

# Dark matter direct searches and scalar strangeness of the nucleon

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## **Collaborators:**

- **Xiu-Lei Ren, Beihang U.**
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- **Hiroshi Toki, Osaka U.**

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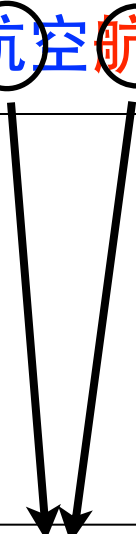
# Beihang University

北航

- **Beijing University (Institute) of Aeronautics and Astronautics (BUAA)**
- 北京航空航天大学

# Beihang University

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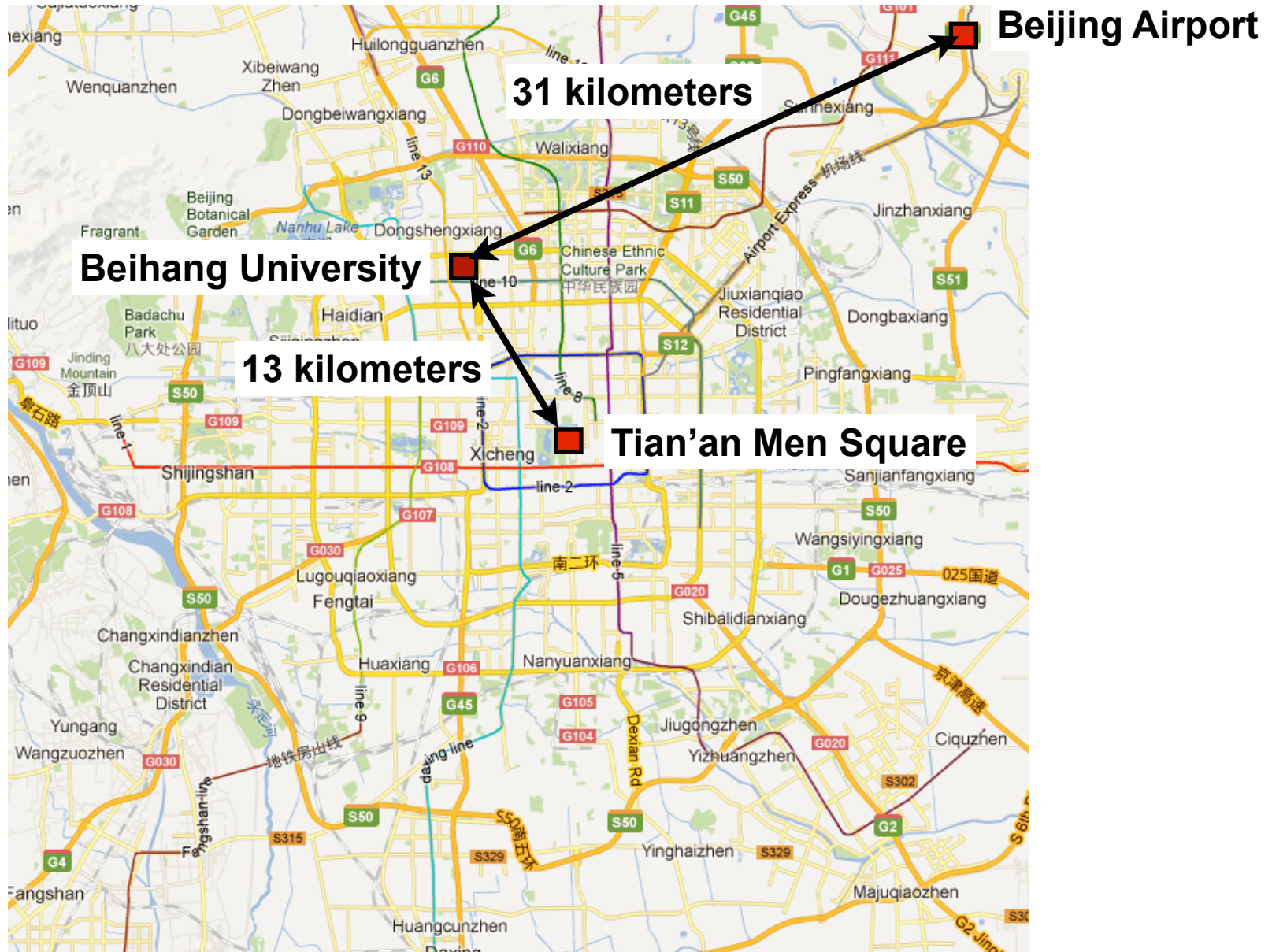
- **Beihang University**
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# Beihang University in China

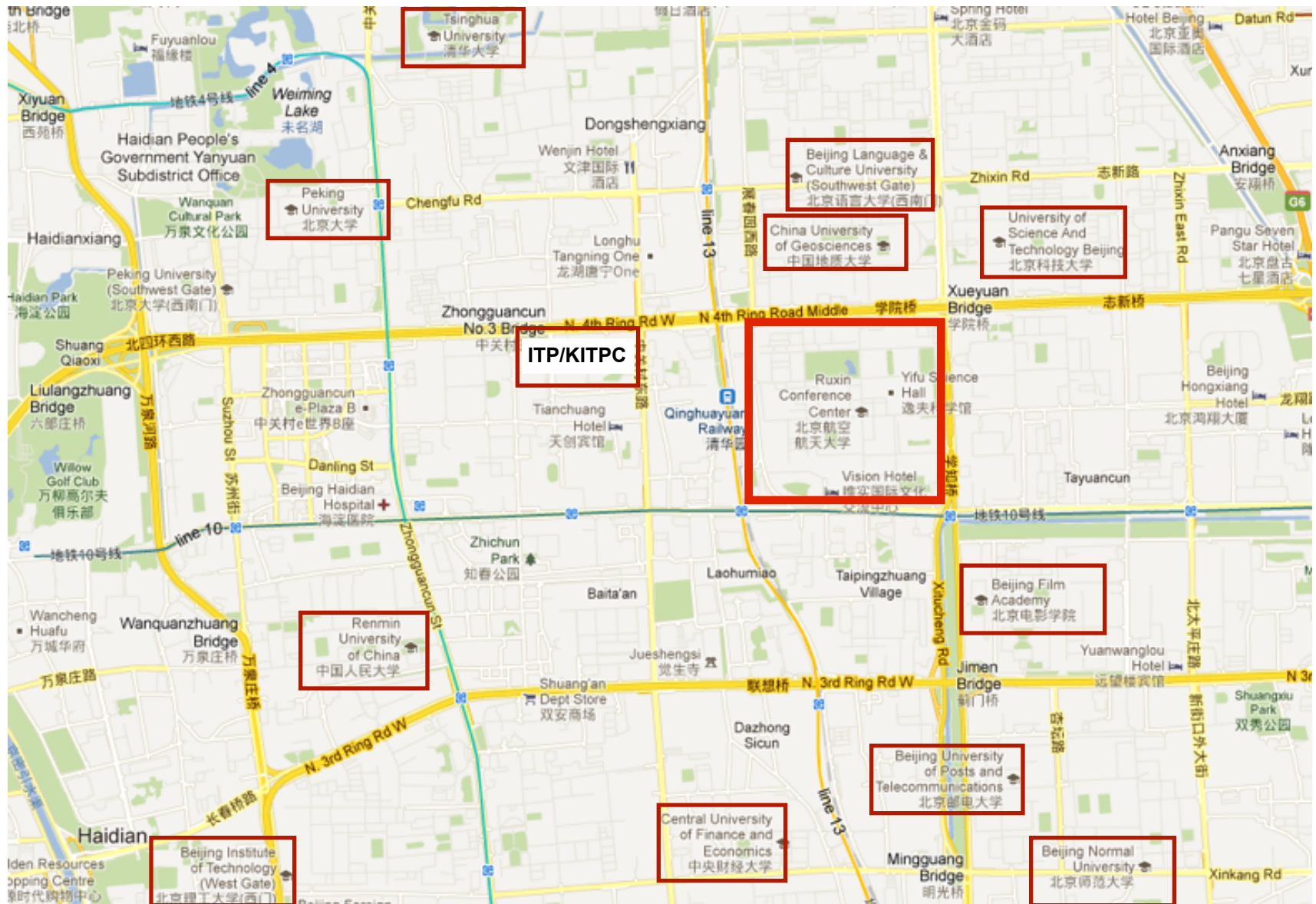


# Beihang University in Beijing





# Beihang University in Xueyuan Road



# School of Physics and Nuclear Energy Engineering



the old main building

East gate

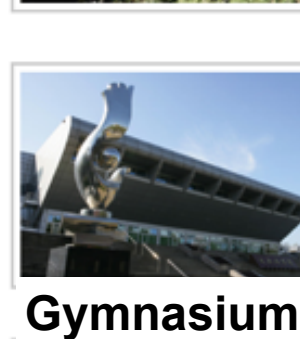




**New Main Building**



**SPNEE**



**Gymnasium**



**Shahe campus**



**Lake/bridge**

# Beihang University: key numbers

- Established in **1952**
- **17** schools and **6** departments,
- More than **3300** faculty and staff, more than **26000** student
- **42** research institutes or interdisciplinary research centers,
- **11** key disciplines of the national level,
- **89** laboratories (including 4 national key laboratories, 5 national specialized laboratories, and 12 provincial or ministerial-level key laboratories.)
- **Top five** Chinese universities in per capita funds raised for scientific research



# School of Physics and Nuclear Energy Engineering



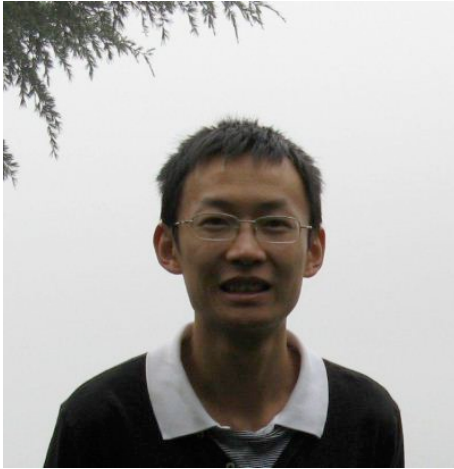
- Founded in **2009**
- Over **90** faculty and staff (four foreign)
- **3** departments:
  - department of physics,
  - department of applied physics,
  - department of nuclear science and technology
- **299** undergraduate students, **111** master students, and **78** Ph.D. students

# 22 faculties in Department of Nuclear Science and Technology

- **8** in the theory group, including **myself**
- **9** in the experimental group
- **4** in the nuclear applications and fusion related condensed matter physics

# Theory group faculties

**Hua-Xing Chen**



**Li-Sheng Geng**



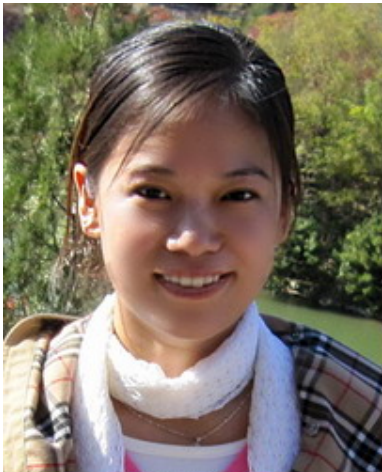
**Jie Meng**



**Dan-Yang Pang**



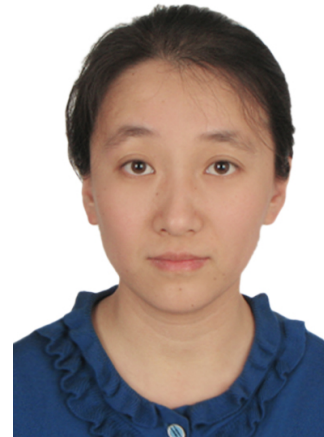
**Chun-Yan Song**



**Wen-Ling Wang**



**Shi-Sheng Zhang**



**Yujie Zhang**



# International collaborations

- HEP (Chengping Shen)



- Nuclear (Isao Tanihata, super FRS )



- Fusion ITER (Guang-Hong Lv)



the way to new energy

International Thermonuclear Experimental Reactor

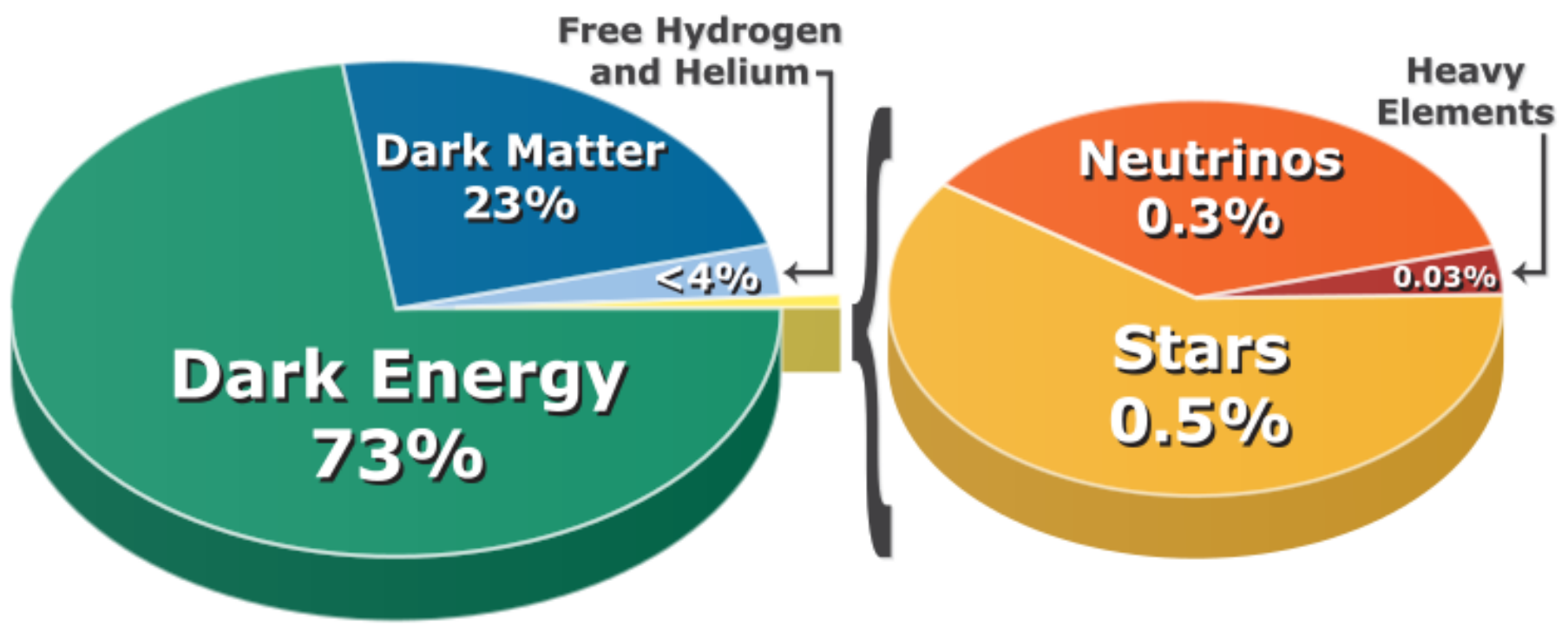
# Contents

- ❖ **Motivation: Baryon Sigma Terms, Dark Matter Direct Detection, and quark-flavor structure of the nucleon**
- ❖ **Lattice QCD and Chiral Perturbation Theory (ChPT)**
  - ✓ A very brief introduction to lattice QCD
  - ✓ ChPT in the one-baryon sector — the power counting breaking problem and its recovery
- ❖ **Octet baryon sigma terms from application of Feynman-Hellmann theorem, LQCD simulations, and Chiral perturbation theory**
- ❖ **Summary**

# Motivation



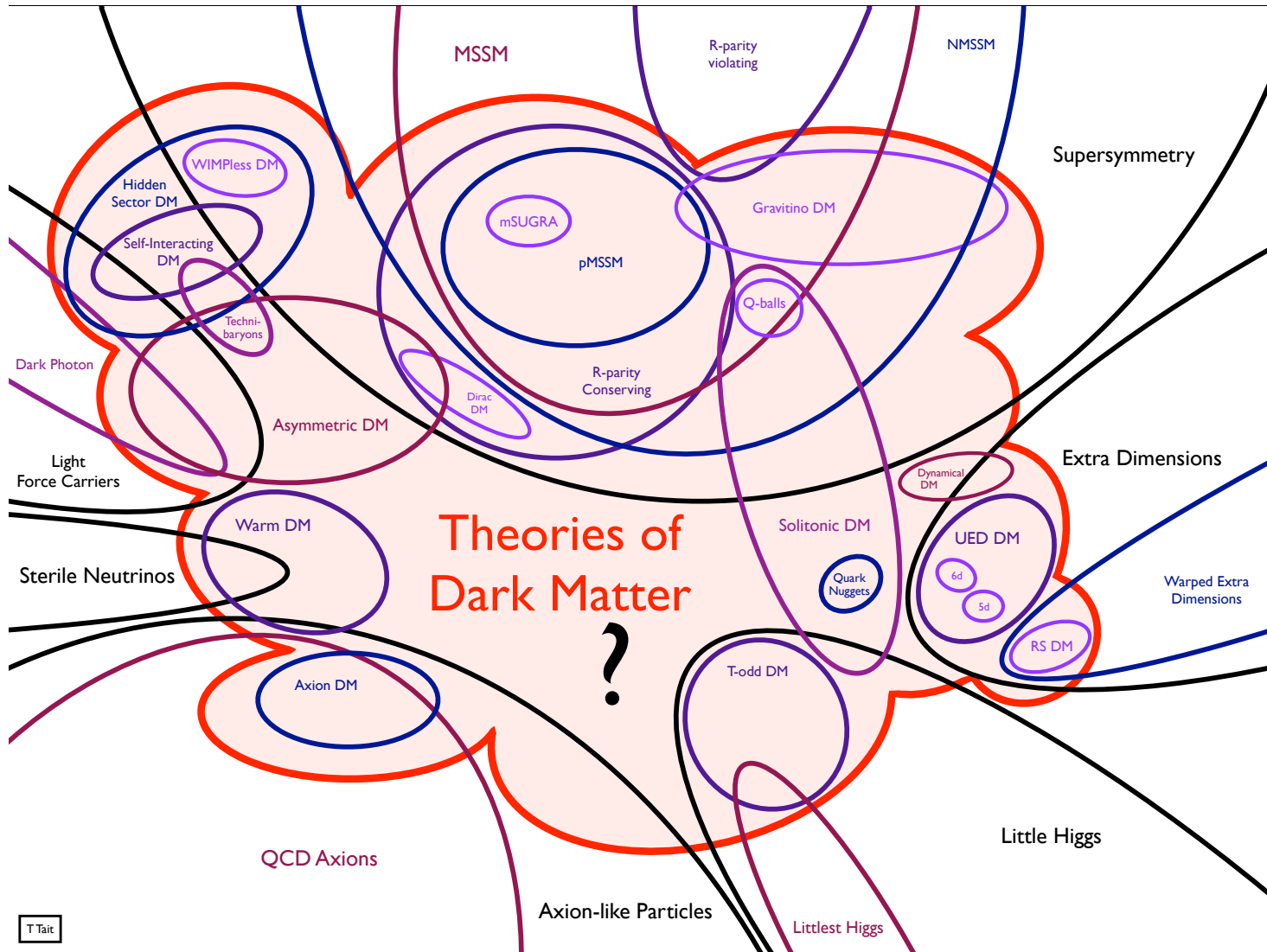
# Energy-matter composition of the universe



# What is dark matter?

- **What we know!**
  - Dark (electric neutral)
  - (Probably) Massive (cold/non-relativistic)
  - Still around today (stable or with a lifetime of the age of the universe)
- **What we do not know!**
  - Mass, spin,... (with some constraints)
  - Couplings: **gravity**, weak Interactions, Higgs, quarks/ gluons, leptons?
- **Questions can only be answered ultimately by experiments, but theories are needed to formulate the questions**

# No lack of theories

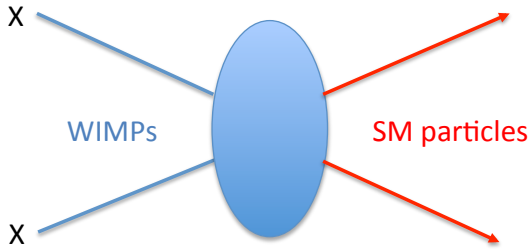


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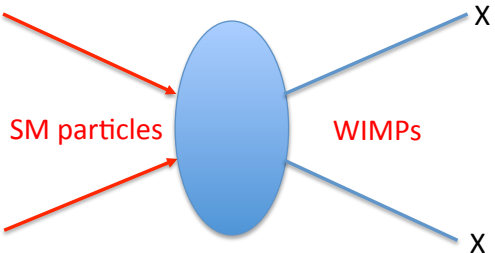
# Some popular candidates

- **Axions, and axion-like particles**
- **Sterile Neutrinos**
- **Dark photons**
- **Weakly Interacting Massive Particles (WIMPS)**
  - WIMPs **naturally** can account for the amount of dark matter we observe in the Universe
  - WIMPs **automatically** occur in many models of physics beyond the Standard Model

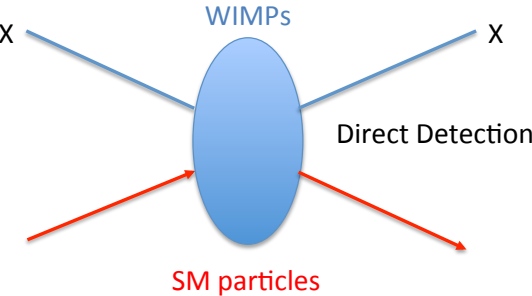
# Particle searches for WIMPs



Indirect Detection



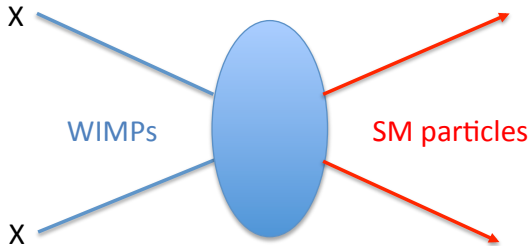
Collider Searches



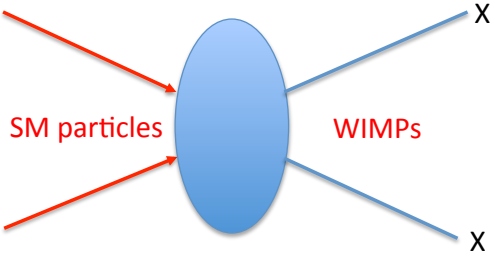
Direct Detection



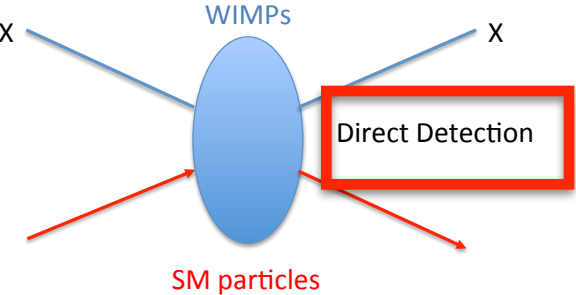
# Particle searches for WIMPs



Indirect Detection

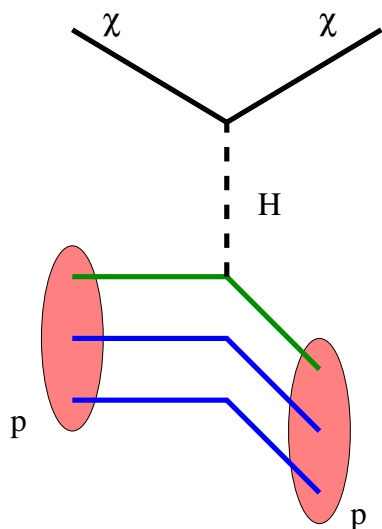


Collider Searches





# Spin-independent neutrino-nucleon scattering



$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi} \chi \rightarrow \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi} \chi$$

$$\lambda_N \rightarrow \sum_{q=1}^6 f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi} [Z f_P + (A - Z) f_N]$$

with

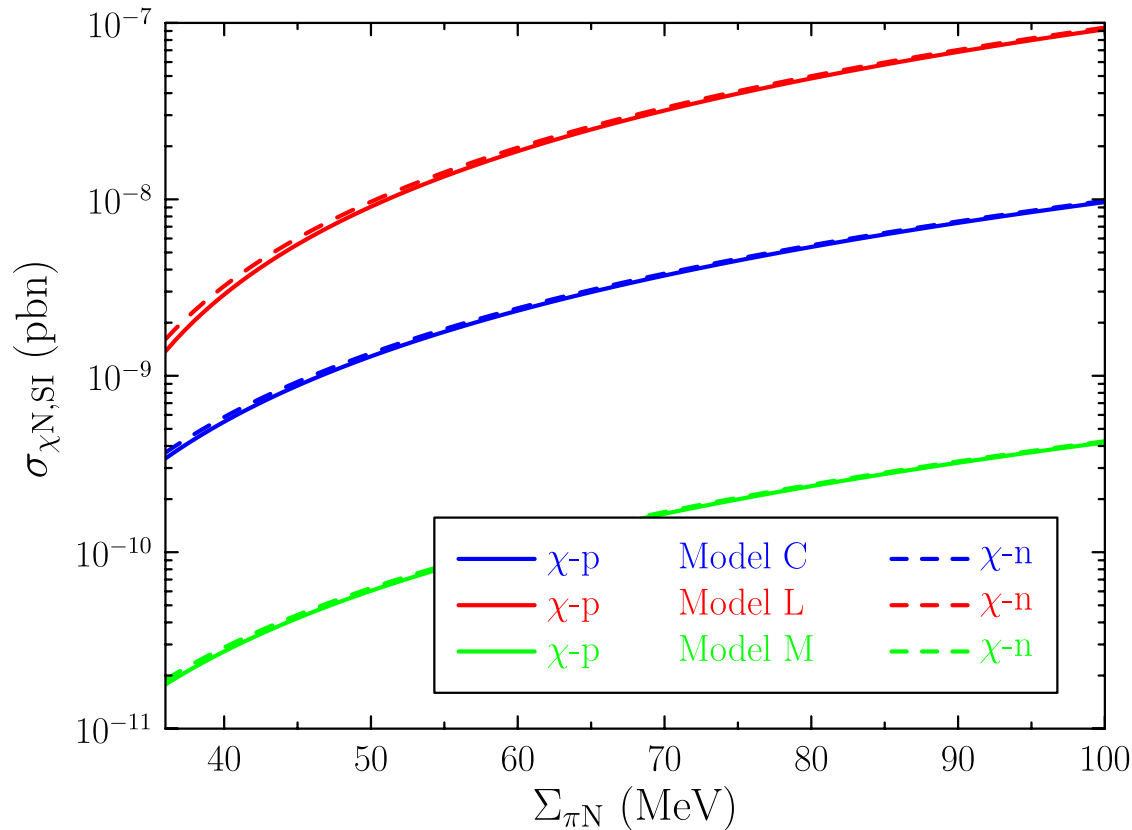
$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

**pion- and strangeness sigma terms (SM physics)**

$$f_{ud}^N M_N = \sigma_{\pi N} = m_q \langle N | u\bar{u} + d\bar{d} | N \rangle$$

$$f_s^N M_N = \sigma_{sN} / 2 = m_s \langle N | s\bar{s} | N \rangle$$

# Strong dependence on the strangeness sigma term



$$\sigma_{\chi p, SI} \sim (\Sigma_{\pi N} - \sigma_0)^2.$$

$$\sigma_{sN} \propto \Sigma_{\pi N} - 36$$

# Determination of the sigma terms

- Experimentally, the pion sigma term can be inferred from pion-nucleon scattering data at Cheng-Dashen point ( $s = u = m_N^2, t = 2M_\pi^2$ )

$$\sigma_{\pi N} = 45 \pm 8 \text{ MeV}$$

J. Gasser et al., PLB253,252

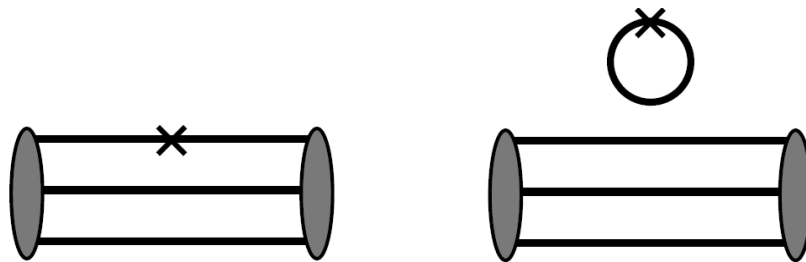
$$\sigma_{\pi N} = (59.1 \pm 1.9 \pm 3.0) \text{ MeV}$$

Hoferichter et al., PRL115, 092301

- Because of lack of kaon-nucleon scattering data, the strangeness-sigma term **cannot** be obtained this way
  - Lattice QCD might be our hope to predict it from first principles

# LQCD determination of sigma terms

- **Direct method**—calculates the 3-point connected and disconnect diagrams



- JLQCD coll., PRD83,114506 (2011)
- R. Babich *et al.*, PRD85,054510 (2012)
- QCDSF coll., PRD85, 054502 (2012)
- ETM coll., JHEP 1208,037(2012)
- M. Engelhardt *et al.*, PRD86, 114510 (2012)
- JLQCD coll., PRD87, 034509 (2013)

- **Spectrum method**-calculates the baryon masses, and relates the sigma terms to their quark mass dependence via the **Feynman Hellman** theorem

$$\sigma_{\pi B} = m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- JLQCD coll., PRD83,114506 (2011)
- R. Babich *et al.*, PRD85,054510 (2012)
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- ETM coll., JHEP 1208,037(2012)
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- JLQCD coll., PRD87, 034509 (2013)

# Why ChPT?

- By themselves, both methods suffer from a number of drawbacks **in addition to the inherent artifacts of LQCD simulations**, e.g.,
  - Method 1 still too time consuming, noise/signal ratio, etc.
  - Method 2 requires calculations at quark masses both larger and smaller than their physical counterparts
- **ChPT can help not only in alleviating some of the drawbacks but also removing the LQCD artifacts**

# Our aim

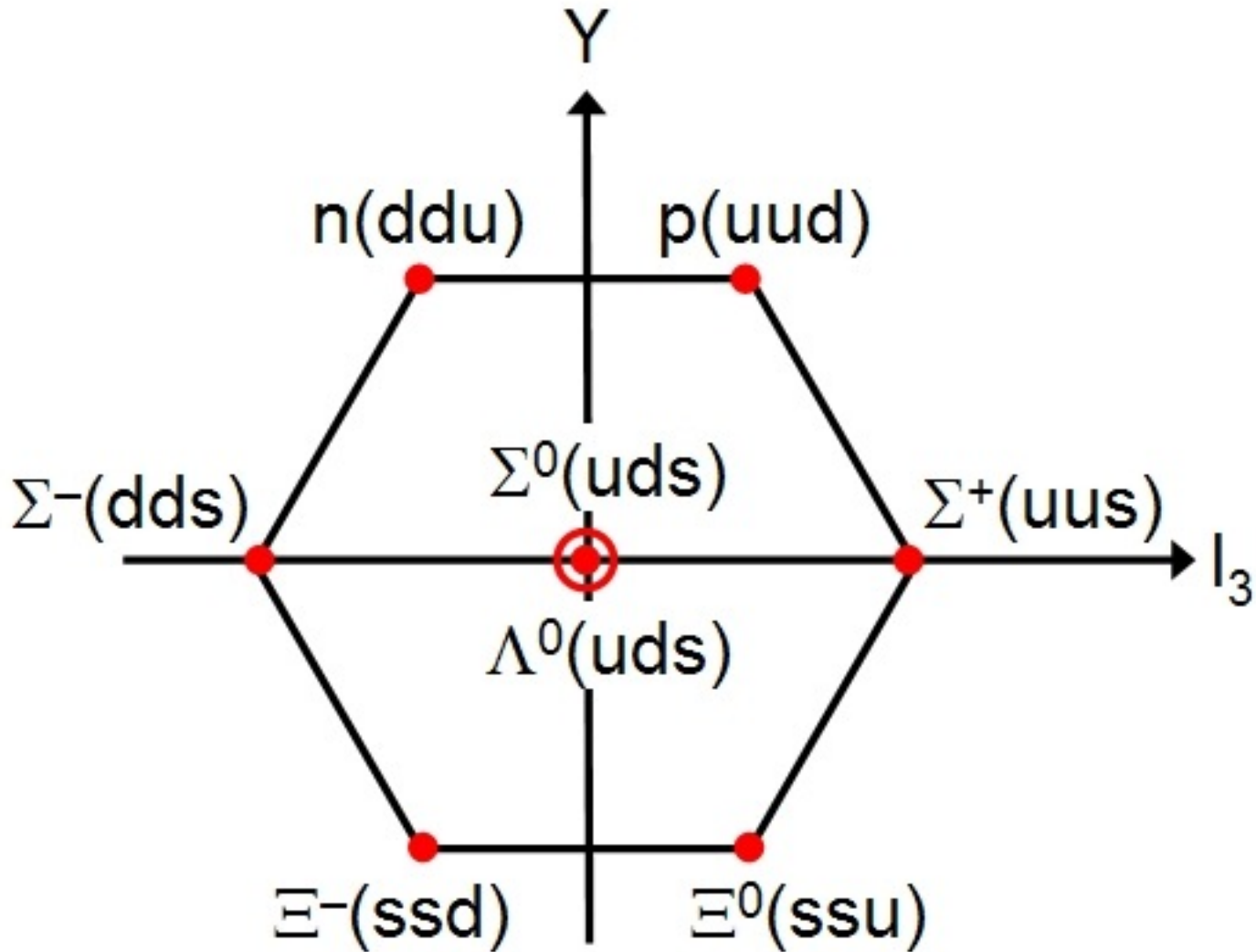
- To apply the **Feynman-Hellmann** theorem to predict the baryon sigma terms using the covariant (EOMS) baryon chiral perturbation theory

$$\sigma_{\pi B} = m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l}$$
$$\sigma_{sB} = m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- To fix the unknown low-energy constants of BChPT, we rely on the IQCD simulations of baryon masses



# Quark-flavor structure of octet baryons



Naive quark model

# Quark-flavor structure of the proton

- Naive quark model—minimal quark contents

$$|p\rangle = |uud\rangle$$

- In reality,

$$|p\rangle = |uud\rangle (1 + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

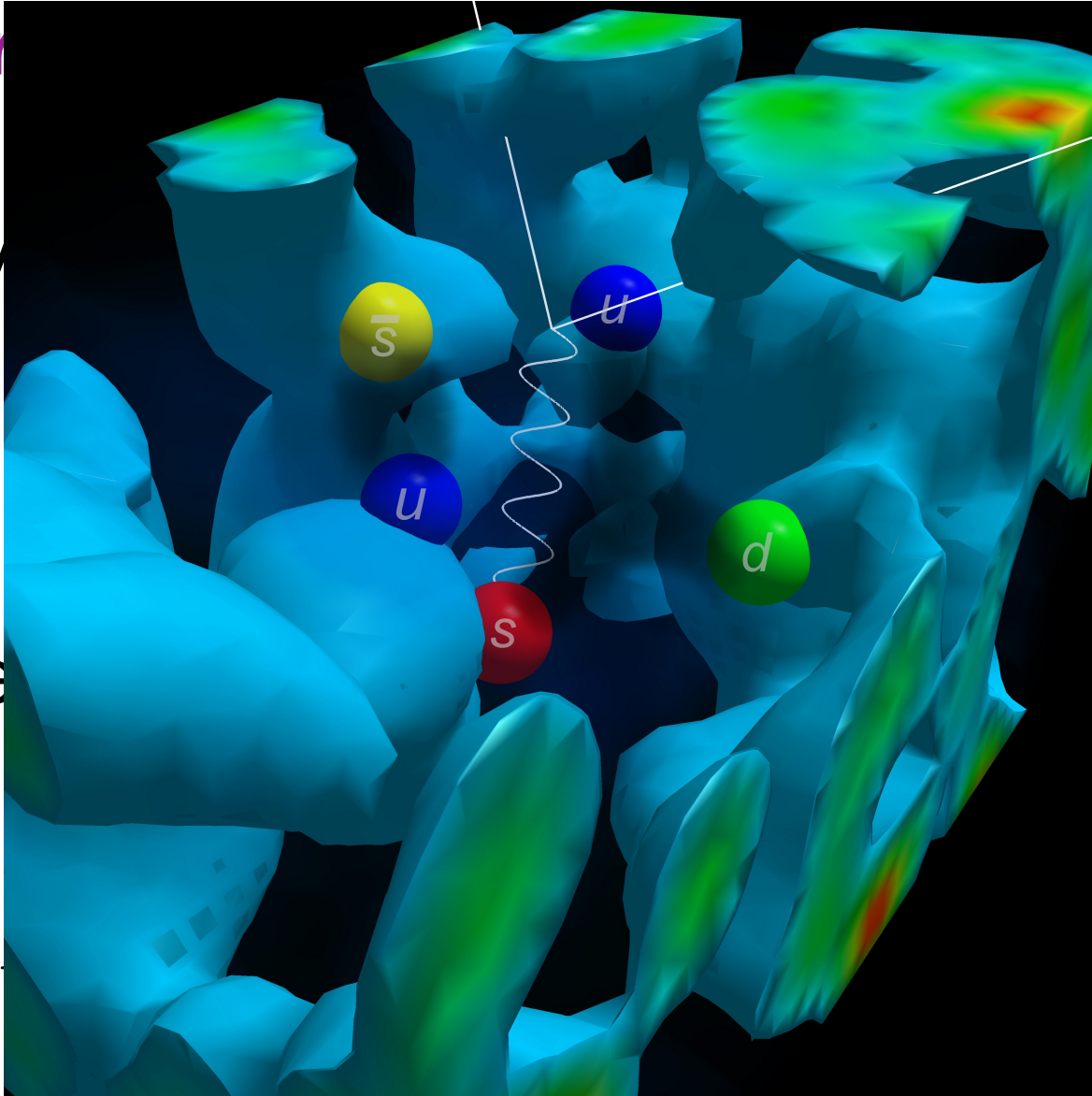
# Quantum

# proton

- Naive

contents

- In re



$$+ |s\bar{s}\rangle)$$

# Strangeness content of the proton

- to the **spin**
  - deep-inelastic lepton scattering
- to the **electromagnetic form factors**
  - parity-violating electron-proton scattering
- to the **mass**
  - scalar strangeness content, cannot be measured directly

$$\langle N | s \bar{s} | N \rangle$$

# Strangeness content of the proton

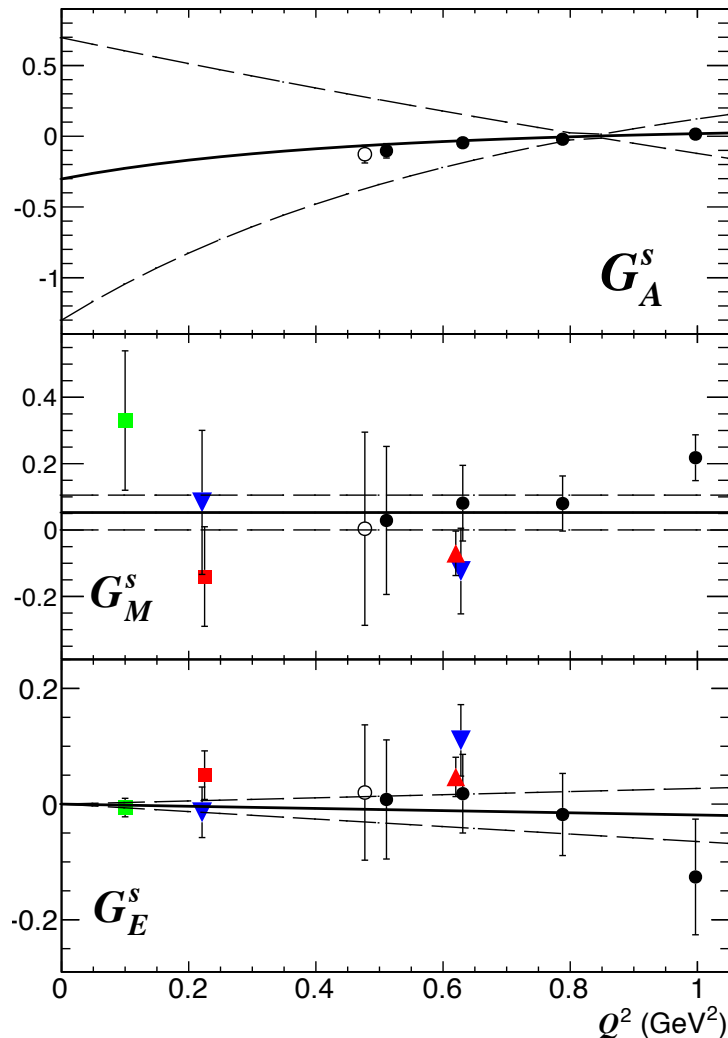
- to the **spin**
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- to the **mass**
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$$\langle N | s \bar{s} | N \rangle$$

How to obtain the scalar strangeness content of the nucleon from the LQCD masses using Chiral Perturbation Theory

# Global fit of the strangeness vector and axial vector form factors of the nucleon

arXiv:1308.5694



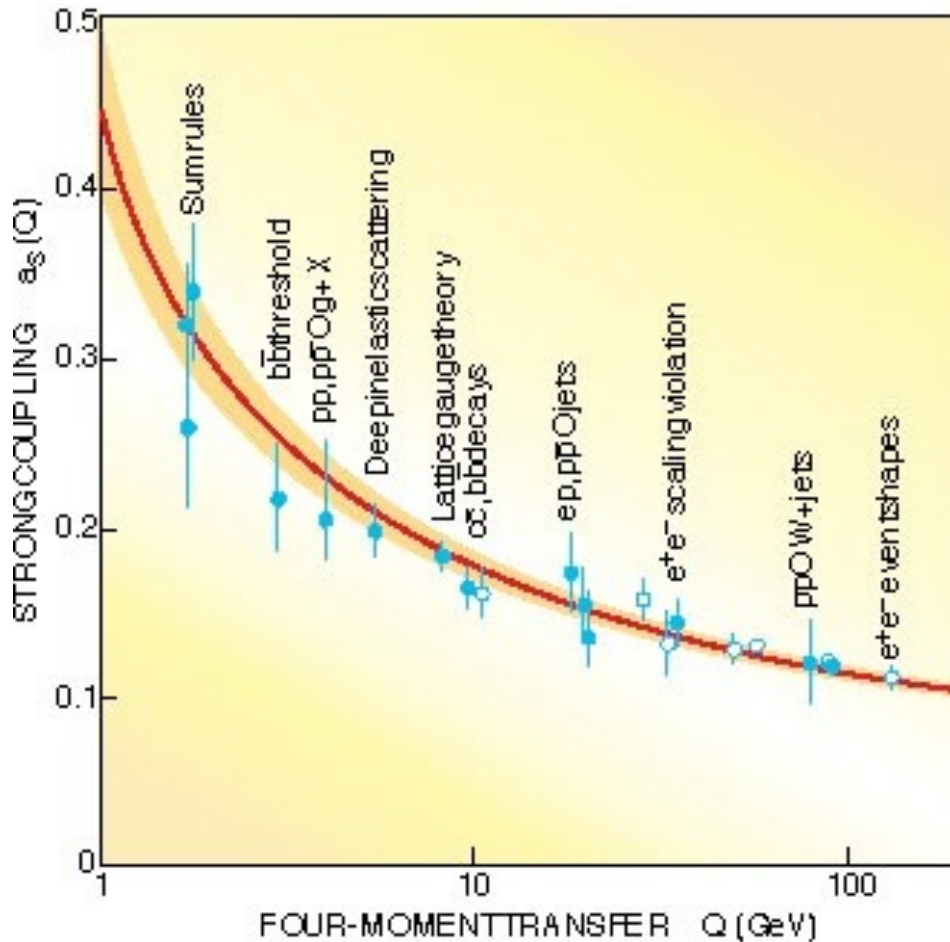
Parameter	Fit value
$\rho_s$	$-0.071 \pm 0.096$
$\mu_s$	$0.053 \pm 0.029$
$\Delta S$	$-0.30 \pm 0.42$
$\Lambda_A$	$1.1 \pm 1.1$
$S_A$	$0.36 \pm 0.50$

- The electric and magnetic form factors are consistent with **zero**, but **not** the axial-vector form factor

# **A few words on LQCD and BChPT, and why BChPT is needed**

# QCD—non-perturbative at low energies

- ❖ Quantum ChromoDynamics—the theory of the strong interaction



Asymptotic freedom—  
Nobel prize in physics 2004

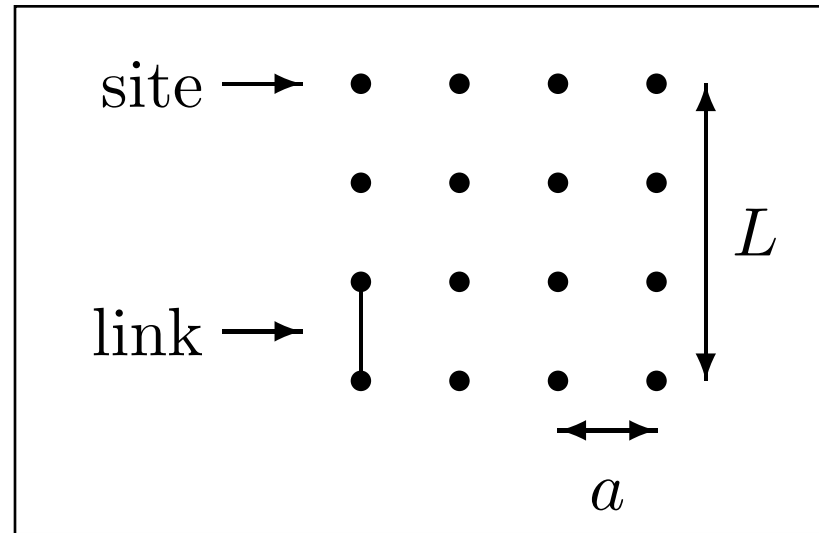
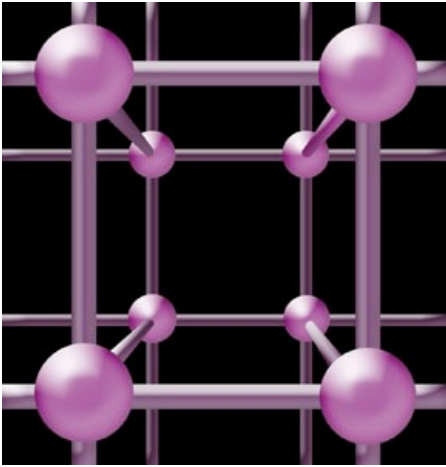
High energy: perturbative QCD  
successful

Low energy: non-perturbative  
problematic

- ✓ Phenomenological models
- ✓ Effective field theories
- ✓ LQCD



# Brute Force: Lattice QCD



**Basic idea: discretize space-time and solve non-perturbative strong interaction physics in a finite hypercube, utilizing monte carlo sampling techniques**

# Calculating path-integral in Euclidean space-time

- Vacuum

$$Z = \int [DU] e^{-S_g(U) + \text{Tr} \ln M[U]}$$

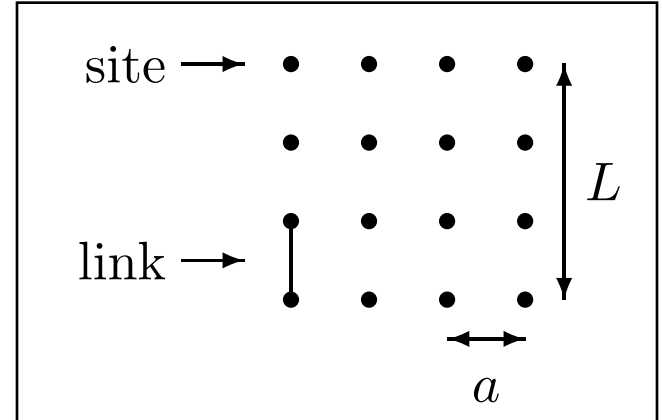
- Observable

$$\langle O \rangle = \int [DU] O(U) e^{-S_g(U) + \text{Tr} \ln M[U]}$$

# Simulation parameters and costs

- light quark masses:  $m_u/m_d$
- lattice spacing:  $a$
- lattice volume:  $V=L^4$

$$\text{cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a}$$



- To reduce cost: employ larger than **physical light quark masses, finite lattice spacing and volume.**
- To obtain **physical** quantities, multiple extrapolations are needed

# Multiple extrapolations

- **Chiral extrapolations:** light quark masses to their physical values

$$m_q \rightarrow m_q(\text{Phys.})$$

- **Finite volume corrections:** infinite space-time

$$L \rightarrow \infty$$

- **Continuum extrapolations:** zero lattice spacing

$$a \rightarrow 0$$

# Why Chiral Perturbation Theory needed?

- All can be performed with the help of Chiral Perturbation Theory
- The low-energy effective field theory of QCD
  - provides a **bridge** to link LQCD simulations to the physical world
  - **helps/guides** to perform the aforementioned extrapolations

# Interplay between ChPT and LQCD Simulations

- As the low-energy EFT of QCD, ChPT provides a model-independent description of low-energy strong interaction phenomena by itself
- At higher orders, which are needed to achieve accuracy at the **few percent** level, there might be **too many unknown** low-energy constants (LECs), which can not easily be determined by experimental data alone
- LQCD simulations provide a **solution** to overcome the above difficulty

# Chiral Perturbation Theory (ChPT) in essence

- Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons

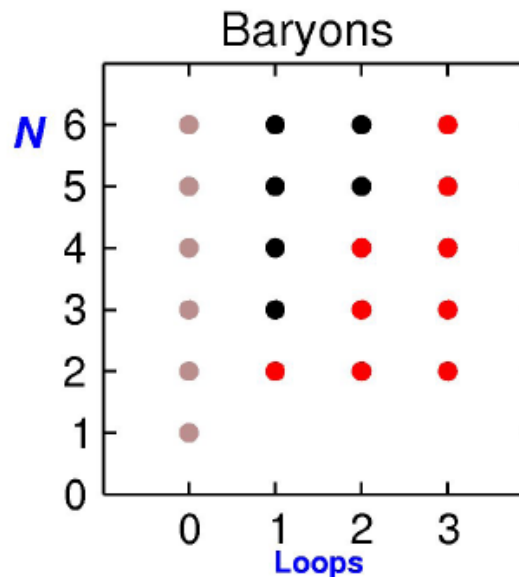
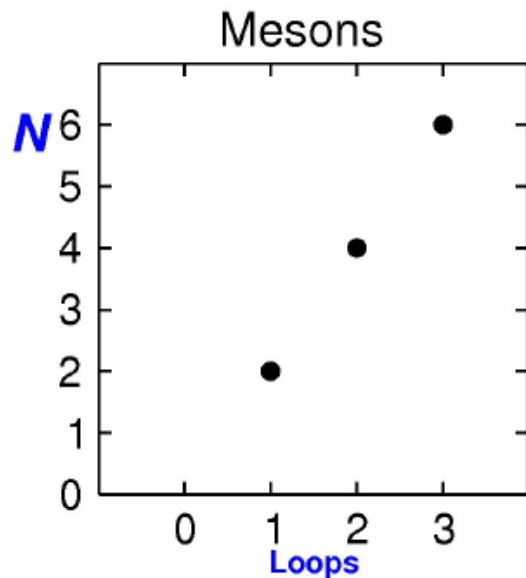
$$\mathcal{L}_{\text{QCD}}[q, \bar{q}; G] \rightarrow \mathcal{L}_{\text{ChPT}}[U, \partial U, \dots, \mathcal{M}, N]$$

- $U$  parameterizes the Nambu-Goldstone bosons
- $\partial U$  vanishes at  $E = \vec{p} = 0$  (Nambu-Goldstone theorem)
- $M$  parameterizes the explicit symmetry breaking
- $N$  denotes interactions with matter fields
- Exact mapping via chiral Ward identities

- ChPT exploits the symmetry of the QCD Lagrangian and its ground state; **in practice, one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses.** (J. Gasser, 2003)

# Power-counting-breaking (PCB) in the one-baryon sector

- ChPT very successful in the study of Nambu-Goldstone boson self-interactions. (at least in SU(2))
- In the one-baryon sector, things become problematic because of the **nonzero (large)** baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., **a systematic power counting is lost!**



*red dots denote possible PCB terms (pion-nucleon scattering)*

*J. Gasser et al., NPB 307, 779(1988)*

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$



# Power-counting-restoration methods

- **Heavy Baryon ChPT**: baryons are treated “semi-relativistically” by a simultaneous expansion in terms of external momenta and  $1/M_N$  (*Jenkins et al., 1993*). It converges slowly for certain observables!
- **Relativistic baryon ChPT**: removing power counting breaking terms but retaining higher-order relativistic corrections, thus, keeping relativity.
  - **Infrared** baryon ChPT (*T. Becher and H. Leutwyler, 1999*)
  - Fully relativistic baryon ChPT–Extended On-Mass-Shell (**EOMS**) scheme (*J. Gegelia et al., 1999; T. Fuchs et al., 2003*)
- IR scheme separates the full integral into the **Infrared** and **Regular** parts:

$$H = \frac{1}{ab} = \int_0^1 dz \frac{1}{[(1-z)a + zb]^2} \equiv I + R = \int_0^\infty \dots dz - \int_1^\infty \dots dz$$

# Power-counting-restoration methods

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$$H = \text{Infrared}$$

# Extended-on-Mass-Shell (EOMS)

- “Drop” the PCB terms

$$\boxed{\text{tree} = M_0 + b m_\pi^2} \quad + \quad \boxed{\text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \dots}$$

$$\Downarrow a = 0; b' = 0$$

$$\boxed{M_N = M_0 + b m_\pi^2 + c m_\pi^3 + \dots \quad (\mathcal{O}(p^3))}$$

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- Equivalent to redefinition of the LECs

$$\boxed{\text{tree} = M_0 + b m_\pi^2} \quad + \quad \boxed{\text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \dots}$$

$$\Downarrow M_0^r = M_0(1 + a M_0^2); b^r = b^0 + b' M_0$$

$$\boxed{M_N = M_0^r + b^r m_\pi^2 + c m_\pi^3 + \dots \quad (\mathcal{O}(p^3))}$$

# Extended-on-Mass-Shell (EOMS)

- “Drop” the PCB terms

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots}$$

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$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots}$$

**ChPT contains all possible terms allowed by symmetries, therefore whatever analytical terms come out from a loop amplitude, they must have a corresponding LEC**

# HB vs. Infrared vs. EOMS

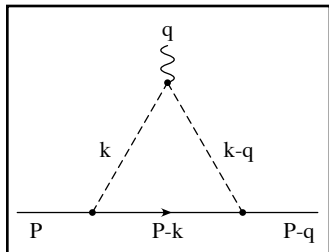
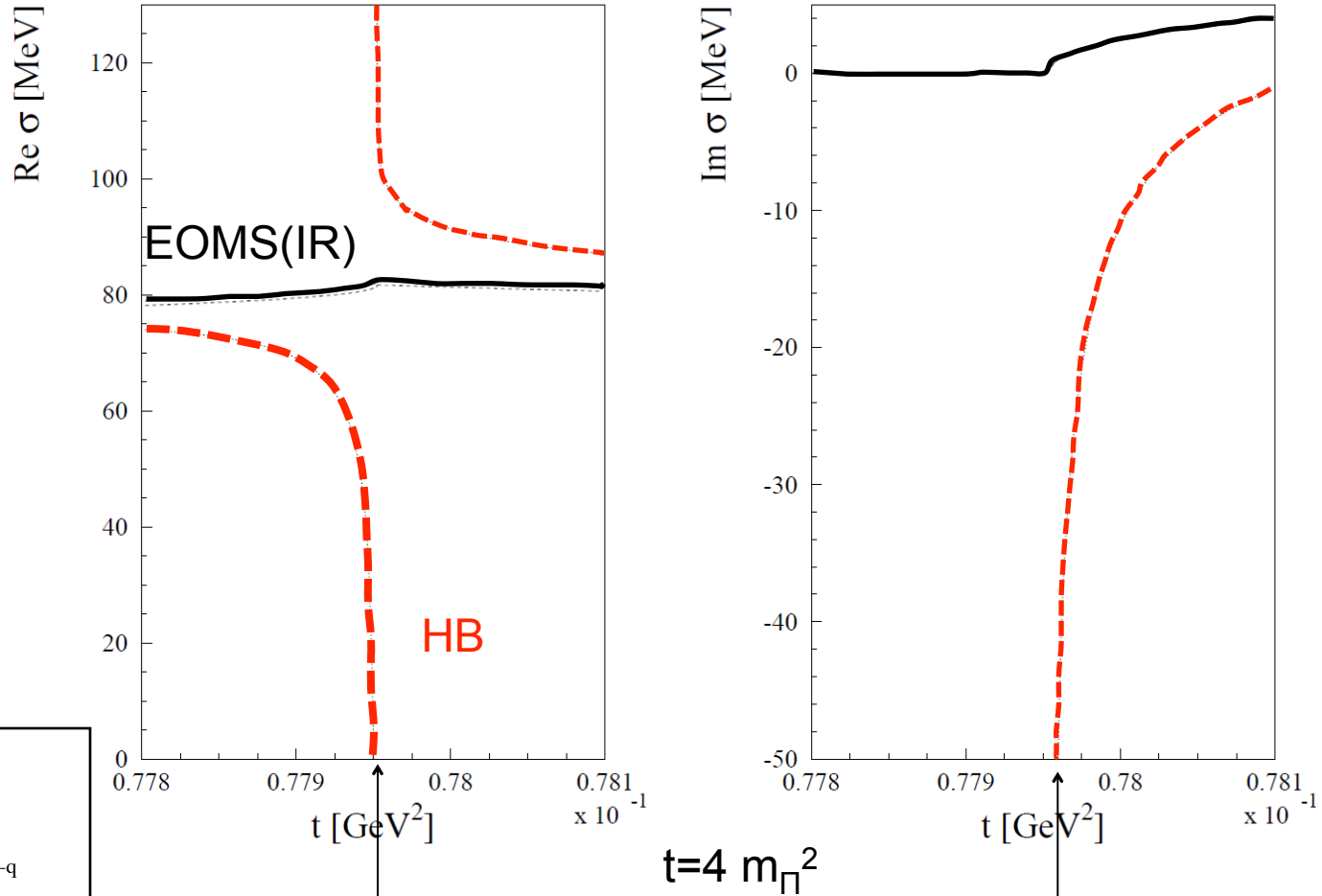
- **Heavy baryon (HB) ChPT**
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- **Infrared BChPT**
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- **Extended-on-mass-shell (EOMS) BChPT**
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

# The nucleon scalar form factor at $q^3$

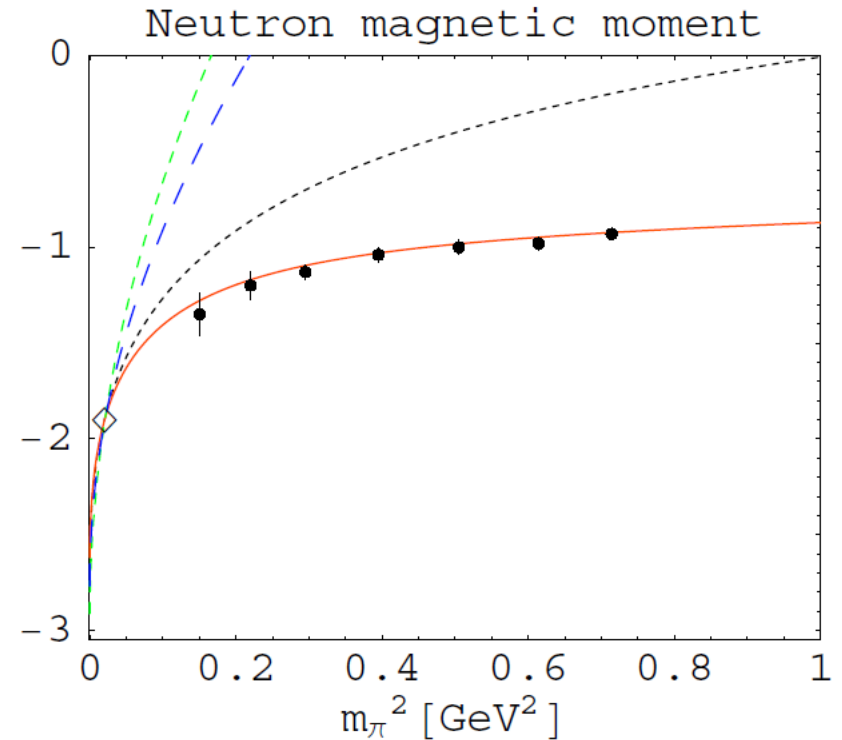
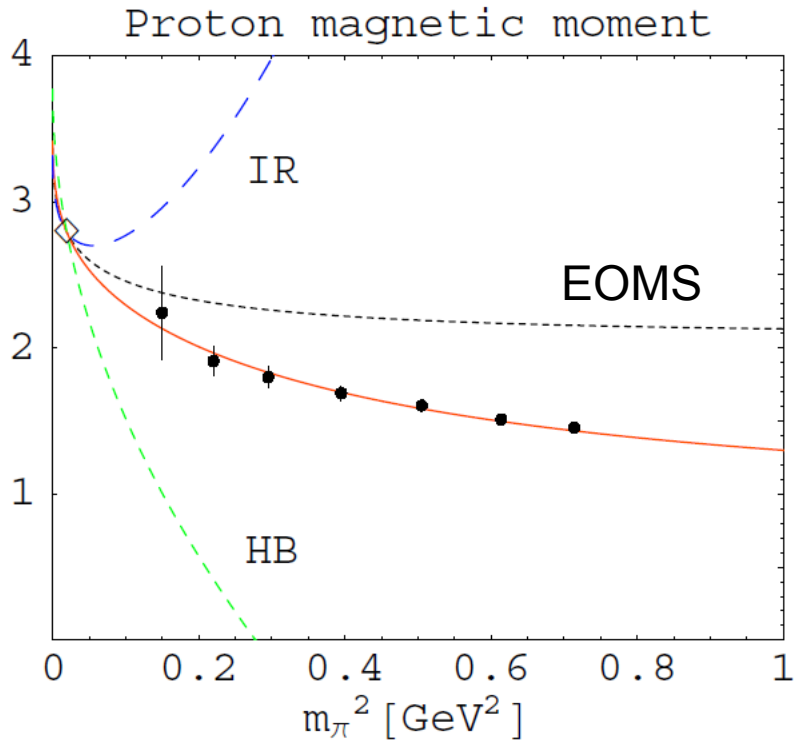
$t$

$$\langle p(p', s') | \mathcal{H}_{sb}(0) | p(p, s) \rangle = \bar{u}(p', s') u(p, s) \sigma(t), \quad t = (p' - p)^2$$

$$\mathcal{H}_{sb} = \hat{m}(\bar{u}u + \bar{d}d)$$



# Proton and neutron magnetic moments: chiral extrapolation



V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.



# Octet baryon magnetic moments at NLO BChPT

$$\chi^2 = \sum (\mu_{th} - \mu_{exp})^2$$

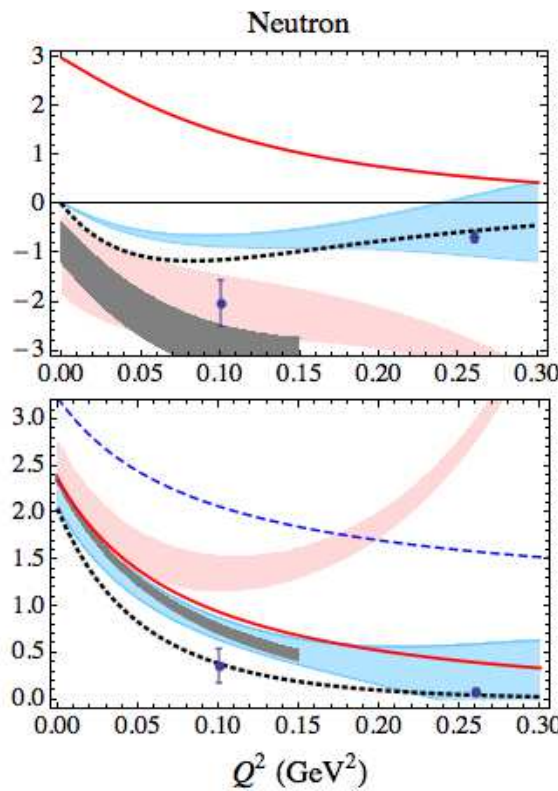
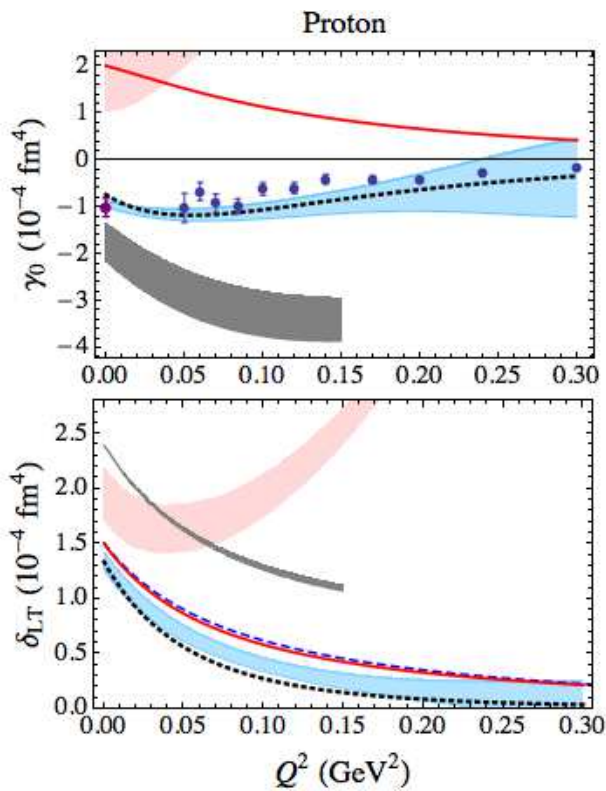
		$p$	$n$	$\Lambda$	$\Sigma^-$	$\Sigma^0$	$\Sigma^+$	$\Xi^-$	$\Xi^0$	$\Lambda\Sigma^0$	$\chi^2$
LO	C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	<b>0.46</b>
	HB	3.01	-2.62	-0.42	-1.35	0.42	2.18	-0.52	-0.70	1.68	<b>1.01</b>
NLO	IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	<b>1.83</b>
	EOMS	2.58	-2.10	-0.66	-1.10	0.66	2.43	-0.95	-1.27	1.58	<b>0.18</b>
	Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—



- Contribution of the chiral series [LO(1+NLO/LO)]:

$$\begin{aligned} \mu_p &= 3.47(1-0.257), & \mu_n &= -2.55(1-0.175), & \mu_\Lambda &= -1.27(1-0.482), \\ \mu_{\Sigma^-} &= -0.93(1+0.187), & \mu_{\Sigma^+} &= 3.47(1-0.300), & \mu_{\Sigma^0} &= 1.27(1-0.482), \\ \mu_{\Xi^-} &= -0.93(1+0.025), & \mu_{\Xi^0} &= -2.55(1-0.501), & \mu_{\Lambda\Sigma^0} &= 2.21(1-0.284). \end{aligned}$$

# Generalized Spin Polarizabilities

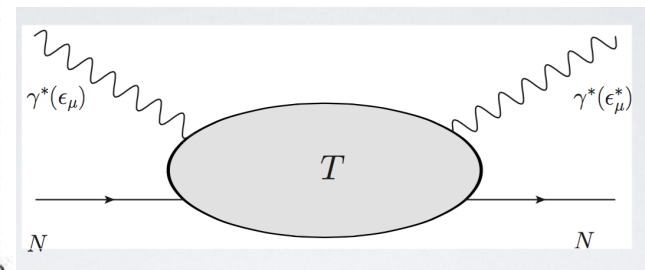


**Grey bands: EOMS + small scale**

**Blue dashed:  $O(p^4)$  HB**

**Red bands: IR calculation**

**Black dotted: MAID2007**



**Solid line: LO EOMS + delta**

**Blue bands: NLO EOMS + delta**

**Phys.Rev. C90 (2014) 055202**

# Problems reported in SU(3) HBChPT (1)

LHPC (A.Walker-Loud et al.), Phys.Rev.D79:054502, 2009.

TABLE XVII. Results from NLO bootstrap  $\chi$  extrapolations of the octet baryon masses, using mixed action (MA) and  $SU(3)$  heavy baryon  $\chi$ PT. C=1.2(2), D=0.715(50), F=0.453(50)

FIT: NLO	Range	$M_0$ (GeV)	$\sigma_M$ (GeV $^{-1}$ )	$\alpha_M$ (GeV $^{-1}$ )	$\beta_M$ (GeV $^{-1}$ )	C	D	F	$\chi^2$	d.o.f.
$M_N, M_\Lambda,$	007–020: MA	1.087(51)	−0.03(5)	−0.72(8)	−0.62(4)	0.15(9)	0.33(4)	0.14(3)	6.0	5
$M_\Sigma, M_\Xi$	007–020: $SU(3)$	1.014(32)	−0.07(4)	−0.77(10)	−0.56(5)	0.18(9)	0.30(6)	0.19(4)	5.5	5
	007–030: MA	1.149(57)	0.01(4)	−0.79(11)	−0.67(7)	0.12(9)	0.38(6)	0.16(3)	14.4	9
	007–030: $SU(3)$	1.091(66)	−0.04(3)	−0.99(28)	−0.73(19)	0.1(1)	0.44(14)	0.24(7)	11.9	9
	007–040: MA	1.147(52)	0.01(3)	−0.78(10)	−0.68(6)	0.13(9)	0.39(6)	0.16(3)	14.9	13
	007–040: $SU(3)$	1.090(61)	−0.04(3)	−0.99(26)	−0.73(18)	0.1(1)	0.45(13)	0.25(6)	12.5	13

TABLE XX. Results from NLO bootstrap  $\chi$  extrapolations of the decuplet masses, using mixed action (MA) and  $SU(3)$  heavy baryon  $\chi$ PT.

FIT: NLO	Range	$M_{T,0}$ (GeV)	$\bar{\sigma}_M$ (GeV $^{-1}$ )	$\gamma_M$ (GeV $^{-1}$ )	C	H	$\chi^2$	d.o.f.
$M_\Delta, M_{\Sigma^*},$	007–020: MA	1.68(10)	−0.04(3)	1.2(3)	0.00(07)	1.2(2)	18.9	7
$M_{\Xi^*}, M_{\Omega^-}$	007–020: $SU(3)$	1.52(05)	−0.20(4)	1.3(3)	0.00(15)	1.4(3)	20.3	7
	007–030: MA	1.64(08)	−0.05(2)	1.1(2)	0.00(07)	1.1(2)	21.0	11
	007–030: $SU(3)$	1.52(04)	−0.19(4)	1.3(3)	0.00(15)	1.4(3)	21.1	11
	007–040: MA	1.73(08)	−0.01(1)	1.2(2)	0.00(06)	1.2(2)	32.8	15
	007–040: $SU(3)$	1.57(04)	−0.18(4)	1.4(3)	0.00(14)	1.6(2)	34.8	15

mixed action heavy baryon chiral perturbation theory. Both the three-flavor and two-flavor functional forms describe our lattice results, although the low-energy constants from the next-to-leading order  $SU(3)$  fits are inconsistent with their phenomenological values. Next-to-next-to-leading order  $SU(2)$  continuum

# Problems reported in SU(3) HBChPT (II)

PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502, 2009.

PACS-CS (S.Aoki et al.), Phys.Rev.D79:034503, 2009.

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing  $D$ ,  $F$ , and  $\mathcal{C}$  at the phenomenological estimate.

	LO	NLO		Phenomenological
		Case 1	Case 2	
$m_B$	0.410(14)	0.391(39)	-0.15(9)	
$\alpha_M$	-2.262(62)	-2.62(62)	-15.3(2.0)	
$\beta_M$	-1.740(58)	-2.6(1.5)	-21.3(3.0)	
$\sigma_M$	-0.53(12)	-0.71(34)	-9.6(1.4)	
$D$		$0.000(16) \times 10^{-8}$	0.80 fixed	0.80
$F$		$0.000(9) \times 10^{-8}$	0.47 fixed	0.47
$\mathcal{C}$		0.36(30)	1.5 fixed	1.5
$\chi^2/\text{dof}$	1.10(63)	1.39(77)	153(82)	

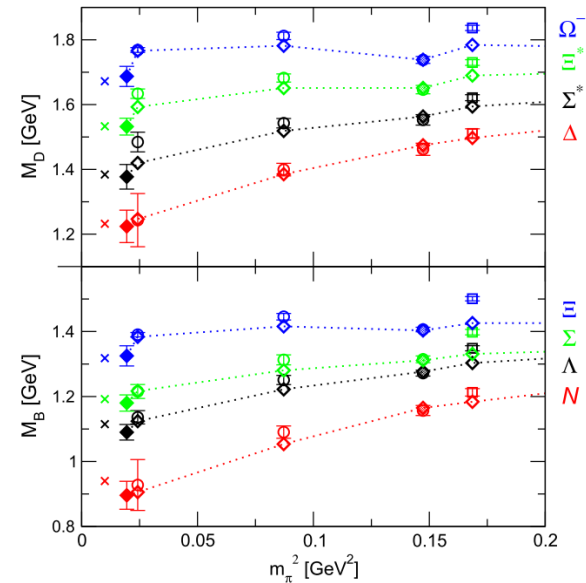
We investigate the quark mass dependence of baryon masses in 2 + 1 flavor lattice QCD using SU(3) heavy baryon chiral perturbation theory up to one-loop order. The baryon mass data used for the analyses are obtained for the degenerate up-down quark mass of 3 to 24 MeV and two choices of the strange quark mass around the physical value. We find that the SU(3) chiral expansion fails to describe both the octet and the decuplet baryon data if phenomenological values are employed for the meson-baryon couplings. The SU(2) case is also examined for the nucleon. We observe that higher order terms are controlled only around the physical point. We also evaluate finite size effects using SU(3) heavy baryon chiral perturbation theory, finding small values of order 1% even at the physical point.

# Covariant ChPT works!

TABLE III: Extrapolation in MeV and values of the LECs from the fits to the PACS-CS results [3] on the baryon masses using B $\chi$ PT up to NLO. The  $\chi^2$  is the estimator for the fits to the IQCD results whereas  $\tilde{\chi}^2$  include also experimental data. See the text for details.

	GMO	HB	Covariant	Expt.
$M_N$	974(22)	767(21)	896(19)(39)	940(2)
$M_\Lambda$	1117(21)	1045(20)	1090(20)(14)	1116(1)
$M_\Sigma$	1168(23)	1213(22)	1180(24)(6)	1193(5)
$M_\Xi$	1286(22)	1394(21)	1325(24)(20)	1318(4)
$M_\Delta$	1321(28)	1267(22)	1224(24)(49)	1232(2)
$M_{\Sigma^*}$	1434(27)	1468(22)	1377(24)(29)	1385(4)
$M_{\Xi^*}$	1548(27)	1624(23)	1532(25)(8)	1533(4)
$M_{\Omega^-}$	1662(27)	1735(24)	1687(28)(13)	1672(1)
$M_{B_0}$ [MeV]	906(40)	513(32)	760(32)	
$b_0$ [GeV $^{-1}$ ]	-0.261(24)	-1.654(19)	-0.979(38)	
$b_D$ [GeV $^{-1}$ ]	0.042(9)	0.368(9)	0.192(25)	
$b_F$ [GeV $^{-1}$ ]	-0.173(7)	-0.824(6)	-0.520(20)	
$M_{D_0}$ [MeV]	1250(48)	1122(32)	957(37)	
$t_0$ [GeV $^{-1}$ ]	-0.11(5)	-0.705(37)	-1.05(8)	
$t_D$ [GeV $^{-1}$ ]	-0.253(10)	-0.738(10)	-0.683(20)	
$\chi^2_{\text{d.o.f.}}$	0.60	9.3	2.2	
$\tilde{\chi}^2_{\text{d.o.f.}}$	4.3	36.4	2.8	

$\chi^2$  : only lattice points  
 $\tilde{\chi}^2$  : lattice points + data



- Unlike HB, EOMS describes the PACS data very well.
- Although LO ChPT describes better than LQCD simulations, **NLO ChPT extrapolations** (at physical light quark masses) are **closer to** the physical data.

*Jorge Martin Camalich, L.S. Geng, M.J. Vicente Vacas,  
 Phys.Rev.D82:074504,2010.*

# Some successful applications of covariant BChPT (in the three-flavor sector)

## ❖ Octet (decuplet) baryon magnetic moments:

Phys.Rev.Lett.101:222002,2008; Phys.Lett.B676:63-68,2009; Phys.Rev.D80:034027,2009

## ❖ Octet and Decuplet baryon masses

Phys.Rev.D82:074504,2010; Phys.Rev.D84:074024,2011; JHEP12(2012)073; Phys.Rev.D87:074001 (2013); Phys.Rev. D89:054034,2014 ; Eur.Phys.J. C74:2754,2014

## ❖ Hyperon vector coupling $f_1(0)$

Phys.Rev.D79:094022,2009;arXiv:Phys.Rev. D89 (2014) 113007

## ● Octet baryon axial coupling

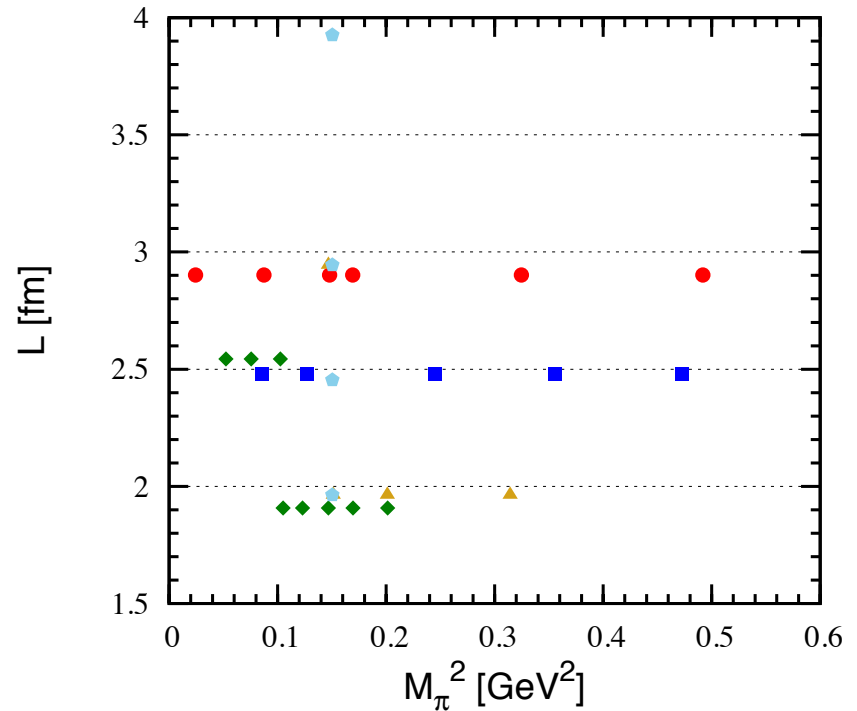
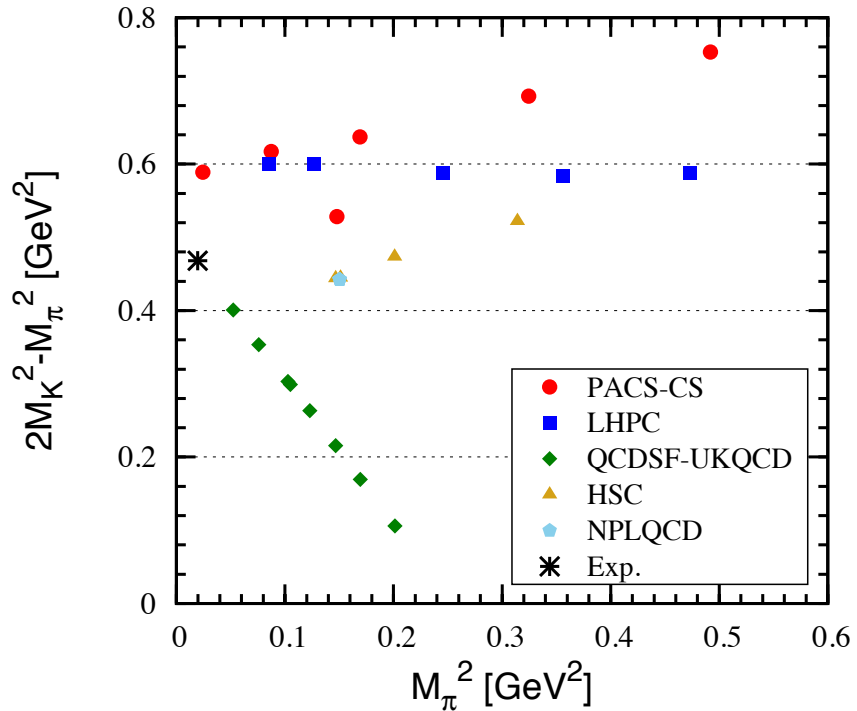
Phys.Rev.D78:014011,2008, Phys.Rev. D90 (2014) 054502



# Two key factors for a reliable determination of the baryon sigma terms

- Lattice QCD simulations of baryon masses at **various** quark masses, volumes, and lattice spacings, and with different fermion/gauge actions
- A reliable formulation of ChPT, which **not only** can well describe the LQCD data, **but also** needs to satisfy all symmetry and analyticity constraints

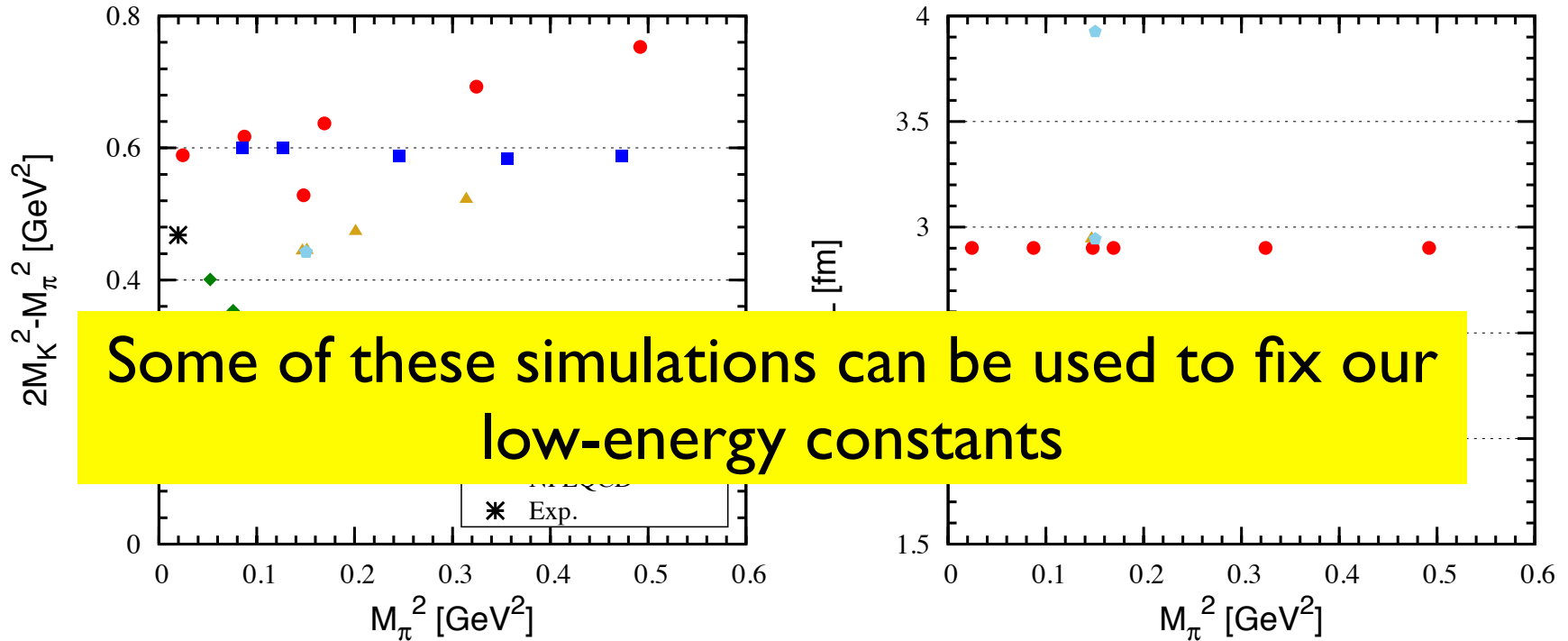
# landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses



## To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum:  $a \rightarrow 0$
- Extrapolate to physical light quark masses:  $m_q \rightarrow m_q(\text{Phys.})$
- Extrapolate to infinite space-time:  $L \rightarrow \infty$

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# Systematic description of the LQCD data with the EOMS BChPT

- NNLO EOMS BChPT study of the PACS-CS and LHPC data: [Camalich, Geng, Vacas, PRD82\(2010\)074504](#)
- Finite volume corrections: [Geng, Ren, Camalich, Weise, PRD84\(2011\)074024](#);
- First systematic study of all publically available LQCD data: [Ren, Geng, Camalich, Meng, Toki, JHEP12\(2012\)073](#);
- Effects of virtual decuplet baryons: [Ren, Geng, Meng, Toki, PRD87\(2013\)074001](#)
- Continuum extrapolations: [Ren, Geng, Meng, Eur.Phys.J. C74:2754,2014](#)

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The EOMS BChPT can be trusted to predict  
the baryon sigma terms

- Effects of virtual decuplet baryons: *Ren, Geng, Meng, Toki, PRD87(2013)074001*
- Continuum extrapolations: *Ren, Geng, Meng, Eur.Phys.J. C74:2754,2014*

# Selection of LQCD data

- All  $n_f=2+1$  LQCD simulations
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD, BWM
  - **BWM**—not publicly available
  - HSC/NPLQCD—Low statistics/single combination of quark masses

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PACS-CS, LHPC, QCDSF-UKQCD



# An accurate determination of baryon sigma terms

- **Scale setting:** mass independent (given by the LQCD simulations or self-consistently determined) vs. mass dependent ( $r_0$ ,  $r_1$ ,  $X_\pi$ )
- **Isospin breaking effects:** better constrain the LQCD LECs—consistent with the latest BWM study [Science 347 (2015) 1452]
- **Theoretical uncertainties caused by truncating chiral expansions:** NNLO vs. N3LO; EOMS vs. FRR

# Scale-setting effects on the determination of baryon sigma terms

*arXiv:1301.3231*

*P.E. Shanahan\*, A.W. Thomas and R.D. Young*

- **Lattice-scale setting**

- PACS-CS data with **mass independent** scale-setting:

$$\sigma_{sN} = 59 \pm 7 \text{ (MeV)}$$

- PACS data with **mass dependent** ( $r_0$ ) scale-setting:

$$\sigma_{sN} = 21 \pm 6 \text{ (MeV)}$$

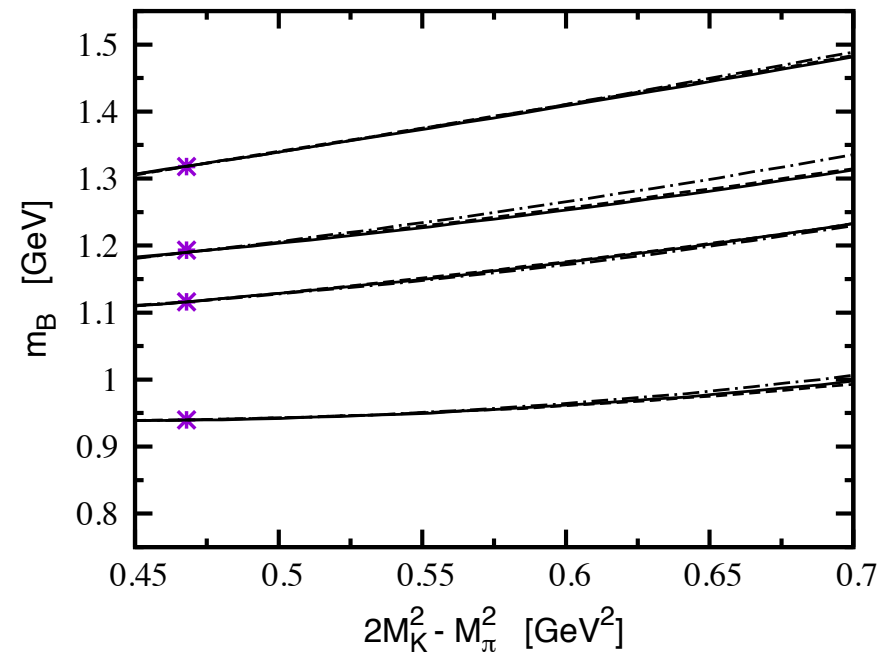
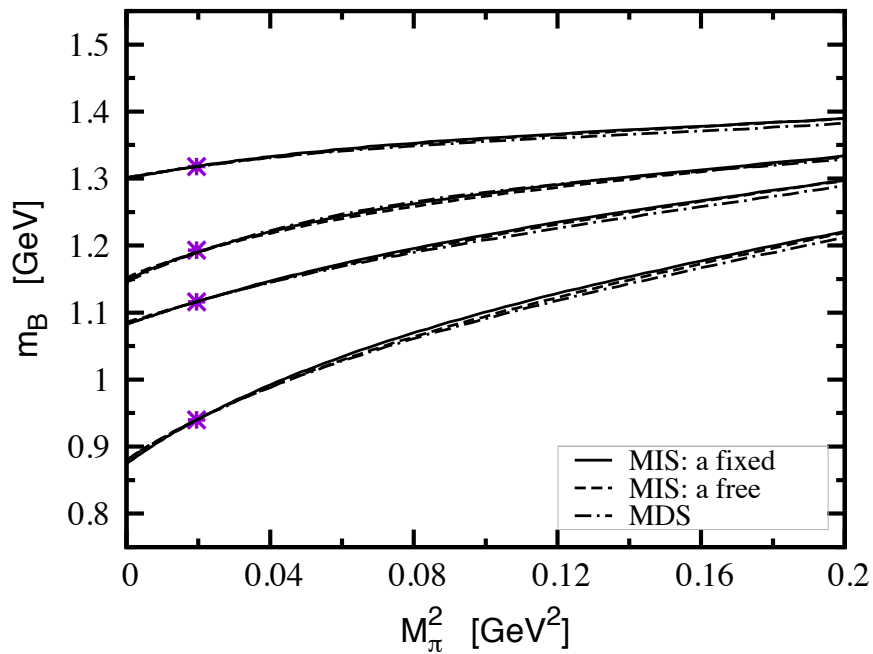
- **Whether other LQCD data will show the same trend?**

# Three different fits at N<sup>3</sup>LO

	MIS		MDS
	<i>a</i> fixed	<i>a</i> free	
$m_0$ [MeV]	884(11)	877(10)	887(10)
$b_0$ [GeV <sup>-1</sup> ]	-0.998(2)	-0.967(6)	-0.911(10)
$b_D$ [GeV <sup>-1</sup> ]	0.179(5)	0.188(7)	0.039(15)
$b_F$ [GeV <sup>-1</sup> ]	-0.390(17)	-0.367(21)	-0.343(37)
$b_1$ [GeV <sup>-1</sup> ]	0.351(9)	0.348(4)	-0.070(23)
$b_2$ [GeV <sup>-1</sup> ]	0.582(55)	0.486(11)	0.567(75)
$b_3$ [GeV <sup>-1</sup> ]	-0.827(107)	-0.699(169)	-0.553(214)
$b_4$ [GeV <sup>-1</sup> ]	-0.732(27)	-0.966(8)	-1.30(4)
$b_5$ [GeV <sup>-2</sup> ]	-0.476(30)	-0.347(17)	-0.513(89)
$b_6$ [GeV <sup>-2</sup> ]	0.165(158)	0.166(173)	-0.0397(1574)
$b_7$ [GeV <sup>-2</sup> ]	-1.10(11)	-0.915(26)	-1.27(8)
$b_8$ [GeV <sup>-2</sup> ]	-1.84(4)	-1.13(7)	0.192(30)
$d_1$ [GeV <sup>-3</sup> ]	0.0327(79)	0.0314(72)	0.0623(116)
$d_2$ [GeV <sup>-3</sup> ]	0.313(26)	0.269(42)	0.325(54)
$d_3$ [GeV <sup>-3</sup> ]	-0.0346(87)	-0.0199(81)	-0.0879(136)
$d_4$ [GeV <sup>-3</sup> ]	0.271(30)	0.230(24)	0.365(23)
$d_5$ [GeV <sup>-3</sup> ]	-0.350(28)	-0.302(50)	-0.326(66)
$d_7$ [GeV <sup>-3</sup> ]	-0.435(10)	-0.352(8)	-0.322(7)
$d_8$ [GeV <sup>-3</sup> ]	-0.566(24)	-0.456(30)	-0.459(33)
$\chi^2/\text{d.o.f.}$	0.87	0.88	0.53

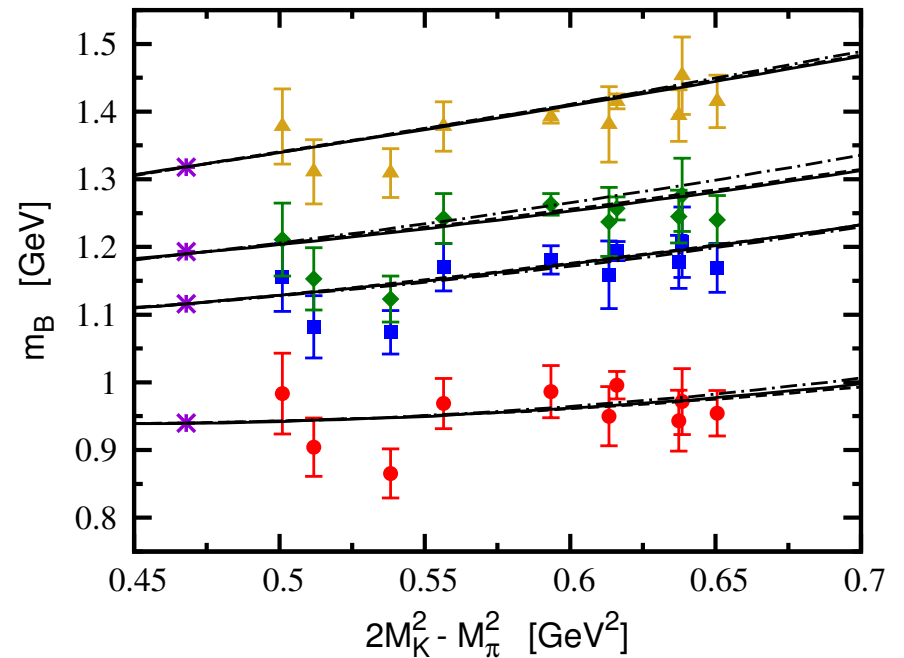
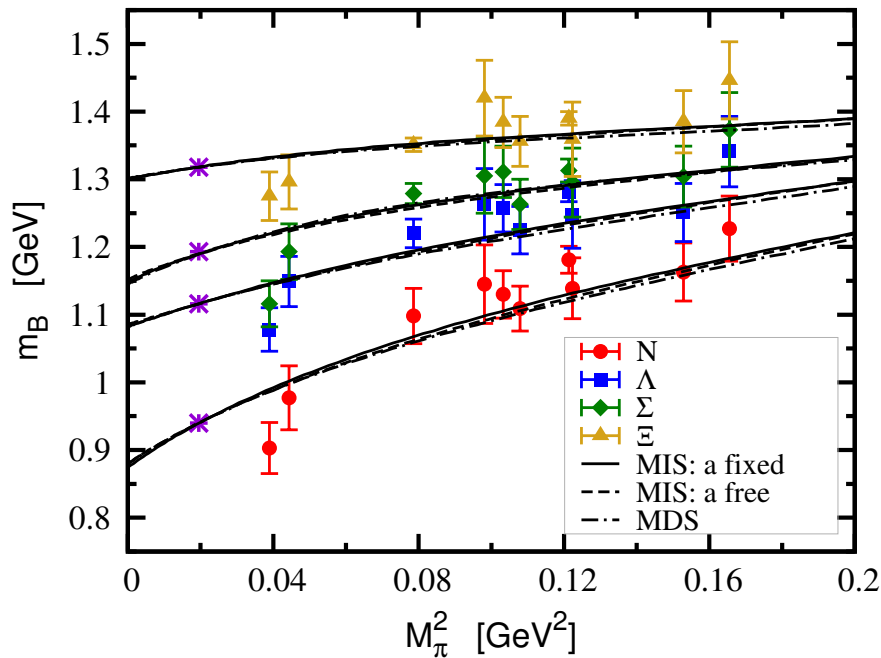
- **Mass independent**
  - Lattice spacing **a fixed** to the published value
  - Lattice spacing a determined **self-consistently**
- **Mass dependent**
  - $r_0$  for PACS-CS
  - $r_1$  for LHPC
  - $X_\pi$  for QCDSF-UKQCD

# Evolution of baryon masses with u/d and s quark masses



Only central values are shown!

# Evolution of baryon masses with u/d and s quark masses in comparison with the BMW data

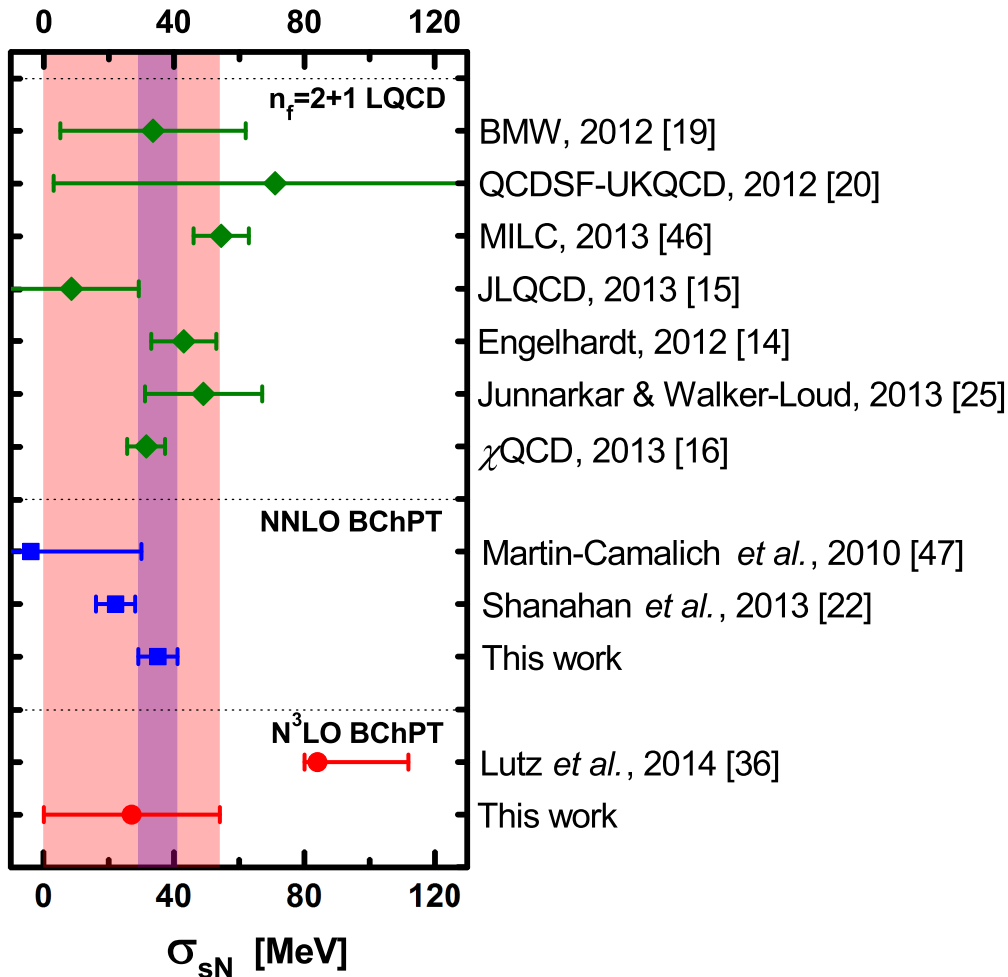


# Baryon sigma terms from N<sup>3</sup>LO BChPT

	MIS		MDS
	<i>a</i> fixed	<i>a</i> free	
$\sigma_{\pi N}$	55(1)(4)	54(1)	51(2)
$\sigma_{\pi \Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi \Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi \Xi}$	16(1)(2)	18(2)	15(3)
$\sigma_{sN}$	27(27)(4)	23(19)	26(21)
$\sigma_{s\Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s\Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s\Xi}$	333(25)(13)	346(15)	340(13)

- All three scale-setting methods yield similar baryon sigma terms

# Comparison with earlier studies



- **Consistent** with most recent LQCD studies and those of NNLO ChPT, e.g., that of Young and Shanahan
- **Uncertainties** at  $N^3$ LO substantially **larger**, because of the extra LECs

Nucleon Strangeness Sigma Term

# Summary



# Summary

- ❖ I have explained how **the baryon sigma terms** (particularly those of the nucleon) are **related** to **dark matter direct searches** and the understanding of the **quark-flavor structure** of the nucleon.

# Summary

- ❖ I have explained how **the baryon sigma terms** (particularly those of the nucleon) are **related** to **dark matter direct searches** and the understanding of the **quark-flavor structure** of the nucleon.
- ❖ I have shown how a combination of **lattice QCD simulations** and baryon **chiral perturbation theory** allows us to make a **reliable prediction** of these terms.

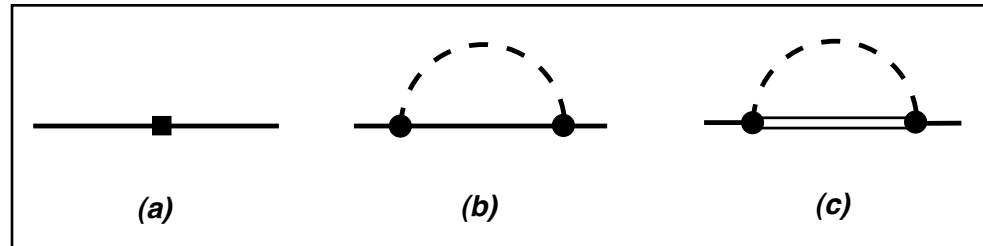
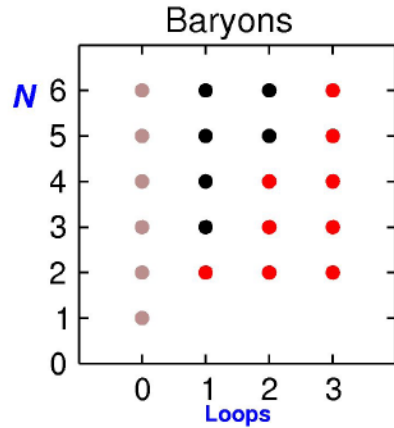
Thank you very much  
for your attention!

# Covariant chiral perturbation theory in heavy-light systems

- **DK interactions and dynamically generated resonances**
  - Phys.Rev. D89 (2014) 054008
  - Phys.Rev. D89 (2014) 014026
  - Phys.Rev. D82 (2010) 054022
- **Decay constants of D/B/D\*/B\* mesons**
  - Phys.Lett. B713 (2012) 453-456
  - Phys.Lett. B696 (2011) 390-395

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$

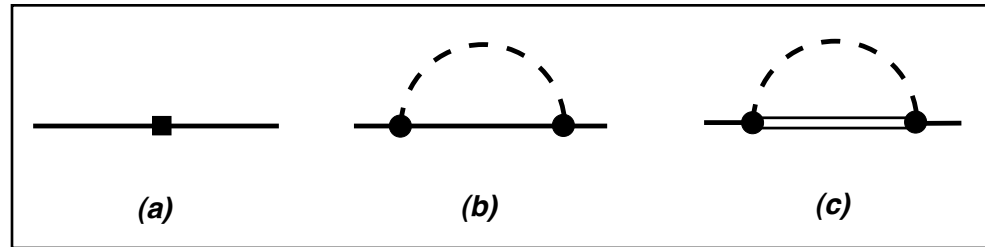
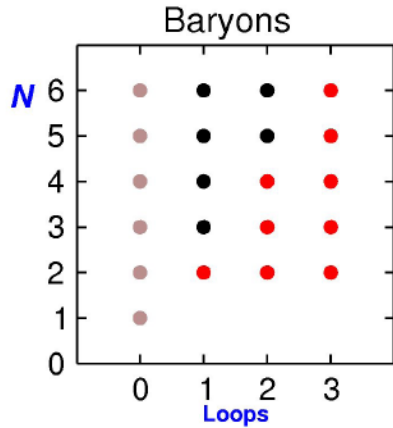
# Nucleon mass up to $O(p^3)$



$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$

# Nucleon mass up to $O(p^3)$



$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

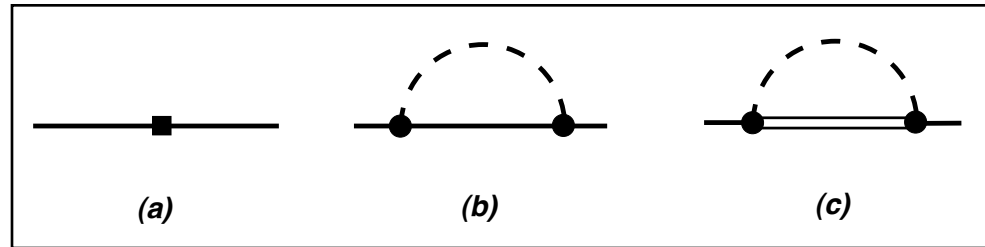
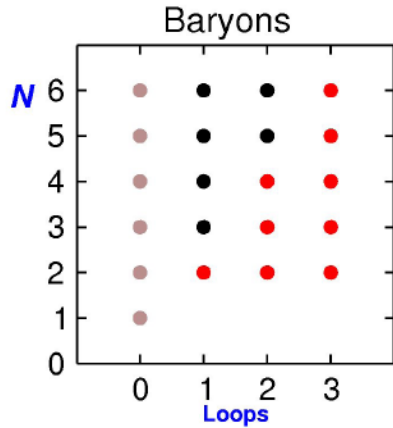
Naively  
(no PCB)

$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \dots)$$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$

# Nucleon mass up to $O(p^3)$



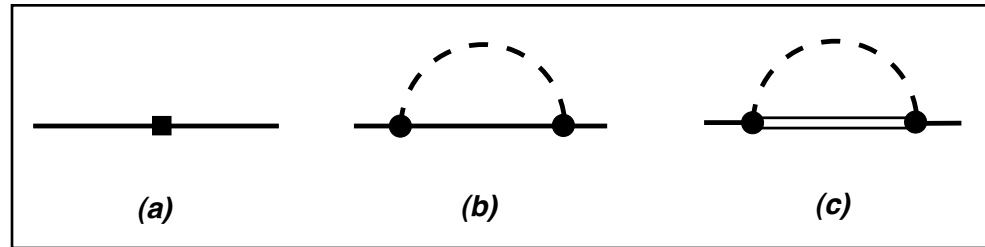
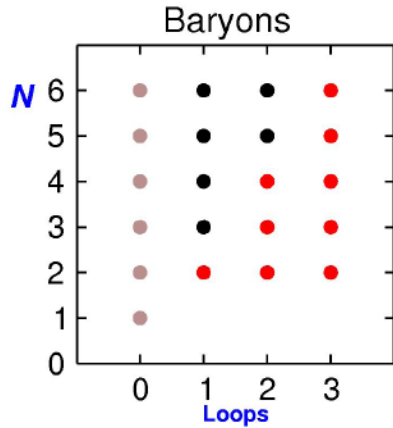
$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

**Naively (no PCB)**  $M_N = M_0 + bm_\pi^2 + \text{loop}$   
 $\text{loop}(= cm_\pi^3 + \dots)$

**However**  $\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots$

$$\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.$$

# Nucleon mass up to $O(p^3)$



$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

**Naively**  
**(no PCB)**

$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \dots)$$

**However**

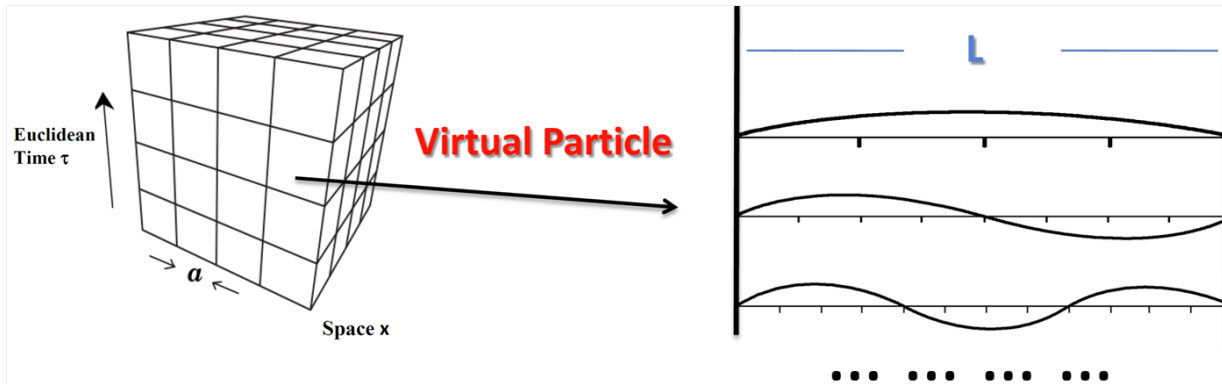
$$\text{loop} = aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \dots$$

No need to calculate, simply recall that  $M_0 \sim O(p^0)$



# Finite volume corrections

- **Physical origin: existence of boundary conditions**

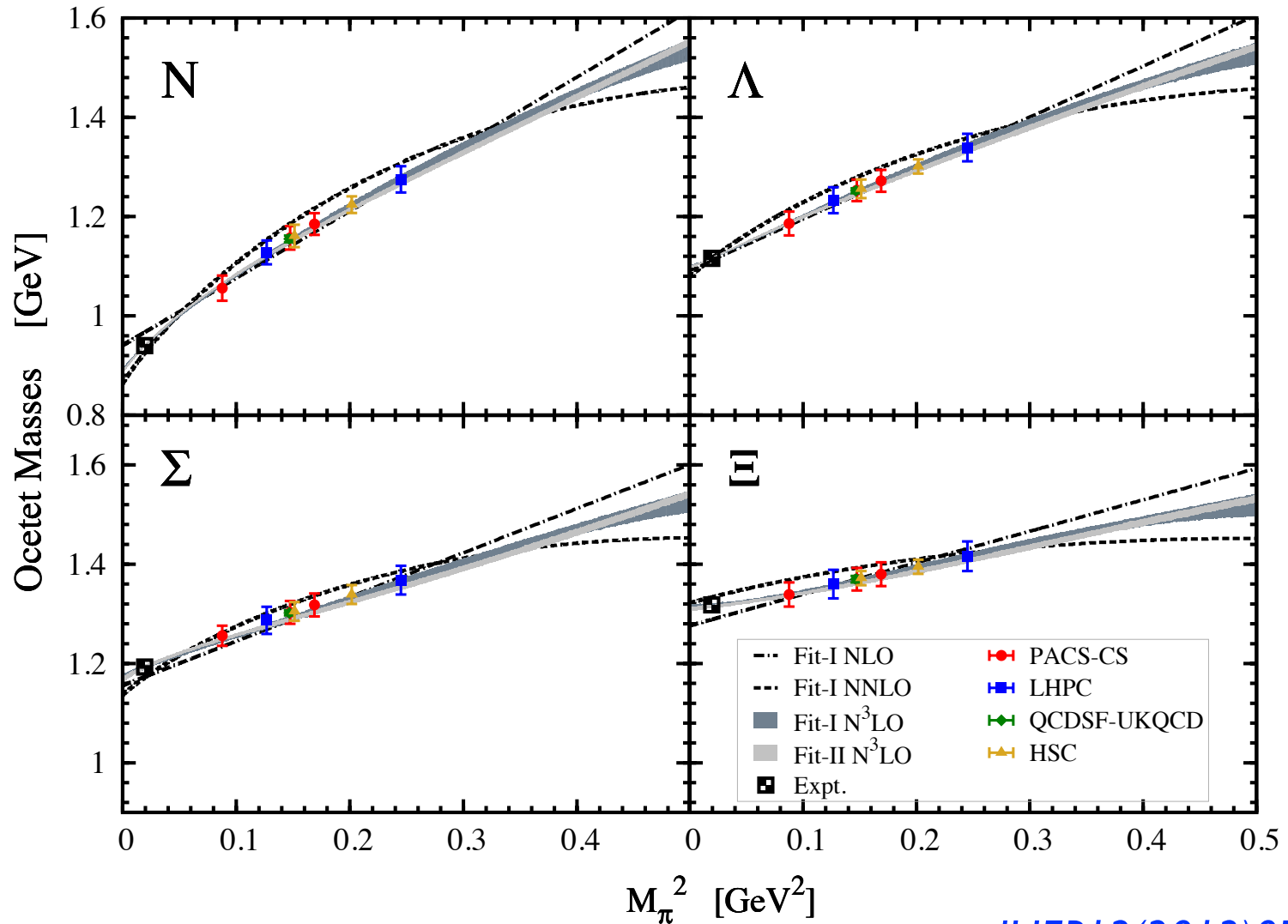


- **Momenta of virtual particles are discretized**

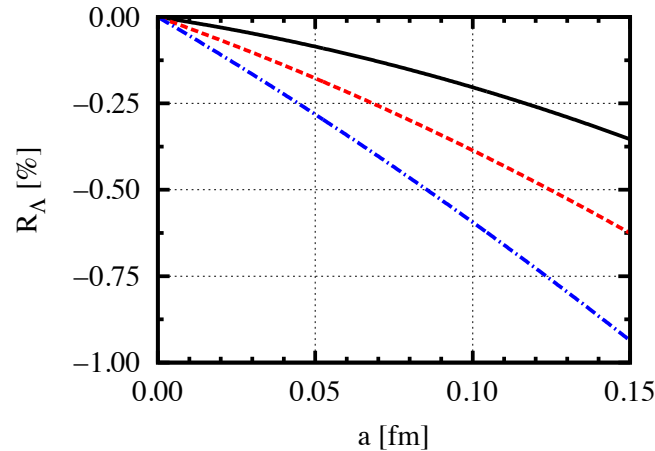
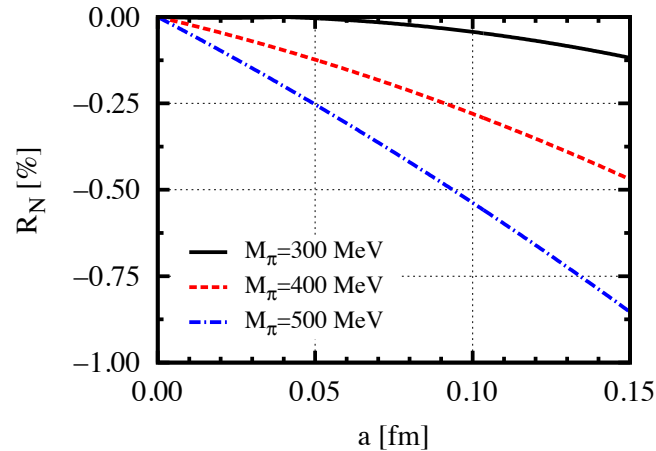
$$k_i = 2\pi \frac{n_i}{L}, \quad (i = 0, 1, 2, 3)$$

$$\int_{-\infty}^{\infty} dk \Rightarrow \sum_{n=-\infty}^{\infty} \left( \frac{2\pi}{L} \right) \cdot n.$$

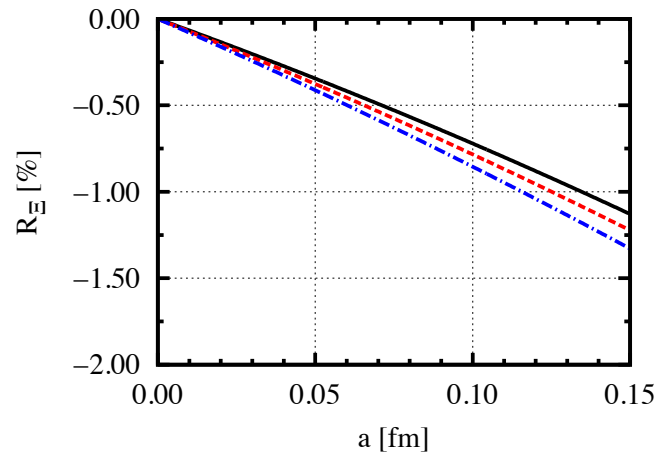
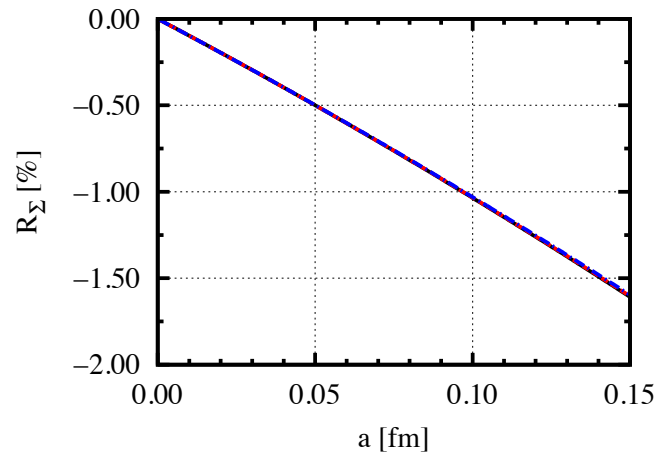
# Chiral extrapolations upto N3LO in BChPT



# Lattice spacing evolutions



$$R_B = m_B^{(a)} / m_B$$



- For LQCD simulations with  $m_\pi < 500$  MeV and  $a < 0.15$  fm, discretization effects are about 1 to 2 percent

# NNLO fits

TABLE I. Values of the LECs obtained from the best fits to the LQCD simulations and the experimental octet baryon masses and the corresponding  $\chi^2/\text{d.o.f.}$ . The underlined numbers denote the values at which they are fixed.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
$m_0$ [MeV]	757(7)	808(1)	829(7)	805(9)
$b_0$ [GeV $^{-1}$ ]	-0.907(6)	-0.710(2)	-0.820(7)	-0.922(20)
$b_D$ [GeV $^{-1}$ ]	0.0582(22)	0.0570(22)	0.101(2)	0.116(3)
$b_F$ [GeV $^{-1}$ ]	-0.508(2)	-0.411(11)	-0.464(2)	-0.510(8)
$f_0$ [GeV]	<u>0.0871</u>	0.105(3)	<u>0.0871</u>	<u>0.0871</u>
$\Lambda$ or $\mu$ [GeV]	<u>1.0</u>	<u>1.0</u>	<u>1.0</u>	1.24(5)
$\chi^2/\text{d.o.f.}$	3.0	1.6	2.4	1.8

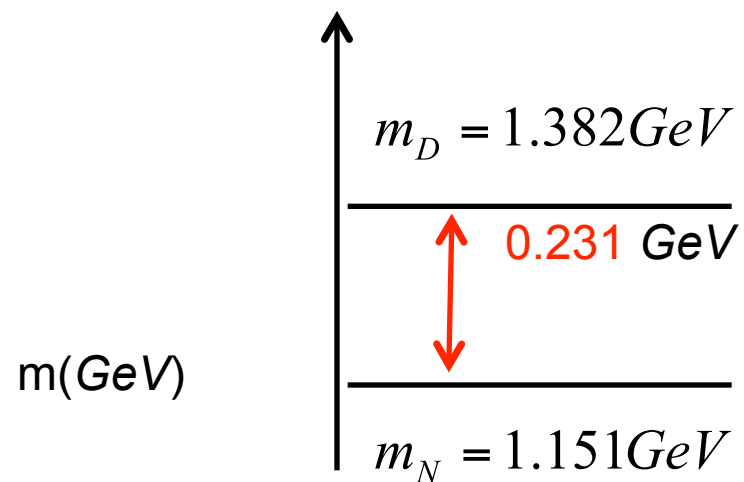
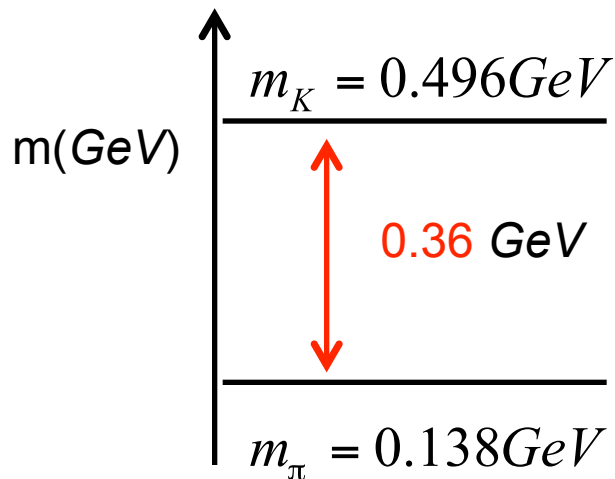
# NNLO sigma terms

TABLE II. Sigma terms of the octet baryons at the physical point, predicted by the NNLO BChPT with the LECs of Table I.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
$\sigma_{\pi N}$ [MeV]	56(0)	47(1)	47(0)	53(1)
$\sigma_{\pi \Lambda}$ [MeV]	35(1)	30(1)	31(1)	34(1)
$\sigma_{\pi \Sigma}$ [MeV]	32(0)	27(1)	25(0)	27(1)
$\sigma_{\pi \Xi}$ [MeV]	13(1)	12(1)	13(1)	13(1)
$\sigma_{s N}$ [MeV]	35(6)	27(7)	21(6)	20(7)
$\sigma_{s \Lambda}$ [MeV]	147(7)	152(7)	162(7)	153(7)
$\sigma_{s \Sigma}$ [MeV]	218(7)	222(7)	226(7)	214(7)
$\sigma_{s \Xi}$ [MeV]	295(7)	313(8)	332(7)	312(8)

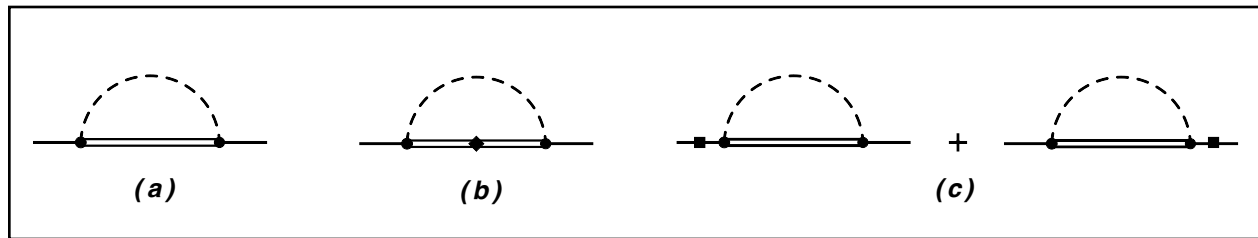
# Effects of dynamical decuplet baryons

- ChPT relies on the assumption that all high-energy degrees of freedom can be integrated out--not necessarily true for SU(3) BChPT



# Feynman diagrams/Lagrangians-no new unknown LECs

- Feynman diagrams



- Lagrangians

- Octet-Decuplet-Pseudoscalar coupling *fixed from decay of a decuplet into an octet baryon and a pseudoscalar*

$$\mathcal{L}_{\phi BT}^{(1)} = \frac{i\mathcal{C}}{m_D F_\phi} \varepsilon^{abc} (\partial_\alpha \bar{T}_\mu^{ade}) \gamma^{\alpha\mu\nu} B_c^e \partial_\nu \phi_b^d + \text{H.c.},$$

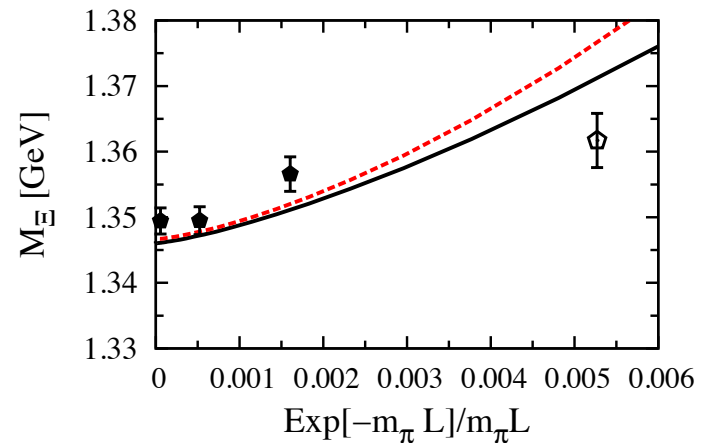
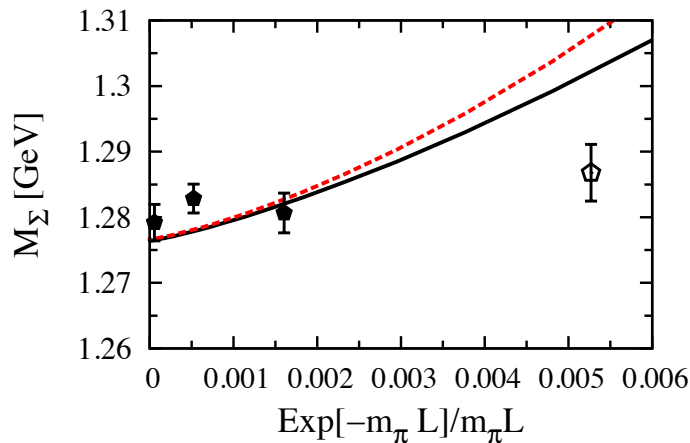
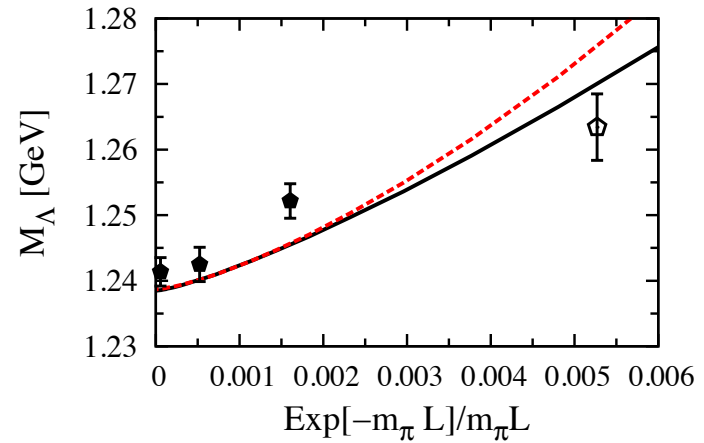
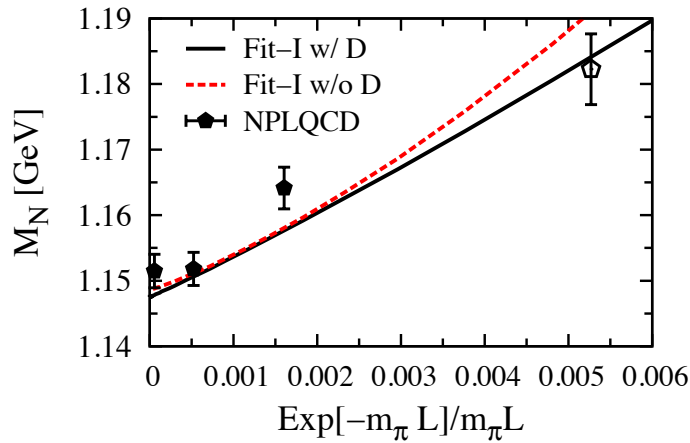
*fixed from decay of a decuplet into an octet baryon and a pseudoscalar*

- mass corrections

$$\mathcal{L}_T^{(2)} = \frac{t_0}{2} \bar{T}_\mu^{abc} g^{\mu\nu} T_\nu^{abc} \langle \chi_+ \rangle + \frac{t_D}{2} \bar{T}_\mu^{abc} g^{\mu\nu} (\chi_+, T_\nu)^{abc},$$

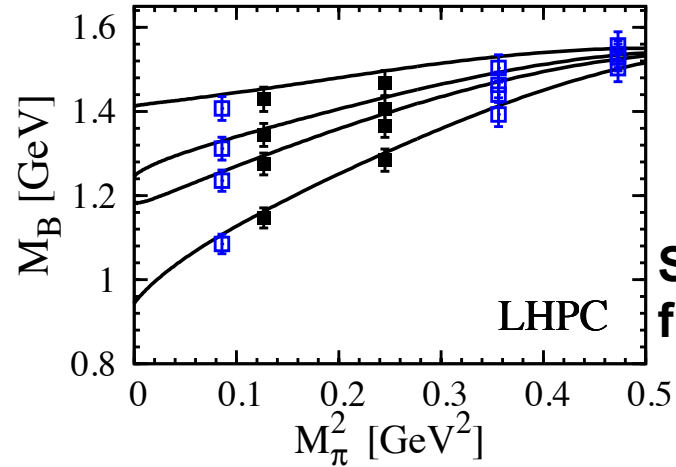
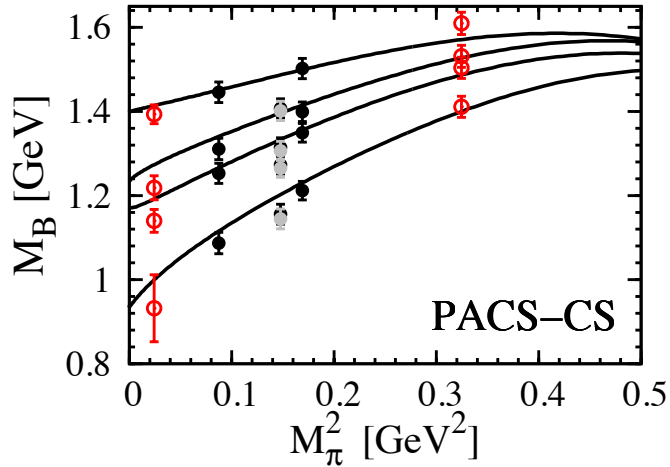
*fixed from the experimental decuplet masses*

# Slightly better description of the volume dependence of the NPLQCD data

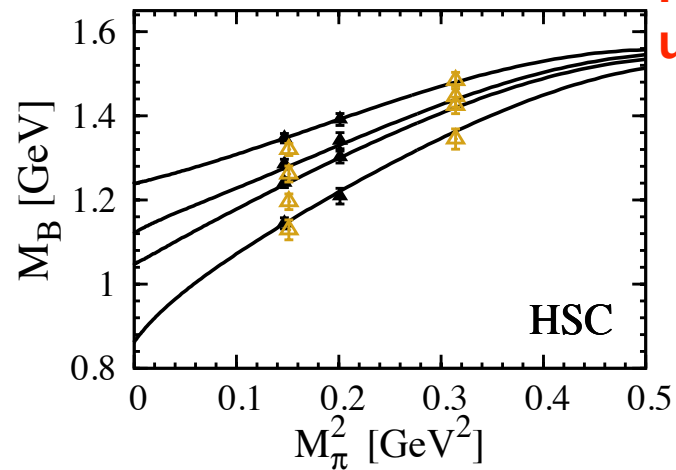
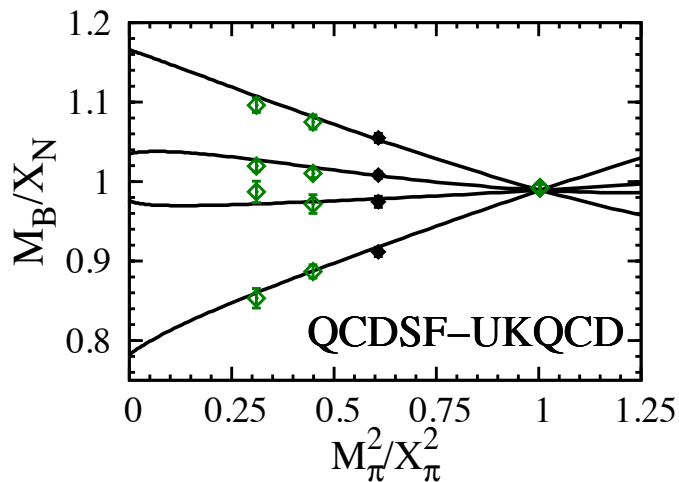




# Unfitted data can also reasonably well described



Solid black:  
fitted



Hollow colored:  
unfitted

# Baryon Pion and Strangeness Sigma terms

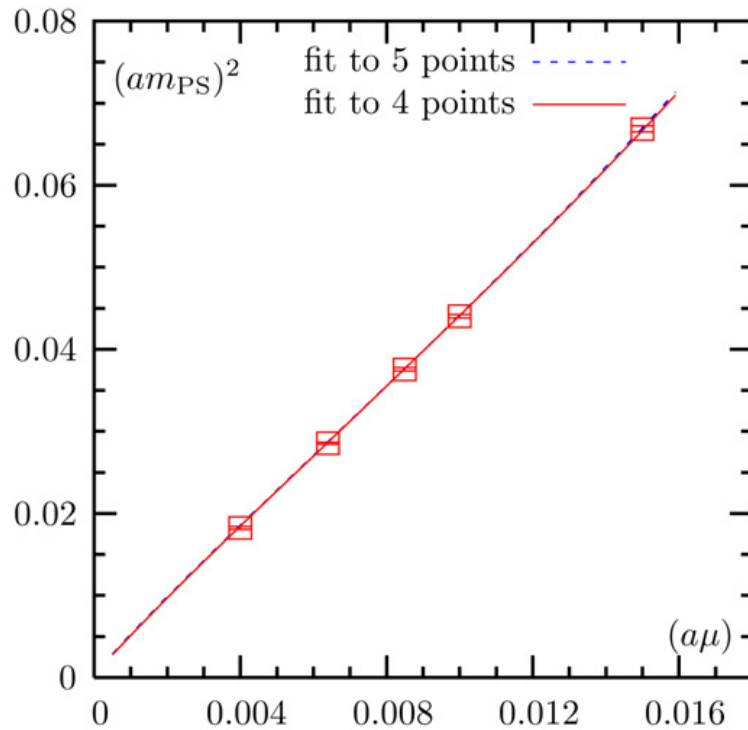
- **Feynman-Hellmann theorem states**

$$\sigma_{\pi B} = m_l \langle B(p) | \bar{u}u + \bar{d}d | B(p) \rangle = m_l \frac{\partial M_B}{\partial m_l}$$
$$\sigma_{sB} = m_s \langle B(p) | \bar{s}s | B(p) \rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

- **Using leading-order ChPT meson masses**

$$\sigma_{\pi B} = \frac{m_{\pi}^2}{2} \left( \frac{1}{m_{\pi}} \frac{\partial}{\partial m_{\pi}} + \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{1}{3m_{\eta}} \frac{\partial}{\partial m_{\eta}} \right) m_B$$
$$\sigma_s = \left( m_K^2 - \frac{m_{\pi}^2}{2} \right) \left( \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{2}{3m_{\eta}} \frac{\partial}{\partial m_{\eta}} \right) m_B$$

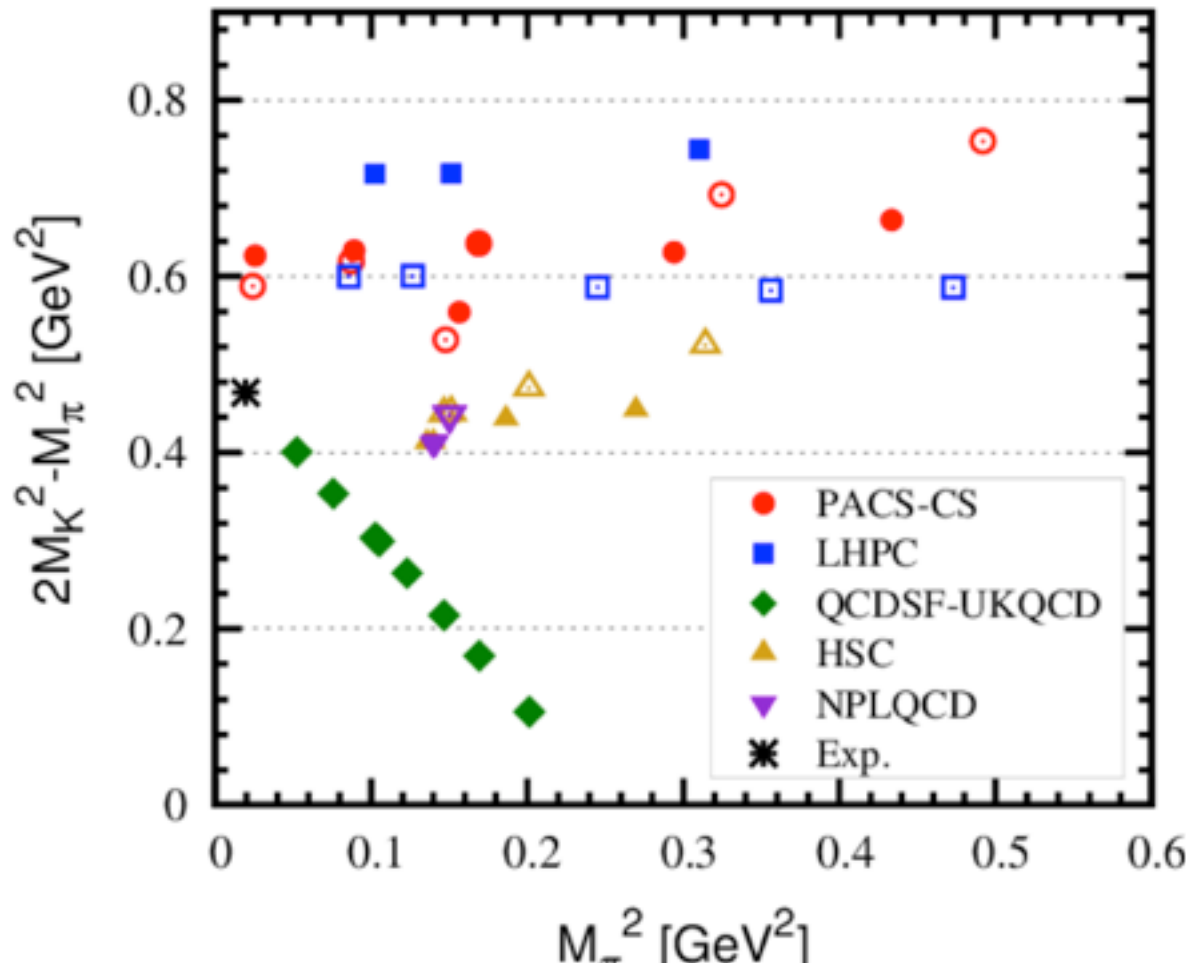
# Pion mass vs. light quark mass



$$m_{\pi}^2 \propto m_q$$

ETM collaboration, hep-lat/0701012

# Scale-setting effects on the octet baryon masses



- **Full symbols:**  
scale dependent
- **Hollow symbols:**  
scale independent