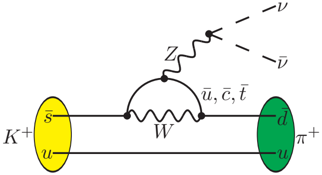
Long-distance contributions to the rare kaon decay $K^+ \to \pi^+ \nu\bar{\nu}$



Xu Feng (Columbia University)

Workshop@INT, 09/28/2015

Collaborators

- on behalf of RBC-UKQCD collaboration
- people involved in this project

UKQCD	RBC
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Antonin Portelli (Southampton)	Christoph Lehner (BNL)
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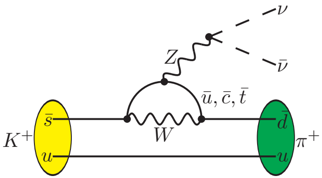
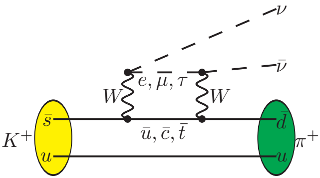
$K^+ \to \pi^+\nu\bar{\nu}$: Experiment vs Standard model

As FCNC process, $K \to \pi\nu\bar{\nu}$ decay through second-order weak interaction



Z-exchange



W-*W*

SM effects highly suppressed in the second order → ideal probes for NP

Past experimental measurement is 2 times larger than SM prediction

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$ arXiv:0808.2459

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = 9.11 \pm 0.72 \times 10^{-11}$$ arXiv:1503.02693

but still consistent with $> 60\%$ exp. error

New experiments

New generation of experiment: NA62 at CERN aims at

- observation of $O(100)$ events in 2-3 years
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$

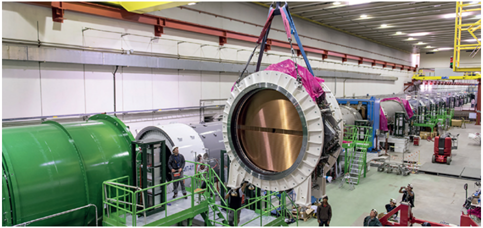


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

$K_L \to \pi^0 \nu \bar{\nu}$

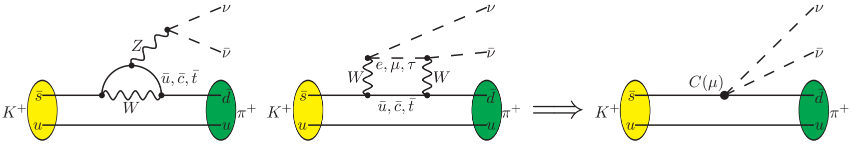
- even more challenging since π^0 decays quickly to two photons
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe K_L decays

Methodology

Top quark contribution

Top quark is very heavy, $m_t = 173$ GeV

- effective Hamiltonian described by a dim-6 operator $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$



- hadronic effects can be given by the hadronic matrix elements, e.g.

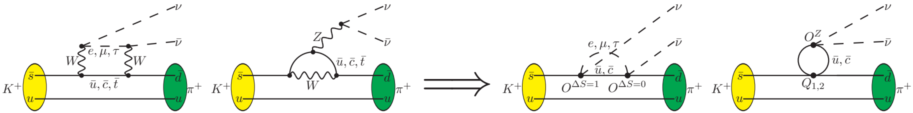
$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle \xleftarrow{\text{isospin rotation}} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$\langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$ is precisely measured in $K^+ \to \pi^0 \ell^+ \nu$

$K \to \pi \nu \bar{\nu}$ decays are theoretically clean

question: how about the charm quark contribution?

Charm quark contribution



- first-order effective field theory

$$\mathcal{H}_{eff} = -i\frac{G_F}{\sqrt{2}}\sum_{q=u,c}\left(V_{qs}^{*}O_{q\ell}^{\Delta S=1} + V_{qd}O_{q\ell}^{\Delta S=0}\right) - i\frac{G_F}{\sqrt{2}}\sum_{q=u,c}\lambda_q O_q^W - i\frac{G_F}{\sqrt{2}}O_\ell^Z$$

W-*W* diagram

$$O_{q\ell}^{\Delta S=1} = (\bar{s}q)_{V-A}(\bar{\nu}\ell)_{V-A}, \quad O_{q\ell}^{\Delta S=0} = (\bar{\ell}\nu)_{V-A}(\bar{q}d)_{V-A}$$

Z-exchange diagram

$$O_q^W = C_1(\mu)Q_{1,q}(\mu) + C_2(\mu)Q_{2,q}(\mu), \quad O_\ell^Z = J_\mu^Z\,\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell$$

2^{nd}-order EFT

- In the 2^{nd}-order EFT, one can construct the bilocal product

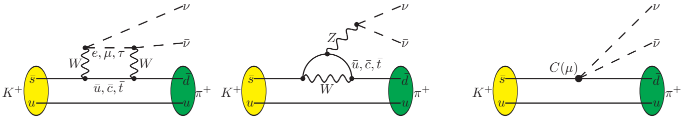
$$\mathcal{B}(y) = \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\frac{\pi^2}{M_W^2}\lambda_c\sum_{\ell=e,\mu,\tau}\mathcal{B}_{WW}(y)+\mathcal{B}_Z(y)$$

$$\mathcal{B}_{WW}(y) = \int d^4x\, T[O_{u\ell}^{\Delta S=1}(x)O_{u\ell}^{\Delta S=0}(y)] - \{u\to c\}$$

$$\mathcal{B}_Z(y) = \int d^4x\, T[O_u^W(x)O_\ell^Z(y)] - \{u\to c\}$$

- Evaluate the hadronic matrix element of bilocal operators

$$\langle\pi\nu\bar{\nu}|\mathcal{B}_{WW}(0)+\mathcal{B}_Z(0)|K^+\rangle + \langle\pi\nu\bar{\nu}|C_W O^{(6)}(0)|K^+\rangle$$



Counter term $C_W O^{(6)}$ is introduced to remove the singularities in $\mathcal{B}_{WW}$ and $\mathcal{B}_Z$. Here $O^{(6)} = (\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$

Generic feature of 2^{nd}-order weak interaction

- Bilocal structure is a generic feature of 2^{nd}-order weak interaction
- Method originally proposed by [N.H. Christ, Lat2010]
- Successfully applied to the lattice calculation of K_L-K_S mass difference [Bai et. al., PRL 113('14) 112003]

$$\int d^4x \langle \bar{K}^0 | T\{H_W(x) H_W(0)\} | K^0 \rangle$$

- A summary of "Long-distance contributions to flavour-changing processes" is given by [C.T. Sachrajda, plenary talk, Lat2014]

Minkowski vs Euclidean

Given a non-local matrix element in Minkowski space

$$\mathcal{T}^M = i\int dt\, \langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s} \frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f+i\epsilon} - \sum_n \frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n+i\epsilon}$$

In Euclidean space

$$\mathcal{T}^E = \sum_{t=-T_a}^{T_b} \langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s} \frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(1-e^{(E_f-E_{n_s})T_b}\right)$$
$$-\sum_n \frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(1-e^{(E_K-E_n)T_a}\right)$$

if $E_n < E_K$, remove exp growing contamination, $\mathcal{T}^E \Rightarrow \mathcal{T}^M$

Infinite volume vs finite volume

- Above two-pion threshold, $\sum_n$ and $\sum_{n_s}$ shall be replaced by $\oint_n$ and $\oint_{n_s}$
- For infinite volume, integral is well defined using principal value

$$\mathcal{I}^{\infty} = \mathcal{P} \oint_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{\infty\,\infty}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

- For finite volume, energy states are always discrete, we still have

$$\mathcal{I}^{L} = \sum_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{LL}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

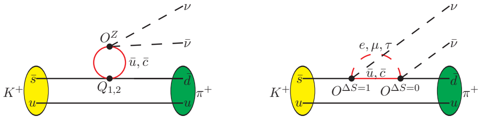
- Finite-volume correction $\mathcal{I}^{\infty} = \mathcal{I}^{L} - \delta\mathcal{I}$

$$\delta\mathcal{I} = \cot(\phi(E)+\delta(E))(\phi'(E)+\delta'(E))\langle f|O^{\Delta S=0}|\pi\pi, E\rangle^{LL}\langle \pi\pi, E|O^{\Delta S=1}|K\rangle\Big|_{E=m_K}$$

  - $\phi(E)$: a known function depending on L; $\delta(E)$: $\pi\pi$ scattering phase
  - $\cot(\phi(E)+\delta(E))$ is singular at $E=E_n$; it cancels the singularity of $\mathcal{I}^L$
- For a complete derivation of $\delta\mathcal{I}$, see [N. Christ, XF, G. Martinelli, C. Sachrajda, arXiv:1504.01170]

Physical cutoff vs lattice cutoff

- By dimensional counting the loop integrals are quadratically divergent



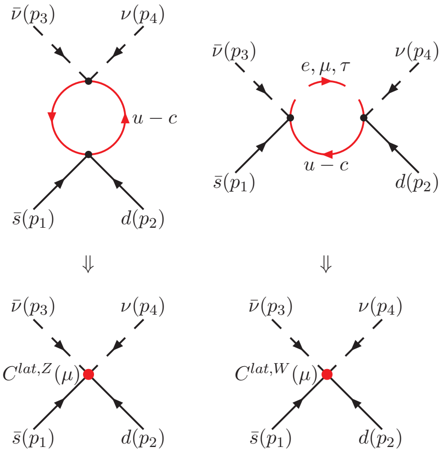
- GIM mechanism reduces the divergence to logarithmic
- In the physical world, the SD divergence is cut off by physical M_W
- In the lattice calculation it is cut off by an energy scale $\Lambda_{lat} \sim \frac{\pi}{a}$
- Correction can be made through $A - A_{SD}^{lat} + A_{SD}^{cont} =$

$$\int d^4x \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu)O^{(6)}|K\rangle + \langle f|C^{cont}(\mu)O^{(6)}|K\rangle$$

- $C^{lat}(\mu)$ is determined non-perturbatively using RI/SMOM approach
- $C^{cont}(\mu)$ can be calculated perturbatively, currently in LO

Short-distance subtraction

- Evaluate off-shell Green's function with $p_i^2 \gg \Lambda_{QCD}^2$
- Energy scale of internal mom. is forced to be larger than $\mu^2 = p_i^2$
- At high energy scale μ, mainly SD contribution to off-shell Green's function
- Correctly represented by a SD operator multiplying with Wilson coefficient $C^{lat}(\mu)$



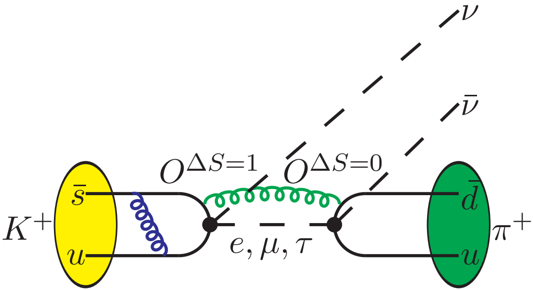
Preliminary lattice results

Lattice setup

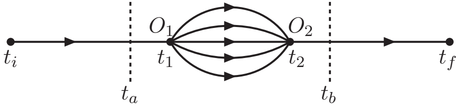
- $16^3 \times 32$, DWF+Iwasaki, $m_\pi \approx 420$ MeV, $m_K \approx 540$ MeV, $a^{-1} = 1.73$ GeV, $m_c = 860$ MeV, 800 configurations
- Construct 4-point correlator $\langle \phi_\pi(t_\pi) O_1(t_1) O_2(t_2) \phi_K^\dagger(t_K) \rangle$
- One can extract the scalar amplitude $F_{BL}^\ell(p_K, p_\nu, p_{\bar{\nu}})$

$$\int dt \, \langle \pi^+ \nu\bar{\nu} | T\{O_1(t) O_2(0)\} | K^+ \rangle = F_{BL}^\ell(p_K, p_\nu, p_{\bar{\nu}}) \, \bar{u}(p_\nu) \not{p}_K (1-\gamma_5) v(p_{\bar{\nu}})$$

W-W diagram: type 1



Double integration



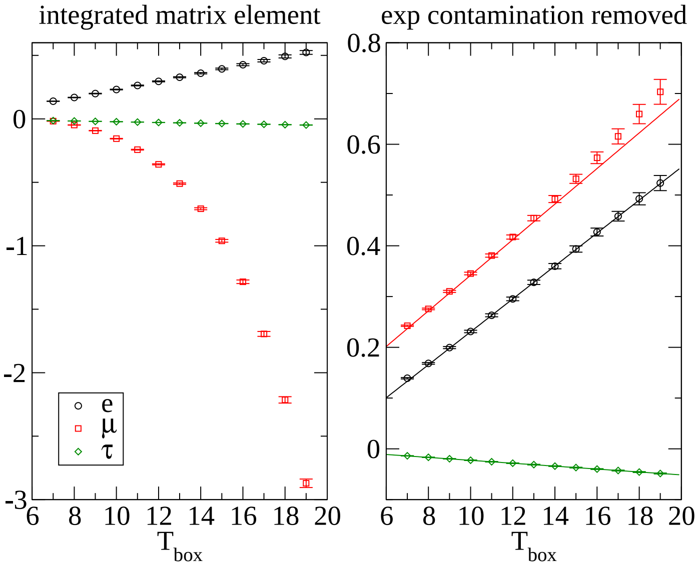
- Perform the double integration to gain a better precision

$$
\begin{aligned}
&\sum_{t_1=t_a}^{t_b}\sum_{t_2=t_a}^{t_b}\langle f|T[O^{\Delta S=1}(t_2)O^{\Delta S=0}(t_1)]|K\rangle\, e^{m_K t_1} e^{-m_f t_1}\\
= \quad &\sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(T_{\text{box}}-\frac{1-e^{(E_f-E_{n_s})T_{\text{box}}}}{E_{n_s}-E_f}\right)\\
&-\sum_{n}\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(T_{\text{box}}+\frac{1-e^{(E_K-E_n)T_{\text{box}}}}{E_K-E_n}\right)
\end{aligned}
$$

here $T_{\text{box}} = t_b - t_a + 1$ is defined as size of the integral window

- Remove the exponential growing contamination, and fit with $a + bT_{box}$, the slope b is what we want

Integrated matrix element



Right figure: the slope of the curve gives $F_{BL}^{\ell}(p_K, p_\nu, p_{\bar{\nu}})$

F^{ℓ}_{BL} for type 1 diagram

F^{ℓ}_{BL}	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

- Vacuum saturation approximation assumes only single-lepton contribution in the intermediate state

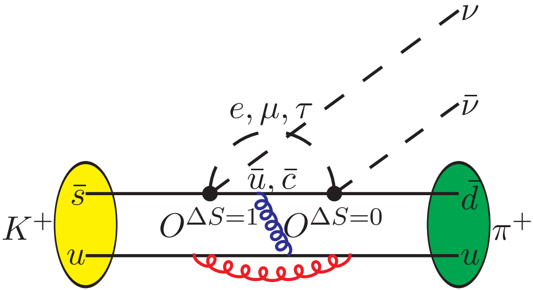
$$f_K p_{K,\mu} \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)\frac{\not{q}}{q^2-m_\ell^2}\gamma_\nu(1-\gamma_5)v(p_{\bar{\nu}})f_\pi p_{\pi,\nu}$$

$$= f_K f_\pi \frac{2q^2}{q^2-m_\ell^2}\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}})$$

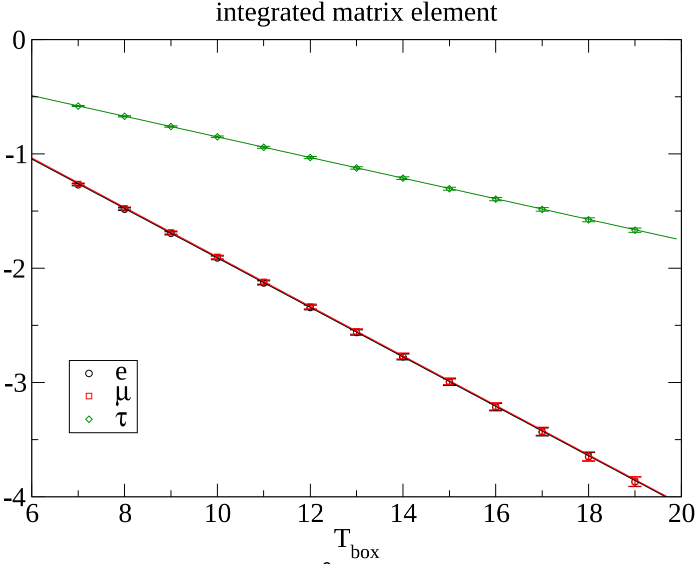
with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$

- In the above table, model results are given by $Z_A^{-2} f_K f_\pi \frac{2q^2}{q^2-m_\ell^2}$

Type 2 diagram



Type 2 diagram



Intermediate state is given by $\ell + \pi^0$, since pion is heavy, we don't observe significant exponential growing effects

Preliminary results

- Type 1 diagram

F_{BL}^{ℓ}	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

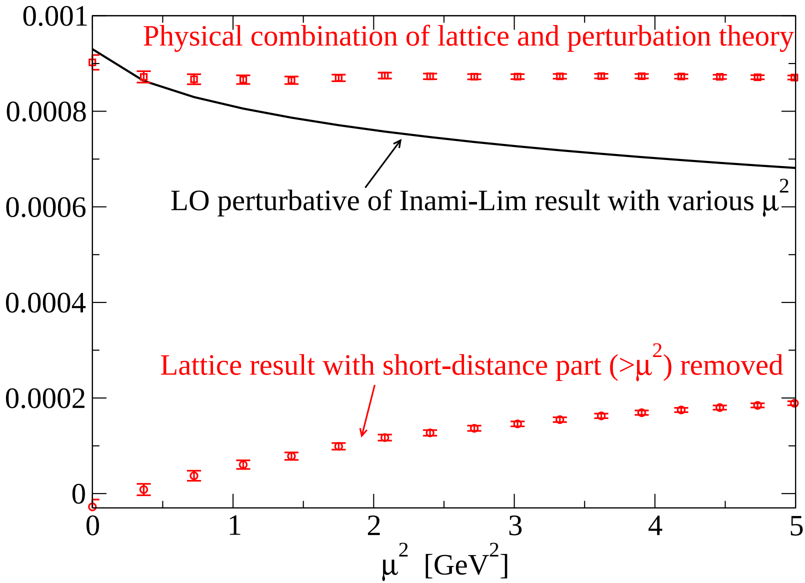
- Type 2 diagram

F_{BL}^{ℓ}	lattice
e	$-2.164(31) \times 10^{-1}$
μ	$-2.164(31) \times 10^{-1}$
τ	$-9.03(14) \times 10^{-2}$

- It seem that type 2 contribution is much larger than type 1, but

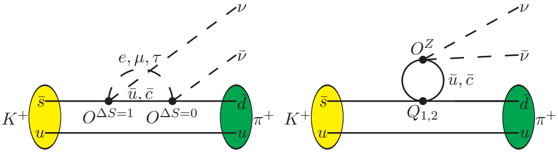
type 2 diagram contains large lattice cutoff effects due to SD divergence

Short-distance matching and correction



Remaining challenge

- Charm quark appears in the GIM mechanism



  - we need the physical charm quark mass
  - $am_c \ll 1$ & $m_\pi L \geq 4 \Rightarrow$ large lattice $(L/a)^4$

- Physical amplitude is given by

$$\langle \pi\nu\bar{\nu}|\mathcal{B}_{WW}(0) + \mathcal{B}_Z(0)|K^+\rangle + \langle \pi\nu\bar{\nu}|C_W O^{(6)}(0)|K^+\rangle$$

need to provide C_W with controlled error

$$C_W = C^{cont}(\mu) - C^{lat}(\mu)$$

  - we now work in LO PT for $C^{cont}(\mu)$, no QCD correction
  - how large the QCD correction to $C^{cont}(\mu)$?

QCD correction to C^{cont}

- No one has calculated QCD correction to C^{cont} at $p_i \neq 0$
- At $p_i = 0$, α_s correction to C^{cont} has been calculated [Buchalla, Buras, NPB, 1994]
- at renormalization scale $\bar{\mu} = m_c$, we have

$$
\begin{aligned}
C^{cont} &\propto X_0(x) + \frac{\alpha_s}{4\pi} X_1(x), \quad x = \frac{m_c^2}{M_W^2} \\
&= -\frac{3}{4} x \ln x - \frac{1}{4} x + \frac{\alpha_s}{4\pi}\left(-2x \ln^2 x - 7x \ln x - \frac{23 + 2\pi^2}{3} x\right) \\
&= (16.32 - 0.66 - 13.05 + 5.54 - 1.37) \times 10^{-4} \\
&= 6.78 \times 10^{-4}
\end{aligned}
$$

  - More than 50% correction from α_s correction
  - This is because $\ln x \sim -8.2$ is large, it compensates $\alpha_s/(4\pi)$

Solution: step scaling

- Solution 1: add QCD correction to the PT calculation of $C^{cont}(\mu)$
- Solution 2: use large external momenta and matching scale μ
  - from scale $\mu = m_c$ to 15 GeV, $\alpha_s(\mu)$ descreases to 25%, $\ln \frac{\mu^2}{M_W^2}$ decrease to 33%. $\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{M_W^2}$ no more significant
  - NPR step scaling
    use superfine lattice spacing a_1, calculate C_W at high energy scale μ_{high}

    $$C_W(a_1) = C^{cont}(\mu_{high}) - C^{lat}(a_1, \mu_{high})$$

    at lower energy scale μ_{low}, we shall have

    $$C_W(a_1) = C^{cont}(\mu_{low}) - C^{lat}(a_1, \mu_{low})$$

    thus $C^{cont}(\mu_{low})$ is known
  - then work at a larger lattice spacing a_2

    $$C_W(a_2) = C^{cont}(\mu_{low}) - C^{lat}(a_2, \mu_{low})$$

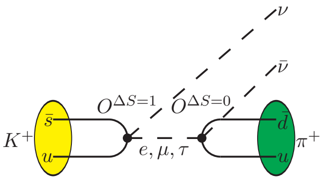
  - step scaling used in B_K, from 3 GeV to 9 GeV [Frison, Boyle, Garron]

Outlook

- Calculation of the non-local matrix element is highly non-trivial
- Our exploratory study sheds light on the feasibility of lattice calculation of $K^+ \to \pi^+ \nu\bar{\nu}$
- We are starting the calculation at $m_\pi = 170$ MeV. In the future, physical charm quark will be included. Matching at higher energy scale is also necessary.
- NA62 will confront SM soon $\Rightarrow$ It's in a timely fashion for lattice QCD to make impact on $K^+ \to \pi^+ \nu\bar{\nu}$

Back up slides

Remove exp growing contamination



$$\langle \pi^+ \nu\bar{\nu} | O_{u\ell}^{\Delta S=0}(0) | \bar{\ell}\nu\rangle \frac{1}{2E_{\bar{\ell}}} \frac{1}{2E_\nu} \langle \bar{\ell}\nu | O_{u\ell}^{\Delta S=1}(t) | K^+\rangle$$

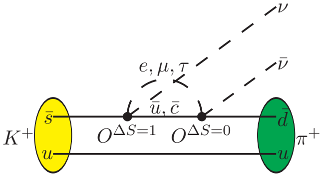
$$= \langle \pi^+ | \bar{u}\gamma_\mu(1-\gamma_5)d(0)|0\rangle\langle 0|\bar{s}\gamma_\nu(1-\gamma_5)u(0)|K^+\rangle$$

$$\times \bar{u}(p_\nu)\gamma_\nu(1-\gamma_5)\frac{-i\not{p}_{\bar{\ell}} + m_{\bar{\ell}}}{2E_{\bar{\ell}}}\gamma_\mu(1-\gamma_5)v(p_{\bar{\nu}})\cdot e^{(E_{\bar{\ell}}+E_\nu-E_K)t}$$

$$= -2f_K f_\pi \bar{u}(p_\nu)\not{p}_K \frac{-i\not{p}_{\bar{\ell}}}{2E_{\bar{\ell}}}\not{p}_\pi(1-\gamma_5)v(p_{\bar{\nu}})\cdot e^{(E_{\bar{\ell}}+E_\nu-E_K)t}$$

$$\equiv c_{t<0}\cdot e^{(E_{\bar{\ell}}+E_\nu-E_K)t}$$

Finite volume correction

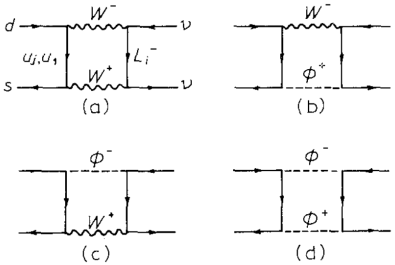


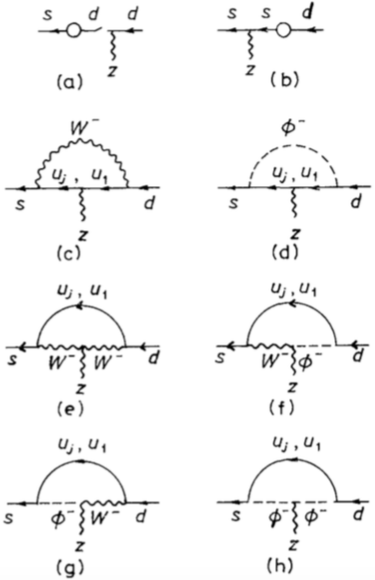
$$T_{WW}^{FV} = \left(\frac{1}{L^3} \sum_{\vec{k}} \int \frac{dk_0}{2\pi} - \mathcal{P} \int \frac{d^4k}{(2\pi)^4} \right)$$

$$\left\{ A_\alpha^{K^+ \to \pi^0}(p_K, k) \frac{1}{k^2 + m_\pi^2} A_\beta^{\pi^0 \to \pi^+}(k, p_\pi) \right\}$$

$$\times \left\{ \bar{u}(p_\nu) \gamma^\alpha (1 - \gamma_5) \frac{-i(\not{P} - \not{k}) + m_\ell}{(P-k)^2 + m_\ell^2} \gamma^\beta (1 - \gamma_5) v(p_{\bar{\nu}}) \right\}$$

Perturbation calculation



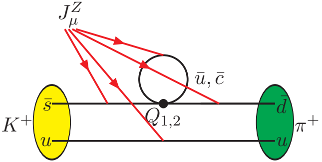


Preliminary results for Z-exchange diagrams

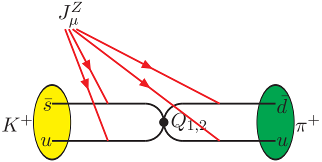
Summary of Z-exchange diagrams

Four classes: Type 1 (Q_1), Type 1 (Q_2), Type 2 (Q_1), Type 2 (Q_2)

- Connected diagrams, J_μ^Z can be inserted into all the possible quark line

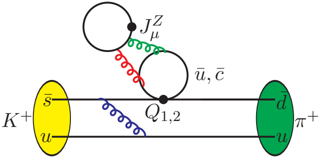


Type 1 diagram

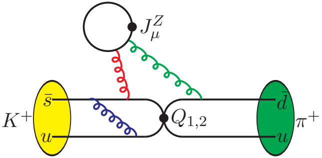


Type 2 diagram

- Disconnected diagrams (usually excluded in lattice calculation since they are noisy and difficult to calculate)

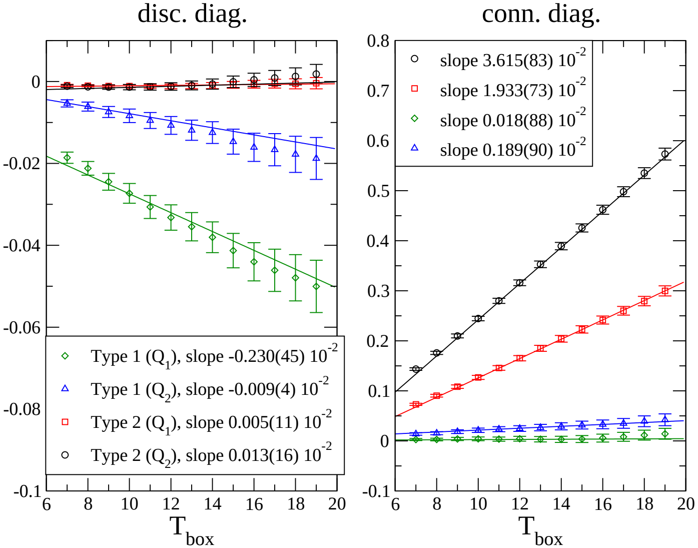


Type 1 diagram



Type 2 diagram

Integrated matrix element



Disc. diag. is relatively noisy, but its contribution is small. Adding the disc. part does not affect the conn. part significantly