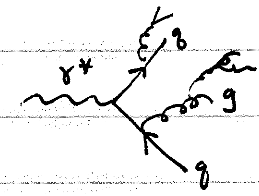
14 OCT 2015 C. Lee, LANL

SCET$_{++}$: A new soft-collinear mode for small-R jets

1509.04287 w/ Y.-T. Chien & A. Hornig

see also 1001.0014 S. Ellis, AH, CL, C. Vermilion, J. Walsh

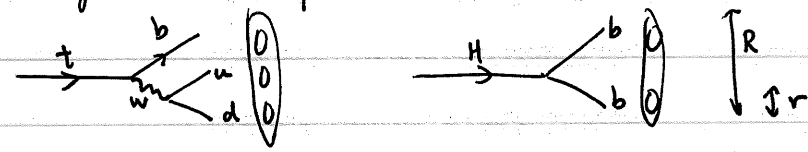
1309.3560 A. von Manteuffel, R. Schabinger, H.X. Zhu



Jets are a feature of QCD and played a key role in establishing the existence of the gluon & the precise predictions of QCD.

Now they are part of signatures of decays of heavy SM & BSM particles like top, Higgs, $\tilde{q}$, $\tilde{g}$, etc.

Use of jet algorithms and "substructure" observables to squeeze info out of jets has exploded in recent years.



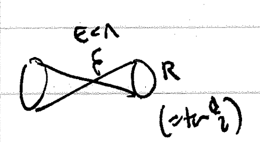
Precise theory predictions of dependence of jet cross sections on jet algorithms / subjet observables a huge challenge.

Today: $e^+e^- \to 2$ jets w/ "thrust cone" algorithm

Predict dependence on small R.

logs of R resummed so far only to LL.

(mostly conceptual, $O(\alpha_s)$, skip over NGLs, alg-dependence, etc.)

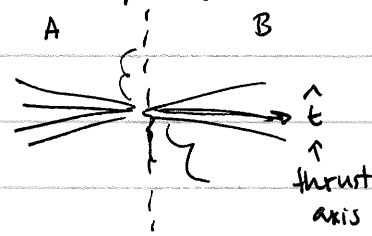


e.g. $\frac{1}{\sigma_0}\sigma_{\text{cone}}(\Lambda,R) = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln\frac{2\Lambda}{Q}\ln R - 3\ln R - \frac{1}{2} + 3\ln 2\right)$

(k_T/S-W: $-\frac{\pi^2}{3} + \frac{5}{2}$)

Compare:

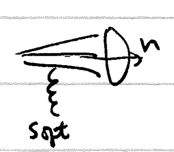
Jet event shapes / mass distributions resummed to high accuracy (N^3LL)



$$m^2 = m_A^2 + m_B^2 = \Big(\sum_{i\in A} p_i\Big)^2 + \Big(\sum_{i\in B} p_i\Big)^2 \equiv Q^2\tau$$ "hemisphere mass"

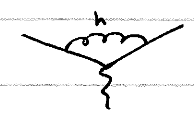
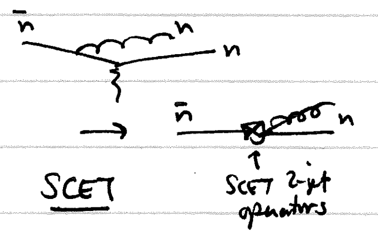
a jet of mass m^2 has two types of degrees of freedom:

light-cone momenta: $p = (\bar{n}\cdot p,\ n\cdot p,\ p_\perp)$ $\quad p^2 = p^+p^- - \vec{p}_\perp^{\,2}$

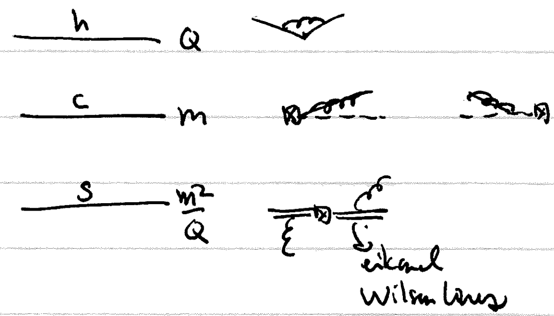


collinear: $\left(Q, \frac{m^2}{Q}, m\right) = Q(1, \lambda^2, \lambda)$ $\quad \lambda \equiv \frac{m}{Q}$

soft: $\left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right) = Q(\lambda^2, \lambda^2, \lambda^2)$

EFT: integrate out hard modes:  or 

match onto theory of collinear & soft modes: <u>SCET</u>



In full QCD, $\sigma(\tau) \equiv \int_0^{Q^2\tau} \frac{1}{\sigma_0}\frac{d\sigma}{dm^{2\prime}}\, dm^{2\prime} = 1 - \frac{\alpha_s(Q)}{2\pi} C_F \left[2\ln^2\tau + 3\ln\tau + 1 - \frac{\pi^2}{3}\right] + \mathcal{O}(\tau) + \mathcal{O}(\alpha_s^2)$

In SCET: $\sigma(\tau) = H(Q^2,\mu)\, J_n(Q\sqrt{\tau},\mu) \otimes J_{\bar{n}}(Q\sqrt{\tau},\mu) \otimes S(Q\tau,\mu)$

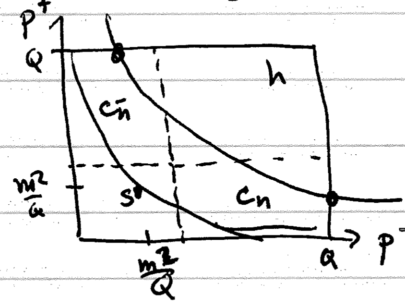
$$H = 1 + \frac{\alpha_s}{4\pi}\left(-8\ln^2\frac{\mu}{Q} - 12\ln\frac{\mu}{Q} + \frac{7\pi^2}{3} - 16\right)C_F \equiv 1 + \frac{\alpha_s}{4\pi}\left(\Gamma_H^0 \ln^2\frac{\mu}{Q} + \gamma_H^0 \ln\frac{\mu}{Q} + c_H^1\right)$$

$$J = 1 + \frac{\alpha_s}{4\pi}C_F\left(8\ln^2\frac{\mu}{Q\sqrt{\tau}} + 6\ln\frac{\mu}{Q\sqrt{\tau}} + 7 - \pi^2\right) = 1 + \frac{\alpha_s}{4\pi}\left(\Gamma_J^0\ln^2\frac{\mu}{Q\sqrt{\tau}} + \gamma_J^0\ln\frac{\mu}{Q\sqrt{\tau}} + c_J^1\right)$$
($\equiv J_{\text{incl}}$)

$$S = 1 + \frac{\alpha_s}{4\pi}C_F\left(-8\ln^2\frac{\mu}{Q\tau} + \frac{\pi^2}{3}\right) = 1 + \frac{\alpha_s}{4\pi}\left(\Gamma_S^0\ln^2\frac{\mu}{Q\tau} + \gamma_S^0\ln\frac{\mu}{Q\tau} + c_S^1\right)$$

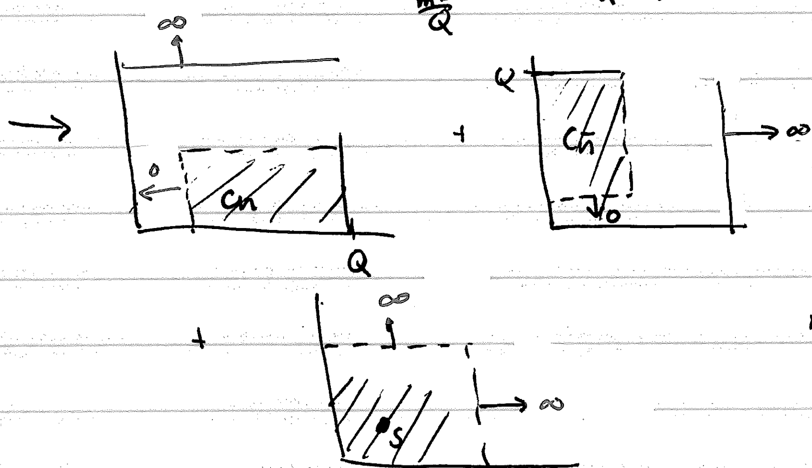
$$\frac{7\pi^2}{3} - 16 + 14 - 2\pi^2 + \frac{\pi^2}{3} = -2 + \frac{2\pi^2}{3} \quad \checkmark$$

Momentum regions:



could use cutoffs

or pure dim reg.



(− "0-bin" subtractions)

regulate in dim.reg.

$\downarrow$

introduce μ

(replaces cutoffs

i.e. mode has to have a scaling that can "see" a boundary

otherwise $\begin{cases} \infty \\ \text{or} \\ 0 \end{cases}$.

Resummation comes from solving RGE's:

$$\mu \frac{d}{d\mu} H = \gamma_H H$$

$$\mu \frac{d}{d\mu} \tilde{J} = \gamma_{\tilde{J}} \tilde{J} \qquad (\tilde{F} = \text{Laplace or Fourier transform})$$

$$\mu \frac{d}{d\mu} \tilde{S} = \gamma_{\tilde{S}} \tilde{S}$$

$$\Rightarrow \quad F(\mu) = F(\mu_0) \exp \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_F(\mu')$$

$F(\mu_0)$: eval where $\ln \frac{\mu_0}{Q_F} \approx 0$

$\exp \int$: resums logs to all orders in α_s

note $\quad \gamma_H + 2\gamma_{\tilde{J}} + \gamma_{\tilde{S}} = 0.$

$(\sigma = \mu \text{ indep.})$

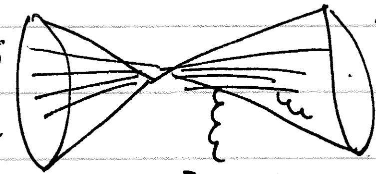
$$\gamma_F[\mu] = \kappa_F \Gamma_{\text{cusp}}[\alpha] \ln \frac{\mu}{Q_F} + \gamma_F[\alpha_s]$$

Γ_{cusp}: "cusp anom dim" $\qquad$ $\gamma_F[\alpha_s]$: "non-cusp"

$\Gamma_{\text{cusp}} \rightarrow$ known to $\mathcal{O}(\alpha_s^3)$ for $\Theta = \pi$

Consider measurement of "jet thrust" (or mass)

τ, or m_J^2.



R $\left(= \tan\frac{\theta}{2}\right)$

$E_{out} < \Lambda$

full QCD $\Rightarrow$ $$\sigma_{cone}(\tau,\Lambda,R) = \theta(R^2-\tau)\left\{1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2\tau - 3\ln\tau - 8\ln R \ln\frac{2\Lambda R}{Q\tau} - 1 + 6\ln\frac{\tau+R^2}{R^2}\right)\right\}$$

$$\left(+\theta(\tau-R^2)\left\{1 + \frac{\alpha_s C_F}{2\pi}\left(-8\ln R\ln\frac{2\Lambda}{Q} - 4\ln R + 4\ln 2 - 1\right)\right\}\right)$$

what are the relevant scales now?

natural to guess:

Q

$Q\sqrt{\tau}$

$Q\tau$ $\longrightarrow$ S_{in} $Q\tau/R$, $p \sim Q\tau\left(\frac{1}{R^2}, 1, \frac{1}{R}\right)$

Λ $\longrightarrow$? S_{out} $2\Lambda\sqrt{R}$?

compute: $\sigma = H(Q^2,\mu)\, J^{cone}(Q\sqrt{\tau},R)^2\, S_{in}^2(Q\tau)\, S_{out}(\Lambda,R)$

find: H $\downarrow$ same

1001.0014

$$J^{cone} = J_{incl} + \Delta J_{cone} = J^{cone}\left(\frac{Q\sqrt{\tau}}{\mu}, R\right)$$

$$\Delta J_{cone} = \frac{\alpha_s C_F}{4\pi}\,\theta(R^2-\tau)\,\frac{6}{(\tau+R^2)}$$

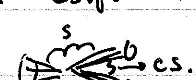
$$S_{in} = 1 + \frac{\alpha_s}{4\pi}\left(-4C_F\ln^2\frac{\mu R}{Q\tau} + \frac{\pi^2}{6}C_F\right) = S_{in}\left(\frac{\mu}{Q\tau/R}\right)$$

$$S_{out} = 1 + \frac{\alpha_s}{4\pi}\left(8C_F\ln R\ln\frac{\mu^2}{4\Lambda^2 R} - \frac{2\pi^2}{3}C_F\right)$$

$\downarrow$

$\mu_{out} = 2\Lambda\sqrt{R}$?

(btw theory? h, hc, $Q\tau$ soft $\longrightarrow$ $Q\tau/R$ "csoft" 1106.6047, $Q\tau$ } "SCET$_+$"



puzzles: • $\mu_{out} = 2\Lambda\sqrt{R}$ does not get rid of large logs in S_{out} at $\mathcal{O}(\alpha_s^2)$

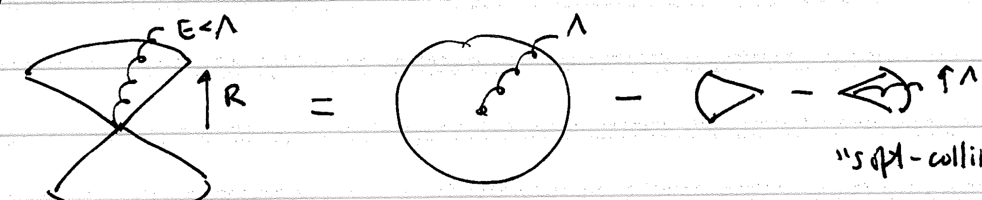
• the $\ln^2 R$ term looks like it should be $\propto \Gamma_{cusp}$, confirmed to $\mathcal{O}(\alpha_s^2)$ [1309.3560] ← computed $S(\tau,\Lambda,R)$, by brute force, no factorization of scales.

• multiple conjectures in lit:

$$\gamma_{out} = \underset{\uparrow}{4\Gamma_{cusp}\ln R} + \underset{\substack{\downarrow \\ 0 \text{ at } \alpha_s}}{\gamma_{out}^{n.c.}}$$

Solution: there is a hidden scale!

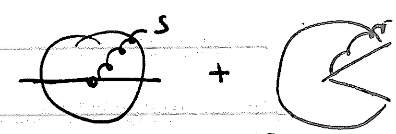
"out" region:



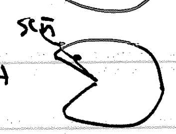
"soft-collinear"

$\text{sc}n \sim 2\Lambda(1, R^2, R)$

or =



soft $(\Lambda,\Lambda,\Lambda)$ cannot see cones of angle R



$\sim 2\Lambda(R^2, 1, R)$

sc_n can see n cone

$\text{sc}_{\bar n}$ can see $\bar n$ cone.

(− sign because $\text{sc} = 0$ in DR)

"SCET$_{++}$"

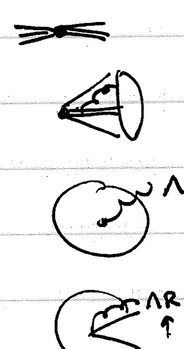
h ——— Q

hc ——— $Q\sqrt{\tau}$

cs ——— $\frac{Q\tau}{R}$

s ——— 2Λ

sc ——— $2\Lambda R$



$S_{out} = S_s \otimes S_{sc}^2$

$\mathcal{O}(\alpha_s)$:

$$\ln R \ln \frac{\mu^2}{4\Lambda^2 R} = \ln^2\frac{\mu}{2\Lambda} - \ln^2\frac{\mu}{2\Lambda R}$$

$$S_{ss} = 1 + \frac{\alpha_s}{4\pi}\left(8C_F \ln^2\frac{\mu}{2\Lambda} - \pi^2 C_F\right)$$

$$S_{sc} = 1 + \frac{\alpha_s}{4\pi}\left(-4C_F\ln^2\frac{\mu}{2\Lambda R} + \frac{\pi^2}{6}C_F\right)$$

→ these have standard form

$$\tilde\gamma_{ss} = 4\Gamma_{cusp}\ln\frac{\mu}{2\Lambda} + \gamma_{ss}^{n.c.}$$

$\to \mathcal{O}(\alpha_s^2)$ ↗ can now extract to $\mathcal{O}(\alpha_s^2)$

$$\tilde\gamma_{sc} = -2\Gamma_{cusp}\ln\frac{\mu}{2\Lambda R} + \gamma_{sc}^{n.c.}$$

in paper:

- used $\mathcal{O}(\alpha_s^2)$ $S_{out}(\Lambda, R, \mu)$ in 1309.3560

  to extract $\gamma^{n.c.}_{ss, sc, in}$ to $\mathcal{O}(\alpha_s^2)$

$$\gamma_{ss}' = -2\gamma_{in}' = -2\gamma_{sc}'$$
$$= C_F \left[\left(\frac{1616}{27} - 56\zeta_3\right) C_A - \frac{448}{27} T_F n_f - \frac{2\pi^2}{3}\beta_0\right]$$

$$\gamma_{in} = \gamma_{hemi}$$
$$\gamma_{ss} = -2\gamma_{sc} = \gamma_{DY} \text{ (timelike soft function)}$$

also known to $\mathcal{O}(\alpha_s^2)$.

- showed relation to total 2-jet rate

$$\sigma^{2\text{-jet}}_{cone}(\Lambda, R) = \sigma_{cone}(\tau, \Lambda, R)\Big|_{\tau \to \tau_{max} = R^2}$$

note $\frac{\text{nc}}{\text{cs}}\ \frac{Q\sqrt{\tau}}{\frac{Q\tau}{R}} \longrightarrow \frac{QR}{}$ at $\tau = R^2$

$$\Rightarrow \quad \sigma_{2\text{-jet}} = H(Q^2)\, J_{unmeas}(QR, \mu)\, S_s(2\Lambda)\, S_{sc}(2\Lambda R)$$
$$J_{unmeas} \equiv J_{cone}(Q\sqrt{\tau}) \otimes S_{in}\left(\frac{Q\tau}{R}\right)\Big|_{\tau \to R^2}$$

$\Rightarrow$ J_{un} to $\mathcal{O}(\alpha_s^2)$ for first time.

- confirmed $\mathcal{O}(\alpha_s^2)$ $\ln R$ enhanced terms with EERAD3.
- established $\ln^2 R$ terms in c.s. $\propto \Gamma_{cusp}$.

  can predict all pure $\ln^n R \ln^m \tau$ terms to NNLL

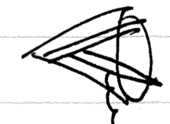
- open questions: – NGLs $\alpha_s^2 \ln^2 \frac{2\Lambda R^2}{Q\tau} = \ln^2\left(\frac{Q\tau}{R} \Big/ 2\Lambda R\right)$ (cs / sc)

  – Soft logs $\alpha_s^2 \ln^2 \frac{2\Lambda}{Q}$ & higher.

  → require new operators for more subjets

cf. MIT group,

Bern group



} no new logs of R though.

s

* all these small-R issues can now be tackled within context of SCET$_{++}$!