New Developments in the Theory of n-$\bar{n}$ Oscillations

Susan Gardner

Department of Physics and Astronomy
University of Kentucky
Lexington, KY 40506

gardner@pa.uky.edu

Based, in part, on SG and Ehsan Jafari (UK), Phys. Rev. D91, 096010 (2015) [arXiv:1408.2264v2] and on work in collaboration with Xinshuai Yan (UK).

UNIVERSITY OF KENTUCKY

Motivation for Studies of n-$\bar{n}$ Oscillations

The Standard Model (SM) leaves many questions unanswered.
Most notably it cannot explain **the cosmic baryon asymmetry, dark matter, or dark energy.**

$\mathcal{B}$ violation plays a role in at least one of these puzzles.

Although $\mathcal{B}$ violation appears in the SM (sphalerons), we know nothing of its pattern at accessible energies.

Do processes occur with $|\Delta\mathcal{B}| = 1$ or $|\Delta\mathcal{B}| = 2$ or both?
The SM conserves $\mathcal{B} - \mathcal{L}$, but does Nature?

Severe limits on nucleon decay ($|\Delta\mathcal{B}| = 1$) exist, but the origin of $|\Delta\mathcal{B}| = 2$ processes can be completely distinct.

[Marshak and Mohapatra, PRL, 1980; Babu and Mohapatra, PLB, 2001 & 2012; Arnold, Fornal, and Wise, PRD, 2013]

If neutron-antineutron oscillations are observed (a “background free” signal!), then $\mathcal{B} - \mathcal{L}$ is **broken**, and **we have discovered physics beyond the SM.**

n-$\bar{n}$ Oscillations: Models and Broader Impacts

It has long been thought that n-$\bar{n}$ oscillations could shed light on the mechanism of

- Baryogenesis [Kuzmin, 1967]
- Neutrino mass [Mohapatra and Marshak, 1980]

In contrast to proton decay, n-$\bar{n}$ probes new physics at "intermediate" energy scales, as they can be generated by **d=6** and **d=9** operators, respectively. Crudely, $\Lambda_{p\,decay} \geq 10^{15}\,\mathrm{GeV}$ and $\Lambda_{n\bar{n}} \geq 10^{5.5}\,\mathrm{GeV}$.

n-$\bar{n}$ oscillations have been discussed in **many** different contexts. Some examples...

- TeV-Scale Seesaw + Q-L Unification (* Post-Sphaleron Baryogenesis) [Babu, Dev, Mohapatra, 2009; Babu, Dev, Fortes, Mohapatra, 2013]
- SO(10) GUT-Scale Seesaw + TeV sextets (* predicted θ_{13}) [Babu, Mohapatra]
- TeV-Scale Extra Dimensions [Dvali, Gabadadze; Nussinov, Shrock, 2002; Winslow, Ng, 2010]
- Supersymmetry/Superstring [Mohapatra and Valle, 1986; Goity, Sher, 1994]

Some are predictive (*) in that they will fail in their purpose if $\tau_{n\bar{n}}$ is larger than some "X" (within envisioned experimental reach! $X \simeq 5 \times 10^{10}$ s!)

on Majorana masses

A massive neutrino could be a Majorana particle, with its mass generated by the $d = 5$ operator $\psi^T C\psi + h.c.$.

[Weinberg, 1979]

- Recall that C is fixed so that the conjugate field $\psi_C \equiv \gamma_0 C\psi^*$ transforms like ψ under Lorentz transformations.
- If $\psi = \psi_C$, then the neutrino can be described by a two-component field.
- Also $\psi^T C\psi$ is Hermitian and can be rewritten as $\bar{\psi}\psi$: it is Lorentz invariant and C, P, and T even.

The observation of neutrinoless $\beta\beta$ decay ($|\Delta\mathcal{L}| = 2$) would reveal that the neutrino is Majorana, that the neutrino is its own antiparticle.

[Schechter and Valle, PRD, 1982]

Recall "$\nu_e \neq \bar{\nu}_e$" is just reflective of the $V - A$ law. [Davis, 1955]

Motivation for n-$\bar{n}$: the Quark Analog of 0$\nu\beta\beta$ Decay

The SM preserves $\mathcal{B} - \mathcal{L}$, so that the observation of either *n*-$\bar{n}$ oscillations ($|\Delta\mathcal{B}| = 2$) or of neutrinoless $\beta\beta$ decay ($|\Delta\mathcal{L}| = 2$) would reveal the existence of dynamics beyond the SM.

However, QCD is a gauge theory in SU(3) color $\leftrightarrow$ **3** $\neq$ **3***.
Thus *n* is distinct from $\bar{n}$, and it has a significant magnetic moment.

Thus the "Majorana dynamics" of quarks should be fundamentally different from the neutrino case....

To explore how, we embed the usual phenomenology of *n*-$\bar{n}$ oscillations in a 4×4 **framework, including the role of spin.** [SG and Jafari, arXiv:1408.2264v2]

Usual Phenomenology of $n - \bar{n}$ Oscillations

A 2×2 effective Hamiltonian framework for n-$\bar{n}$ mixing

[Marshak and Mohapatra, PLB, 1980; Cowsik and Nussinov, PLB, 1981; Phillips II et al. [NNbar Collaboration], arXiv:1410.1100]

$$\mathcal{H} = \begin{pmatrix} M_n - \mu_n B & \delta \\ \delta & M_n + \mu_n B \end{pmatrix},$$

yields

$$P_{n\to\bar{n}}(t) \simeq \frac{\delta^2}{2(\mu_n B)^2}\left[1 - \cos\left(2\mu_n B t\right)\right]\exp(-\lambda t)$$

so that unless $t \ll 1/(2\mu_n B)$, a nonzero **B** "quenches" n-$\bar{n}$ oscillations.

There have been many studies of n-$\bar{n}$ in "elixir" magnetic fields, all in the 2×2 framework.

[Arndt, Prasad, Riazuddin, PRD 1983; Pusch, Nuov. Cim. 1983; Krsticć, Komarov, Janen, Zenko, PRD 1988; Dubbers, NIM 1989; Kinkel, Z. Phys. C 1992]

Experimentally magnetic fields have been mitigated (to great expense), yielding $P_{n\to\bar{n}}(t) \simeq \delta^2 t^2$ and $\tau_{n\bar{n}} \equiv 1/\delta$ with $\tau_{n\bar{n}} \geq 0.85 \times 10^8\,\mathrm{s}$ at 90% C.L.

[Baldo-Ceolin et al., ZPC, 1994 (ILL)]

However, the n, $\bar{n}$ system in a magnetic field has four (not two!) physical degrees of freedom; we consider them explicitly.

n-$\bar{n}$ Oscillations and Nuclear Stability

n-$\bar{n}$ oscillations can be studied in bound or free systems.

New limits on dinucleon decay in nuclei have also recently been established.

[Gustafson et al., Super-K Collaboration, arXiv:1504.0104.]

$^{16}\mathrm{O}(pp) \to {}^{14}\mathrm{C}\,\pi^+\pi^+$ has $\tau > 7.22 \times 10^{31}$ years at 90% CL.
$^{16}\mathrm{O}(pn) \to {}^{14}\mathrm{N}\,\pi^+\pi^0$ has $\tau > 1.70 \times 10^{32}$ years at 90% CL.
$^{16}\mathrm{O}(nn) \to {}^{14}\mathrm{O}\,\pi^0\pi^0$ has $\tau > 4.04 \times 10^{32}$ years at 90% CL.
Note $\tau_{NN} = T_{\rm nuc}\tau_{n\bar{n}}^2$ with $T_{\rm nuc} \sim 1.1 \times 10^{25}\mathrm{s}^{-1}$

Large suppression factors appear in all such nuclear studies, making free searches more effective.

In the case of bound n-$\bar{n}$ the suppression is set by

$$\frac{\delta^2}{(V_n - V_{\bar{n}})^2}$$

the difference in nuclear optical potentials. [Dover, Gal, and Richard; Friedman and Gal, 2008]

Now $^{16}\mathrm{O}(n\!-\!\bar{n})$ has $\tau > 1.9 \times 10^{32}$ years at 90% CL,
yielding $\tau_{n\bar{n}} > 2.7 \times 10^8$ s.($100\times$ better!) [Abe et al., Super-K Collaboration, arXiv:1109.4227.]
Cf. free limit: $\tau_{n\bar{n}} \geq 0.85 \times 10^8\,\mathrm{s}$ at 90% C.L. [Baldo-Ceolin et al., ZPC, 1994 (ILL)]
with future improvements expected.

The nuclear suppression dwarfs that from magnetic fields.

Effective Hamiltonian Framework

A 4×4 **matrix describes** $\mathcal{H}$ **in this case.**
Hermiticity as well as CP and T invariance limit its form.
$\mathcal{H}_{ij}$ with $i, j = 1, \ldots 4$ maps to $n(\mathbf{p}, +)$, $\bar{n}(\mathbf{p}, +)$, $n(\mathbf{p}, -)$, and $\bar{n}(\mathbf{p}, -)$.
Translate **CP** and **T** to QM by noting (at low energies)

$$\mathbf{b}^\dagger(\mathbf{p}, s)|0\rangle = |n(\mathbf{p}, s)\rangle \quad ; \quad \mathbf{d}^\dagger(\mathbf{p}, s)|0\rangle = |\bar{n}(\mathbf{p}, s)\rangle \,,$$

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \left\{ b(\mathbf{p}, s) u(\mathbf{p}, s) e^{-ip\cdot x} + d^\dagger(\mathbf{p}, s) v(\mathbf{p}, s) e^{ip\cdot x} \right\} \,,$$

with spinors defined as

$$u(\mathbf{p}, s) = \mathcal{N} \begin{pmatrix} \chi^{(s)} \\ \frac{\sigma\cdot\mathbf{p}}{E+M}\chi^{(s)} \end{pmatrix} \quad ; \quad v(\mathbf{p}, s) = \mathcal{N} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{E+M}\chi'^{(s)} \\ \chi'^{(s)} \end{pmatrix} \,,$$

noting $\chi'^{(s)} = -i\sigma^2\chi^{(s)}$, $\chi^+ = \binom{1}{0}$, $\chi^- = \binom{0}{1}$, and $\mathcal{N} = \sqrt{E+M}$. This yields

$$\mathbf{CP}\,\mathbf{b}(\mathbf{p}, s)\,(\mathbf{CP})^\dagger = \mathbf{d}(-\mathbf{p}, s) \quad ; \quad \mathbf{CP}\,\mathbf{d}(\mathbf{p}, s)\,(\mathbf{CP})^\dagger = -\mathbf{b}(-\mathbf{p}, s)$$

and

$$\mathbf{T}\,\mathbf{b}(\mathbf{p}, s)\,(\mathbf{T})^{-1} = s\mathbf{b}(-\mathbf{p}, -s) \quad ; \quad \mathbf{T}\,\mathbf{d}(\mathbf{p}, s)\,(\mathbf{T})^{-1} = s\mathbf{d}(-\mathbf{p}, -s)$$

Effective Hamiltonian Framework

At low energies, in the **absence of magnetic fields**, under CPT, Lorentz invariance, and Hermiticity

$$\mathcal{H} = \begin{pmatrix} M & \delta & 0 & 0 \\ \delta^* & M & 0 & 0 \\ 0 & 0 & M & -\delta \\ 0 & 0 & -\delta^* & M \end{pmatrix}$$

N.B. the two sectors decouple

In a **static** magnetic field $\mathbf{B}_0$ [$\omega_0 \equiv -\mu_n B_0$]

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & 0 & 0 \\ \delta^* & M-\omega_0 & 0 & 0 \\ 0 & 0 & M-\omega_0 & -\delta \\ 0 & 0 & -\delta^* & M+\omega_0 \end{pmatrix}$$

ω_0 real

Here the magnetic field "quenches" n-$\bar{n}$ oscillations.

The impact of magnetic fields on n-$\bar{n}$ oscillations changes once the two 2×2 sectors are coupled.

This claim is sensitive to the assumptions we have taken.
[cf. Berezhiani and Vainshtein, 2015]

Operator Analysis

For the neutron we can construct a Majorana field ψ_M such that $\gamma_0 C \psi_M^* = \psi_M$ by writing

$$\psi_M \equiv \frac{1}{\sqrt{2}} (\psi_n + \psi_n^c) \,,$$

where $\psi_n^c = \gamma_0 C \psi_n^*$.

The mass term then takes the form $\psi_M^T C \psi_M$ and thus can be rewritten as $\bar{\psi}_M \psi_M$ — it is Lorentz invariant, and C, P, and T even.

However the piece $\psi_n^T C \psi_n$ is odd under P, but C and T even. The last also follows from rotational invariance. **It is CPT odd, but Lorentz invariant!**

It is "nonlocal" because it is only a piece of the $\psi_M^T C \psi_M$ operator; note Greenberg's theorem "CPT violation implies the violation of Lorentz invariance" only established for local quantum field theories.

[Greenberg, PRL, 2002]

A nonlocal exception to the CPT theorem has already been noted.

[Chaichian, Dolgov, Novikov, Tureanu, PLB, 2011]

Operator Analysis

CPT-even $|\Delta B| = 2$ $(n - \bar{n})$ operators of Dirac fields are also possible. Here are some examples.

- $\psi_n^T C \gamma_5 \not\partial \psi_n + \text{h.c.}$
  (This is also CP odd and T odd.)
- $\psi_n^T C \sigma_{\mu\nu} \psi_n F^{\mu\nu}$
  (N.B. would appear to vanish from fermion antisymmetry...)
- $\psi_n^T C \sigma_{\mu\nu} \gamma_5 \psi_n F^{\mu\nu}$
  (N.B. would appear to vanish from fermion antisymmetry...)
- $\psi_n^T C (\gamma^\mu \partial^\nu - \gamma^\nu \partial^\mu) \psi_n F^{\mu\nu}$
- $\psi_n^T C \partial_\mu \psi_n \partial_\nu F^{\mu\nu} \psi_n$
- ...

Effective Hamiltonian Framework

In the SM, a **time-dependent** magnetic field **perpendicular** to $\mathbf{B}_0$ can flip the spin. [$\omega_1 \equiv -\mu_n B_1$ and $\mathbf{B}_1 \perp \mathbf{B}_0$]

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 & -\delta & M+\omega_0 \end{pmatrix}$$

If $|\omega_1| \sim |\omega_0|$, the "quenching" disappears!

Alternatively, or additionally, BSM operators can act

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & 0 & \varepsilon_1 \\ \delta & M-\omega_0 & -\varepsilon_1 & 0 \\ 0 & -\varepsilon_1^* & M-\omega_0 & -\delta \\ \varepsilon_1^* & 0 & -\delta & M+\omega_0 \end{pmatrix}$$

The term ε_1 can have different sources.

The effect of ε_1 is independent of magnetic fields. However, if ε_1 itself is independent of magnetic field, then $\varepsilon_1 \neq 0$ violates angular momentum conservation. [SG and Jafari, arXiv:1408.2264v2]

Such terms can be nonzero if Lorentz symmetry is violated (LV) and can be constrained without magnetic field mitigation. [Babu, Mohapatra, arXiv:1504.01176]

However, if ε_1 depends on external fields, then $\varepsilon_1 \neq 0$ w/o LV. Enter a n-$\bar{n}$ transition magnetic moment.... [SG and Jafari, arXiv:1408.2264v2]

Effective Field Theory Analysis

The leading-dimension (spin-independent) n-$\bar{n}$ operators (in the $E \ll \Lambda_{n\bar{n}}$ limit) have been explicitly constructed; there are 18 six-quark operators (of mass dimension 9) invariant under SU(3)$_{\mathrm{C}}$ and U(1)$_{\mathrm{em}}$.

[Rao and Shrock, 1982, 1984]

They contain the quark-level building blocks $q^{\mathrm{T}\,\alpha}_{i\,\chi} C q^{\beta}_{f\,\chi}$.

- q_i^α is a quark field of flavor $i \in u, d$ and color α; "T" means transpose.
- Note $\chi \in \mathrm{L}, \mathrm{R}$, so that, e.g., $u^\alpha_{\substack{\mathrm{L}\\\mathrm{R}}} \equiv (1 \mp \gamma_5) u^\alpha$.
- The n-$\bar{n}$ operators can contain different color structures but are color singlets.

The leading operators do not change the spin of the particles on which they act.

At first glance a n-$\bar{n}$ transition magnetic moment operator could contain

$$[u^{T\,\rho}_{\chi_3} C \sigma^{\mu\nu} d^{\sigma}_{\chi_2}] F_{\mu\nu}$$

as one of the three "building blocks" — so that there are many possibilities.

These n-$\bar{n}$ operators are of dimension 11.

Comparison to Flavor-Spin ν Oscillations

4×4 **effective Hamiltonia with transition magnetic moments have been previously analyzed in the context of flavor-spin ν oscillations of solar (and supernova) neutrinos.**

[Okun, Voloshin, Vysotsky, 1986; Lim and Marciano, 1988]

E.g., $\nu_e \to \bar{\nu}_\mu$ or $\bar{\nu}_\tau$ for Majorana neutrinos

Resonant enhancements occur (in matter) when the two neutrino states have the same energy.

N.B. the flavor-diagonal ν transition magnetic moment vanishes due to the antisymmetry of fermion exchange.

[Schechter and Valle, 1981; Nieves, 1982; Kayser, 1982; Shrock, 1982; Davidson, Gorbahn, Santamaria, 2006]

That is, the Majorana ν transition magnetic moment is

$$\mu_{\alpha\beta}\nu_\alpha^T C\sigma^{\mu\nu} P_L \nu_\beta F_{\mu\nu} + h.c.$$

with $C = i\gamma^2\gamma^0$ and $P_L = (1-\gamma_5)/2$

Since the right-handed Majorana neutrino is the antineutrino, this vanishes for $\alpha = \beta$.

This cancellation can be evaded in the neutron case.

On the Majorana transition magnetic moment

Generally we can find suitable (CPT even) $n - \bar{n}$ operators by inserting different operator structures in the $q_\chi^T C q_\chi$ "building blockings" in the operators enumerated by Rao and Shrock. This is under study.

As for the Majorana transition magnetic moment, we can study the quark-level operators by inserting either $\sigma \cdot F$ or $\gamma_5 \sigma \cdot F$ in one of the $q_\chi^T C q_\chi$ "building blockings" in the operators of Rao and Shrock.

However, since the operator is local, this matrix element vanishes — at least in the (rigid) bag model.

One could replace $\sigma \cdot F$ with $(\gamma^\mu \partial^\nu - \gamma^\nu \partial^\mu) F_{\mu\nu}$. Since the ∂ operator is minimally non-local, the zero from fermion antisymmetry should be evaded. This is under study.

In what follows we consider the SM mechanism to couple the two 2×2 sectors exclusively.

Solving the Hamiltonian in our $\{|n_i\rangle\}$ basis, we find normalized eigenvectors $\{|u_i\rangle\}$, each with associated eigenvalue λ_i, with $i \in 1, \ldots, 4$.

The time evolution of a state of the Hamiltonian is thus given by

$$|\psi(t)\rangle = \sum_{i=1}^{4} e^{-i\lambda t} \langle u_i|\psi(0)\rangle \, |u_i\rangle \, .$$

Letting $|\psi(0)\rangle = |n_k\rangle$ and defining $a_{ij} \equiv \langle n_j|u_i\rangle$, we find

$$\mathcal{P}_{n_k \to n_j} = \left| \sum_{i=1}^{4} e^{-i\lambda_i t} a_{ij} a_{ik}^* \right|^2 \, .$$

Our solutions are exact.

Examples

Case (i): a static transverse field $\mathbf{B}_1$ is suddenly applied at $t = 0$.

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 & -\delta & M+\omega_0 \end{pmatrix}$$

Noting $|\delta| \ll |\omega_0|, |\omega_1|$,

$$\begin{aligned} \mathcal{P}_{n+\to\bar{n}+}(t) &= \delta^2 \left[\frac{\omega_1^4 t^2}{(\omega_0^2+\omega_1^2)^2} \cos^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) + \frac{\omega_0^4}{(\omega_0^2+\omega_1^2)^3} \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) \right. \\ &\quad \left. + \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \right] + \mathcal{O}(\delta^3); \\ \mathcal{P}_{n+\to\bar{n}-}(t) &= \delta^2 \left[\frac{\omega_1^2 t^2}{\omega_0^2+\omega_1^2} - \frac{\omega_1^4 t^2}{(\omega_0^2+\omega_1^2)^2} \cos^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) \right. \\ &\quad \left. + \frac{\omega_0^2\omega_1^2}{(\omega_0^2+\omega_1^2)^3} \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) - \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \sin\left(2t\sqrt{\omega_0^2+\omega_1^2}\right) \right] \\ &\quad + \mathcal{O}(\delta^3) . \end{aligned}$$

Consider $|\omega_0| \sim |\omega_1|$. There's no "quenching"!

Examples

Note $\mathcal{P}_{n+\to\bar{n}-}(t) = \mathcal{P}_{n-\to\bar{n}+}(t)$ and $\mathcal{P}_{n+\to\bar{n}+}(t) = \mathcal{P}_{n-\to\bar{n}-}(t)$.

The unpolarized n-$\bar{n}$ transition probability is

$$\begin{aligned}\mathcal{P}_{n\to\bar{n}}(t) &= \delta^2\left[\frac{\omega_1^2 t^2}{\omega_0^2+\omega_1^2} + \frac{\omega_0^2}{(\omega_0^2+\omega_1^2)^2}\sin^2(t\sqrt{\omega_0^2+\omega_1^2})\right.\\ &+ \left.\frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}}\left(1-\sin\left(2t\sqrt{\omega_0^2+\omega_1^2}\right)\right)\right] + \mathcal{O}(\delta^3)\,,\end{aligned}$$

— and the first term is of $\mathcal{O}(1)$ in magnetic fields!

For reference,

$$\mathcal{P}_{n+\to n-}(t) = \frac{\omega_1^2}{\omega_0^2+\omega_1^2}\sin(t\sqrt{\omega_0^2+\omega_1^2}) + \mathcal{O}(\delta^2)$$

Note the common pre-factor in magnetic fields.

Examples

Case (ii): a time-dependent, transverse magnetic field $\mathbf{B}_1$

— as employed in magnetic resonance studies [cf. the Rabi formula!]

Here $\hat{H}_n(t) = \omega_0\,\sigma_z + \omega_1(\cos\omega t\,\sigma_x + \sin\omega t\,\sigma_y)$.

$$\mathcal{H}(t) = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 e^{-i\omega t} & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 e^{i\omega t} & -\delta & M+\omega_0 \end{pmatrix} .$$

To solve $i\partial_t\psi = \mathcal{H}\psi$ with $\psi = \{a_+(t), \bar{a}_+(t), a_-(t), \bar{a}_-(t)\}$ use $\overset{(-)}{a}_\pm \equiv \overset{(-)}{b}_\pm \exp(\mp i\omega t/2)$ to yield $i\partial_t\tilde{\psi} = \tilde{\mathcal{H}}\tilde{\psi}$ with $\tilde{\psi} = \{b_+(t), \bar{b}_+(t), b_-(t), \bar{b}_-(t)\}$ and

$$\tilde{\mathcal{H}} = \begin{pmatrix} M-\Delta\omega_- & \delta & \omega_1 & 0 \\ \delta & M-\Delta\omega_+ & 0 & -\omega_1 \\ \omega_1 & 0 & M+\Delta\omega_- & -\delta \\ 0 & -\omega_1 & -\delta & M+\Delta\omega_+ \end{pmatrix}$$

with $\Delta\omega_\pm \equiv \omega/2 \pm \omega_0$. $\Delta\omega_+ \neq \Delta\omega_-$?!

Examples

$\Delta\omega_+ \neq \Delta\omega_-$?!

However, magnetic resonance experiments are typically realized via a linearly polarized radio frequency (rf) field, so that if $\Delta\omega_+ = 0$, then $\Delta\omega_- = 0$ also. Thus under usual experimental conditions the largest contributions have $\Delta\omega_+ = -\Delta\omega_-$. On resonance, for which $\Delta\omega_\pm = 0$, we have

$$\begin{aligned}
\mathcal{P}_{n+\,\to\bar{n}+}(t) &\approx \delta^2 t^2 \cos^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) + \mathcal{O}(\delta^3); \\
\mathcal{P}_{n+\,\to\bar{n}-}(t) &\approx \delta^2 t^2 \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) + \mathcal{O}(\delta^3)\,,
\end{aligned}$$

neglecting contributions controlled by $|\omega|/2+\omega_0$. [Ramsey, 1956]

Finally $\mathcal{P}_{n\,\to\bar{n}}(t) \approx \delta^2 t^2 + \mathcal{O}(\delta^3)$.

Magnetic fields do not quench the *n*-$\bar{n}$ transition rate.

Implications of a "Majorana neutron"

The existence of n-$\bar{n}$ oscillations would connote that the neutron and antineutron can be rewritten in terms of Majorana states. [Marciano]

A Majorana state $|\Psi_M\rangle$ transforms into itself under **C**, up to a global phase. We have $[\mathbf{C}b(\mathbf{p},s)\mathbf{C}^\dagger = d(\mathbf{p},s)]$

$$|\Psi_M^\pm(\mathbf{p},s)\rangle = \frac{1}{\sqrt{2}}\left(|\bar{n}(\mathbf{p},s)\rangle \pm |n(\mathbf{p},s)\rangle\right) ,$$

$\Longrightarrow$ **two distinct Majorana states, each with $s = \pm$, exist.**

The neutron and antineutron are distinct (note $e^\mp$ under semileptonic decay), and the Majorana basis must account for four degrees of freedom.

In contrast, we would have a two-component Majorana neutrino.

In the absence of magnetic fields, we have indeed found that pairs of eigenvectors can be expressed in terms of (one of) the Majorana states.

Summary

Although many have studied the impact of external magnetic fields on n-$\bar{n}$ oscillations, our work is the first to incorporate spin in a fundamental way.

We find, in constrast to earlier studies, that magnetic field mitigation is unnecessary. That is, "quenching" can be avoided without a fine-tuned configuration of magnetic fields. This should greatly enable future experimental studies in search of n-$\bar{n}$ oscillations (and Majorana dynamics). In this we assume the $n - \bar{n}$ transition operator to be CPT even. Remarkably $n^T C n$ is CPT odd.

In the presence of n-$\bar{n}$ oscillations, the mass eigenstates become entangled combinations of Majorana states.

Certain subleading BSM n-$\bar{n}$ operators can also be enhanced through the application of external magnetic fields....