New perspectives on $|\Delta B|=2$ dynamics

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Reasons to believe that *B* is not a fundamental symmetry of Nature:

- matter-antimatter asymmetry of the Universe
- > nonperturbative *B* violation in the SM
- grand unification

 $\Delta B=1$ processes:

$$\mathcal{O}_6 \sim \frac{q \, q \, q \, l}{\Lambda^2}$$

probe physics up to the GUT scale ~ 10¹⁶ GeV

 $\Delta B=1$ processes:

$$\mathcal{O}_6 \sim \frac{q \, q \, q \, l}{\Lambda^2}$$

probe physics up to the GUT scale ~ 10¹⁶ GeV

 $\Delta B=2$ processes:

$$\mathcal{O}_9 \sim \frac{q \, q \, q \, q \, q \, q}{\Lambda^5}$$

 probe a lower energy scale ~ hundreds of TeV (not necessarily!)

|∆B|=2 processes

Violating baryon number by two units:

- new physics!
- closely related to physics behind neutrino masses if *B-L* is a fundamental symmetry
- probe physics in the TeV GUT region
- hope for baryogenesis



$$n \leftrightarrow \bar{n}$$
Current limit: $\tau_{n\bar{n}} \gtrsim 2.7 \times 10^8 s$
[Abe et al., Super-K Collaboration (2011)]
$$\longrightarrow \Lambda^5 \gtrsim (500 \text{ TeV})^5$$

|∆B|=2 processes

Neutron-antineutron oscillations

$$\leftrightarrow \bar{n}$$
Current limit: $\tau_{n\bar{n}} \gtrsim 2.7 \times 10^8 s$
[Abe et al., Super-K Collaboration (2011)]
$$\longrightarrow \Lambda^5 \gtrsim (500 \text{ TeV})^5$$

• Dinucleon decays

 \mathcal{N}

$$p \, p \to K^+ K^+$$

$$n p \to e^+ \nu$$

imit:
$$\tau_{16}_{O \to 14}_{CK^+K^+} > 1.7 \times 10^{32} \text{ yr}$$

[Litos et al., Super-K Collaboration (2014)]

imit:
$$\tau_{np \to e^+ \nu} > 2.6 \times 10^{32} \text{ yr}$$

[Takhistov et al., Super-K Collaboration (2015)]

$$\longrightarrow \Lambda^8 \gtrsim (10 \text{ TeV})^8$$

Models with $|\Delta B|=2$

1 SO(10) GUT scale seesaw with TeV scalars [Babu, Mohapatra (2012)]

② TeV scale seesaw with quark-lepton unification

[Mohapatra, Marshak (1980), Babu, Dev, Mohapatra (2009); Babu, Dev, Fortes, Mohapatra (2013)]

③ TeV scale extra dimensions

[Dvali, Gabadadze (2002); Nussinov, Shrock (2002); Winslow, Ng (2010)]

- Supersymmetric and superstring models
 [Zwirner (1983), Mohapatra, Valle (1986); Goity, Sher (1995)]
- **5 SM or MSSM with additional multiplets** [Ajaib, Gogoladze, Mimura, Shafi (2009); Gu, Sarkar (2011); Arnold, BF, Wise (2013), Herrmann (2014)]

1 SO(10) GUT scale seesaw + TeV scalars [Babu, Mohapatra (2012)]

- natural framework for small neutrino masses
- explains the neutrino mixing pattern
- SM gauge group up to SO(10) breaking scale
- diquarks (6,1,1/3), (6,1,-2/3) and (6,1,4/3) ~ TeV contained in the 126-plet or 54-plet Higgs
- gauge coupling unification at ~ 3 x 10¹⁵ GeV
- proton lifetime ~ 10³⁴ years

(1) SO(10) and neutron-antineutron oscillations [Babu, Mohapatra (2012)]

• Lagrangian:

$$\mathcal{L}_{\Delta B\neq 0} = f_{dd} d^{c} d^{c} \Delta_{d^{c} d^{c}} + \frac{f_{ud}}{\sqrt{2}} \left(u^{c} d^{c} + d^{c} u^{c} \right) \Delta_{u^{c} d^{c}} + f_{uu} u^{c} u^{c} \Delta_{u^{c} u^{c}} + \lambda v_{BL} \left(\Delta_{u^{c} d^{c}} \Delta_{u^{c} d^{c}} \Delta_{d^{c} d^{c}} + \Delta_{d^{c} d^{c}} \Delta_{d^{c} d^{c}} \Delta_{u^{c} u^{c}} \right) + h.c.$$

• Amplitude for nnbar oscillations:

$$G_{N-\bar{N}} \simeq \frac{\eta f_{ud}^2 f_{dd} \lambda v_{BL}}{M_{\Delta_{u^c d^c}}^4 M_{\Delta_{d^c d^c}}^2}$$

Combined with a successful baryogenesis:

$$\tau_{n\bar{n}} \sim 10^8 - 10^{11} s$$

2 TeV scale seesaw with quark-lepton unification

[Mohapatra, Marshak (1980), Babu, Dev, Mohapatra (2009); Babu, Dev, Fortes, Mohapatra (2013)]

 $|\tau_{n\bar{n}} < 5 \times 10^{10} \ s$

- Pati-Salam unification (including gauged *B-L*) above the seesaw scale
- TeV-scale color sextet scalars mediating neutronantineutron oscillations
- Iow-scale baryogenesis needed to make predictions for nnbar oscillation frequency
- post-sphaleron baryogenesis requires

(5) Simplified models

[Arnold, BF, Wise (2013)]

- Search for the simplest models which violate baryon number but do not give rise to tree-level proton decay:
 - minimal new particle content
 - no global symmetries imposed
 - only renormalizable couplings

Possible renormalizable operators

operator	SU(3) imes SU(2) imes U(1) rep. of X	В	L
XQQ, Xud	$\left(ar{6},1,-1/3 ight),\left(3,1,-1/3 ight)$	-2/3	0
XQQ	$\left(\overline{6},3,-1/3 ight) ,\left(3,3,-1/3 ight)$	-2/3	0
Xdd	$(3,1,2/3)$, $ig(ar{6},1,2/3ig)$	-2/3	0
Xuu	$\left(ar{6},1,-4/3 ight),\left(3,1,-4/3 ight)$	-2/3	0
ХQĪ	$\left(3,1,-1/3 ight),\left(3,3,-1/3 ight)$	1/3	1
Хūē	(3, 1, -1/3)	1/3	1
Xdē	(3, 1, -4/3)	1/3	1
$Xar{Q}e, XLar{u}$	(3,2,7/6)	1/3	-1
ХĪd	$\left(ar{3},2,-1/6 ight)$	-1/3	1
XLL	(1, 1, 1), $(1, 3, 1)$	0	-2
Xee	(1, 1, 2)	0	-2

Tree-level proton decay





Tree-level proton decay







- *B*-violation arises from the scalar potential
- only two types of scalar interactions which may violate baryon number without proton decay:



Tree-level proton decay





$$p \to \pi^+ \pi^+ e^- \nu \, \nu$$

Scalar representations without proton decay

operator	SU(3) imes SU(2) imes U(1) rep. of X	В	L
XQQ, Xud	$\left(ar{6},1,-1/3 ight),$	-2/3	0
XQQ	$\left({ar 6}, 3, -1/3 ight),$	-2/3	0
Xdd	$(3,1,2/3)$, $(\overline{6},1,2/3)$	-2/3	0
Xuu	$\left(ar{6},1,-4/3 ight),$	-2/3	0
ХQĪ	,	1/3	1
Хūē		1/3	1
Xdē		1/3	1
$Xar{Q}e, XLar{u}$		1/3	-1
ХĪd		-1/3	1
XLL	(1, 1, 1), $(1, 3, 1)$	0	-2
Xee	(1, 1, 2)	0	-2

Model 1. Model 2. Model 3. Model 4. Model 5. Model 6. Model 7. Model 8. Model 9.

 $X_1 = (\bar{6}, 1, -1/3), X_2 = (\bar{6}, 1, 2/3)$ $X_1 = (\bar{6}, 3, -1/3), X_2 = (\bar{6}, 1, 2/3)$ $X_1 = (\bar{6}, 1, 2/3), X_2 = (\bar{6}, 1, -4/3)$ $X_1 = (3, 1, 2/3), X_2 = (\overline{6}, 1, -4/3)$ $X_1 = (\bar{6}, 1, -1/3), X_2 = (1, 1, 1)$ $X_1 = (\bar{6}, 3, -1/3), X_2 = (1, 1, 1)$ $X_1 = (\overline{6}, 3, -1/3), X_2 = (1, 3, 1)$ $X_1 = (\overline{6}, 1, 2/3), X_2 = (1, 1, -2)$ $X_1 = (3, 1, 2/3), X_2 = (1, 1, -2)$

 $X_{1} = (\overline{6}, 1, -1/3), \quad X_{2} = (\overline{6}, 1, 2/3), \quad X_{1} = (\overline{6}, 3, -1/3), \quad X_{2} = (\overline{6}, 1, 2/3), \quad X_{1} = (\overline{6}, 1, 2/3), \quad X_{2} = (\overline{6}, 1, 2/3), \quad X_{1} = (\overline{6}, 1, 2/3), \quad X_{2} = (\overline{6}, 1, -4/3), \quad X_{1} = (3, 1, 2/3), \quad X_{2} = (\overline{6}, 1, -4/3)$ **Model 5.** $X_1 = (\overline{6}, 1, -1/3), X_2 = (1, 1, 1)$ $X_1 = (\overline{6}, 3, -1/3), X_2 = (1, 1, 1)$ Model 6. $X_1 = (\overline{6}, 3, -1/3), X_2 = (1, 3, 1)$ Model 7. $X_1 = (\overline{6}, 1, 2/3), X_2 = (1, 1, -2)$ Model 8. $X_1 = (3, 1, 2/3), X_2 = (1, 1, -2)$ Model 9.

$$\begin{array}{c} X_{1} \\ X_{2} \\ X_{1} \\$$





The model

• New scalars:

$$X_1 = (\bar{6}, 1, -1/3)$$
 and $X_2 = (\bar{6}, 1, 2/3)$

• Interactions:

$$\mathcal{L} = -g_{1}^{ab}X_{1}^{\alpha\beta}\left(Q_{L\alpha}^{a}\epsilon Q_{L\beta}^{b}\right) - g_{2}^{ab}X_{2}^{\alpha\beta}\left(d_{R\alpha}^{a}d_{R\beta}^{b}\right) -g_{1}^{\prime ab}X_{1}^{\alpha\beta}\left(u_{R\alpha}^{a}d_{R\beta}^{b}\right) + \lambda X_{1}^{\alpha\alpha'}X_{1}^{\beta\beta'}X_{2}^{\gamma\gamma'}\epsilon_{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'}$$

The model

• New scalars:

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Transition matrix element

$$\Delta m = \langle \bar{n} | \mathcal{H}_{\mathrm{eff}} | n \rangle$$

Neglecting g_1 the effective $|\Delta B|=2$ Hamiltonian:

$$\begin{split} \mathcal{H}_{\mathrm{eff}} &= -\frac{(g_{1}^{\prime 11})^{2}g_{2}^{11}\lambda}{4M_{1}^{4}M_{2}^{2}}d_{Ri}^{\dot{\alpha}}d_{Ri'}^{\dot{\beta}}u_{Rj}^{\dot{\gamma}}d_{Rj'}^{\dot{\delta}}u_{Rk}^{\dot{\lambda}}d_{Rk'}^{\dot{\chi}}\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon_{\dot{\gamma}\dot{\delta}}\epsilon_{\dot{\lambda}\dot{\chi}} \\ &\times \left(\epsilon_{ijk}\epsilon_{i'j'k'} + \epsilon_{i'jk}\epsilon_{ij'k'} + \epsilon_{ij'k}\epsilon_{i'jk'} + \epsilon_{ijk'}\epsilon_{i'jk'}\right) + \mathrm{h.c.} \end{split}$$



$$\mathcal{L} = -g_1^{ab} X_1^{\alpha\beta} \left(Q_{L\alpha}^a \epsilon Q_{L\beta}^b \right) - g_2^{ab} X_2^{\alpha\beta} (d_{R\alpha}^a d_{R\beta}^b) - g_1^{\prime ab} X_1^{\alpha\beta} (u_{R\alpha}^a d_{R\beta}^b) + \lambda X_1^{\alpha\alpha'} X_1^{\beta\beta'} X_2^{\gamma\gamma'} \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}$$

Using the vacuum insertion approximation and the general form of the hadronic matrix element:

$$\langle 0 | d_{Ri}^{\dot{\alpha}} d_{Rj}^{\dot{\beta}} u_{Rk}^{\dot{\gamma}} | n(p, s) \rangle$$

= $-\frac{1}{18} \beta \epsilon_{ijk} (\epsilon^{\dot{\alpha} \dot{\gamma}} u_R^{\dot{\beta}}(p, s) + \epsilon^{\dot{\beta} \dot{\gamma}} u_R^{\dot{\alpha}}(p, s)).$

where

β

$$\simeq 0.01 \text{ GeV}^3$$

[Tsutsui et al., CP-PACS and JLQCD Collaboration (2004)]

we find:

$$\Delta m| = 2\lambda\beta^2 \frac{|(g_1'^{11})^2 g_2^{11}|}{3M_1^4 M_2^2}$$

Matrix element

$$|\Delta m| = 2\lambda\beta^2 \frac{|(g_1'^{11})^2 g_2^{11}|}{3M_1^4 M_2^2}$$

Current limit:

$$|\Delta m| < 2 imes 10^{-33} {
m ~GeV}$$

[Abe et al., Super-K Collaboration (2011)]

Future sensitivity:

$$|\Delta m|\simeq 7 imes 10^{-35}~{
m GeV}$$

[Kamyshkov (2012), unpublished]

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[Abe et al., Super-K Collaboration (2011)]

Future sensitivity: $|\Delta m| \simeq 7 \times 10^{-35} \text{ GeV}$

[Kamyshkov (2012), unpublished]

Regime (1):
$$M_1 = M_2 \equiv M$$
, and $g_1'^{11} = g_2^{11} = 1$, $\lambda = M$,
Regime (2): $M_2 > M_1$ $M_1 = 5$ TeV $\lambda = M_2$

Regime (1):

$$M_1 = M_2 \equiv M$$

Current limit:

$$M\gtrsim 500~{
m TeV}$$

Future sensitivity:

$$M\gtrsim 1000~{
m TeV}$$

Regime (2):

$$M_1 = 5 \text{ TeV}$$
 $M_2 > M_1$

Current limit:

$$M_2\gtrsim 5 imes 10^{13}~{
m GeV}$$

Future sensitivity:

$$M_2\gtrsim 1.5 imes 10^{15}~{
m GeV}$$

Experimental constraints

• LHC single or pair production of X_1

$$M_1 \gtrsim 1 \text{ TeV}$$

[Richardson, Winn (2012)]

Neutron electric dipole moment



Neutral kaon mixing



Neutron EDM



$$\begin{aligned} d_d &| \simeq \frac{m_t}{6\pi^2 M_1^2} \log\left(\frac{M_1^2}{m_t^2}\right) \left| \text{Im}[g_1^{31}(g'_1^{31})^*] \right| \ e \ \text{cm} \\ \\ d_n &\simeq \frac{4}{3} d_d \\ \\ d_n^{\exp} &< 2.9 \times 10^{-26} \ e \ \text{cm} \end{aligned}$$

[Baker et al. (2006)]

Neutron EDM



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[Baker et al. (2006)]

$$\begin{split} M_1 \simeq 500 \ {\rm TeV} & \longrightarrow \ \left| {\rm Im}[g_1^{31}(g'_1^{31})^*] \right| \lesssim 6 \times 10^{-3} \\ M_1 \simeq 5 \ {\rm TeV} & \longrightarrow \ \left| {\rm Im}[g_1^{31}(g'_1^{31})^*] \right| \lesssim 10^{-6} \end{split}$$

Neutral kaon mixing



$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{g_2^{22} \left(g_2^{11}\right)^*}{M_2^2} (s_{R\alpha} s_{R\beta}) (d_R^{*\alpha} d_R^{*\beta}) \\ \rightarrow & \frac{g_2^{22} \left(g_2^{11}\right)^*}{2M_2^2} (\bar{d}_R^{\alpha} \gamma^{\mu} s_{R\alpha}) (\bar{d}_R^{\alpha} \gamma_{\mu} s_{R\alpha}) \end{aligned}$$

Constraints: [Isidori, Nir, Perez (2010)]

$$\left| \operatorname{Re} \left[g_2^{22} \left(g_2^{11} \right)^* \right] \right| < 1.8 \times 10^{-6} \left(\frac{M_2}{1 \text{ TeV}} \right)^2$$

$$\left| \operatorname{Im} \left[g_2^{22} \left(g_2^{11} \right)^* \right] \right| < 6.8 \times 10^{-9} \left(\frac{M_2}{1 \text{ TeV}} \right)^2$$

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$$\mathcal{H}_{\text{eff}} = \frac{g_2^{22} (g_2^{11})^*}{M_2^2} (s_{R\alpha} s_{R\beta}) (d_R^{*\alpha} d_R^{*\beta})$$
$$\rightarrow \frac{g_2^{22} (g_2^{11})^*}{2M_2^2} (\bar{d}_R^{\alpha} \gamma^{\mu} s_{R\alpha}) (\bar{d}_R^{\alpha} \gamma_{\mu} s_{R\alpha})$$

Constraints: [Isidori, Nir, Perez (2010)]

For M_2 =500 TeV:

1.7 x 10⁻³

= 0.45

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$$\left| \operatorname{Re} \left[g_2^{22} \left(g_2^{11} \right)^* \right] \right| < 1.8 \times 10^{-6} \left(\frac{M_2}{1 \text{ TeV}} \right)^2$$

$$\left| \operatorname{Im} \left[g_2^{22} \left(g_2^{11} \right)^* \right] \right| < 6.8 \times 10^{-9} \left(\frac{M_2}{1 \text{ TeV}} \right)^2$$

Decays of X_2 :





Assumption:



Heavy scalar decay rates:

$$\Gamma(X_2 \to \overline{X}_1 \overline{X}_1) = \frac{3\lambda}{8\pi M_2}$$

$$\times \left[\lambda - \tilde{\lambda} \frac{M_2^2}{4\pi (M_2^2 - \tilde{M}_2^2)} \operatorname{Im}(\operatorname{Tr}(\varsigma^{\dagger} \tilde{g}_2)) \right],$$

$$\Gamma(\overline{X}_2 \to X_1 X_1) = \frac{3\lambda}{8\pi M_2}$$

$$\times \left[\lambda + \tilde{\lambda} \frac{M_2^2}{4\pi (M_2^2 - \tilde{M}_2^2)} \operatorname{Im}(\operatorname{Tr}(\varsigma^{\dagger} \tilde{g}_2)) \right].$$



Branching ratios and final state baryon numbers:

Decay	Br	B _f
$X_2 ightarrow \overline{X}_1 \overline{X}_1$	r	4/3
$X_2 ightarrow ar{d}_R ar{d}_R$	1-r	-2/3
$\overline{X}_2 ightarrow X_1 X_1$	ī	-4/3
$\overline{X}_2 \rightarrow d_R d_R$	$1-\overline{r}$	2/3

Net baryon number:

$$\Delta n_B = \frac{6}{\pi \operatorname{Tr}(g_2^{\dagger} g_2)} \frac{1}{\tilde{M}_2^2 - M_2^2} \operatorname{Im}\left[\lambda \,\tilde{\lambda}^* \operatorname{Tr}(g_2^{\dagger} \,\tilde{g}_2)\right]$$



Branching ratios and final state baryon numbers:

Decay	Br	B _f
$X_2 ightarrow \overline{X}_1 \overline{X}_1$	r	4/3
$X_2 ightarrow \overline{d}_R \overline{d}_R$	1-r	-2/3
$\overline{X}_2 ightarrow X_1 X_1$	r	-4/3
$\overline{X}_2 \to d_R d_R$	$1-\overline{r}$	2/3

Net baryon number:

$$\Delta n_B = \frac{6}{\pi \operatorname{Tr}(g_2^{\dagger} g_2)} \frac{1}{\tilde{M}_2^2 - M_2^2} \operatorname{Im}\left[\lambda \,\tilde{\lambda}^* \operatorname{Tr}(g_2^{\dagger} \,\tilde{g}_2)\right]$$

Washout effects important! [Herrmann (2014)]

Conclusions

- Processes with ΔB=2 provide complimentary searches for baryon number violation to those at the LHC and proton decay experiments.
- Many models require that a ΔB=2 signal should be seen in the upgraded searches.
- A discovery would have a profound impact on our understanding of the Universe.

Thank you!

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• Model 2:

$$X_1 = (\bar{6}, 3, -1/3), X_2 = (\bar{6}, 1, 2/3),$$

$$\mathcal{L} = -g_1^{ab} X_1^{\alpha\beta A} (Q_{L\alpha}^a \epsilon \tau^A Q_{L\beta}^b) - g_2^{ab} X_2^{\alpha\beta} (d_{R\alpha}^a d_{R\beta}^b) + \lambda X_1^{\alpha\alpha' A} X_1^{\beta\beta' A} X_2^{\gamma\gamma'} \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}.$$

• Model 3:

$$X_1 = (\bar{6}, 1, 2/3), X_2 = (\bar{6}, 1, -4/3),$$

$$\mathcal{L} = -g_1^{ab} X_1^{\alpha\beta} (d_{R\alpha}^a d_{R\beta}^b) - g_2^{ab} X_2^{\alpha\beta} (u_{R\alpha}^a u_{R\beta}^b) + \lambda X_1^{\alpha\alpha'} X_1^{\beta\beta'} X_2^{\gamma\gamma'} \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}.$$

• Model 4:

$$X_1 = (3, 1, 2/3), X_2 = (\overline{6}, 1, -4/3),$$

$$\mathcal{L} = -g_1^{ab} X_{1\alpha} (d^a_{R\beta} d^b_{R\gamma}) \epsilon^{\alpha\beta\gamma} - g_2^{ab} X_2^{\alpha\beta} (u^a_{R\alpha} u^b_{R\beta}) + \lambda X_{1\alpha} X_{1\beta} X_2^{\alpha\beta}.$$

• Model 5:

$$X_1 = (\bar{6}, 1, -1/3), X_2 = (1, 1, 1),$$

$$\begin{aligned} \mathcal{L} &= -g_1^{ab} X_1^{\alpha\beta} (Q_{L\alpha}^a \epsilon Q_{L\beta}^b) \\ &- g_2^{ab} X_2 (L_L^a \epsilon L_L^b) - g_1^{\prime ab} X_1^{\alpha\beta} (u_{R\alpha}^a d_{R\beta}^b) \\ &+ \lambda X_1^{\alpha\alpha'} X_1^{\beta\beta'} X_1^{\gamma\gamma'} X_2 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}. \end{aligned}$$

• Model 6:

$$X_1 = (\overline{6}, 3, -1/3), X_2 = (1, 1, 1)$$

$$\mathcal{L} = -g_1^{ab} X_1^{\alpha\beta A} (Q_{L\alpha}^a \epsilon \tau^A Q_{L\beta}^b) - g_2^{ab} X_2 (L_L^a \epsilon L_L^b) + \lambda X_1^{\alpha\alpha' A} X_1^{\beta\beta' B} X_1^{\gamma\gamma' C} X_2 \epsilon^{ABC} \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}.$$

• Model 7:

$$X_1 = (\overline{6}, 3, -1/3), X_2 = (1, 3, 1),$$

$$\begin{split} \mathcal{L} &= -g_1^{ab} X_1^{\alpha\beta A} (Q_{L\alpha}^a \epsilon \tau^A Q_{L\beta}^b) - g_2^{ab} X_2^A (L_L^a \epsilon \tau^A L_L^b) \\ &+ \lambda X_1^{\alpha\alpha' A} X_1^{\beta\beta' B} X_1^{\gamma\gamma' C} X_2^D \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'} \\ &\times (\delta^{AB} \delta^{CD} + \delta^{AC} \delta^{BD} + \delta^{AD} \delta^{BC}). \end{split}$$

• Model 8:

$$X_1 = (\overline{6}, 1, 2/3), X_2 = (1, 1, -2),$$

$$\mathcal{L} = -g_1^{ab} X_1^{\alpha\beta} (d_{R\alpha}^a d_{R\beta}^b) - g_2^{ab} X_2 (e_R^a e_R^b) + \lambda X_1^{\alpha\alpha'} X_1^{\beta\beta'} X_1^{\gamma\gamma'} X_2 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'}.$$

• Model 9:

$$X_1 = (3, 1, 2/3), X_2 = (1, 1, -2),$$

$$\mathcal{L} = -g_1^{ab} X_{1\alpha} (d^a_{R\beta} d^b_{R\gamma}) \epsilon^{\alpha\beta\gamma} - g_2^{ab} X_2 (e^a_R e^b_R) + \lambda X_{1\alpha} X_{1\beta} X_{1\gamma} X_2 \epsilon^{\alpha\beta\gamma}.$$