

QCD, Effective theories and dark matter

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INT, Seattle

2 October, 2015

based on work with M.P. Solon: (Sakurai thesis award)

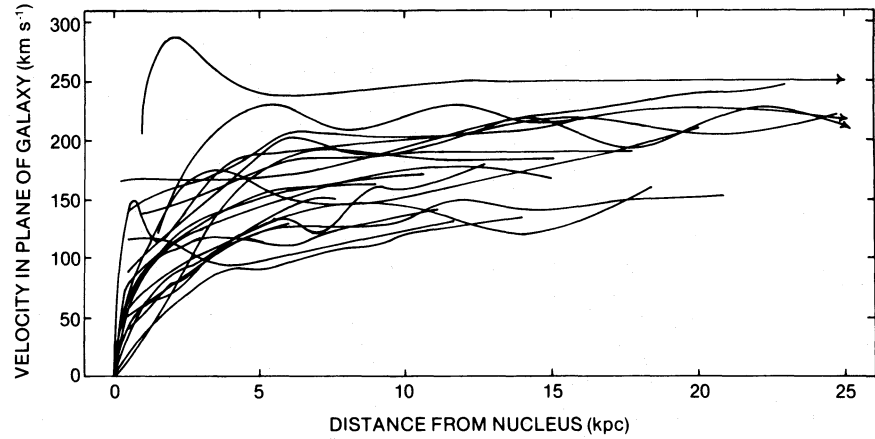
Heavy WIMP Effective Theory [1111.0016](#), [1309.4092](#), PRL

Standard Model Anatomy of WIMP Direct Detection I, II [1401.3339](#), [1409.8290](#), PRD

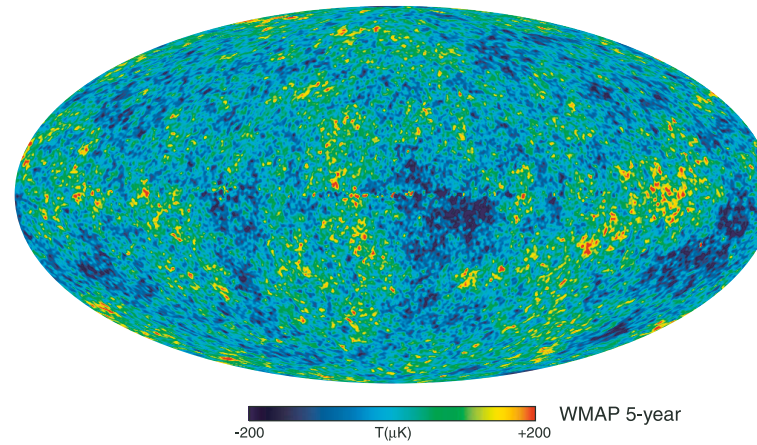
Thanks to co-organizers and participants of MITP program “Effective Theories and Dark Matter”, March 2015: <https://indico.mitp.uni-mainz.de/conferenceDisplay.py?ovw=True&confId=25>

See also INT workshop “Nuclear aspects of DM searches”, December 2014: <http://www.int.washington.edu/PROGRAMS/14-57w/>

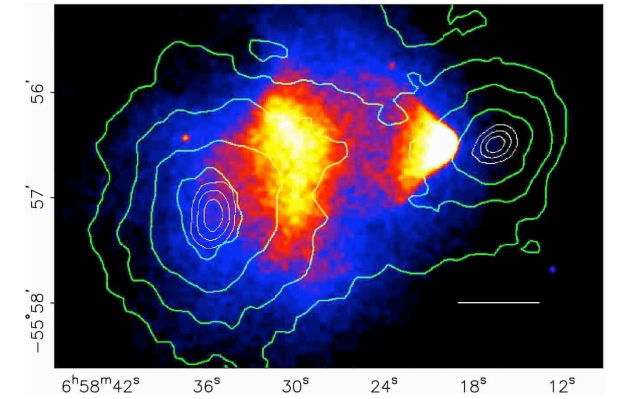
The dark matter problem



modification of galactic rotation curves

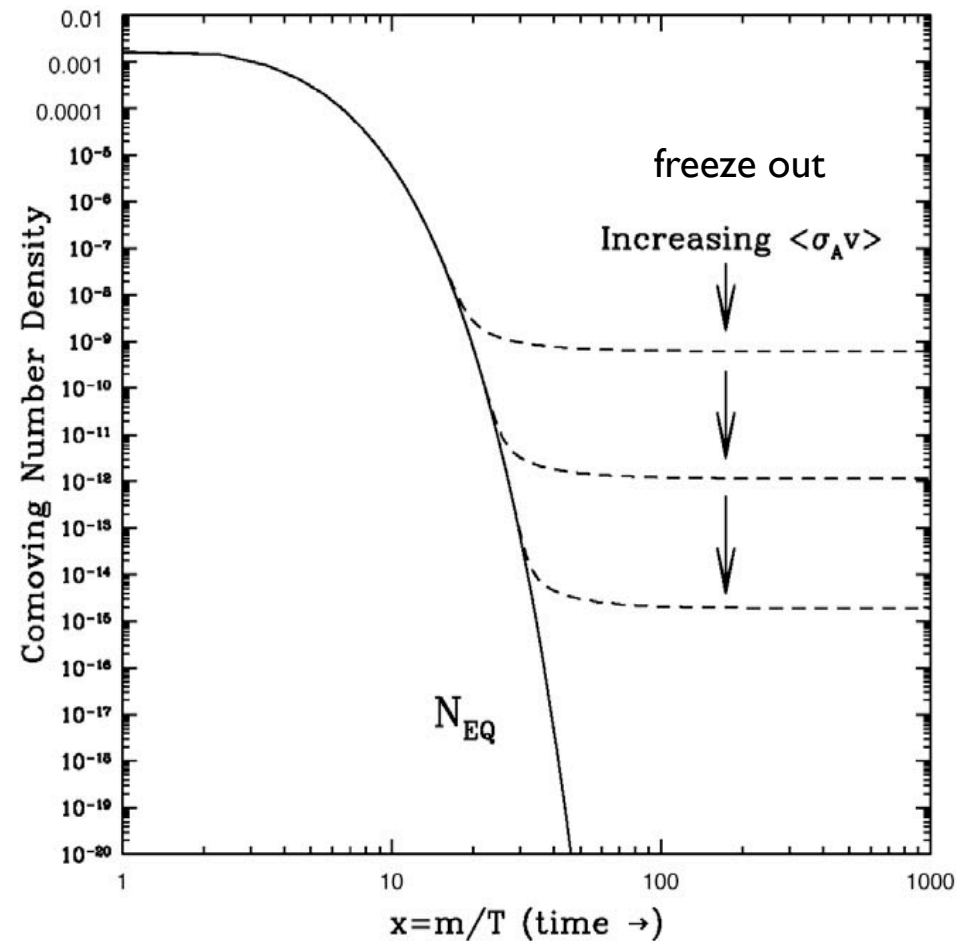


imprints on microwave background

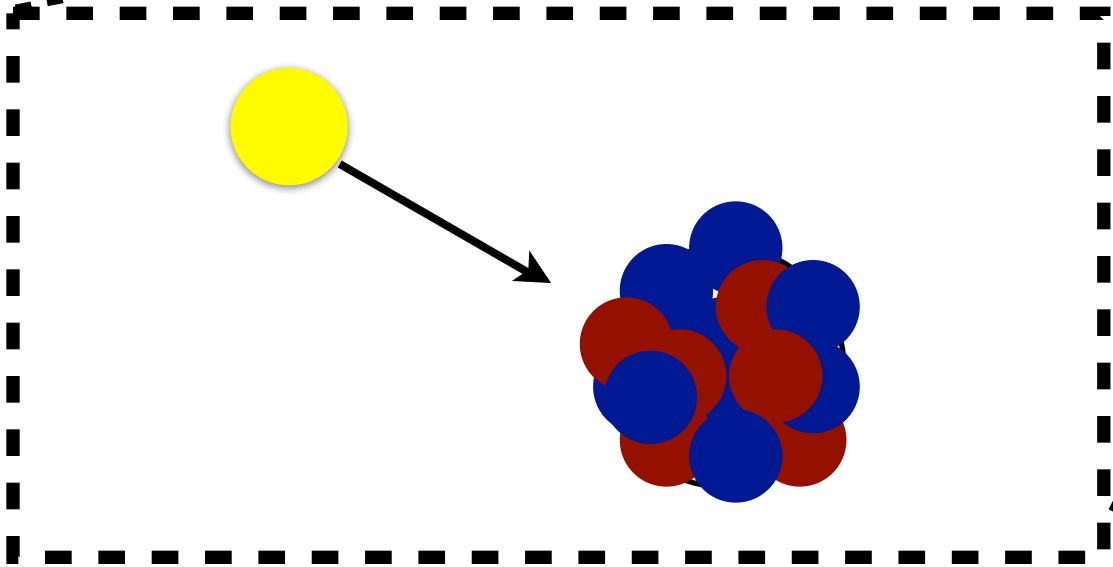
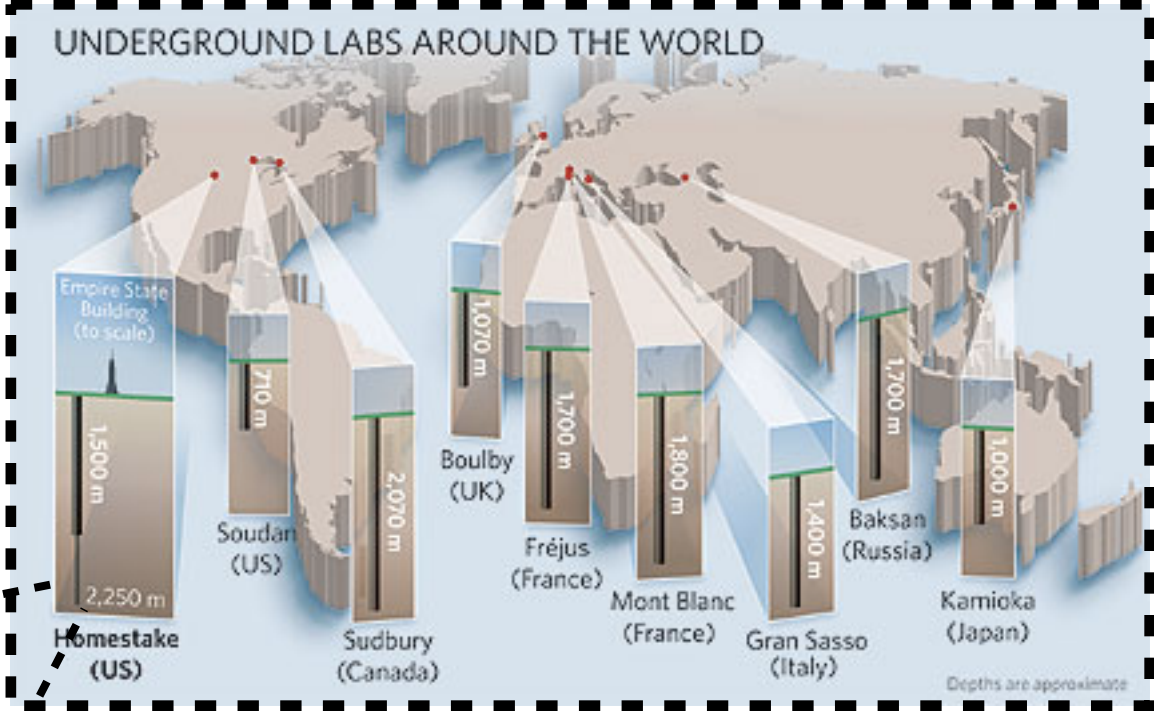
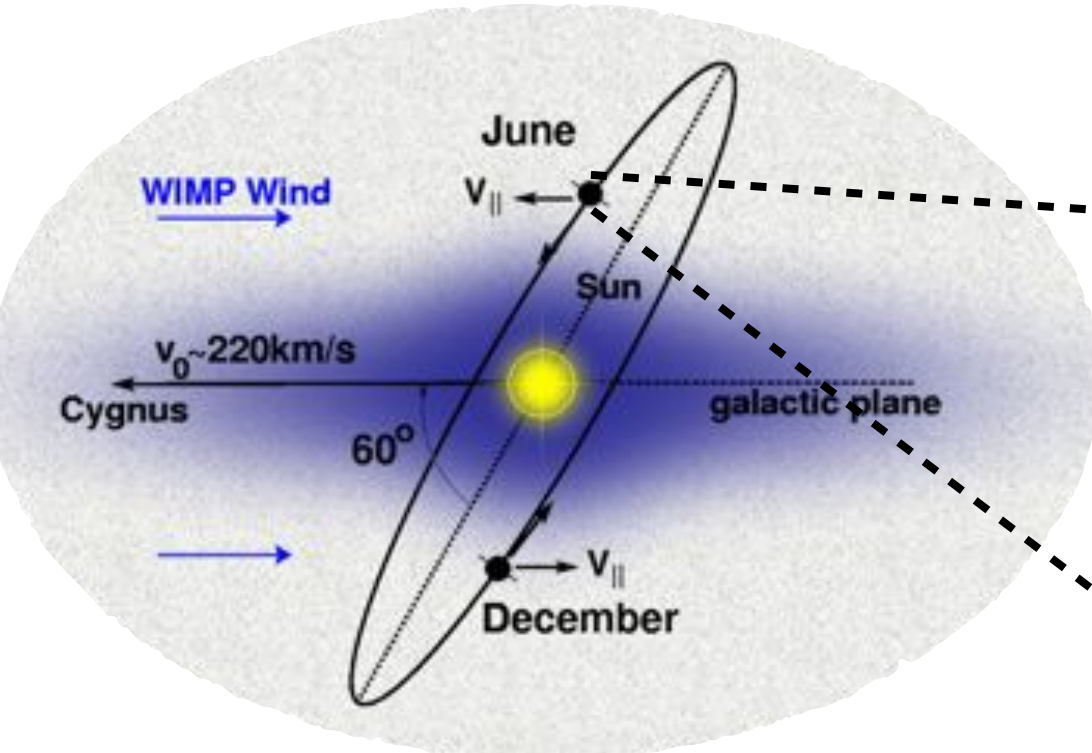


apparent extra collisionless matter from lensing measurements

*perhaps a thermal relic
Weakly Interacting
Massive Particle
(WIMP)*



The dark matter problem



Three motivations for studying QCD & DM

- important, sometimes dramatic, impact on discovery potential
- post-discovery interpretation and/or anomaly debunking
- new field theory tools

Some themes in the contemporary particle physics:

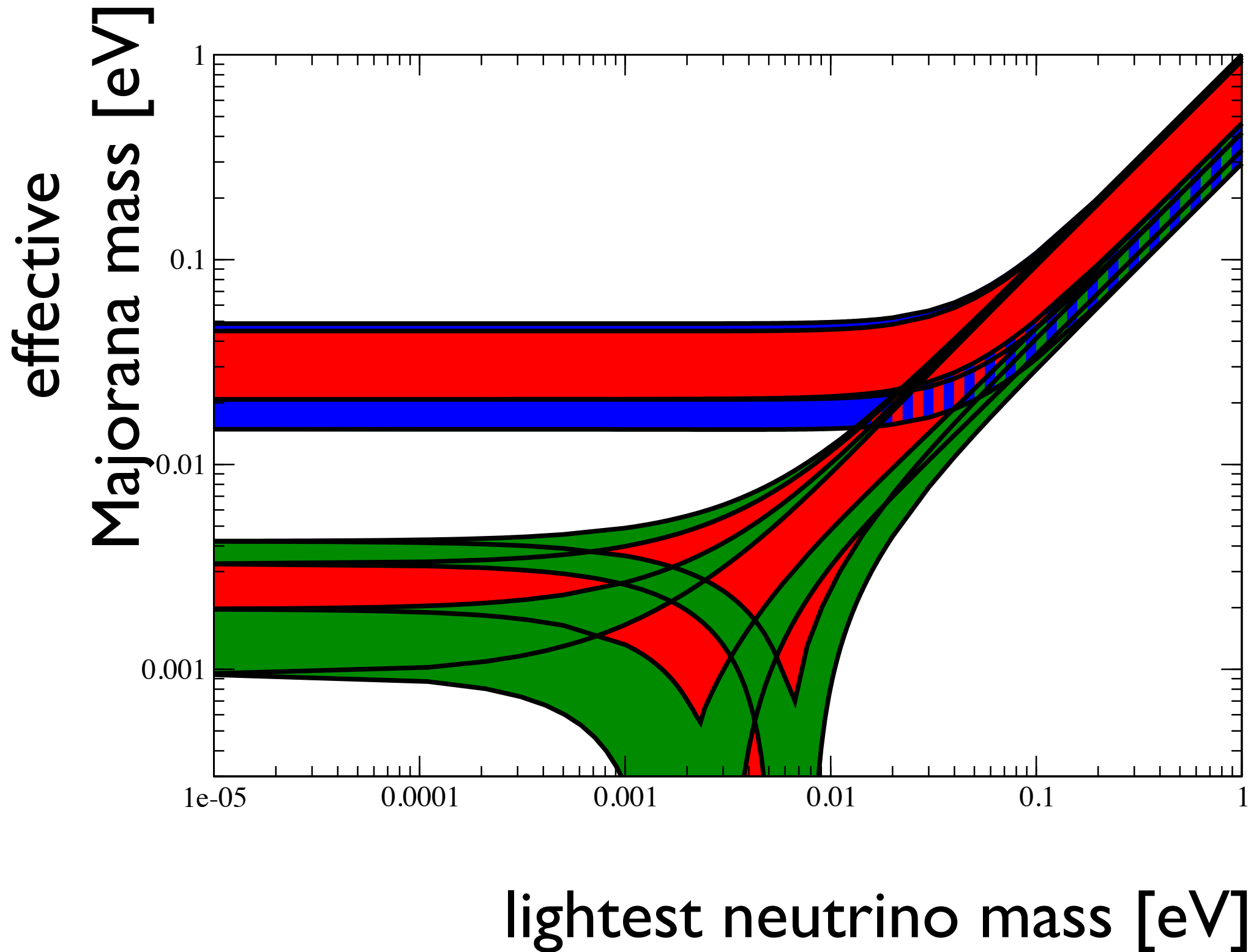
- $\Lambda_{\text{New Physics}} > m_W$ (maybe $\gg m_W$)
- *interplay of particle, astro, nuclear*
- *lattice QCD and baryon matrix elements*
- interplay of radiative corrections and hadronic structure

Compelling physics questions demand analysis outside the comfort zone of any one field.

Dark Matter applications a prime example

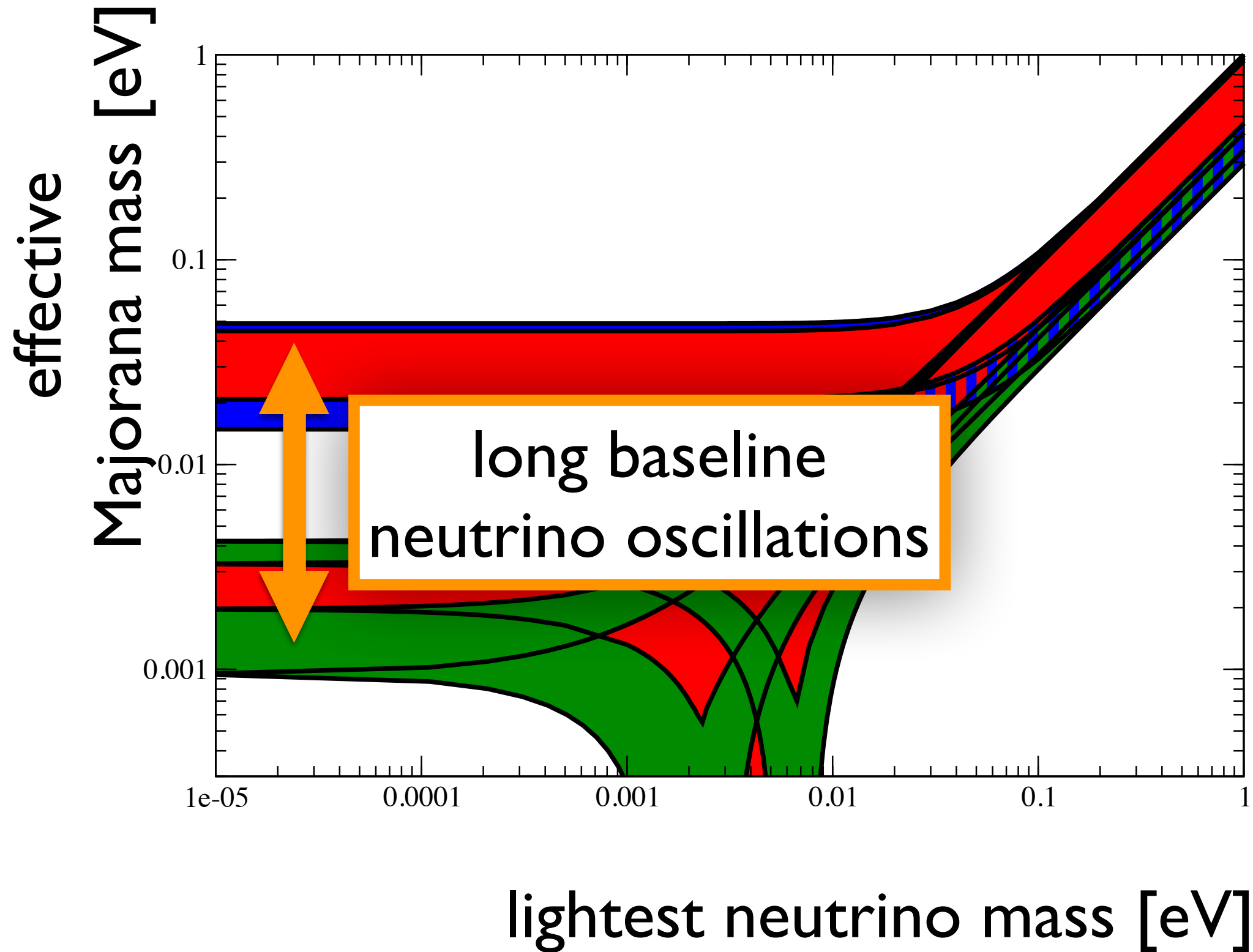


Interplay of particle-, nuclear-, astro-physics/cosmology



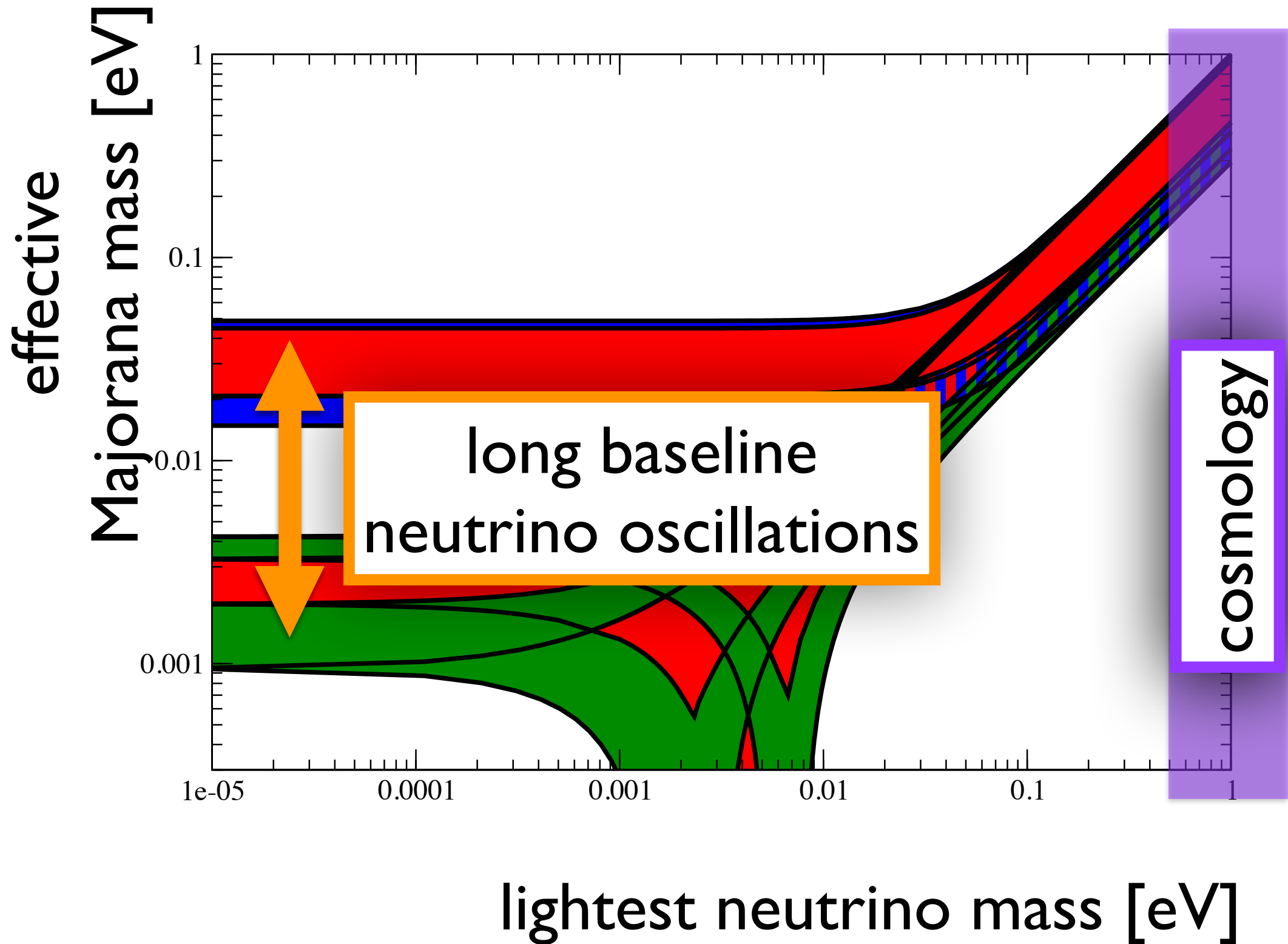
- Observability of lepton number violation depends on presently unknown neutrino mass ordering

Interplay of particle-, nuclear-, astro-physics/cosmology



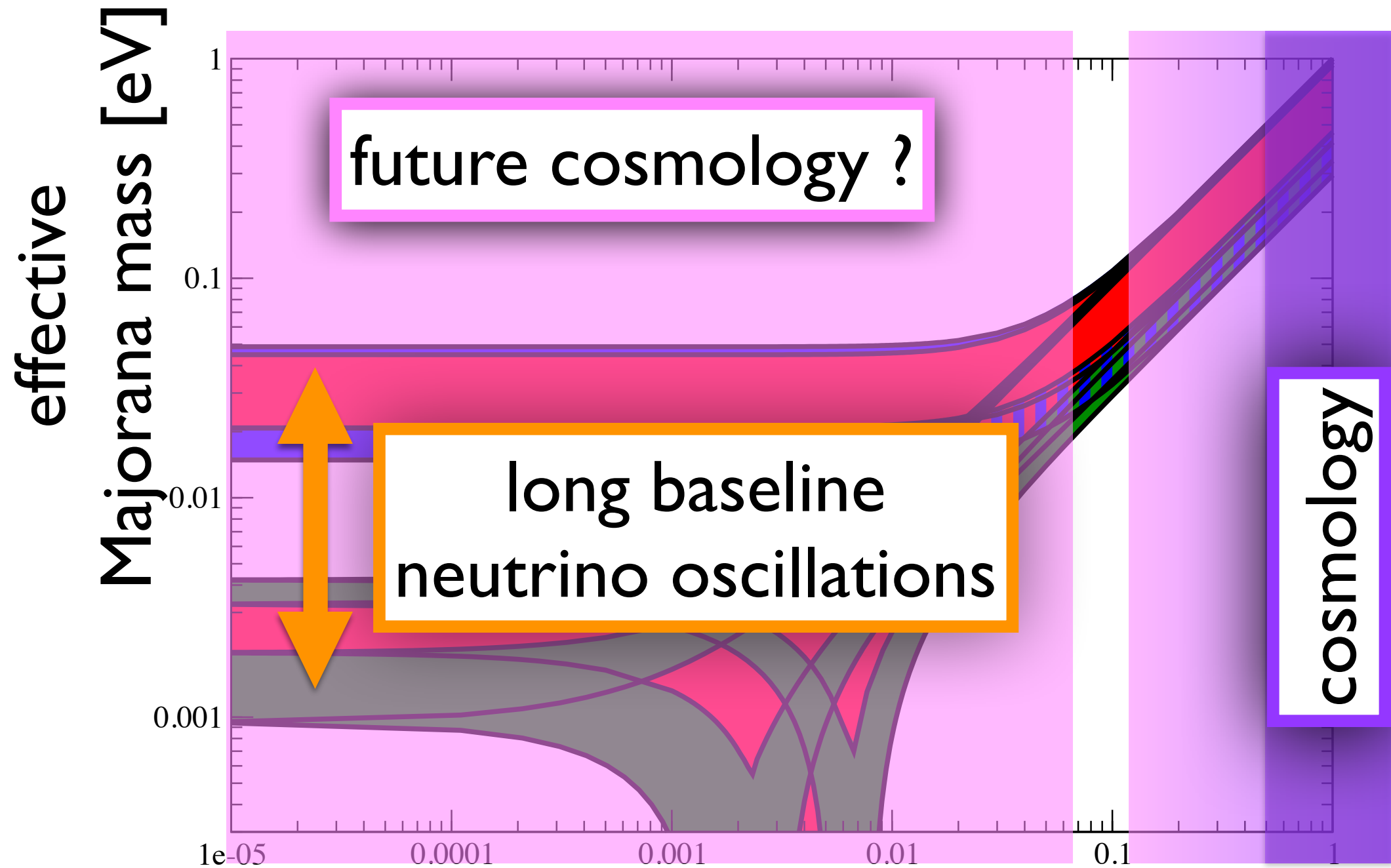
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Interplay of particle-, nuclear-, astro-physics/cosmology



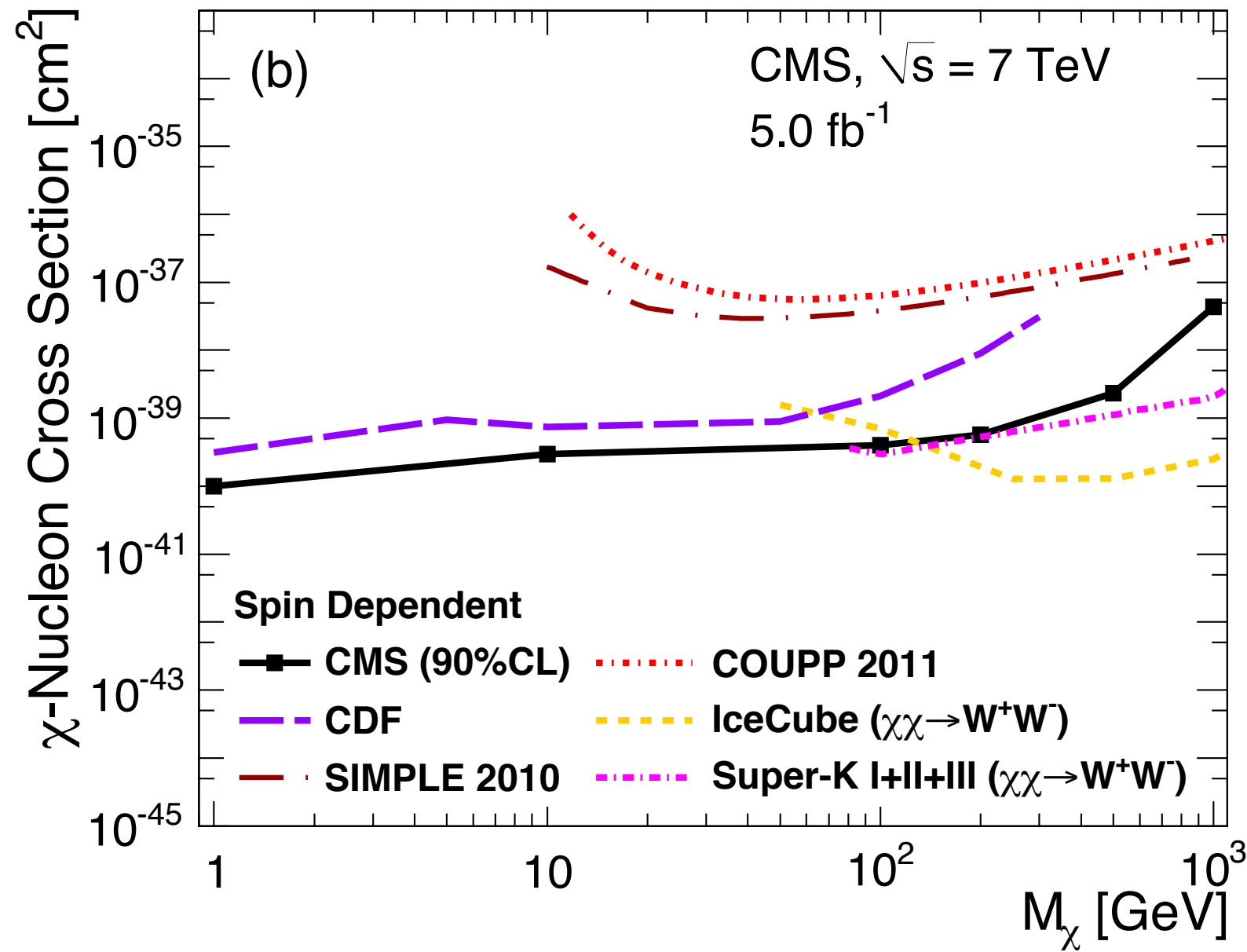
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Interplay of particle-, nuclear-, astro-physics/cosmology



- Observability of lepton number violation depends on presently unknown neutrino mass ordering

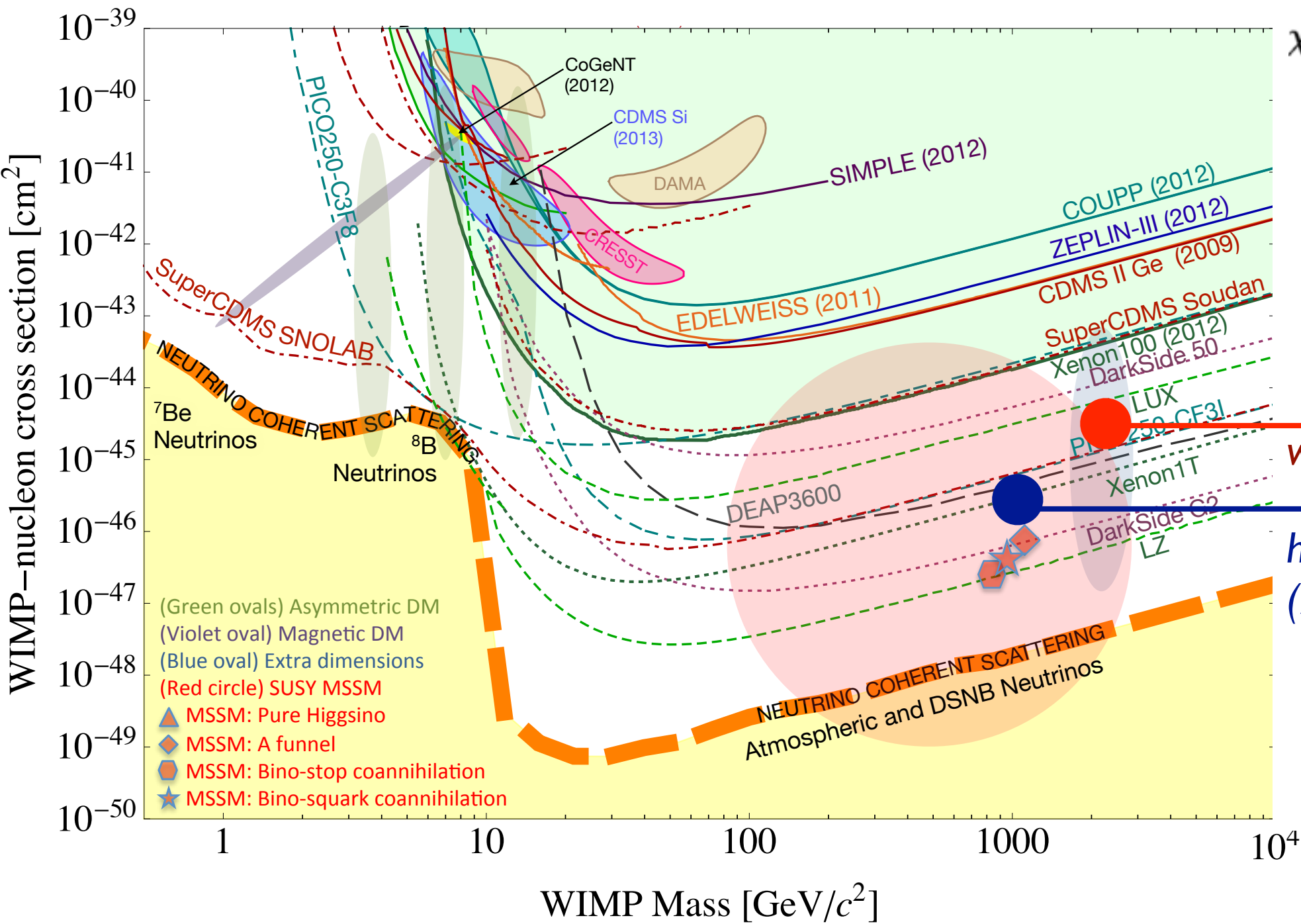
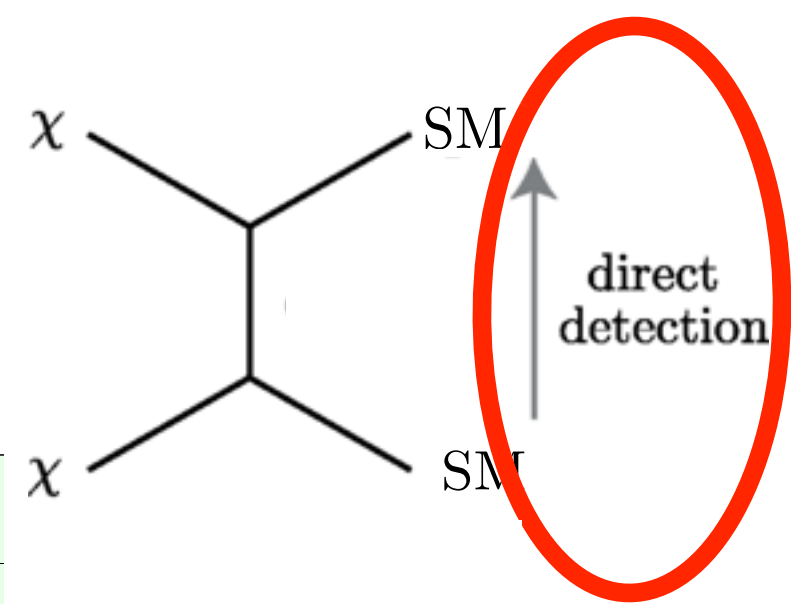
Interplay of particle-, nuclear-, astro-physics/cosmology



Axial-vector quark contact interaction with Dirac fermion WIMP

- Wide range of searches with overlapping constraints

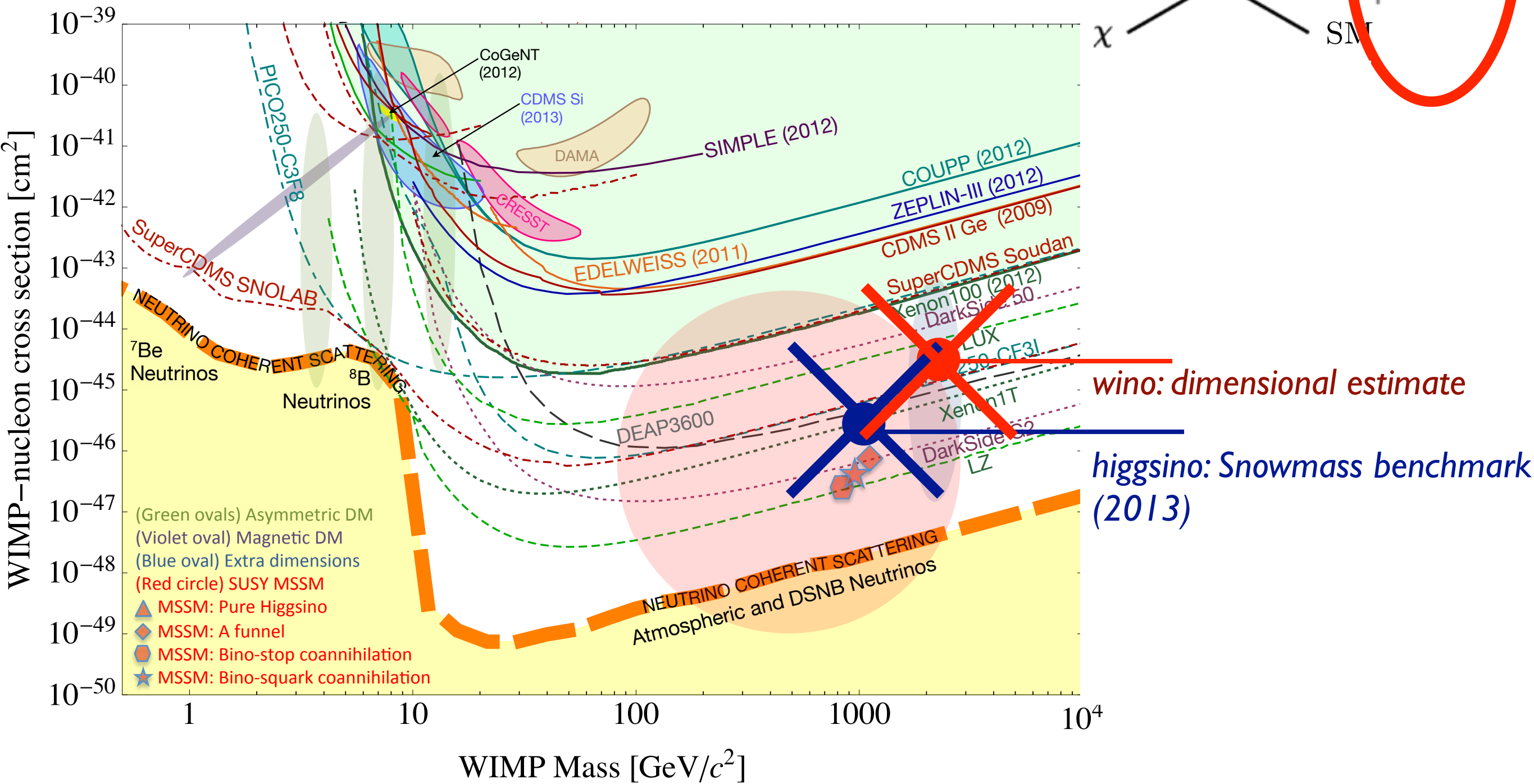
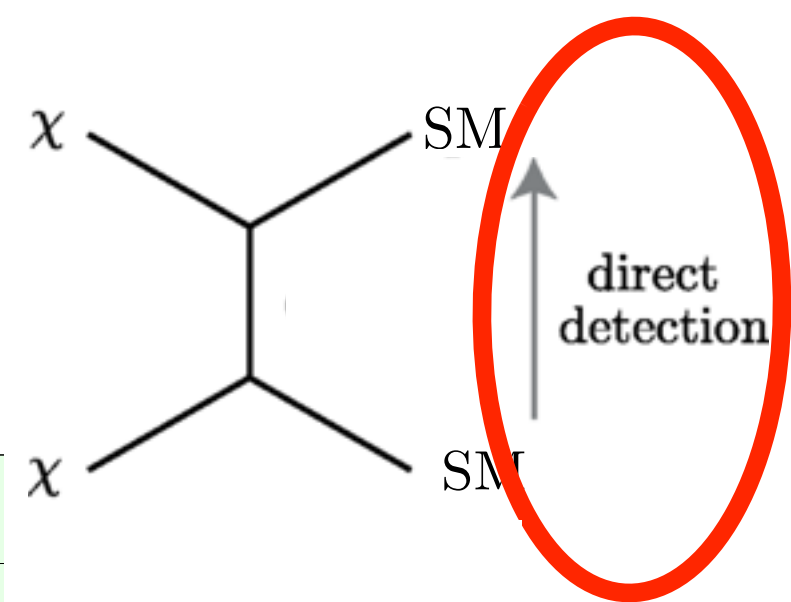
Not quibbling about percents (example I: heavy WIMP scattering)



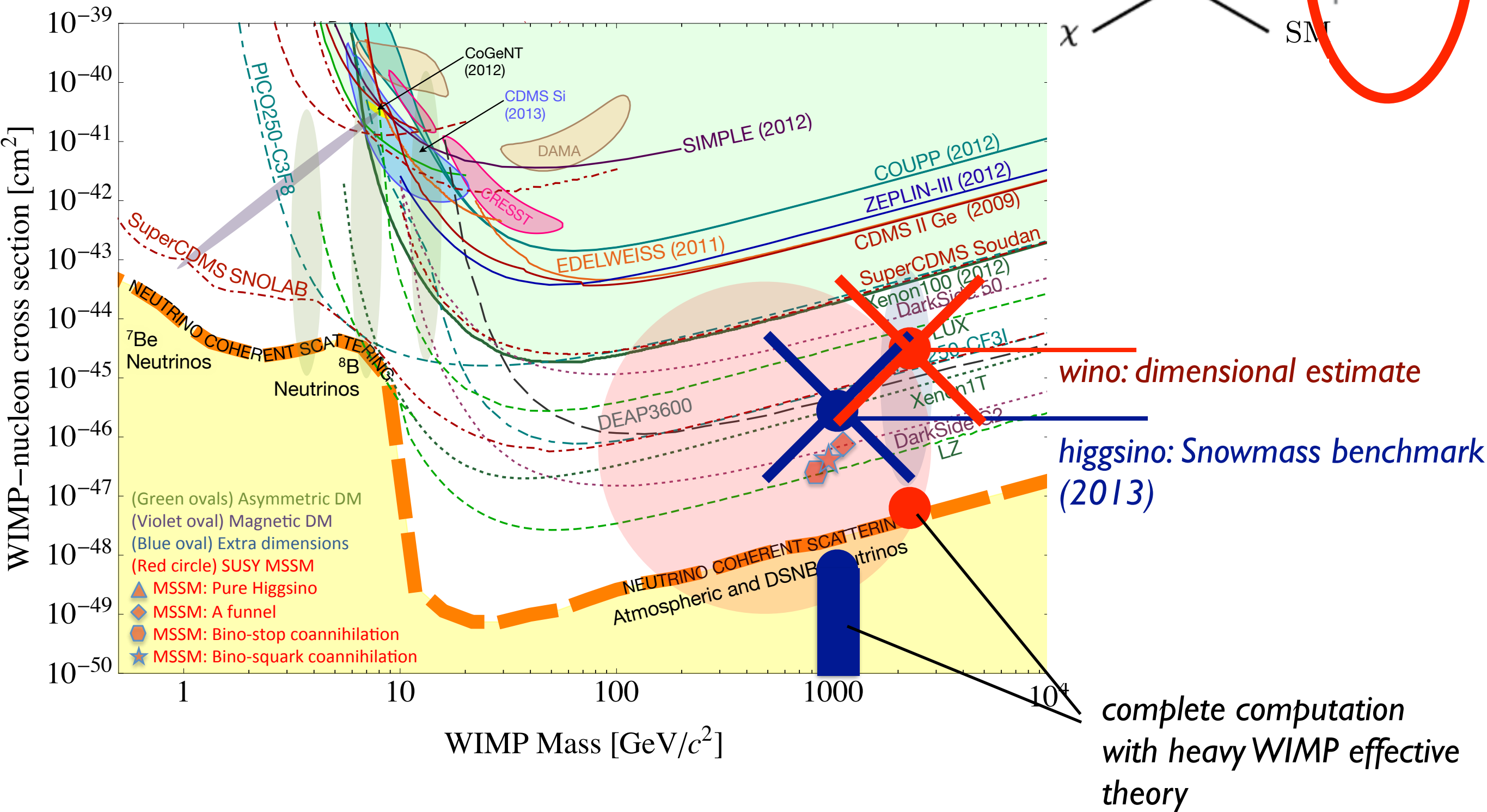
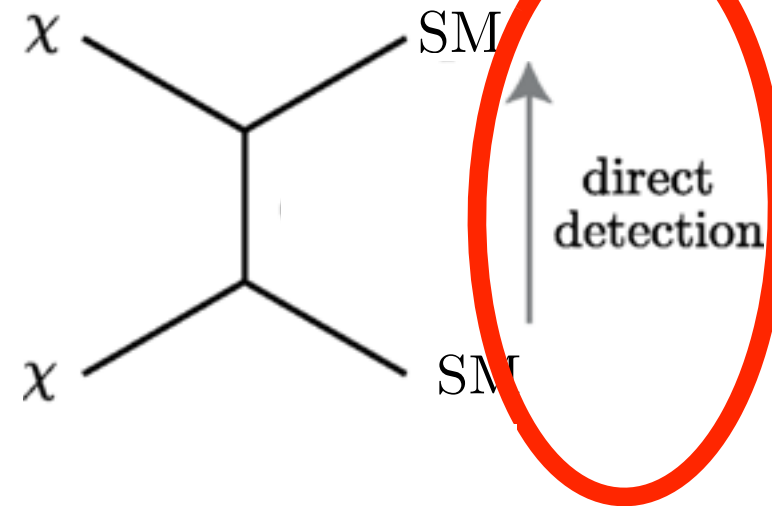
wino: dimensional estimate

higgsino: Snowmass benchmark (2013)

Not quibbling about percents (example I: heavy WIMP scattering)

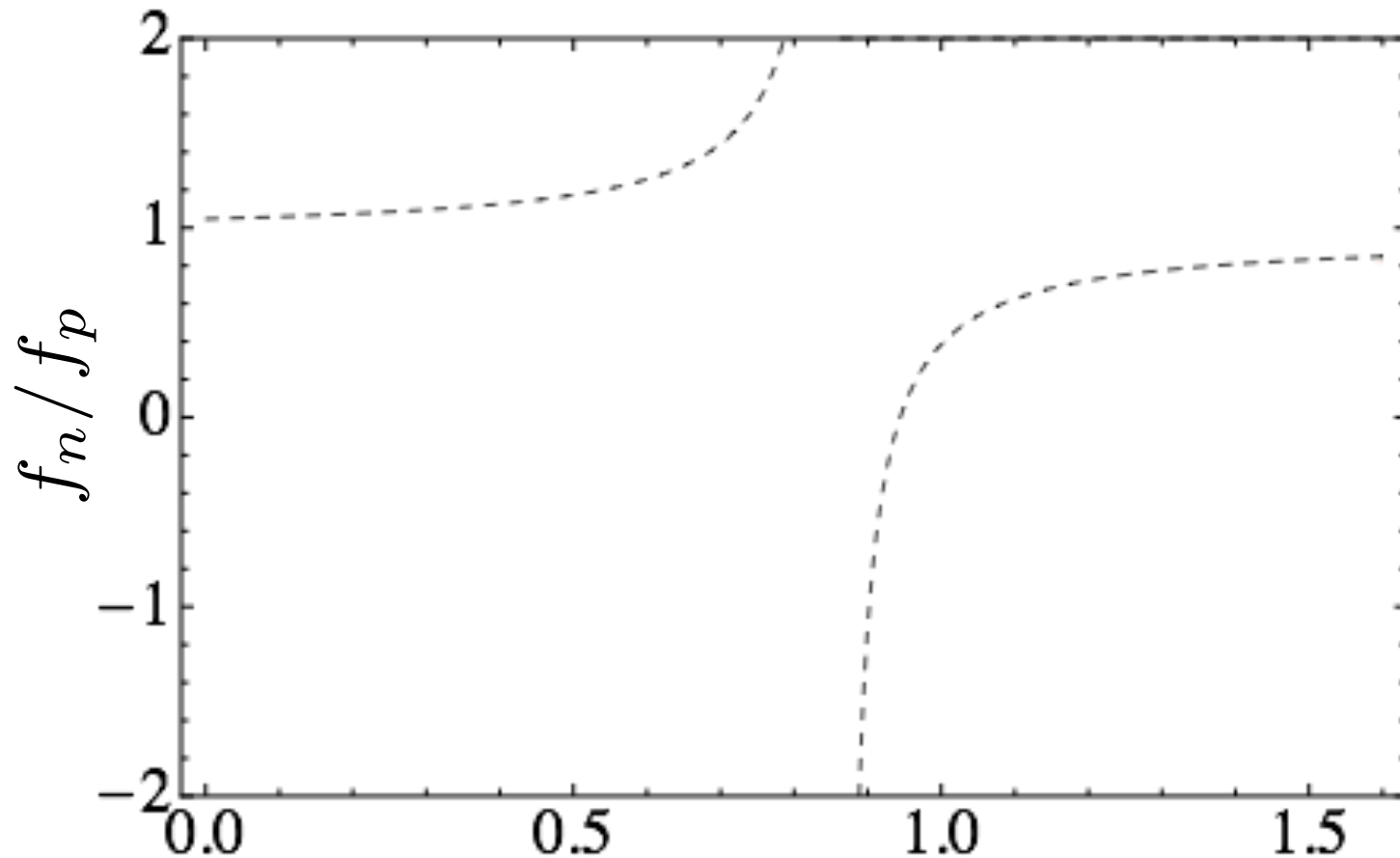
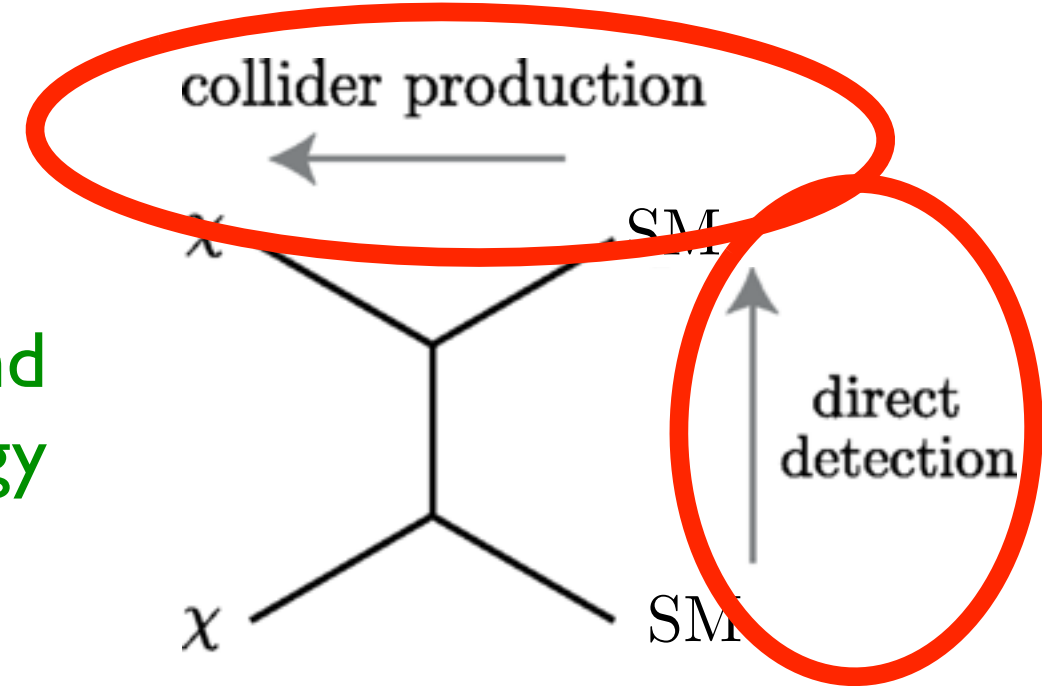


Not quibbling about percents (example I: heavy WIMP scattering)



Not quibbling about percents (example 2: light WIMPs)

DM complementarity: connect direct detection and collider phenomenology



f_n/f_p = ratio of SI nucleon amplitudes for WIMP-nucleon scattering

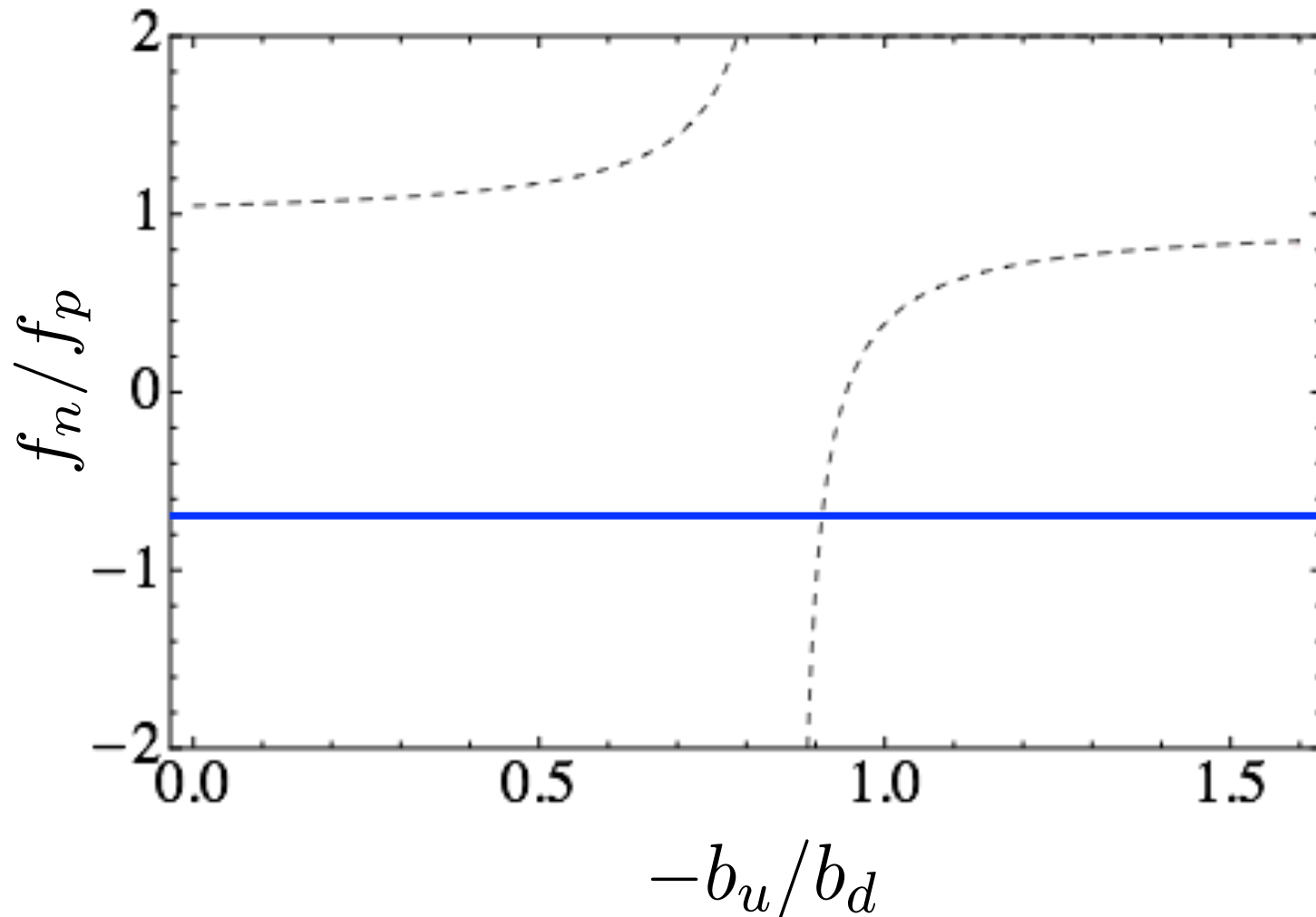
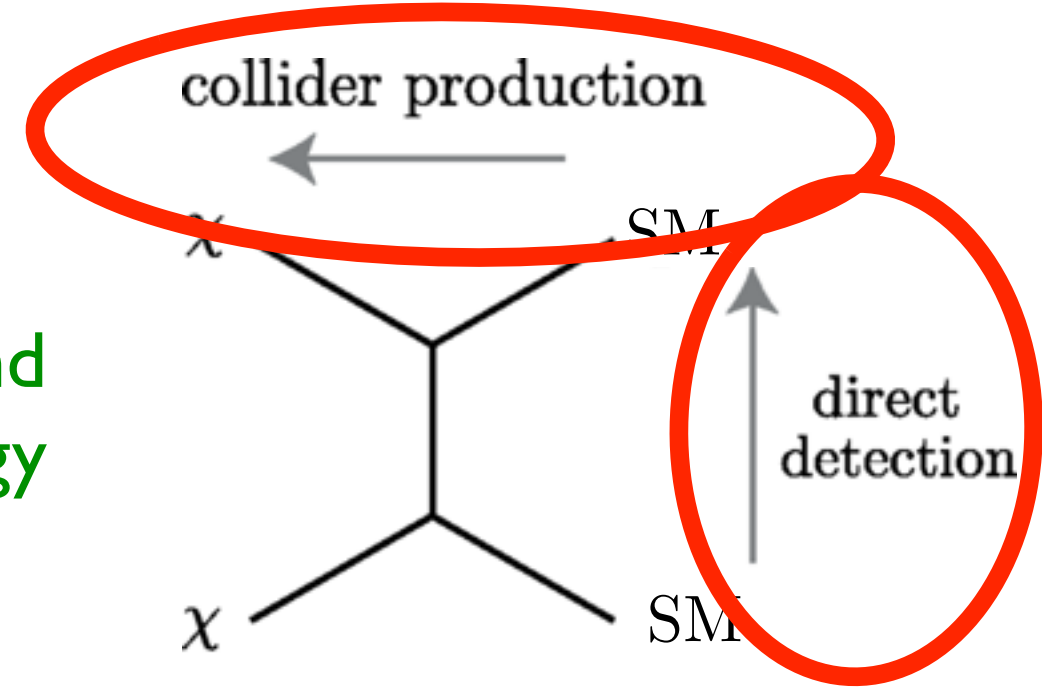
$$-b_u/b_d$$

$$\mathcal{L}_{\chi, \text{SM}} = \bar{\chi}\chi \left[b_u \bar{u}u + b_d \bar{d}d \right]$$

four-fermion interactions constrained by collider bounds on missing energy signatures

Not quibbling about percents (example 2: light WIMPs)

DM complementarity: connect direct detection and collider phenomenology



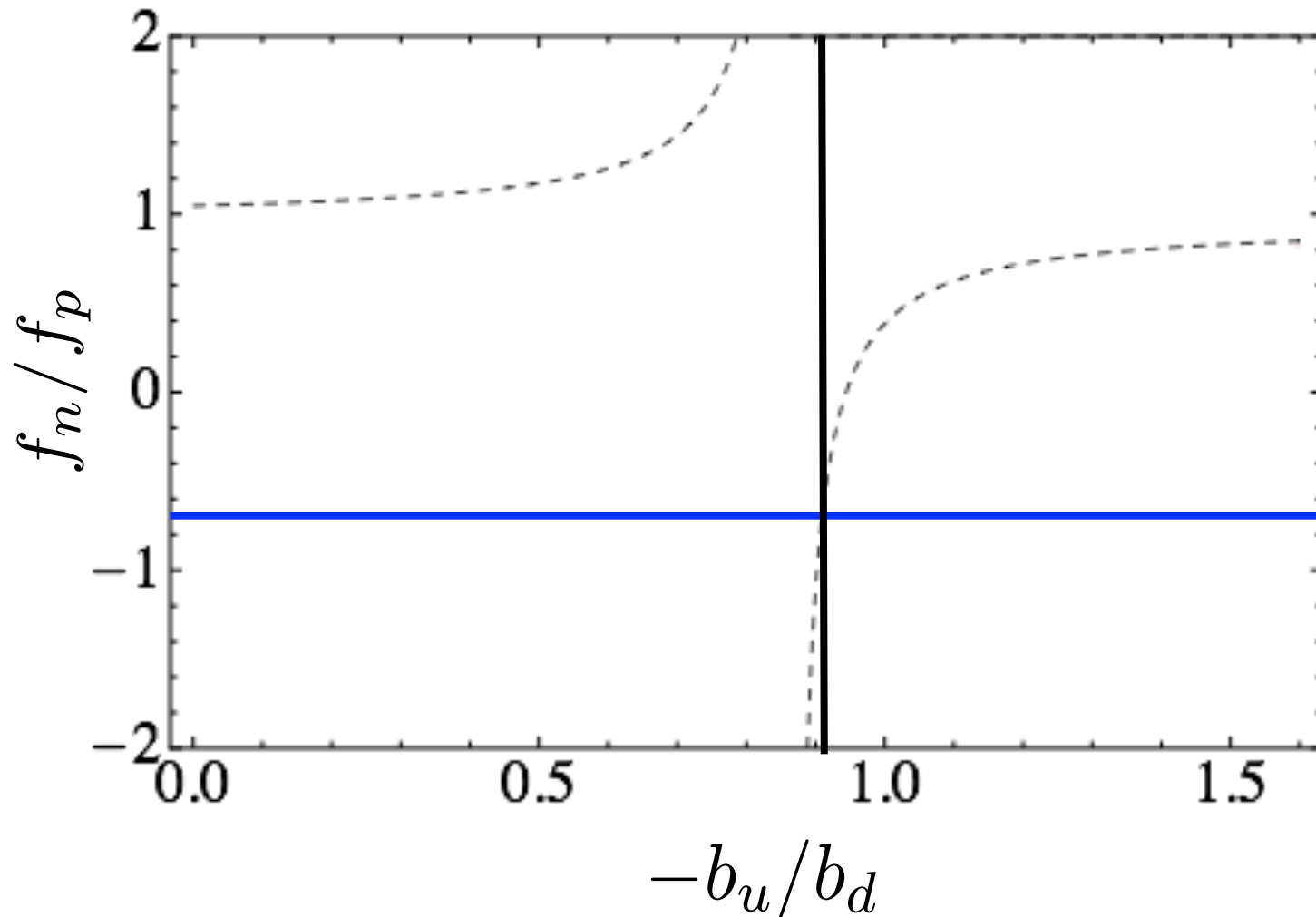
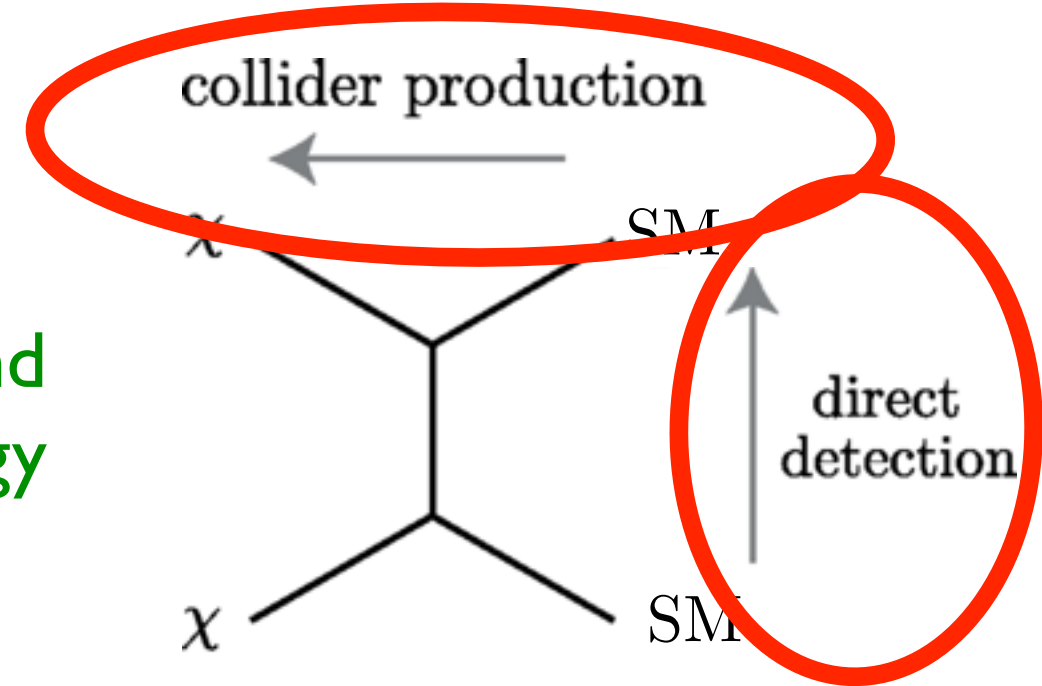
$$f_n/f_p \approx -Z/(A-Z) \approx -0.7$$

engineered to reconcile DAMA
with results from Xe and other
nuclei

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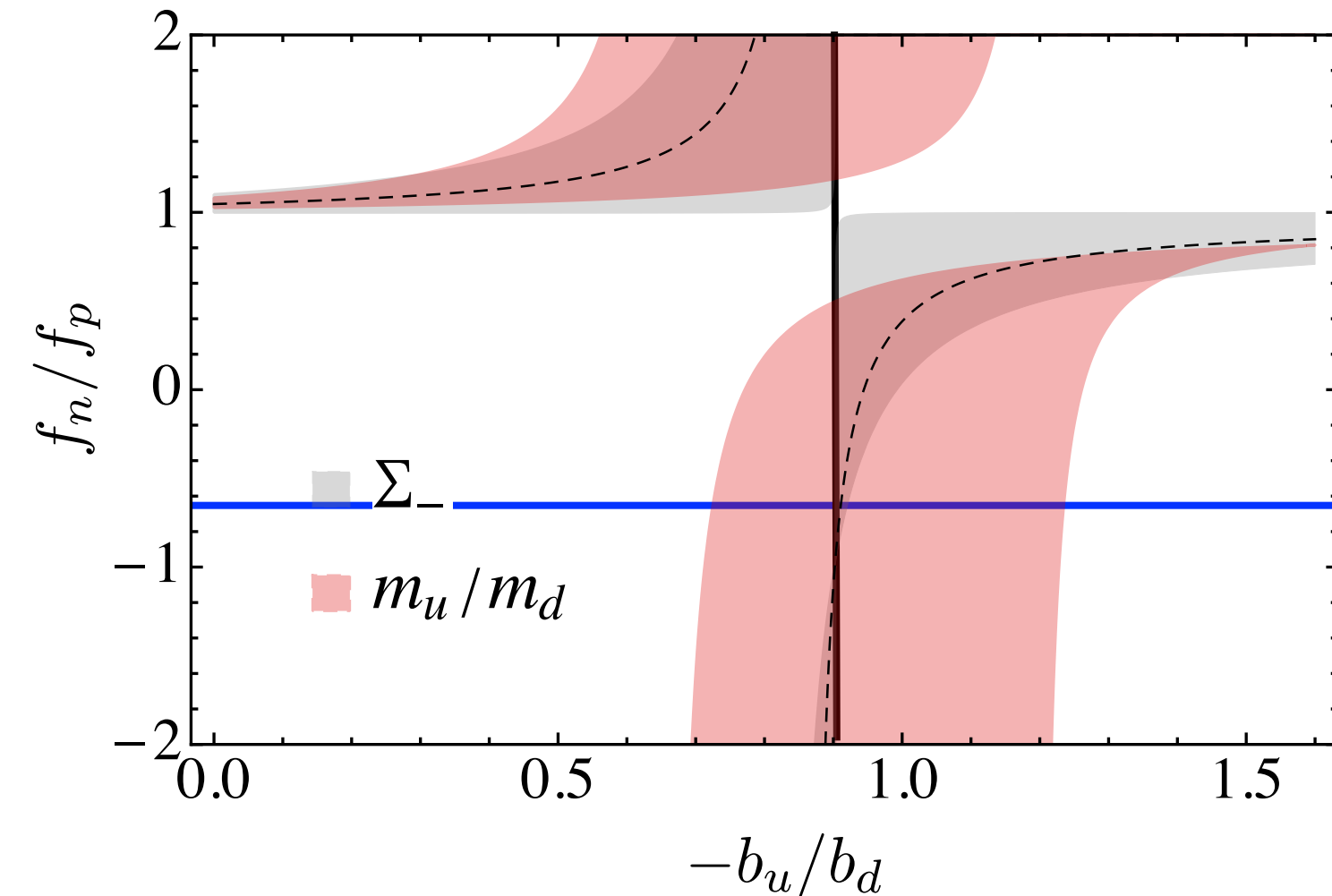
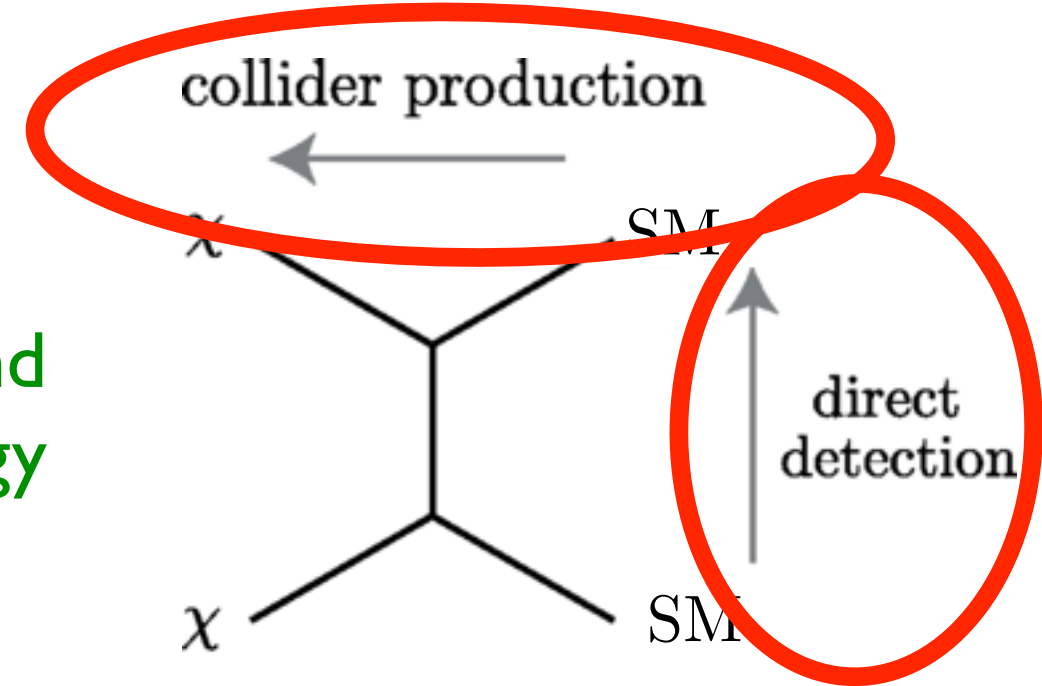
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Solution: $b_u/b_d = -0.9$

However, must account for uncertainties (hadronic and renormalization scale)

Not quibbling about percents (example 2: light WIMPs)

DM complementarity: connect direct detection and collider phenomenology



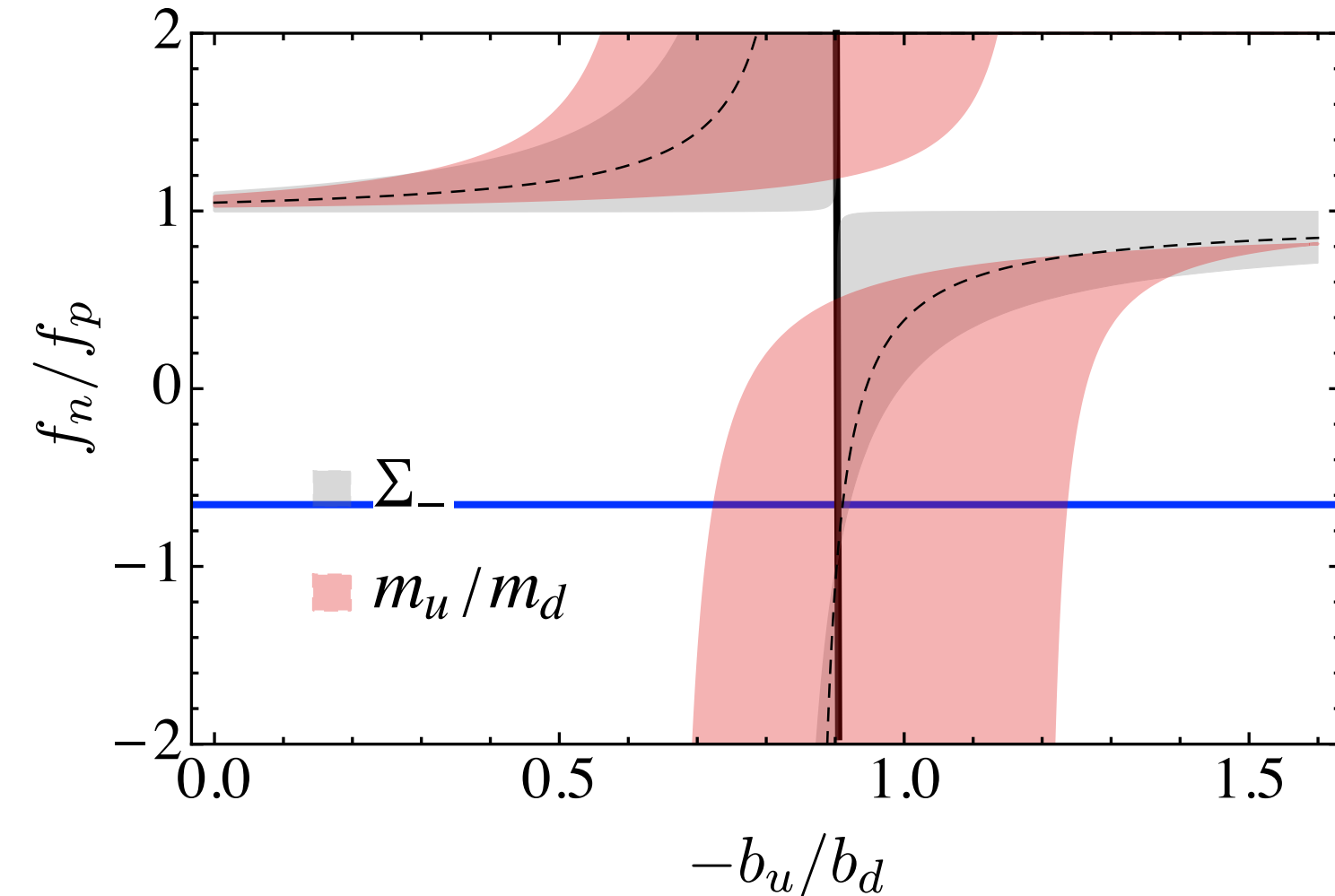
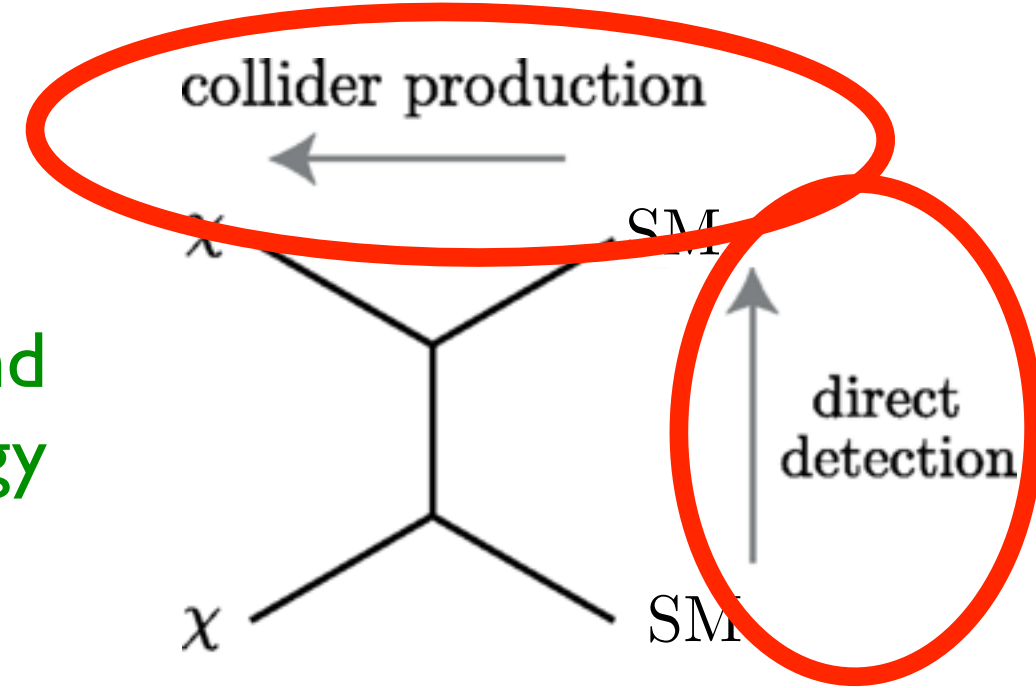
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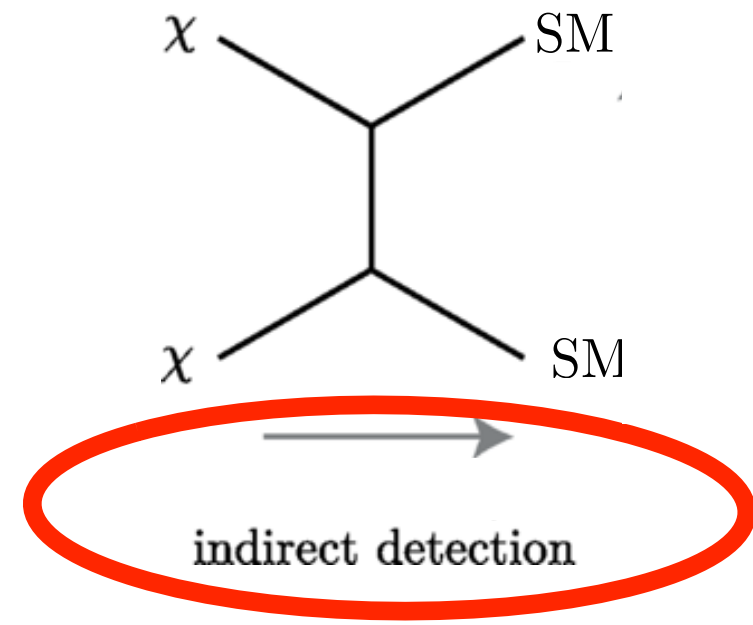
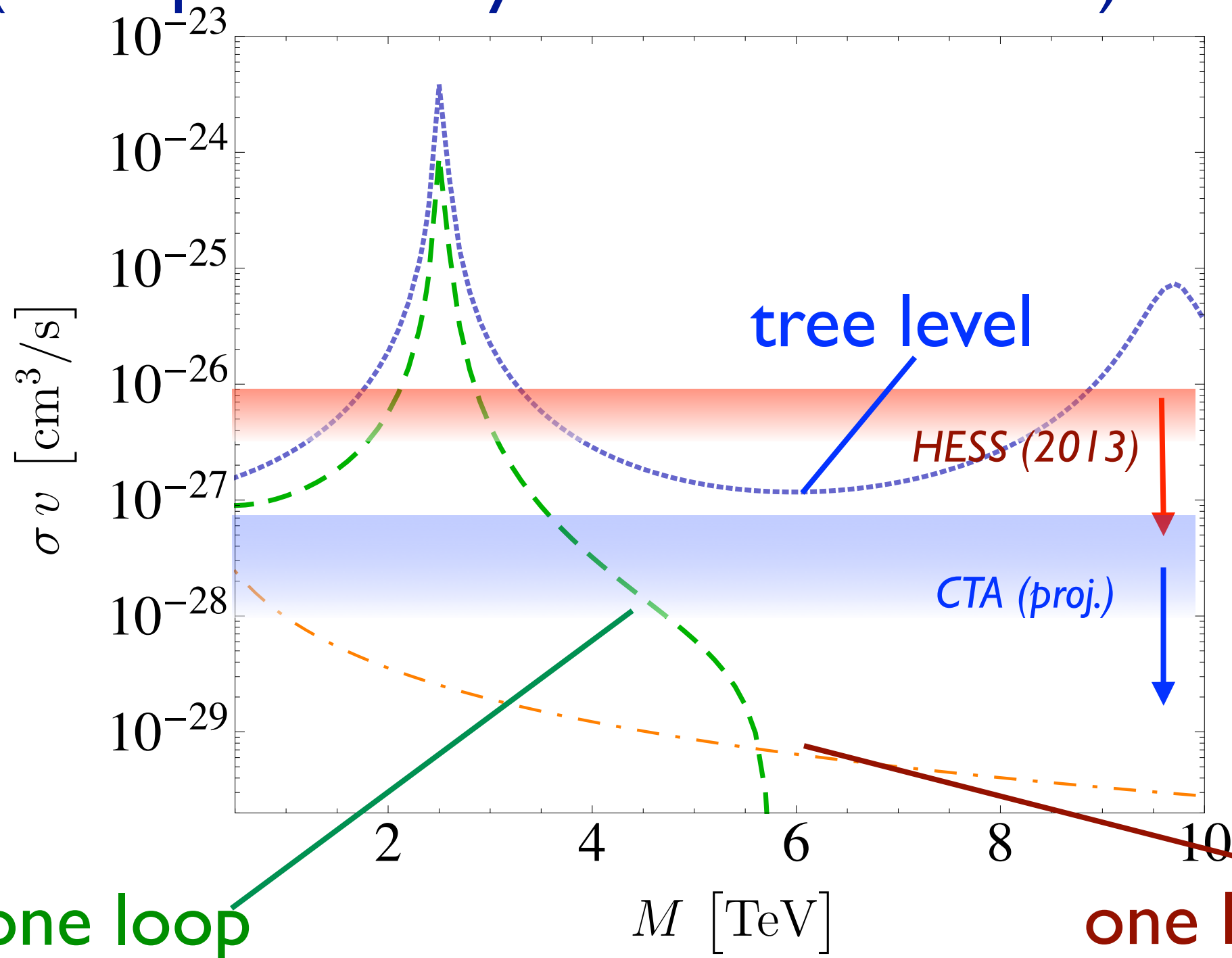
$f_n/f_p \approx -Z/(A-Z) \approx -0.7$
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cf. $b_u/b_d = -1.08$ from “isospin-violating” DM

Assumed one-to-one mapping between b_u/b_d and f_n/f_p invalid

Nontrivial mapping from colliders to direct detection

Not quibbling about percents (example 3: heavy WIMP annihilation)



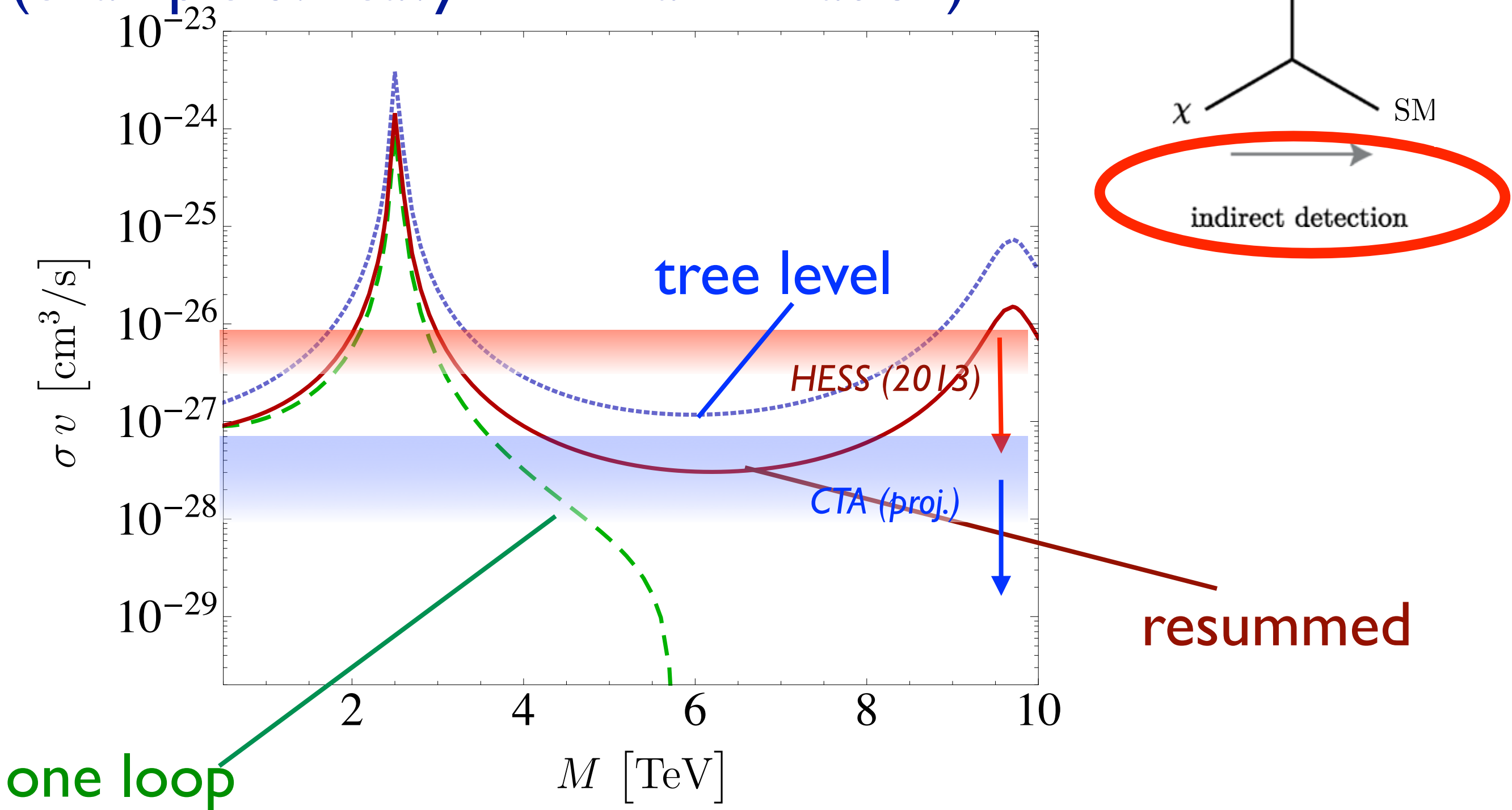
Photon line signal
for “wino”
annihilation

one loop

one loop, neglect
wavefunction enhancement

Multi-scale field theory problem, breakdown of naive
perturbation theory

Not quibbling about percents (example 3: heavy WIMP annihilation)

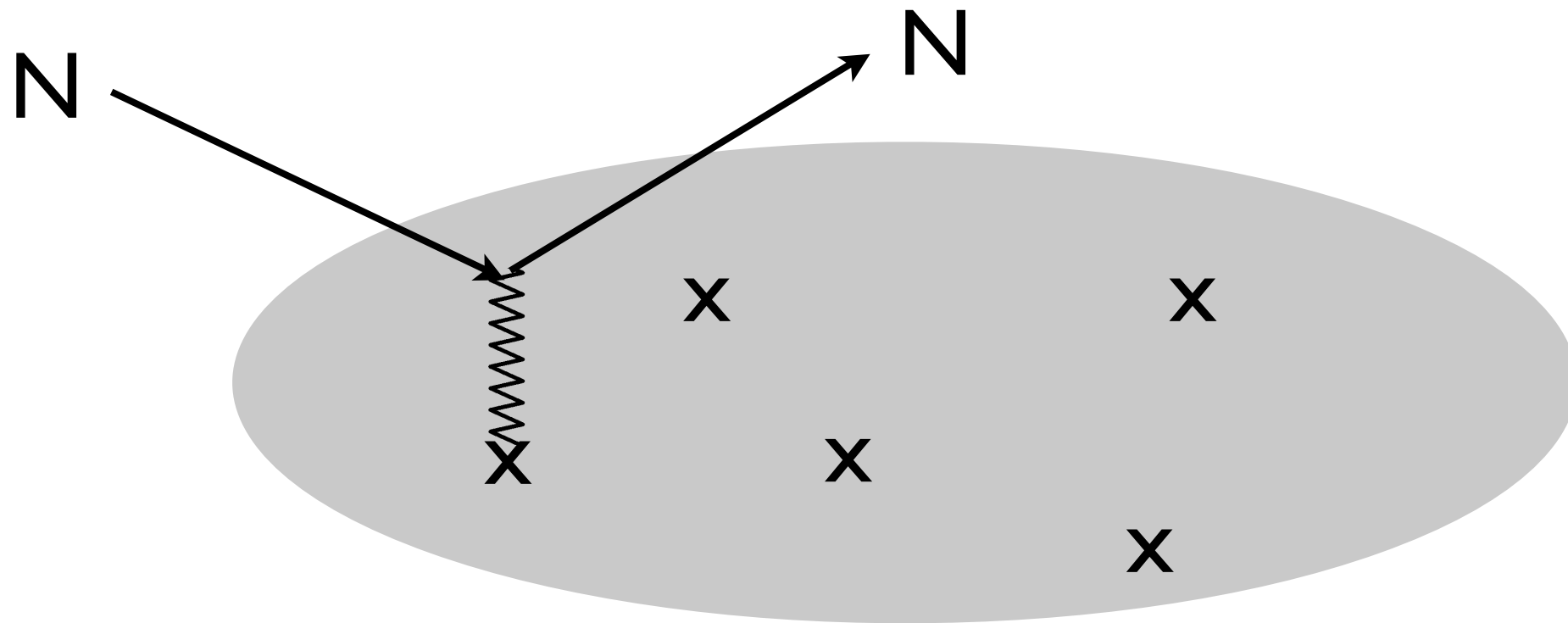


Multi-scale field theory problem, breakdown of naive perturbation theory

Heavy WIMP effective theory

Mechanisms versus models

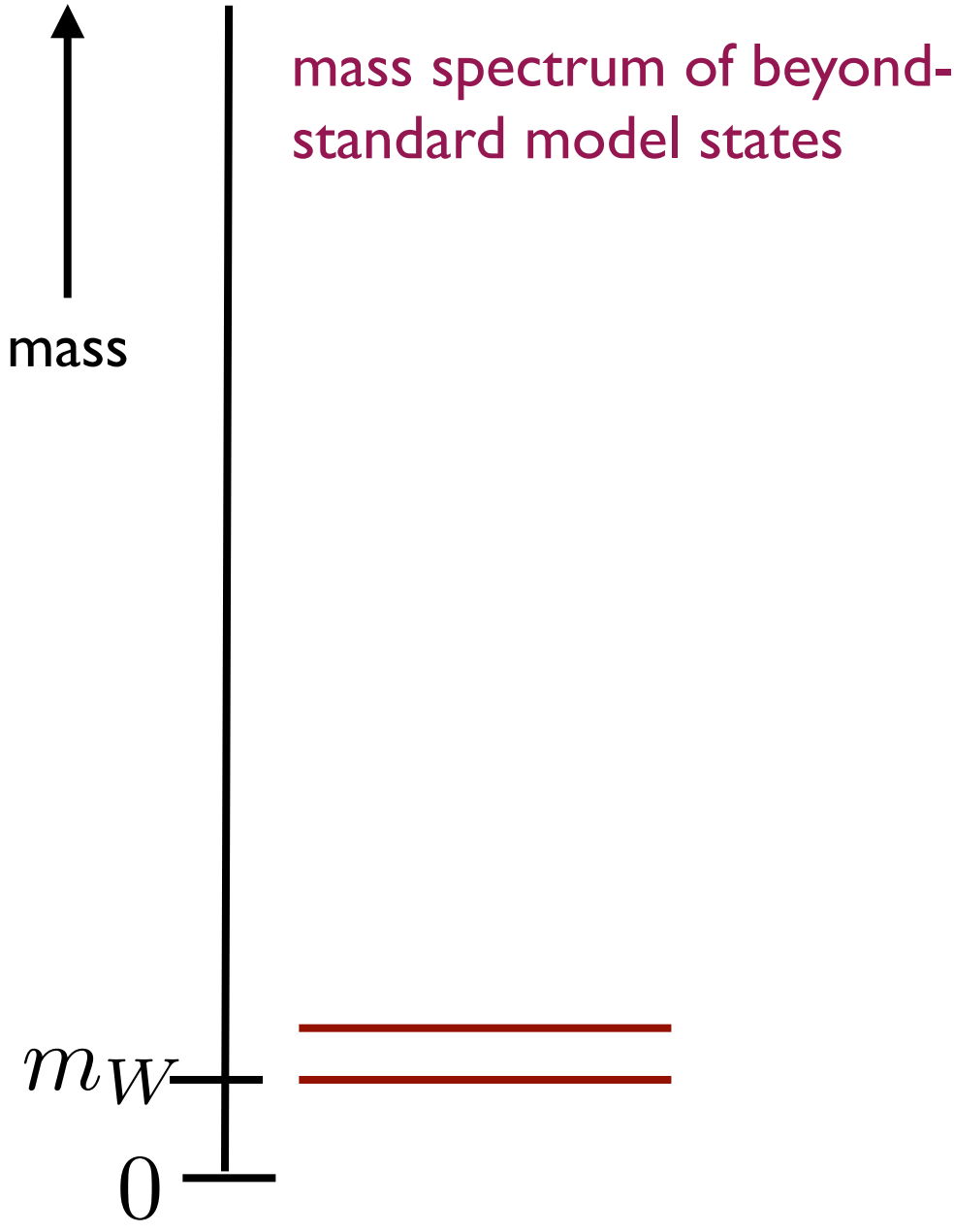
Electroweak charged WIMP Mechanism versus WIMP Model



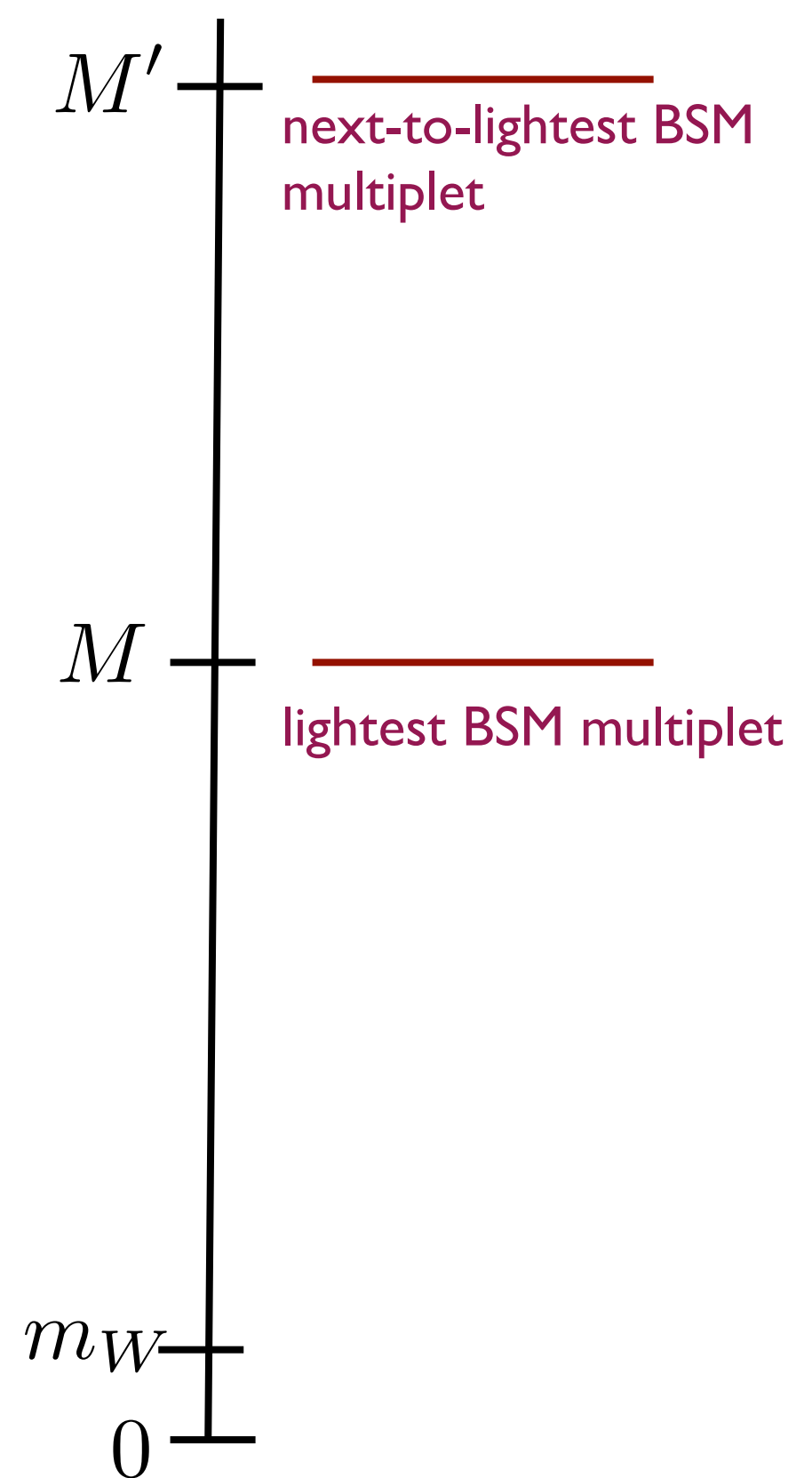
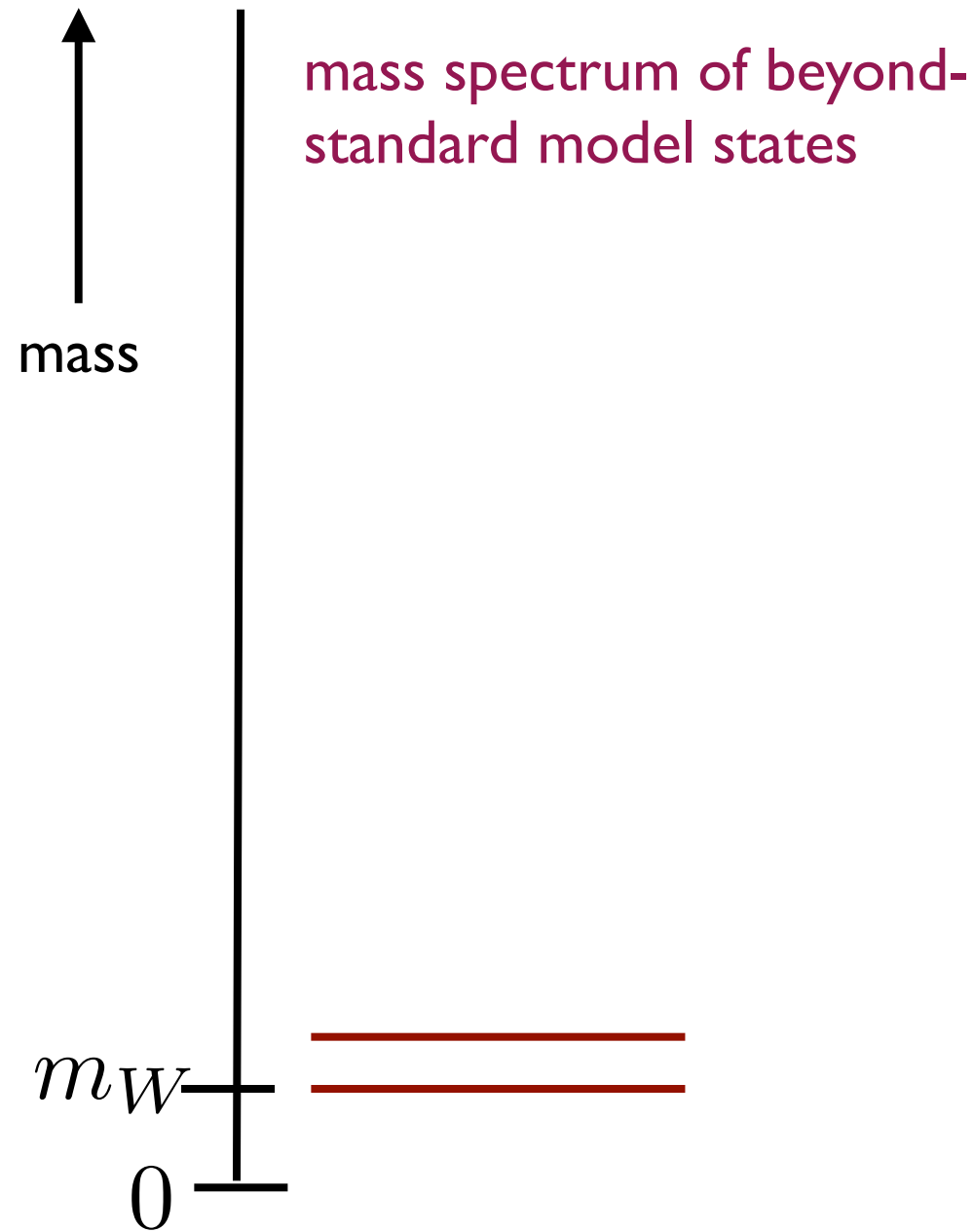
Focus on self-conjugate $SU(2)$ triplet. Could be:

- SUSY wino
- Weakly Interacting Stable Pion
- Minimal Dark Matter
- ...

Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale



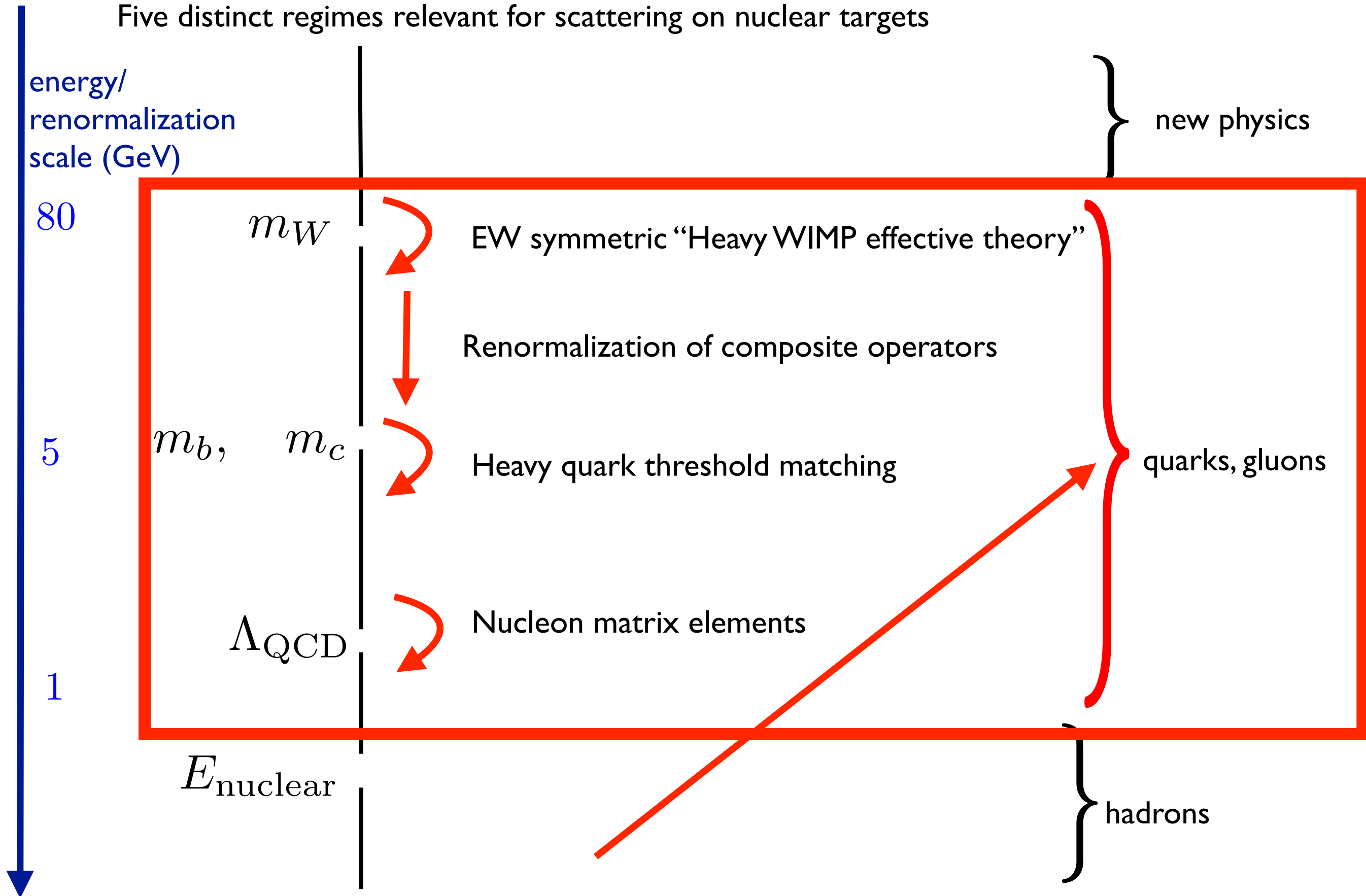
Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale



If WIMP mass $M \gg m_W$, isolation ($M' - M \gg m_W$) becomes generic. Expand in m_W/M , $m_W/(M' - M)$

Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes

Five distinct regimes relevant for scattering on nuclear targets



“SM anatomy” of interactions between weak and hadronic scales

Scale separation:

dark sector
d.o.f.

SM
d.o.f.

params.
(beyond mass)

M	$\chi^{(+,-,0)}$	$Q, A_\mu^a, W_\mu^i, B_\mu$	0
	$\chi_v^{(+,-,0)}$	$Q, A_\mu^a, W_\mu^i, B_\mu$	0
m_W	$\chi_v^{(0)}$	u, d, s, c, b, A_μ^a	12
m_b, m_c	$\chi_v^{(0)}$	u, d, s, A_μ^a	8
Λ_{QCD}	$\chi_v^{(0)}$	N, π	3
m_π	$\chi_v^{(0)}$	n, p	2
$l/R_{nucleus}$	$\chi_v^{(0)}$	\mathcal{N}	1

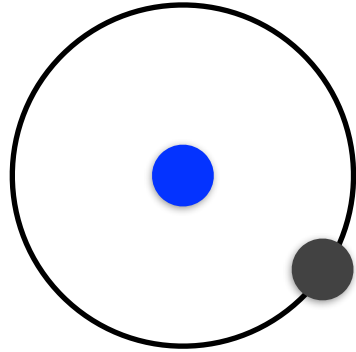
Many manifestations of heavy particle symmetry:

prediction:

small parameter:

- hydrogen/deuterium spectroscopy

$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \quad (m_e Z\alpha) \ll m_e$$



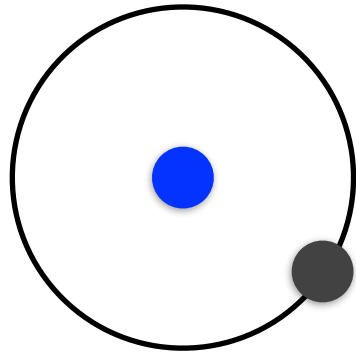
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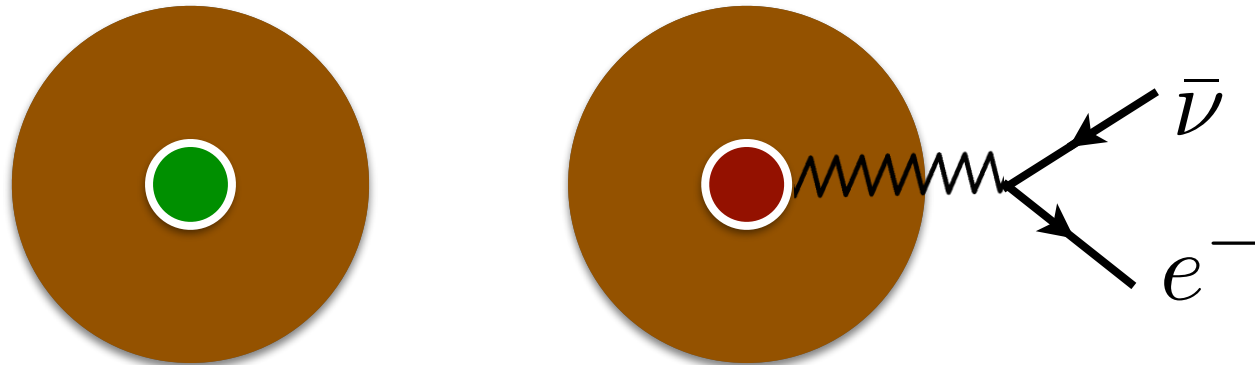
$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \quad (m_e Z\alpha) \ll m_e$$



- heavy meson transitions

$$F^{B \rightarrow D}(v' = v) = 1 + \dots$$

$$\Lambda_{\text{QCD}} \ll m_{b,c}$$



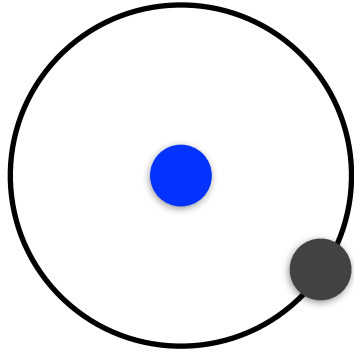
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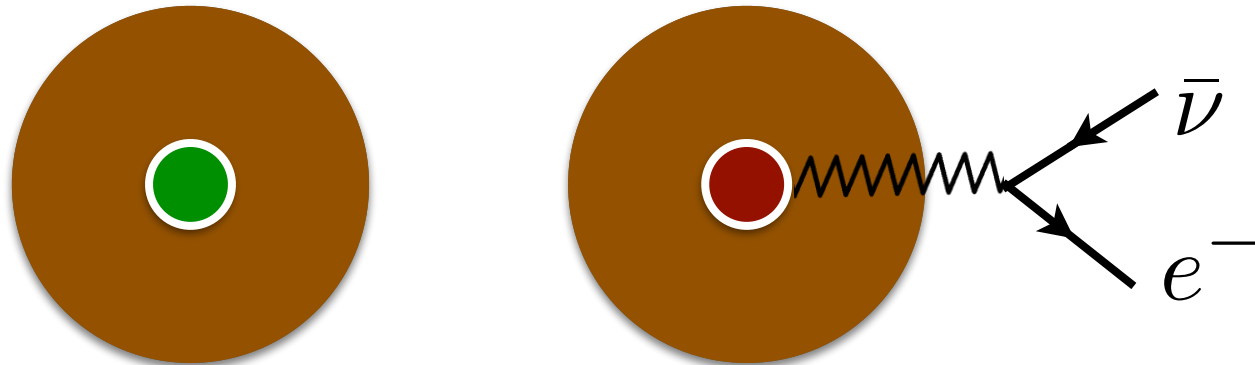
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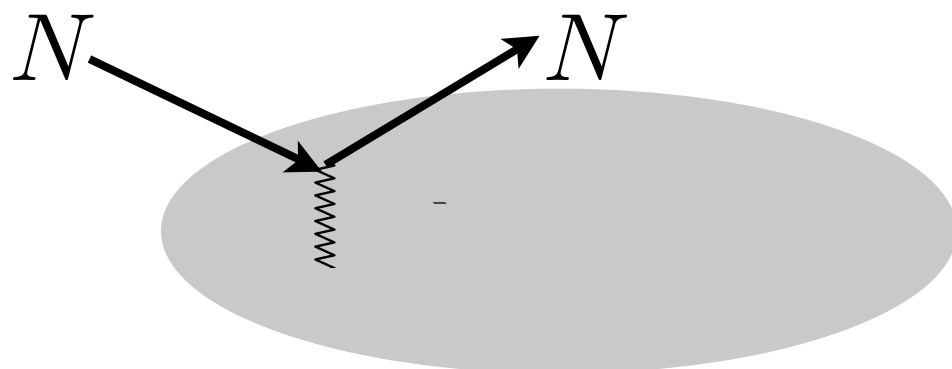
$$\Lambda_{\text{QCD}} \ll m_{b,c}$$



- DM interactions

$$\sigma(\chi N \rightarrow \chi N) = ?$$

$$m_W \ll m_\chi$$

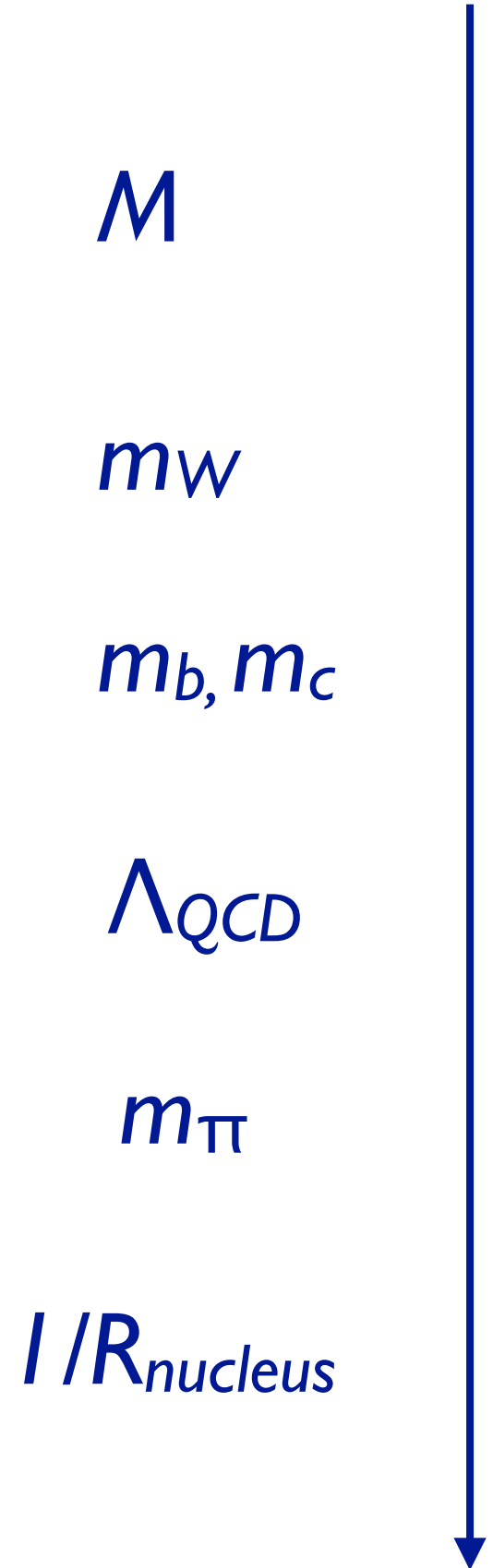


Scale separation:

dark sector
d.o.f.

SM
d.o.f.

params.
(beyond mass)



M

$\chi^{(+,-,0)}$

$Q, A_\mu^a, W_\mu^i, B_\mu$

0

m_W

$\chi_v^{(+,-,0)}$

$Q, A_\mu^a, W_\mu^i, B_\mu$

0

m_b, m_c

$\chi_v^{(0)}$

u, d, s, c, b, A_μ^a

12

$\chi_v^{(0)}$

u, d, s, A_μ^a

8

Λ_{QCD}

$\chi_v^{(0)}$

N, π

3

m_π

$\chi_v^{(0)}$

n, p

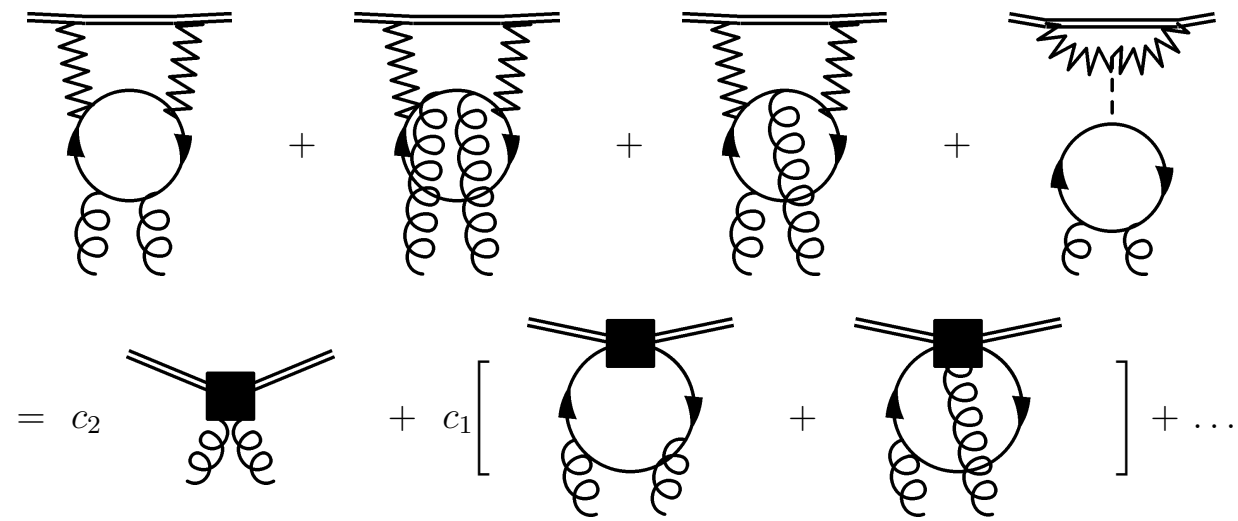
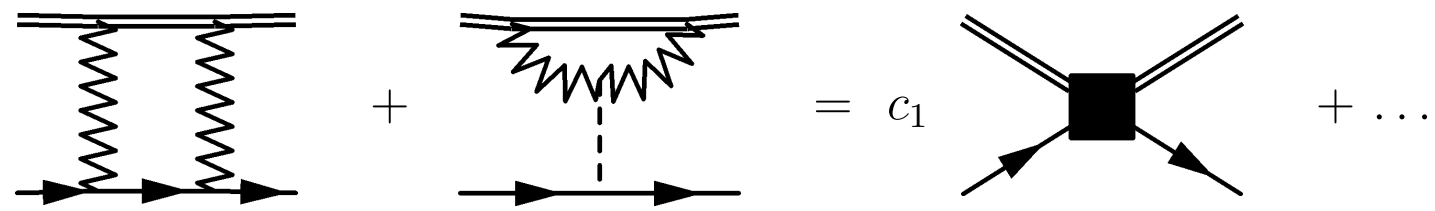
2

$1/R_{nucleus}$

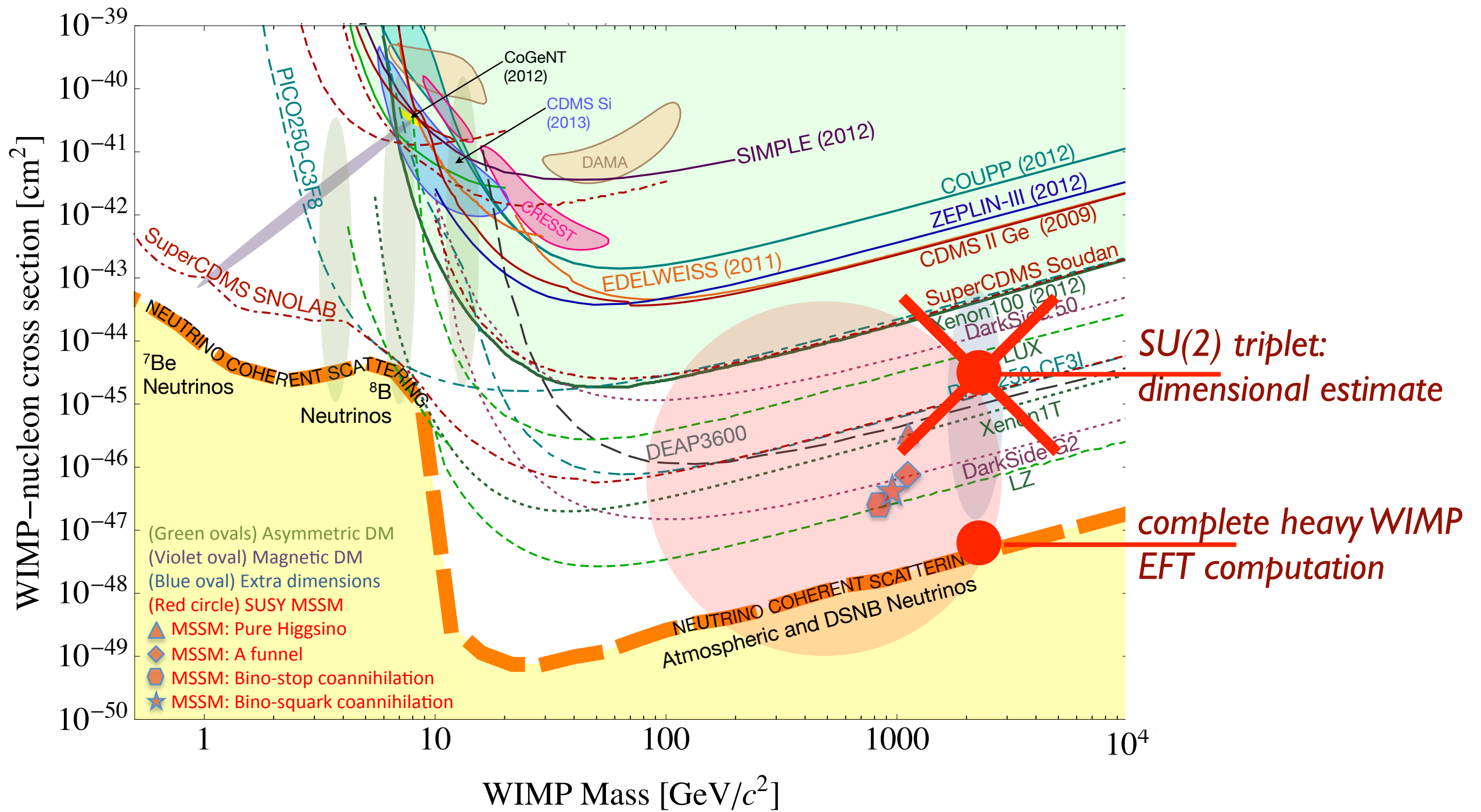
$\chi_v^{(0)}$

\mathcal{N}

1



- the heavy lifting is necessary




Perturbative QCD

Scale separation:

dark sector
d.o.f.

SM
d.o.f.

params.
(beyond mass)



M	$\chi^{(+,-,0)}$	$Q, A_\mu^a, W_\mu^i, B_\mu$	0
	$\chi_v^{(+,-,0)}$	$Q, A_\mu^a, W_\mu^i, B_\mu$	0
m_W	$\chi_v^{(0)}$	u, d, s, c, b, A_μ^a	12
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Λ_{QCD}	$\chi_v^{(0)}$	N, π	3
m_π	$\chi_v^{(0)}$	n, p	2
$l/R_{nucleus}$	$\chi_v^{(0)}$	\mathcal{N}	1

Dark matter - Standard Model interactions

$$\mathcal{L} = \frac{1}{\Lambda^n} O_{\text{DM}} \times O_{\text{SM}}$$

d	Fermion	d	Scalar	d	Heavy particle
3	$\bar{\psi}[1, i\gamma_5, \gamma^\mu\gamma_5, \{\gamma^\mu, \sigma^{\mu\nu}\}]\psi$	2	$ \phi ^2$	3	$\bar{\chi}_v[1, \{\sigma_{\perp}^{\mu\nu}\}]\chi_v$
4	$\bar{\psi}[\{1, i\gamma_5, \gamma^\mu\gamma_5\}, \gamma^\mu, \sigma^{\mu\nu}]i\partial_{\perp}^{\rho}\psi$	3	$\{\phi^*i\partial_{\perp}^{\mu}\phi\}$	4	$\bar{\chi}_v[\{1\}, \sigma_{\perp}^{\mu\nu}]i\partial_{\perp}^{\rho}\chi_v$

d	QCD operator basis
3	$V_q^{\mu} = \bar{q}\gamma^{\mu}q$ $A_q^{\mu} = \bar{q}\gamma^{\mu}\gamma_5q$
4	$T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5q$ $O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^A G_{\rho\sigma}^A$ $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_{\perp}^{\nu\}} - \frac{g^{\mu\nu}}{4}i\not{D}_{\perp}\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_{\perp}^{\nu\}}\gamma_5q$

complete
QCD basis
for $d \leq 7$

Renormalization and matching (sample):

$$\mathcal{L}_{\phi_0, \text{SM}} = \frac{1}{m_W^3} \phi_v^* \phi_v \left\{ \sum_q \left[c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_\mu v_\nu O_{1q}^{(2)\mu\nu} \right] + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_\mu v_\nu O_2^{(2)\mu\nu} \right\} + \dots$$

$m_q \bar{q}q$: $G_{\mu\nu}^A G^{A\mu\nu}$

focus on spin-0 (evaluate spin-2 at weak scale)

Renormalization group evolution from weak scale to hadronic scales, with perturbative corrections at heavy quark mass thresholds

$$c_i(\mu_Q) = M_{ij}(\mu_Q) c'_j(\mu_Q).$$

$$M(\mu_Q) = \left(\begin{array}{ccc|cc} & & & M_{qQ} & M_{qg} \\ & \mathbb{1}(M_{qq} - M_{qq'}) + \mathcal{J}M_{qq'} & & \vdots & \vdots \\ & & & M_{qQ} & M_{qg} \\ \hline M_{gq} & \dots & M_{gq} & M_{gQ} & M_{gg} \end{array} \right)$$

Can show that:

$$M_{qq} \equiv 1, \quad M_{qq'} \equiv 0, \quad M_{gq} \equiv 0$$

M_{gQ} and M_{qQ} known through

3 loops:

Chetyrkin et al. (1997)

New results for gluon-induced decoupling relations

$$M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6} \log \frac{\mu_Q}{m_Q} + \frac{1}{9} \log^2 \frac{\mu_Q}{m_Q}$$

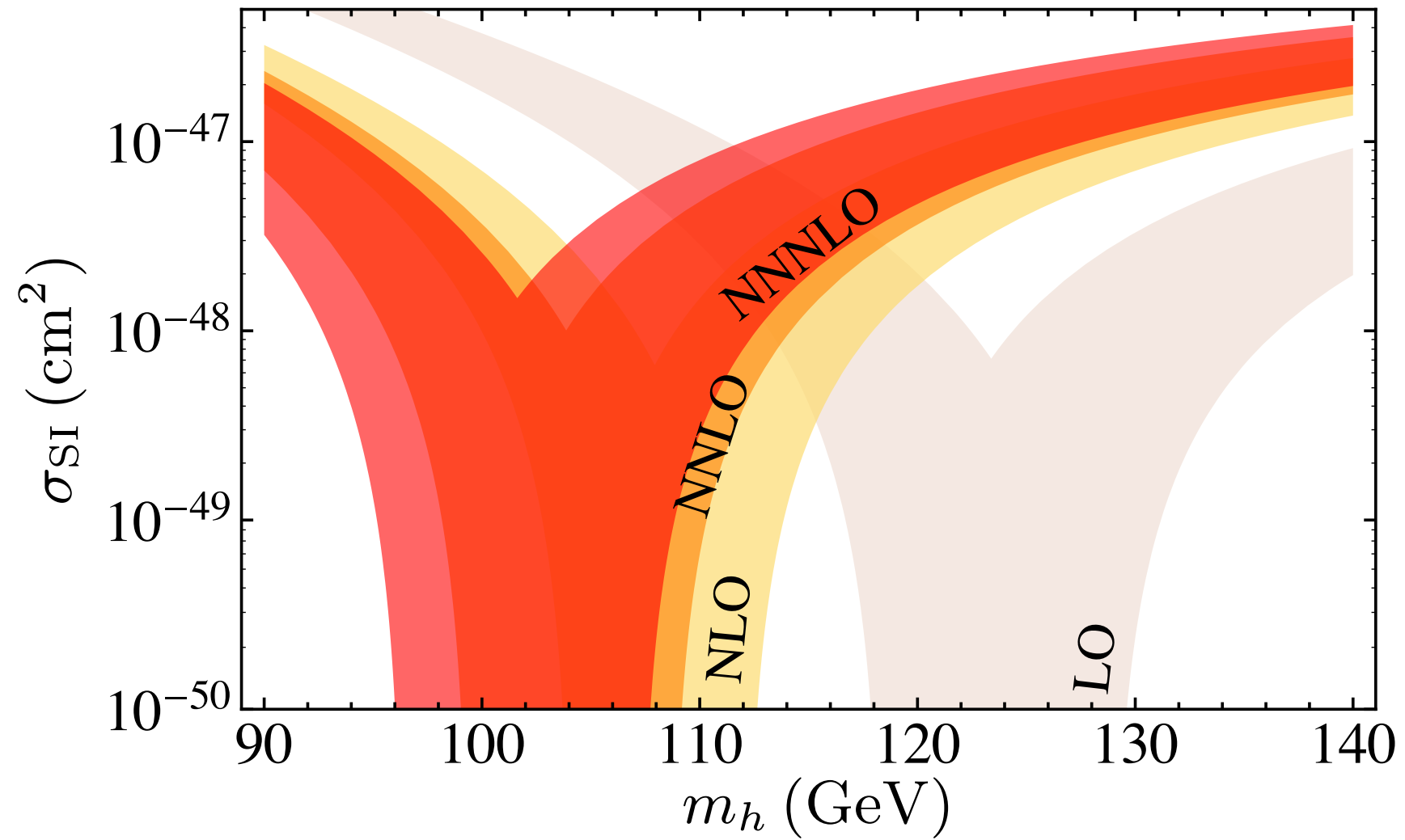
$$M_{gg}^{(3)} = \frac{564731}{41472} - \frac{2821}{288} \log \frac{\mu_Q}{m_Q} + \frac{3}{16} \log^2 \frac{\mu_Q}{m_Q} - \frac{1}{27} \log^3 \frac{\mu_Q}{m_Q} - \frac{82043}{9216} \zeta(3) \\ + n_f \left[-\frac{2633}{10368} + \frac{67}{96} \log \frac{\mu_Q}{m_Q} - \frac{1}{3} \log^2 \frac{\mu_Q}{m_Q} \right],$$

$$M_{qg}^{(2)} = -\frac{89}{54} + \frac{20}{9} \log \frac{\mu_Q}{m_Q} - \frac{8}{3} \log^2 \frac{\mu_Q}{m_Q}.$$

Hill, Solon (2014)



- the heavy lifting is necessary




Hadronic matrix elements

Scale separation:

dark sector
d.o.f.

SM
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params.
(beyond mass)



M	$\chi^{(+,-,0)}$	$Q, A_\mu^a, W_\mu^i, B_\mu$	0
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m_W			
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m_b, m_c			
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Λ_{QCD}			
	$\chi_v^{(0)}$	N, π	3
m_π			
	$\chi_v^{(0)}$	n, p	2
$1/R_{nucleus}$			
	$\chi_v^{(0)}$	\mathcal{N}	1

d	QCD operator basis
3	$V_q^\mu = \bar{q}\gamma^\mu q$ $A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$
4	$T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5 q$ $O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5 q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^A G_{\rho\sigma}^A$ $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4}i\cancel{D}_-\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G_{\lambda}^{A\nu} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_-^{\nu\}}\gamma_5 q$

complete
QCD basis
for $d \leq 7$

- For canonical example (heavy electroweak multiplet), scalar operators
- Selected other examples

d	QCD operator basis
3	$V_q^\mu = \bar{q}\gamma^\mu q$ $A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$
4	$T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5 q$ $O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5 q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^A G_{\rho\sigma}^A$ <div style="border: 2px solid red; padding: 5px;"> $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD^{\nu\}} - \frac{g^{\mu\nu}}{4}i\not{D}\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ </div> $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD^{\nu\}}\gamma_5 q$

- **C-even spin-2: determined by PDF moments**

$$\langle N|O^{(2)\mu\nu}|N\rangle = k^\mu k^\nu \int_0^1 dx x[q(x) + \bar{q}(x)]$$

d	QCD operator basis
3	$V_q^\mu = \bar{q}\gamma^\mu q$ $A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$
4	$T_a^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5 q$ <div style="border: 2px solid red; padding: 5px; display: inline-block; margin: 10px 0;"> $O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ </div> $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5 q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^A G_{\rho\sigma}^A$ $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4}i\cancel{D}_-\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}_\lambda + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_-^{\nu\}}\gamma_5 q$

- C-even spin-0: nucleon sigma terms (nucleon mass sum rule for gluon operator)

$$m_N = (1 - \gamma_m) \sum_q \langle N | m_q \bar{q}q | N \rangle + \frac{1}{2} \beta \langle N | (G_{\mu\nu}^a)^2 | N \rangle$$

- up, down quarks & isospin-violating dark matter

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2} \langle N | (\bar{u}u + \bar{d}d) | N \rangle$$

$$= 44(13) \text{ MeV}$$

Durr et al. 1109.4265

$$\Sigma_- = (m_d - m_u) \langle N | (\bar{u}u - \bar{d}d) | N \rangle$$

$$= \pm 2(2) \text{ MeV}$$

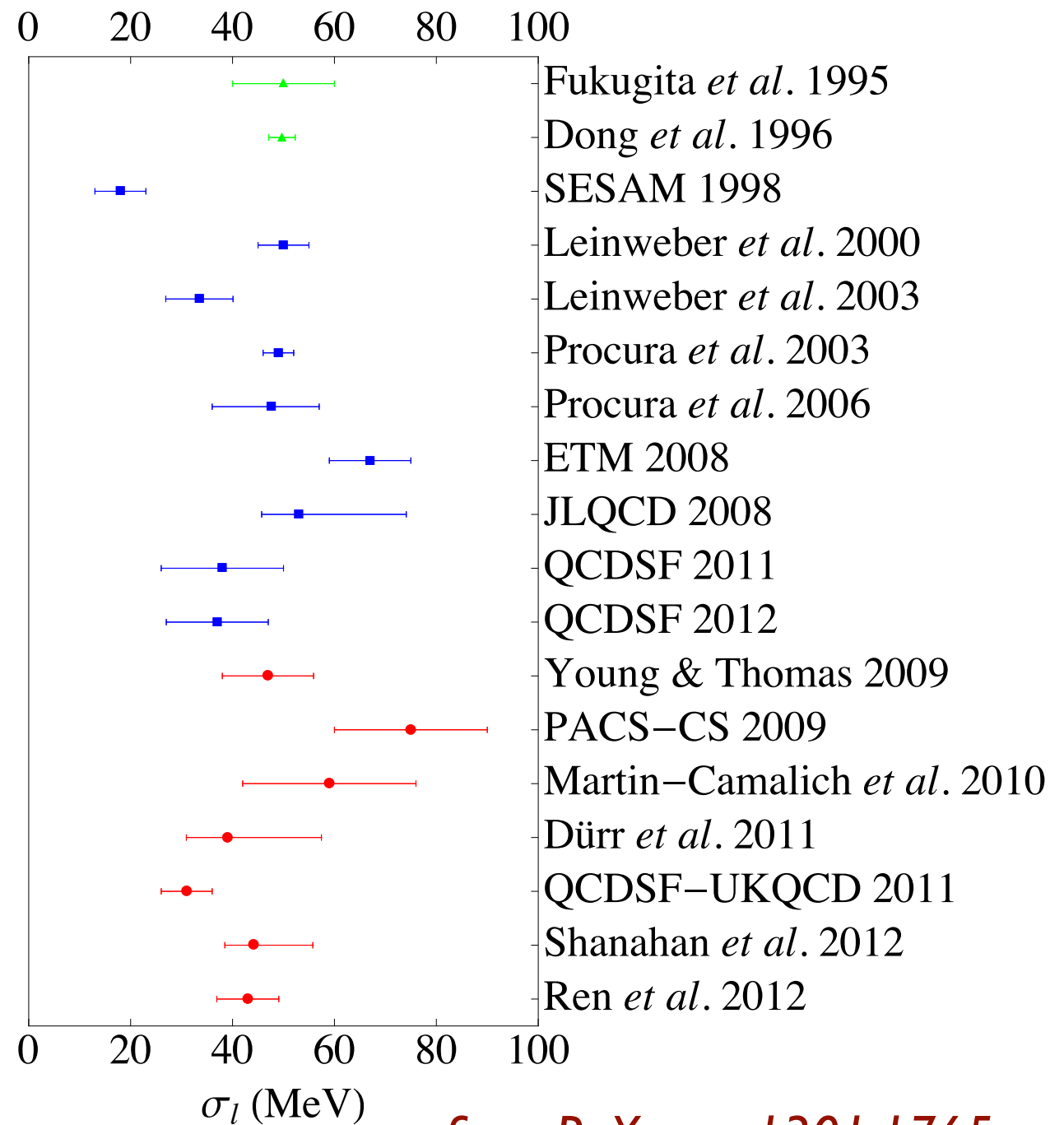
Gasser, Leutwyler (1982)

$$= \pm 2(1) \text{ MeV}$$

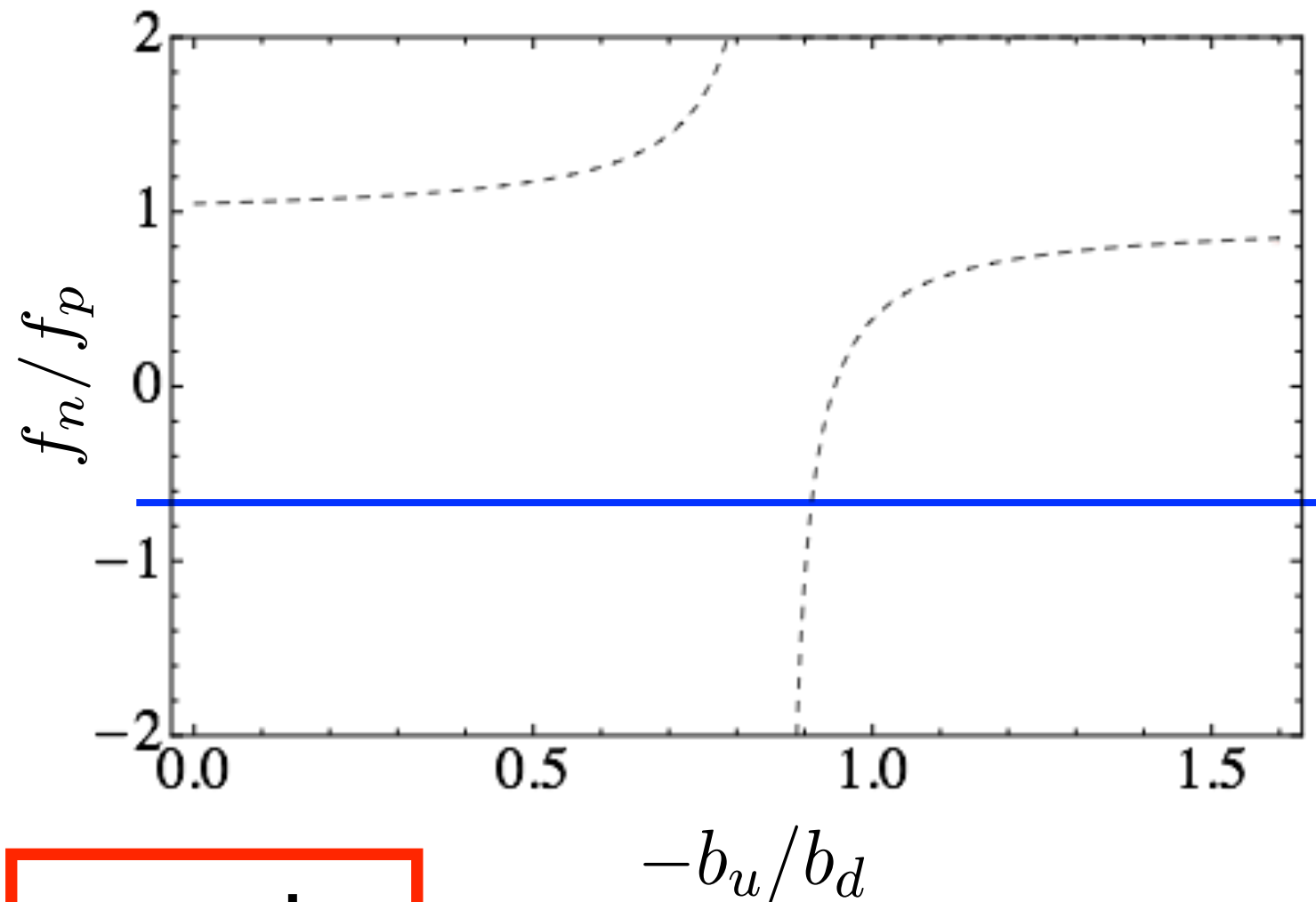
Crivellin, Hoferichter, Procura (2014)

$$\frac{m_u}{m_d} = 0.49 \pm 0.13$$

PDG



- up, down quarks & isospin-violating dark matter



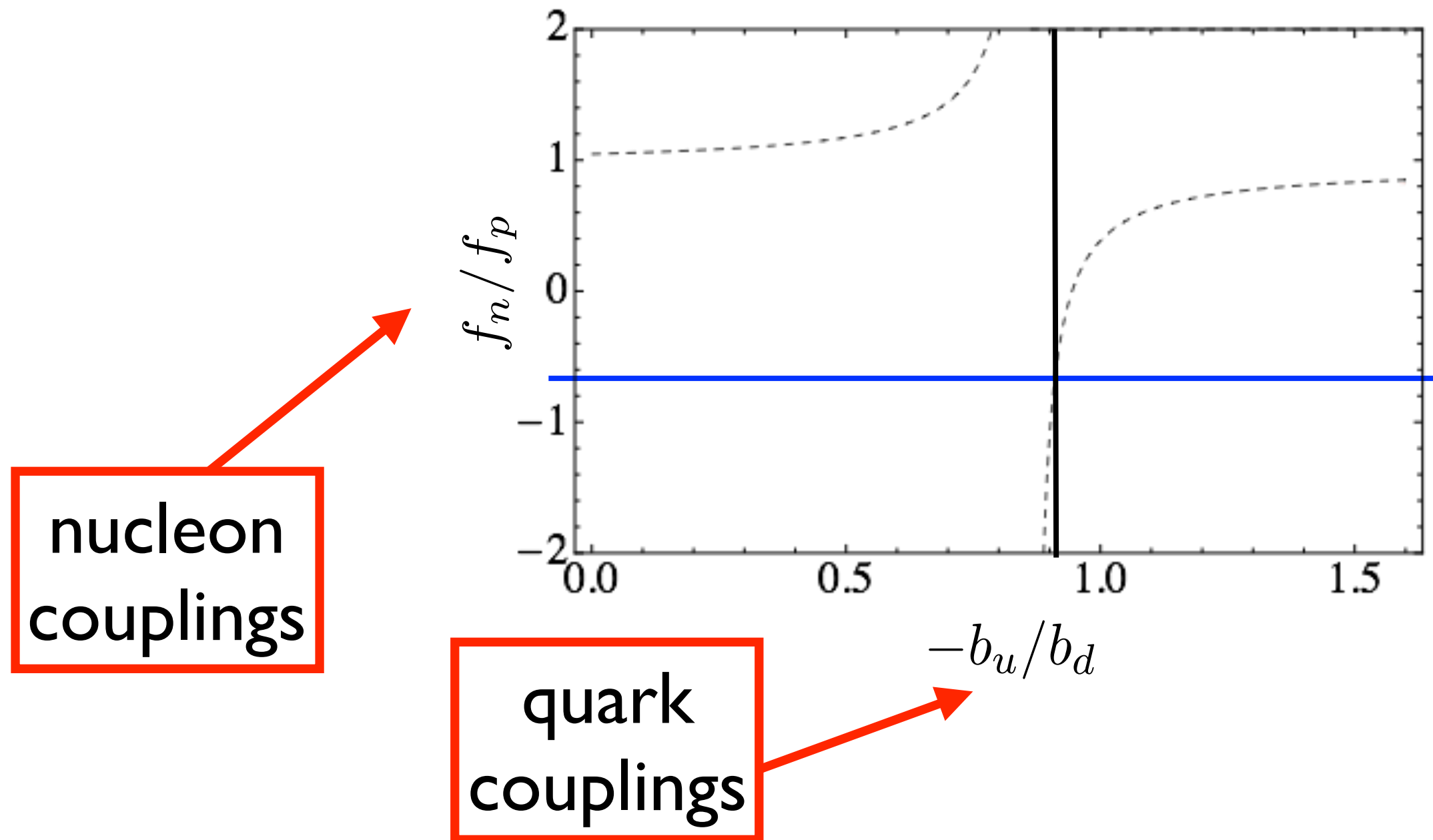
nucleon
couplings

quark
couplings

$-b_u/b_d$

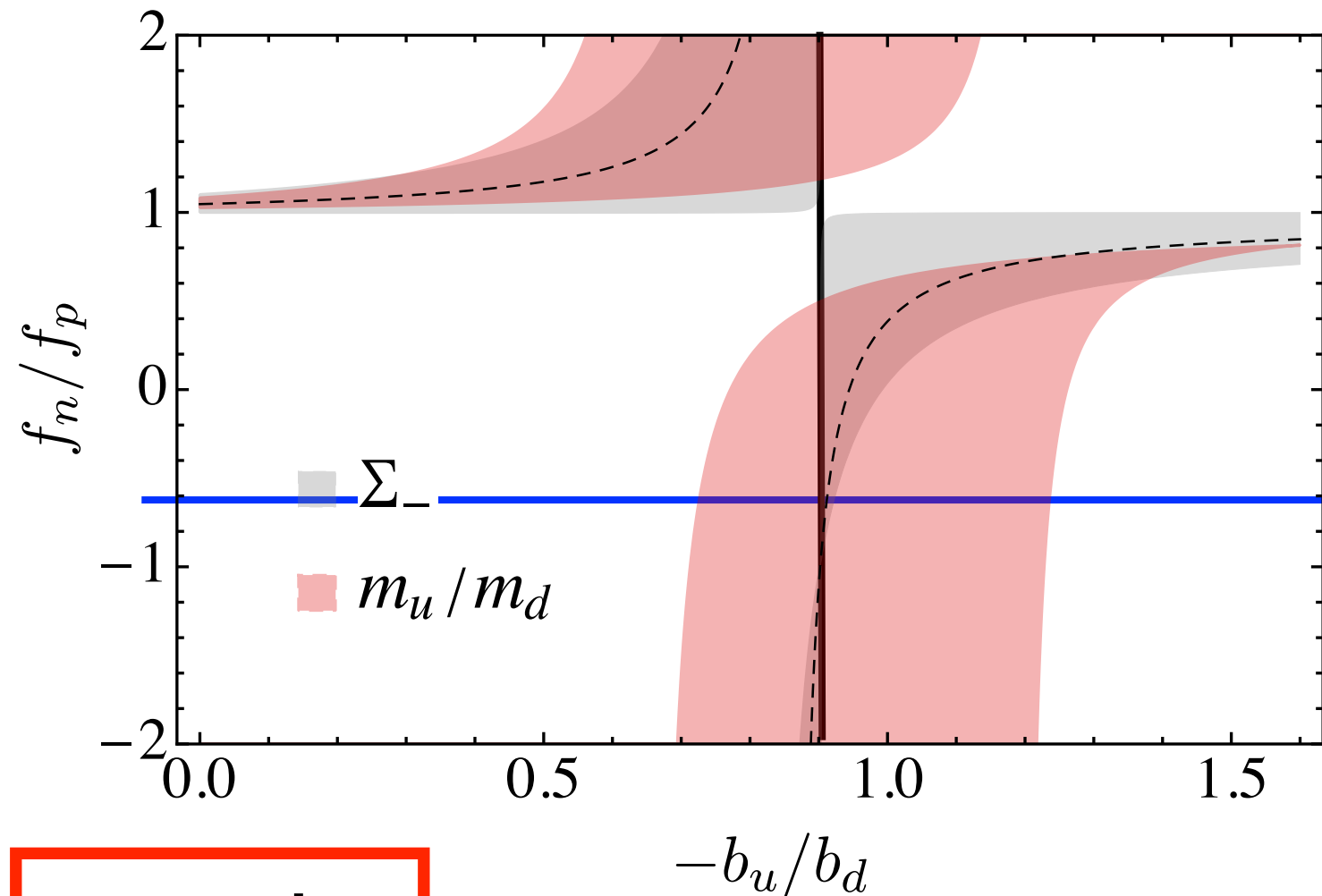
hadronic uncertainties important for determining
viability of models for potential signals

- up, down quarks & isospin-violating dark matter



hadronic uncertainties important for determining viability of models for potential signals

- up, down quarks & isospin-violating dark matter



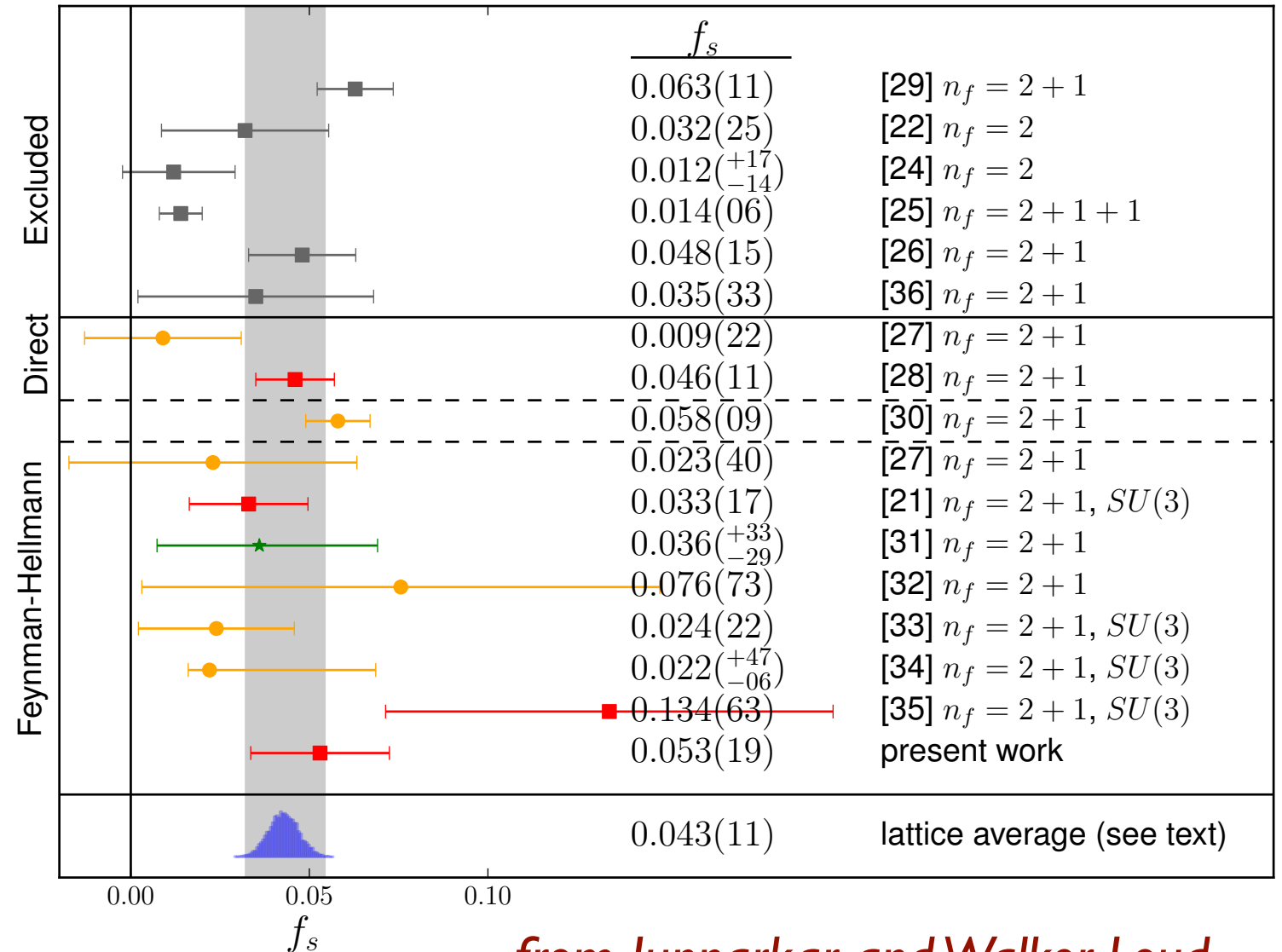
nucleon
couplings

quark
couplings

hadronic uncertainties important for determining
viability of models for potential signals

- strange quarks & heavy wino dark matter

$$\begin{aligned}\Sigma_s &= \langle N | \bar{s}s | N \rangle \\ &= 40 \pm 20 \text{ MeV}\end{aligned}$$



from Junnarkar and Walker-Loud,
1301.1114

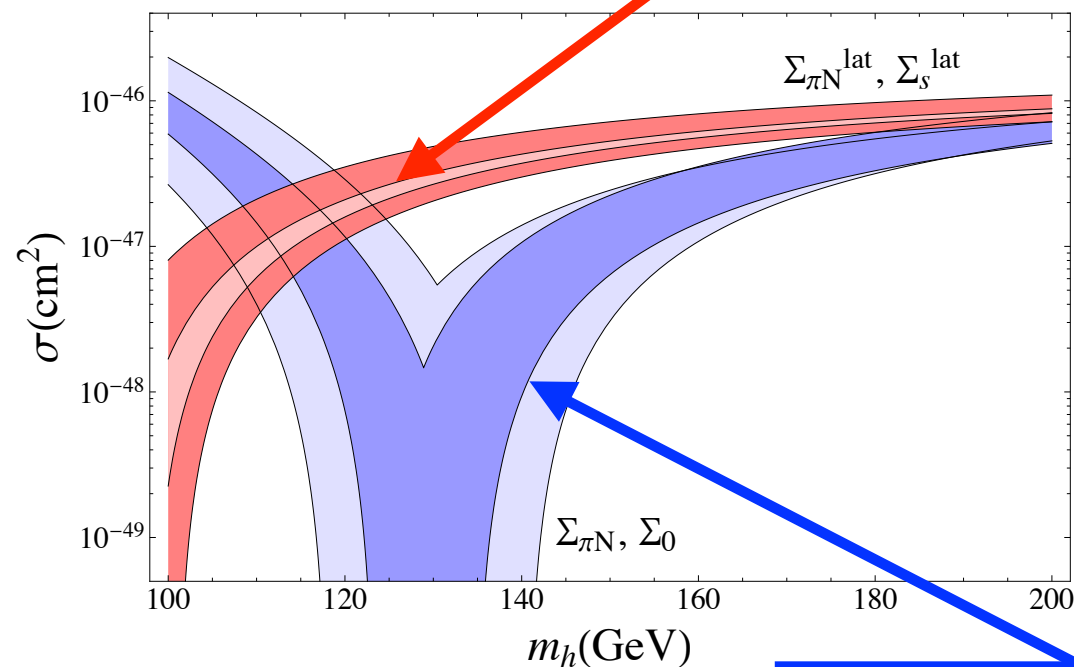


- strange quarks & heavy wino dark matter

lattice QCD inputs

$$\Sigma_{\pi N} = 47(9) \text{ MeV}$$

$$\Sigma_s = 50(8) \text{ MeV}$$



baryon spectroscopy
inputs

Pavan et al. hep-ph/0111066

Borasoy and Meissner, hep-ph/9607432

determines if cross section is above or below neutrino background for direct detection

- charm quarks & heavy higgsino dark matter

$$\Sigma_c = m_c \langle N | \bar{c}c | N \rangle$$

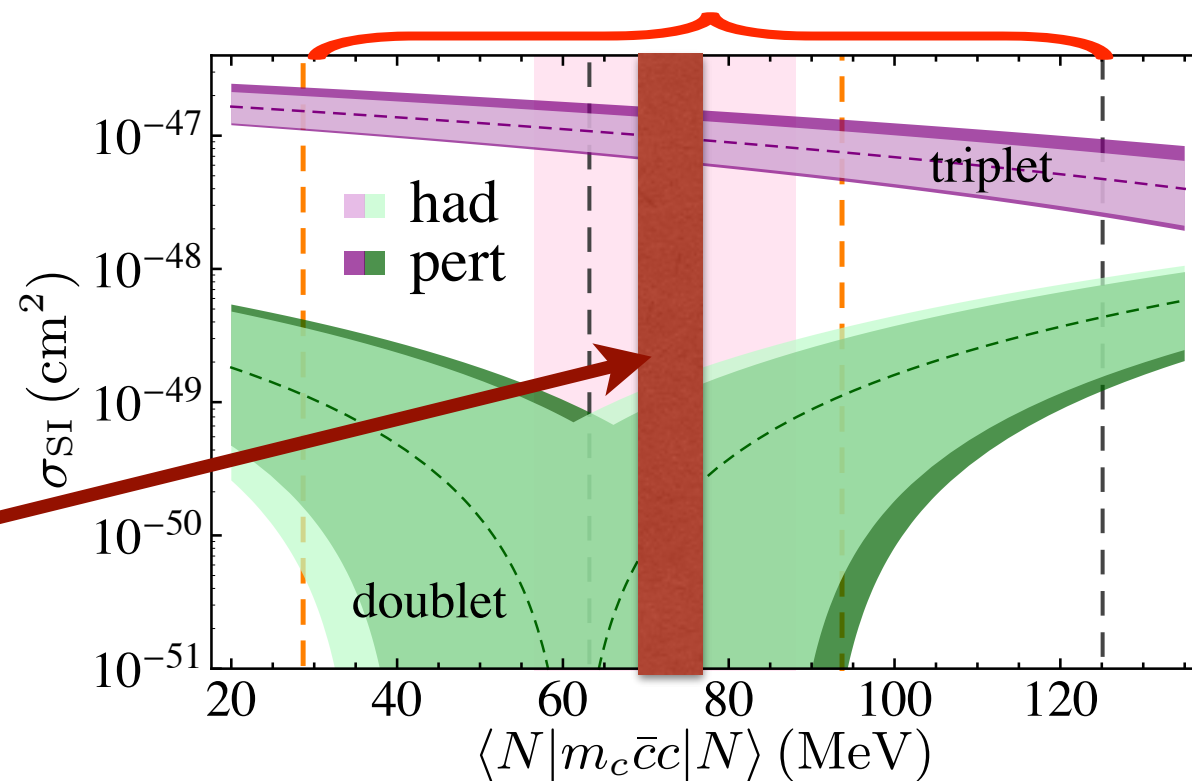
$$= m_N \begin{cases} 0.073(3) & p\text{QCD RJH, Solon 2014} \\ 0.10(3) & \text{Freeman et al [MILC] 1204.3866} \\ 0.07(3) & \text{Gong et al. 1304.1194} \end{cases}$$



- charm quarks & heavy higgsino dark matter

present lattice QCD range

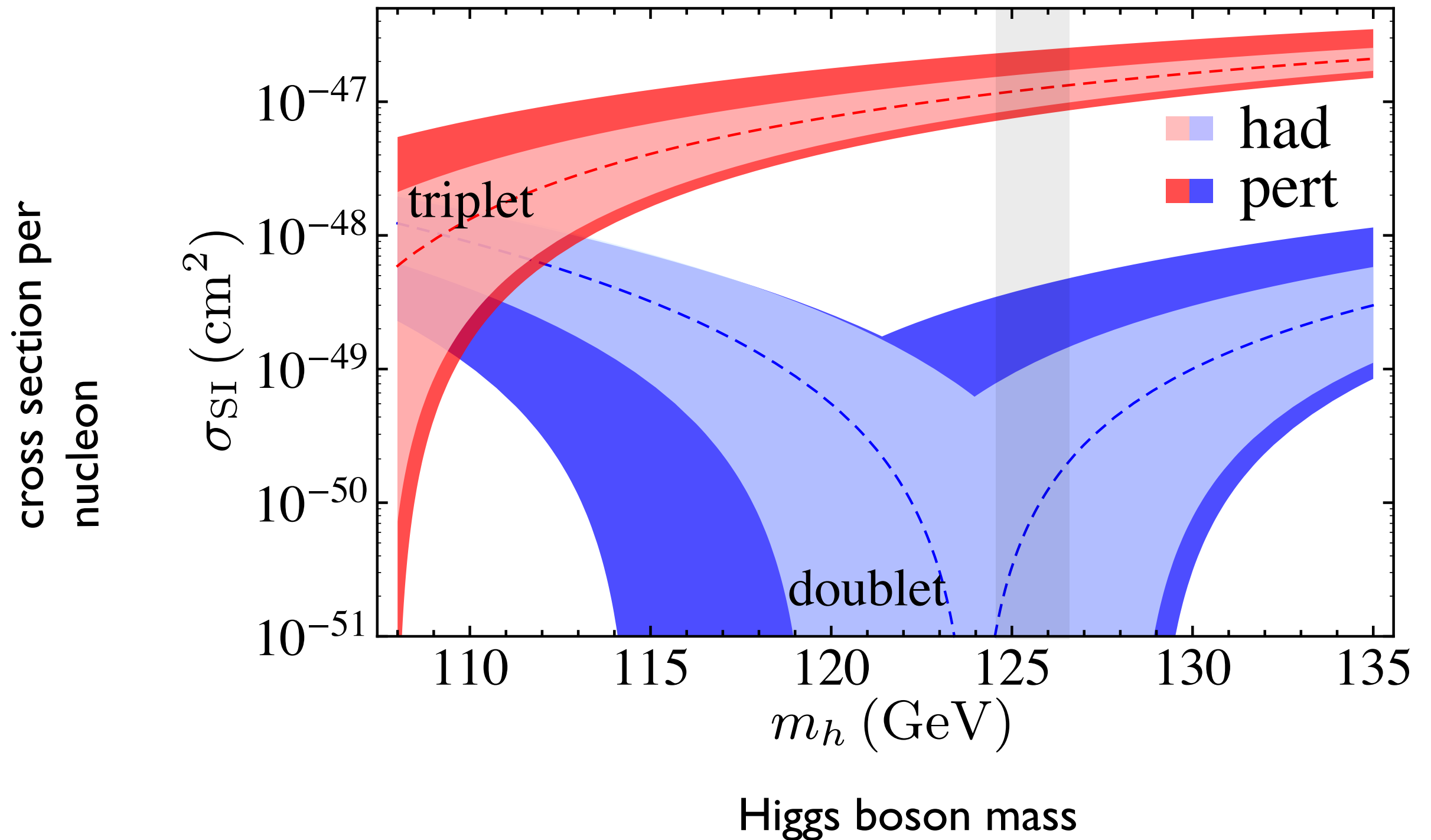
pQCD



$1/m_c$ could potentially shift cancellation region

summary results for heavy electroweak charged WIMP scattering

Solon, RJH, PRL 2014



**other illustrative
examples**

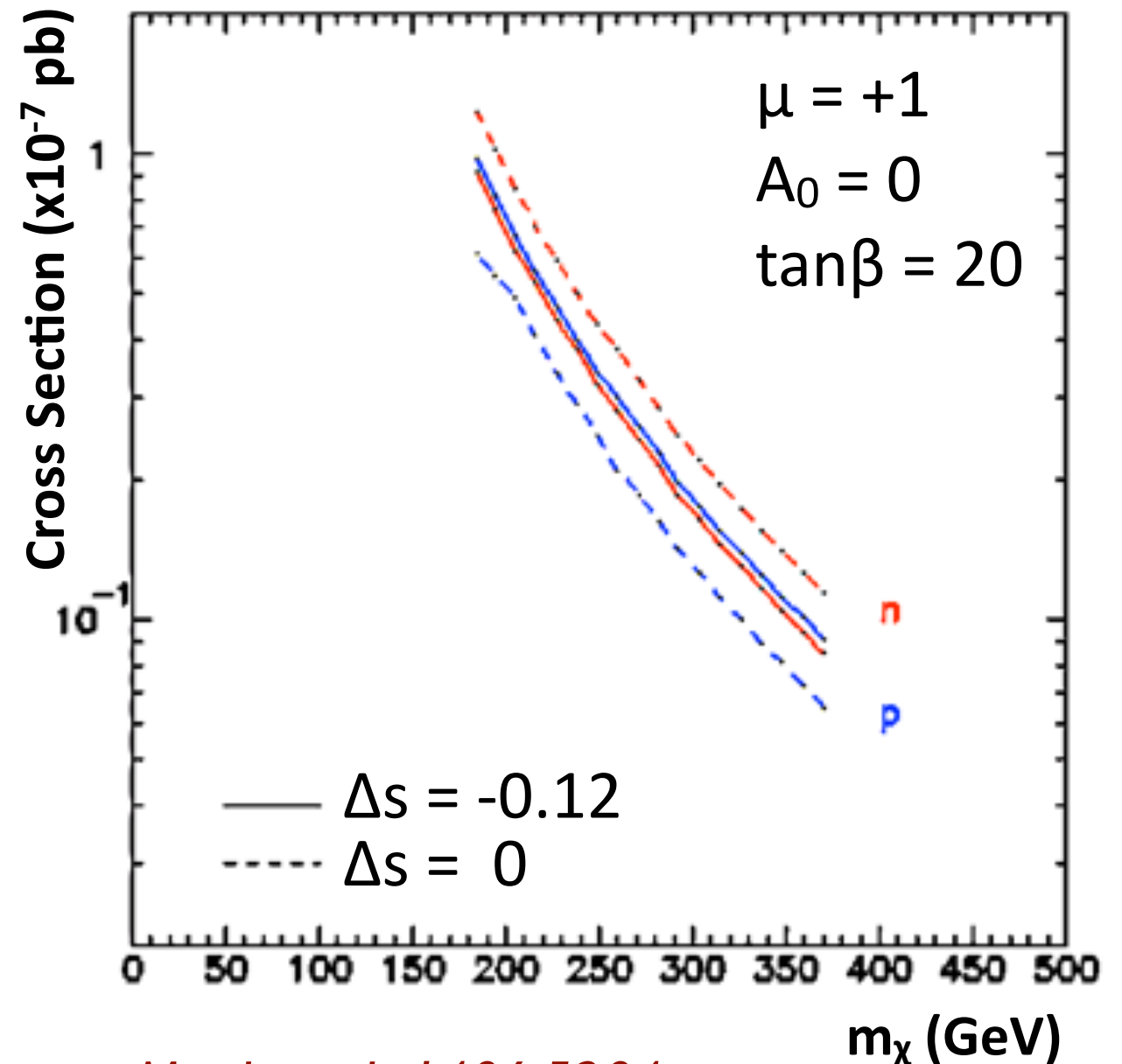
d	QCD operator basis
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- strange component of nucleon spin & spin-dependent neutralino direct detection

- strange component of nucleon spin & spin-dependent neutralino direct detection

$$\langle N | \bar{s} \gamma^\mu \gamma_5 s | N \rangle$$

$$F_A^s(q^2 = 0) = \Delta s$$



Miceli et al., 1406.5204

Relevant, especially post-discovery for spin-dependent cross sections

d	QCD operator basis
3	$V_q^\mu = \bar{q}\gamma^\mu q$ $A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$
4	$T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5 q$ $O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ <div style="border: 2px solid red; padding: 5px; display: inline-block;"> $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5 q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^A G_{\rho\sigma}^A$ </div> $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4}i\cancel{D}_-\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}_\lambda + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_-^{\nu\}}\gamma_5 q$

- flavor singlet pseudoscalar & low-mass WIMPs

- flavor singlet pseudoscalar & low-mass WIMPs

$$\sum_{q=u,d,s} \langle N(k') | \bar{q} i \gamma_5 q | N(k) \rangle \equiv \kappa(q^2, \mu) \bar{u}(k') i \gamma_5 u(k)$$

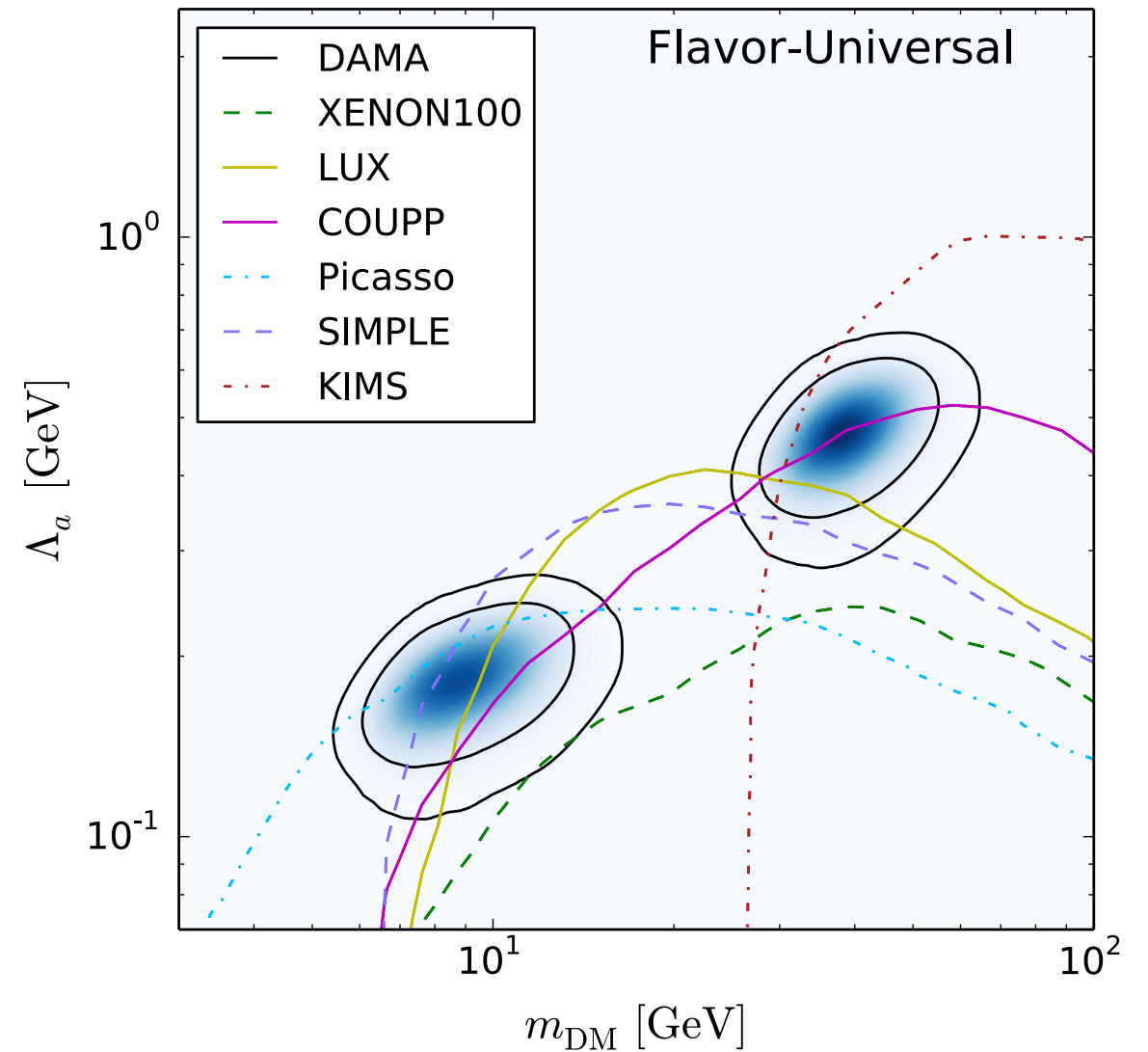
$$\kappa \sim 0?$$

$$\mathcal{L} = g_\chi a \bar{\chi} i \gamma_5 \chi + \sum_q g_f a \bar{q} i \gamma_5 q$$



$$\mathcal{L} \sim \frac{1}{\Lambda^2} \sum_{N=p,n} g_N \bar{\chi} \gamma_5 \chi \bar{N} \gamma_5 N$$

$$|g_p/g_n| \sim 15 - 45$$



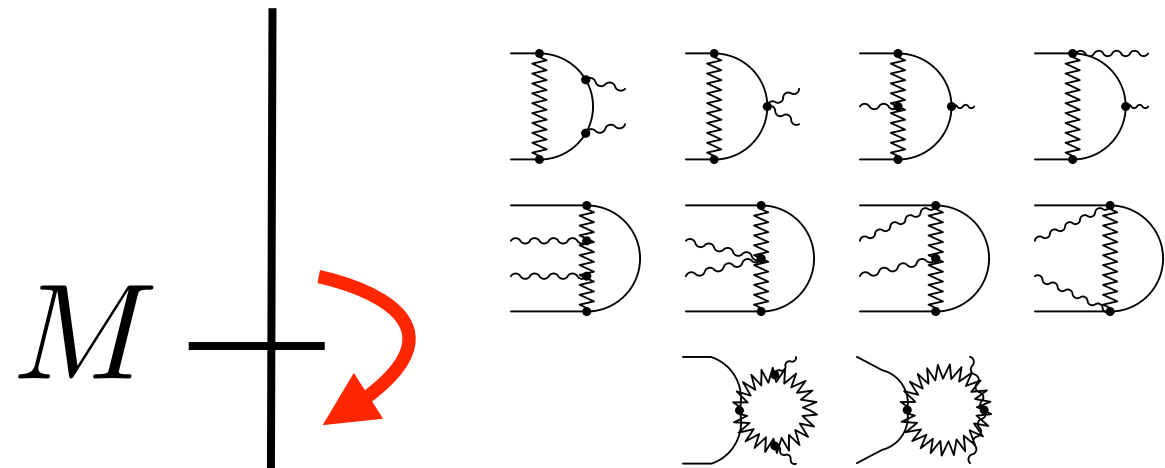
Arina et al. 1406.5542

Impacts tension between experiments

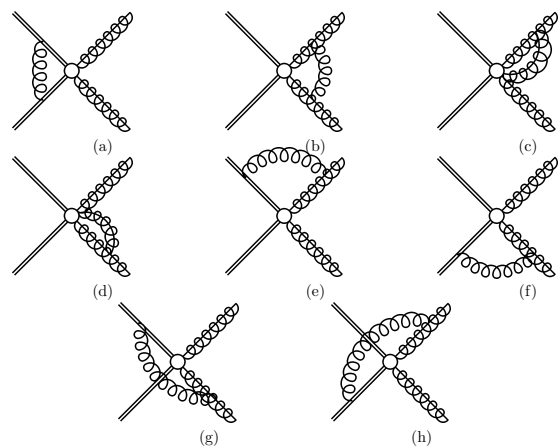
Field theory tools

Extend Heavy WIMP Effective Theory to describe annihilation.

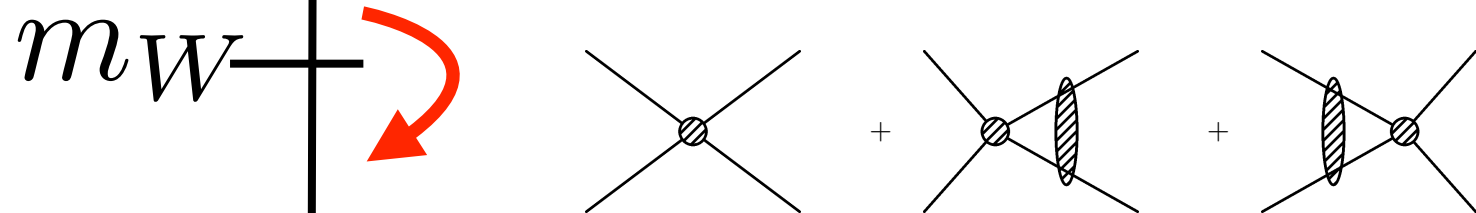
Worked example: SU(2) triplet annihilation to photons



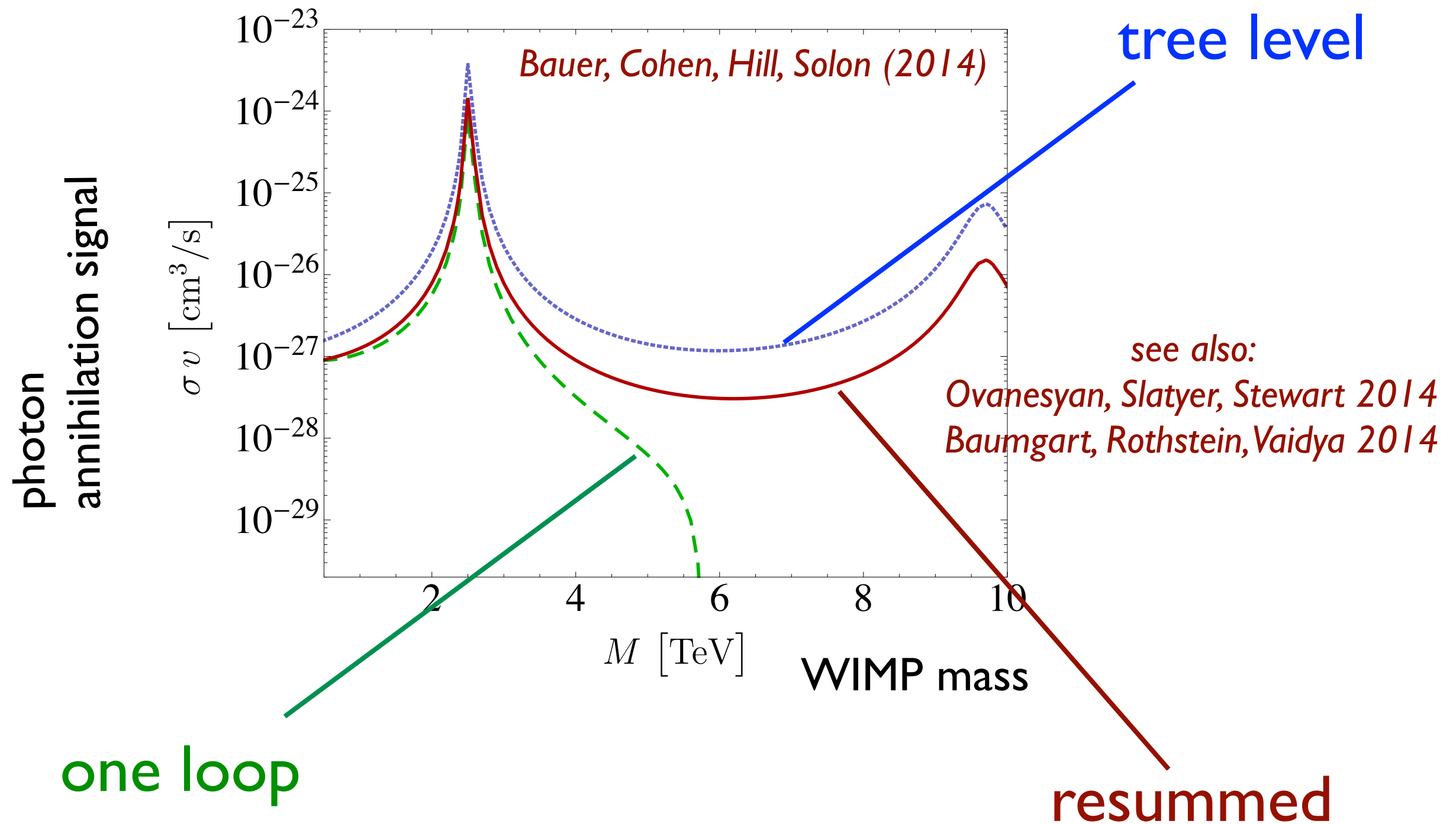
hard annihilation
(makes it happen)



Sudakov suppression
(makes it slower)



Sommerfeld enhancement
(makes it faster)

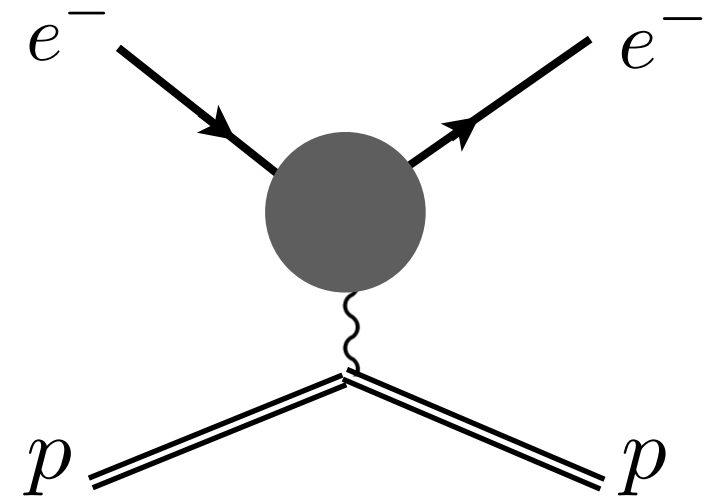


General framework in which to reliably compute annihilation signals for heavy WIMPs.

Novel field theory tools for DM have broad application

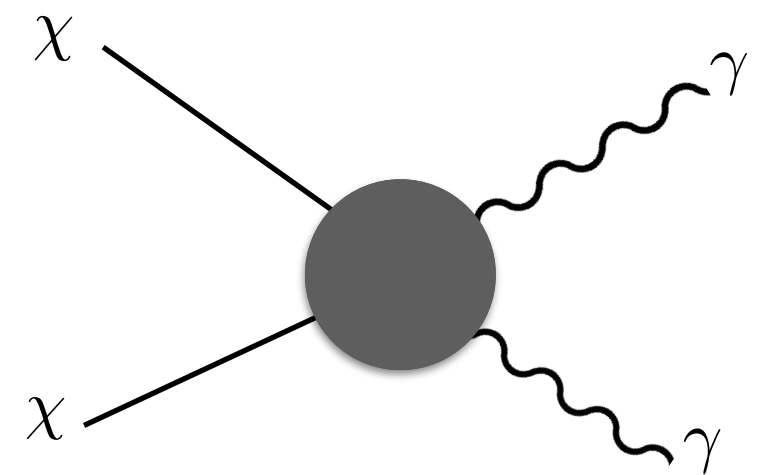
$$\alpha \log^2 \frac{Q^2}{m_e^2} \Big|_{Q^2 = \text{GeV}^2} \approx 1$$

radiative corrections to e-p scattering (proton radius puzzle)



$$\alpha_W \log^2 \frac{M_{\text{DM}}^2}{m_W^2} \Big|_{M_{\text{DM}} = \text{TeV}} \approx 1$$

heavy WIMP annihilation



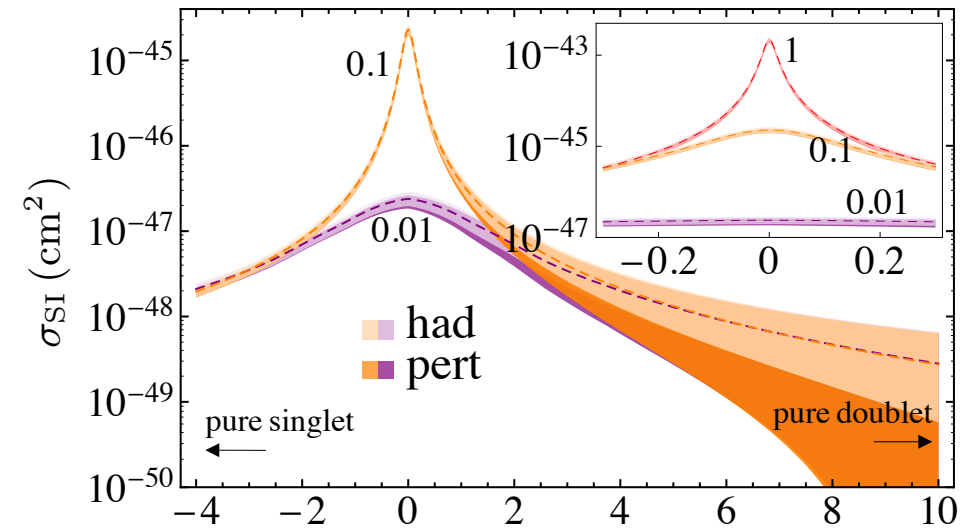
Summary

- QCD corrections are important to dark matter searches
 - determine discovery potential (heavy wino)
 - determine compatibility of potential signals between experiments
- interplay with perturbative and nonperturbative QCD
 - lattice matrix elements
 - high-order decoupling relations
 - novel nuclear responses
- has motivated new field theory tools for particle and nuclear physics

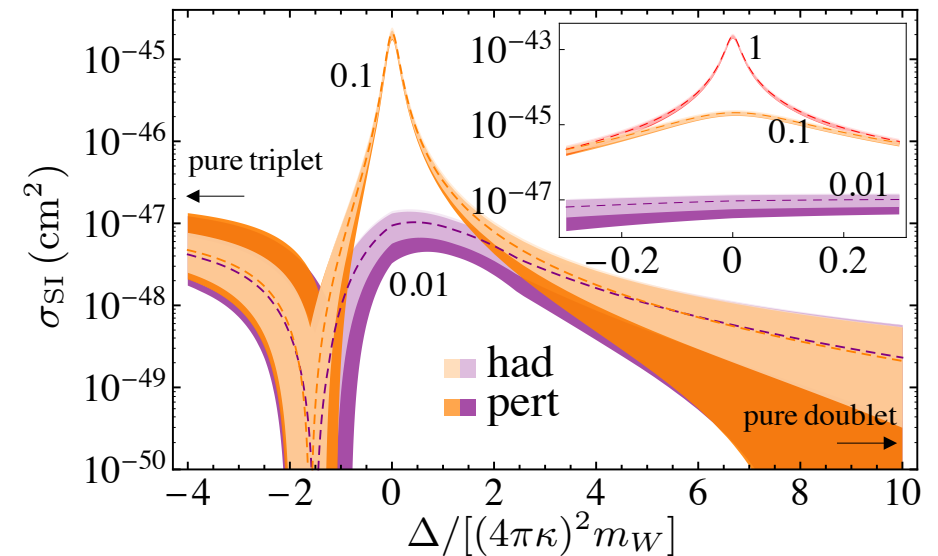
back up

Additional states in the dark sector

singlet-doublet (e.g., bino-higgsino)



triplet-doublet (e.g., wino-higgsino)



Δ : mass splitting of multiplets, in units where tree/loop crossover occurs at ~ 1

interplay of mass-suppressed (tree level) and loop suppressed contributions

Single-nucleon operators

$$\begin{aligned}
\mathcal{L}_{N\chi,PT} = & \frac{1}{m_N^2} \left\{ d_1 N^\dagger \sigma^i N \chi^\dagger \sigma^i \chi + d_2 N^\dagger N \chi^\dagger \chi \right\} + \frac{1}{m_N^4} \left\{ d_3 N^\dagger \partial_+^i N \chi^\dagger \partial_+^i \chi + d_4 N^\dagger \partial_-^i N \chi^\dagger \partial_-^i \chi \right. \\
& + d_5 N^\dagger (\partial^2 + \overleftarrow{\partial}^2) N \chi^\dagger \chi + d_6 N^\dagger N \chi^\dagger (\partial^2 + \overleftarrow{\partial}^2) \chi + id_8 \epsilon^{ijk} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \partial_+^k \chi \\
& + id_9 \epsilon^{ijk} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \partial_-^k \chi + id_{11} \epsilon^{ijk} N^\dagger \partial_+^k N \chi^\dagger \sigma^i \partial_-^j \chi + id_{12} \epsilon^{ijk} N^\dagger \partial_-^k N \chi^\dagger \sigma^i \partial_+^j \chi \\
& + d_{13} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \sigma^i \partial_+^j \chi + d_{14} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^i \partial_-^j \chi + d_{15} N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_+ N \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_+ \chi \\
& + d_{16} N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_- N \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_- \chi + d_{17} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^j \partial_-^i \chi \\
& + d_{18} N^\dagger \sigma^i (\partial^2 + \overleftarrow{\partial}^2) N \chi^\dagger \sigma^i \chi + d_{19} N^\dagger \sigma^i (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) N \chi^\dagger \sigma^j \chi \\
& \left. + d_{20} N^\dagger \sigma^i N \chi^\dagger \sigma^i (\partial^2 + \overleftarrow{\partial}^2) \chi + d_{21} N^\dagger \sigma^i N \chi^\dagger \sigma^j (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) \chi \right\} + \mathcal{O}(1/m_N^6), \quad (')
\end{aligned}$$

Lorentz invariance:

$$\begin{aligned}
rd_4 + d_5 = \frac{d_2}{4}, \quad d_5 = r^2 d_6, \quad 8r(d_8 + rd_9) = -rd_2 + d_1, \quad 8r(rd_{11} + d_{12}) = -d_2 + rd_1 \\
rd_{14} + d_{18} = \frac{d_1}{4}, \quad d_{18} = r^2 d_{20}, \quad 2rd_{16} + d_{19} = \frac{d_1}{4}, \quad r(d_{16} + d_{17}) + d_{19} = 0, \quad d_{19} = r^2 d_{21},
\end{aligned}$$

Light WIMP+ SM

$$\begin{aligned}
\mathcal{L}_{\psi, \text{SM}} = & \frac{c_{\psi 1}}{m_W} \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} + \frac{c_{\psi 2}}{m_W} \bar{\psi} \sigma^{\mu\nu} \psi \tilde{F}_{\mu\nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\psi 3,q}}{m_W^2} \bar{\psi} \gamma^\mu \gamma_5 \psi \bar{q} \gamma_\mu q + \frac{c_{\psi 4,q}}{m_W^2} \bar{\psi} \gamma^\mu \gamma_5 \psi \bar{q} \gamma_\mu \gamma_5 q \right. \\
& + \frac{c_{\psi 5,q}}{m_W^2} \bar{\psi} \gamma^\mu \psi \bar{q} \gamma_\mu q + \frac{c_{\psi 6,q}}{m_W^2} \bar{\psi} \gamma^\mu \psi \bar{q} \gamma_\mu \gamma_5 q + \frac{c_{\psi 7,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} q + \frac{c_{\psi 8,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} q \\
& + \frac{c_{\psi 9,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi 10,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi 11,q}}{m_W^3} \bar{\psi} i \partial_-^\mu \psi \bar{q} \gamma_\mu q \\
& + \frac{c_{\psi 12,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^\mu \psi \bar{q} \gamma_\mu q + \frac{c_{\psi 13,q}}{m_W^3} \bar{\psi} i \partial_-^\mu \psi \bar{q} \gamma_\mu \gamma_5 q + \frac{c_{\psi 14,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^\mu \psi \bar{q} \gamma_\mu \gamma_5 q \\
& \left. + \frac{c_{\psi 15,q}}{m_W^3} \bar{\psi} \sigma_{\mu\nu} \psi m_q \bar{q} \sigma^{\mu\nu} q + \frac{c_{\psi 16,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\psi} \sigma^{\mu\nu} \psi m_q \bar{q} \sigma^{\rho\sigma} q \right\} + \frac{c_{\psi 17}}{m_W^3} \bar{\psi} \psi G_{\alpha\beta}^A G^{A\alpha\beta} \\
& + \frac{c_{\psi 18}}{m_W^3} \bar{\psi} i \gamma_5 \psi G_{\alpha\beta}^A G^{A\alpha\beta} + \frac{c_{\psi 19}}{m_W^3} \bar{\psi} \psi G_{\alpha\beta}^A \tilde{G}^{A\alpha\beta} + \frac{c_{\psi 20}}{m_W^3} \bar{\psi} i \gamma_5 \psi G_{\alpha\beta}^A \tilde{G}^{A\alpha\beta} + \dots,
\end{aligned}$$

Majorana:

$c_{\psi n}$ with $n = 1, 2, 5, 6, 11, 12, 13, 14, 15, 16$ vanish,

Heavy WIMP + SM

$$\begin{aligned}
\mathcal{L}_{\chi_v, \text{SM}} = & \frac{c_{\chi 1}}{m_W} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \chi_v F_{\mu\nu} + \frac{c_{\chi 2}}{m_W} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \chi_v \tilde{F}_{\mu\nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\chi 3,q}}{m_W^2} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_v \sigma_{\perp}^{\nu\rho} \chi_v \bar{q} \gamma^{\sigma} q \right. \\
& + \frac{c_{\chi 4,q}}{m_W^2} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_v \sigma_{\perp}^{\nu\rho} \chi_v \bar{q} \gamma^{\sigma} \gamma_5 q + \frac{c_{\chi 5,q}}{m_W^2} \bar{\chi}_v \chi_v \bar{q} \psi q + \frac{c_{\chi 6,q}}{m_W^2} \bar{\chi}_v \chi_v \bar{q} \psi \gamma_5 q + \frac{c_{\chi 7,q}}{m_W^3} \bar{\chi}_v \chi_v m_q \bar{q} q \\
& + \frac{c_{\chi 8,q}}{m_W^3} \bar{\chi}_v \chi_v \bar{q} \psi i v \cdot D_{-} q + \frac{c_{\chi 9,q}}{m_W^3} \bar{\chi}_v \chi_v m_q \bar{q} i \gamma_5 q + \frac{c_{\chi 10,q}}{m_W^3} \bar{\chi}_v \chi_v \bar{q} \psi \gamma_5 i v \cdot D_{-} q \\
& + \frac{c_{\chi 11,q}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} i \partial_{-\mu}^{\perp} \chi_v \bar{q} \gamma_{\nu} q + \frac{c_{\chi 12,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} i \partial_{-\rho}^{\perp} \chi_v \bar{q} \gamma^{\sigma} q + \frac{c_{\chi 13,q}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} i \partial_{-\mu}^{\perp} \chi_v \bar{q} \gamma_{\nu} \gamma_5 q \\
& + \frac{c_{\chi 14,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} i \partial_{-\rho}^{\perp} \chi_v \bar{q} \gamma^{\sigma} \gamma_5 q + \frac{c_{\chi 15,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_v \sigma_{\perp}^{\nu\rho} \chi_v \bar{q} (\psi i D_{-}^{\sigma} + \gamma^{\sigma} i v \cdot D_{-}) q \\
& + \frac{c_{\chi 16,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} v^{\mu} \bar{\chi}_v \sigma_{\perp}^{\nu\rho} \chi_v \bar{q} (\psi i D_{-}^{\sigma} + \gamma^{\sigma} i v \cdot D_{-}) \gamma_5 q + \frac{c_{\chi 17,q}}{m_W^3} \bar{\chi}_v i \partial_{-}^{\perp \mu} \chi_v \bar{q} \gamma_{\mu} q \\
& + \frac{c_{\chi 18,q}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \partial_{+\mu}^{\perp} \chi_v \bar{q} \gamma_{\nu} q + \frac{c_{\chi 18,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \partial_{+}^{\perp \rho} \chi_v \bar{q} \gamma^{\sigma} q + \frac{c_{\chi 20,q}}{m_W^3} \bar{\chi}_v i \partial_{-}^{\perp \mu} \chi_v \bar{q} \gamma_{\mu} \gamma_5 q \\
& + \frac{c_{\chi 21,q}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \partial_{+\mu}^{\perp} \chi_v \bar{q} \gamma_{\nu} \gamma_5 q + \frac{c_{\chi 22,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \partial_{+}^{\perp \rho} \chi_v \bar{q} \gamma^{\sigma} \gamma_5 q + \frac{c_{\chi 23,q}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \chi_v m_q \bar{q} \sigma_{\mu\nu} q \\
& + \left. \frac{c_{\chi 24,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \chi_v m_q \bar{q} \sigma^{\rho\sigma} q \right\} + \frac{c_{\chi 25}}{m_W^3} \bar{\chi}_v \chi_v G_{\alpha\beta}^A G^{A\alpha\beta} + \frac{c_{\chi 26}}{m_W^3} \bar{\chi}_v \chi_v G_{\alpha\beta}^A \tilde{G}^{A\alpha\beta} \\
& + \frac{c_{\chi 27}}{m_W^3} \bar{\chi}_v \chi_v v_{\mu} v_{\nu} G_{\alpha}^{A\mu} G^{A\nu\alpha} + \frac{c_{\chi 28}}{m_W^3} \bar{\chi}_v \sigma_{\perp}^{\mu\nu} \chi_v \epsilon_{\mu\nu\alpha\beta} v^{\alpha} v^{\gamma} G^{A\beta\delta} G_{\gamma\delta}^A + \dots, \tag{7}
\end{aligned}$$

Lorentz:

$$\frac{m_W}{M} c_{\chi 3} + 2c_{\chi 12} = \frac{m_W}{M} c_{\chi 4} + 2c_{\chi 14} = \frac{m_W}{M} c_{\chi 5} - 2c_{\chi 17} = \frac{m_W}{M} c_{\chi 6} - 2c_{\chi 20} = c_{\chi 11} = c_{\chi 13} = 0,$$

Majorana:

$c_{\chi n}$ vanish for $n=1, 2, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24$.