Flavor Physics and Lattice QCD in the Precision Era

Aida X. El-Khadra
(University of Illinois)

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Flavor Physics and Lattice QCD in the Precision Era

Thanks to better methods (algorithms, formalism/theoretical understanding) and significant increases in computational resources we now have a growing number of results for

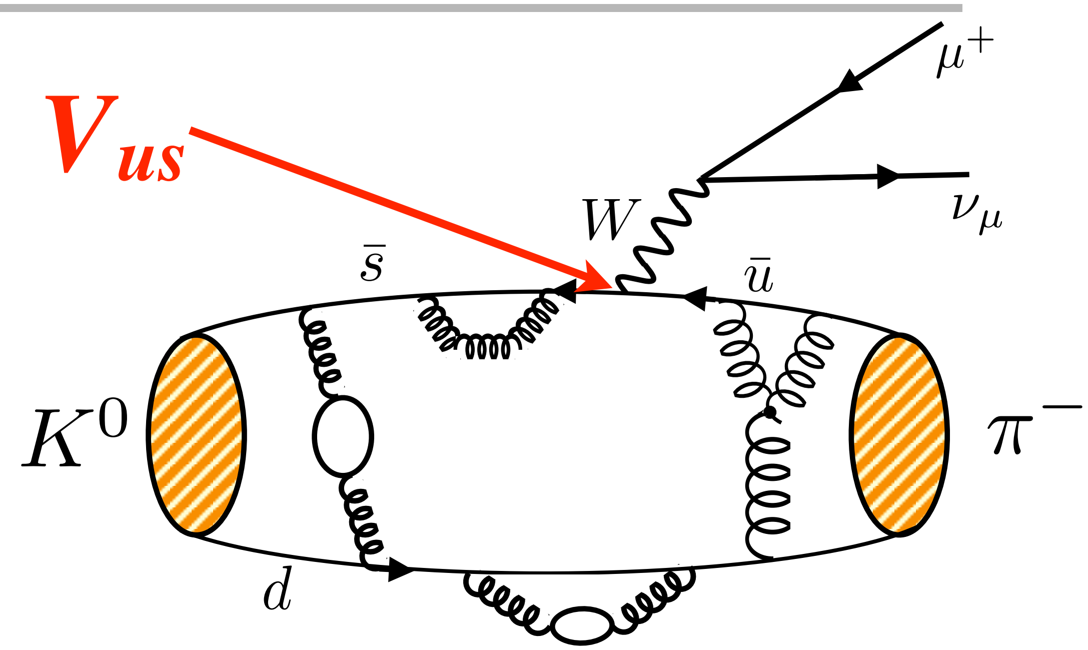
- simple meson quantities with unprecedented precision
- new quantities (two hadron systems, resonances, ...) with control over systematic errors

Outline

- Motivation and introduction
- Simple quantities with single, stable hadrons
  - low-lying QCD spectrum
  - weak decays (leptonic, semileptonic, mixing)
    - → CKM, BSM phenomenology
  - high precision → including QED
- Conclusions & Outlook

Why Lattice QCD?

example: $K^0 \to \pi^- \ell^+ \nu_\ell$



generic EW process involving hadrons:

(experiment) = (known) x (**CKM element**) x (had. matrix element)

$\Gamma_{K\ell 3}, \Gamma_{K\ell 2}, \ldots$

$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2}, \frac{d\Gamma(D\to K\ell\nu)}{dq^2}, \ldots$

$\Delta m_{d(s)}$

$\vdots$

Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

Introduction to Lattice QCD

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

systematic error analysis

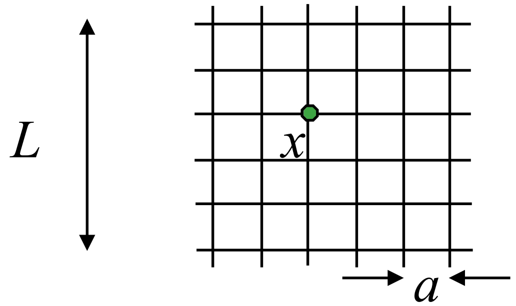
...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➙ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

systematic error analysis

discretization effects

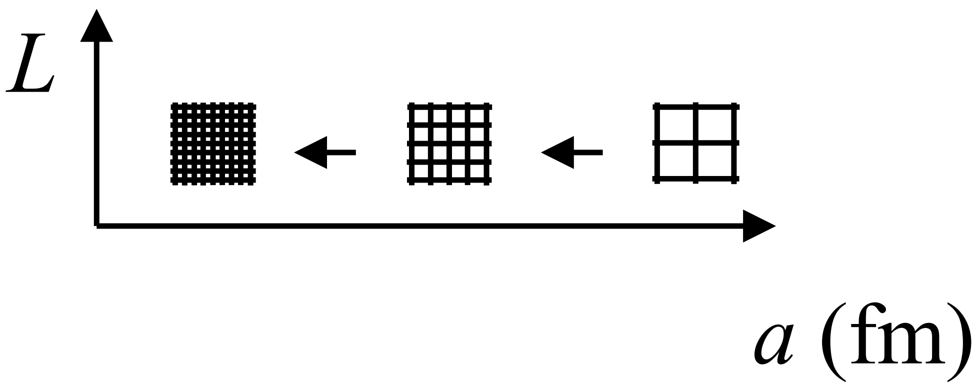


discrete space-time → discrete QCD action

Symanzik EFT: $\langle \mathcal{O} \rangle^{\rm lat} = \langle \mathcal{O} \rangle^{\rm cont} + O(ap)^n$

p is the typical momentum scale associated with $\langle \mathcal{O} \rangle$

for light quark systems, $p \sim \Lambda_{\rm QCD}$

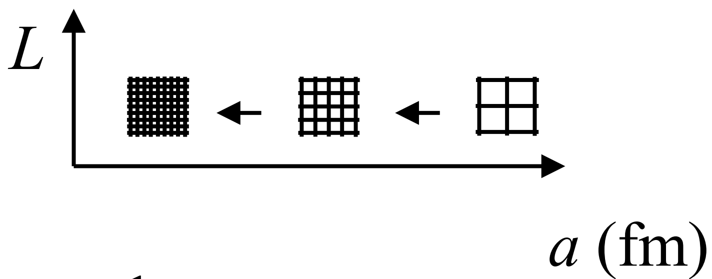


The form of $O(ap)^n$ depends on the details of the lattice action.

All modern light-quark actions start at $n = 2$
(improved Wilson, twisted-mass Wilson, asqtad, HISQ, Domain Wall, Overlap, ...).

systematic error analysis

discretization effects for b quarks



- If we use light quark actions for heavy quarks, discretization errors ~ $O(am_h)^2$, with currently available lattice spacings
  for charm $am_c \sim 0.15\text{-}0.6$ and for b: $am_b > 1$

➡ need effective field theory methods for b quarks
for charm lattice spacings are sufficiently small so that we can use improved light quark methods

- avoid errors of $(am_b)^2$ by using EFT in the formulation/matching of lattice action/currents:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use the same improved light quark action as for charm (HISQ, twisted mass Wilson, NP imp. Wilson, Overlap, ...)
  - ✦ keep $am_h < 1$
  - ✦ use HQET and/or static limit to extrapolate to the physical b quark mass

systematic error analysis

light quark mass effects

Simulations with $m_{\text{light}} = 1/2\,(m_u + m_d)$ **at the physical u/d quark masses are now available**, but many results still have

$$m_{\text{light}} > 1/2\,(m_u + m_d)_{\text{phys}}$$

χPT can be used to extrapolate/interpolate to the physical point.

- Can include discretization effects (for example, staggered χPT)
- It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

systematic error analysis

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in CPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(or if there are two or more intermediate state hadrons)

see talks by: X. Feng and S. Sharpe later today
M. Buchoff (Wed)
W. Detmold (Fri)

systematic error analysis

other effects

- ✓ statistical errors: from monte carlo integration
  consider/include systematic errors from correlator fit procedure

- ✓ n_f dependence: realistic sea quark effects: use $n_f = 2+1$ or $n_f = 2+1+1$
  Note: $n_f = 2$ (effects due to quenching the strange quark appear to be small)

- ❖ renormalization (and matching):
  - ⇒ with lattice perturbation theory: need to include PT errors
  - ⇒ nonperturbative methods
  - ⇒ use absolutely normalized currents where possible

systematic error analysis

...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➙ ab initio

The EFT description:

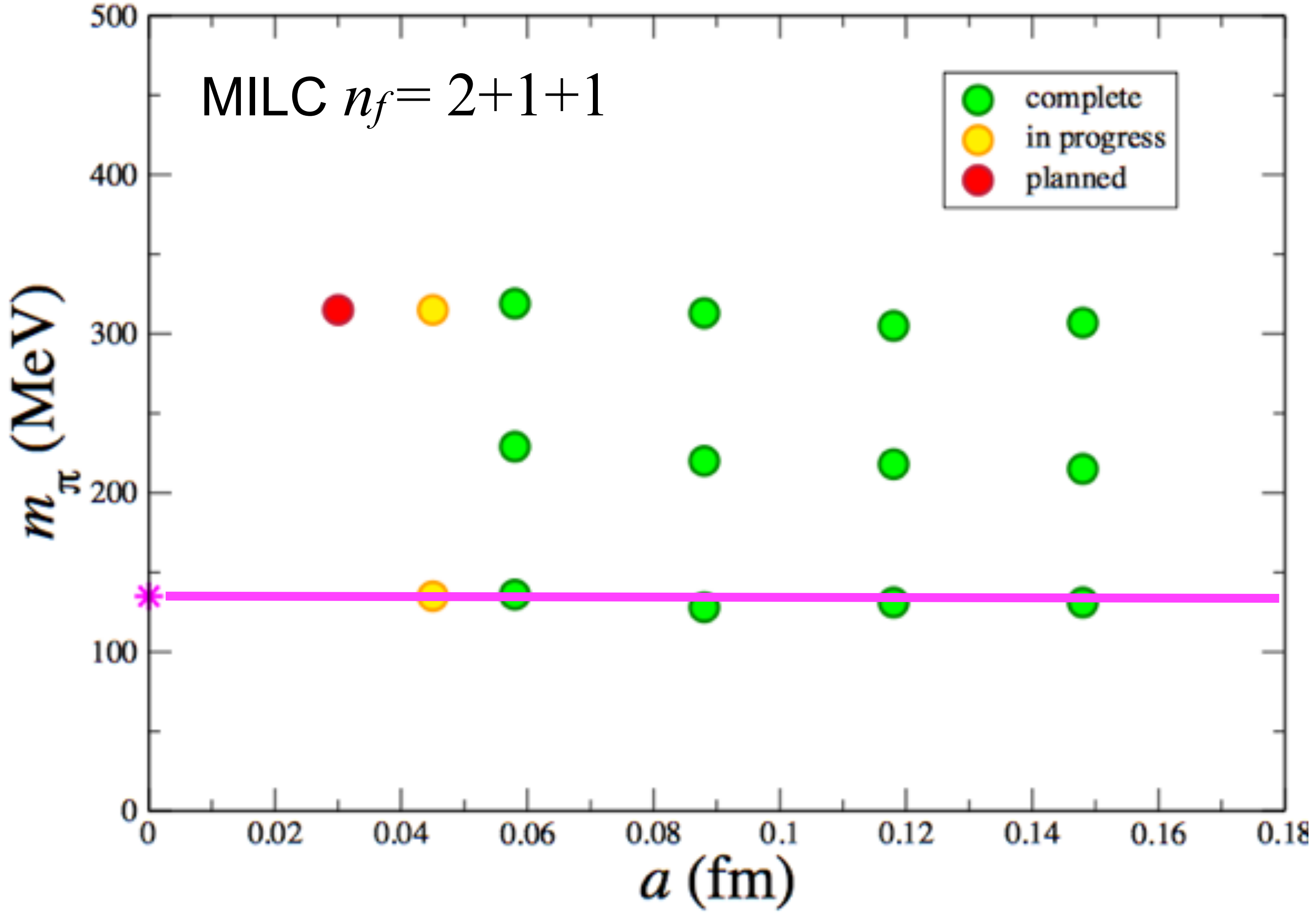
- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...

systematic error analysis

For example, set of ensembles by MILC collaboration

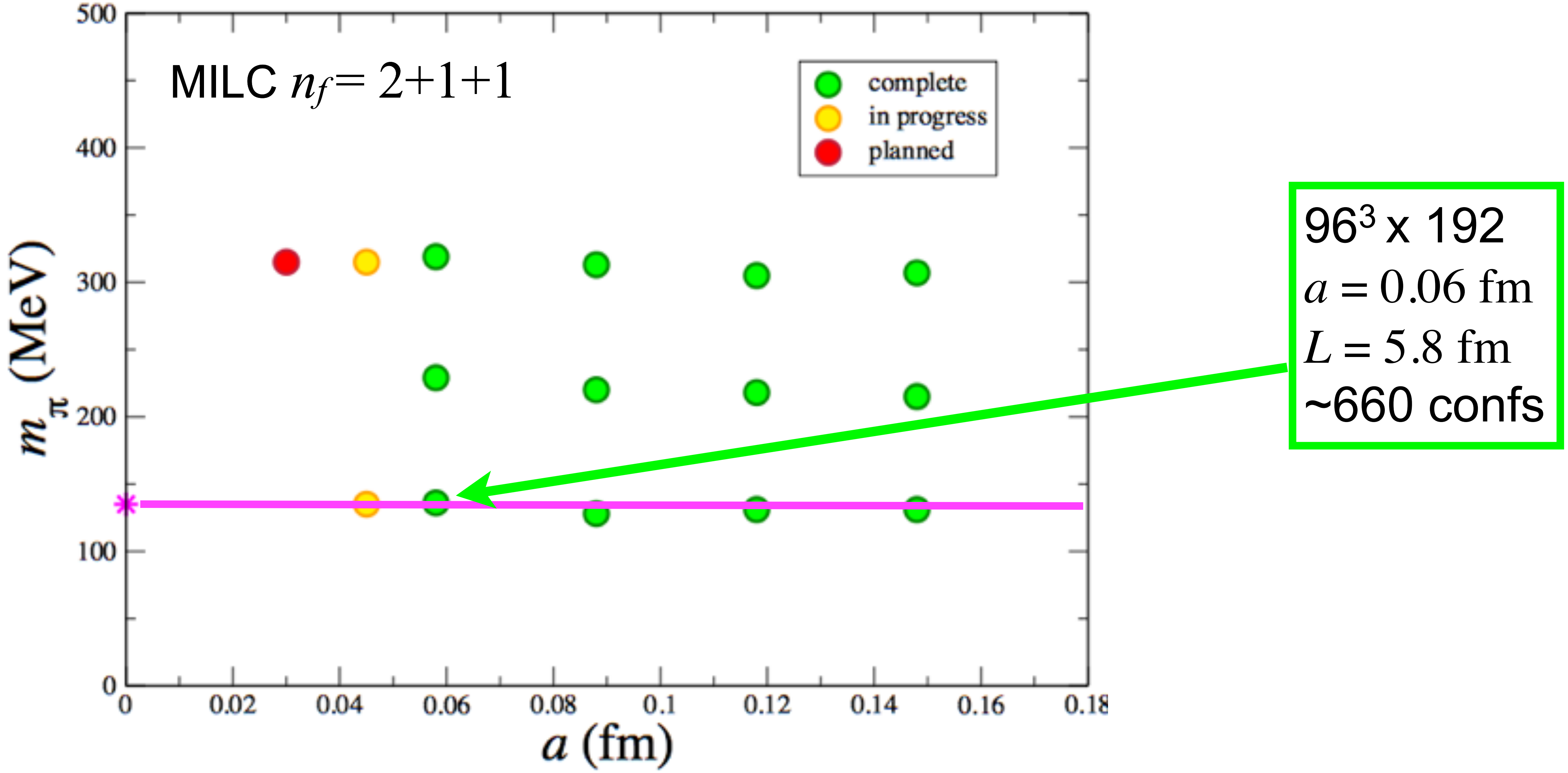


Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

systematic error analysis

For example, set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

Strategy

- Lattice QCD action has the same free parameters as continuum QCD:
  quark masses and α_s

- use experimentally measured hadron masses as input, for example:
  π, K, D_s, B_s mesons for u, d, s, c, b quark masses

- need an experimental input to determine the lattice spacing (a) in GeV:
  2S-1S splitting in Υ system, f_π, Ω, Ξ mass, …

- lattice QCD calculations of all other quantities should agree with experiment …

Simple quantities in LQCD

Stable (under the strong interaction) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- π, K, D, D_s, B, B_s mesons

  spectrum, decay constants, weak matrix elements for mixing, semileptonic and rare decay form factors

- charmonium and bottomonium (η_c, J/ψ, h_c, …, η_b, $\Upsilon(1S)$, $\Upsilon(2S)$, ..)

  states below open D/B threshold

  spectrum, leptonic widths, electromagnetic matrix elements

- stable baryons

  spectrum, matrix elements of local operators

This list includes low-lying hadron spectrum and most of the important quantities for CKM physics.

Excluded are ρ, K^* mesons and other resonances.

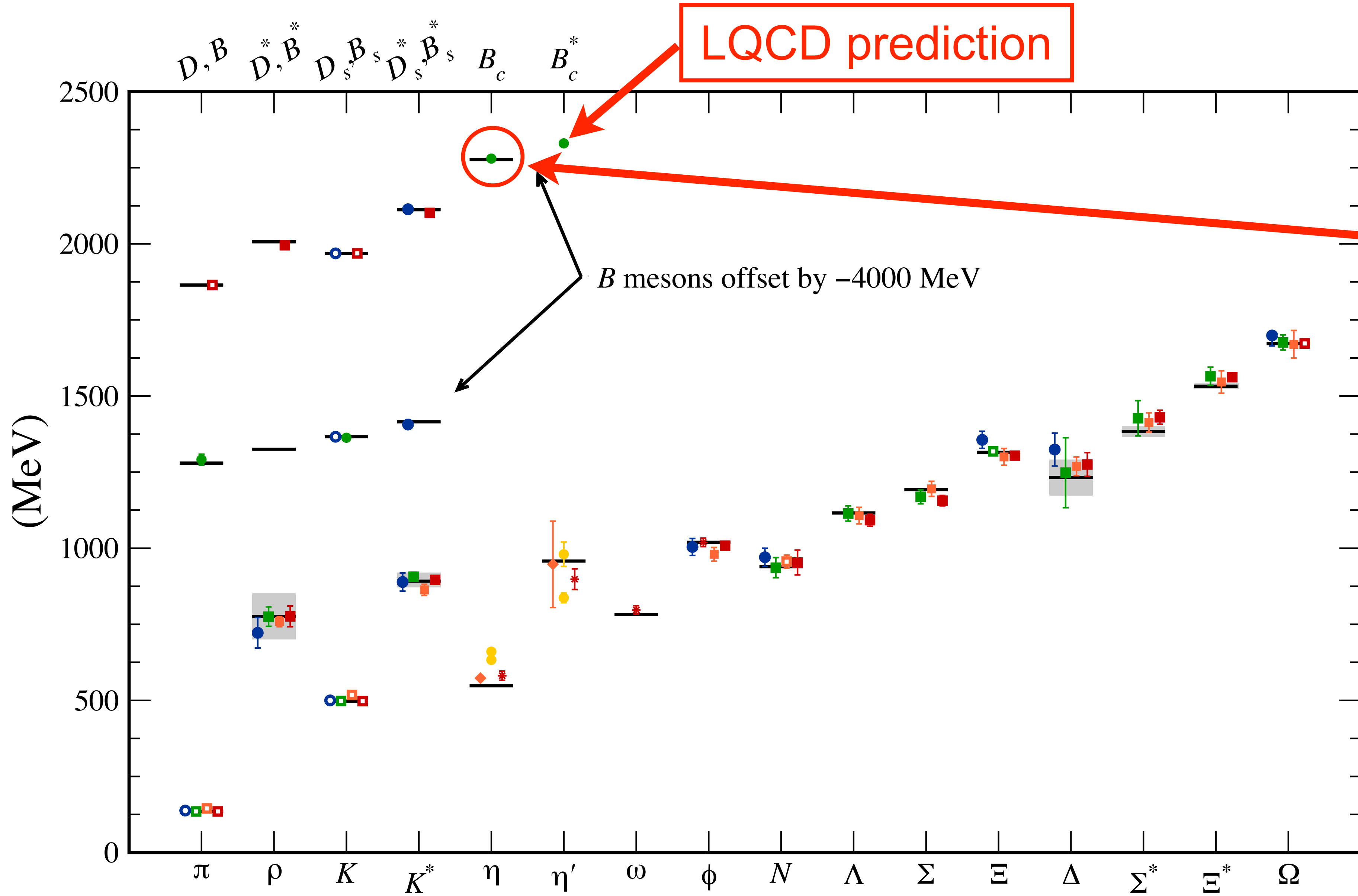
Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

- low-lying hadron spectrum
- weak decays (leptonic, semileptonic, mixing)
  → CKM, BSM phenomenology
- high precision → including QED

Low-lying hadron spectrum

A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204, updated)



predicted by LQCD (HPQCD + FNAL 2004) before exp. measurement (CDF)

new results for charmed baryons shown in appendix

π…Ω: BMW, MILC, PACS-CS, QCDSF; η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, *B*: Fermilab, HPQCD, Mohler-Woloshyn

Simple quantities in LQCD

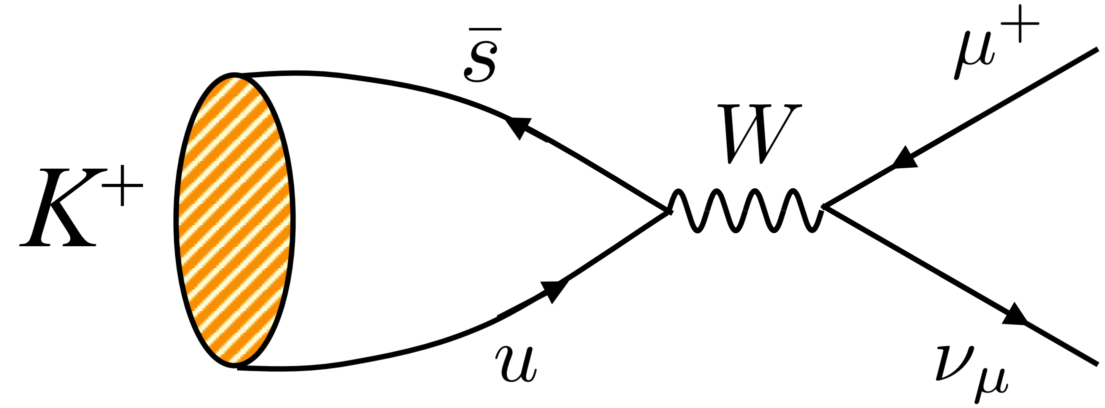
Focus on results with complete error budgets and reliable systematic error estimates.

- ☆ low-lying hadron spectrum
- ☆ weak decays - leptonic, semileptonic, mixing
  - ✦ Kaons
  - ✦ D mesons
  - ✦ B mesons

  → CKM, BSM phenomenology
- ☆ high precision → including QED

Leptonic K, D, B decays

example: $K^+ \to \mu^+ \nu_\mu$

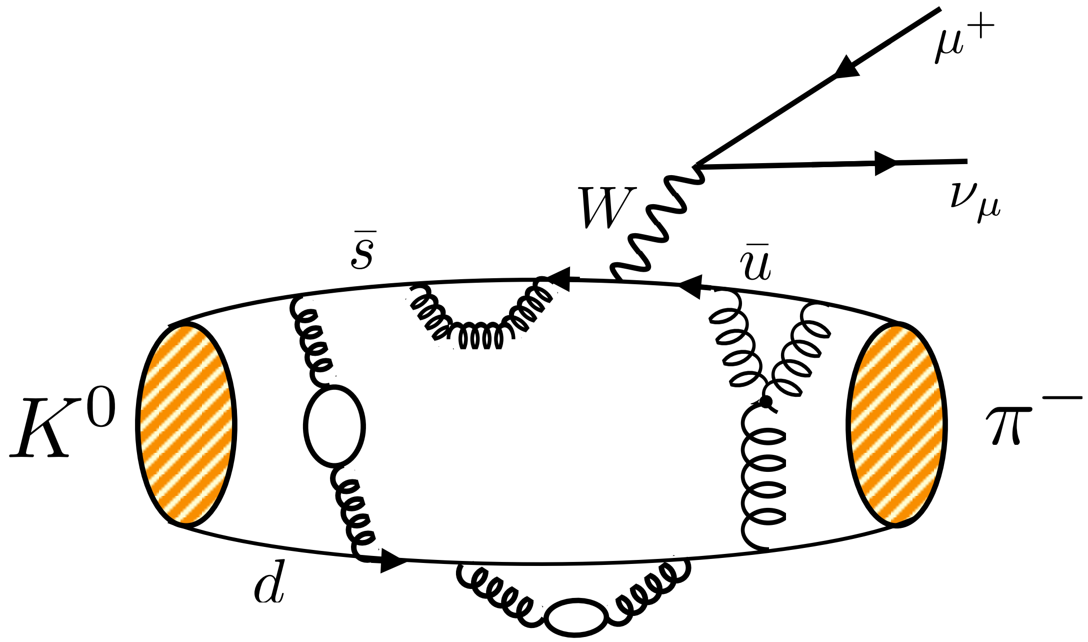


$$\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right) = (\text{known}) \times (1 + \delta^{\ell}_{\text{EM}}) \times |V_{us}|^2 \times f^2_{K^+}$$

- use experiment + LQCD input for determination of CKM element
- similar for B ($|V_{ub}|$) and $D_{(s)}$ ($|V_{cd(s)}|$) mesons
- **ratios** for example f_{K^+}/f_{π^+}: statistical and systematic errors tend to cancel.
- $\delta^{\ell}_{\text{EM}}$ includes structure dependent EM corrections. It is needed to relate the "pure QCD" decay constant to experiment and is currently estimated phenomenologically.

semileptonic K, D, B decays

example: $K^0 \to \pi^- \ell^+ \nu_\ell$

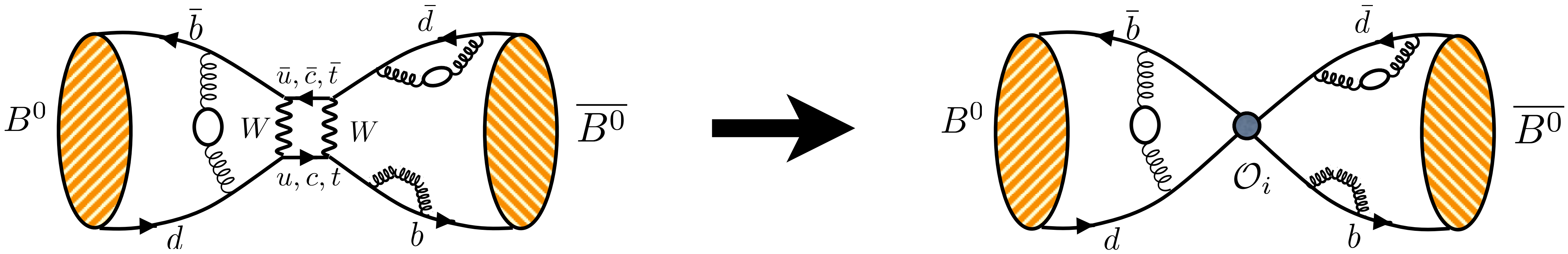


$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

Needed to relate "pure QCD" form factor to experiment. Currently estimated phenomenologically.

Neutral K, B mixing

Standard Model



SM: $\Delta M_q = (\text{known}) \times |V^*_{tq} V_{tb}|^2 \times \langle \bar{B}^0_q | \mathcal{O}_1 | B^0_q \rangle$

also:

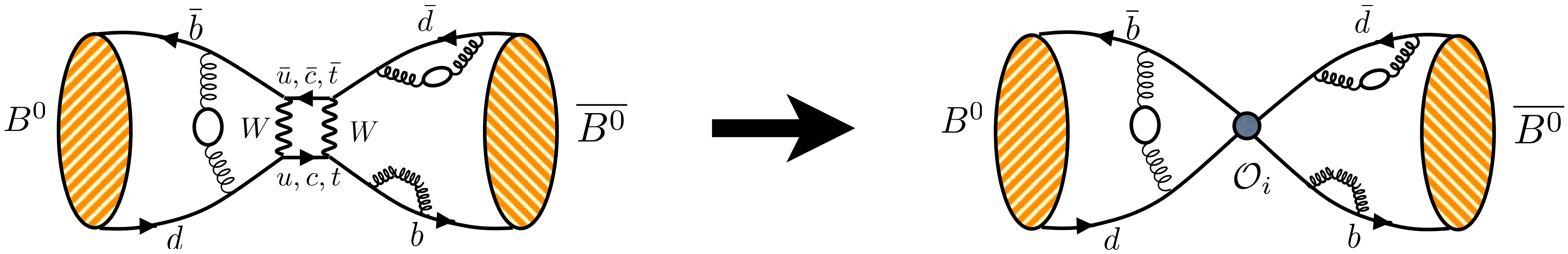
$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2$$ with $$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \, \langle \bar{B}^0_q | \mathcal{O}_1 | B^0_q \rangle + G_3 \, \langle \bar{B}^0_q | \mathcal{O}_3 | B^0_q \rangle \right] \cos\phi_q + O(1/m_b)$$

$$\epsilon_K = (\text{known}) \times B_K \kappa_\epsilon \times |V_{cb}|^2 \times \bar{\eta} \times f(\bar{\rho}, \bar{\eta}, V_{cb}, \eta_i)$$

Neutral K, B mixing

Standard Model



In general :

$$\mathcal{H}_{\rm eff} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

Recent and ongoing LQCD calculations of K, D, and B mixing quantities now include results for hadronic matrix elements of all five operators.

Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

- low-lying hadron spectrum
- weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - B mesons

  → CKM, BSM phenomenology

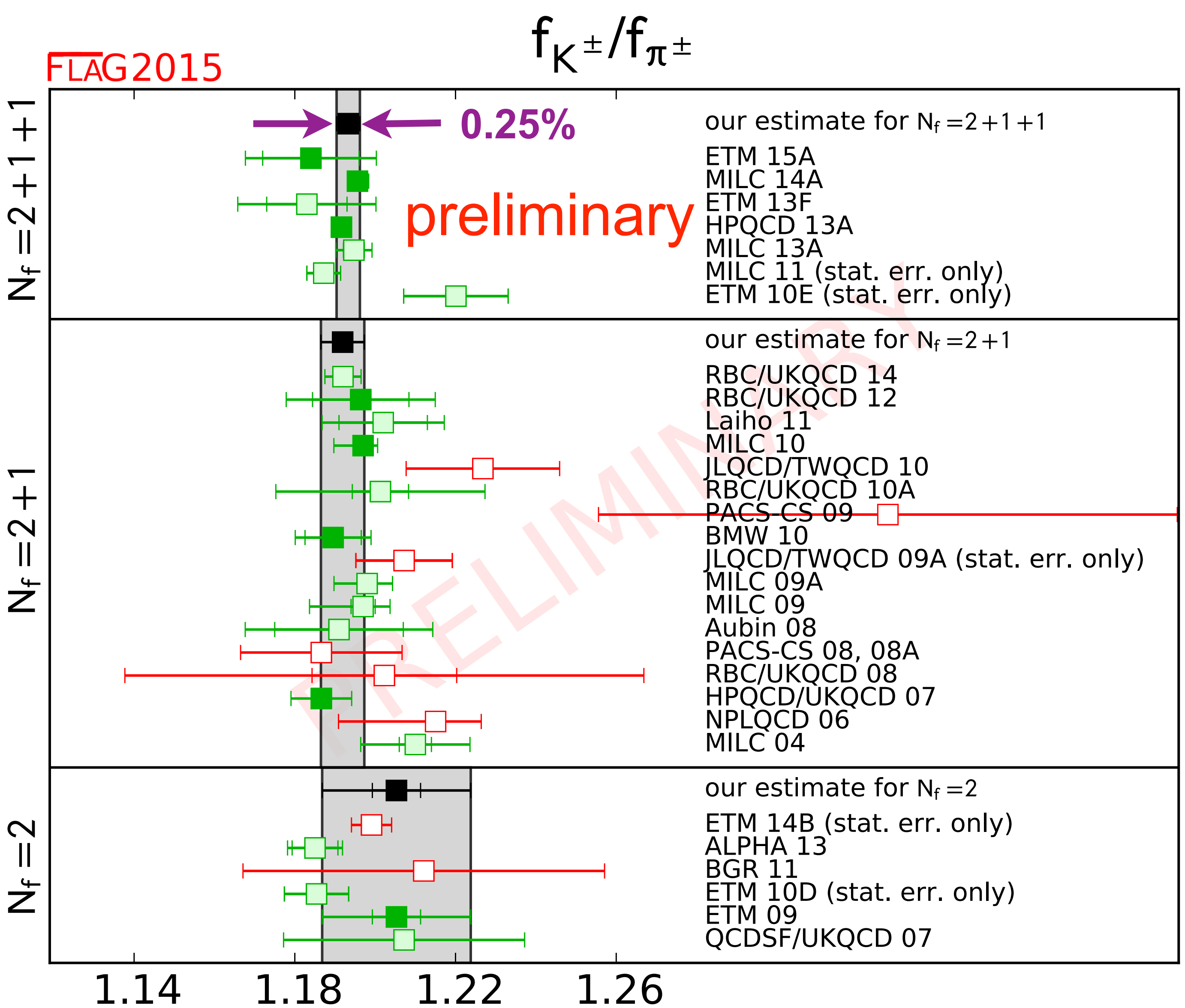
- high precision → including QED

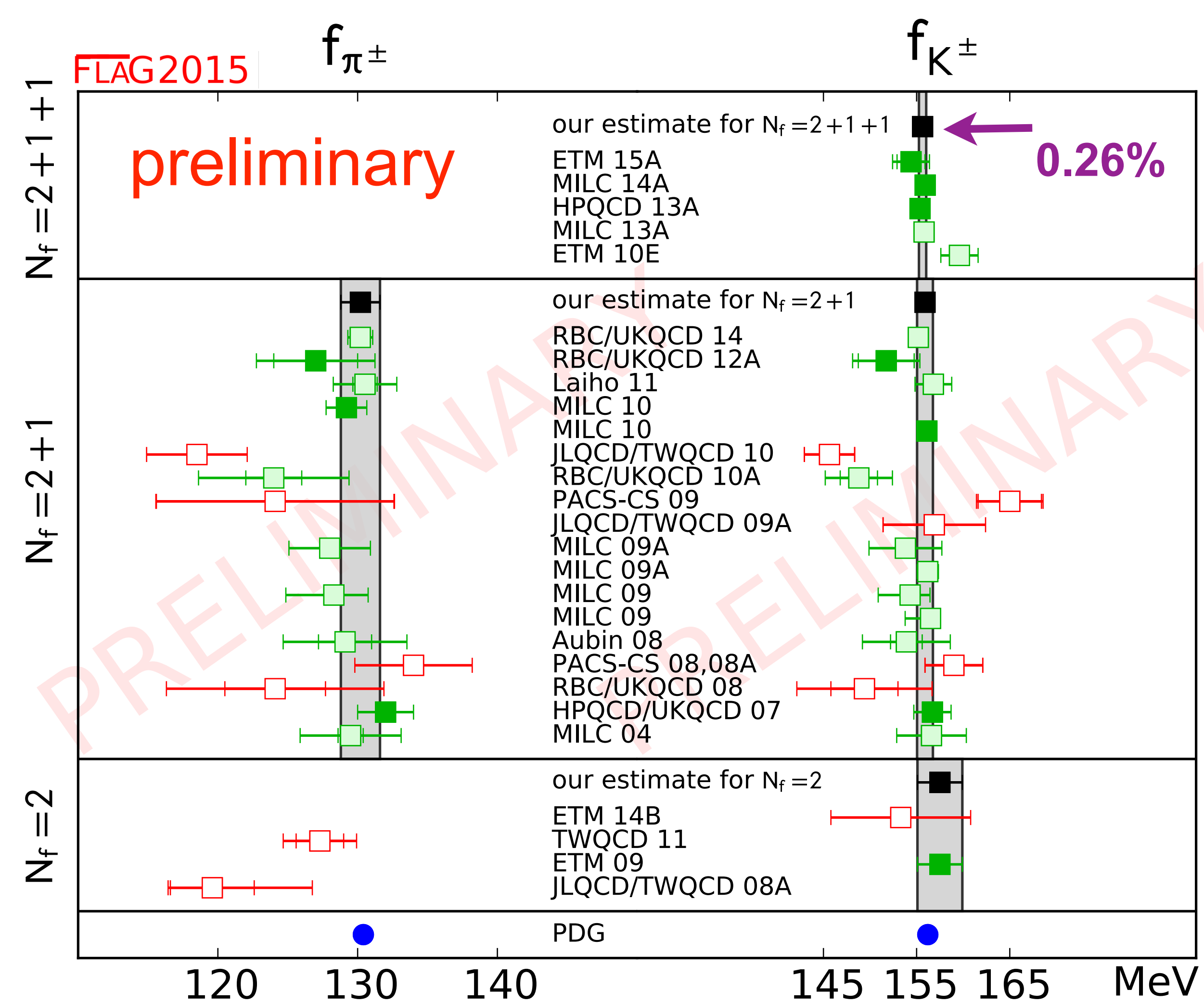
Kaon summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)

courtesy of S. Simula (FLAG-3, V_{us} working group)

status as of mid 2015



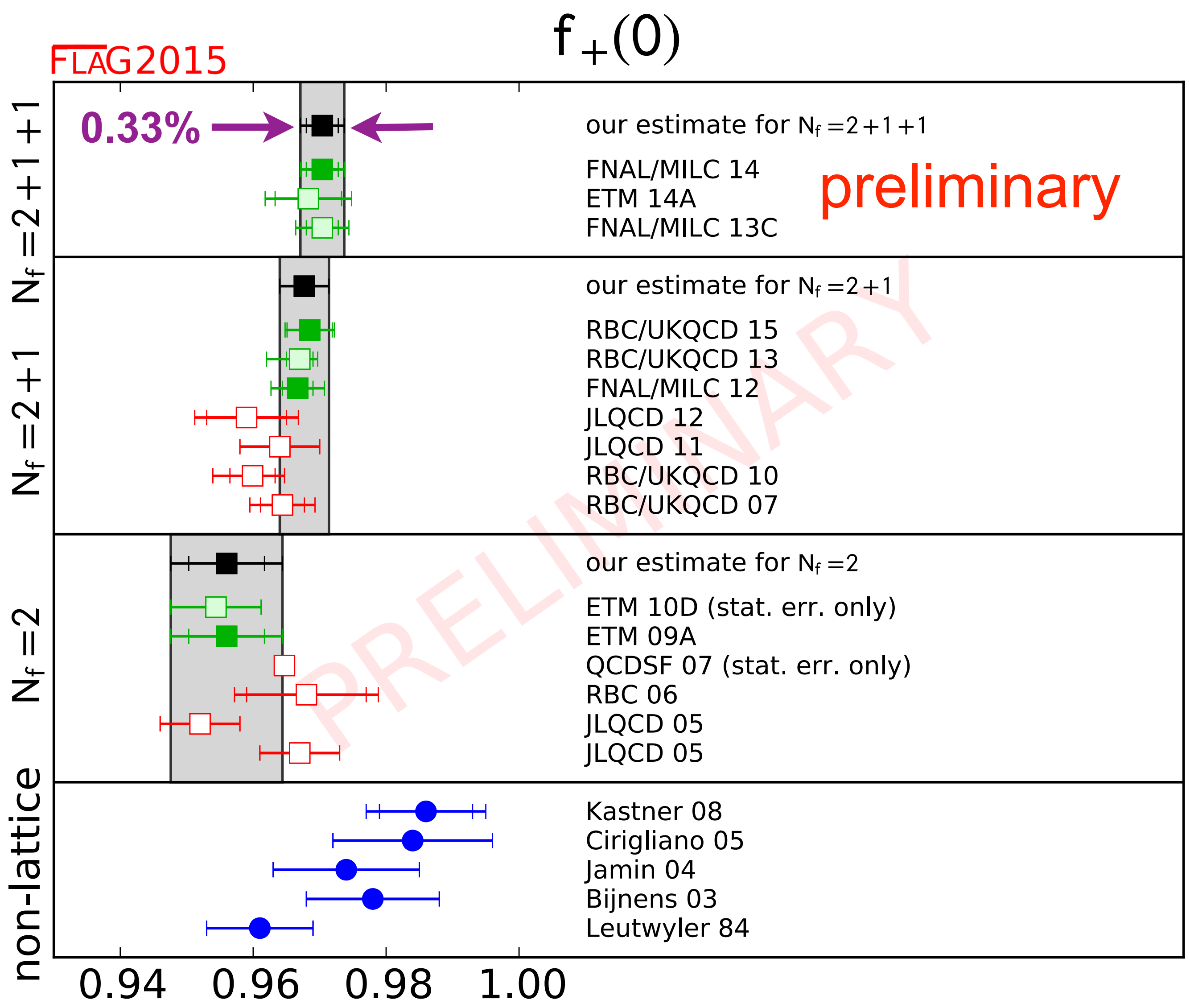


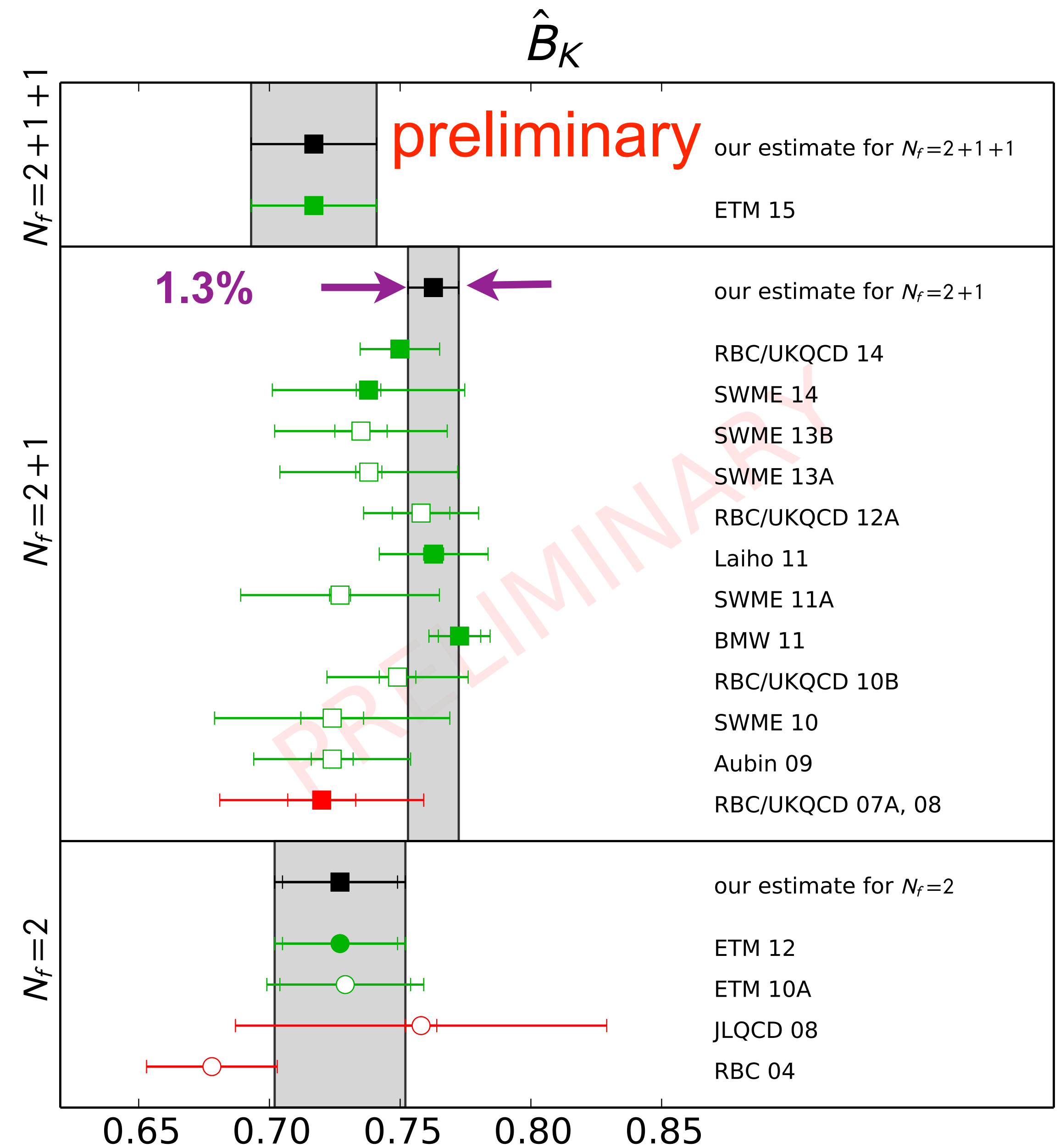
Kaon summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)

courtesy of S. Simula (V_{us} working group) and H. Wittig (B_K working group)

status as of mid 2015

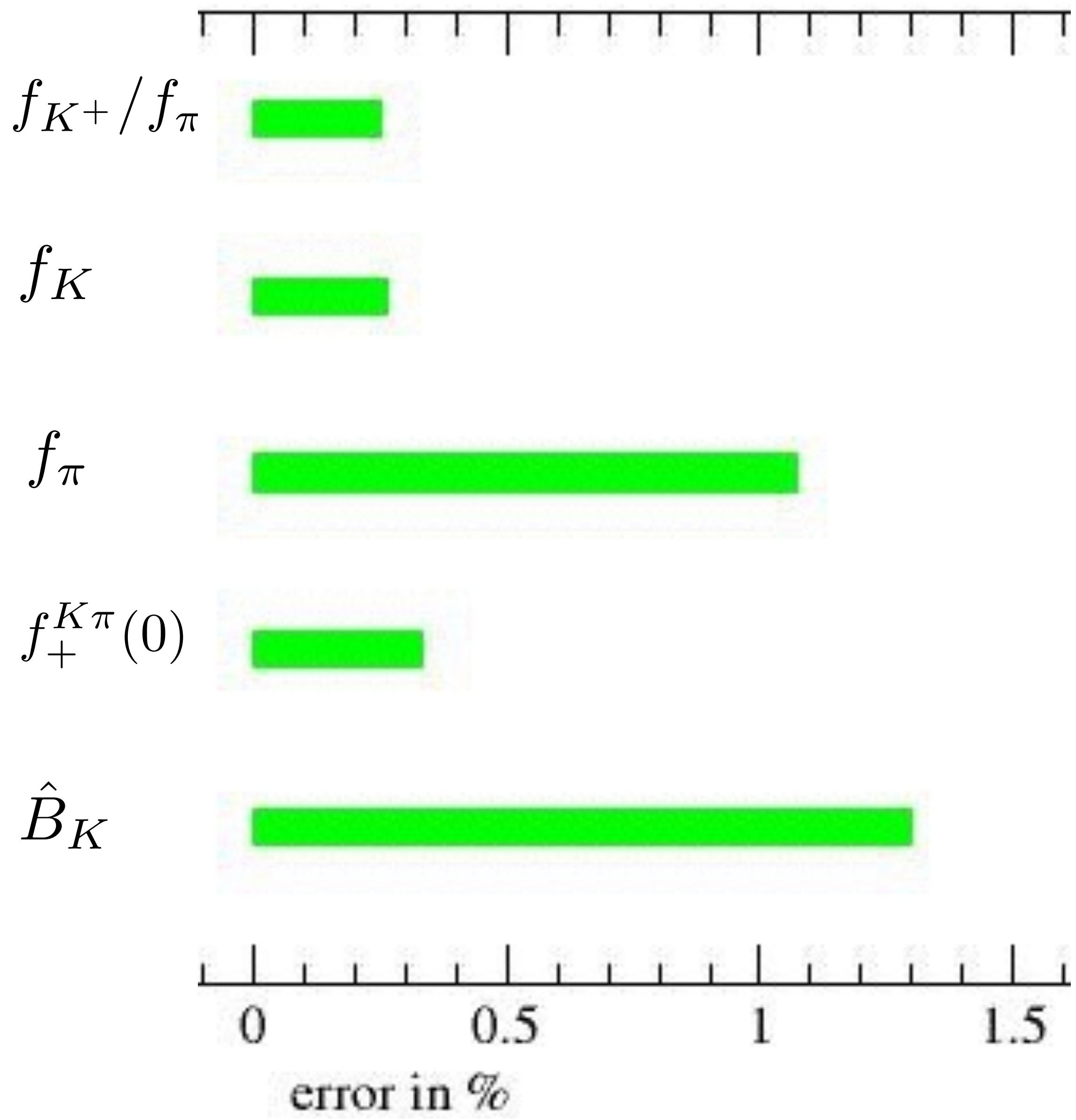




Kaon summary

For all quantities there are results that use **physical mass** ensembles

errors (in %) preliminary **FLAG-3 averages**



independent results (different methods)

small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ ensembles with small lattice spacings
- ✦ NPR or no renormalization

Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

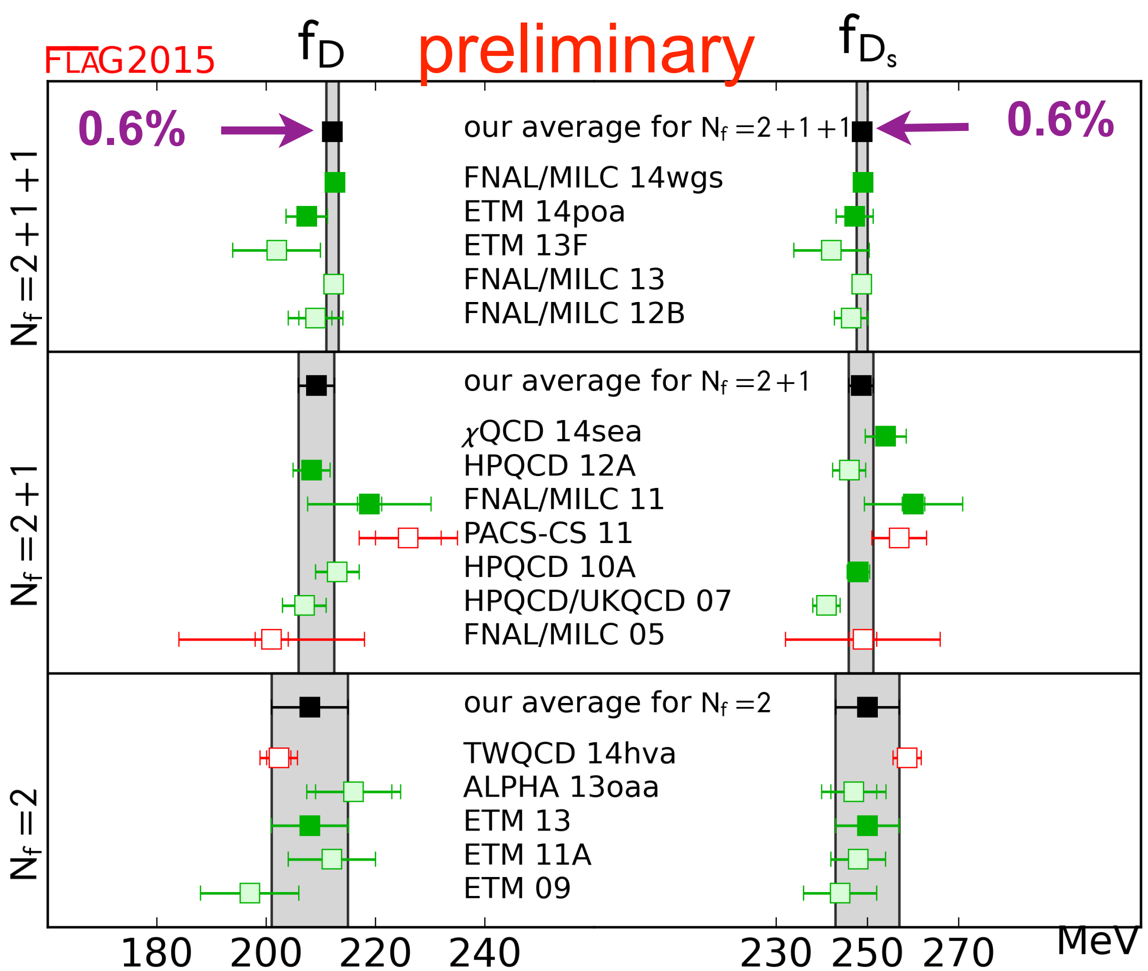
- low-lying hadron spectrum → quark masses
- weak decays - leptonic, semileptonic, mixing
  - Kaons
  - ✦ D mesons
  - B mesons

  → CKM, BSM phenomenology
- high precision → including QED

D meson summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)

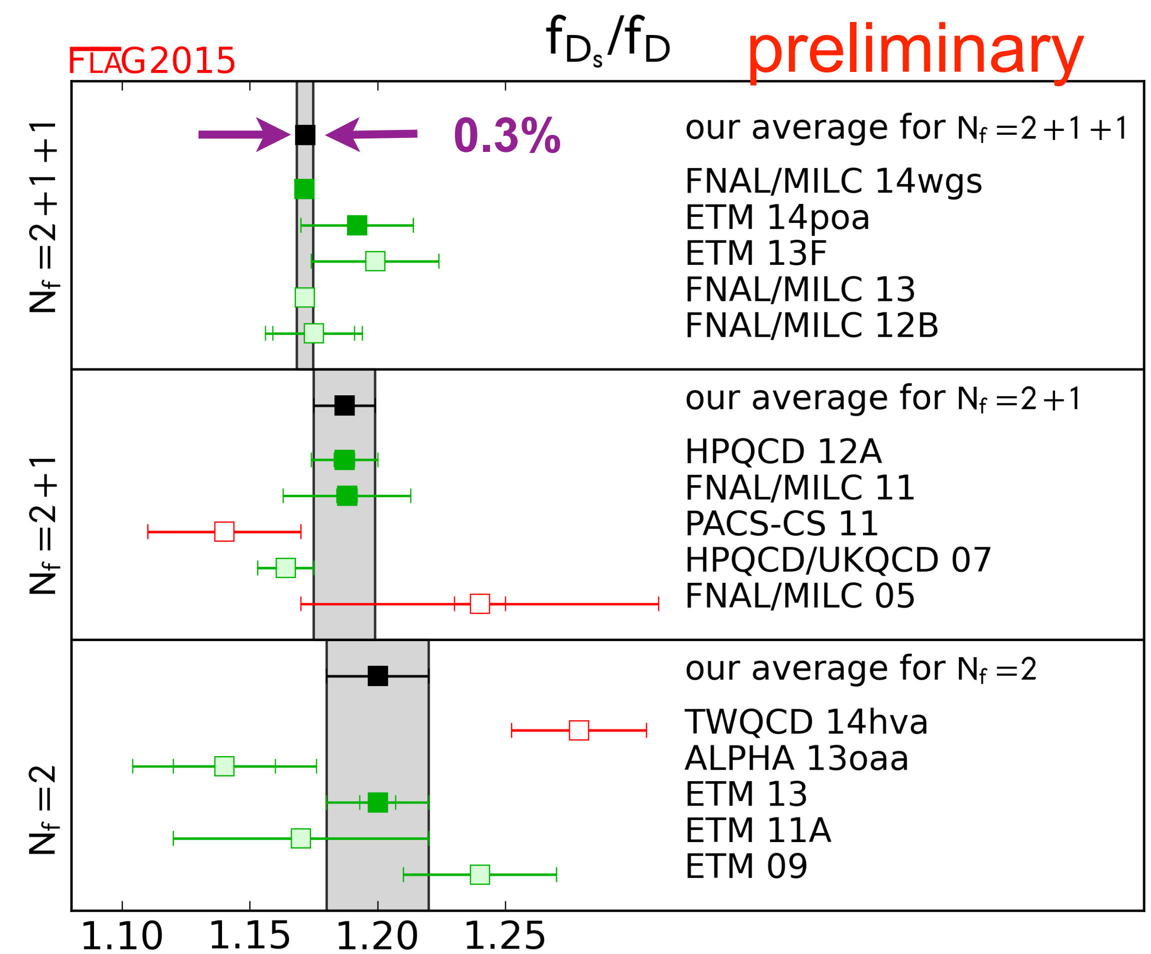
courtesy of M. Della Morte (HQ working group)

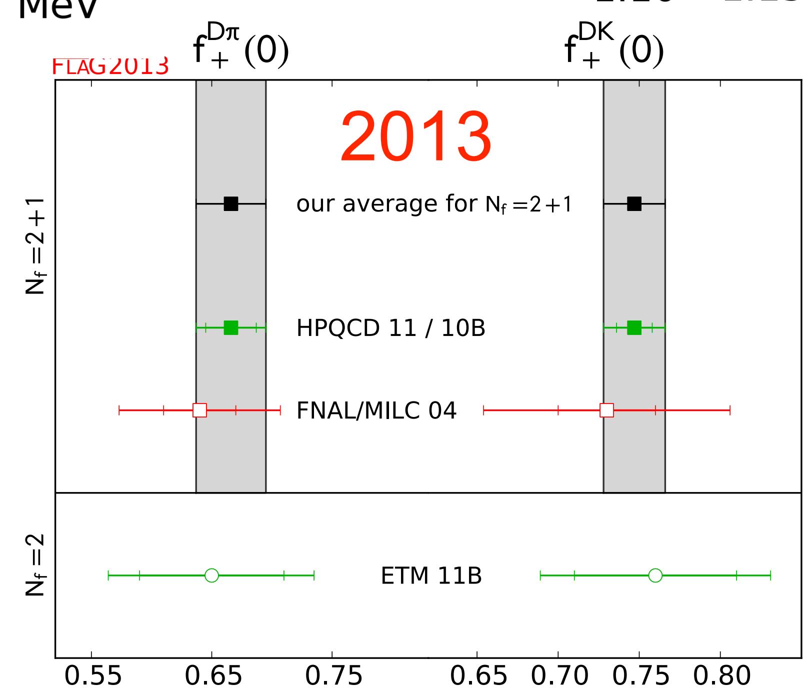


preliminary

status

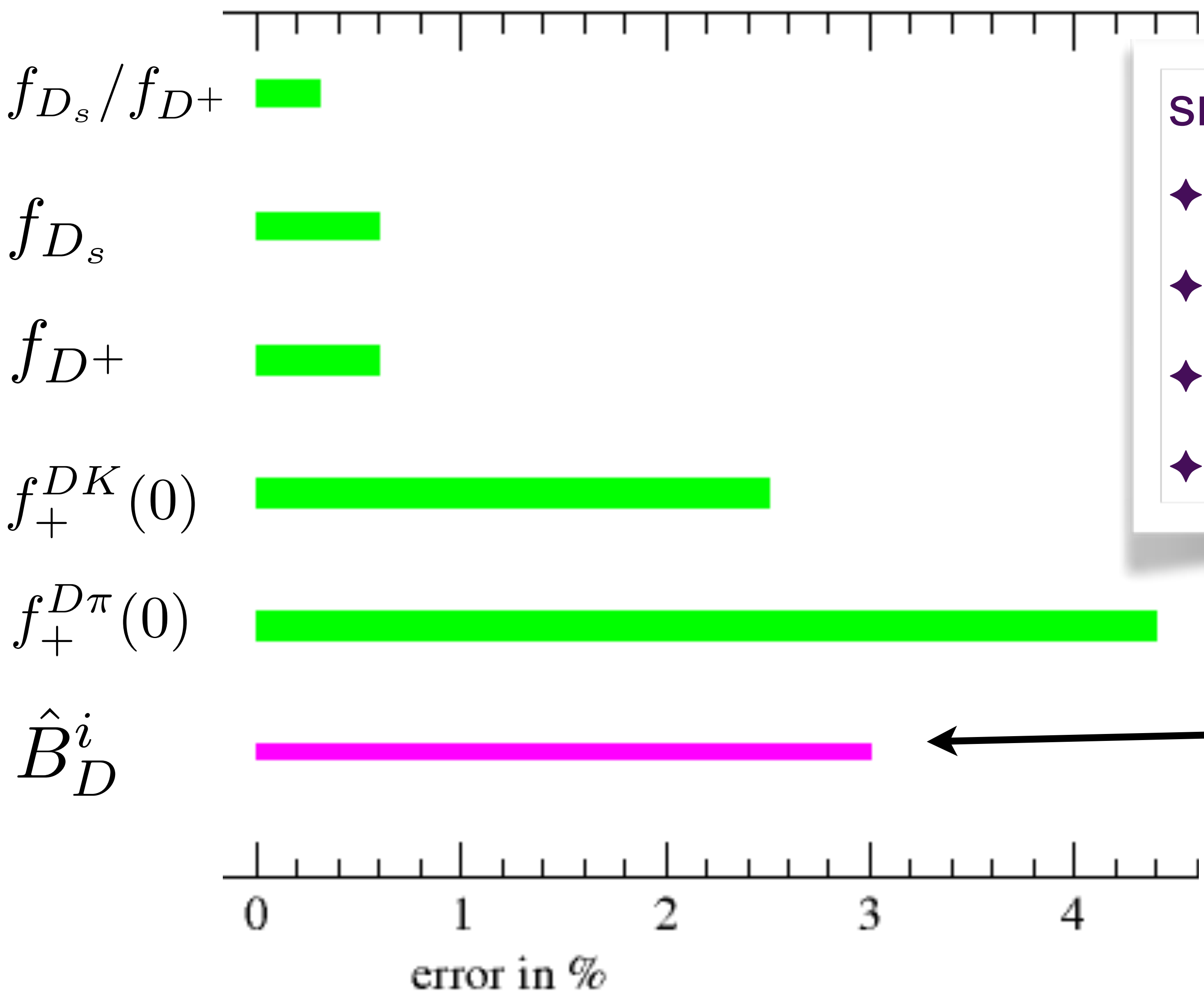
mid 2015





D meson summary

errors (in %) **(preliminary) FLAG-3 averages** + **new results**



small errors due to

- ✦ **physical light quark masses** ($f_{D(s)}$)
- ✦ improved charm-quark action
- ✦ ensembles with small lattice spacings
- ✦ PCAC or NPR

- **First results for D mixing bag parameters** (all five) of local operators by ETM (2013, 2014) $n_f = 2, 2{+}1{+}1$
- work in progress: FNAL/MILC (J. Chang thesis), see backup slides

Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

- ☆ low-lying hadron spectrum → quark masses
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - ✦ B mesons

→ CKM, BSM phenomenology

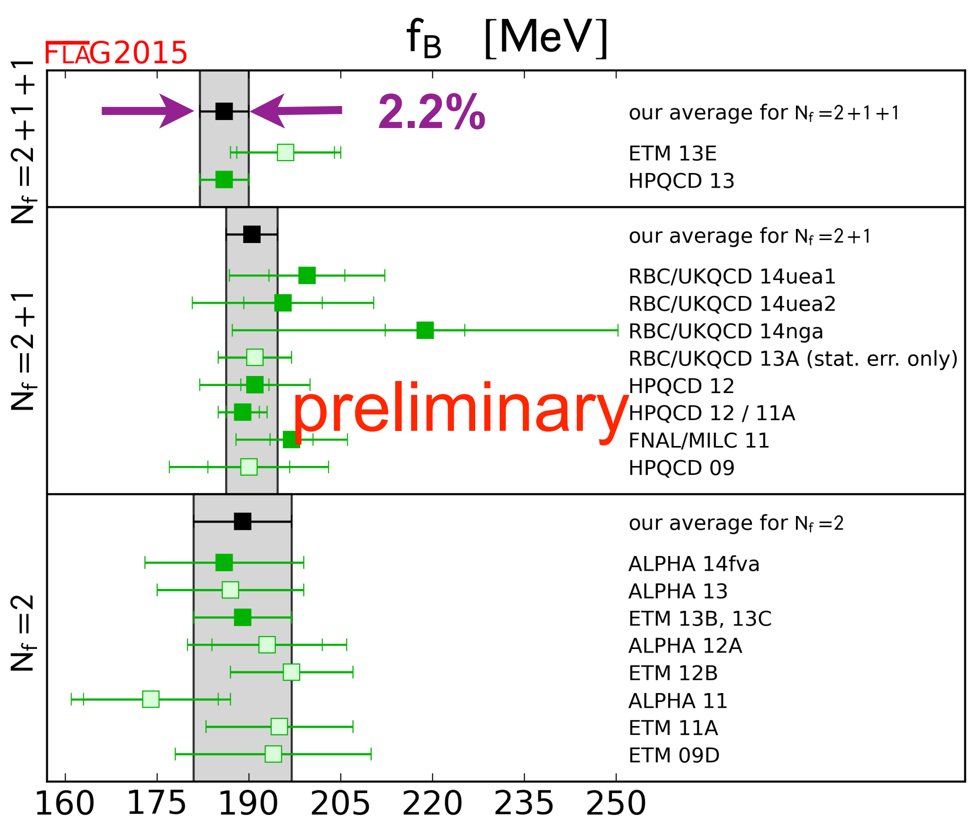
- ☆ high precision → including QED

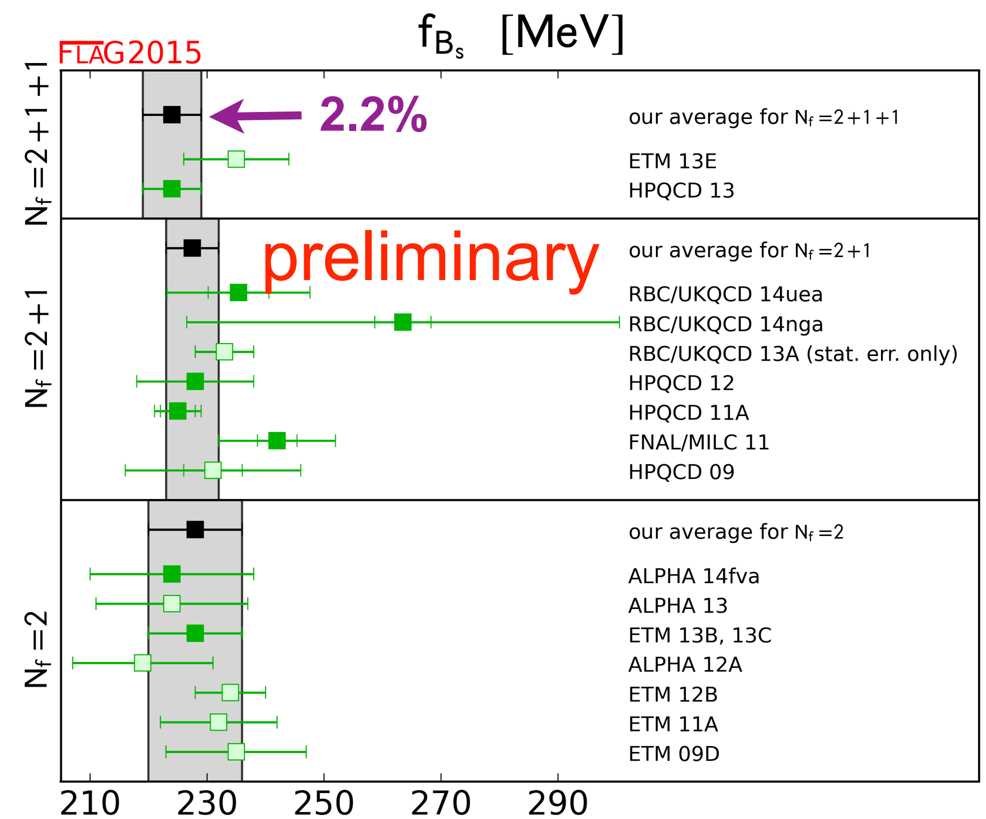
B meson summary

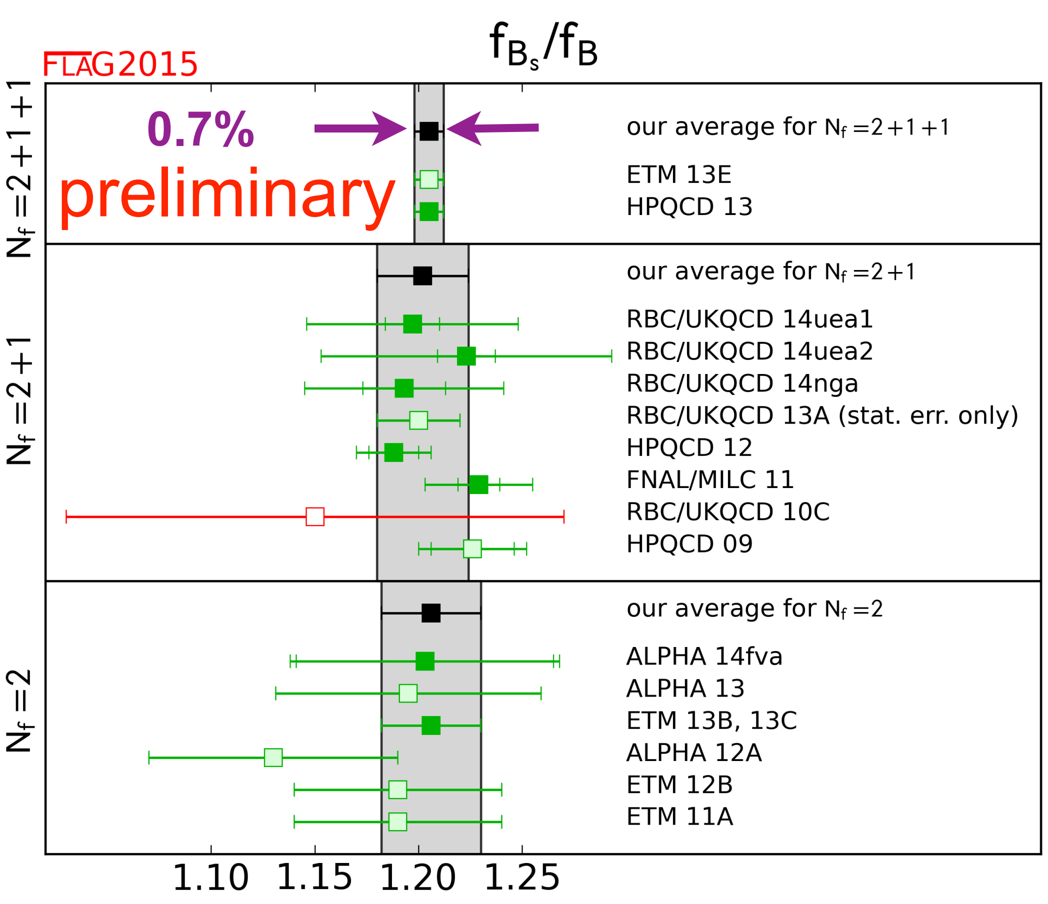
S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)

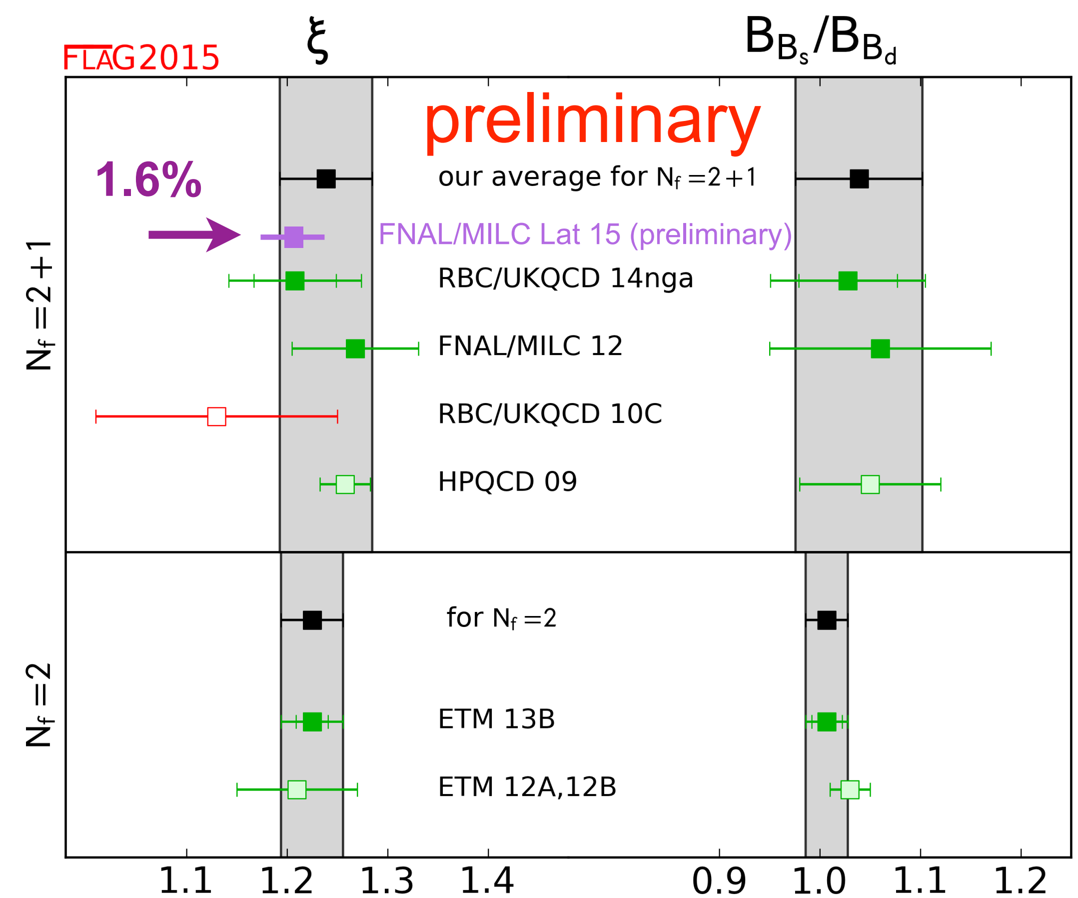
courtesy of
M. Della Morte
(HQ working group)

preliminary
status
mid 2015









Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

$$\frac{d\Gamma(B\to D^*\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2-1)^{1/2} |\mathcal{F}(\omega)|^2$$

$$\frac{d\Gamma(B\to D\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2-1)^{3/2} |\mathcal{G}(\omega)|^2$$

at zero recoil (HFAG 2014):

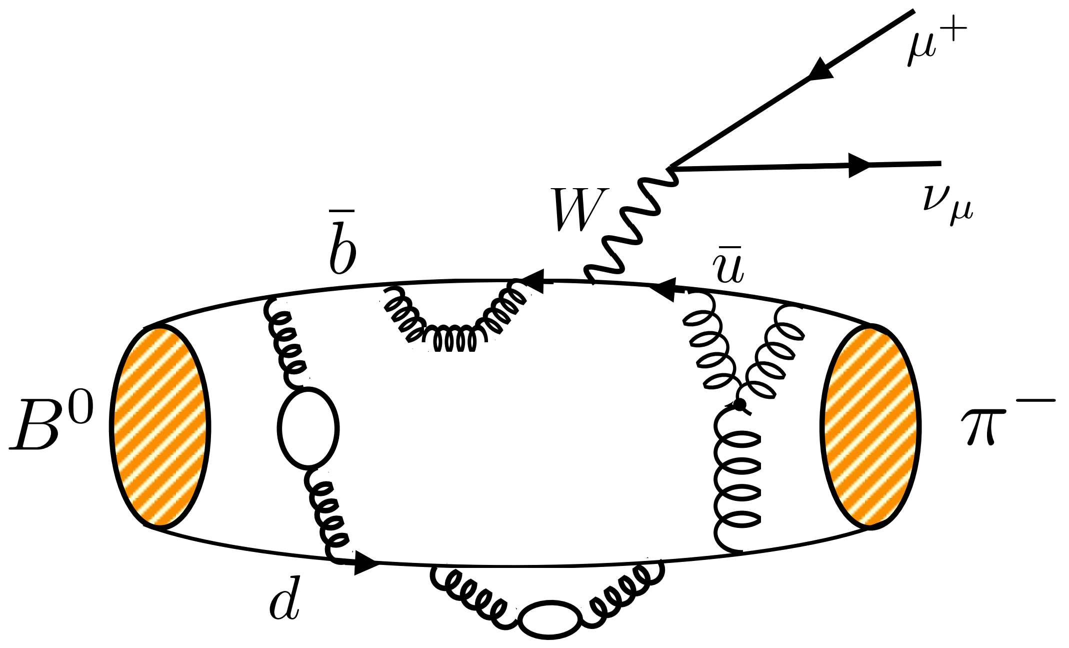
$$B \to D^*\ell\nu : \eta_{\rm EW}|V_{cb}|\mathcal{F}(1) = (35.81 \pm 0.11 \pm 0.44)\ 10^{-3}$$

$$B \to D\ell\nu : \ \eta_{\rm EW}|V_{cb}|\mathcal{G}(1) = (42.65 \pm 0.71 \pm 1.35)\ 10^{-3}$$

- need form-factors at non-zero recoil for shape comparison, R(D)

Semileptonic B-meson decay to light hadrons

Example: $B \to \pi \ell \nu$

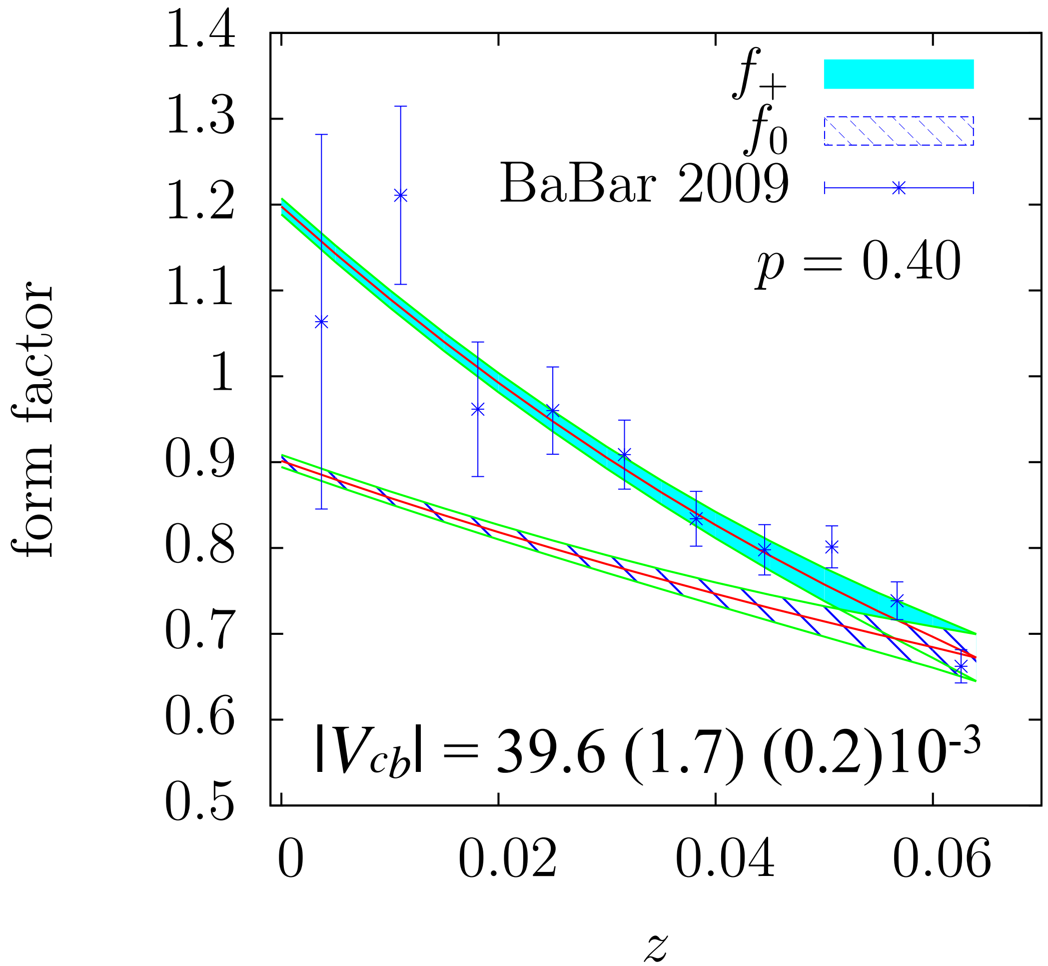


$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

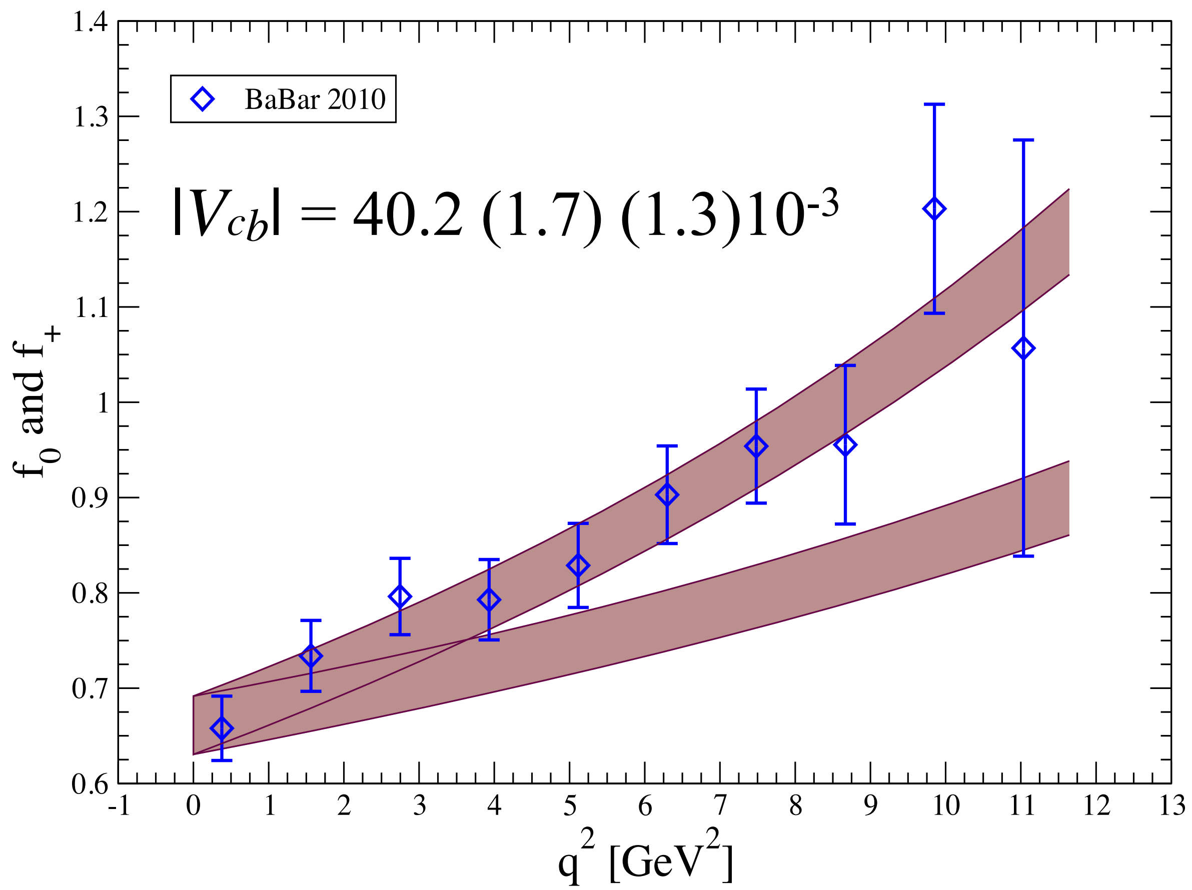
- ★ shape for semileptonic B decays:
  use z-expansion for model-independent parameterization of q^2 dependence (see back-up slide)
- ★ calculate all form factors, $f_+(q^2), f_0(q^2)$ (and $f_T(q^2)$ for the corresponding rare decay)
- ★ LQCD predictions of $B_s \to K \ell \nu$ form factors exist (HPQCD, RBC/UKQCD) and more are in progress (FNAL/MILC).

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

FNAL/MILC (arXiv:1503.07237, PRD 2015)



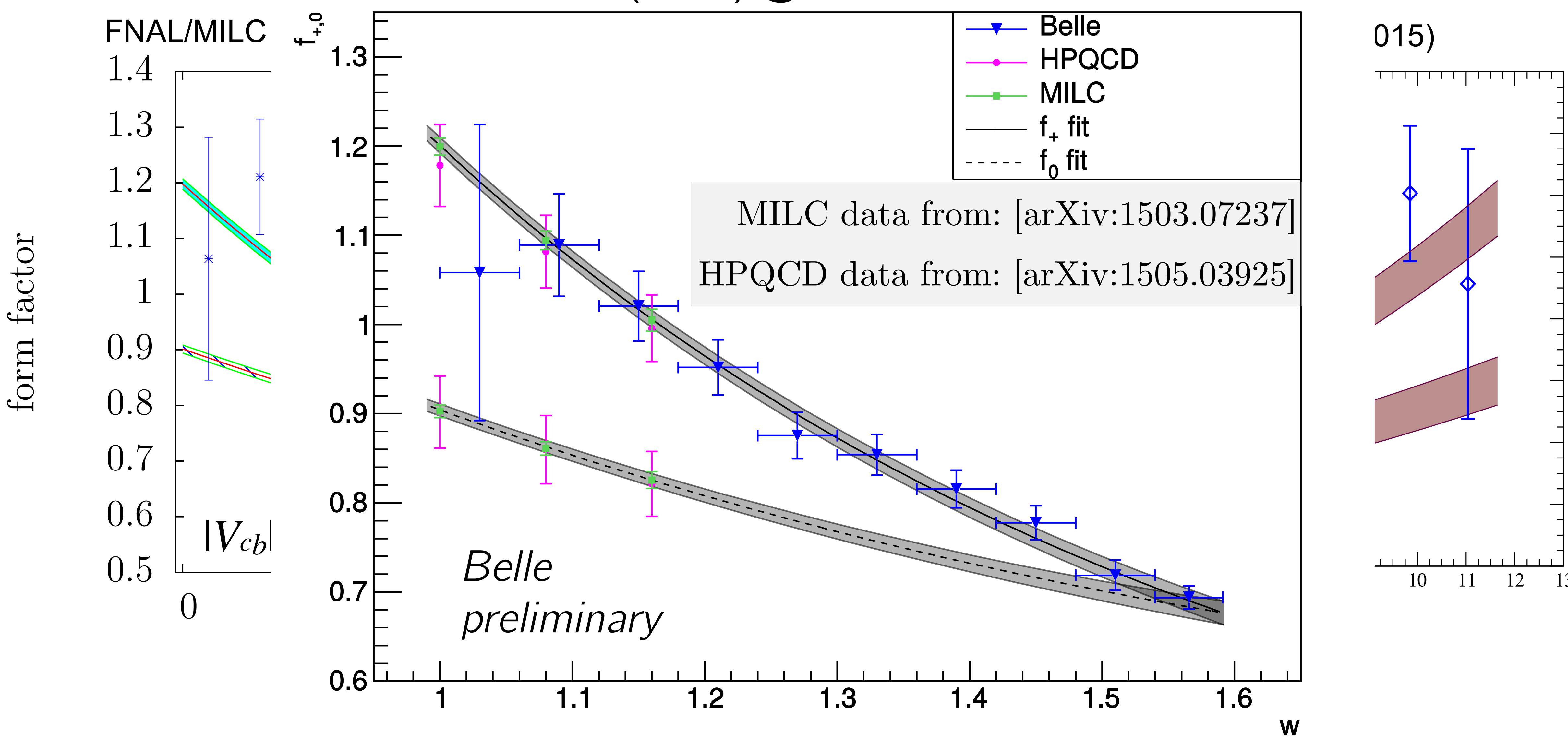
HPQCD (arXiv:1505.03925, PRD 2015)



- combined fit to LQCD form factors + BaBar data.
- LQCD form factor errors (~1.2%) smaller than experiment.

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

R. Glattauer (Belle) @ EPS 2015



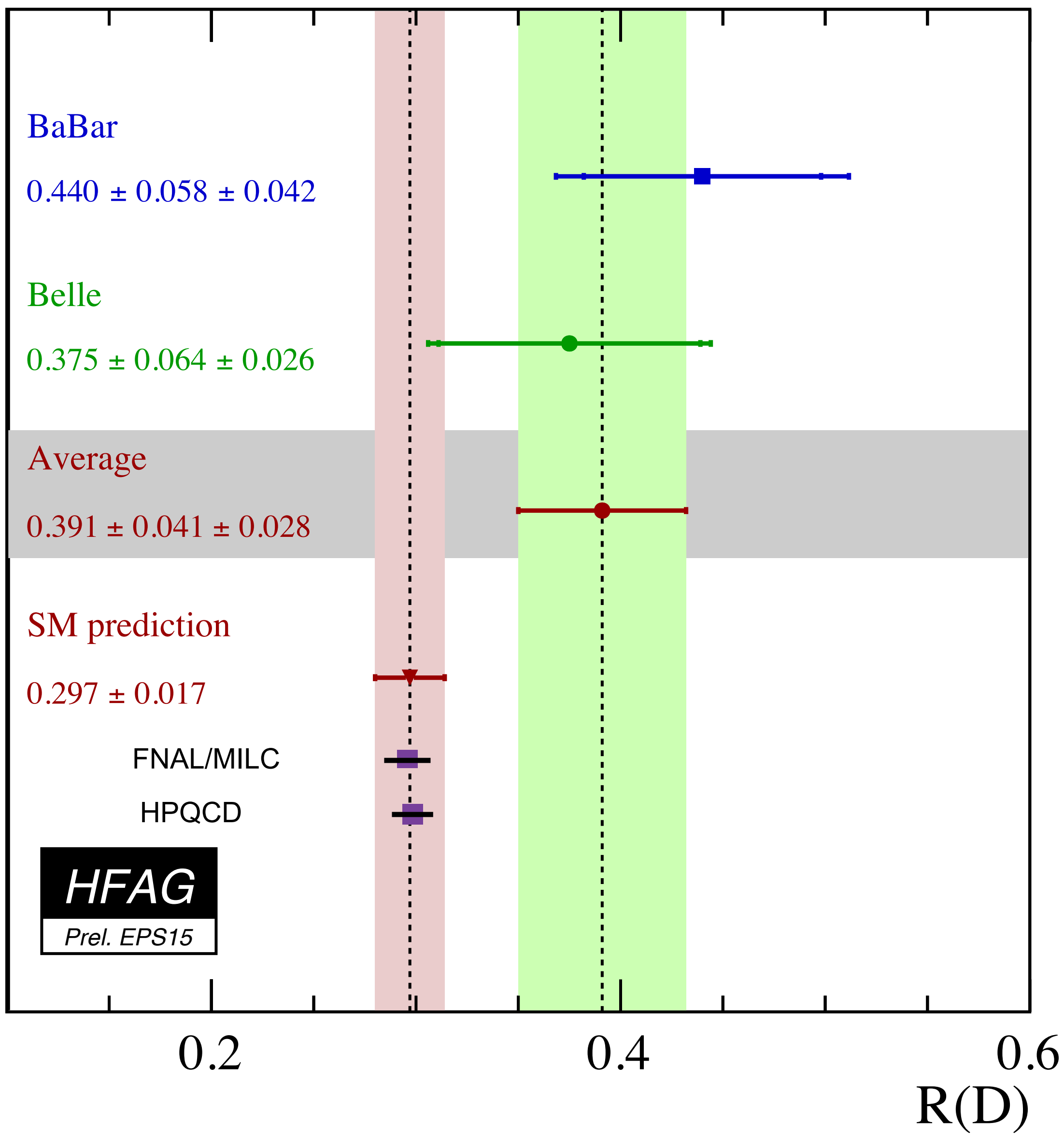
P. Gambino, global fit (Belle + BaBar + HPQCD + FNAL/MILC) @ EPS 2015:

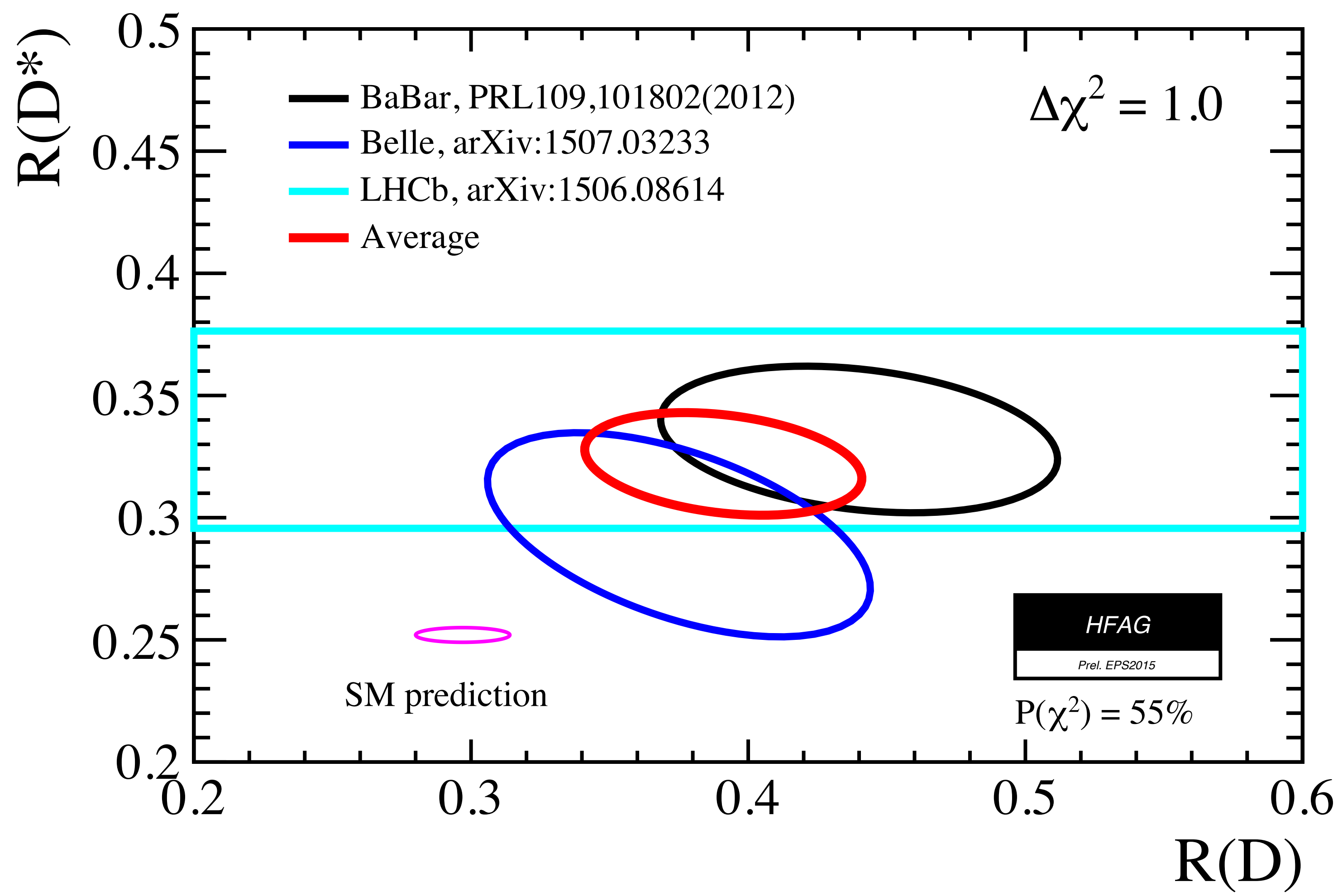
$$|V_{cb}| = 41.09\ (95)\ 10^{-3}$$

The ratio $R(D^{(*)})$

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$

HFAG average for EPS 2015

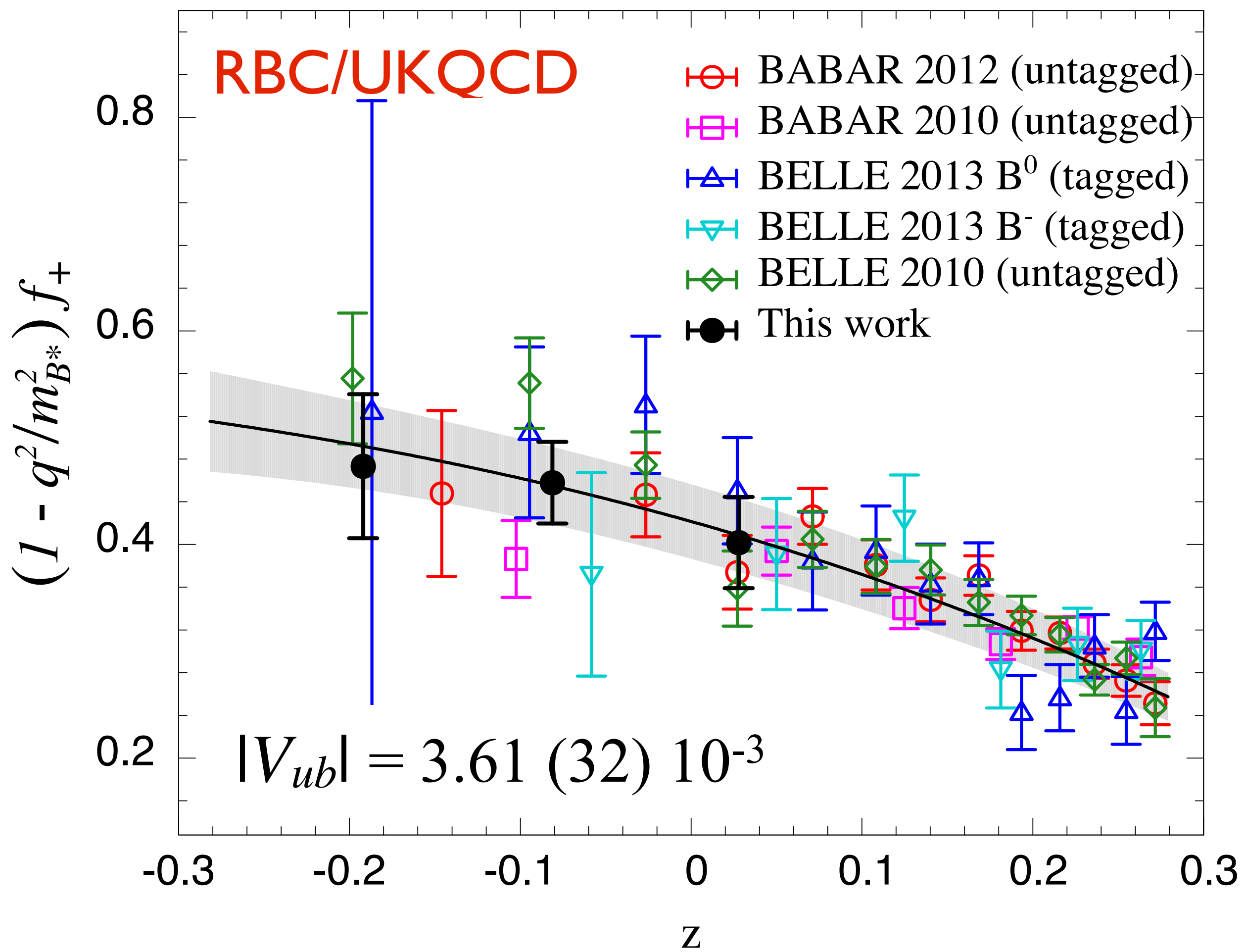




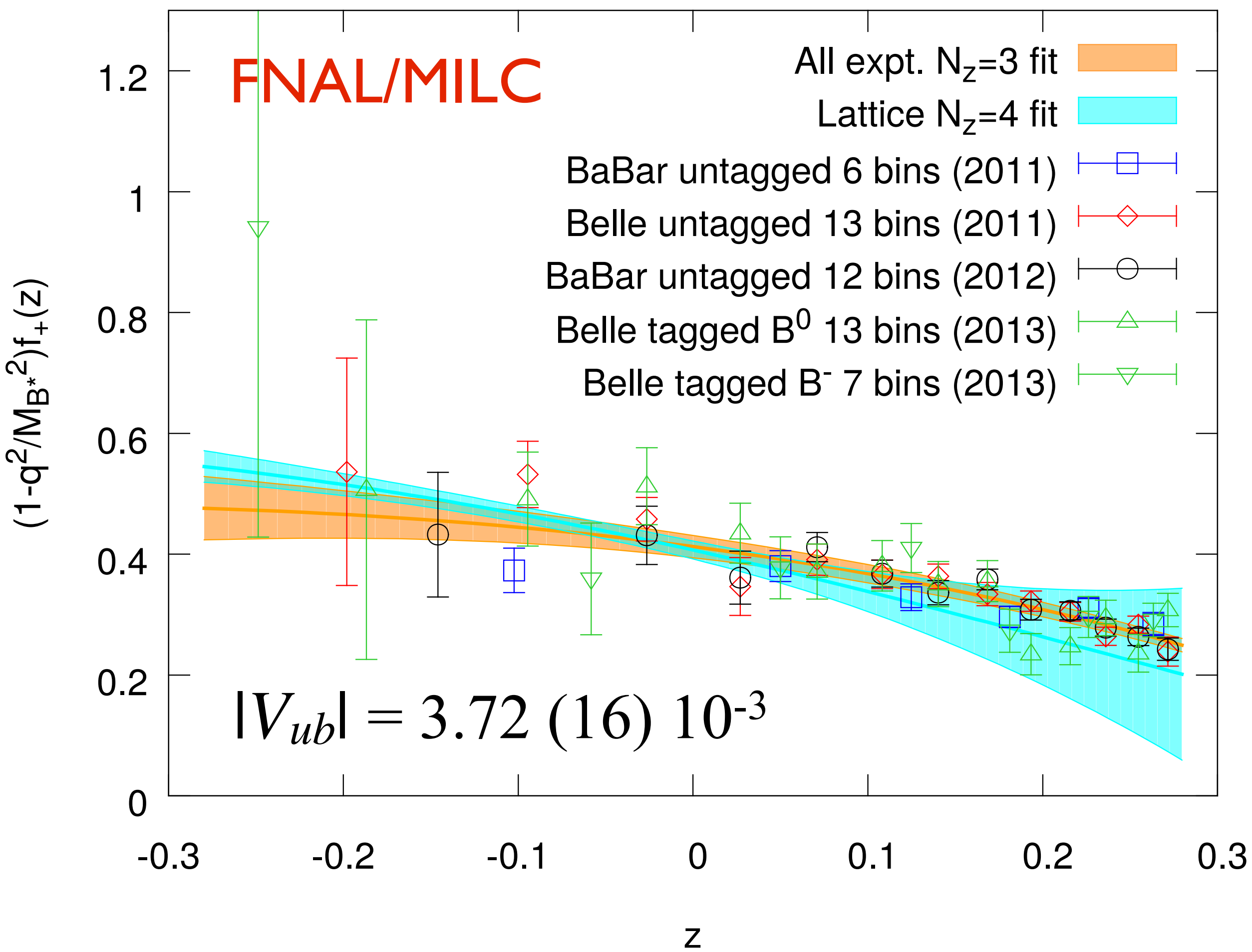
HFAG average: combined 3.9σ excess

Form factor for $B \to \pi \ell \nu$ & V_{ub}

RBC/UKQCD (arXiv:1501.05373, PRD 2015)



FNAL/MILC (arXiv:1503.07839, PRD 2015)



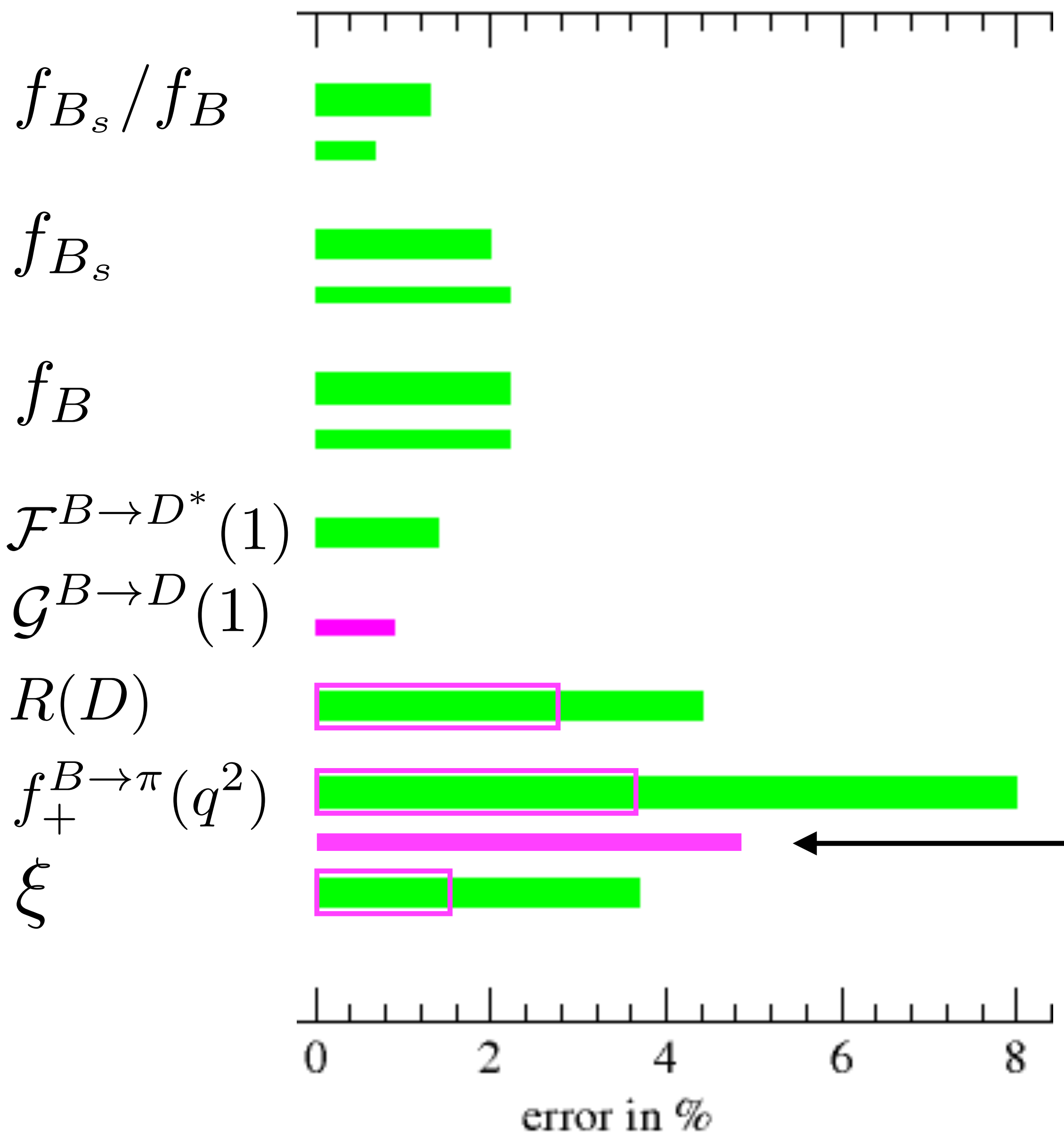
New: First determination of $|V_{ub}/V_{cb}|$ from **baryon** decay!

(Detmold et al, arXiv:1503.01421, PRD 2015) + LHCb (arXiv:1504.01568, Nature 2015)

$$R_{FF} = \frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15\mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to p \mu \nu)}{dq^2} dq^2}{\int_{7\mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to \Lambda_c \mu \nu)}{dq^2} dq^2} = 1.471 \pm 0.094 \pm 0.109$$

B meson summary

errors (in %) **(preliminary) FLAG-3 averages** + **new results**



New: form factors for $\Lambda_b \to p/\Lambda_b \to \Lambda_c$

(Detmold et al,arXiv:1503.01421, PRD 2015)

Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

- ☆ low-lying hadron spectrum → quark masses
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - B mesons

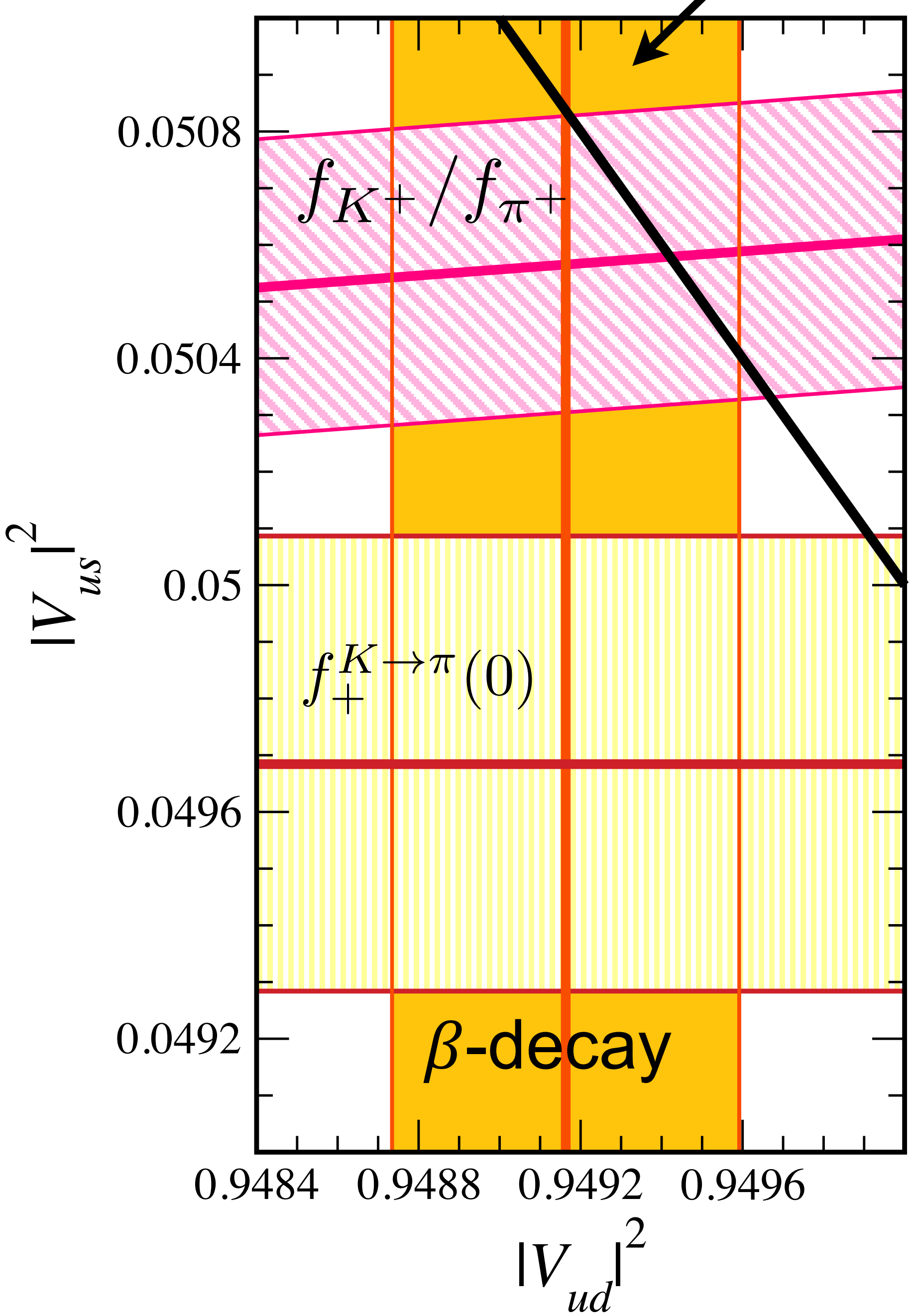
→ CKM, BSM phenomenology

- ☆ high precision → including QED

Implications for the 1st row of the CKM Matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$



Constraining $|V_{us}|$ using FLAG-3 averages for K_{l3} form factor or for f_{K^+}/f_{π^+}

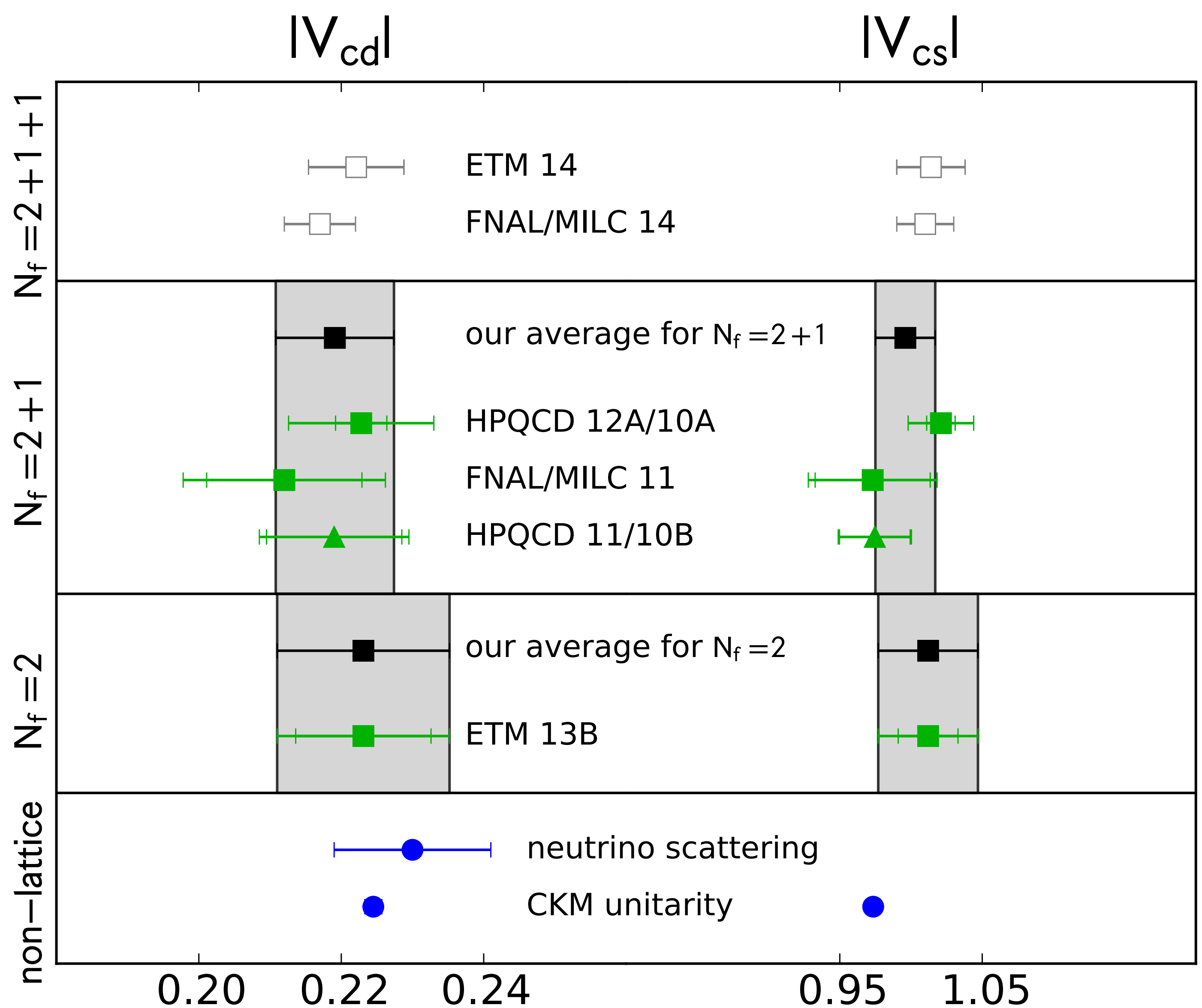
The uncertainty on $|V_{us}|^2$ is slightly smaller than the uncertainty on $|V_{ud}|^2$

Time to revisit the uncertainty on $|V_{ud}|$?

Slight tension between K_{l2} and K_{l3} and for K_{l3} with unitarity prediction.

Implications for the 2nd row of the CKM Matrix

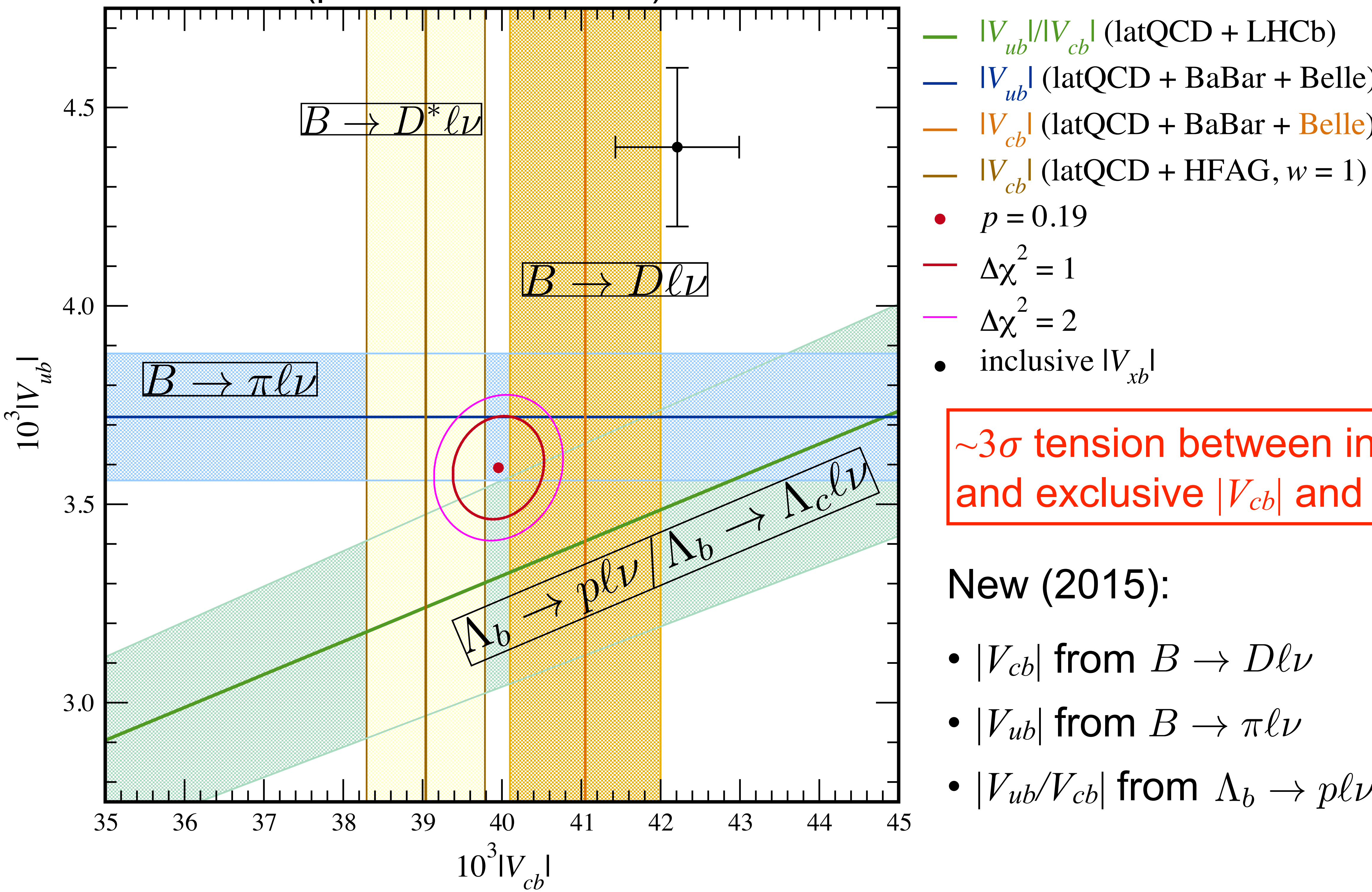
C. Pena review @ Lattice 2015



Slight tension for $|V_{cs}|$ from leptonic decay with CKM unitarity

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



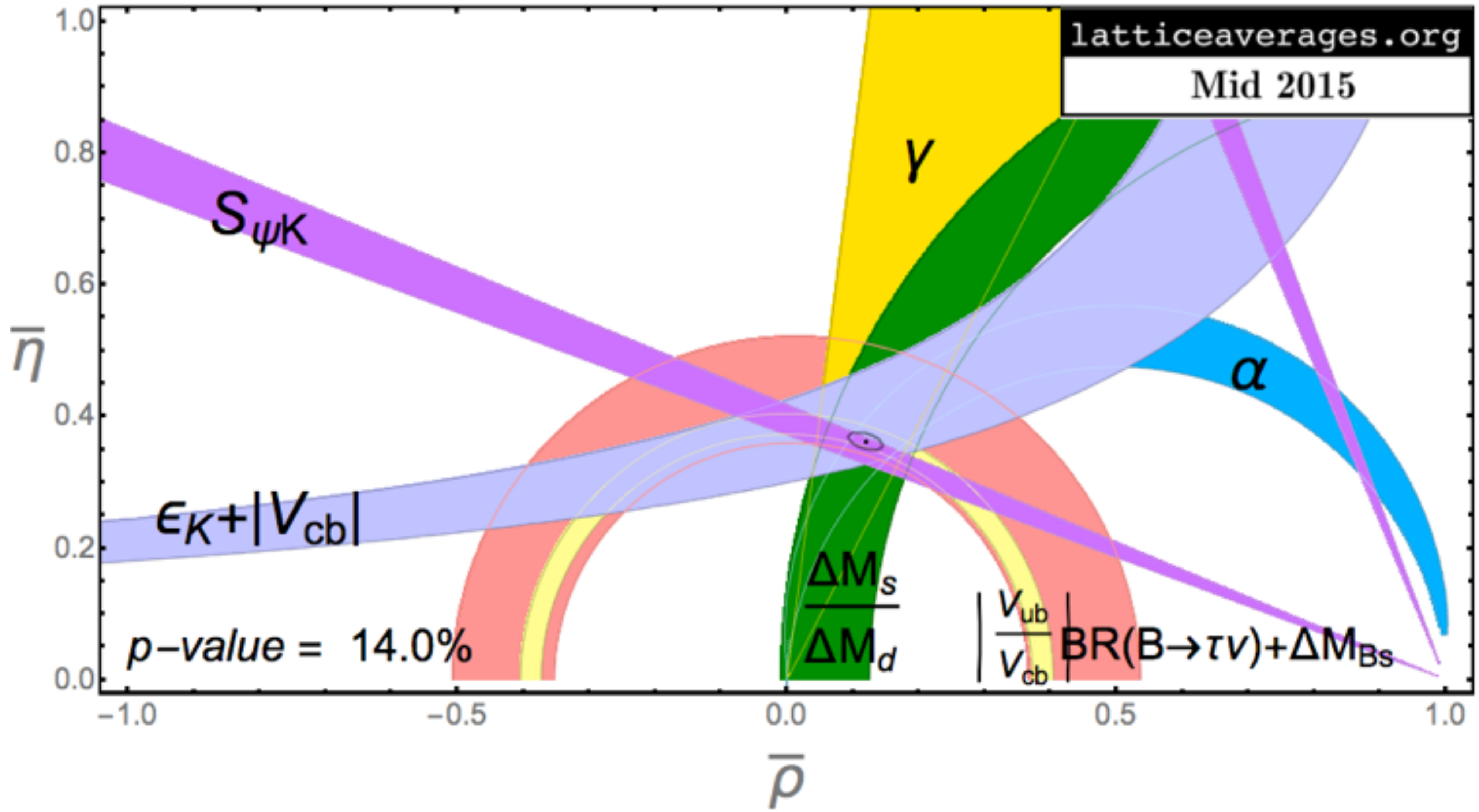
$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

New (2015):

- $|V_{cb}|$ from $B \to D\ell\nu$
- $|V_{ub}|$ from $B \to \pi\ell\nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\ell\nu/\Lambda_b \to \Lambda_c\ell\nu$

UT analysis

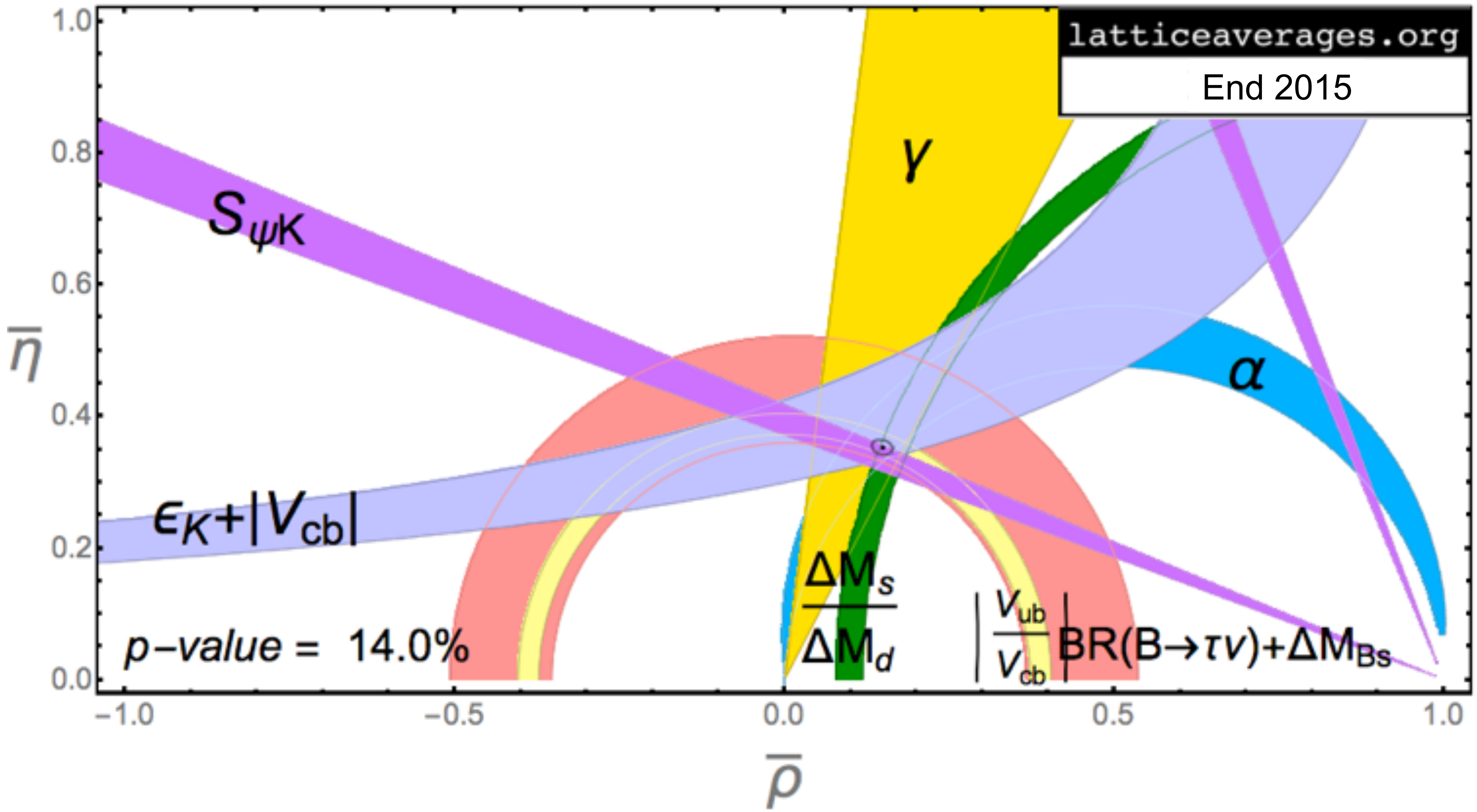
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



Exclusive $|V_{cb}|$, $|V_{ub}|$ (Kronfeld average)

UT analysis

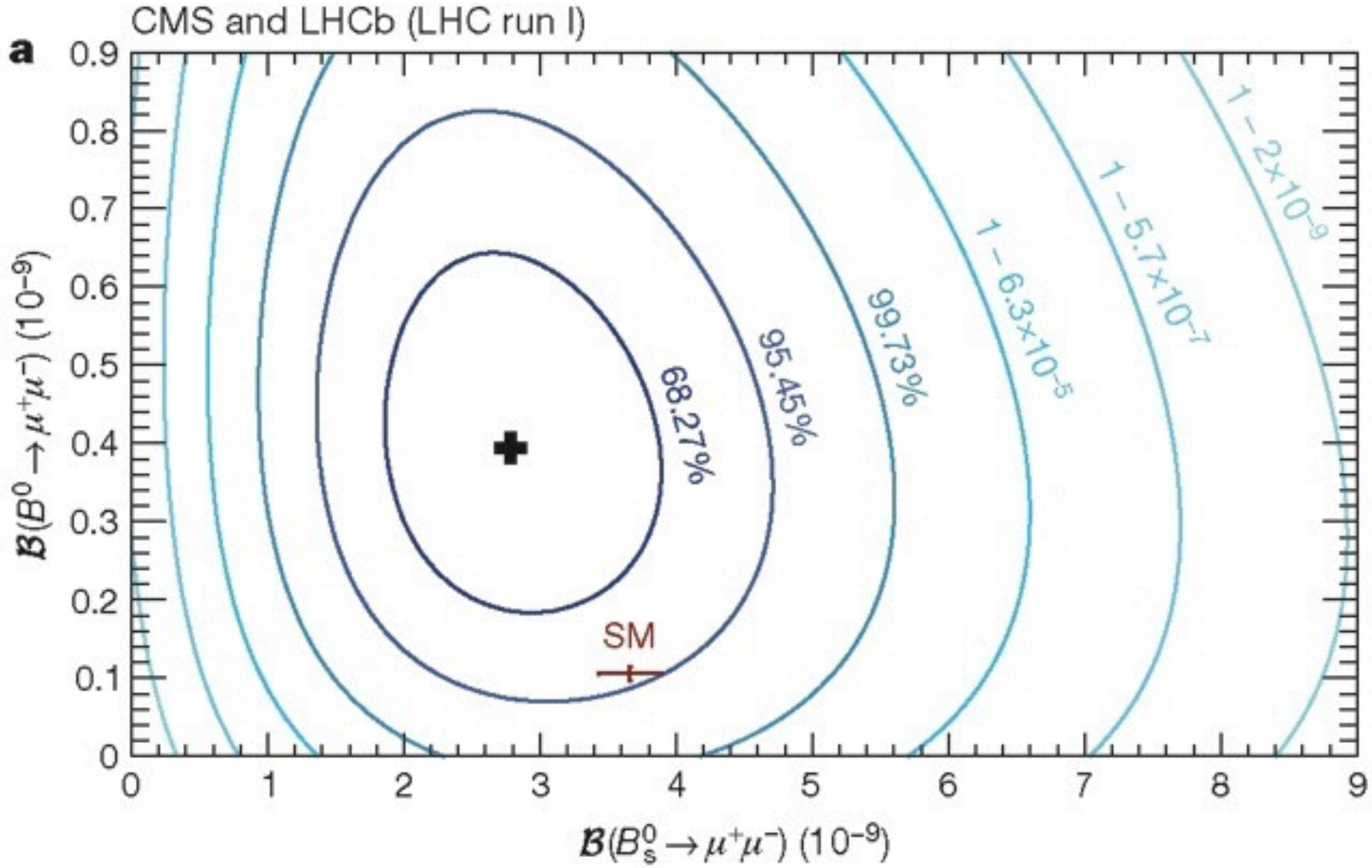
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



Exclusive $|V_{cb}|$, $|V_{ub}|$ (Kronfeld average)
+ preliminary FNAL/MILC Lattice 15 ξ

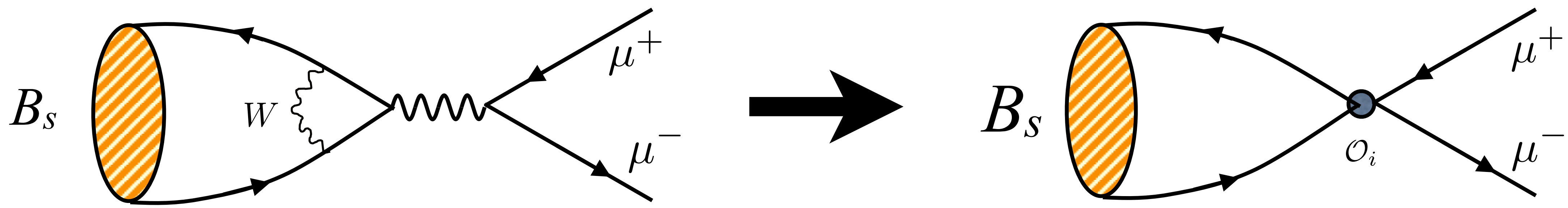
BSM phenomenology $B_s \to \mu^+\mu^-$

CMS+LHCb combined (Nature 2015)

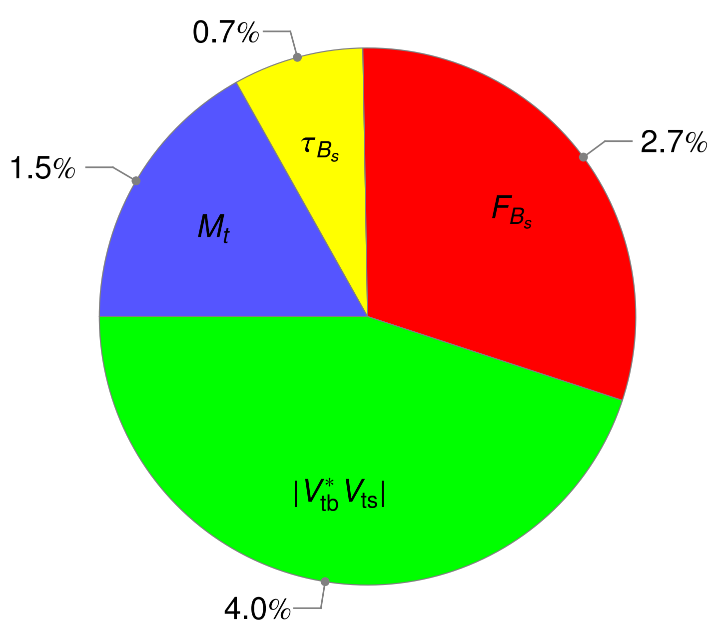


SM predictions depend on $f_{B(s)}$ or $\hat{B}_{B_s}$

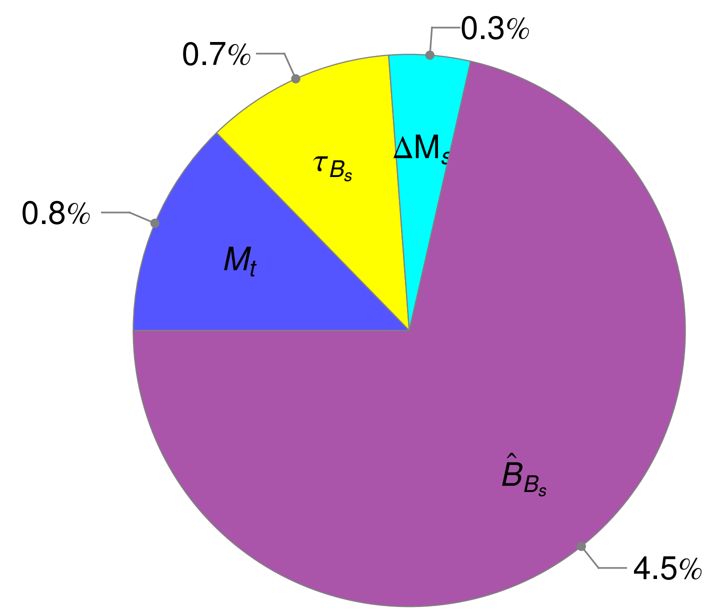
BSM phenomenology $B_s \to \mu^+ \mu^-$



Standard Model prediction: Buras, et al (arXiv:1303.3820, JHEP 2013), Bobeth, et al (arXiv:1311.0903, PRL 2014)

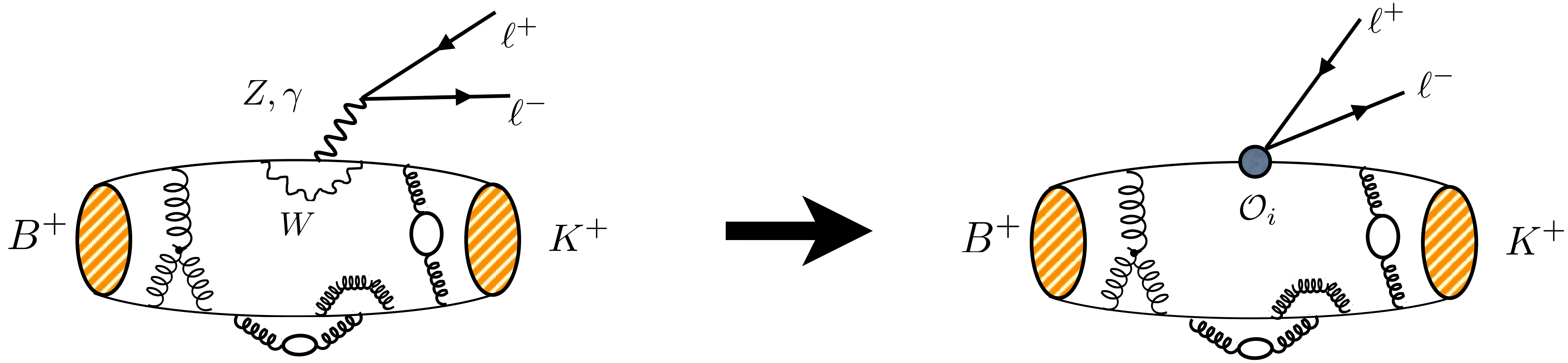


uses f_{B_s} from HPQCD 13



uses $\hat{B}_{B_s}$ from HPQCD 09

Form factors for $B \to K, \pi\ \ell^+\ell^-$



$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i \left(C_i O_i + C_i' O_i' \right)$$

Need 3 form factors: $f_{+,0,T}(q^2)$ ⟵

- low recoil (high q^2) OPE
- high recoil (low q^2) SCET
- compare theory with exp.

HPQCD for $B \to K$
(arXiv:1306.0434, 1306.2384, PRL 2013)

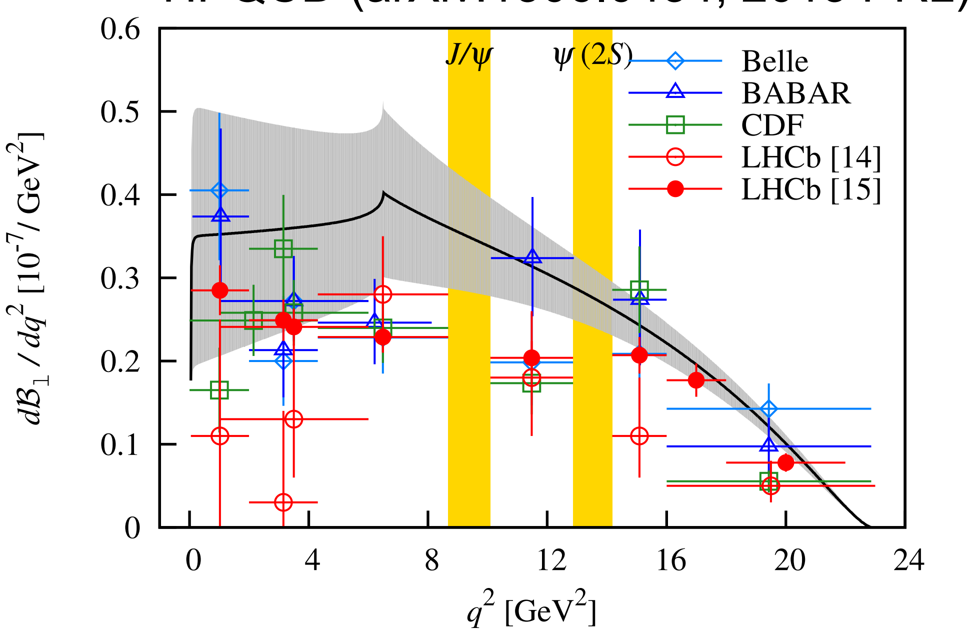
FNAL/MILC for $B \to K, B \to \pi$
(arXiv:1509.06235, 1507.01618, PRL 2015)

also:
Cambridge group for $B \to K^*, B_s \to \phi$
(arXiv:1310.3722, 1310.3887, PRL 2014)

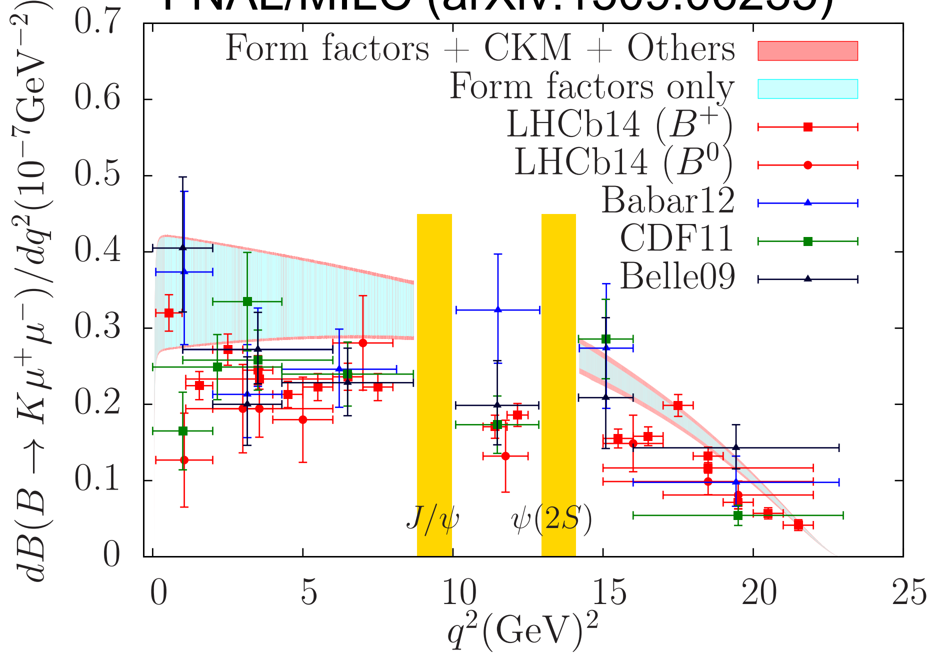
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. Theory

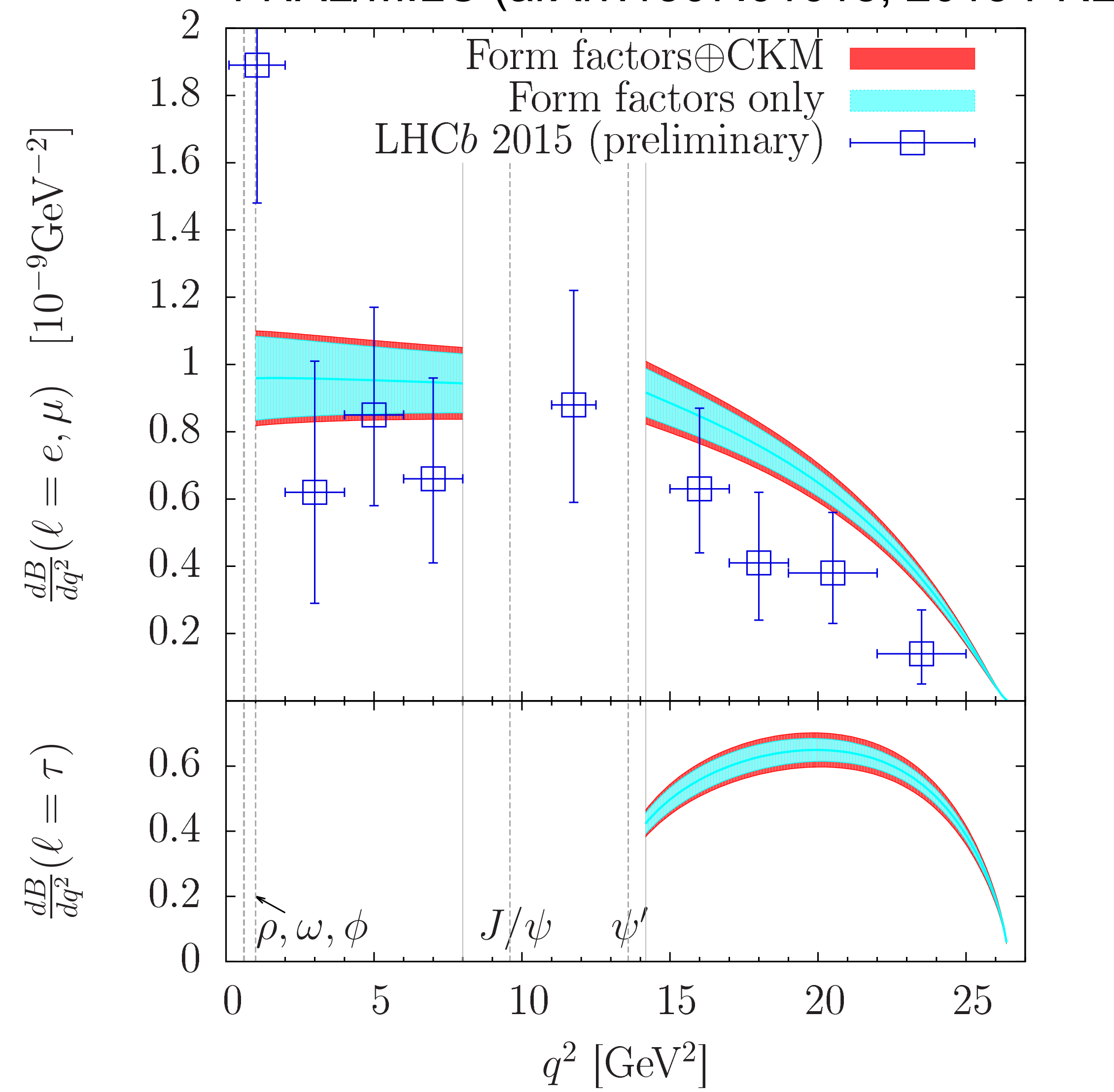
HPQCD (arXiv:1306.0434, 2013 PRL)



FNAL/MILC (arXiv:1509.06235)



FNAL/MILC (arXiv:1507.01618, 2015 PRL)



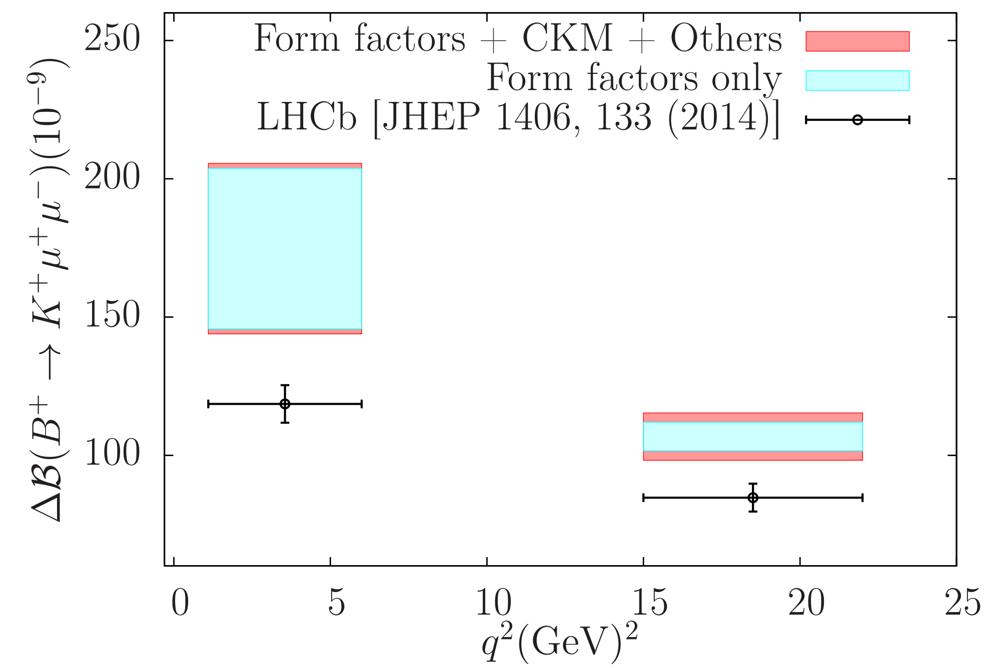
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. theory

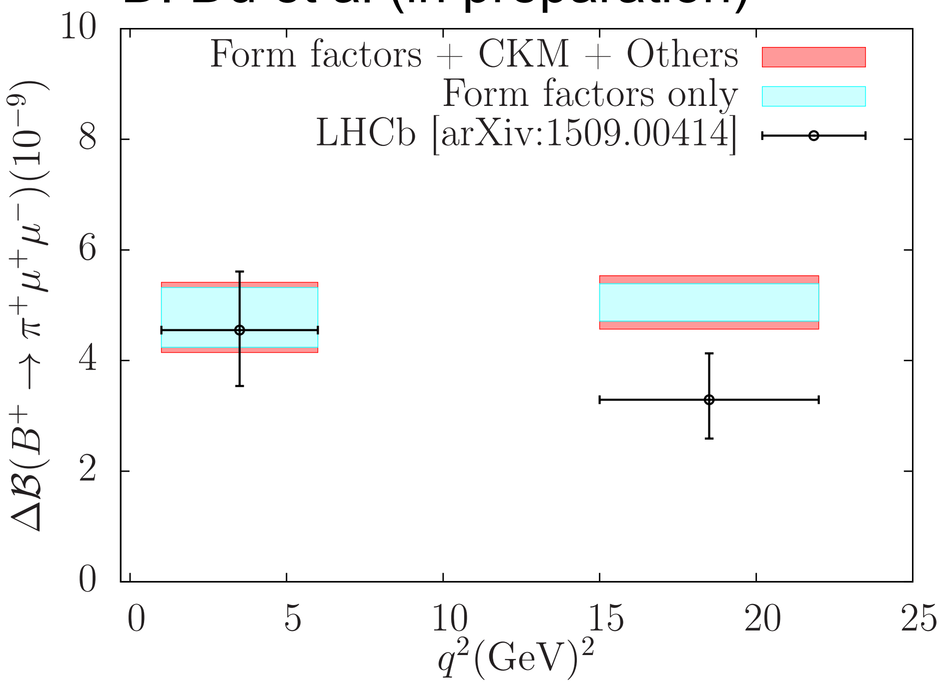
- LHCb data + FNAL/MILC form factors (arXiv:1509.00414,1403.8044, JHEP 2014)
- focus on large bins above and below charmonium resonances
- theory error commensurate with experiment
- yields $\sim$1-2σ tensions
- $\Rightarrow$ determine $|V_{td}/V_{ts}|, |V_{td}|, |V_{ts}|$

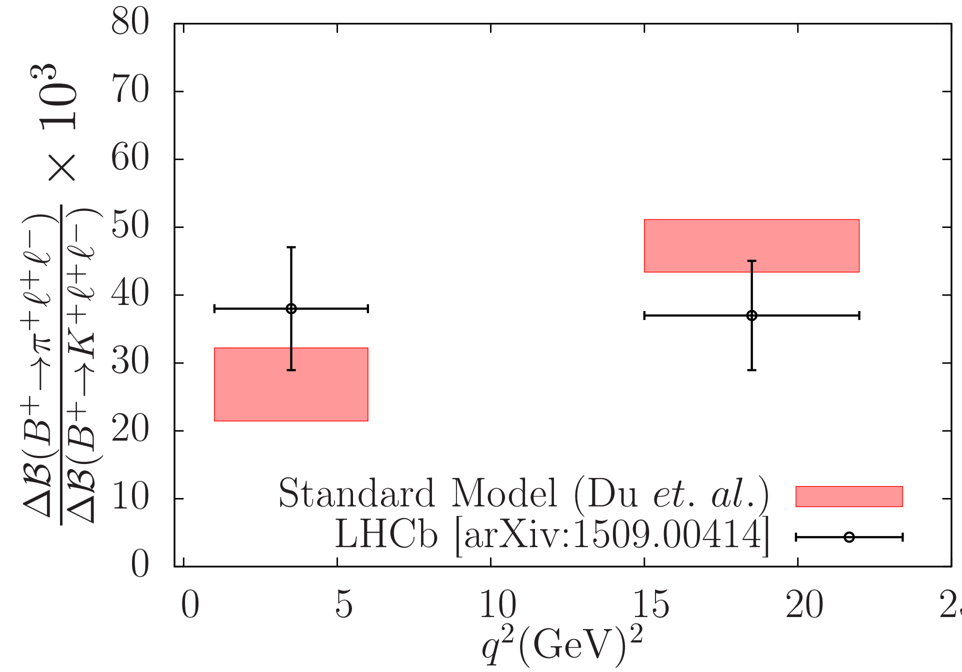
  or constrain Wilson coefficients

D. Du et al (in preparation)



D. Du et al (in preparation)





Summary

- simple quantities:

  kaons: ~**0.2-0.3%** for SU(3) breaking ratios

  ~ **1%** for other quantities

  D,D_s-mesons: ~ **0.3%** for SU(3) breaking ratio f_{Ds}/f_D

  ~ **0.6%** for decay constants

  ~ **3-5%** for other quantities

  B,B_s-mesons: ~ **0.7%** for SU(3) breaking ratio f_{Bs}/f_B

  ~ **2%** for decay constants, **1.4%** for $B \to D^*$

  $\lesssim$ **5%** for other quantities

  → precision will continue to improve

- for B: leverage high precision D results with B/D ratios

- LQCD calculations of simple quantities have been (are being) extended to include rare decays, BSM mixing parameters, baryon decays, ...

Conclusions & Outlook

- Lattice QCD is needed to quantify nonperturbative QCD effects.

- Precise LQCD results now exist for a few quantities with errors that are commensurate with experimental uncertainties.

- Better precision is still needed in order to maximize the impact of precision frontier experiments.
  ⇒ constrain/discover/understand New Physics

- Recent breakthrough in LQCD: availability of ensembles with physical light quark masses. Previously dominant systematic error now subdominant (smaller than statistical errors).

- Sub-percent precision: we need to do include QED effects (has already started).

- LQCD calculations are being performed for many other quantities, including the study of resonances, $K \to \pi\pi$, hadronic contributions to muon g-2, ...

Farah Willenbrock

Backup slides

Simple quantities in LQCD

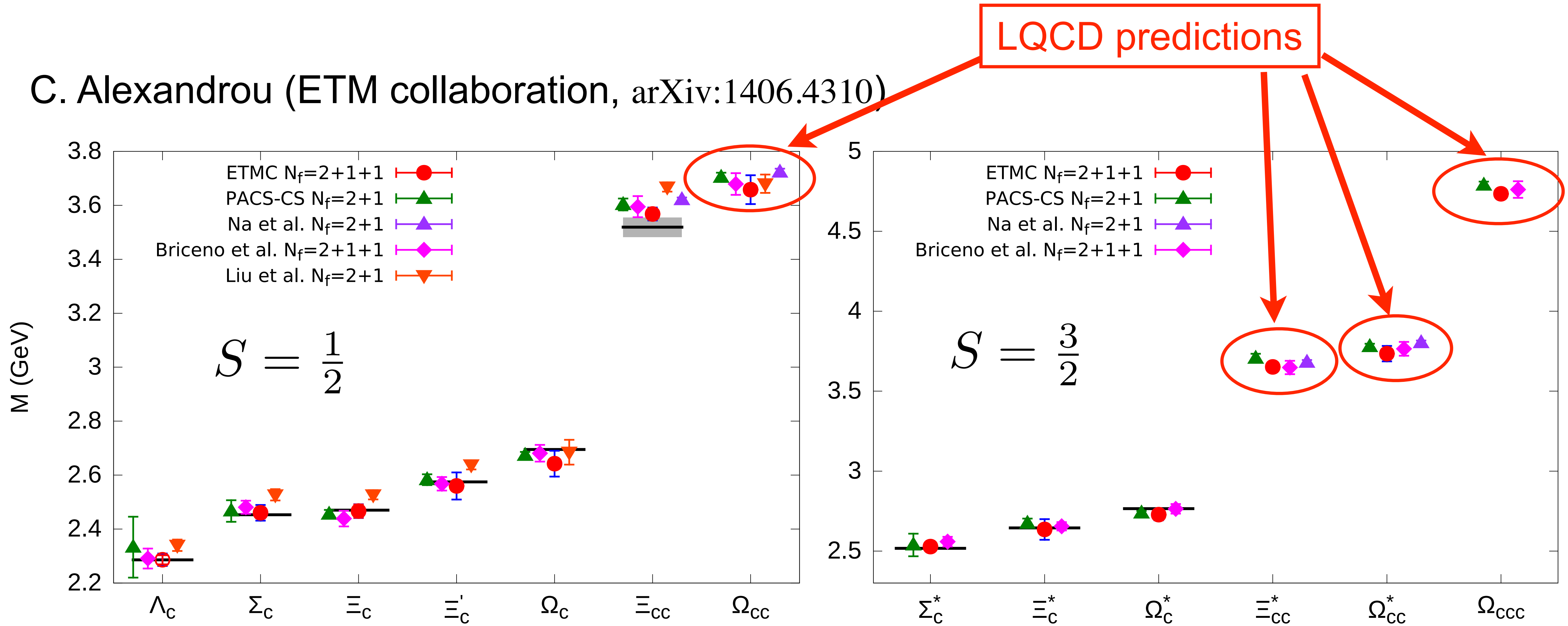
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \mu\nu & K \to \pi \ell\nu \;\; K \to \mu\nu & B \to \pi \ell\nu,\; B_s \to K \ell\nu \;\; \Lambda_b \to p \ell\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \pi \ell\nu \;\; D \to \ell\nu & D \to K \ell\nu \;\; D_s \to \ell\nu & B_{(s)} \to D_{(s)}, D^*_{(s)} \ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ B^0 - \overline{B^0} & B_s^0 - \overline{B_s^0} & \end{pmatrix}$$

$$(\rho, \eta) \quad K^0 - \overline{K^0}$$

Low-lying hadron spectrum

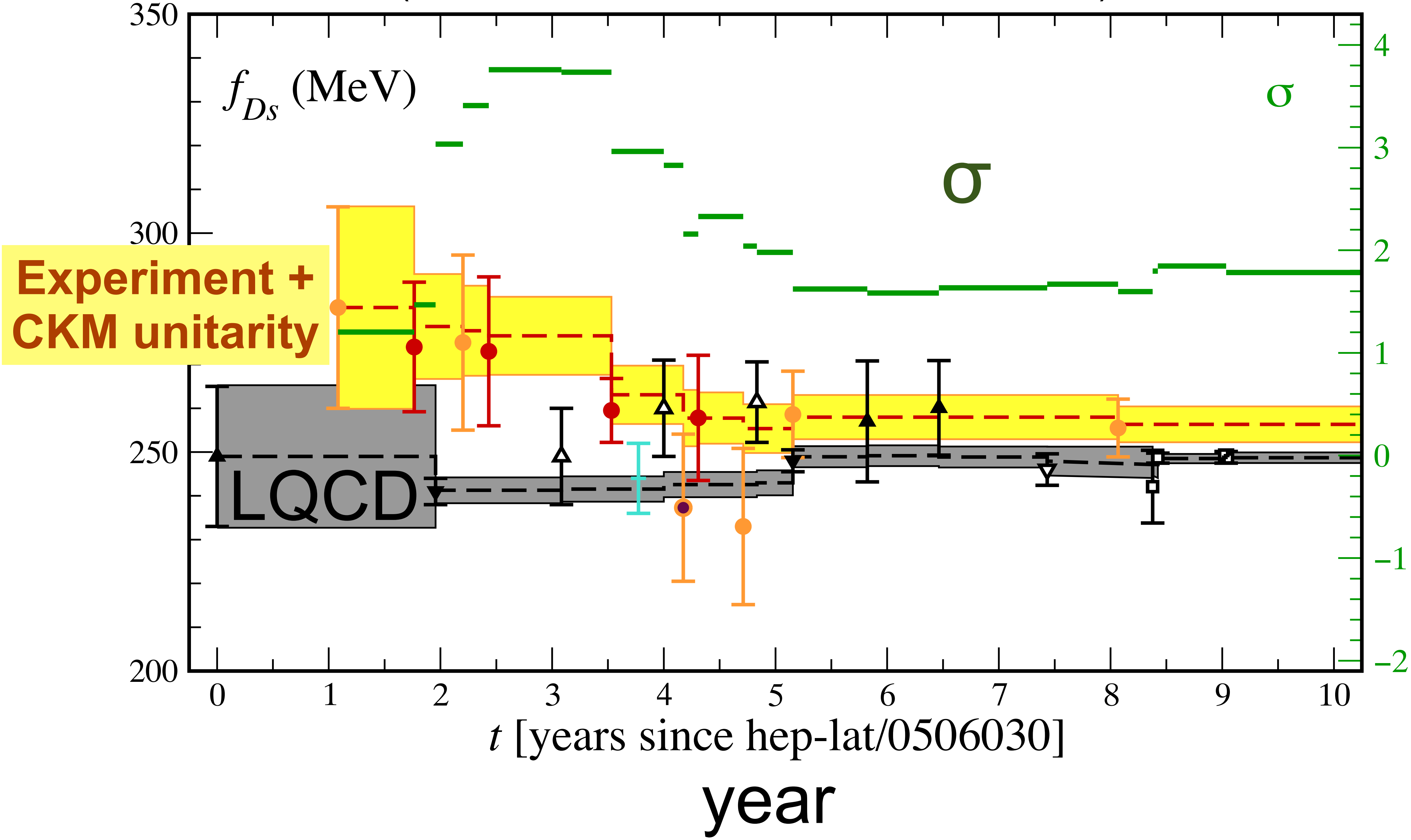
new results for the charmed baryon spectrum:

C. Alexandrou (ETM collaboration, arXiv:1406.4310)



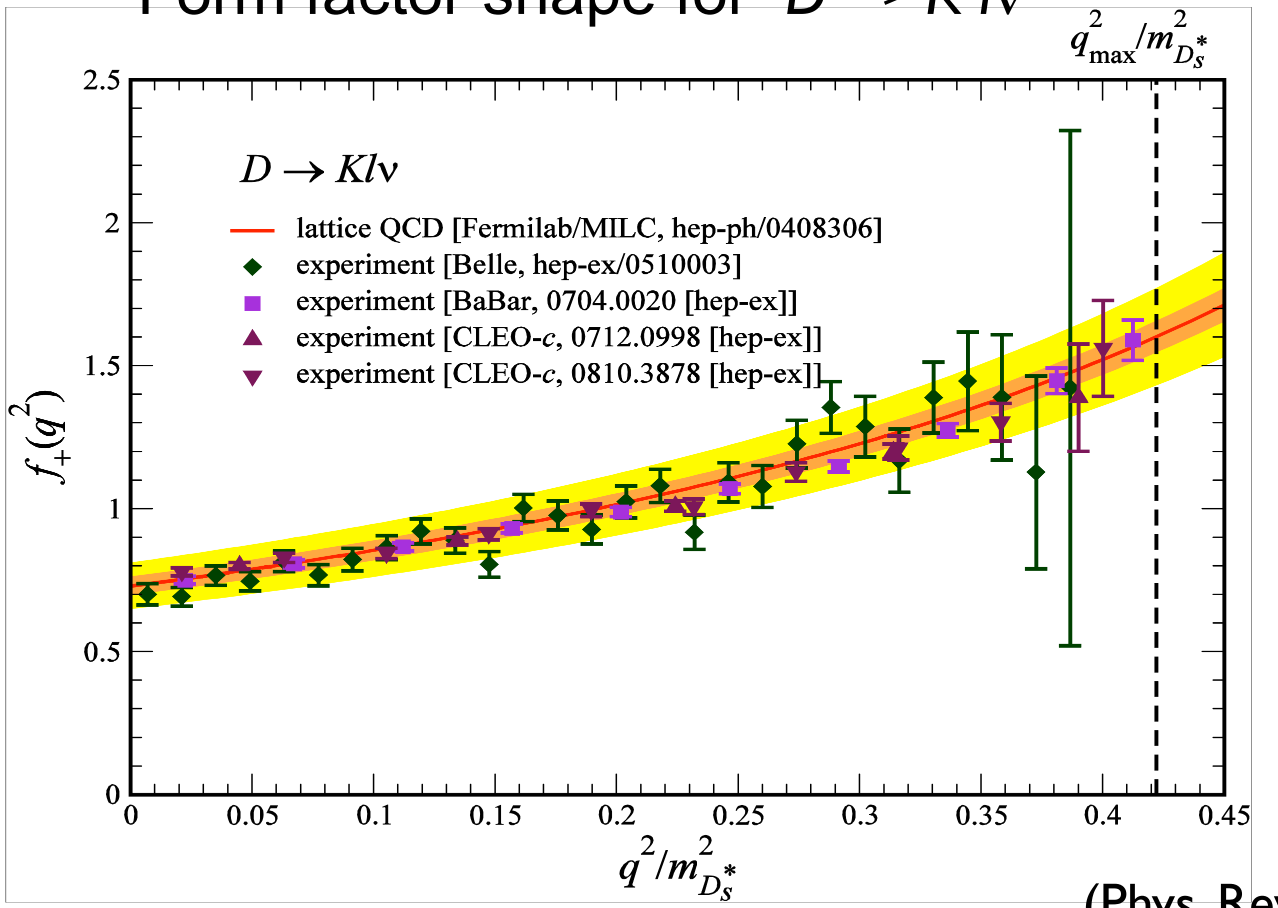
LQCD Achievements: f_{Ds} time history

A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204)

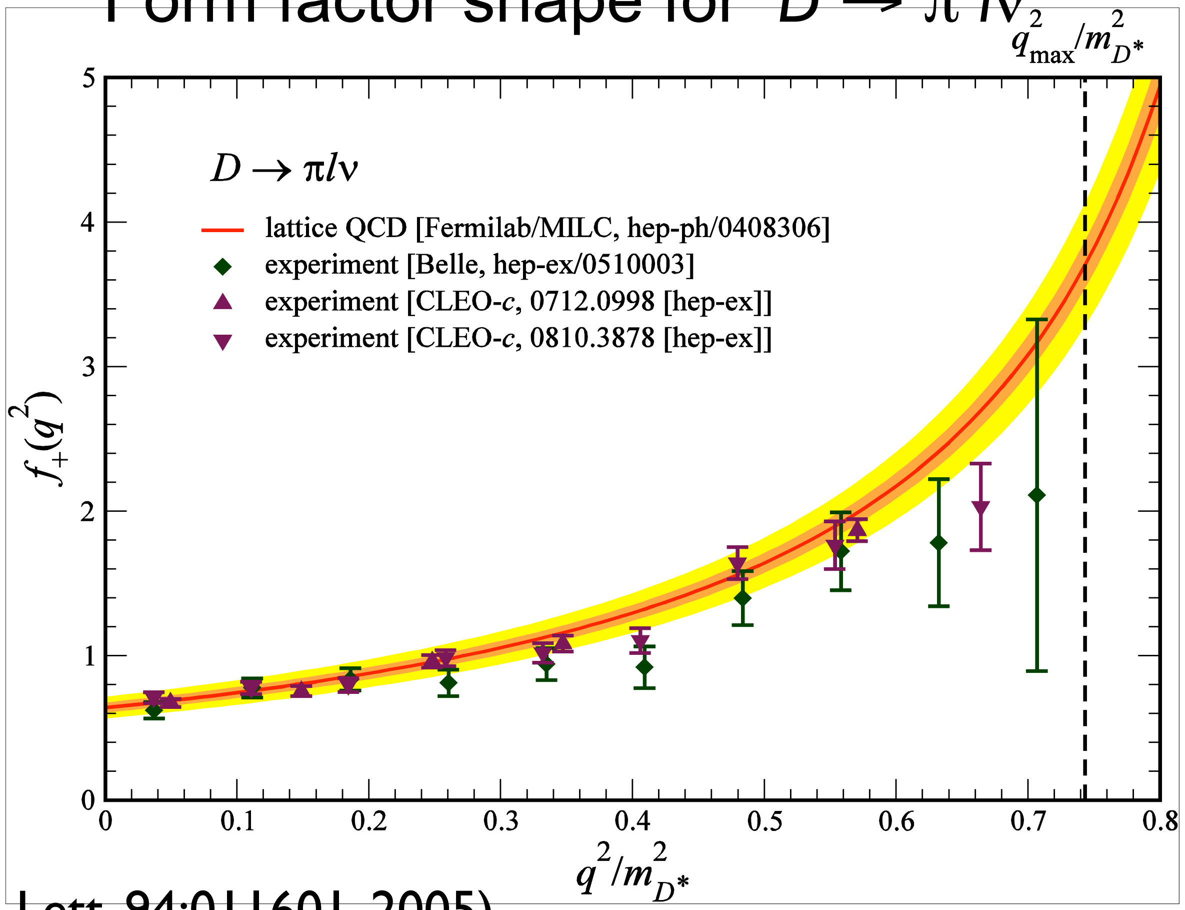


LQCD Achievements: Predictions

Form factor shape for $D \to K\, l\nu$



Form factor shape for $D \to \pi\, l\nu$



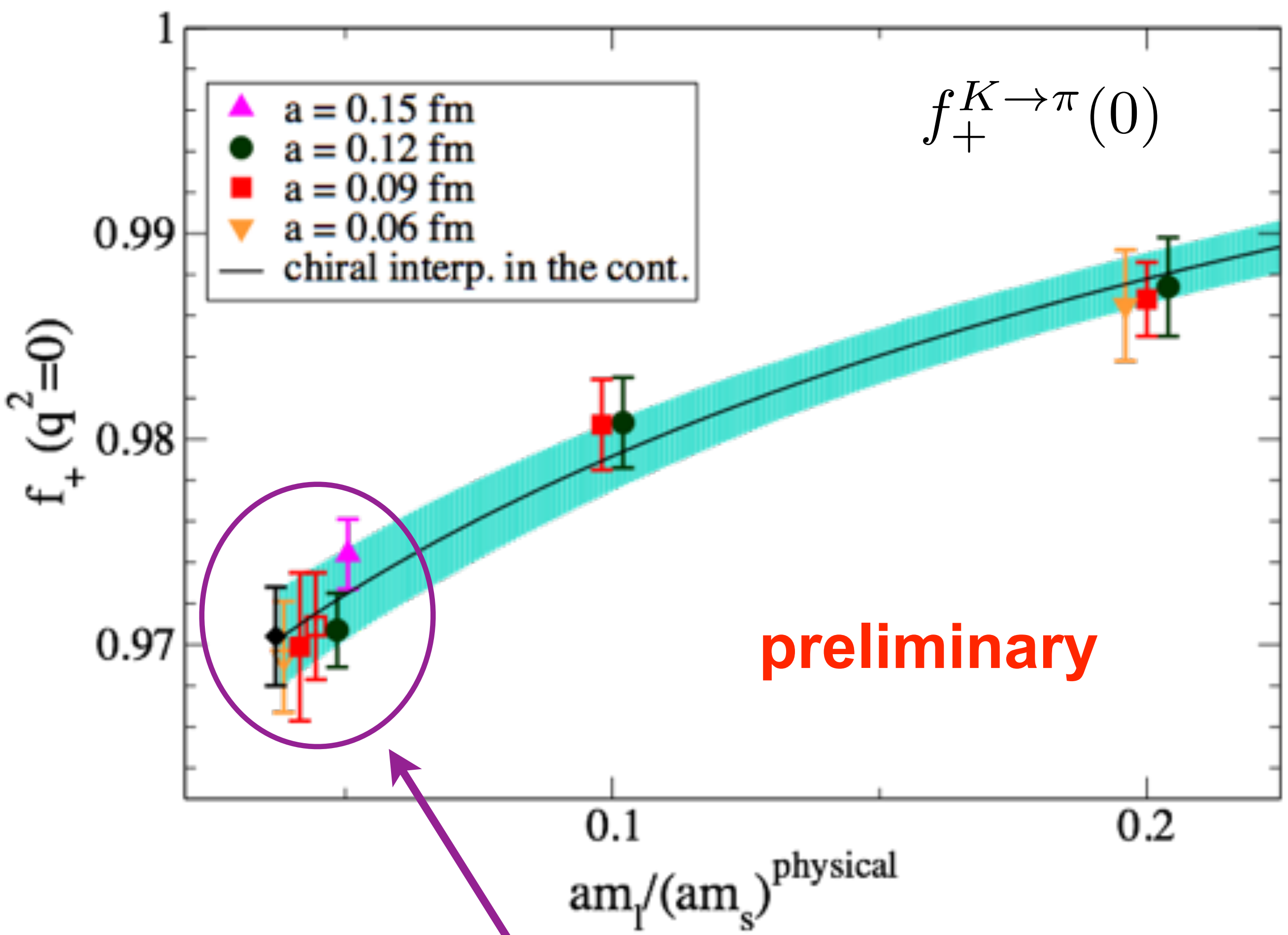
(Phys. Rev. Lett. 94:011601, 2005)

- Normalization agrees with experiment plus CKM unitarity
- *Prediction* of the shape

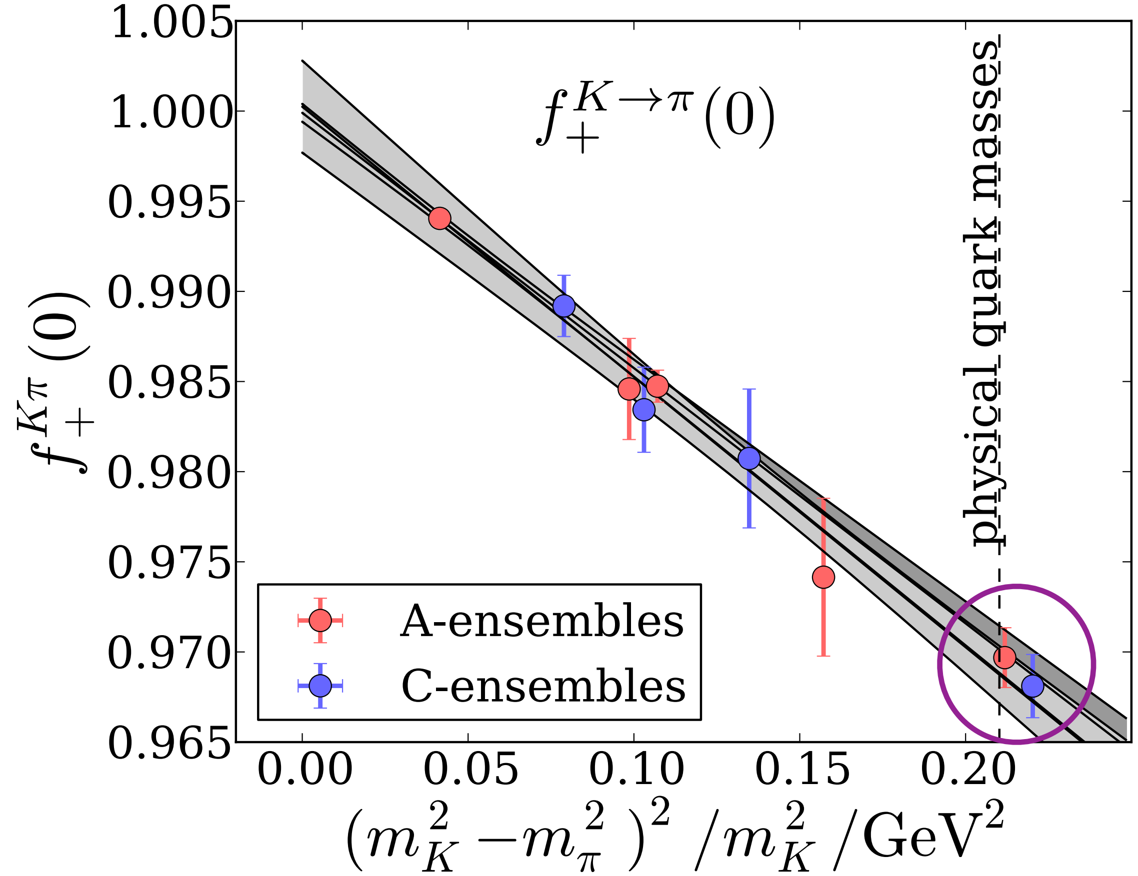
also: B_c mass prediction (HPQCD+FNAL PRL 2005, hep-lat/0411027)

Kaon summary: K_{l3} example

T. Primer (FNAL/MILC) @ Lattice 2014
(update of arXiv:1312.1228)



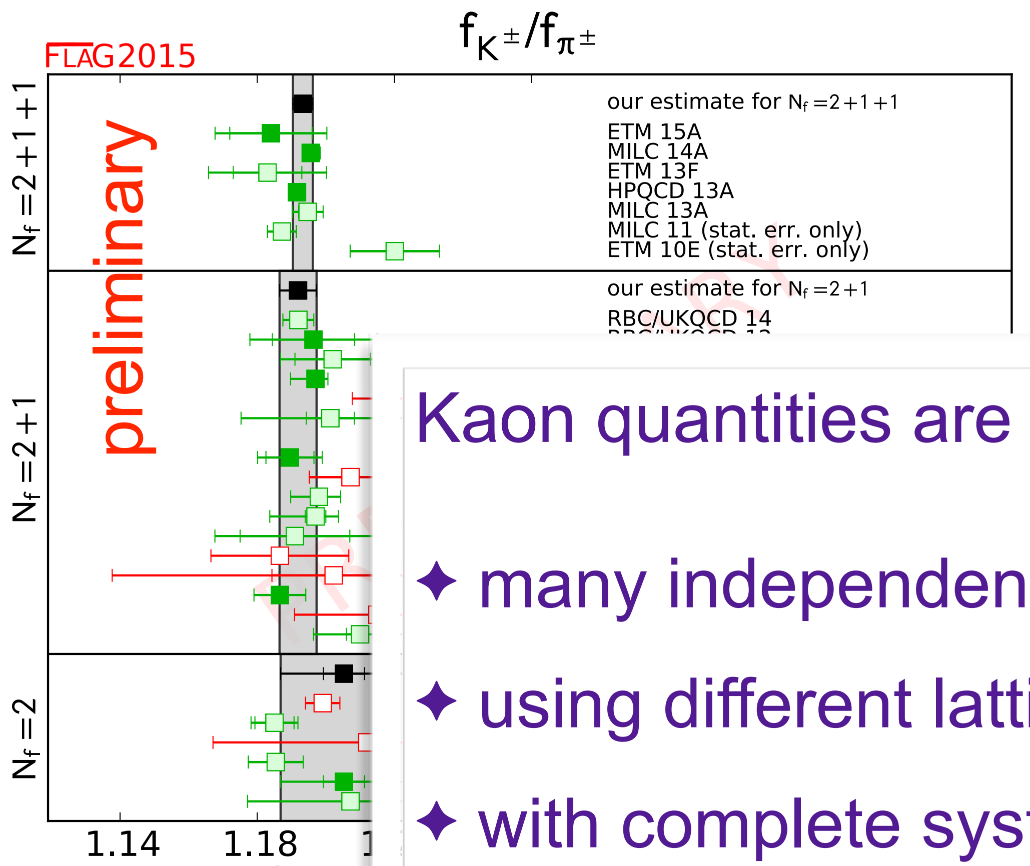
RBC/UKQCD (1504.01692, JHEP 2015)



data at the physical point (offset horizontally)

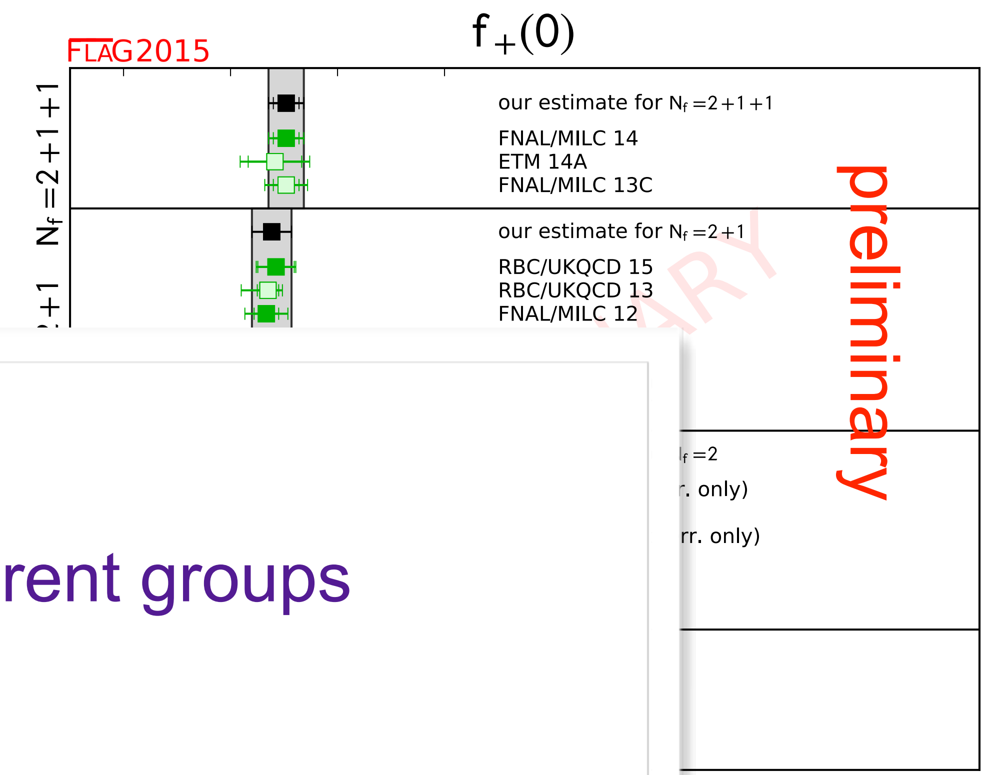
Kaon summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555, FLAG-3 update)



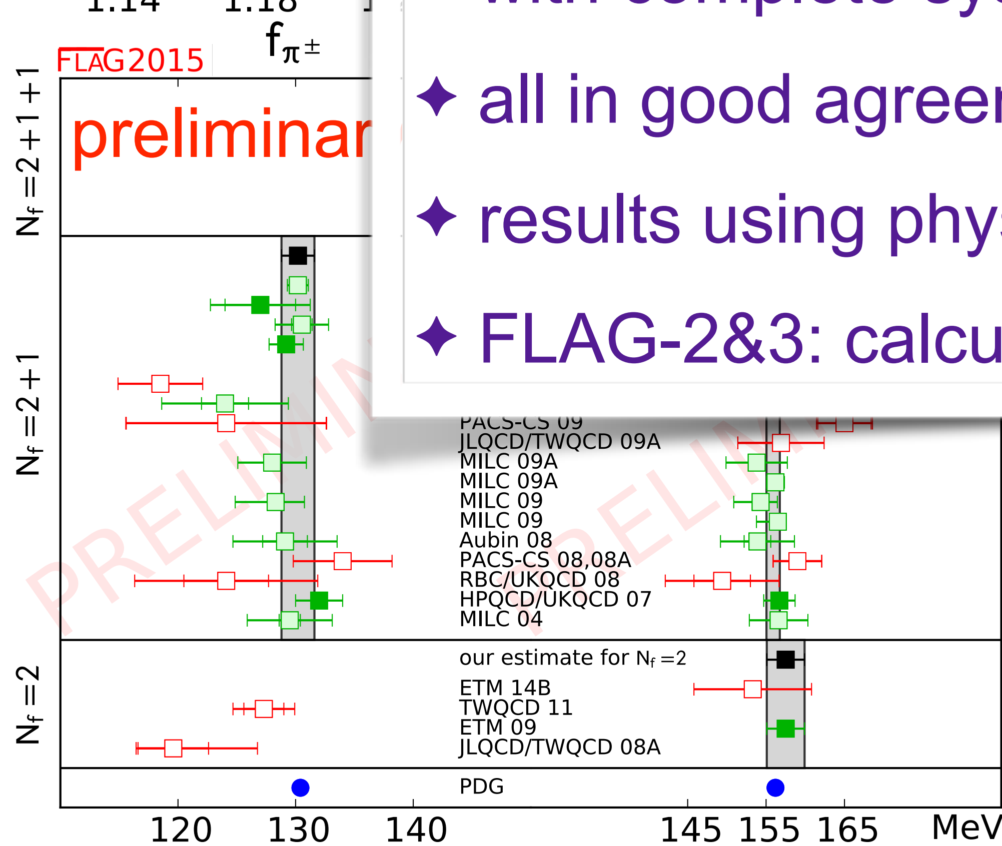
preliminary

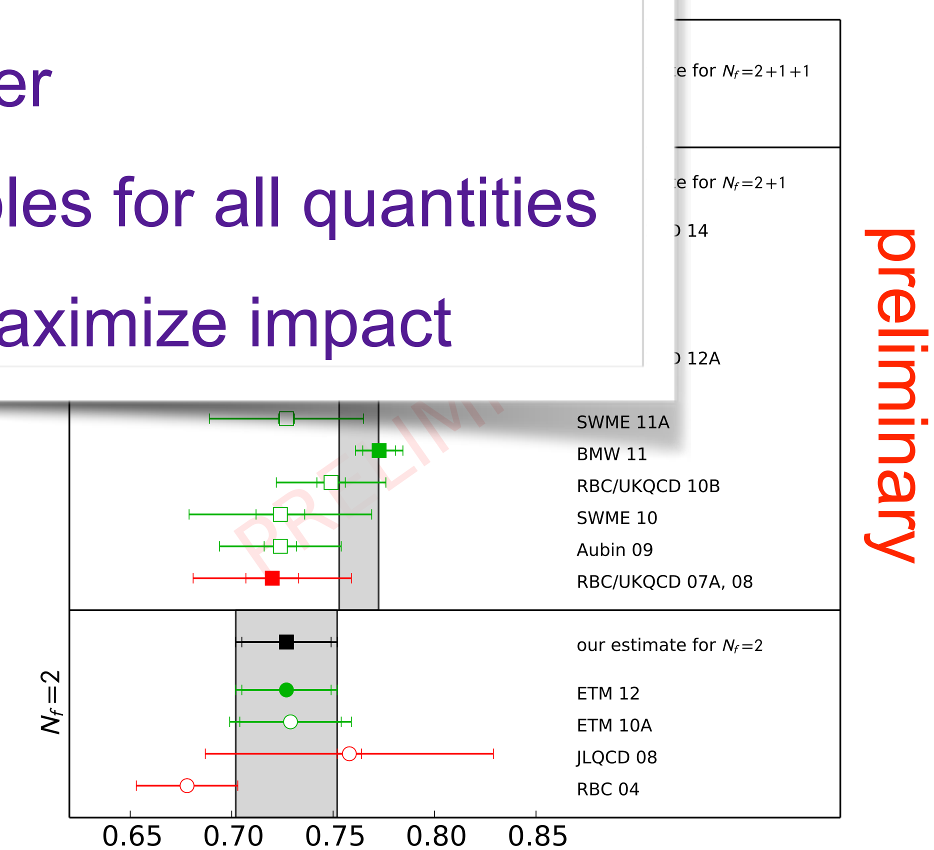
status
mid 2015



Kaon quantities are well studied:

- many independent results from different groups
- using different lattice methods
- with complete systematic error budgets and small errors
- all in good agreement with each other
- results using physical mass ensembles for all quantities
- FLAG-2&3: calculate averages to maximize impact





Neutral D-meson mixing

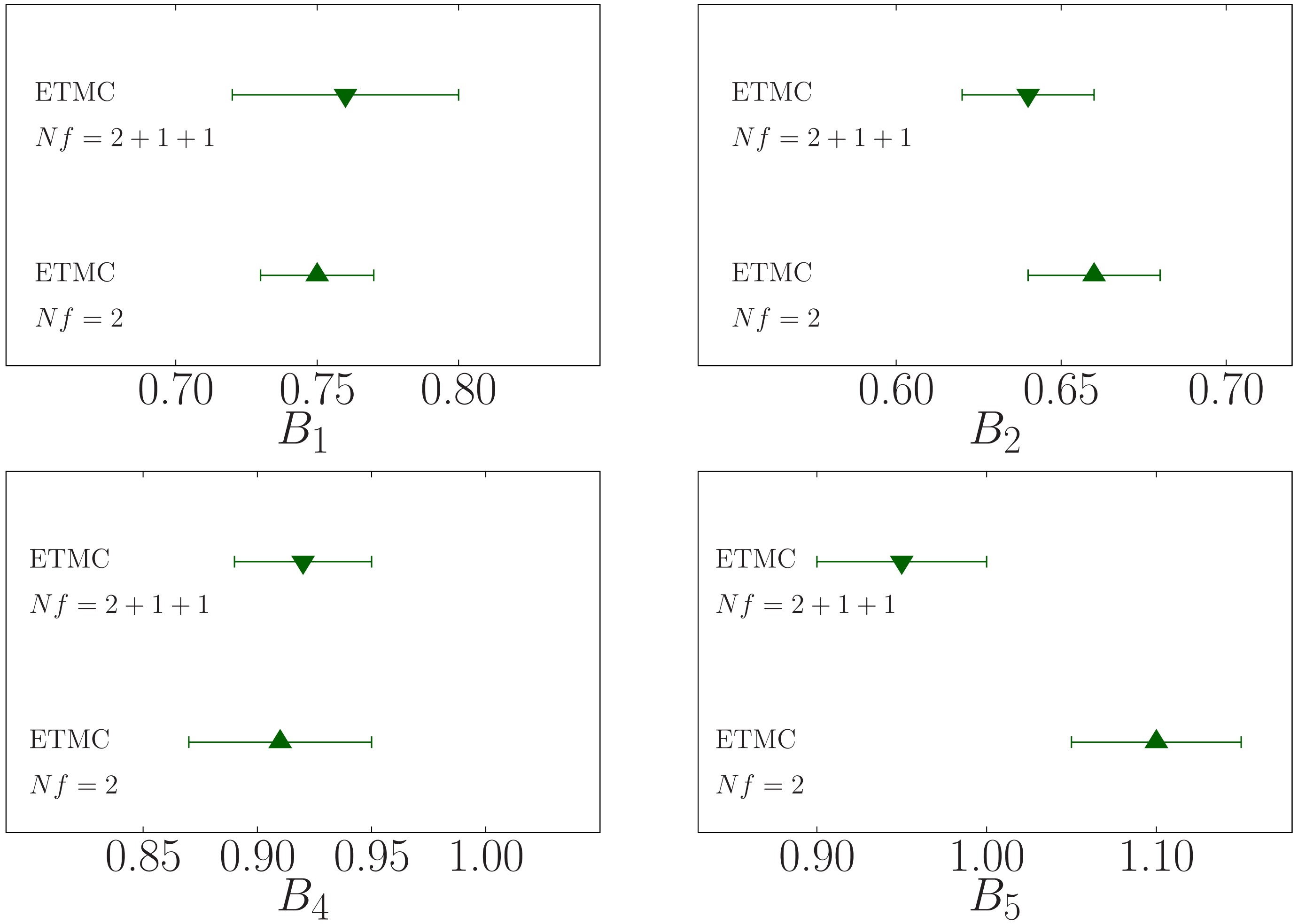
N. Carrasco @ ICHEP 2014

First unquenched LQCD calculation by ETM in 2013
short-distance operators only

- **ETMC:** OS/MTM Mixed action

$N_f = 2$, (N. Carrasco et al. arxiv 1403.7302, To be published in Phys. Rev. D)

$N_f = 2+1+1$ (N. Carrasco et al. PoS LATTICE2013 393, arxiv 1310:5461)



3-5% precision

Neutral D-meson mixing

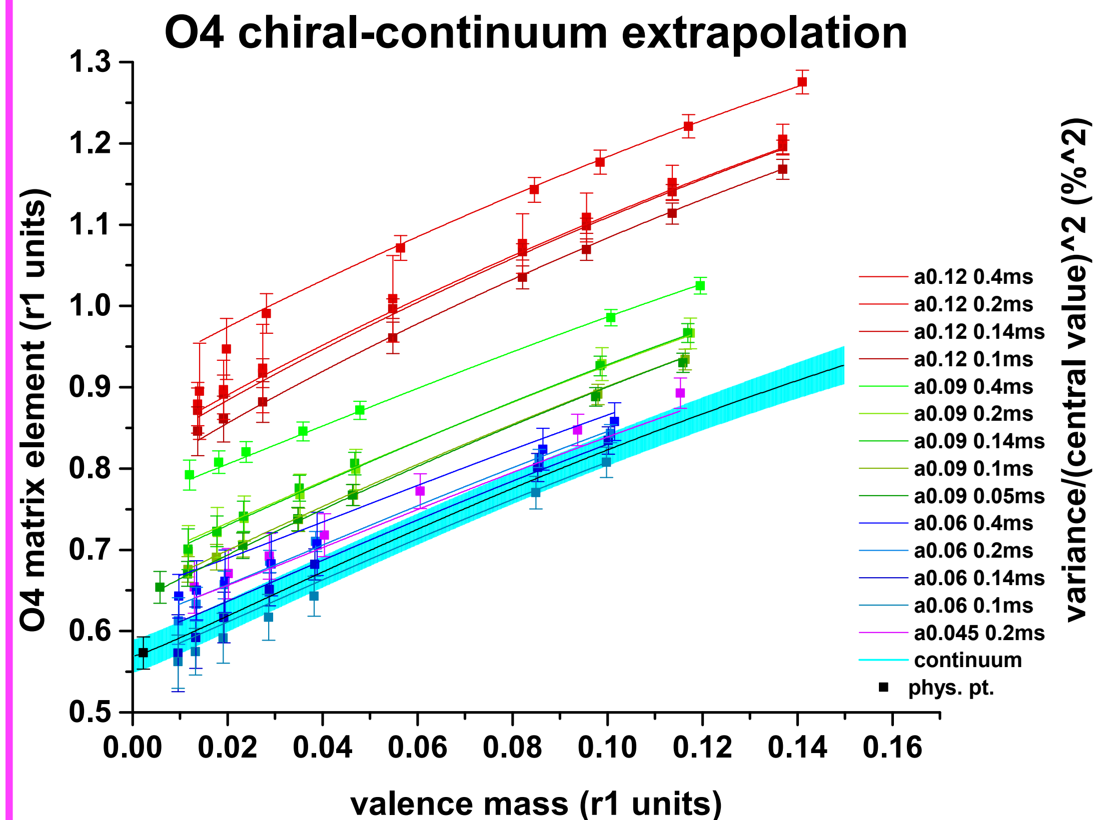
review by C. Bouchard @ Lattice 2014

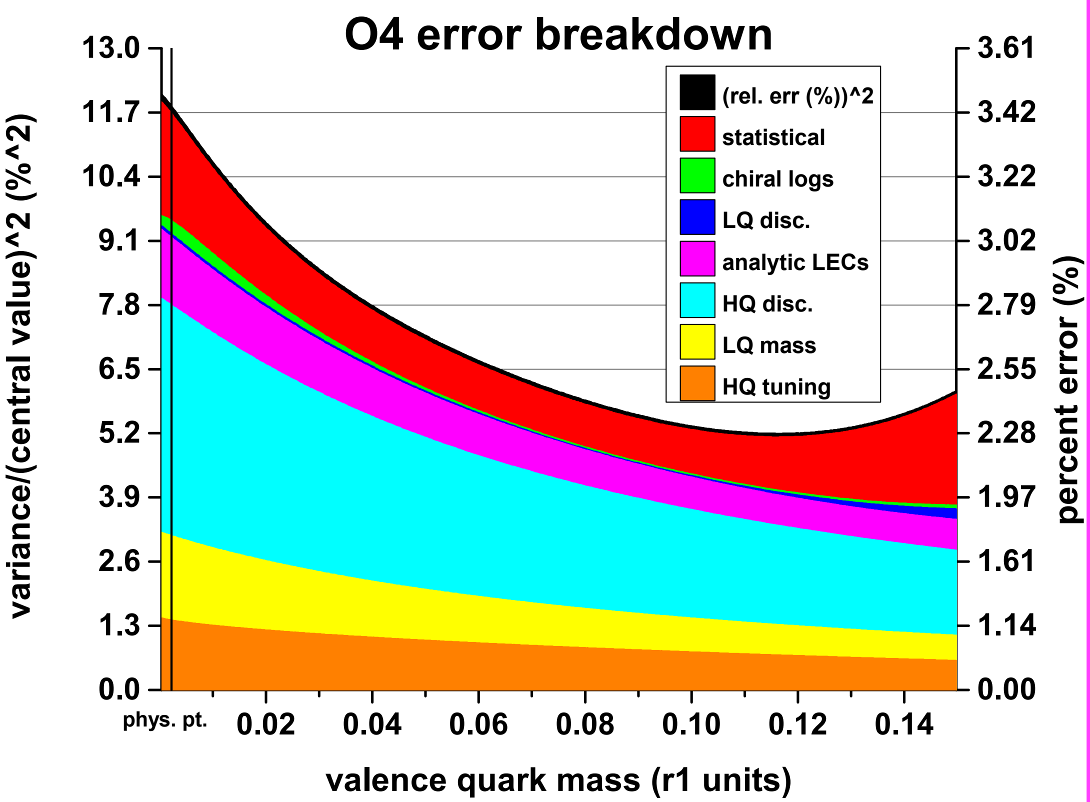
Short Distance D^0 Mixing

FNAL/MILC

MILC Nf=2+1 asqtad configurations
FNAL charm and asqtad light valence
a: 0.045 - 0.125 fm
Mpi: 177 – 559 MeV

Chia Cheng Chang; 25th @ 12:50; sess. 6

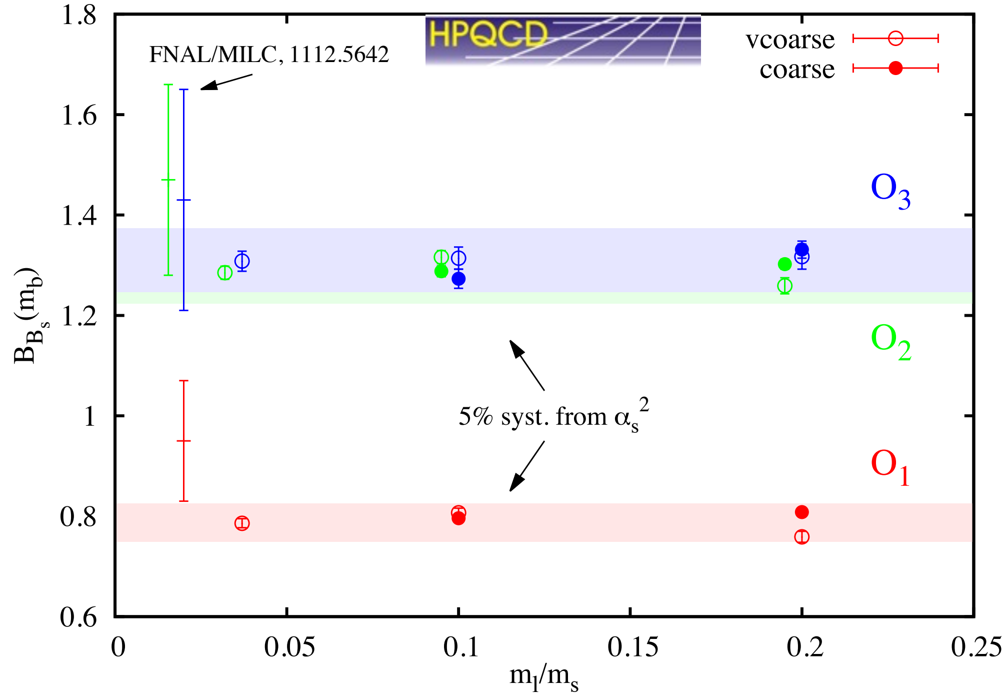




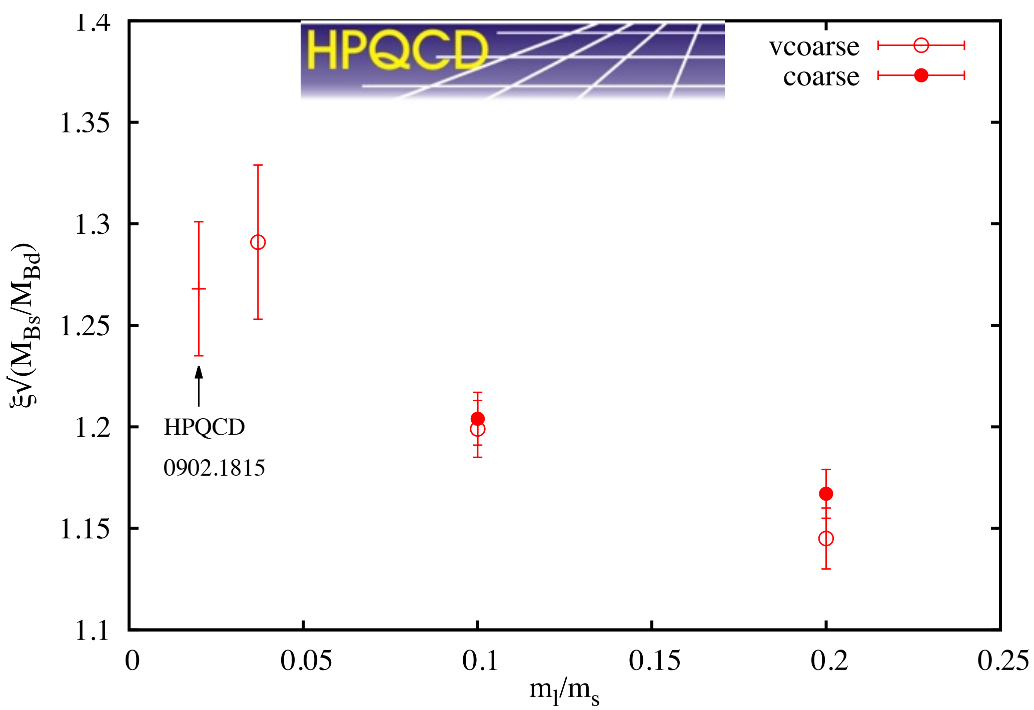
61

Neutral B-meson mixing

C. Davies (HPQCD) @ Lattice 2014

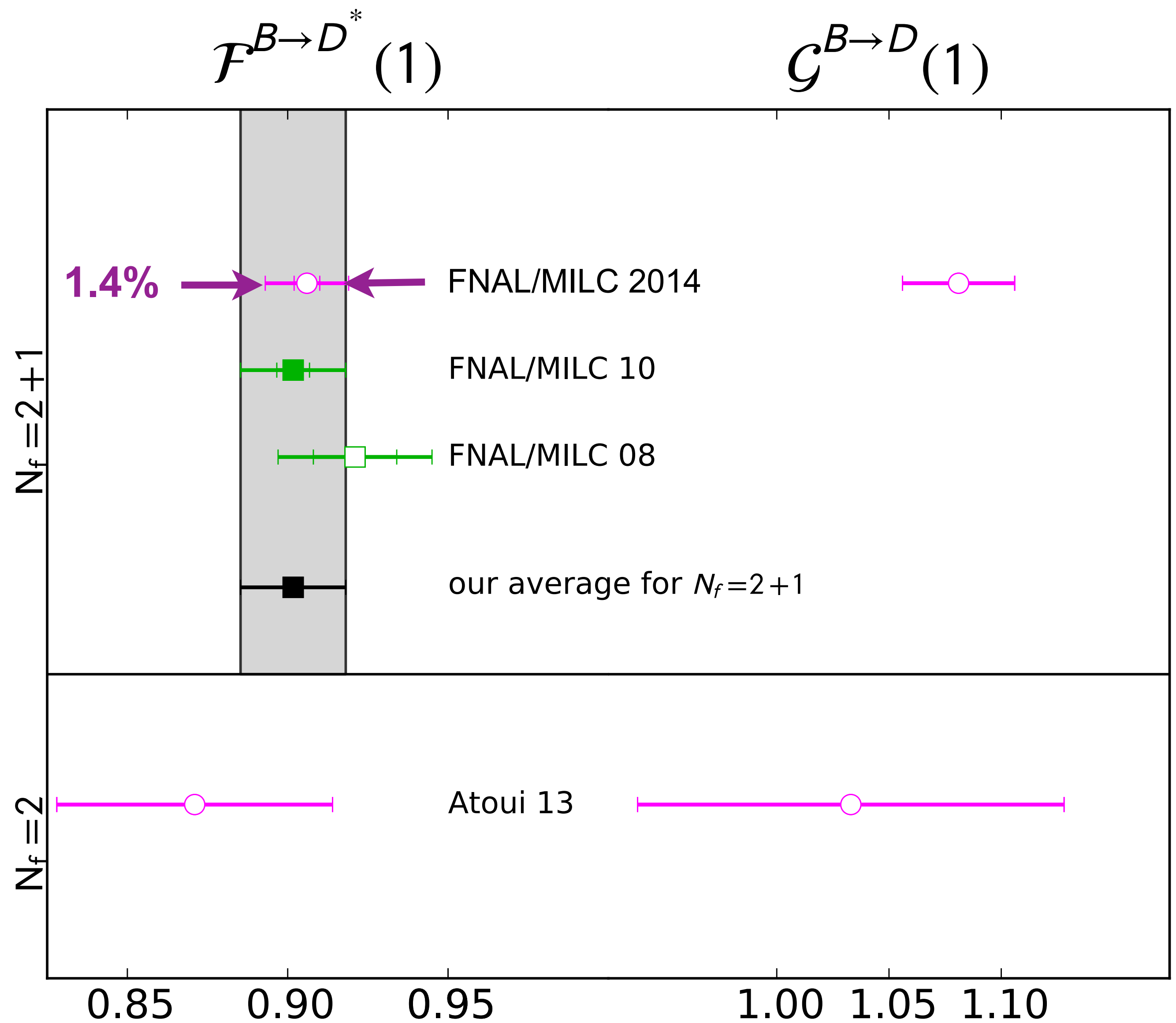


- physical mass ensembles (a first for B mixing)
- NRQCD b quarks
- MILC HISQ $n_f = 2+1+1$ ensembles at 3 a's
- Renormalization: 1-loop LPT dominates error for bag parameters
- calculation still ongoing



- RBC/UKQCD (arXiv:1406:6192) static limit
- also ongoing work by ETM and RBC (rel HQ)
- FNAL/MILC Lattice 2015: ~1.6% error on ξ

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}



For determinations of V_{cb} from $B \to D$ decay, combine exp. differential decay rates with lattice form factors over entire kinematic range.

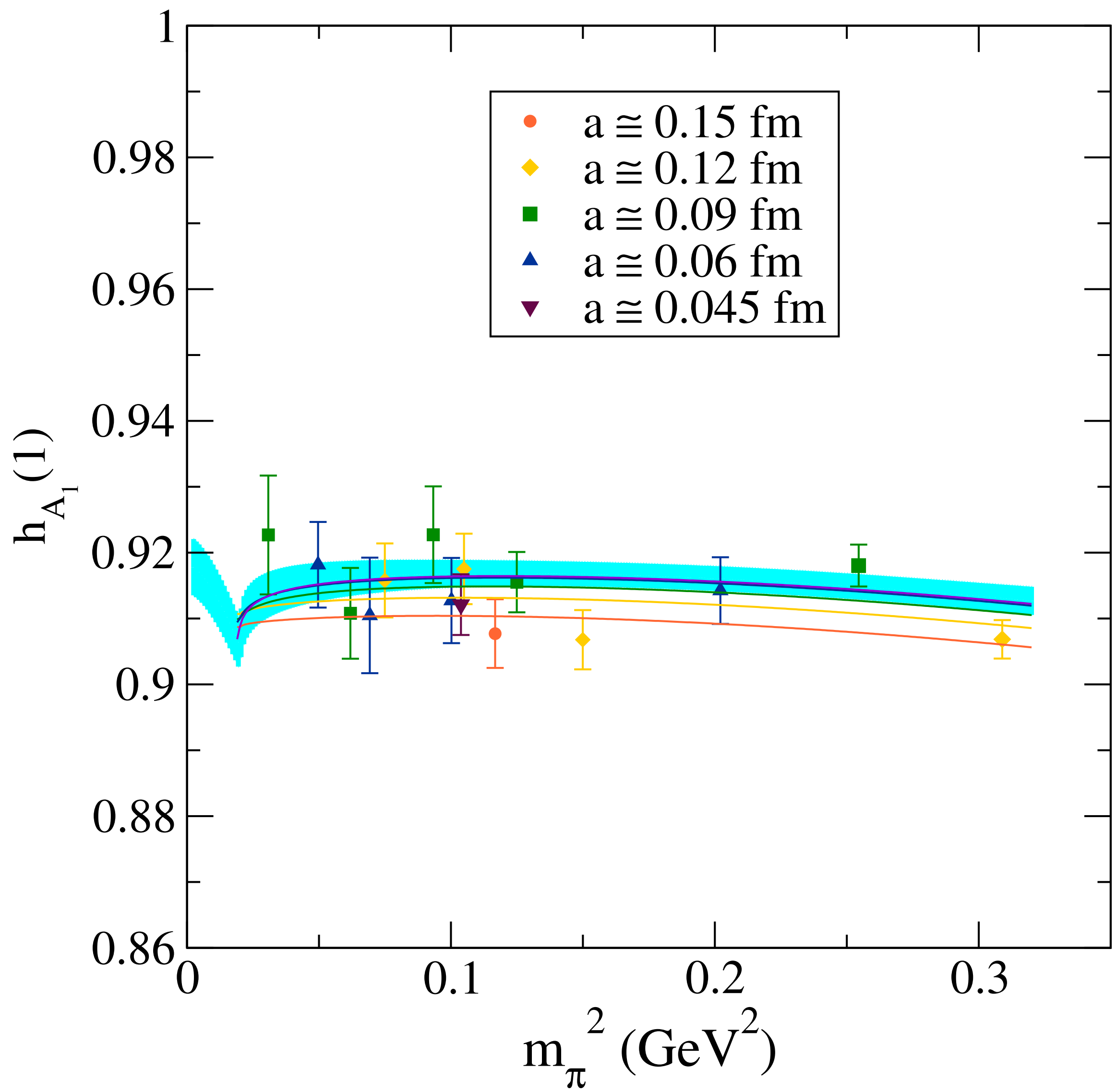
results reported by
- ✦ FNAL/MILC (2014 & 2015)
- ✦ Orsay group using ETM ratio method
- ✦ HPQCD (2015) using NRQCD-HISQ quarks

✦ work in progress:
Bailey et al (SWME) using OK action

Also recent work on $B_s \to D_s^{(*)}$ form factors

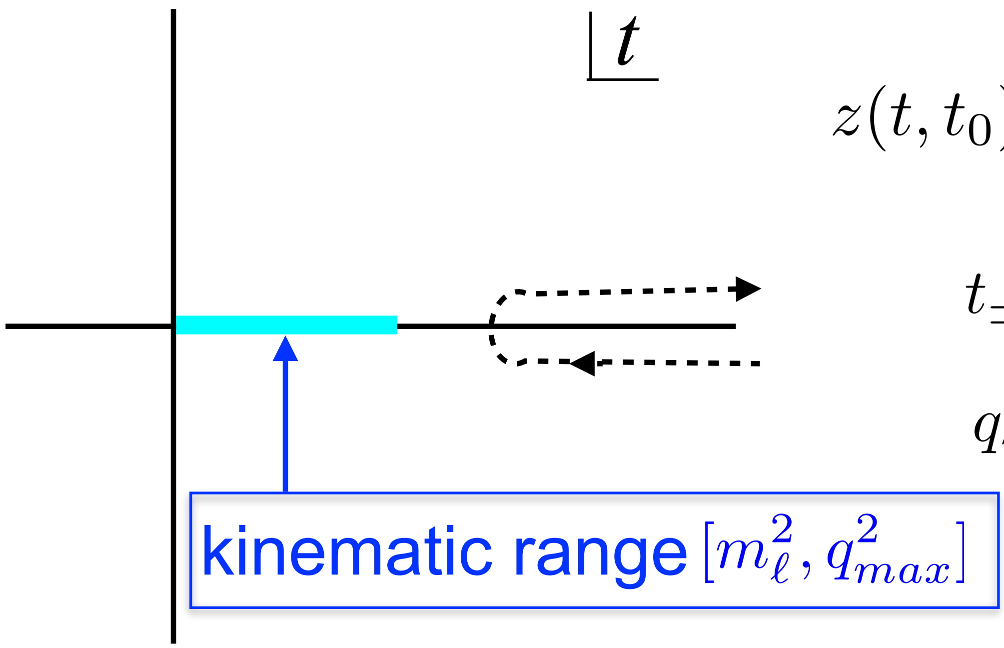
Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

FNAL/MILC (J. Bailey et al, 1403.0625, PRD 2014)



small errors due to
- ✦ use of ratios
- ✦ 2014:
  5 a's, 12 ensembles
  min. $m_\pi \sim 174$ MeV
- ✦ lattice error now same size as exp. error

The z-expansion

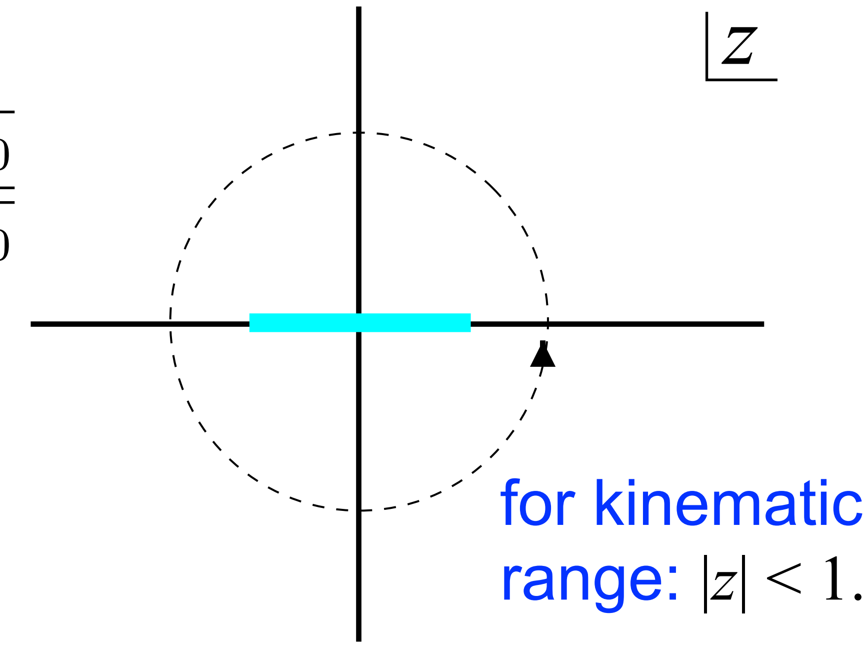


$$t = q^2$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd et al (hep-ph/9412324,PRL 95)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

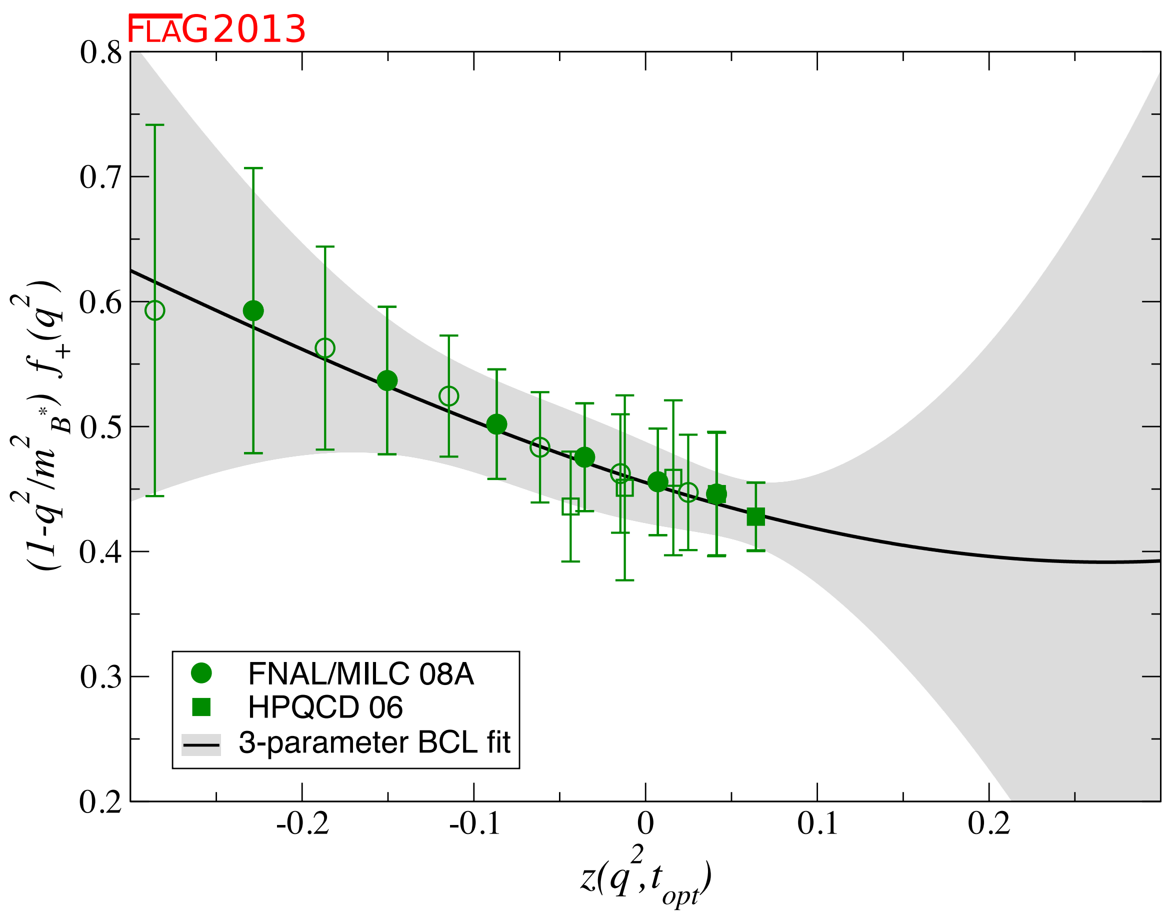
The form factor can be expanded as:

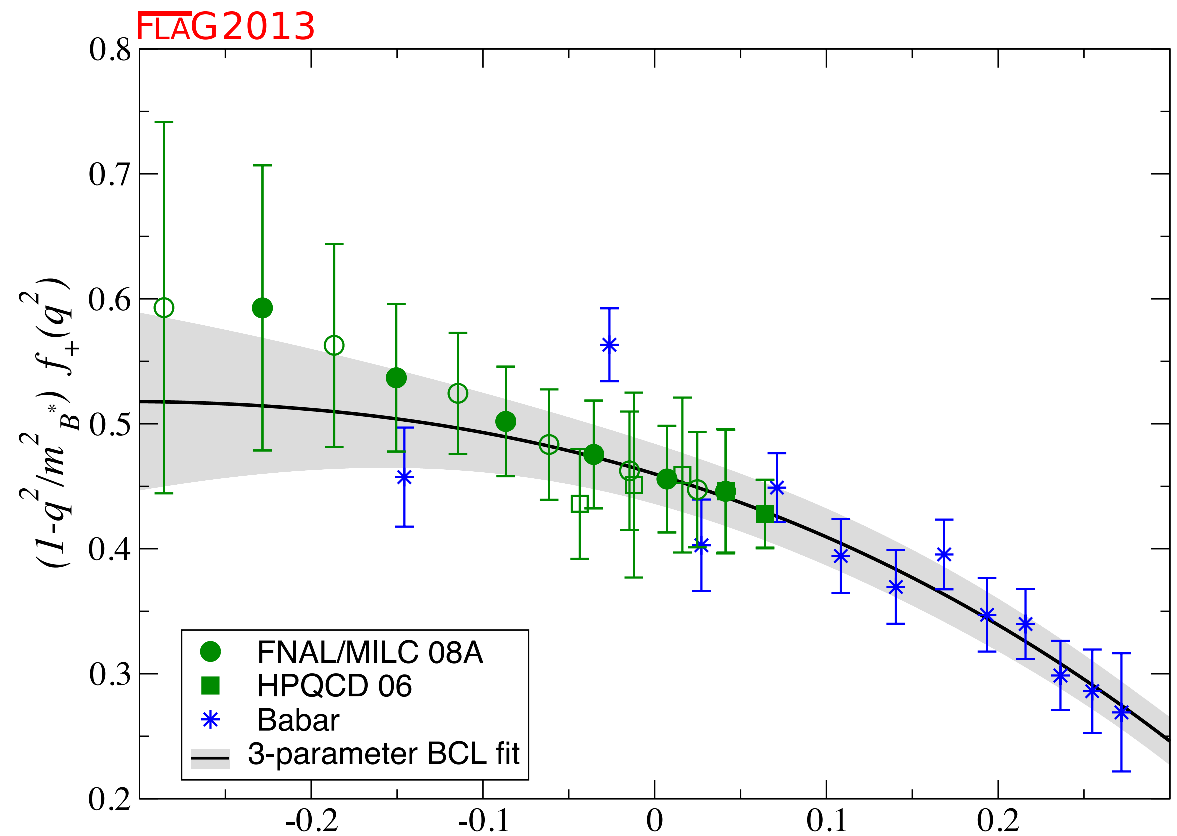
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0} a_k(t_0) z(t,t_0)^k$$

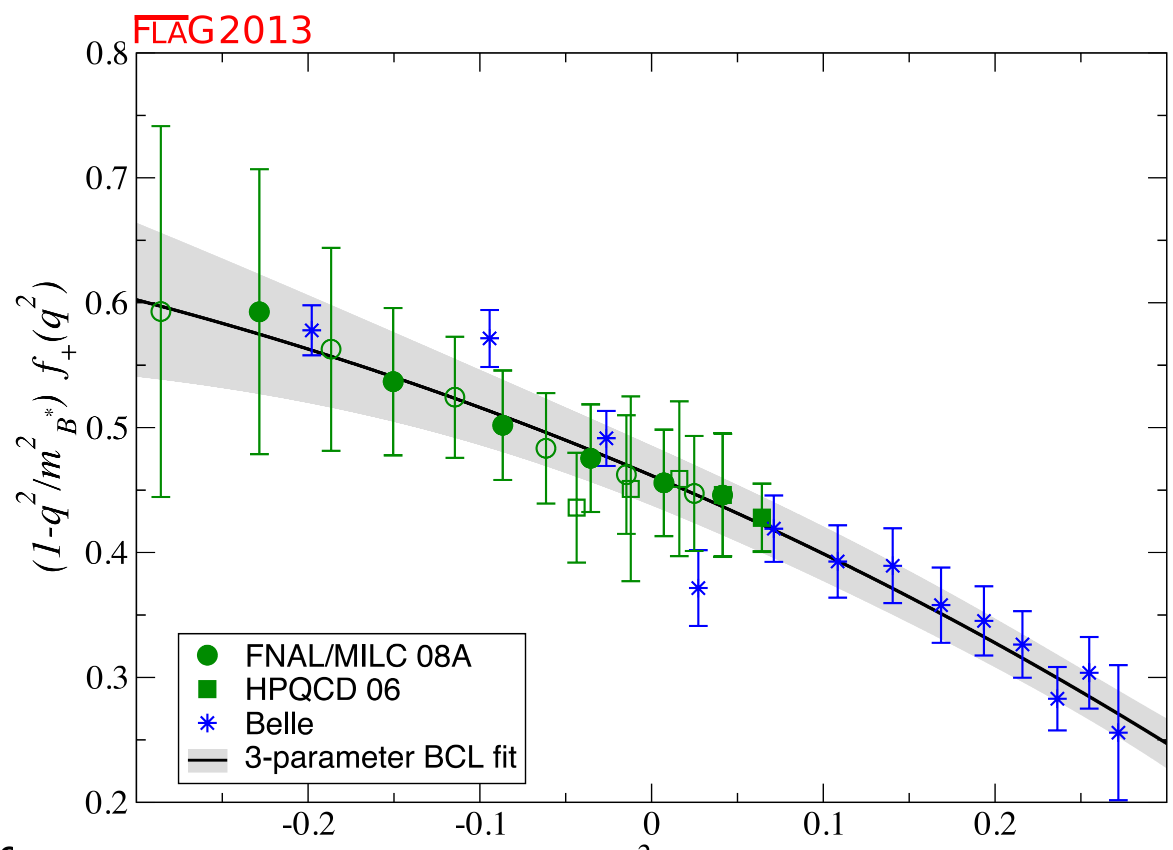
- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

Form factor for $B \to \pi \ell \nu$ & V_{ub}

S. Aoki et al (FLAG-2 review, arXiv:1310.8555)

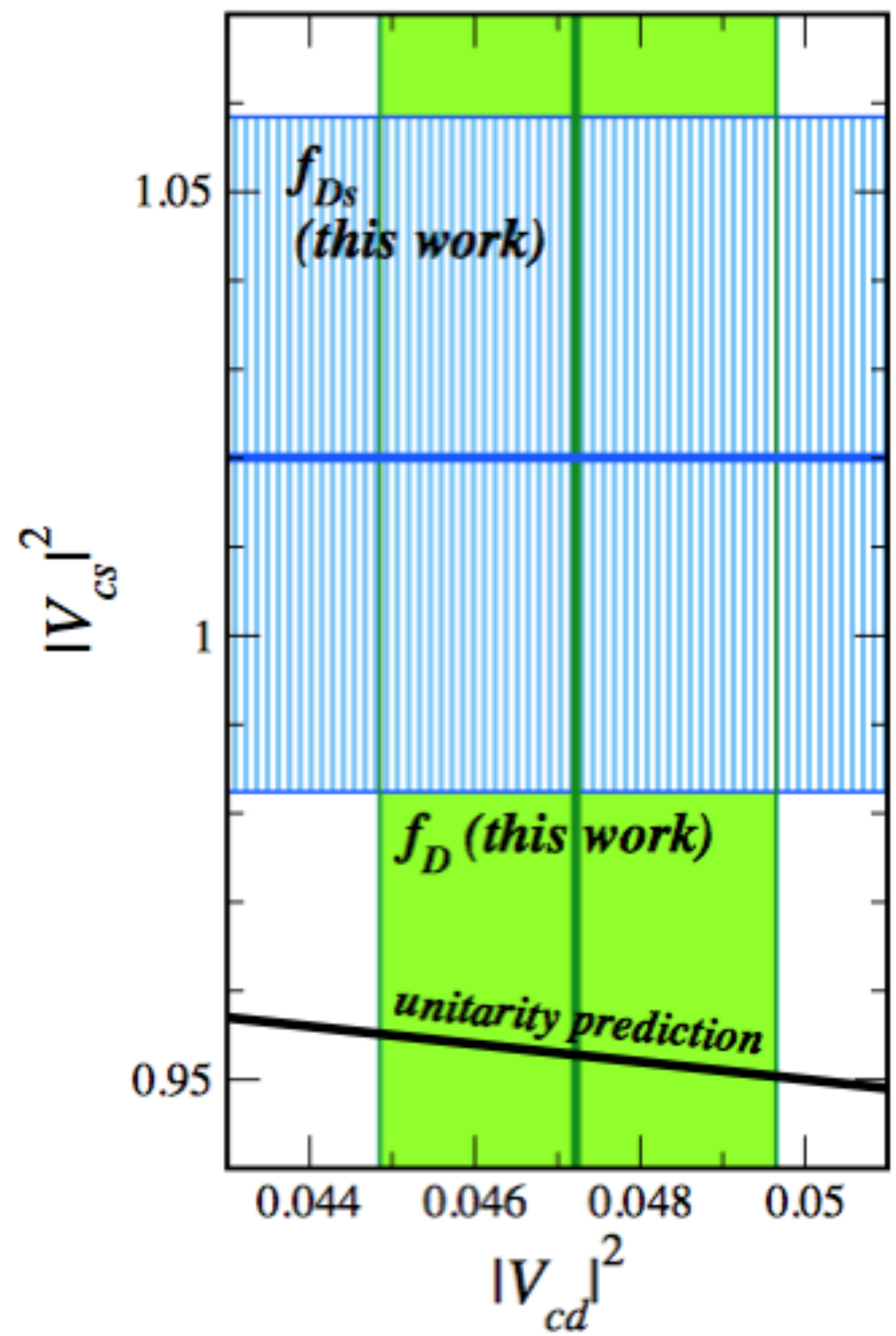






Implications for the 2nd row of the CKM Matrix

J. Komijani @ Lattice 2014 (FNAL/MILC, arXiv:1407.3772, PRD 2014)



$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$|V_{cb}| \approx 4 \times 10^{-2} \approx 0$$

Slight tension with unitarity prediction

BSM phenomenology

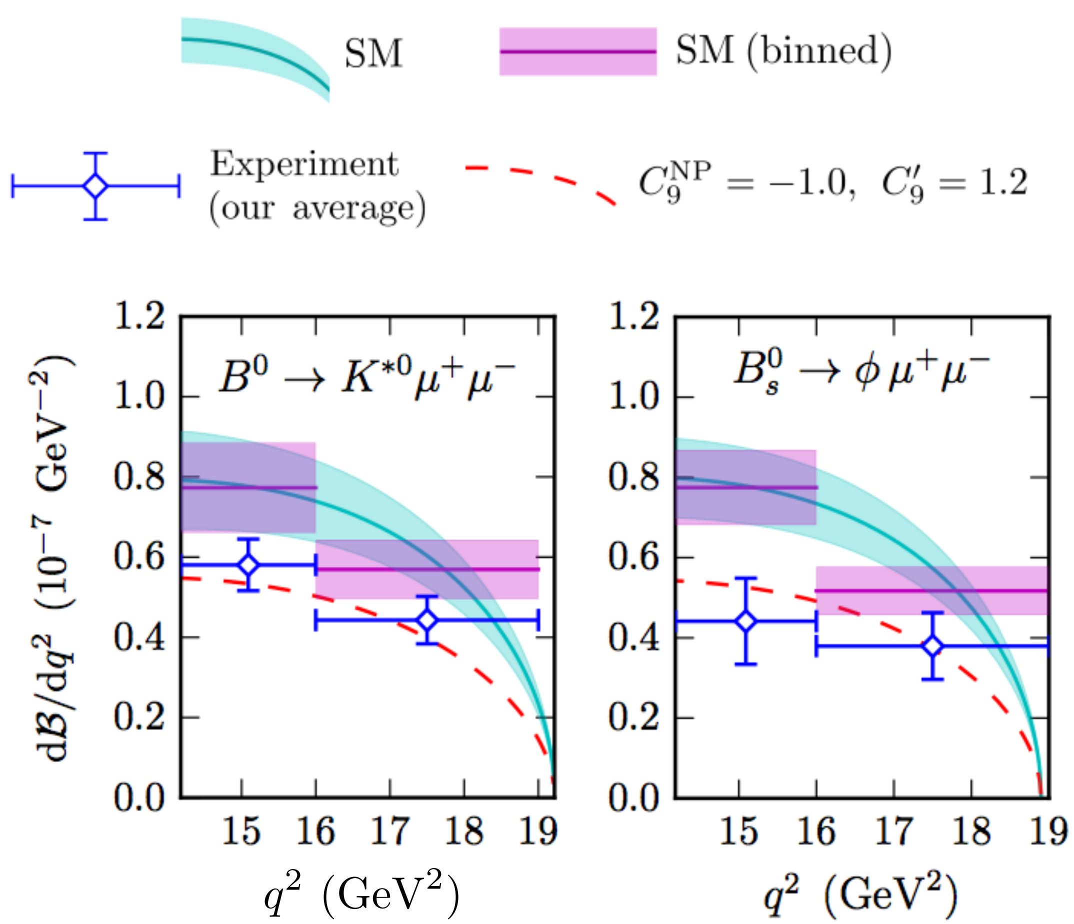
review by C. Bouchard @ Lattice 2014

$B \to K^{(*)}\ell\ell,\ B_s \to \phi\ell\ell$

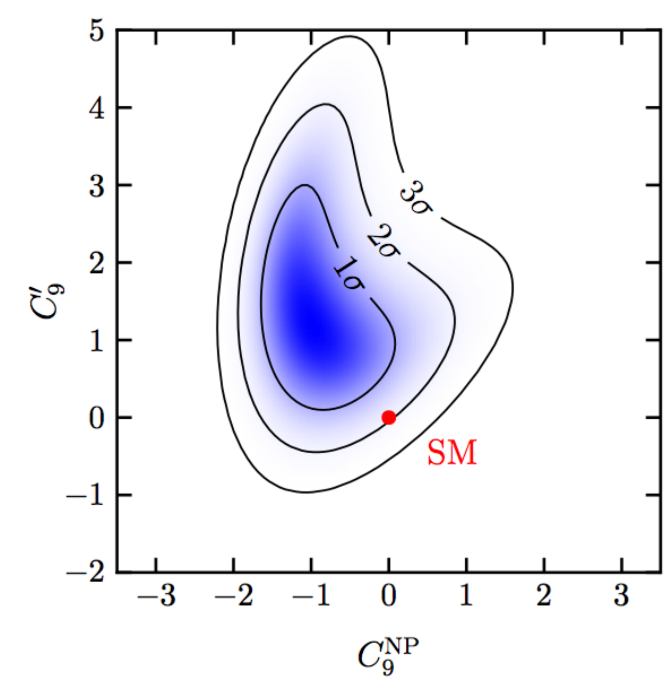
Horgan et al., PRL 112, 212003 (2014); PRD 89, 094501 (2014)

MILC 2+1 asqtad gauge fields
NRQCD b with asqtad light/strange valence
a: 0.09, 0.12 fm
Mpi: 313 – 519 MeV

caveat:
K^*, ϕ treated as stable
(narrow width approximation)
unstable K^*, ϕ: beyond simple



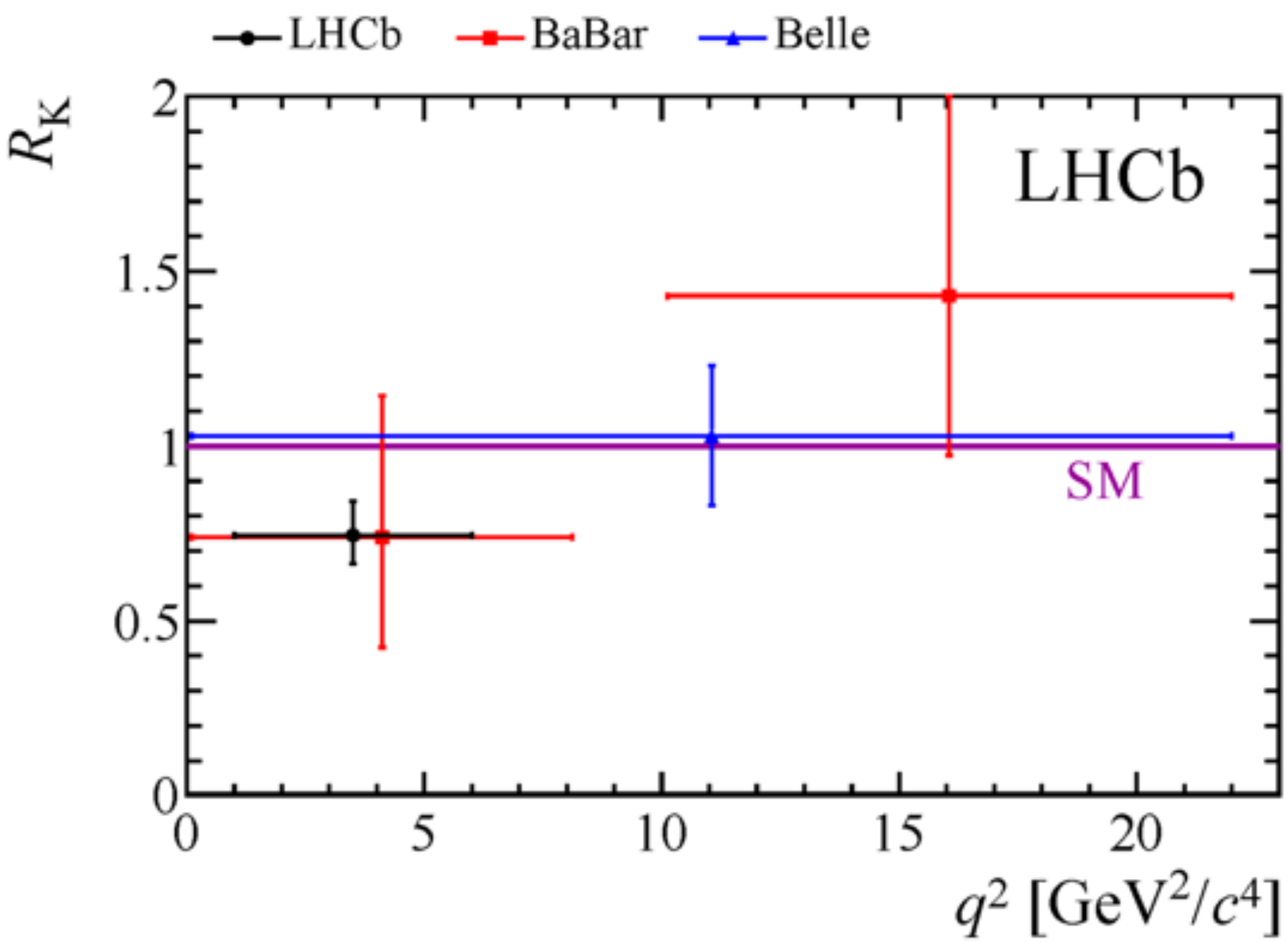
Combined fit to $B \to K^*\mu\mu$ and $B_s \to \phi\mu\mu$ data.



53

BSM phenomenology

Lepton universality test: $B \to K\mu^+\mu^- / B \to K e^+ e^-$



LHCb (arXiv:1406.6482):

$$R_K = 0.745\,\binom{90}{74}(36)$$

SM prediction using LQCD form factors calculated by HPQCD

(C. Bouchard et al, arXiv:1303.0434, PRL 2013):

$$R_K(1\,\mathrm{GeV}^2, 6\,\mathrm{GeV}^2) = 1.00081(38)$$

$\sim 2.6\,\sigma$ tension between LHCb measurement and SM prediction

Simple quantities in LQCD

- low-lying hadron spectrum
- weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - B mesons

  ➙ CKM, BSM phenomenology

- high precision ➙ including QED

Including QED

- Need to consider strong isospin breaking effects together with EM effects
- Strong isospin breaking in the sea is a subdominant effect (~NNLO in ChPT)
  currently: isospin symmetric u,d sea: $m_u = m_d$
- QCD + quenched QED (electro quenched):
  sea quarks neutral, valence quarks charged
- Can use spectrum results from QCD + quenched QED in pure QCD calculations by adjusting the valence quark masses to physical m_u , m_d . With this the leading strong and EM isospin breaking effects can be included.
- To connect LQCD calculations of weak matrix elements to experiment, need to account for structure dependent EM radiative corrections:
  K, π decay: estimated phenomenologically using CHPT
  (see for example, Cirigliano, et al, arXiv:1107.6001)
- We now need similar phenomenological estimates for weak D and B decays

Including QED

review by A. Portelli @ Lattice 2014 and WG 1, Wednesday

- **new**: full QCD+QED simulations used in spectrum calculations:
  BMW ($n_f = 1+1+1+1$) at multiple lattice spacings, light quark masses
  QCDSF ($n_f = 1+1+1$)
  RBC/UKQCD ($n_f = 2+1$)
  PACS-CS ($n_f = 1+1+1$)
  similar plans by other groups (MILC, RBC/UKQCD, ...)

- EM radiative corrections :

$$\Gamma\left(\pi^+ \to \ell^+ \nu_\ell(\gamma)\right) = \Gamma\left(\pi^+ \to \ell^+ \nu_\ell\right) + \Gamma\left(\pi^+ \to \ell^+ \nu_\ell \gamma\right)$$

Proposal by Carrasco et al (arXiv:1502.00257, 2015 PRD) to calculate corrections at $O(\alpha)$.

Including QED

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Including QED

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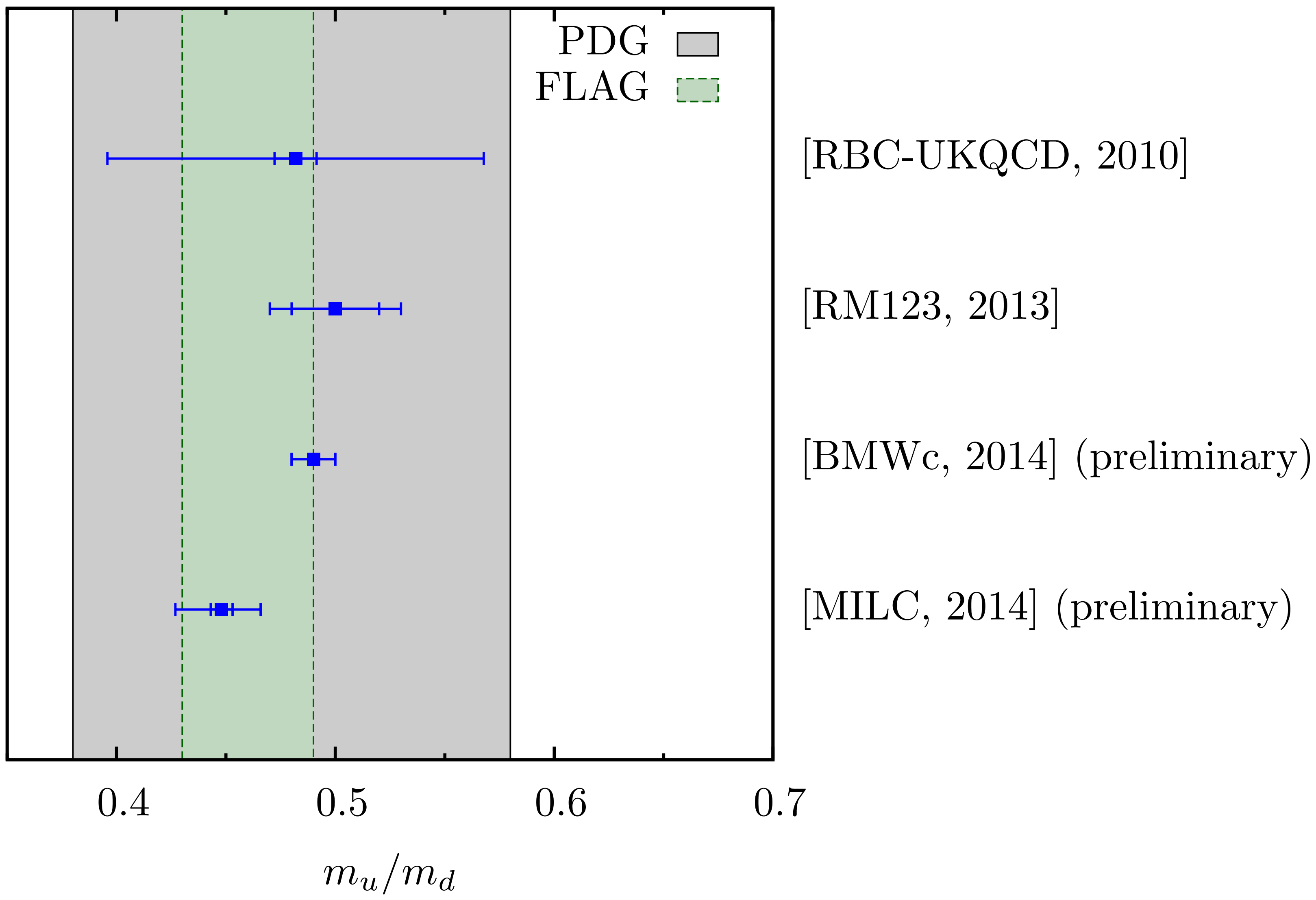

Including QED

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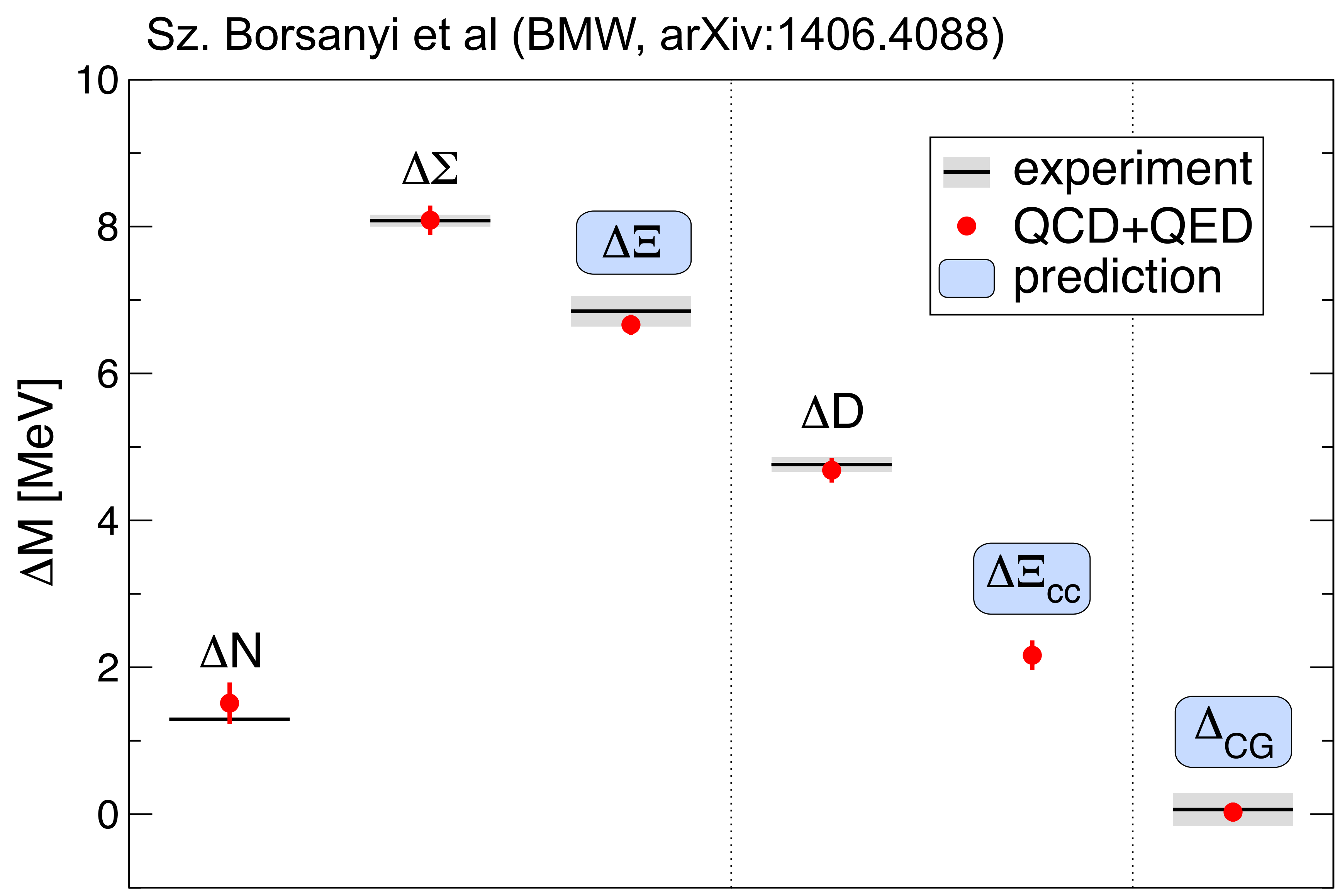

Including QED

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