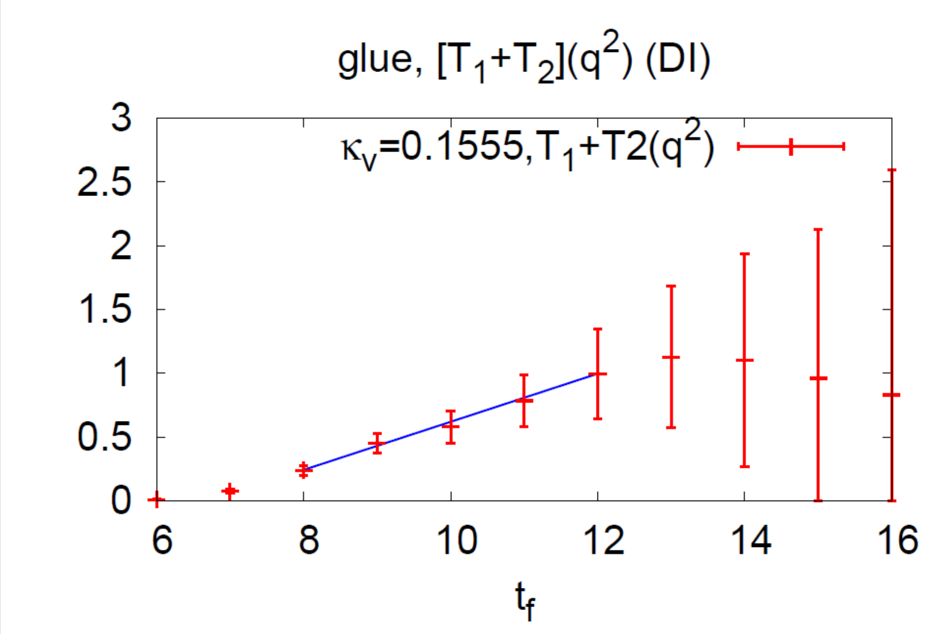
$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_\lambda (c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2\right]}{(\sum_\mu s_\mu^2 + [\rho + \sum_\nu (c_\nu - 1)]^2)^{3/2}}$$

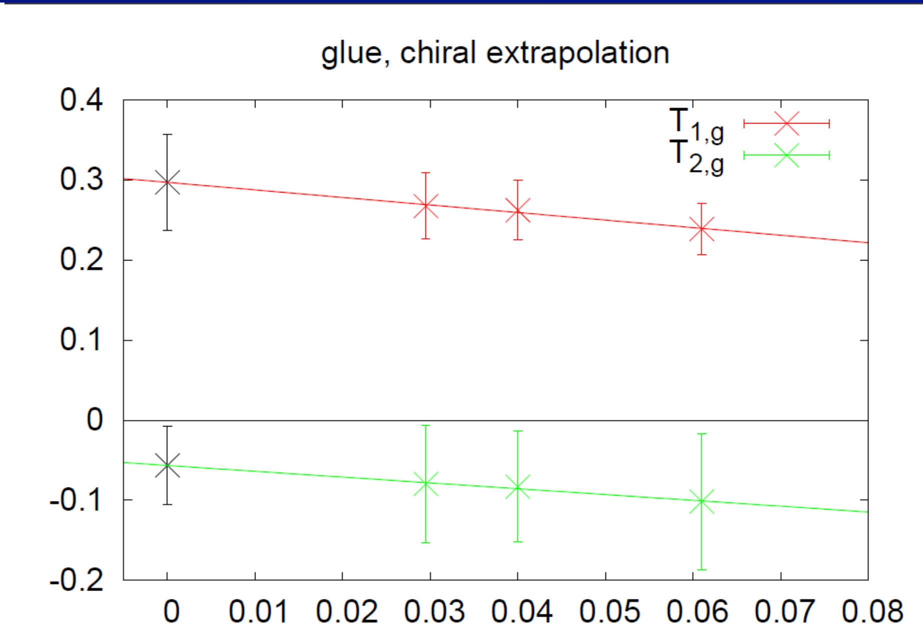
where, $r = 1, \quad \rho = 1.368, \quad c^T = 0.11157$

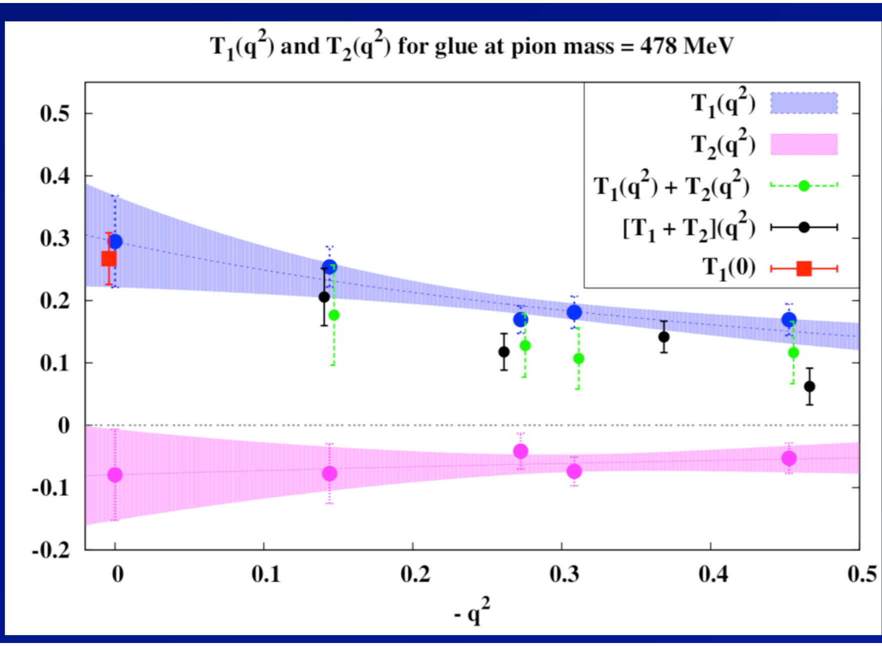
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

- Noise estimation $D_{ov}(x,x) \rightarrow \langle \eta_x^\dagger (D_{ov}\eta)_x \rangle$
  with Z_4 noise with color-spin dilution and some dilution in space-time as well.

Glue T_1 (q^2) and T_2 (q^2)







M. Deka et al., PRD (2015), 1312.4816 (χ QCD Collaboration)

Quenched 16^3 x 24 lattice, β=6.0, $m_\pi \geq$ 478 MeV, 500 configurations

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1,$$

$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \quad \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

M. Glatzmaier, KFL
arXiv:1403.7211

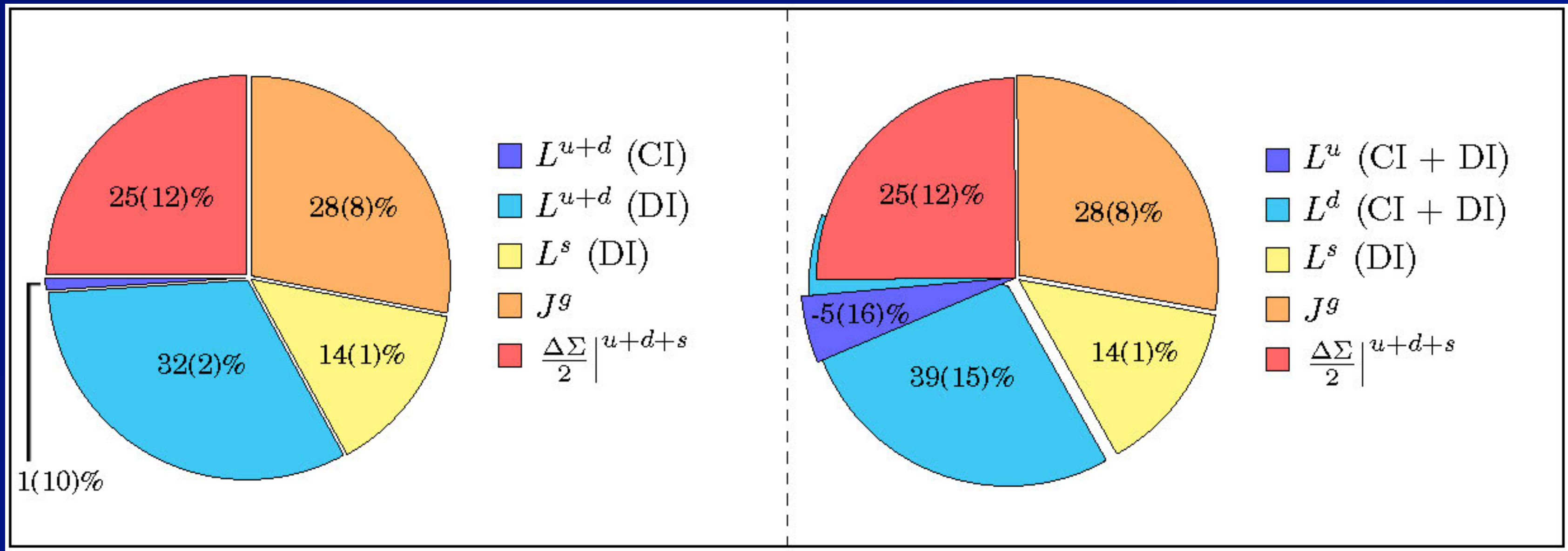
Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

$\overline{MS}$ (2 GeV)

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.416 (40)	0.151 (20)	0.567 (45)	0.037 (7)	0.023 (6)	0.334 (56)
$T_2(0)$	0.283 (112)	-.217 (80)	0.061 (22)	-0.002 (2)	-.001 (3)	-0.056 (52)
2J	0.704 (118)	-.070 (82)	0.629 (51)	0.035 (7)	0.022 (7)	0.278 (76)
g_A	0.91 (11)	-0.30 (12)	0.62 (9)	-0.12 (1)	-0.12 (1)	
2 L	-0.21 (16)	0.23 (15)	0.01 (10)	0.16 (1)	0.14 (1)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum (M. Deka *et al*, 1312.4816, PRD)

pizza cinque stagioni



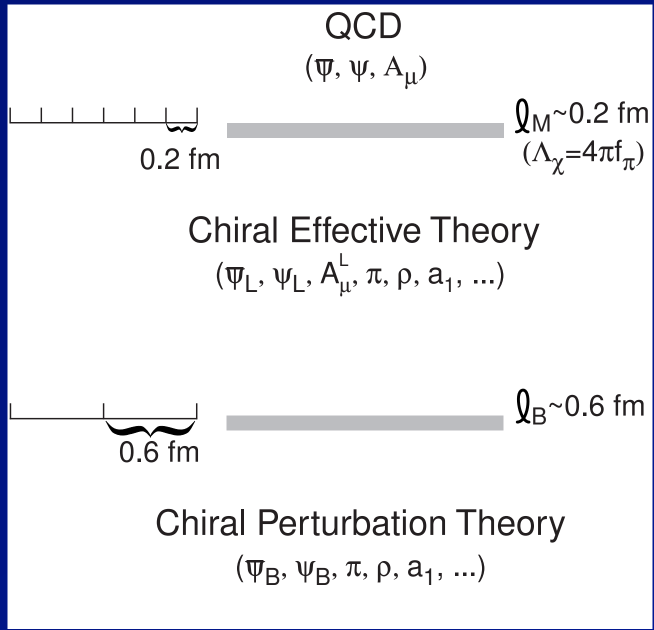
$\Delta q \approx 0.25;$

$2\,L_q \approx 0.47\ (0.01(\mathrm{CI})+0.46(\mathrm{DI}));$

$2\,J_g \approx 0.28$

These are quenched results so far.

Chiral effective theory of baryons



Manohar and Georgi (1984)

KFL (2000)

$$\psi = \psi_L + \psi_S, \qquad A^\mu = A^\mu_L + A^\mu_S$$

$$L_{\chi QCD} = L_{QCD'}(\bar{\psi}_L, \psi_L, A^\mu_L) + L_M(\pi, \rho, G, ...)$$
$$+ L_{\sigma q}(\bar{\psi}_L, \psi_L, \pi, \rho, G, ...)$$

Little bag model (Brown, Rho)
Clouding bag model (Thomas, Theberge, Miller)
Chiral quark-soliton model (Wakamatsu)

Orbital Angular Momentum

skyrmion

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#

Quark Spin Calculation with Axial-vector current

- Recent calculation of strange quark spin with dynamical fermions

Leader et al., 1410.1657

$$\Delta s = -0.106(23)$$

  - R. Babich et al. (1012.0562)

$$\Delta s = -0.019(11)$$

  - QCDSF (G. Bali et al. 1206.4205) gives

$$\Delta s = -0.020(10)(4)$$

  - M. Engelhardt (1210.0025)

$$\Delta s = -0.031(17)$$

  - C. Alexandrou et al. (arXiv:1310.6339)

$$\Delta s \sim -0.0227(34)$$

  - A.J. Chambers et al. (arXiv:1508.06856)

$$\Delta s \sim -0.018(6)$$

Quark Spin from Anomalous Ward Identify

- Calculation of the point axial-vector in the DI is not sufficient.
- AWI needs to be satisfied. $\partial_\mu A_\mu^0 = i2mP + \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
- Unrenormalized AWI for overlap fermion for point current

$$\kappa_A \partial_\mu A_\mu^0 = i2mP + iN_f 2q(x)$$

Renormaliztion and mixing:

$$Z_A \kappa_A \partial_\mu A_\mu^0 = i2Z_m m Z_P P + iN_f 2(Z_q q(x) + \lambda \partial_\mu A_\mu^0)$$

  - Overlap fermion --> mP is RGI ($Z_m Z_P$=1)
  - Overlap operator for $q(x) = -1/2 \, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ has no multiplicative renormalization.
  - Espriu and Tarrach (1982) $Z_A(2-\mathrm{loop}) = 1 - \left(\frac{\alpha}{\pi}\right)^2 \frac{3}{8} C_2(R) N_f \frac{1}{\varepsilon},$

$$\lambda = -\left(\frac{\alpha}{\pi}\right)^2 \frac{3}{16} C_2(R) \frac{1}{\varepsilon}$$

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.137 fm

($O(a^2)$ extrapolation)

La ~ 2.8 fm
m_π ~ 330 MeV

24^3 x 64, a =0.115 fm

→

La ~ 5.5 fm
m_π ~ 140 MeV

48^3 x 96, a =0.115 fm

La ~ 2.7 fm
m_π ~ 295 MeV

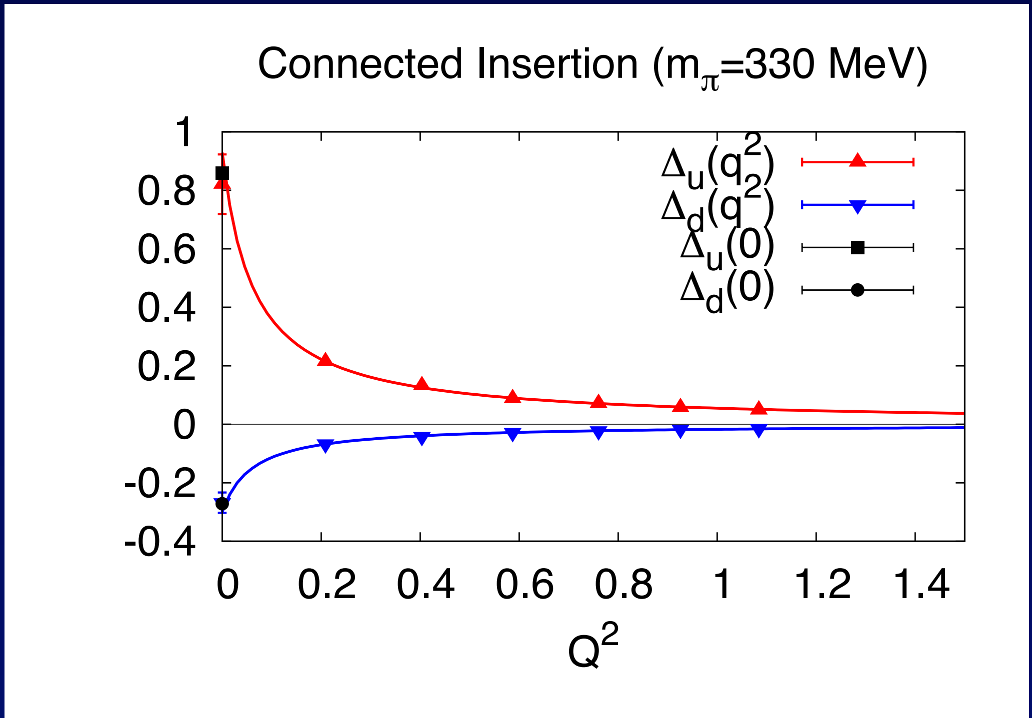
32^3 x 64, a =0.085 fm

→

La ~ 5.5 fm
m_π ~ 140 MeV

64^3 x 128, a =0.085 fm

Connected Insertion from Ward Identity



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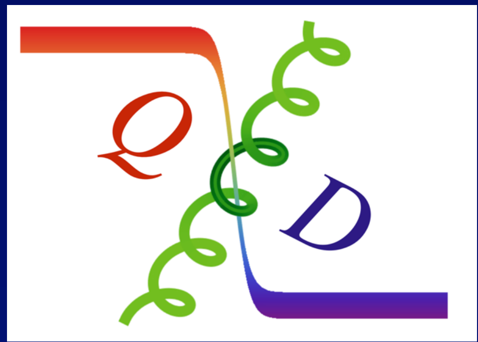
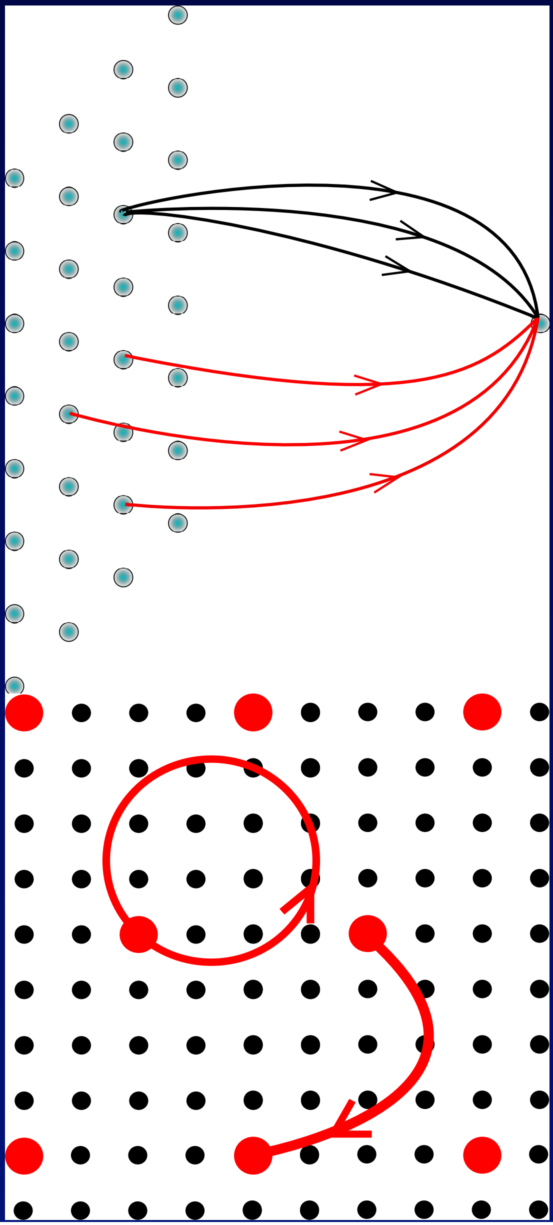


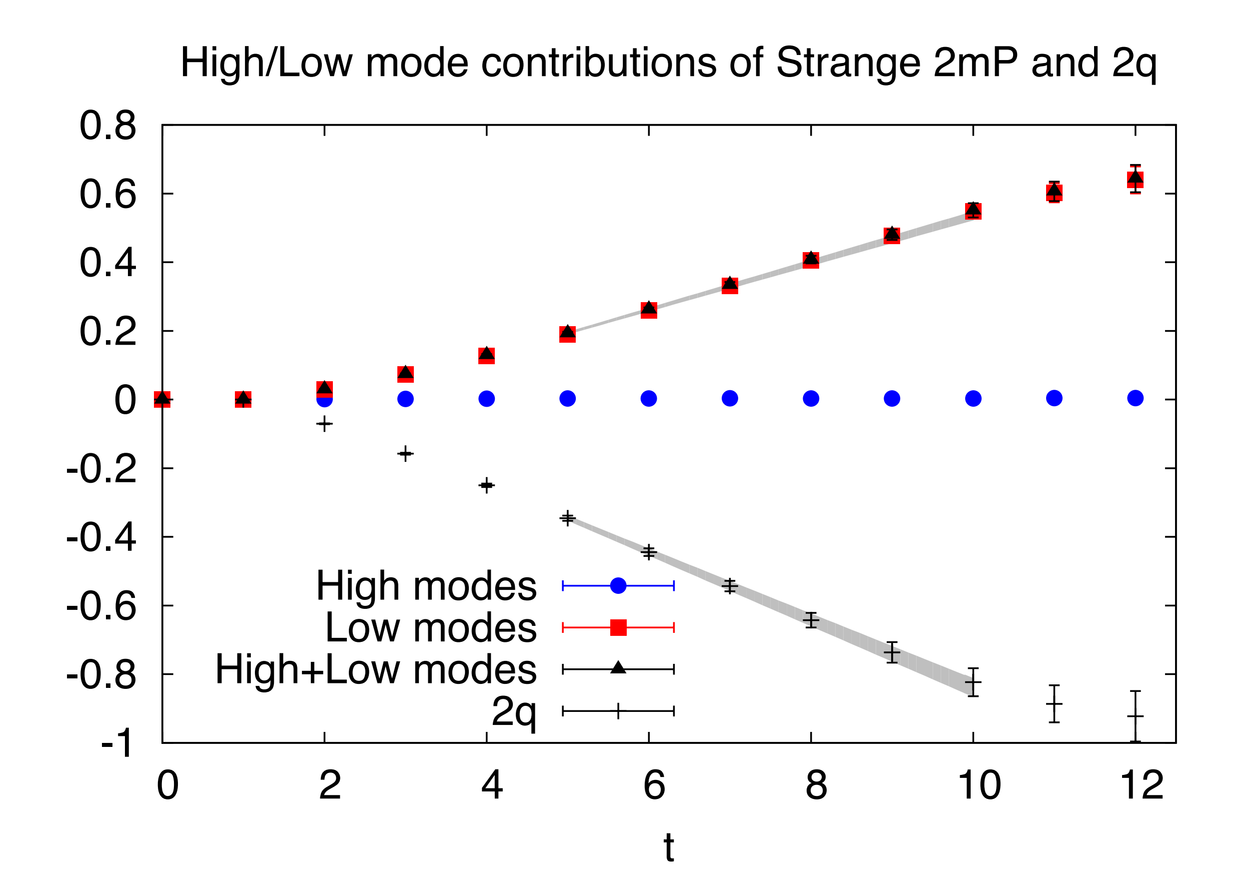
Y. Yang, M. Gong *et al*

Numerical improvement for DI calculation

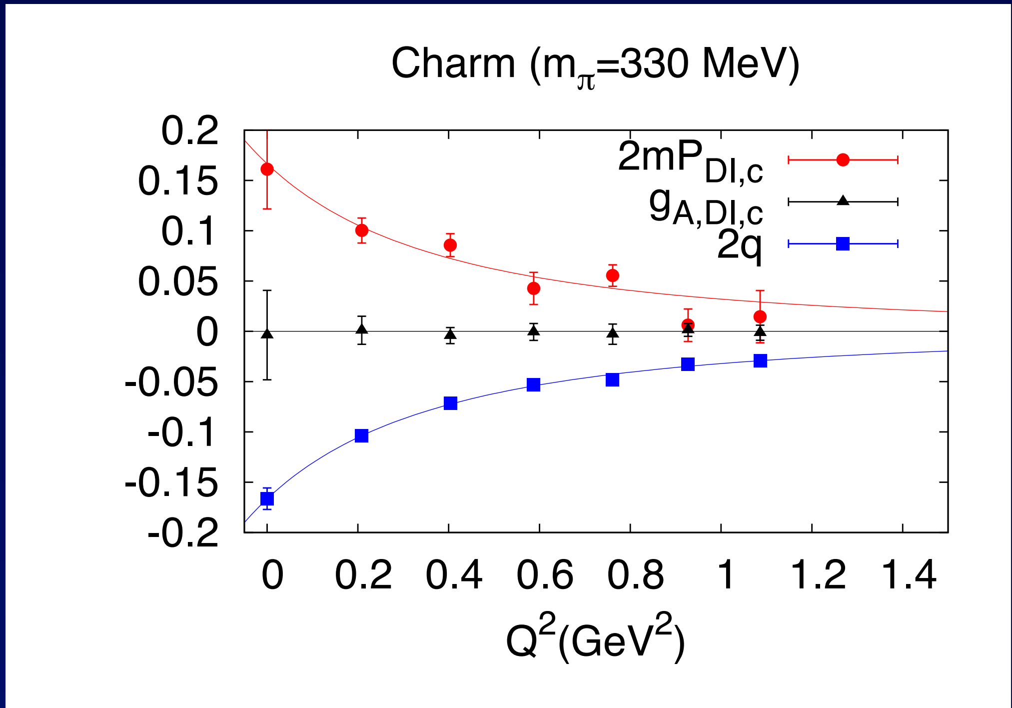


- 24^3 x 64 DWF lattice with m_π = 330 MeV 200 configurations
- 8 grid smeared source with Z_3 noise with low-mode substitution → gain factor ~ 5.6
- Sources on 32 time slices → gain ~ 6 (low precision inversion ~ 1/3 time)
- Low-mode average for the quark loop with Z_4 noise on grid (odd-even) for the high modes (4 such noises/configuration)
- 32 grid Z_4 noise (2^4 grids x odd-even) for q(x)

Low-mode average for quark loop

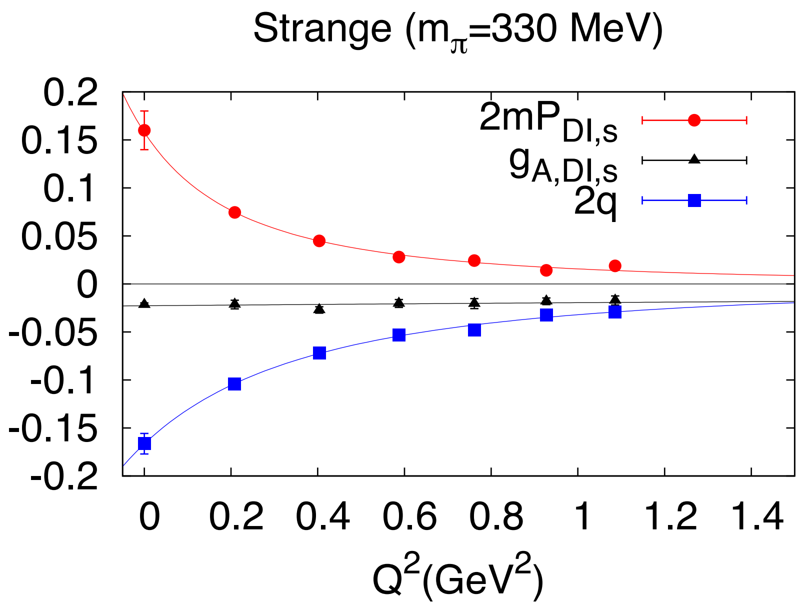


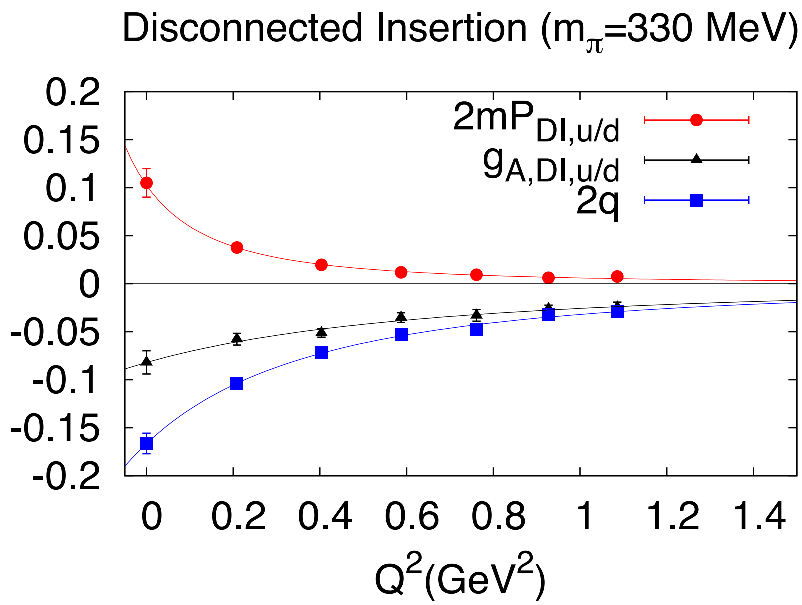
Disconnected Insertion for the Charm Quark



- Topological term is large and negative
- Pseudoscalar term and the topological term cancel

Disconnected Insertion for the Strange and u/d Quarks





Strange

u/d (DI)

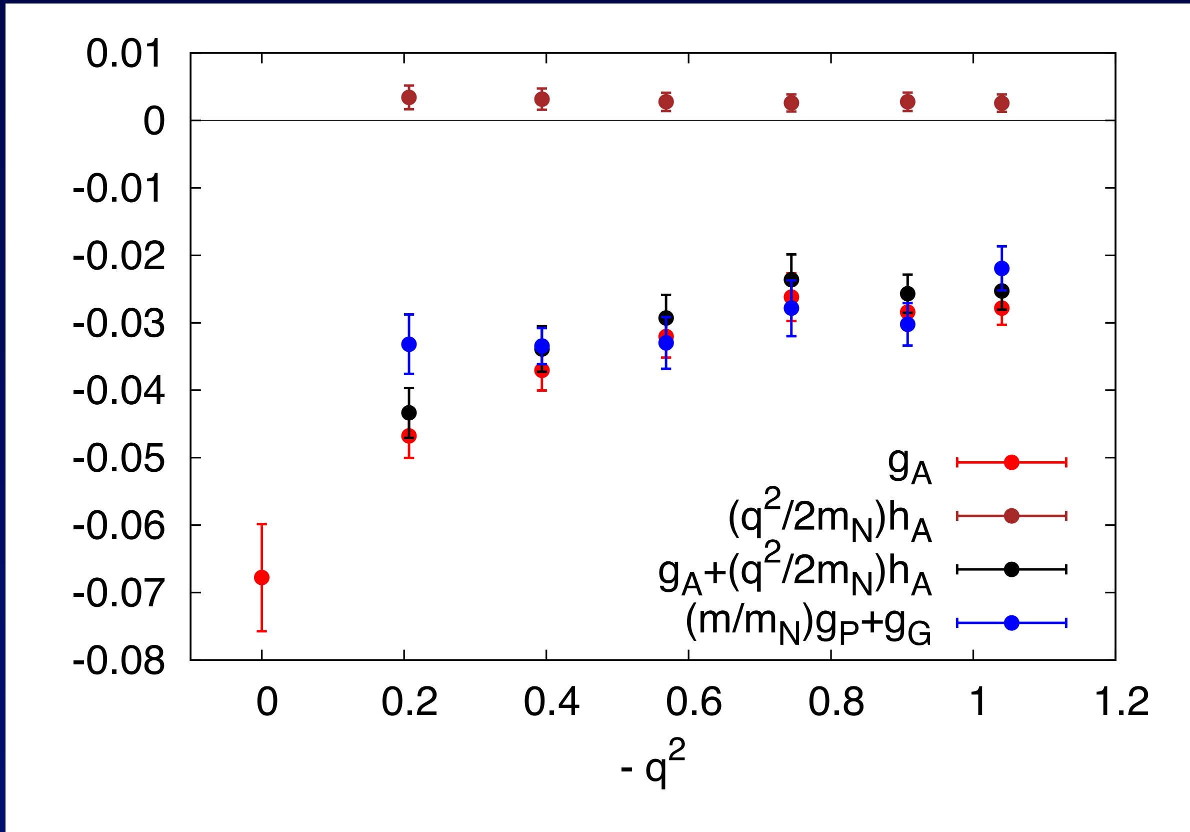
Anomalous Ward Identity in nucleon at finite q^2

$$\kappa_A \langle p',s | A_\mu | p,s \rangle = \lim_{q\to 0} \frac{i|s|}{\vec{q}\cdot\vec{s}} \langle p',s | 2\sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q | p,s \rangle$$

$$2M_N \kappa_A g_A^0(q^2) + q^2 \kappa_{h_A} h_A^0(q^2) = 2m g_P^0(q^2) + 2M_N g_G(q^2)$$

- Calculate $g_A(q^2)$, $h_A(q^2)$, $g_P(q^2)$, $g_G(q^2)$ for q^2 from 0.2 to 1.1 GeV^2 and fit κ_A and κ_{hA} for the local axial-vector operator.
- For the 24^3 x 64 lattice, $1/\kappa_A = 0.39(3)$, $1/\kappa_{hA} = 12(6)$
- This can explain why the previous calculation of Δs with axial-vector currents are small.

Renormalized Δs



$\Delta s = - 0.068(8)$

Quark Spin from AWI

Overlap fermion on 2+1 flavor $24^3\times 64$ DWF lattice (L=2.8 fm)

g_A^0 comp	m_π=330 MeV ($m_V=m_{sea}$)
Δu+Δd (CI)	0.57(2)
Δc	~0
Δs	-0.068(8)
Δu(DI)=Δd(DI)	-0.09(2)
g_A^0	0.32(5)

$\Delta_S = -\ 0.073(8)$

at $m_\pi = 140$ MeV

The triangle anomaly (topological charge) is responsible for the smallness of quark spin in the proton (`proton spin crisis).

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering

  In Minkowski space

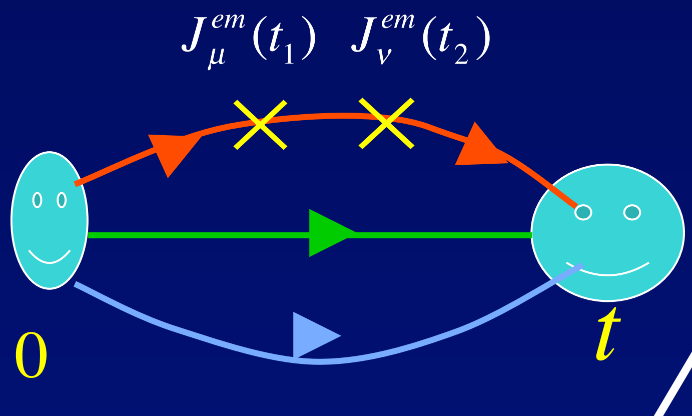
$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

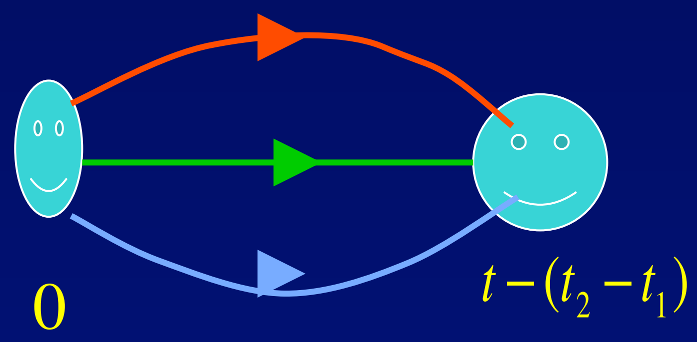
$$T_{\mu\nu}(q^2,\nu) = \frac{1}{2M_N}\int \frac{d^4x}{(2\pi)^4}\, e^{iq\cdot x} < P \,|\, T[J_\mu^{em}(x) J_\nu^{em}(0)] \,|\, P >,$$

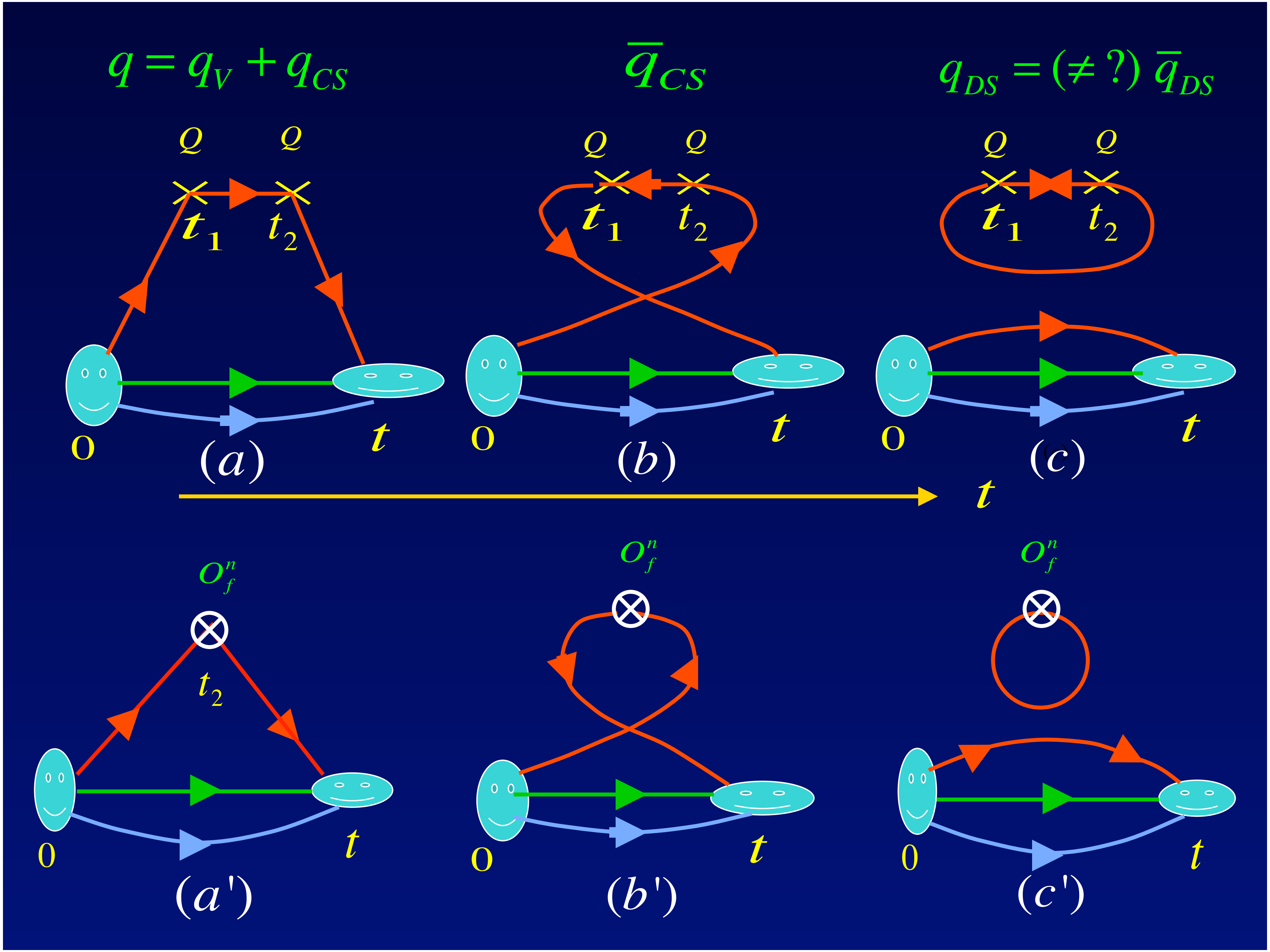
$$W_{\mu\nu}(q^2,\nu) = \frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} = \frac{(2\pi)^3}{2M_N}\sum_n \delta^4(p_n - p + q) < P \,|\, J_\mu^{em} \,|\, n >< n \,|\, J_\nu^{em} \,|\, P >$$

- Euclidean path-integral

K.F. Liu, PRD 62, 074501 (2000)

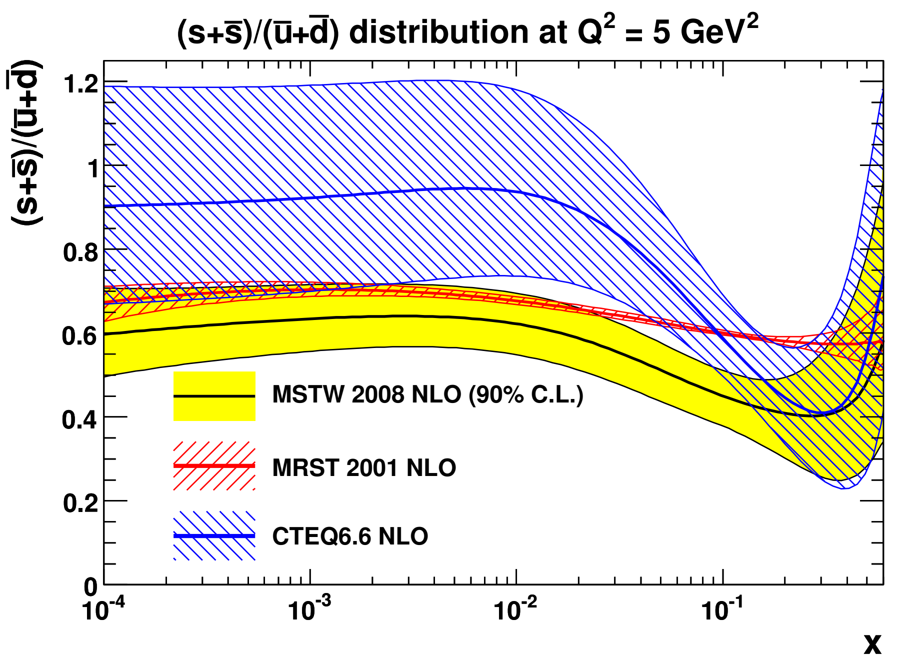


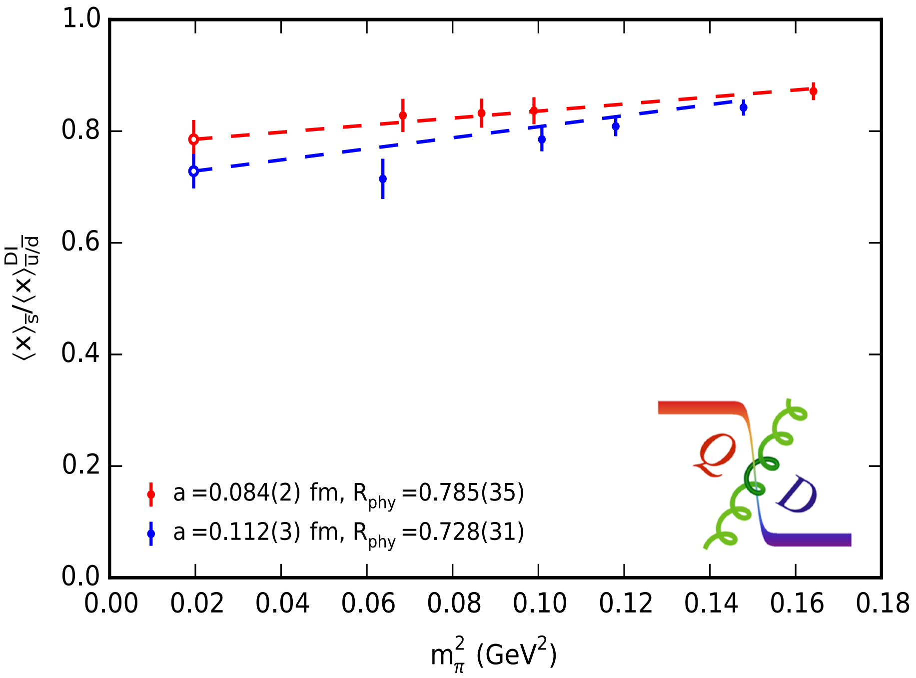




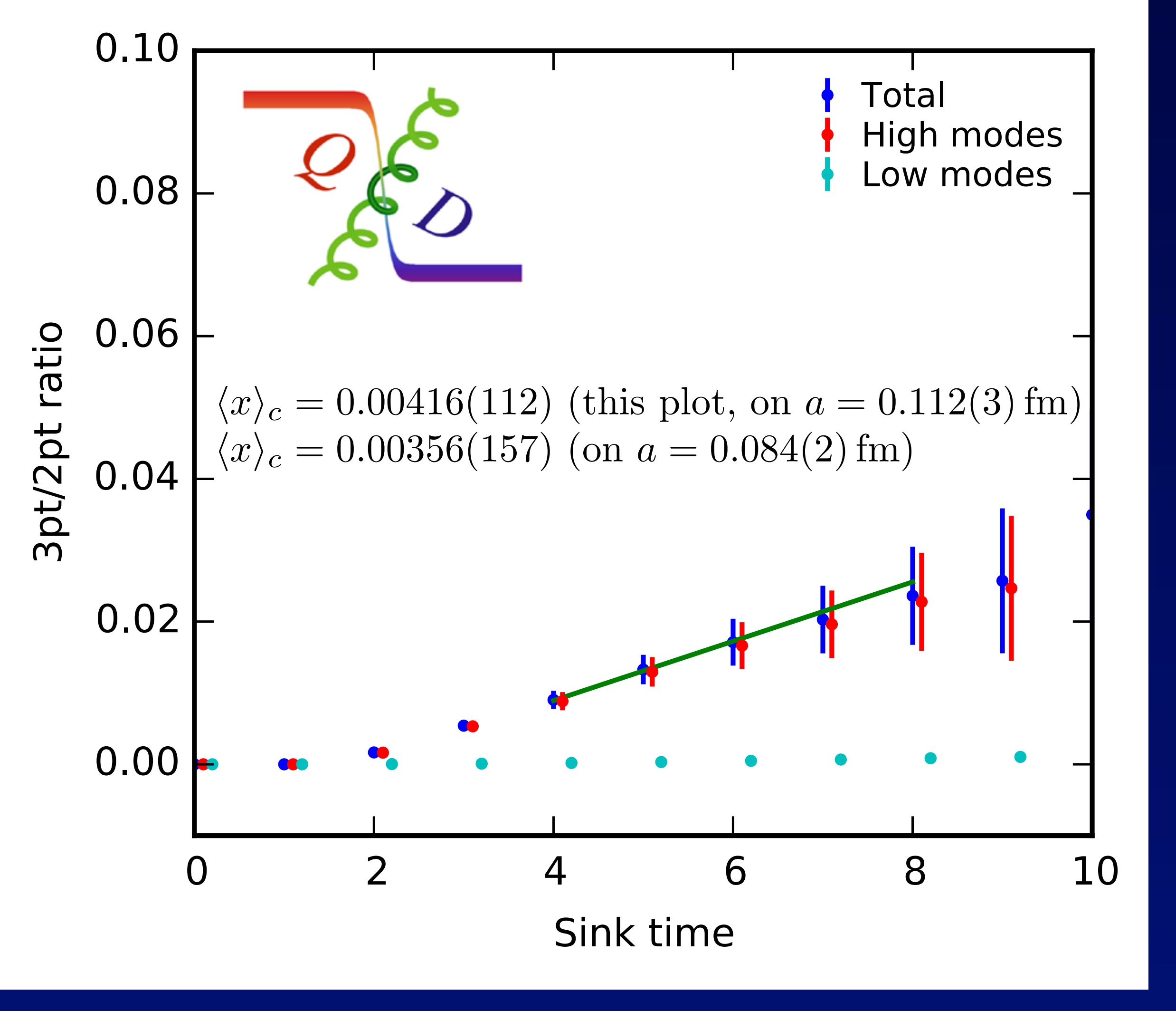
$q = q_V + q_{CS}$
$\bar{q}_{CS}$
$q_{DS} = (\neq ?)\ \bar{q}_{DS}$
Q
Q
t_1
t_2
0
t
(a)
(b)
(c)
t
O_f^n
t_2
(a')
(b')
(c')

Ratio of $\langle x \rangle_s / \langle x \rangle_{u/d}$ (DI)





$$\langle x \rangle_s = 0.027(6)$$



0.10
0.08
0.06
0.04
0.02
0.00
3pt/2pt ratio
0
2
4
6
8
10
Sink time
Total
High modes
Low modes
$\langle x \rangle_c = 0.00416(112)$ (this plot, on $a = 0.112(3)$ fm)
$\langle x \rangle_c = 0.00356(157)$ (on $a = 0.084(2)$ fm)

Glue Spin and Helicity ΔG

- Jaffe and Manohar -- spin sum rule on light cone

$$S_g = \int d^3x\, \vec{E}\times\vec{A} \text{ in light-cone gauge } (A^+ = 0) \text{ and IMF frame.}$$

  - Not gauge invariant
  - Light cone not accessible on the Euclidean lattice.

- Manohar – gauge invariant light-cone distribution

$$\Delta g(x)\, S^+ = \frac{i}{2xP^+}\int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \langle PS \,|\, F_a^{+\alpha}(\xi^-) L^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0) \,|\, PS\rangle$$

  - After integration of x, the glue helicity operator is

$$\mathrm{H}_g(0) = \vec{E}^a(0)\times\left(\vec{A}^a(0) - \frac{1}{\nabla^+}\left(\vec{\nabla}A^{+,b}\right)L^{ba}(\xi^-,0)\right)$$

  - Non-local and on light cone

Glue Spin and Helicity ΔG

- X.S. Chen, T. Goldman, F. Wang; Wakamatsu; Hatta, etc.

Gauge invariant decomposition $\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$

$$S_g = \int d^3x \,\mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys}), \quad A^\mu = A^\mu_{phys} + A^\mu_{pure}, \quad F^{\mu\nu}_{pure} = 0;$$

$$A^\mu_{phys} \to g^\dagger A^\mu_{phys} g, \quad A^\mu_{pure} \to g^\dagger A^\mu_{pure} g - \frac{i}{g} g^\dagger \partial^\mu g$$

$$D^i A^i_{phys} = \partial^i A^i_{phys} - ig\,[A^i, A^i_{phys}] = 0$$

  - Gauge invariant but frame dependent

- X. Ji, J.H. Zhang, Y. Zhao; Y. Hatta, X. Ji, Y. Zhao

Infinite momentum frame

$$\vec{E}^a(0) \times \vec{A}^a_{phys} \xrightarrow[p_z \to \infty]{} \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+} \left(\vec{\nabla} A^{+,b} \right) L^{ba}(\xi^-, 0) \right)$$

Glue Spin and Helicity ΔG

- Large momentum limit

$$S_g = \frac{\langle PS | \int d^3x \ \mathrm{Tr}\,(\vec{E} \times \vec{A}_{phys})_z \,| PS \rangle}{2E_P} \xrightarrow[P_Z \to \infty]{} \Delta G$$

  - Calculate S_g at finite P_z
  - Match to MS-bar scheme at 2 GeV
  - Large momentum effective theory to match to IMF
  - Similar proof for the quark and glue orbital angular momenta which are related to form factors in generalized TMD (GTMD)

  (Y. Zhao, KFL, and Y. Yang, arXiv:1506.08832)

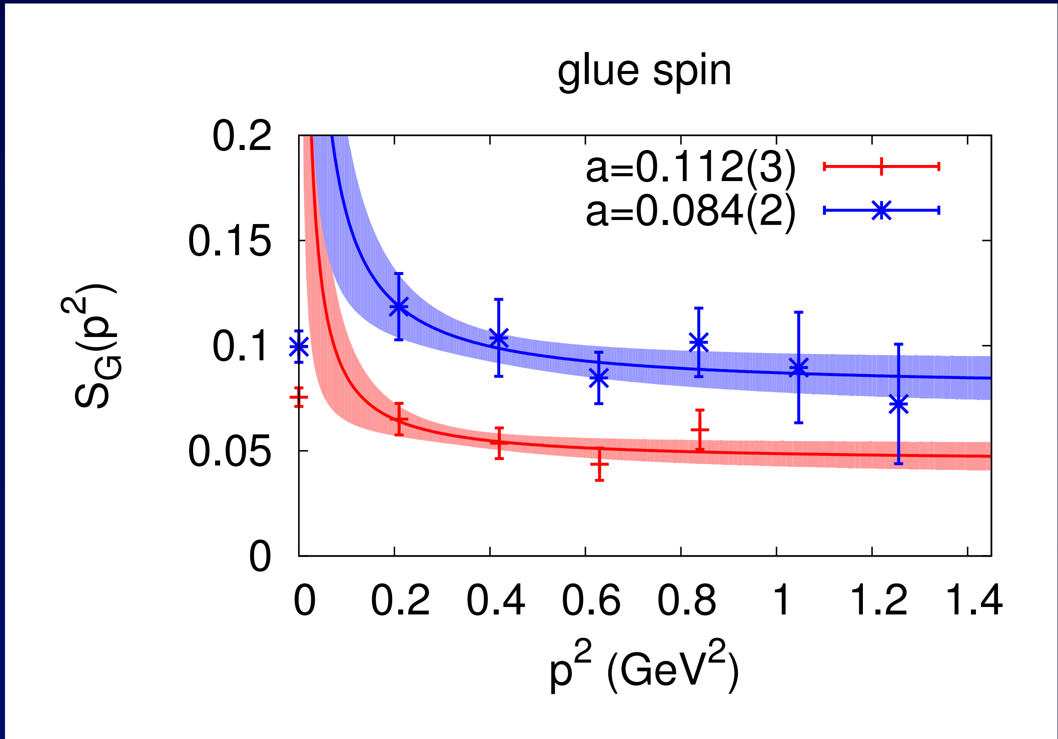
- Solution of A_{phys} -- related to A in Coulomb gauge

$$U^\mu(x) = g_c(x) U_c^\mu(x) g_c^{-1}(x + a\hat{\mu}),$$

$$U_{pure}^\mu(x) \equiv g_c(x) g_c^{-1}(x + a\hat{\mu}),$$

$$A_{phys}^\mu(x) \equiv \frac{i}{a g_0}\left(U^\mu(x) - U_{pure}^\mu(x)\right) = g_c(x) A_c(x) g_c^{-1}(x) + O(a).$$

S_G in Coulomb gauge at p^2 = 0 to 1.24 GeV^2 on the 24^3 x 64 and 32^3 x 64 lattices



$$a+\frac{b}{P^2}$$

Y. Yang (χQCD), Preliminary

S_G = 0.13(3)

$$Tr(\vec{E}\times\vec{A}_{phys}) = Tr(\vec{E}\times g_C^{-1}\vec{A}_C g_C) = Tr(\vec{E}_C\times\vec{A}_C)$$

Summary and Challenges

- Decomposition of proton spin and hadron masses into quark and glue components on the lattice is becoming feasible. Large momentum frame for the proton to calculate glue helicity remians a challenge.

- `Proton Spin Crisis' is likely to be the second example of observable U(1) anomaly.

- Continuum limit at physical pion mass and large lattice volume (5.5 fm) with chiral fermions are being carried out.