QCD, Effective theories and dark matter

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INT, Seattle 2 October, 2015

based on work with M.P. Solon: (Sakurai thesis award) Heavy WIMP Effective Theory 1111.0016, 1309.4092, PRL Standard Model Anatomy of WIMP Direct Detection I, II 1401.3339, 1409.8290, PRD

Thanks to co-organizers and participants of MITP program "Effective Theories and Dark [Matter", March 2015: https://indico.mitp.uni-mainz.de/conferenceDisplay.py?](https://indico.mitp.uni-mainz.de/conferenceDisplay.py?ovw=True&confId=25) ovw=True&confId=25

[See also INT workshop "Nuclear aspects of DM searches", December 2014:](http://www.int.washington.edu/PROGRAMS/14-57w/) http:// www.int.washington.edu/PROGRAMS/14-57w/

The dark matter problem $\sqrt{1}$ ⇥ $\sqrt{1 \ G \ M_{gal} \ m_{*} \ \ }}$ GM_{gal} m_* m_{\ast} v^2 v_c^2 VELOCITY IN PLANE OF GALAXY (km s⁻¹) $\frac{1}{\sqrt{G}}\frac{M(r)}{2}$ $\overline{\mathcal{Z}}$ r_{*} *r*2 *r* 250 200 VIMP $1/6$ ⇥ $\widehat{\bar{\mathbb{M}}}$ ⇥ $g_{al} m_*$ p _{*g*} \leq *n* R T _g *^m*⇥ *^v*² \equiv $\frac{1}{2}$ is light and $\frac{1}{2}$ is a massive neutrino: produced when $\frac{1}{2}$ is an interval pair and $\frac{1}{2}$ is an interval pair and $\frac{1}{2}$ is a massive neutrino: produced when $\frac{1}{2}$ is a massive neutrino: \frac 2 2

rownic hackground. i Owave-Dachgi Ouliu most notable improvements are the measurements of the dark imprints on microwave background apparent extra collisionless modification of galactic rotation imprints on misrowayo basksround $\frac{d\mu g}{d\sigma} = \frac{p_g \sigma \mu g_d \sin \rho g}{2}$ imprints on inicrowave

WMAP 5-year

apparent extra collisionless contours show the positions of the property collisions of the corresponding to 68.3%, show the blue to 68.3%, α matter from lensing ma sculing constants. In the scaling constant of the scaling constant in the scaling constant of the scaling c measurements

creation. Reaction maintains thermal equilibrium.

 r_{*}

 $\frac{\partial}{\partial g} G = M g_{ab}$

*r*2

curves

dp^g

dr

 ω_l interacting τ

 $\frac{r}{\sqrt{r}}$

 \mathcal{L} are a directed ellipticities are a directed ellipticities are a directed ellipticity. 2

τ **The dark matter problem**

Three motivations for studying QCD & DM

• important, sometimes dramatic, impact on discovery potential

• post-discovery interpretation and/or anomaly debunking

• new field theory tools

Some themes in the contemporary particle physics:

- Λ*New Physics > mW (maybe >> mW)*
- *interplay of particle, astro, nuclear*
- *lattice QCD and baryon matrix elements*
- interplay of radiative corrections and hadronic structure

Compelling physics questions demand analysis outside the comfort zone of any one field.

Dark Matter applications a prime example

s/cosmology *Interplay of particle-, nuclear-, astro-physics/cosmology*

Figure 13.10: The effective Mass Including mass FeVI α is a function of minimental method α . lightest neutrino mass [eV]

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. Interplay of particle_s nuclear-, astro-physics/cosmol *Interplay of particle-, nuclear-, astro-physics/cosmology*

 $s \in \mathcal{S}$ experience in published with published in the public set of the preliminary contribution of th $s \in \mathcal{S}$ experience in angle of set • Wide range of searches with overlapping constraints

your favorite curve is not here!

Tia\$Miceli Tia\$Miceli 4 4

Not quibbling about percents collider production (example 2: light WIMPs) **SM** DM complementarity: connect direct detection and direct 0.5 collider phenomenology detection \mathbb{R}^5 0.0 SM $\sim m_t$ 1 $\emptyset.0$ $f^{-0.5}$
*f*_{*n*} -6.5 -0.5 $\mu \sim m_t$ \mathbf{e} $f_n/f_p \approx -Z/(A-2) \approx -0.7$ f_n/f_p $\}$ 7_A .0 engineered to Peconcile DAMA S*mu* ^ê*md* -1.5 -1 M eamd_cother with results from X e and other -1.5 nuclei $\mu \sim m_c$ -2.0 $\frac{1}{2.0}$ $\frac{\mu \sim m_c}{1.5}$ + 1.0 $-2\frac{1}{0.0}$ 0.5 1.0 1.5 $\begin{array}{ccc} 0.0 & \hspace{1.5cm} 0.5 & \hspace{1.5cm} 1.0 & \hspace{1.5cm} 1.5 \ -b_u/b_d & \end{array}$ -2.0 -1.5 -1.0 $-b_u/b_d$ -2.0 -1.5 -1.0 *bu/b^d b^g* $\mathcal{L}_{\chi, \text{SM}} = \bar{\chi} \chi \left[b_u \bar{u} u + b_d \bar{d} d \right]$ Figure 1: The ratio *fn/f^p* of the e↵ective WIMP-neutron (*fn*) and WIMP-proton (*fp*) couplings in

Solution: $b_u/b_d = -0.9$

France 1: The vertex of the respective interest in the extendior of the energy of the equation ($\frac{1}{2}$ terms of the parameters *bⁱ* in Eq. (91). For *b^g* = 0 (left panel), *fn/f^p* is independent of ⇤ and depends *Figure 1: The ratio are allowed the anti-dividence with and ferromanization searcy* However, must account for uncertainties (hadronic and renormalization scale)


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Solution: b_u/b_d = -0.9
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However, must account for uncertainties (hadronic and renormalization scale)

cf. b_u/b_d =-1.08 from "isospin-violating" DM

Assumed one-to-one manning between b. /b. and f. /f. invalid terms of the parameters *bⁱ* in Eq. (91). For *b^g* = 0 (left panel), *fn/f^p* is independent of ⇤ and depends Assumed one-to-one mapping between b_u/b_d and f_n/f_p invalid

Nontrivial mapping from colliders to direct detection and the ratio *Rud* = *mu/m^d* (red), with ranges given in (58) and (60). We illustrate the e↵ect of

 M_{U} the first non-vanishing *^O*(↵4) contribution to *^w*00, is shown in dashed green. The LL resummed result including one-loop matching one-loop matching coefficients at the high and weak scales and results and \mathbf{r} contribution, is shown in solid red. 10 Multi-scale field theory problem, breakdown of naive perturbation theory

Heavy WIMP effective theory

Mechanisms versus models

Electroweak charged WIMP Mechanism versus WIMP Model

Focus on self-conjugate SU(2) triplet. Could be:

- SUSY wino

- ...

- Weakly Interacting Stable Pion
- Minimal Dark Matter

Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale

If WIMP mass M \gg m_W, isolation (M'-M \gg m_W) becomes generic. Expand in m_W/M, m_W/(M'-M)

Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes

14

Many manifestations of heavy particle symmetry:

prediction: small parameter:

- hydrogen/deuterium spectroscopy $E_n(H) = -\frac{1}{2}$

 $\overline{}$

$$
E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \qquad (m_e Z\alpha) \ll m_e
$$

Many manifestations of heavy particle symmetry:

- hydrogen/deuterium spectroscopy *En*(*H*) = ¹ 2 *me*(*Z*↵) ² + *...* (*meZ*↵) ⌧ *m^e* prediction: small parameter: *e ^F ^B*!*^D*(*v*⁰ - heavy meson transitions ⁼ *^v*)=1+ *...* ⇤QCD ⌧ *^mb,c* ⌫¯

 $\overline{}$

Many manifestations of heavy particle symmetry:

\n prediction: \n $E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots$ \n $(m_e Z\alpha) \ll m_e$ \n\n
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MANAVY

• the heavy lifting is necessary

Perturbative QCD

Dark matter - Standard Model interactions

$$
\mathcal{L} = \frac{1}{\Lambda^n} O_{\rm DM} \times O_{\rm SM}
$$

} complete QCD basis for d≤7

Renormalization and matching (sample): <u>then of manzacion and macenting</u> (sample). *<u>Renormalization and</u>* <u>Kenormalization and match</u> The sum rule relating matrix elements ^h*O*0(*S*) *ⁱ* i in a theory with *n^f* + 1 flavors has the analogous form. enormalization and matching (sample): which is a system of the system of the system of the equations which is \sim on the matrices *R*(0) and *M*(0). In the following, we drop the superscript (0) for brevity. In the case Ronormalization and matching (cample): evaluate matrix elements (surreshold materials through the conditions of the c

$$
\mathcal{L}_{\phi_0,\text{SM}} = \frac{1}{m_W^3} \phi_v^* \phi_v \Bigg\{ \sum_q \left[c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_\mu v_\nu O_{1q}^{(2)\mu\nu} \right] + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_\mu v_\nu O_2^{(2)\mu\nu} \Bigg\} + \dots
$$
\n
$$
m_q \bar{q} q
$$
\nfocus on spin-0 (evaluate spin-2 at weak scale)

 ^g^µ⇧*iD/* ⇥ ⇥*{^µiD*⇧*}* ¹ focus on spin-0 (evaluate spin-2 at weak scale) us on spin-0 (evaluate spin-2 at weak scale) $\overline{}$ p_P matrix of p_P in terms of p_P and p_P are matching conditions to match p_P

d d A^{*B*} *A*^{*B*} *<i>P*_{**B**} $\frac{1}{2}$ *A*^{*B*} + *A* larization with *d* = 4 2⇤ the spacetime dimension. We use the background field method Renormalization group evolution from weak scale to had
perturbative corrections at heavy quark mass thresholds $\overline{}$ 2 *q*¯ *^gµ*⌫ ⁴ *iD/ ^g* ⁼ *GAµG^A*⌫ ⁴ (*G^A* ⁺ *^gµ*⌫ ↵)² Renormalization group evolution from weak scale to hadronic scales, with 2
2
2 ˜(*µ*) *Rgg* = ˜(*µh*) *, Rqg* ² ˜(*µ*) \mathbf{m} weak scale to ˜(*µh*) ⇥ 1 *m*(*µh*) ⇤ *.* (32) *i*h*O*⁰ *ⁱ*i = *ci*h*Oi*i + *O*(1*/mQ*)*,* (26) where prime design and the contract of the contract of the contract of the second theories, respectively. 8 Let

$$
c_i(\mu_Q) = M_{ij}(\mu_Q) c'_j(\mu_Q)
$$

$$
M(\mu_Q) = \begin{pmatrix} 1 & M_{qq} - M_{qq'} \\ \frac{1}{M_{qq} - M_{qq'}} & M_{qq'} \end{pmatrix} \begin{pmatrix} M_{qq} & M_{qg} \\ \vdots & \vdots \\ M_{qq} & M_{gg} \end{pmatrix}
$$

stymmetrization, and the subscript \mathcal{L} denotes an active quark flavor. The antisymmetric tensor current c Can show that:

$$
M_{qq} \equiv 1, \quad M_{qq'} \equiv 0, \quad M_{gq} \equiv 0
$$

M_{gQ} and M_{qQ} known through coupling constant in the M_{gQ} 3 loops: *^s*) results for *MgQ* and *MqQ* from Ref. [48], we may solve the relations in Eq. (1997) order by order by order. $Chetyrkin$ et al. (1997) M s and M s known through theory. Employing this ↵*^s* counting and the *^O*(↵⁴ s loops: ϵ order by ϵ order by ϵ and ϵ (1007) *M*_g and *M*^g and *M*_g and *M* M_{gQ} and M_{qQ} known through 3 loops: *Chetyrkin et al. (1997)*

 $\frac{1}{2}$ found in the set of the mass *mass induced decoupling relations* the through NLO, we recover the recovered for σ New results for gluon-induced decoupling relations

$$
M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6} \log \frac{\mu_Q}{m_Q} + \frac{1}{9} \log^2 \frac{\mu_Q}{m_Q}
$$

$$
M_{gg}^{(3)} = \frac{564731}{41472} - \frac{2821}{288} \log \frac{\mu_Q}{m_Q} + \frac{3}{16} \log^2 \frac{\mu_Q}{m_Q} - \frac{1}{27} \log^3 \frac{\mu_Q}{m_Q} - \frac{82043}{9216} \zeta(3)
$$

+
$$
n_f \left[-\frac{2633}{10368} + \frac{67}{96} \log \frac{\mu_Q}{m_Q} - \frac{1}{3} \log^2 \frac{\mu_Q}{m_Q} \right],
$$

$$
M_{gg}^{(2)} = -\frac{89}{54} + \frac{20}{9} \log \frac{\mu_Q}{m_Q} - \frac{8}{3} \log^2 \frac{\mu_Q}{m_Q}.
$$

(*n^f* + 1)-flavor theory in terms of those in the *n^f* -flavor theory, up to power corrections. Employing

• the heavy lifting is necessary

Hadronic matrix elements

- pseudoscalar *n* For canonical example (heavy electro D *weak* mult (*AµB*⌫ *^A*⌫*Bµ*)*/*2 and *^A{µB*⌫*}* ⌘ (*AµB*⌫ ⁺ *^A*⌫*Bµ*)*/*2 respectively denote antisymmetrization and scalar operators and the subscript α *T^q* and the quark pseudoscalar operator *O*(0) • For canonical example (heavy electroweak multiplet),
- Selected other examples basis that is closed under renormalization and contains all operators not forbidden by symmetry.

d
\nQCD operator basis
\n3
\n
$$
V_q^{\mu} = \bar{q}\gamma^{\mu}q
$$
\n
$$
A_q^{\mu} = \bar{q}\gamma^{\mu}\gamma_5q
$$
\n4
\n
$$
T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5q
$$
\n6
\n
$$
O_q^{(0)} = m_q\bar{q}q, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}
$$
\n
$$
O_q^{(0)} = m_a\bar{q}i\gamma_5q, \quad O_g^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^A G^{A\mu}
$$
\n
$$
O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}i}D^{\nu\}}_{} - \frac{g^{\mu\nu}}{4}i\rlap{\,/}{D}_{-}\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2
$$
\n
$$
O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}i}D^{\nu\}}_{}^{}\gamma_5q
$$

• C-even spin-2[·] determined by PDF moments *a*_[*n*</sup> *, O*(2) *a*_{*a*}₂ *c*_o₁ *c*₁ *c*₁ *a*₂ • C-even spin-2: determined by PDF moments

$$
\langle N|O^{(2)\mu\nu}|N\rangle = k^{\mu}k^{\nu}\int_0^1 dx\,x[q(x) + \bar{q}(x)]
$$

d	QCD operator basis
3	$V_{q}^{\mu} = \bar{q}\gamma^{\mu}q$
4	$A_{q}^{\mu} = \bar{q}\gamma^{\mu}\gamma_{5}q$
4	$T_{q}^{\mu\nu} = im_{q}\bar{q}\sigma^{\mu\nu}\gamma_{5}q$
$O_{q}^{(0)} = m_{q}\bar{q}q, \quad O_{g}^{(0)} = G_{\mu\nu}^{A}G^{A\mu\nu}$	
$O_{5q}^{(0)} = m_{q}\bar{q}i\gamma_{5}q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^{A}G_{\rho\sigma}^{A}$	
$O_{q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}i}D_{-}^{\nu\}} - \frac{g^{\mu\nu}}{4}i\rlap{\,/}D_{-}\right)q, \quad O_{g}^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^{A})^{2}$	
$O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}i}D_{-}^{\nu\}}\gamma_{5q}$	

• C-even spin-0: nucleon sigma terms (nucleon ma pseudoscalar $\sum_{i=1}^{n}$ $\frac{1}{2}$ $\frac{1}{2}$ *O*(2) (*AµB*⌫ *^A*⌫*Bµ*)*/*2 and *^A{µB*⌫*}* ⌘ (*AµB*⌫ ⁺ *^A*⌫*Bµ*)*/*2 respectively denote antisymmetrization and • C-even spin-0: nucleon sigma terms (nucleon mass sum rule for gluon operator)

$$
m_N=(1-\gamma_m)\sum_q \langle N|m_q\bar{q}q|N\rangle +\frac{1}{2}\beta \langle N|(G_{\mu\nu}^a)^2|N\rangle
$$

• up, down quarks & isospin-violating dark matter *^s*) contributions to ˜. In Sec. 6, we will see that corrections to the leading \bullet up, down quarks $\&$ isospin-violating dark matter $\frac{1}{2}$ up. down guarks & isospin-vio

⌃⇡*^N* = *m^u* + *m^d* ² ^h*N|*(¯*uu* ⁺ ¯ *dd*)*|N*i = 44(13) MeV *,* ⌃ = (*m^d ^mu*)h*N|*(¯*uu* ¯ *dd*)*|N*i = *±*2(2) MeV *,* (58) where the upper (lower) sign in ⌃ is for the proton (neutron) [70]. The numerical value for the pion-nucleon sigma term ⌃⇡*^N* is the lattice result from Ref. [71] with errors symmetrized. For the strange scalar nucleon matrix element, we use the updated lattice result *m^N f*(0) *s,N* = 40*±*20 MeV from Ref. [72], where we assume a conservative 50% uncertainty compared to their estimate of 25%. For models with identical couplings to up and down quarks, it is sucient to take as input ⁼ ⌃⇡*^N* ⌃*/*² ⇡ ⌃⇡*^N* , neglecting the small contribution from ⌃. For general applications requiring separately the up and down quark scalar matrix elements let us write *m^N* 1 1 + *Rud m^N* (1 ⇠)*,* ⇠ = 1 + *Rud* 1 *Rud* ⌃ 21 *dd*)*|N*i = 44(13) MeV *, dd*)*|N*i = *±*2(2) MeV *,* (58) where the upper (lower) sign in ⌃ is for the proton (neutron) [70]. The numerical value for the pion-nucleon sigma term ⌃⇡*^N* is the lattice result from Ref. [71] with errors symmetrized. For the strange scalar nucleon matrix element, we use the updated lattice result *m^N f*(0) *s,N* = 40*±*20 MeV from Ref. [72], where we assume a conservative 50% uncertainty compared to their estimate of 25%. For models with identical couplings to up and down quarks, it is sucient to take as input ⁼ ⌃⇡*^N* ⌃*/*² ⇡ ⌃⇡*^N* , neglecting the small contribution from ⌃. For general applications requiring separately the up and down quark scalar matrix elements let us write ⌃⇡*^N* (1 ⇠)*,* ⇠ = 1 + *Rud* 1 *Rud* ⌃ 2⌃⇡*^N ,* (59) *Durr et al. 1109.4265* Ú Ú ‡ ‡ ‡ ‡ ‡ ‡ ‡ ‡ ‡ Ê Ê Ê Ê Ê Ê Ê 0 20 40 60 80 100 0 20 40 60 80 100 Fukugita *et al.* 1995 Dong *et al.* 1996 SESAM 1998 Leinweber *et al.* 2000 Leinweber *et al.* 2003 Procura *et al.* 2003 Procura *et al.* 2006 ETM 2008 JLQCD 2008 QCDSF 2011 QCDSF 2012 Young & Thomas 2009 PACS-CS 2009 Martin-Camalich *et al*. 2010 Dürr *et al*. 2011 QCDSF-UKQCD 2011 Shanahan *et al*. 2012 Ren *et al*. 2012 ^s*^l* ^HMeV^L *from R. Young, 1301.1765 ^s*) contributions to ˜. In Sec. 6, we will see that corrections to the leading order relation are numerically important in the case of electroweak-charged WIMPs. We may extract the up and down quark scalar nucleon matrix elements from the scale-invariant *m^u* + *m^d* ² ^h*N|*(¯*uu* ⁺ ¯ *dd*)*|N*i = 44(13) MeV *,* ⌃ = (*m^d ^mu*)h*N|*(¯*uu* ¯ *dd*)*|N*i = *±*2(2) MeV *,* (58) where the upper (lower) sign in ⌃ is for the proton (neutron) [70]. The numerical value for the pion-nucleon sigma term ⌃⇡*^N* is the lattice result from Ref. [71] with errors symmetrized. For the strange scalar nucleon matrix element, we use the updated lattice result *m^N f*(0) *s,N* = 40*±*20 MeV from Ref. [72], where we assume a conservative 50% uncertainty compared to their estimate of 25%. For models with identical couplings to up and down quarks, it is sucient to take as input ⁼ ⌃⇡*^N* ⌃*/*² ⇡ ⌃⇡*^N* , neglecting the small contribution from ⌃. For general applications requiring separately the up and down quark scalar matrix elements let us write ⌃⇡*^N* (1 + ⇠)*, f*(0) 1 + *Rud* ⌃⇡*^N m^N* (1 ⇠)*,* ⇠ = 1 + *Rud* ⌃ *,* (59) *q,N* , the scale dependence is implicit, and the second equality is obtained by *^s*) contributions to ˜. In Sec. 6, we will see that corrections to the leading order relation are numerically important in the case of electroweak-charged WIMPs. We may extract the up and down quark scalar nucleon matrix elements from the scale-invariant *dd*)*|N*i = 44(13) MeV *, dd*)*|N*i = *±*2(2) MeV *,* (58) where the upper (lower) sign in ⌃ is for the proton (neutron) [70]. The numerical value for the pion-nucleon sigma term ⌃⇡*^N* is the lattice result from Ref. [71] with errors symmetrized. For the strange scalar nucleon matrix element, we use the updated lattice result *m^N f*(0) *s,N* = 40*±*20 MeV from Ref. [72], where we assume a conservative 50% uncertainty compared to their estimate of 25%. For models with identical couplings to up and down quarks, it is sucient to take as input ⁼ ⌃⇡*^N* ⌃*/*² ⇡ ⌃⇡*^N* , neglecting the small contribution from ⌃. For general applications requiring separately the up and down quark scalar matrix elements let us write *Gasser, Leutwyler (1982)* = *±*2(1)MeV *Crivellin, Hoferichter, Procura (2014) q f*(0) *q,p f*(0) *u* 0.016(5)(3)(1) 0*.*014(5)(+2 *d* 0.029(9)(3)(2) 0*.*034(9)(+3 Table 10: Scale independent scalar form factors for the proton and neutron for light quark flavors *u, d, s*. The first, second and third uncertainties are from ⌃⇡*^N* , *mu/m^d* and ⌃, respectively. As discussed below Eq. (60), the parameterization in Eq. (59) leads to highly correlated uncertainties in where we employ the quark mass ratios adopted from PDG values [68] (symmetrizing errors), *m^u m^d* = 0*.*49 *±* 0*.*13 *, Rsd* ⌘ *m^s m^d* = 19*.*5 *±* 2*.*5 *.* (60) The resulting numerical values for the light quark scalar matrix elements are collected in Table 10. *PDG*

1 + *Rud*

⌃

d, No. 2007, and for a property of a property and for a property of a property of

 $\overline{f_{\mu\nu}}$

namical flavours of quarks: green is *Nf* = 0, blue *Nf* = 2 and red *Nf* 2 + 1. References: Fukugita *et*

• up, down quarks & isospin-violating dark matter *mu* ^ê*md* $-2\frac{1}{0.0}$ 0.5 1.0 1.5 \mathbf{e}^{\dagger} -2.0 -1.5 -1.0 $-0.$ 0.0 $\overline{0}$. $-b_u/b_d$ S- $\begin{array}{cccc} 0.0 & & 0.5 & & 1.0 & & 1.5 \ & & & -b_u/b_d & & \ \end{array}$ $^{-1}$ $\}$ 1 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 -2.0 -1.5 -1.0 \overline{a} *fn/fp* $fn/$ nucleon couplings quark couplings

hadronic uncertainties important for determining viability of models for potential signals on only the ratio *b dels* for potential signals are from variation of the matrix element \overline{a} and the ratio $\frac{1}{2}$ **r** $\frac{1}{2}$ (red), with ranges $\frac{1}{2}$ terms of the parameters *bⁱ* in Eq. (91). For *b^g* = 0 (left panel), *fn/f^p* is independent of ⇤ and depends on only the ratio *bu/bd*. The uncertainty bands are from variation of the matrix element ⌃ (gray)

Figure 1: The ratio *fn/f^p* of the e↵ective WIMP-neutron (*fn*) and WIMP-proton (*fp*) couplings in

• up, down quarks & isospin-violating dark matter

hadronic uncertainties important for determining viability of models for potential signals on only the ratio *b dels* for potential signals are from variation of the matrix element \overline{a} and the ratio $\frac{1}{2}$ **r** $\frac{1}{2}$ (red), with ranges $\frac{1}{2}$ terms of the parameters *bⁱ* in Eq. (91). For *b^g* = 0 (left panel), *fn/f^p* is independent of ⇤ and depends on only the ratio *bu/bd*. The uncertainty bands are from variation of the matrix element ⌃ (gray) • up, down quarks & isospin-violating dark matter

hadronic uncertainties important for determining viability of models for potential signals non-zero *b^g* in the right panel, with *b^d* = *b^u* = 0*.*01 and ⇤ = 400 GeV. The solid (dashed) line is the • strange quarks & heavy wino dark matter

*.*00 0*.*05 0*.*10 *fs* Feynman-Hellmann *.*053(19) present work *.*134(63) [35] *n^f* =2+1, *SU*(3) *.*022(+47) [34] *ⁿ^f* =2+1, *SU*(3) *.*024(22) [33] *n^f* =2+1, *SU*(3) *.*076(73) [32] *n^f* =2+1 *.*036(+33) [31] *ⁿ^f* =2+1 *.*033(17) [21] *n^f* =2+1, *SU*(3) *.*023(40) [27] *n^f* =2+1 *.*058(09) [30] *n^f* =2+1 *.*046(11) [28] *n^f* =2+1 *.*009(22) [27] *n^f* =2+1 *.*035(33) [36] *n^f* =2+1 *.*048(15) [26] *n^f* =2+1 *.*014(06) [25] *n^f* =2+1+1 *.*012(+17) [24] *ⁿ^f* = 2 *.*032(25) [22] *n^f* = 2 *.*063(11) [29] *n^f* =2+1 *fs .*043(11) lattice average (see text) Direct Excluded *from Junnarkar and Walker-Loud,* ⌃*^s* = h*N|ss*¯ *|N*i = 40 *±* 20 MeV

FIG. 8: Comparison and average of lattice QCD calculations of *f^s* as described in the text. Only *1301.1114*

values that have been extrapolated to the physical quark masses are used. Results that quote

• strange quarks & heavy wino dark matter

in Table 1 and Table 2 are added in quadrature. We have ignored power corrections appearing **determines if cross section is** determines if cross section is above or below neutrino \blacksquare due to a partial cancellation between spin-0 and spin-2 matrix elements, the total cross section and the fractional error depend sensitive corrections and sensitive corrections and perturbative correct background for direct detection

• charm quarks & heavy higgsino dark matter

$$
\Sigma_c = m_c \langle N | \bar{c}c | N \rangle
$$
\n
$$
= m_N \begin{cases}\n0.073(3) & \text{pQCD RJH, Solon 2014} \\
0.10(3) & \text{Freeman et al [MILC] 1204.3866} \\
0.07(3) & \text{Gong et al. 1304.1194}\n\end{cases}
$$

• charm quarks & heavy higgsino dark matter

 t_{max} *proton as a function* of $\frac{1}{n}$ H/III_C could po represent 1 uncertainty from pQCD (hadronic inputs). $\text{t.t. } \mathbf{b}$ is the proton, \mathbf{b} and \mathbf{b} \mathbf{c} as a \mathbf{c} \mathbf{a} \mathbf{c} as a \mathbf{c} I/mc could potentially shift cancellation region cases indicated. The pink region corresponds to charmed the pink region corresponds to charmed the pink region

summary results for heavy electroweak charged WIMP scattering 30 and 30 and

Solon, RJH, PRL 2014

cross section per

cross section per

Fig. 2: Si constant for low-velocity sections in the section of the section \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} are set \mathbf{r} and \mathbf{r} and \mathbf{r} are set \mathbf{r} and \mathbf{r} are set \mathbf{r} and \mathbf Higgs boson mass

other illustrative examples

d	QCD operator basis
3	$V_{\theta}^{\mu} = \bar{q}\gamma^{\mu}q$
$A_{q}^{\mu} = \bar{q}\gamma^{\mu}\gamma_{5}q$	
4	$T_{q}^{\mu\nu} = im_{q}\bar{q}\sigma^{\mu\nu}\gamma_{5}q$
$O_{q}^{(0)} = m_{q}\bar{q}q$, $O_{g}^{(0)} = G_{\mu\nu}^{A}G^{A\mu\nu}$	
$O_{5q}^{(0)} = m_{q}\bar{q}i\gamma_{5}q$, $O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^{A}G_{\rho\sigma}^{A}$	
$O_{q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}i}D_{-}^{\nu\}{} - \frac{g^{\mu\nu}}{4}i\rlap{\,/}{\psi}\right)q$, $O_{g}^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^{A})^{2}$	
$O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}i}D_{-}^{\nu\}}\gamma_{5}q$	

pseudoscalar *O*(0) ⁵*^q , O*(0) 5*g* α ² *Component of nucleon spin-2* . Here *^A*[*µB*⌫] ⌘ (*AµB*⌫ *^A*⌫*Bµ*)*/*2 and *^A{µB*⌫*}* ⌘ (*AµB*⌫ ⁺ *^A*⌫*Bµ*)*/*2 respectively denote antisymmetrization and neutralino direct detection *T^q* and the quark pseudoscalar operator *O*(0) • strange component of nucleon spin & spin-dependent • strange component of nucleon spin & spin-dependent neutralino direct detection

<u>meutralino</u> direct detection

osposially post discovery for spin dependent (b) 95% CL Limits on ADD parameters. The contract of the contract of the contract on ADD parameters. The contract of the contr (b) 95% CL Limits on ADD parameters. The contract of the contract of the contract on ADD parameters. The contract of the contr Relevant, especially post-discovery for spin-dependent cross sections

d
\nQCD operator basis
\n3
\n
$$
V_q^{\mu} = \bar{q}\gamma^{\mu}q
$$
\n
$$
A_q^{\mu} = \bar{q}\gamma^{\mu}\gamma_5q
$$
\n4
\n
$$
T_q^{\mu\nu} = im_q\bar{q}\sigma^{\mu\nu}\gamma_5q
$$
\n6
\n
$$
O_q^{(0)} = m_q\bar{q}q, \quad O_q^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}
$$
\n
$$
O_q^{(0)} = m_q\bar{q}i\gamma_5q, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^A G_{\rho\sigma}^A
$$
\n
$$
O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_{-}^{\nu\}} - \frac{g^{\mu\nu}}{4}i\rlap{\,/}{D_{-}}\right)q, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_{\lambda} + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2
$$
\n
$$
O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}i}D_{-}^{\nu\}}\gamma_5q
$$

⁵*^q , O*(0) 5*g g* 5*q* . Here *^A*[*µB*⌫] ⌘ *•* flavor singlet pseudoscalar & low-mass WIMPs symmetrization, and the subscript *q* denotes an active quark flavor. The antisymmetric tensor current • flavor singlet pseudoscalar & low-mass WIMPs

the matrix equidation of the matrix in the non-sector contain-
Container and the singlet area of the additional with the additional matrix in Eq. (49), to discuss the additional matrix of the additional matrix of the addit • flavor singlet pseudoscalar & low-mass WIMPs

$$
\sum_{q=u,d,s} \langle N(k')|\bar{q}i\gamma_{5}q|N(k)\rangle \equiv \kappa(q^{2},\mu)\bar{u}(k')i\gamma_{5}u(k)
$$
\n
$$
\kappa \sim 0
$$
\n
$$
\mathcal{L} = g_{\chi}a\bar{\chi}i\gamma_{5}\chi + \sum_{q} g_{f}a\bar{q}i\gamma_{5}q
$$
\n
$$
\mathcal{L} \sim \frac{1}{\Lambda^{2}} \sum_{N=p,n} g_{N}\bar{\chi}\gamma_{5}\chi\bar{N}\gamma_{5}N
$$
\n
$$
|g_{p}/g_{n}| \sim 15 - 45
$$
\n
$$
\sum_{n=1, n=1, n=0}^{\infty} \mathcal{L} \sim \frac{1}{\Lambda^{2}} \mathcal{L} \sim 15 - 45
$$
\n
$$
\sum_{n=1, n=1, n=0}^{\infty} \mathcal{L} \sim \frac{1}{\Lambda^{2}} \mathcal{L
$$

⁵*s,p*(0) = Impacts tension between experiments FIG. 1. 2-dimensional credible regions for DAMA (shaded/black solid, 90% and 99% CL) and exclusion limits (99*S*% CL) in

Field theory tools

xample. JU(Z) triple noted by subscript "NR"), and we have introduced the reduced amplitude, *M* []*ⁱ* ! (✏)(✏⁰ + Z *d*3*p* <u>)</u>X Worked example: SU(2) triplet annihilation to photons ✓ *k*² $\mathbf{1}$ **Extend Heavy WIMP Effective Theory to describe annihilation.**

hard annihilation (makes it happen)

Sudakov suppression (makes it slower)

Sommerfeld enhancement (makes it faster)

Figure 10 shows the Sommerfeld enhanced annihilation cross section to line photons for three General framework in which to reliably compute annihilation signals for heavy WIMPs.

Novel field theory tools for DM have broad application *M* + 5 ⇡*i* log

$$
\alpha \log^2 \frac{Q^2}{m_e^2}\Big|_{Q^2 = \text{GeV}^2} \approx 1
$$

itive corrections to e-p
ering (proton radius p

$$
\begin{array}{c}\n\text{road application} \\
\begin{array}{c}\n\hline\ne \\
\hline\n\end{array}\n\end{array}
$$

e dius $\overset{r o_e}{\text{corrections}}$ to ing *M*2 *±* $\mathbf d$ *Z* 2 *Z*(radiative corrections to e-p scattering (proton radius puzzle)

$$
\alpha_W \log^2 \frac{M_{\rm DM}^2}{m_W^2}\bigg|_{M_{\rm DM} = \text{TeV}} \approx 1
$$

 $\frac{n_{W}}{n_{\rm DM} = \text{TeV}}$ heavy WIMP annihilation charged WIMPs.level contribution.

Summary

- QCD corrections are important to dark matter searches
	- determine discovery potential (heavy wino)
	- determine compatibility of potential signals between experiments
- interplay with perturbative and nonperturbative QCD
	- lattice matrix elements
	- high-order decoupling relations
	- novel nuclear responses
- has motivated new field theory tools for particle and nuclear physics

back up

 -0.2 0 0.2

 Ω

0.1

0.01

 10^{-48}

 10^{-4}

 10^{-47}

 10^{-46}

 10^{-45}

 $\frac{1}{2}$ 10^{-40}

 \overline{l}

 $\mathbf{1}$

 $\frac{1}{2}$

 10^{-50}

even Higgs of mass *m^H > m^h* = 126 GeV, aris-

 10^{-45}

 \sim 10

 $40-$

 $\frac{10}{3}$

0.01

 10^{-47}

 10

 $10₅₅$

 \widehat{a}

 \overline{a}

 10^{-46}

 10^{-45}

 $0.1\bigwedge \qquad 10^{-43}$ \bigwedge 1

singlet-doublet (e.g., bino-higgsino)

Additional states in the dark sector

Single-nucleon operators
\n
$$
\mathcal{L}_{N\chi,PT} = \frac{1}{m_N^2} \Big\{ d_1 N^{\dagger} \sigma^i N \chi^{\dagger} \sigma^i \chi + d_2 N^{\dagger} N \chi^{\dagger} \chi \Big\} + \frac{1}{m_N^4} \Big\{ d_3 N^{\dagger} \partial_+^i N \chi^{\dagger} \partial_+^i \chi + d_4 N^{\dagger} \partial_-^i N \chi^{\dagger} \partial_-^i \chi
$$
\n
$$
+ d_5 N^{\dagger} (\partial^2 + \overleftarrow{\partial}^2) N \chi^{\dagger} \chi + d_6 N^{\dagger} N \chi^{\dagger} (\partial^2 + \overleftarrow{\partial}^2) \chi + i d_8 \epsilon^{ijk} N^{\dagger} \sigma^i \partial_-^j N \chi^{\dagger} \partial_+^k \chi
$$
\n
$$
+ i d_9 \epsilon^{ijk} N^{\dagger} \sigma^i \partial_+^j N \chi^{\dagger} \partial_-^k \chi + i d_{11} \epsilon^{ijk} N^{\dagger} \partial_+^k N \chi^{\dagger} \sigma^i \partial_-^j \chi + i d_{12} \epsilon^{ijk} N^{\dagger} \partial_-^k N \chi^{\dagger} \sigma^i \partial_+^j \chi
$$
\n
$$
+ d_{13} N^{\dagger} \sigma^i \partial_+^j N \chi^{\dagger} \sigma^i \partial_+^j \chi + d_{14} N^{\dagger} \sigma^i \partial_-^j N \chi^{\dagger} \sigma^i \partial_-^j \chi + d_{15} N^{\dagger} \sigma \cdot \partial_+ N \chi^{\dagger} \sigma \cdot \partial_+ \chi
$$
\n
$$
+ d_{16} N^{\dagger} \sigma \cdot \partial_- N \chi^{\dagger} \sigma \cdot \partial_- \chi + d_{17} N^{\dagger} \sigma^i \partial_-^j N \chi^{\dagger} \sigma^j \partial_-^i \chi
$$
\n
$$
+ d_{18} N^{\dagger} \sigma^i (\partial^2 + \overleftarrow{\partial}^2) N \chi^{\dagger} \sigma^i \chi + d_{19} N^{\dagger} \sigma^i (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) N \chi^{\dagger} \sigma^j \chi
$$
\n
$$

$$

L Lorentz symmetry is imposed by enforcing invariance under the infinitesimal boost \mathcal{L} *^N* ! *^eim^N* ⌘*·^x* Lorentz invariance:

$$
rd_4 + d_5 = \frac{d_2}{4}, \quad d_5 = r^2 d_6, \quad 8r(d_8 + rd_9) = -rd_2 + d_1, \quad 8r(rd_{11} + d_{12}) = -d_2 + rd_1
$$

$$
rd_{14} + d_{18} = \frac{d_1}{4}, \quad d_{18} = r^2 d_{20}, \quad 2rd_{16} + d_{19} = \frac{d_1}{4}, \quad r(d_{16} + d_{17}) + d_{19} = 0, \quad d_{19} = r^2 d_{21},
$$

coecients *cⁿ* vanish for *n* = 3*,* 4. Since the relativistic fermion case we have the following interactions, $\frac{1}{2}$ Light WIMP+ SM *L ,*SM = <u>r</u> $\frac{1}{2}$

$$
\begin{split} \textcolor{red}{\underbrace{\text{light WIMP+SM}}_{m_W} = \frac{c_{\psi 1}}{m_W} \bar{\psi} \sigma^{\mu \nu} \psi F_{\mu \nu} + \frac{c_{\psi 2}}{m_W} \bar{\psi} \sigma^{\mu \nu} \psi \tilde{F}_{\mu \nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\psi 3,q}}{m_W^2} \bar{\psi} \gamma^{\mu} \gamma_5 \psi \bar{q} \gamma_{\mu q} + \frac{c_{\psi 4,q}}{m_W^2} \bar{\psi} \gamma^{\mu} \gamma_5 \psi \bar{q} \gamma_{\mu q} + \frac{c_{\psi 5,q}}{m_W^2} \bar{\psi} \gamma^{\mu} \psi \bar{q} \gamma_{\mu q} + \frac{c_{\psi 6,q}}{m_W^2} \bar{\psi} \gamma^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_5 q + \frac{c_{\psi 7,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} q + \frac{c_{\psi 8,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} q \\ + \frac{c_{\psi 9,q}}{m_W^3} \bar{\psi} \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi 10,q}}{m_W^3} \bar{\psi} i \gamma_5 \psi m_q \bar{q} i \gamma_5 q + \frac{c_{\psi 11,q}}{m_W^3} \bar{\psi} i \partial_-^{\mu} \psi \bar{q} \gamma_{\mu q} \\ + \frac{c_{\psi 12,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^{\mu} \psi \bar{q} \gamma_{\mu q} + \frac{c_{\psi 13,q}}{m_W^3} \bar{\psi} i \partial_-^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_5 q + \frac{c_{\psi 14,q}}{m_W^3} \bar{\psi} \gamma_5 \partial_-^{\mu} \psi \bar{q} \gamma_{\mu} \gamma_5 q \\ + \frac{c_{\psi 15,q}}{m_W^3} \bar{\psi} \sigma_{\mu \nu} \psi m_q \bar{q} \sigma^{\mu \nu} q + \frac{c_{\psi 16,q}}{m_W^3} \epsilon_{\mu \nu \rho \sigma} \bar{\psi} \sigma^{\mu \nu} \psi m_q \bar{q}
$$

Majorana: \blacksquare of dime \blacksquare

c ⁿ with *n* = 1*,* 2*,* 5*,* 6*,* 11*,* 12*,* 13*,* 14*,* 15*,* 16 vanish, leaving ten types of operators through dimension $c_{\psi n}$ with $n = 1, 2, 5, 6, 11$ For the case of DM with mass *M* & *m^W* , we have the following interactions,⁴ $c_{\psi n}$ with $n = 1, 2, 5, 6, 11, 12, 13, 14, 15, 16$ vanish,

c ⁿ with *n* = 1*,* 2*,* 5*,* 6*,* 11*,* 12*,* 13*,* 14*,* 15*,* 16 vanish, leaving ten types of operators through dimension seven as considered in Ref. [15]. Heavy WIMP + SM ✏*µ*⌫⇢¯*vµ*⌫ $\mathbf{u}_{\alpha\alpha\beta\gamma\delta\gamma}$ *m*³ *W* ¯*vvG^A* ↵*GA*↵ +

$$
\mathcal{L}_{xy,3M} = \frac{c_{x1}}{m_W} \bar{\chi}_{\alpha} \sigma_1^{\alpha \nu} \chi_{\nu} F_{\mu \nu} + \frac{c_{x2}}{m_W} \bar{\chi}_{\alpha} \sigma_1^{\alpha \nu} \chi_{\nu} \bar{F}_{\mu \nu} + \sum_{\eta = \pi, d, x, r, l} \begin{cases} \frac{c_{x3,d}}{m_W^2} \epsilon_{\mu \nu \rho \sigma} v^{\alpha} \chi_{\alpha} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\rho} \bar{g} + \frac{c_{x3,d}}{m_W^2} \bar{\chi}_{\nu} \chi_{\nu} \bar{g}^{\rho} \bar{g}^{\alpha} \chi_{\nu} \chi_{\sigma} \bar{g}^{\rho} \chi_{\nu} \chi_{\sigma} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\sigma} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \chi_{\nu} \chi_{\nu} \bar{g}^{\alpha \rho} \
$$

of dimension eight and higher involving quarks and gluons. In each of (5), (6) and (7) we have Lorentz: where the ellipsis denotes terms higher order in 1*/M*. Working through *^O*(*M*1) for photon operators and *^O*(*M*3) for quark and gluon operators, we find that the variation of Eq. (7) under the boost parity and time-reversal symmetry, to the seven operators describing nucleon-lepton interactions in

$$
\frac{m_W}{M}c_{\chi 3} + 2c_{\chi 12} = \frac{m_W}{M}c_{\chi 4} + 2c_{\chi 14} = \frac{m_W}{M}c_{\chi 5} - 2c_{\chi 17} = \frac{m_W}{M}c_{\chi 6} - 2c_{\chi 20} = c_{\chi 11} = c_{\chi 13} = 0,
$$

^M ⁺ *^O*(*q*2)*.* (8) **d| OI dIId.** c_{new} vanish for $n=1, 2, 5, 6, 15, 16, 17, 18, 19, 21$ \mathbf{a} **r** idjoid did. $c_{\chi n}$ vanish for $n=1, 2, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.$ Majorana: $c_{\chi n}$ vanish for $n=1, 2, 5, 6, 15, 16, 17,$ Eq. (3) or Eq. (4); in particular we find that the coecients *cⁿ* vanish for *n*=1, 2, 5, 6, 15, 16, 17,