Nuclei in Lattice QCD: Prospects and Progress

William Detmold, MIT

Intersections of BSM Phenomenology and QCD for New Physics Searches INT, Oct 2nd 2015

Nuclei in Lattice QCD: Prospects and Progress

I. Nuclear effects in searches for BSM Physics

II. Magnetic Properties of Nuclei and $np \rightarrow d\gamma$

[NPLQCD PRL 113, 252001 (2014), 1506.05518, PRL 115, 132001 (2015)]

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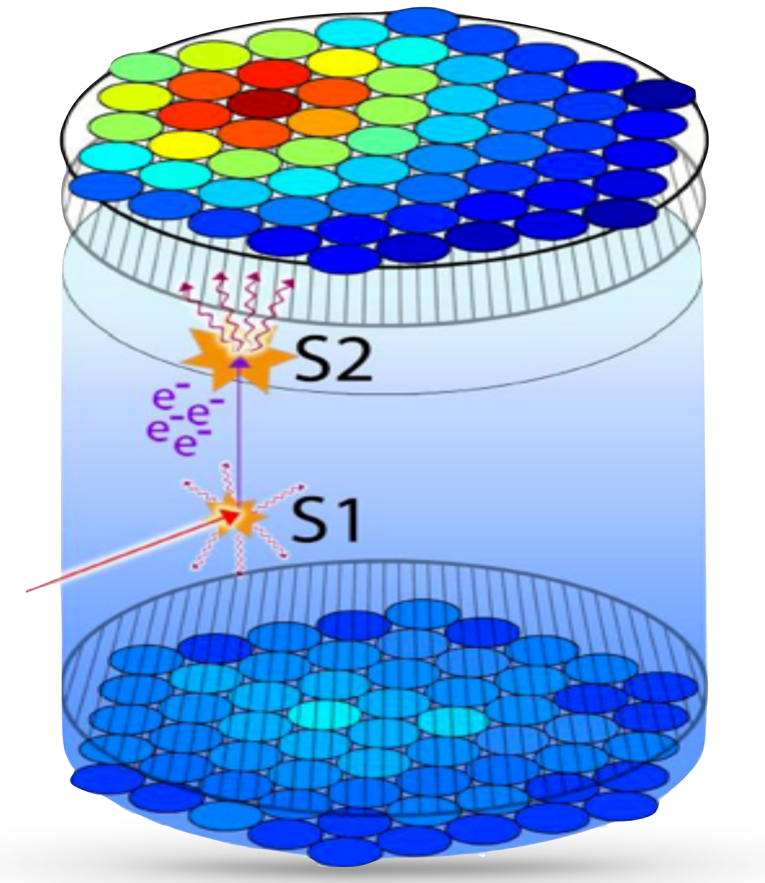
The intensity frontier

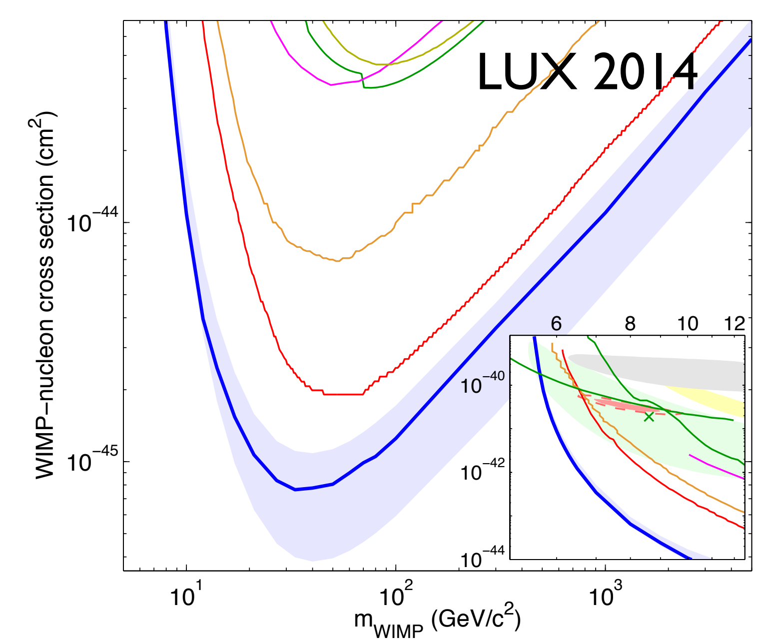
- Seek new physics through quantum effects
- Precise experiments
  - Sensitivity to probe the rarest interactions of the SM
  - Look for effects where there is no SM contribution
- Important focus of HEP(NP) experimental program
  - Dark matter direct detection
  - Neutrino physics
  - Charged lepton flavour violation, EDMs, proton decay, neutron-antineutron oscillations…
- ***Major component is nuclear targets***

The intensity frontier

- Dark matter direct detection: nuclear recoils in large bucket of nuclei as signal
  - Detection rate/bounds depends on dark matter properties/dynamics and x-sec on nucleus
- 😍 Positive signals would be unambiguous
- 😟 Post-detection: precise nuclear x-sec (with quantified uncertainties) to discern underlying dynamics
- Potentially understand seemingly conflicting positive and negative signals
- Inform experimental design and backgrounds

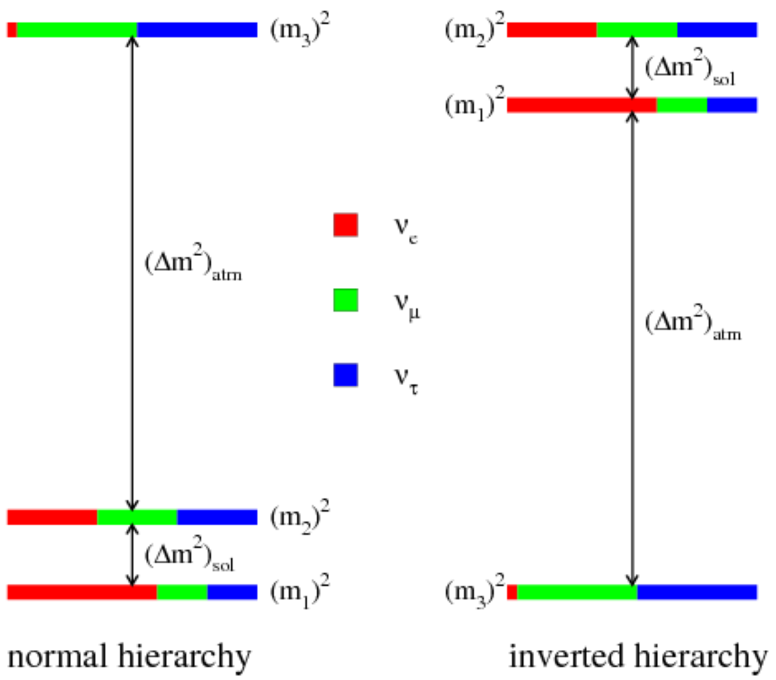


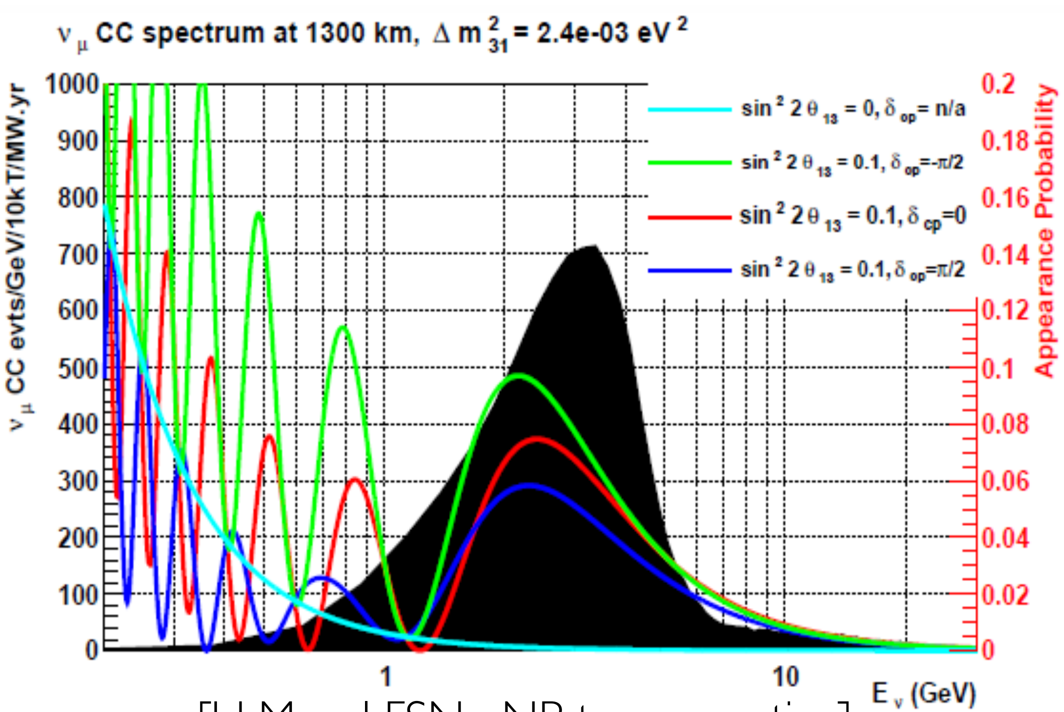
http://www.hep.ucl.ac.uk/darkMatter/



The intensity frontier

- Important goal of LBNF/DUNE: extraction of neutrino mass hierarchy and precise mixing parameters
- Neutrino scattering on argon target
- Requires knowing energies/fluxes to high accuracy
  - Nuclear axial & transition form factors
  - Resonances
  - Neutrino-nucleus DIS
- ~10% uncertainty on oscillation parameters [C Mariani, INT workshop 2013]

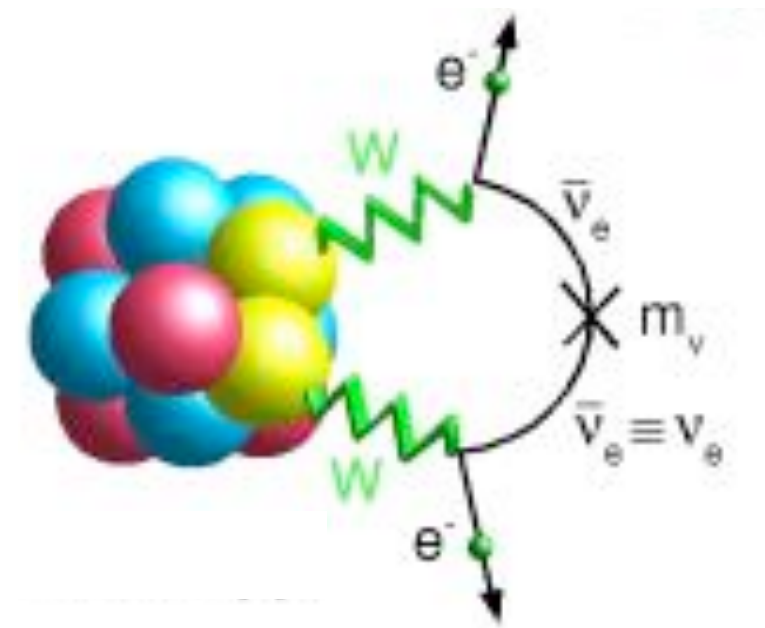


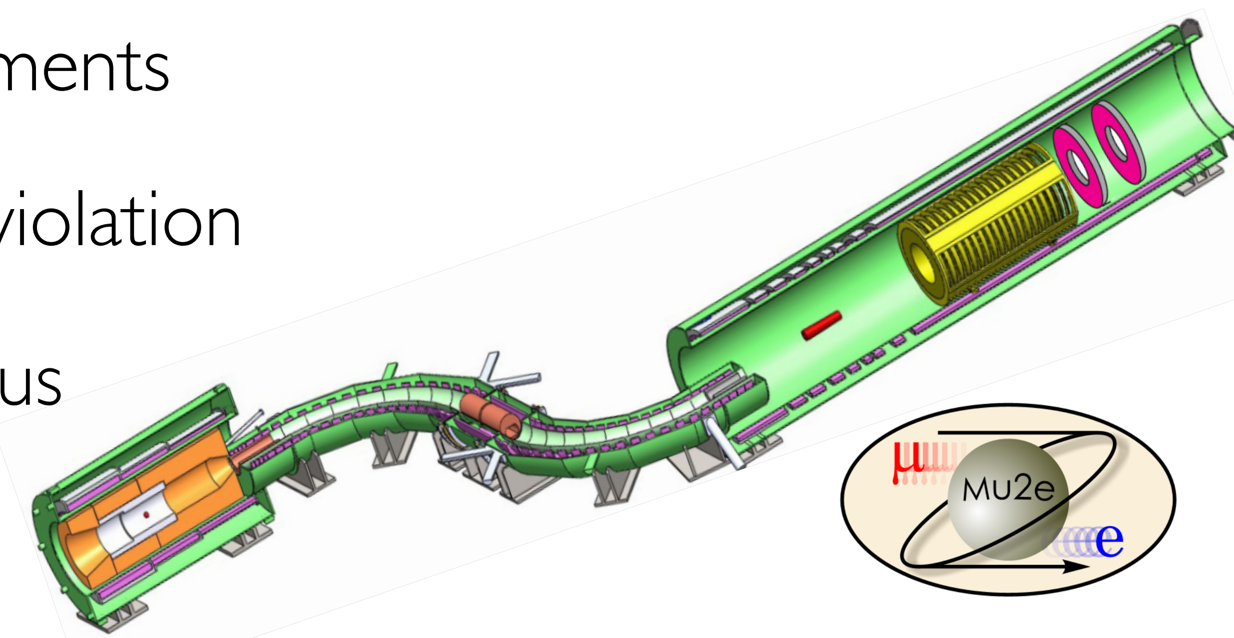


[U Mosel FSNu NP town meeting]

The intensity frontier

- EDMs: potential light nuclear EDM experiments offer complementary handles on CPV
  - Many LECs to constrain in EFT
- $0\nu\beta\beta$ decay: fundamental nature of neutrinos
  - Rates depend on nuclear matrix elements
- μ2e: search for charged lepton flavour violation
  - $\mu \rightarrow e$ conversion in field of Al nucleus





😍 Positive signals would be unambiguous

😟 Post-detection: precise nuclear matrix elements (with quantified uncertainties) to discern underlying dynamics

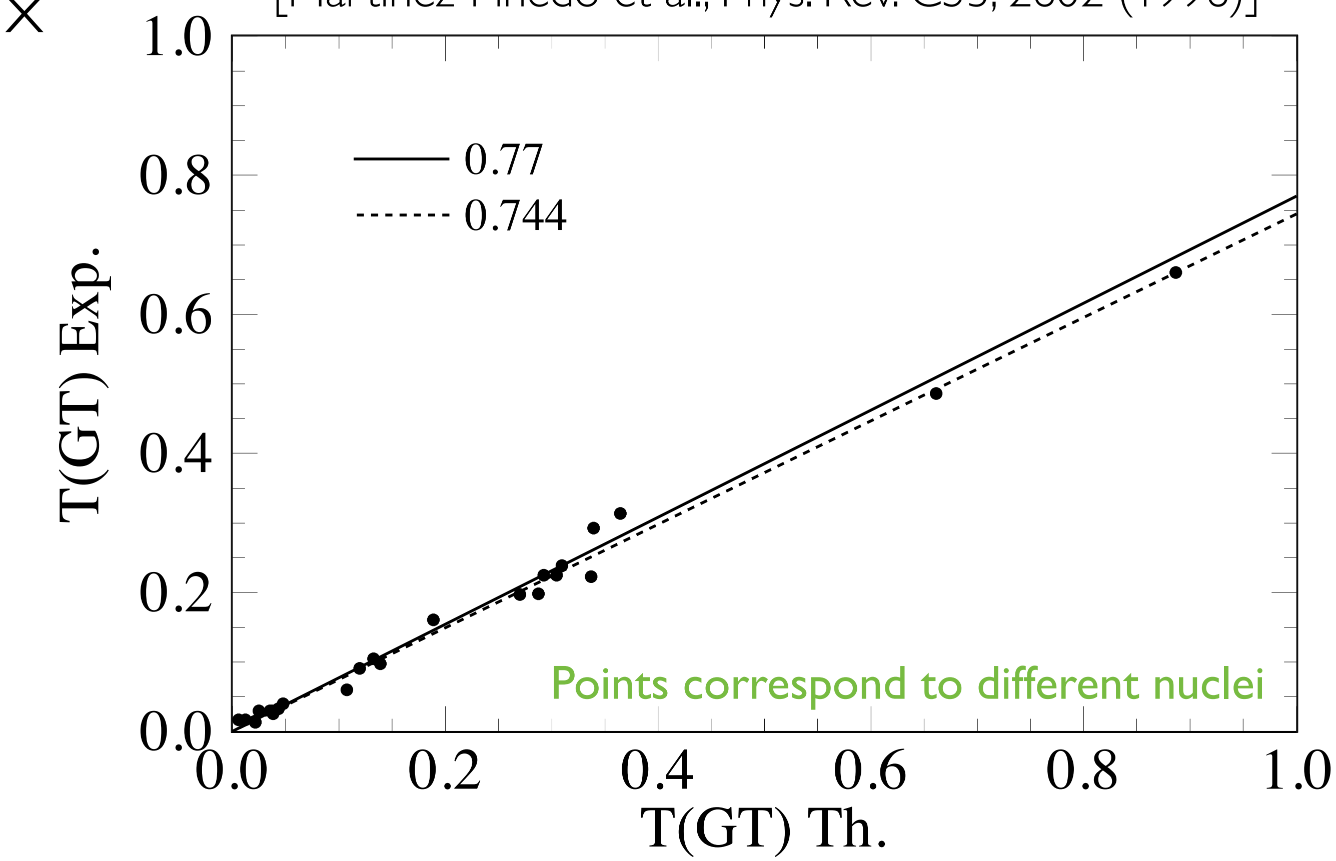
Nuclear uncertainties

- How well do we know nuclear matrix elements?

Stark example of problems: Gamow-Teller transitions in nuclei

- Well measured for large range of nuclei (30<A<60)
- Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
- Matrix elements systematically off by 20–30%
- "Correct" by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]



$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

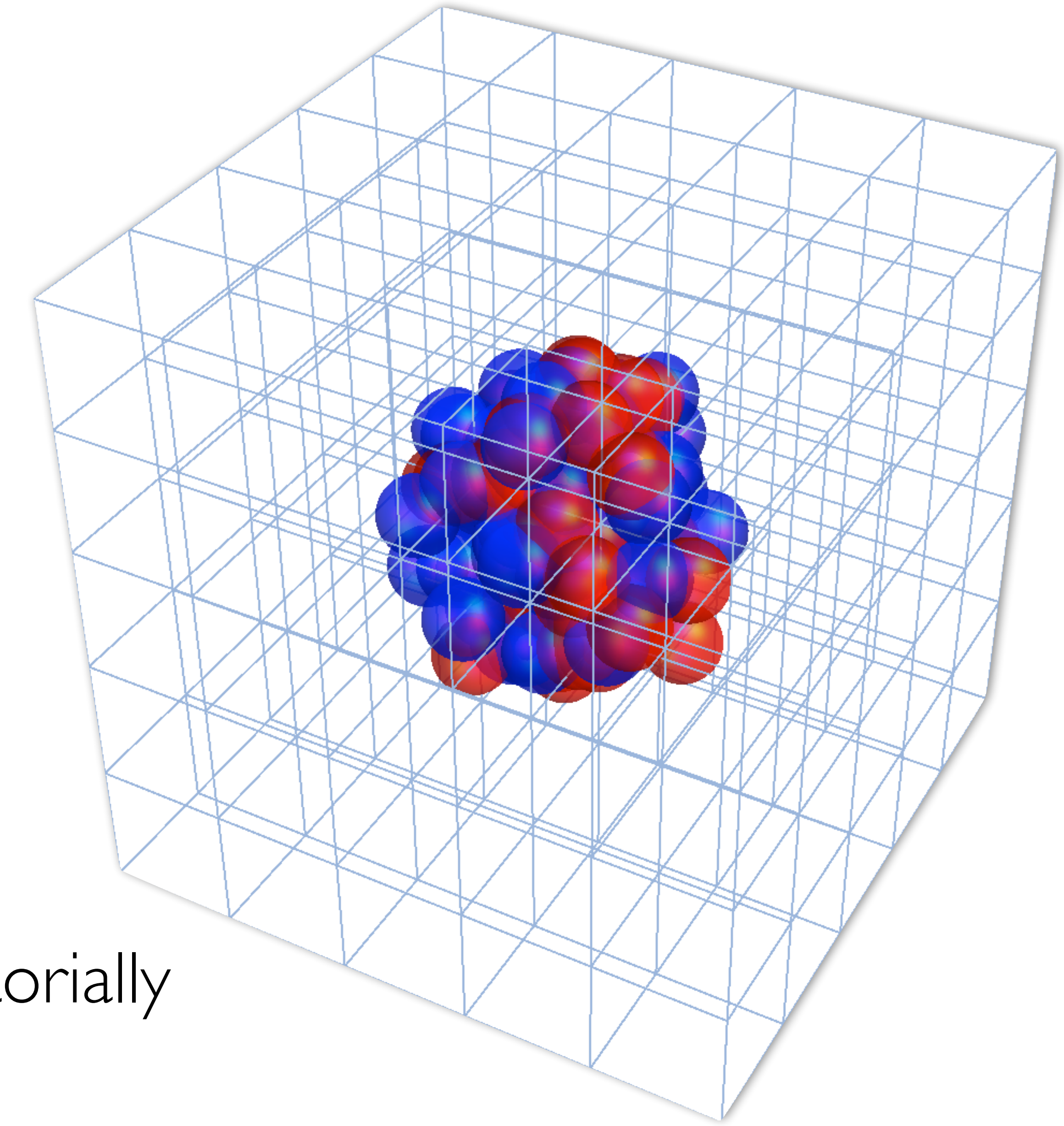
$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}^k_\pm || i \rangle}{\sqrt{2J_i + 1}}$$

Nuclear theory at the intensity frontier

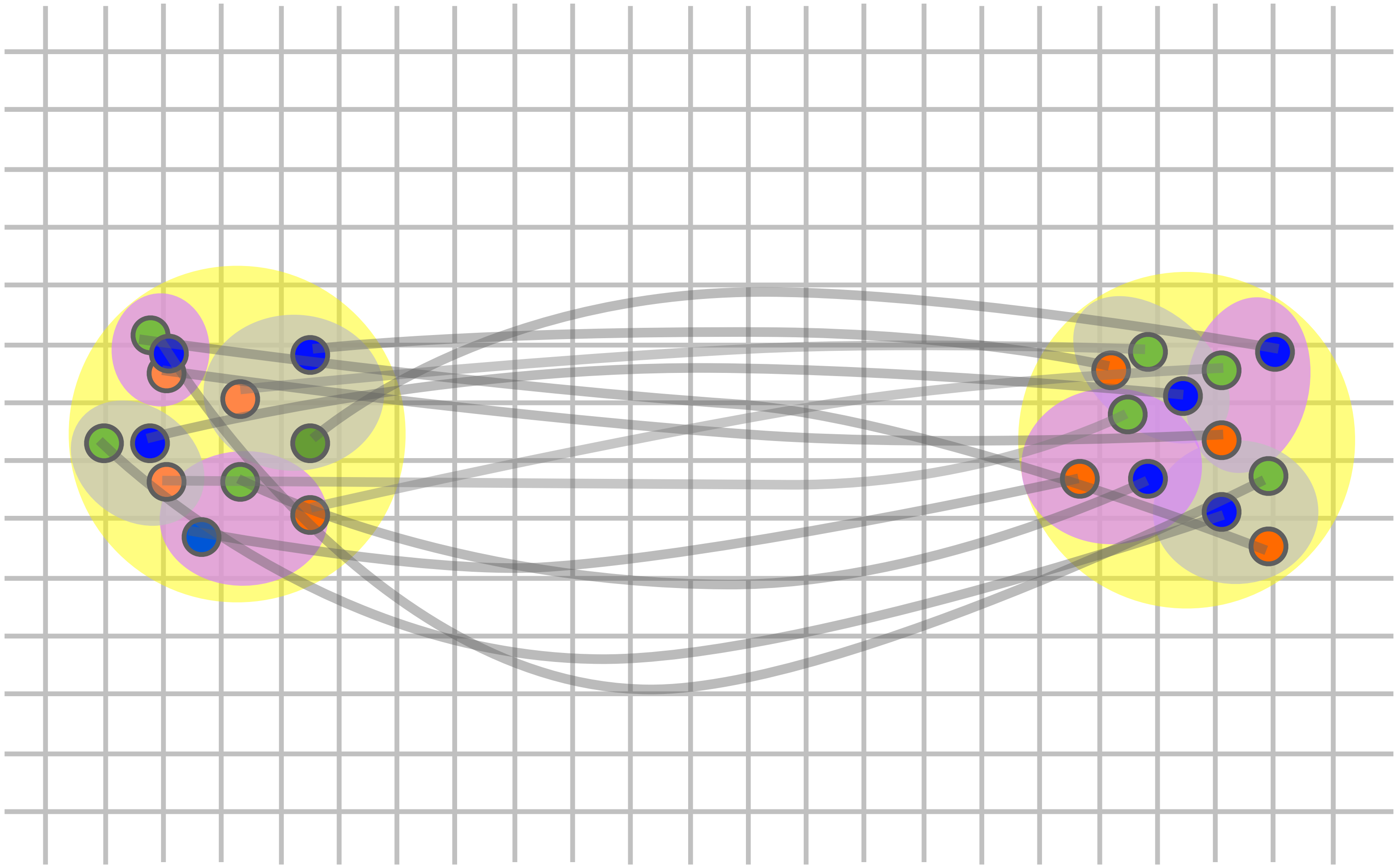
- Definitive need for precision determinations of nuclear matrix elements
  - Must be based on the Standard Model (no hand-waving)
  - Must have fully quantified uncertainties
  - Timeframe and precision goals set by experiment
- Current state is far from this
- *Nuclear physics is the new flavour physics!*
  - Develop appropriate tools

QCD for Nuclear Physics

- Nuclear physics from lattice QCD
- In practice: a hard problem
- At least two exponentially difficult challenges
  - Noise: probabilistic method so statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially



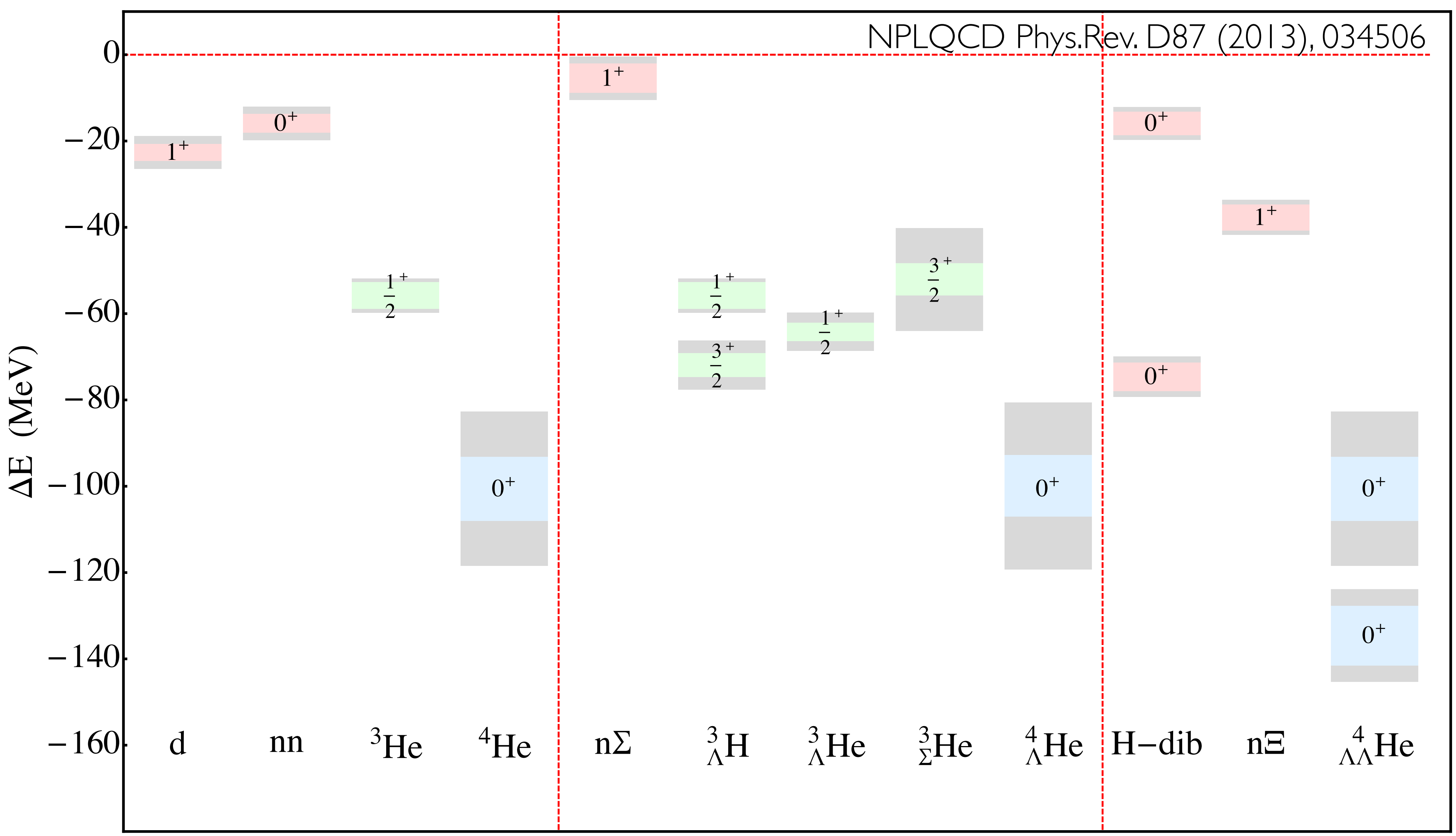
- Quarks need to be tied together in all possible ways
  - $N_{contractions} = N_u! N_d! N_s!$



- Managed using algorithmic trickery [WD & Savage, WD & Orginos; Doi & Endres]
  - Study up to N=72 pion systems, A=5 nuclei

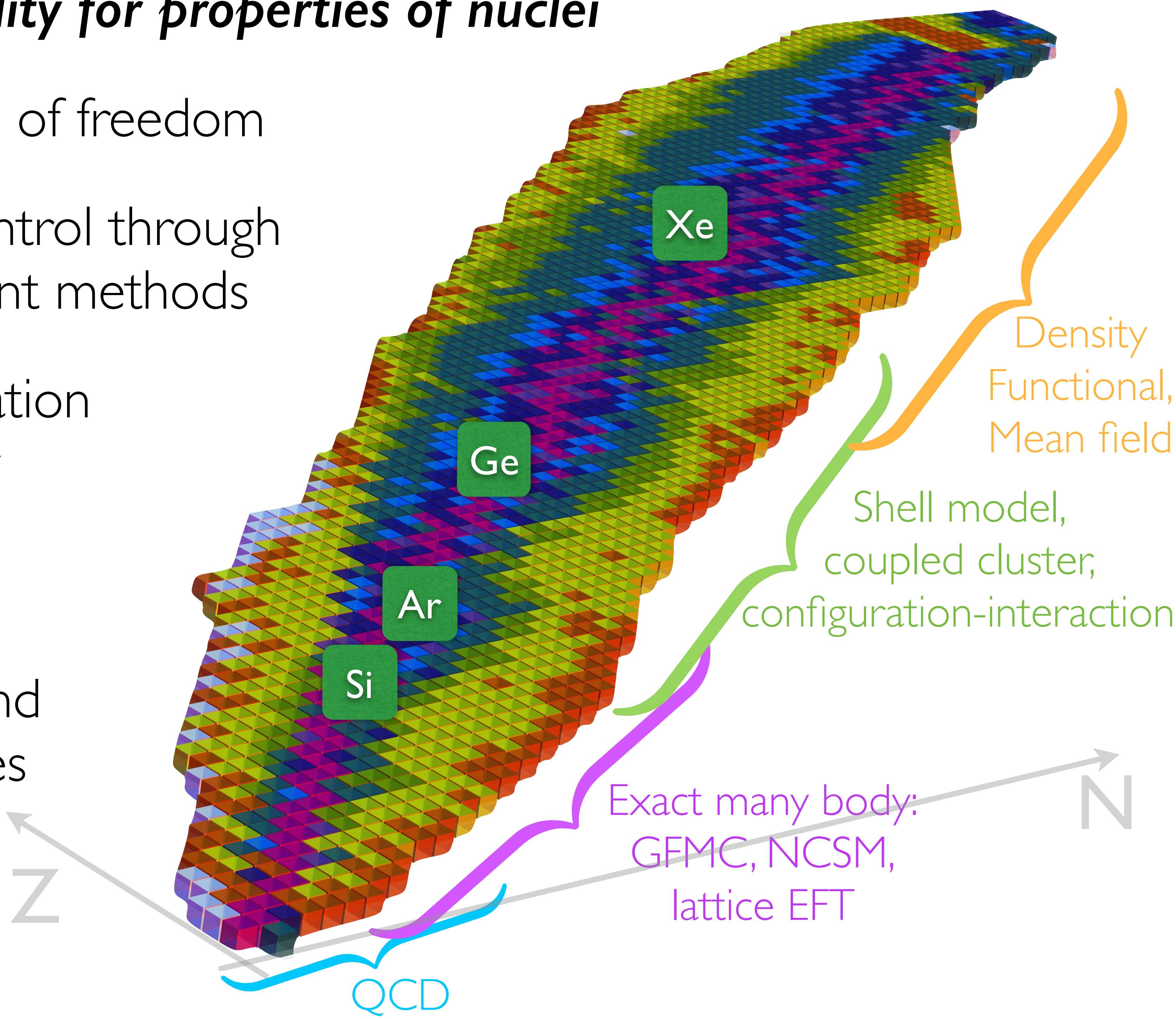
Light nuclei

- Light hypernuclear binding energies @ m_π=800 MeV



Precision Nuclear Physics

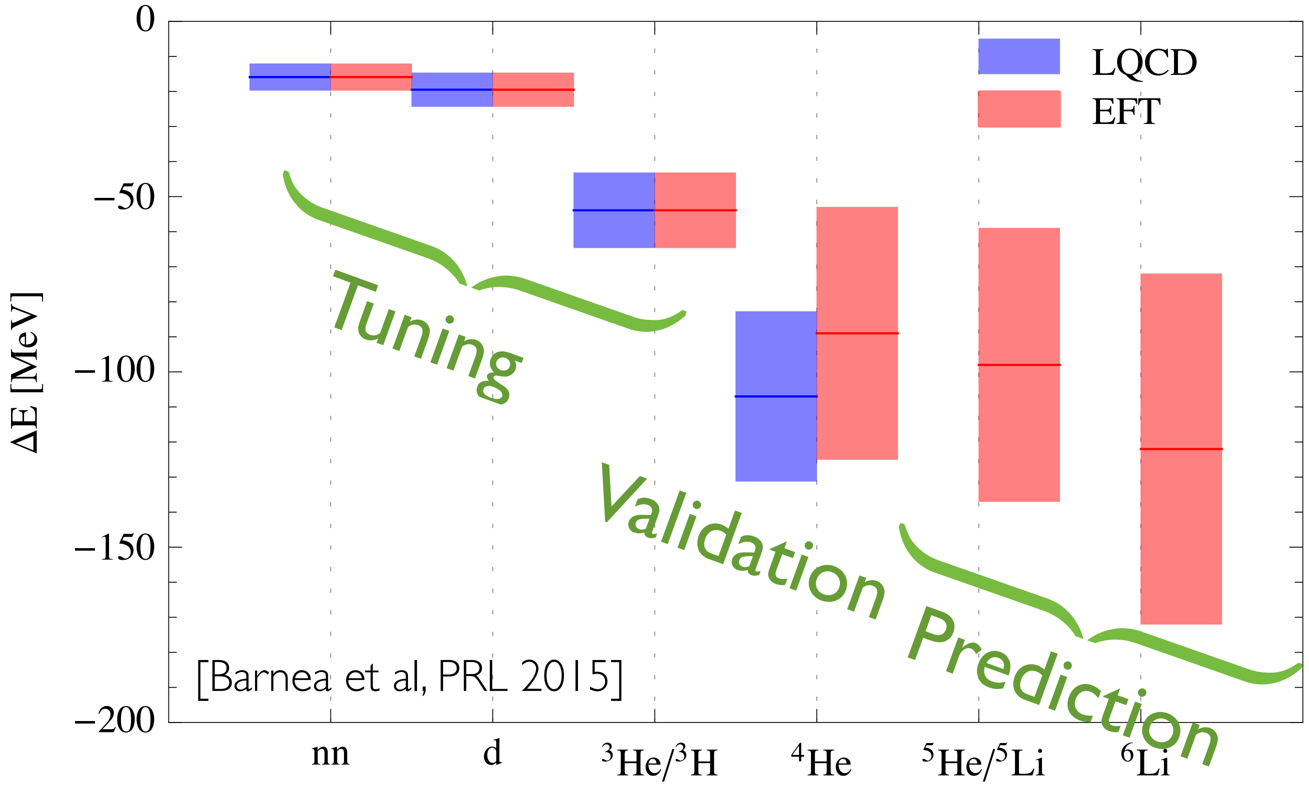
- ***Goal: Predictive capability for properties of nuclei***
- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
  - QCD forms a foundation determines few body interactions & matrix elements
  - Match existing EFT and many body techniques onto QCD



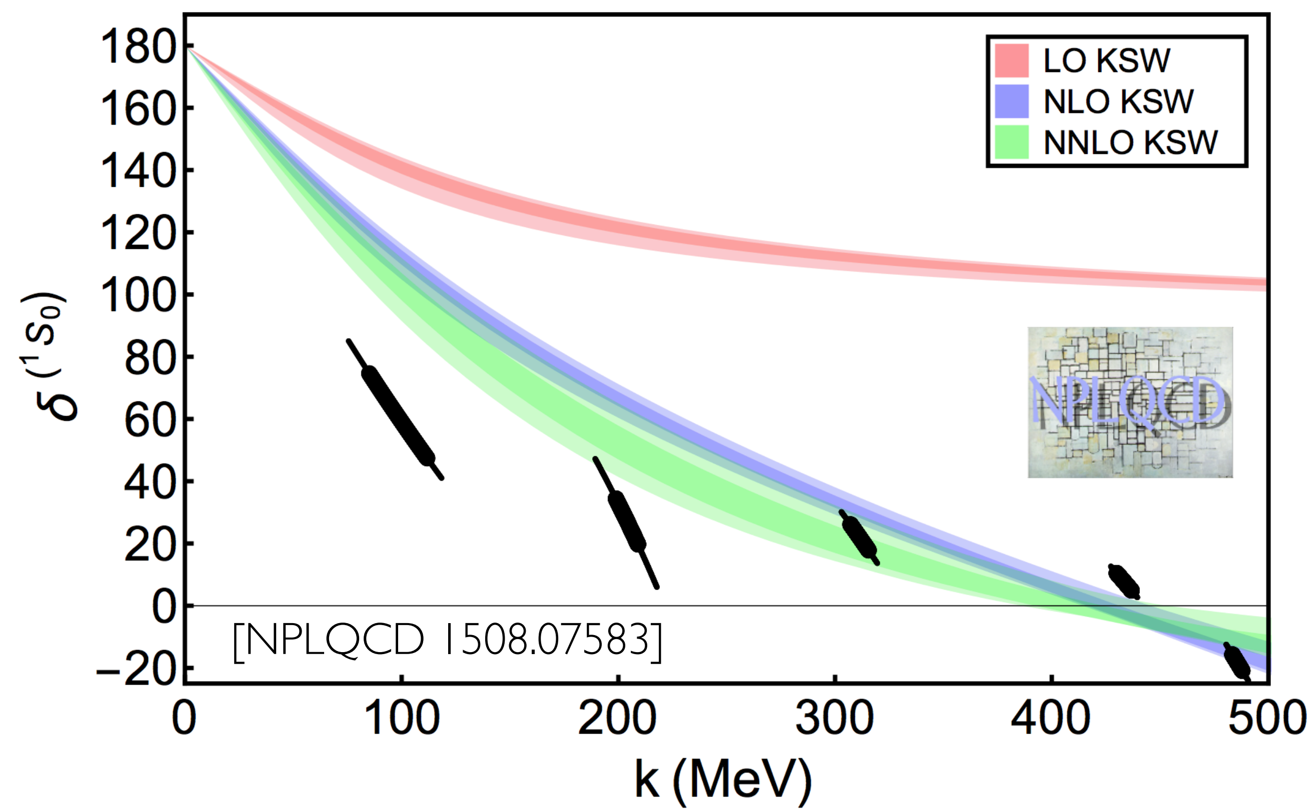
Heavy quark universe

- Combining LQCD and nuclear EFT
- Heavy quarks: even spectro/scattering requires QCD matching

@ m_π = 800 MeV



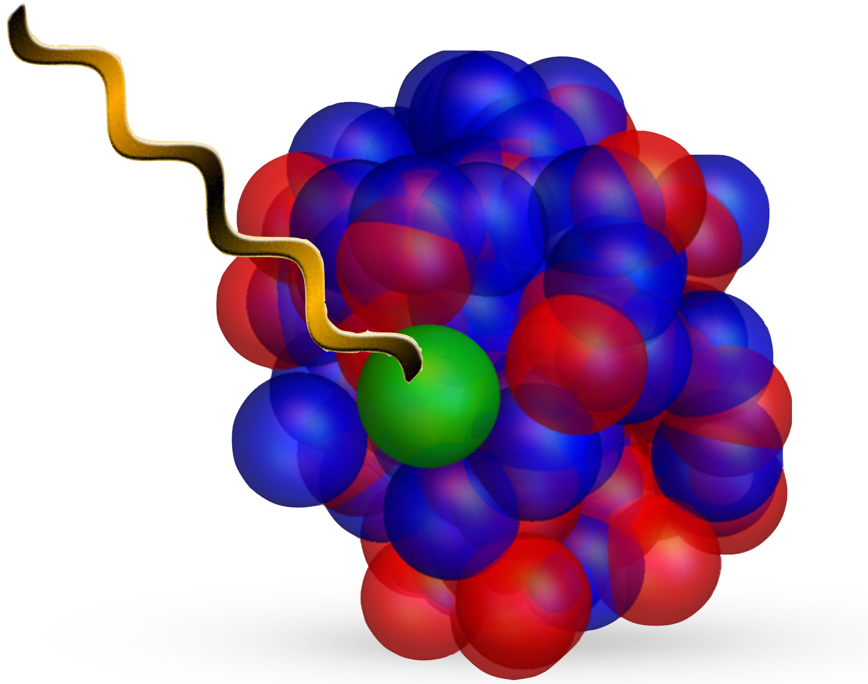
@ m_π = 450 MeV

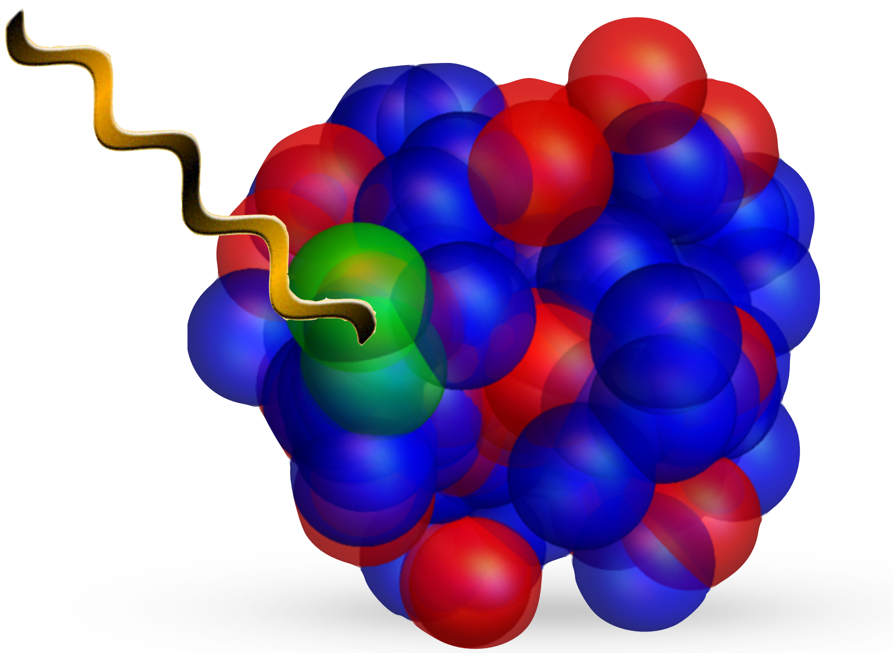


- Equally important for matrix elements

External currents and nuclei

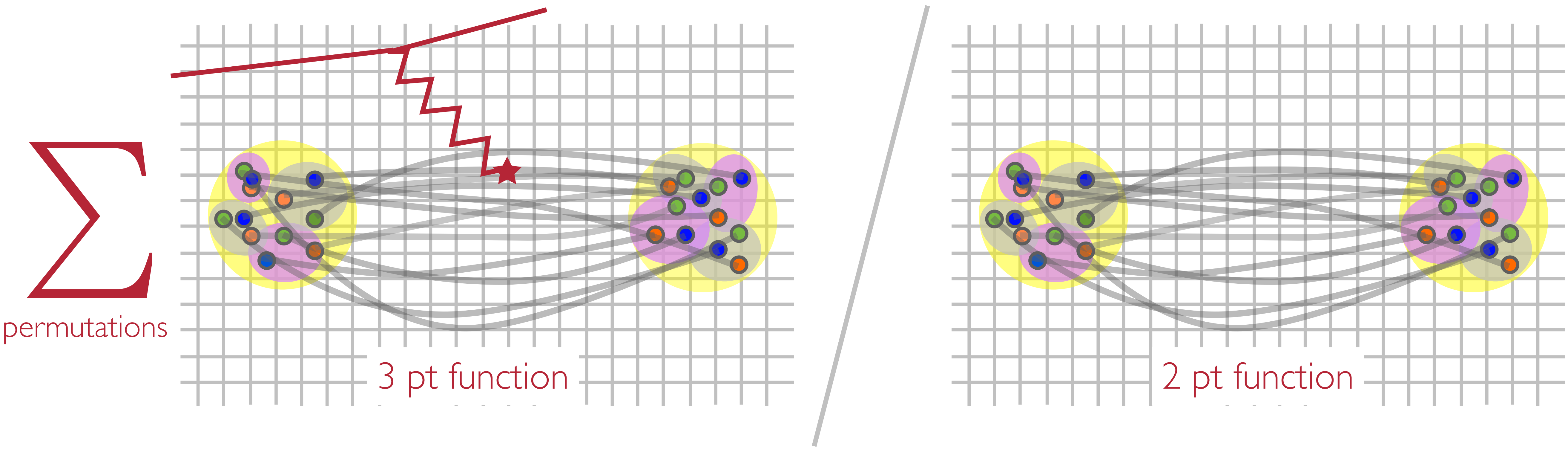
- Xe in LQCD not likely any time soon
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- LQCD: determine one body current from single nucleon
- LQCD: determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei





Nuclear matrix elements

- For deeply bound nuclei, use the techniques as for single hadron matrix elements



  - At large time separations gives ground-state matrix element of current

- For near threshold states, need to be careful with volume effects

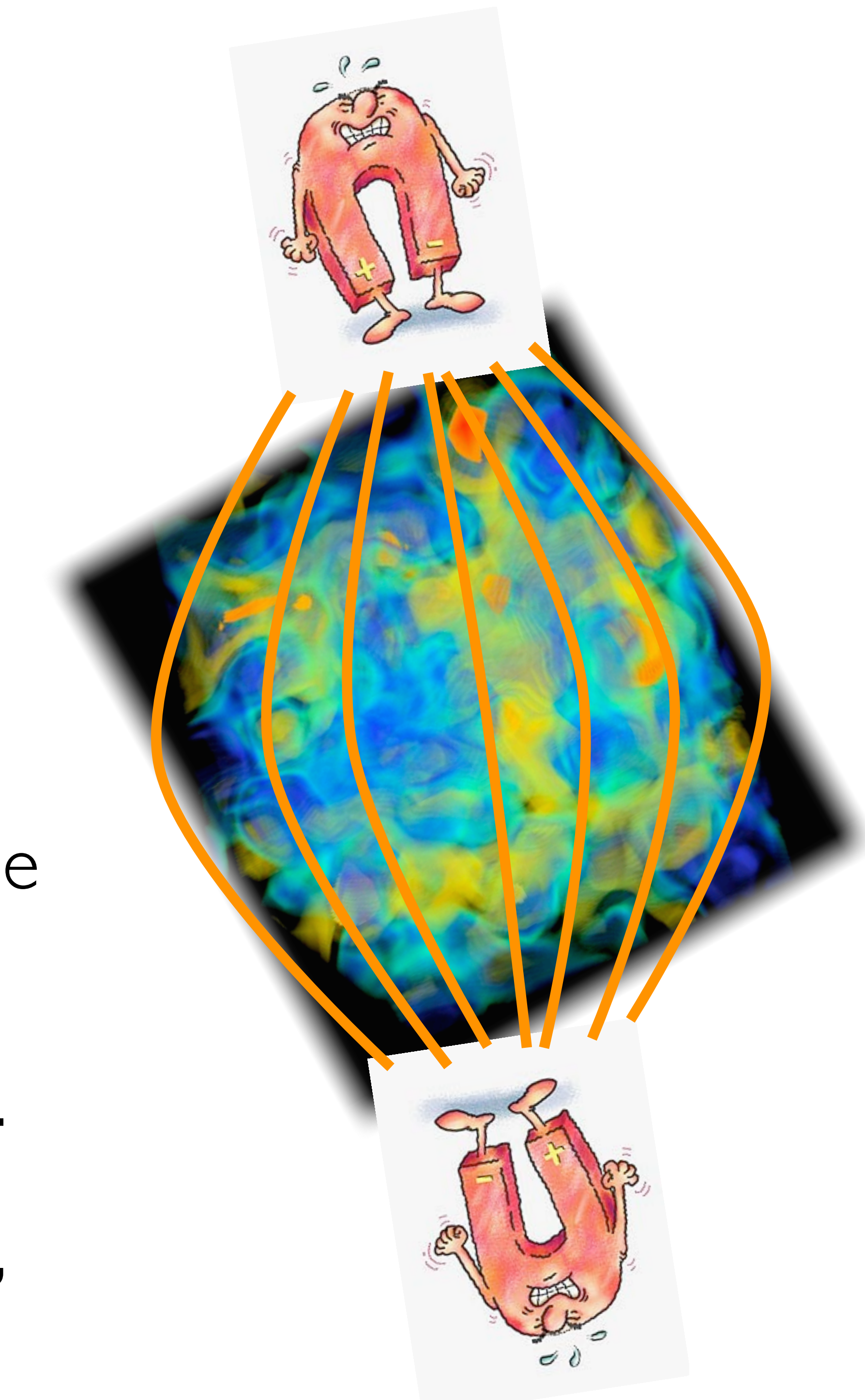
- Calculations of matrix elements of currents in light nuclei just beginning for A<5

Background field methods

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} \\ - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Eg: magnetic moments, polarisabilities, …
  - Not restricted to simple EM fields (axial, twist-2,…)



Magnetic moments of nuclei

- Magnetic field in z-direction (quantised n)

$$U_\mu^{\mathrm{QCD}} \longrightarrow U_\mu^{\mathrm{QCD}} \cdot U_\mu^{(Q)}$$

$$U_\mu^{(Q)}(x) \;=\; e^{i\frac{6\pi Q_q \tilde{n}}{L^2} x_1 \delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L} x_2 \delta_{\mu,1}\delta_{x_1,L-1}}$$

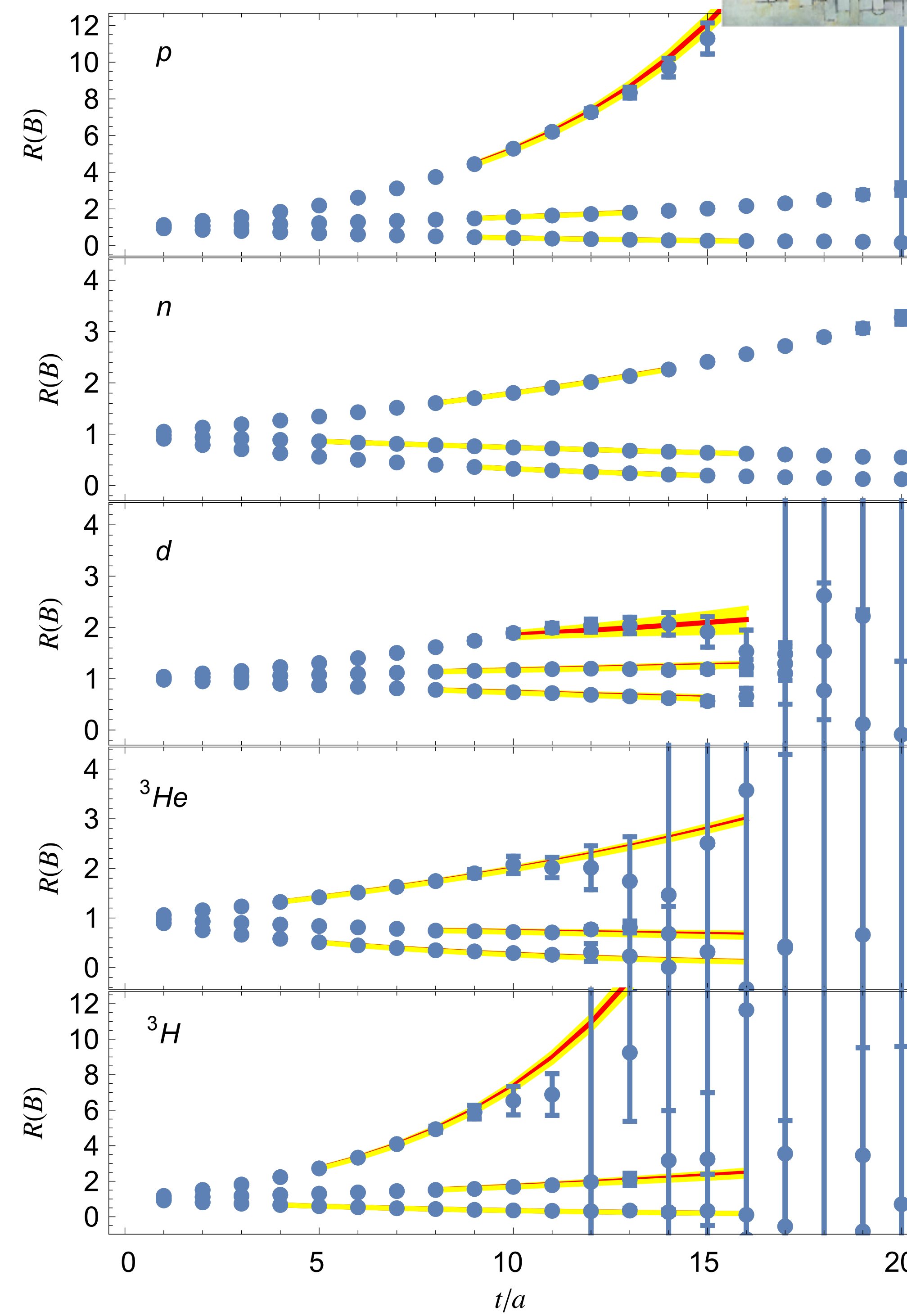
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \;\stackrel{t\to\infty}{\longrightarrow}\; Z e^{-\delta E^{(B)} t}$$

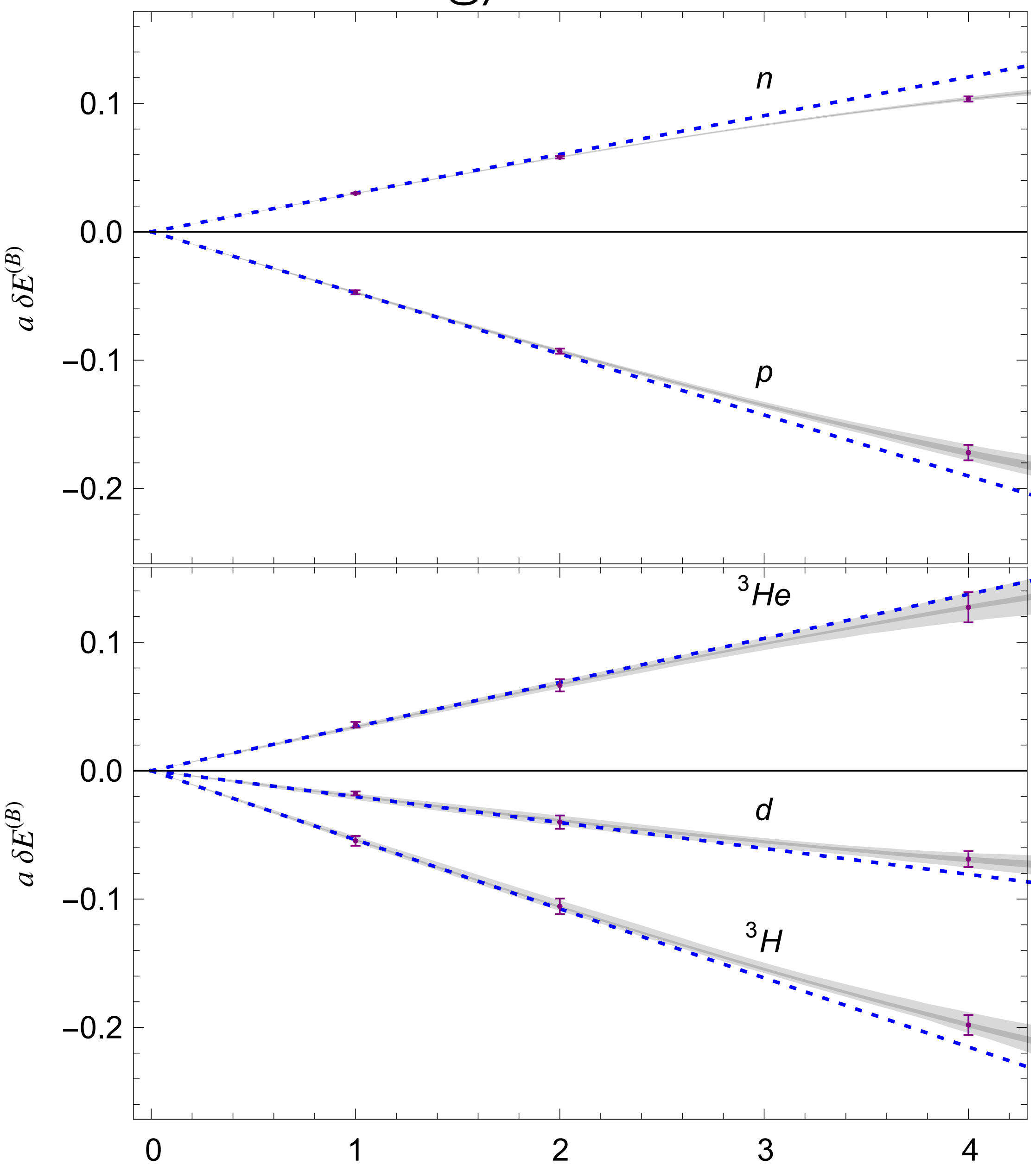
- Careful to be in single exponential region of each correlator



[NPLQCD 1409.3556]

Magnetic moments of nuclei

Energy shift vs B

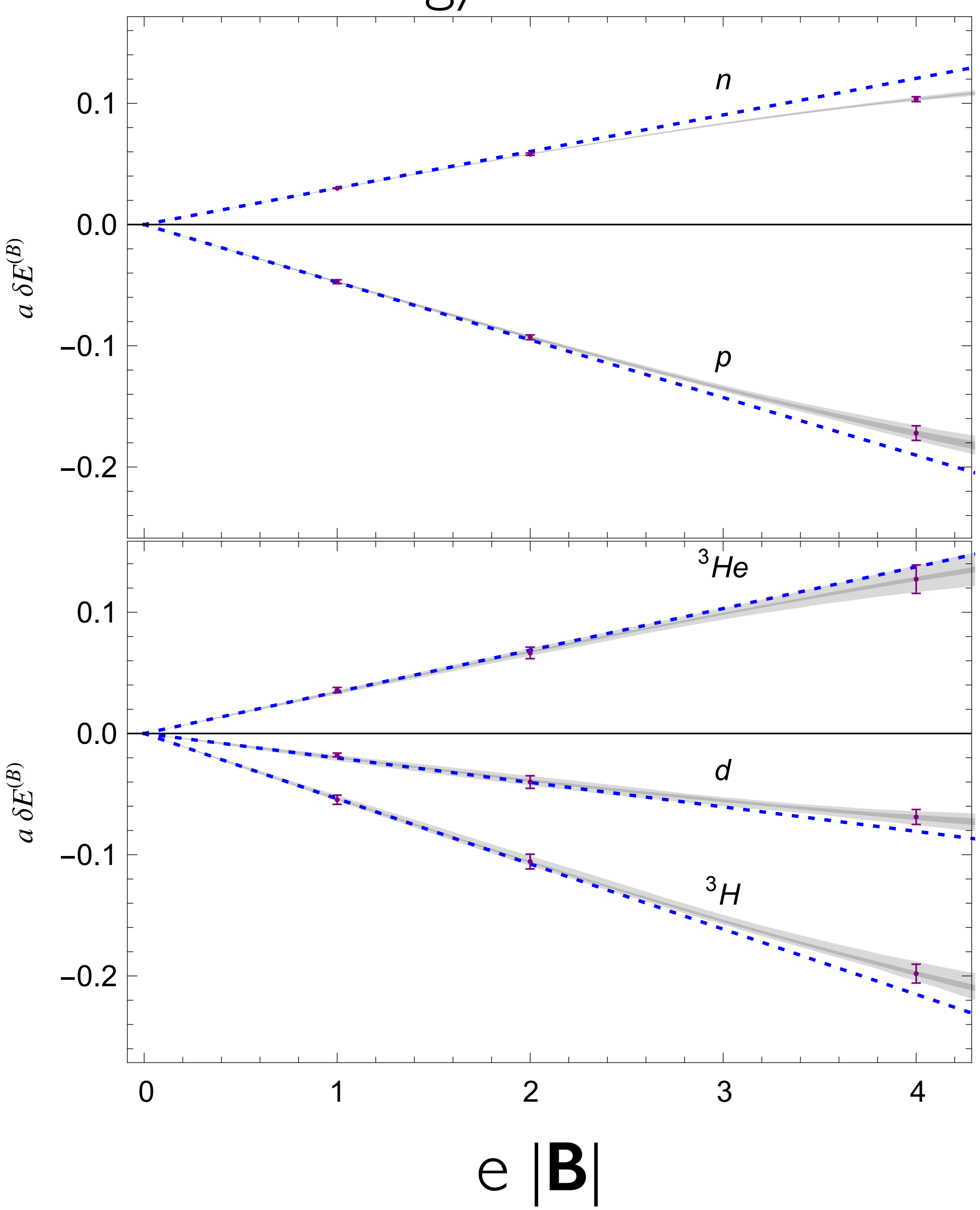


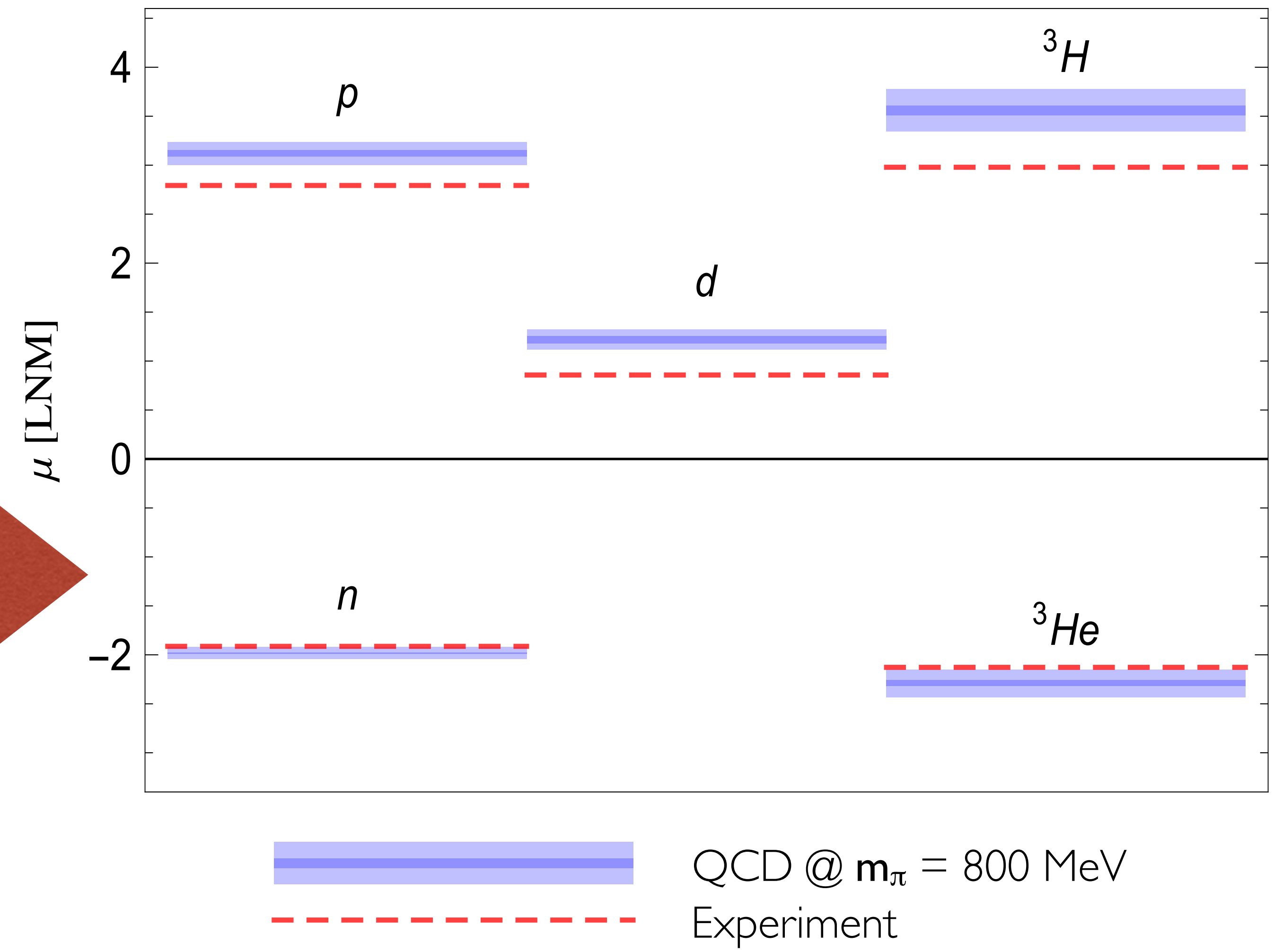
e |**B**|

[NPLQCD PRL 2014]

Magnetic moments of nuclei

Energy shift vs B





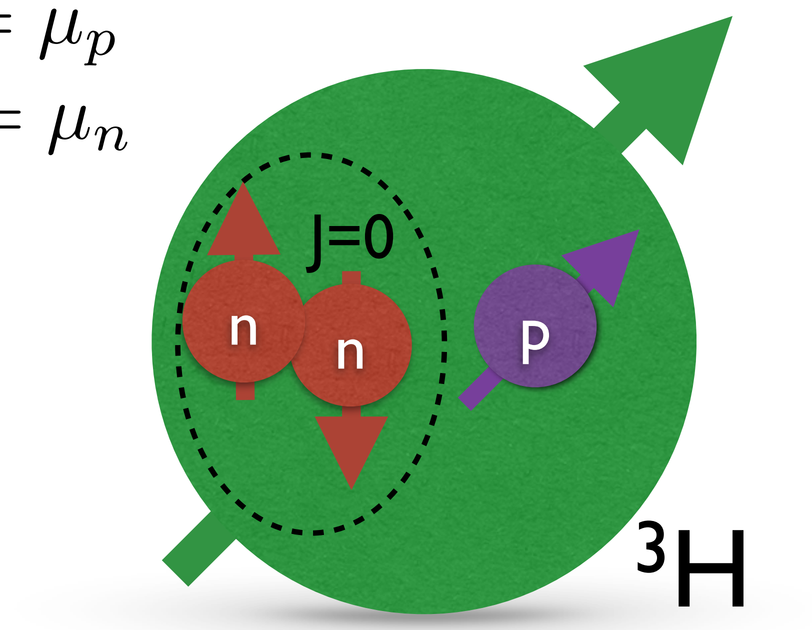
	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

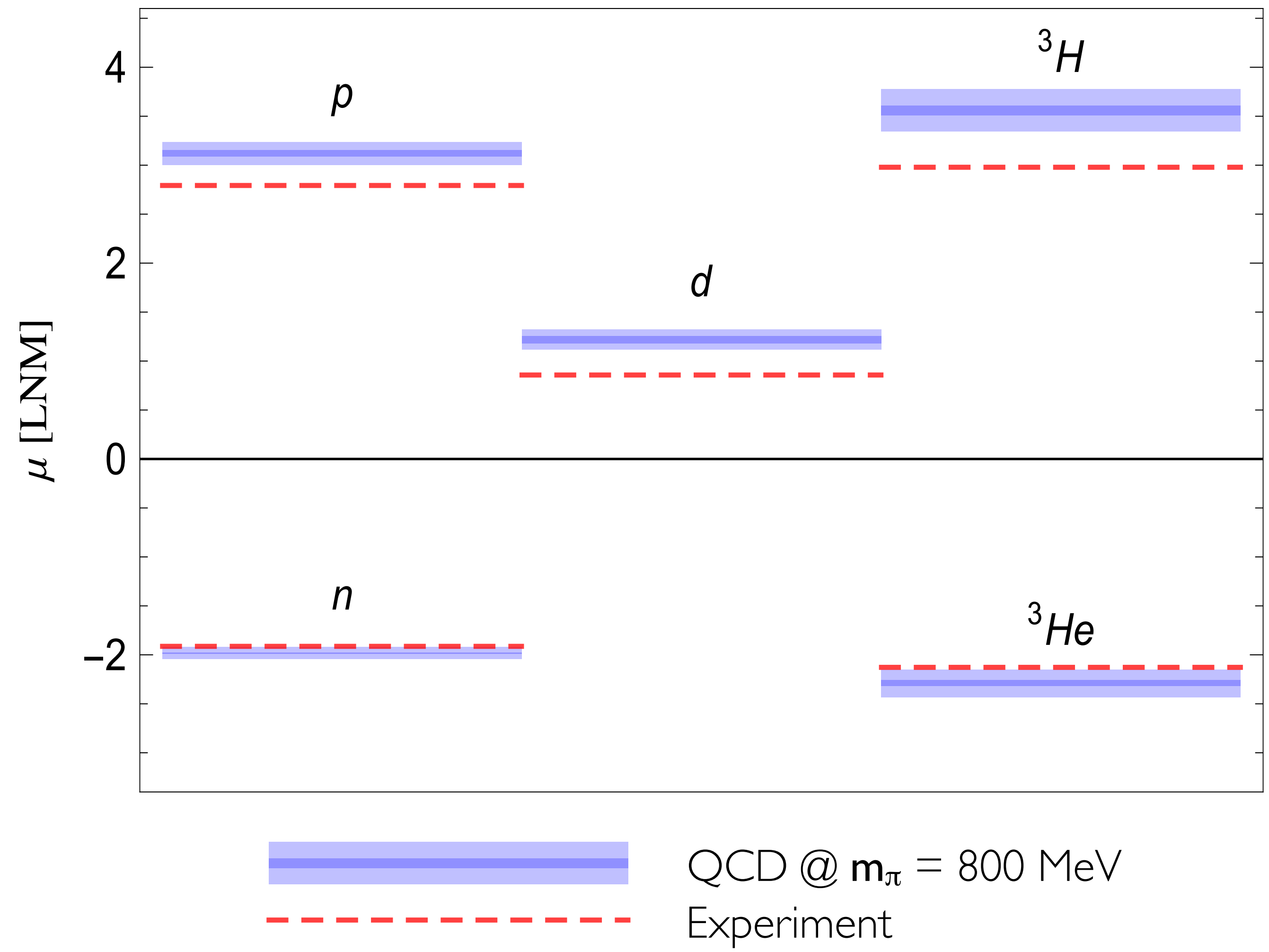
[NPLQCD PRL 2014]

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



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In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL 2014]

Magnetic Polarisabilities

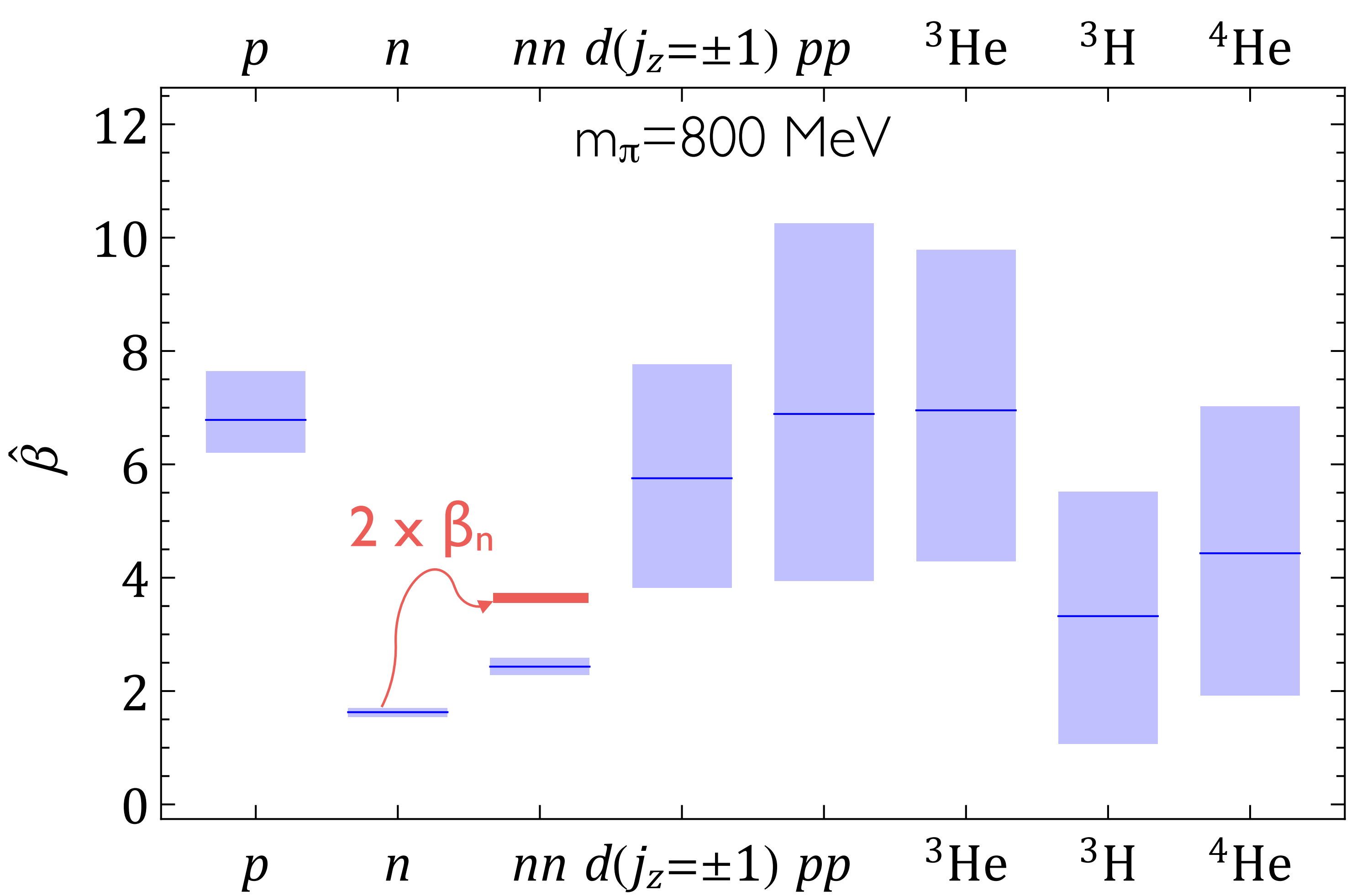
[NPLQCD 1506.05518]

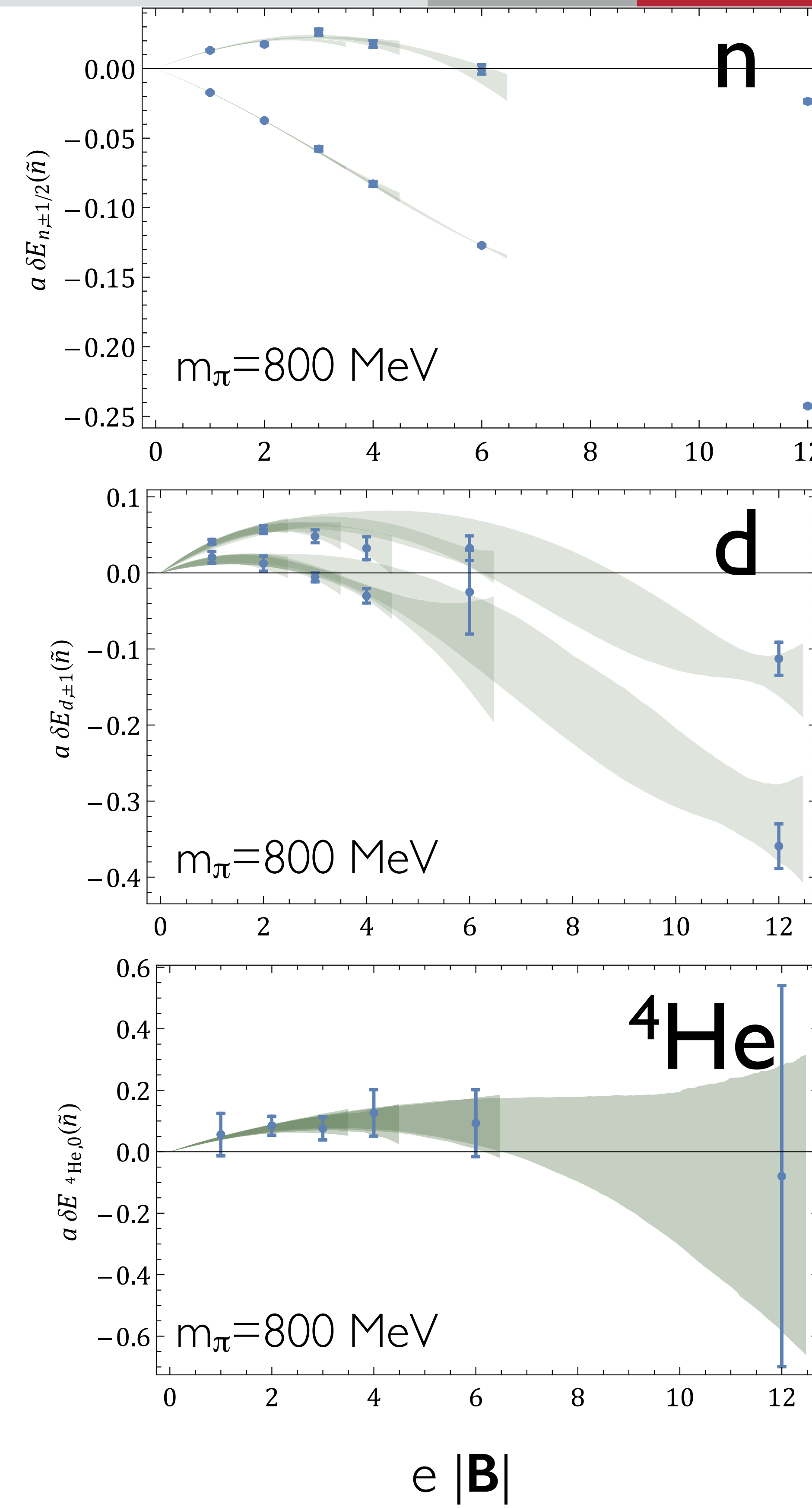
- Second order shifts

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- Care required with Landau levels

- Polarisabilities (dimensionless units)

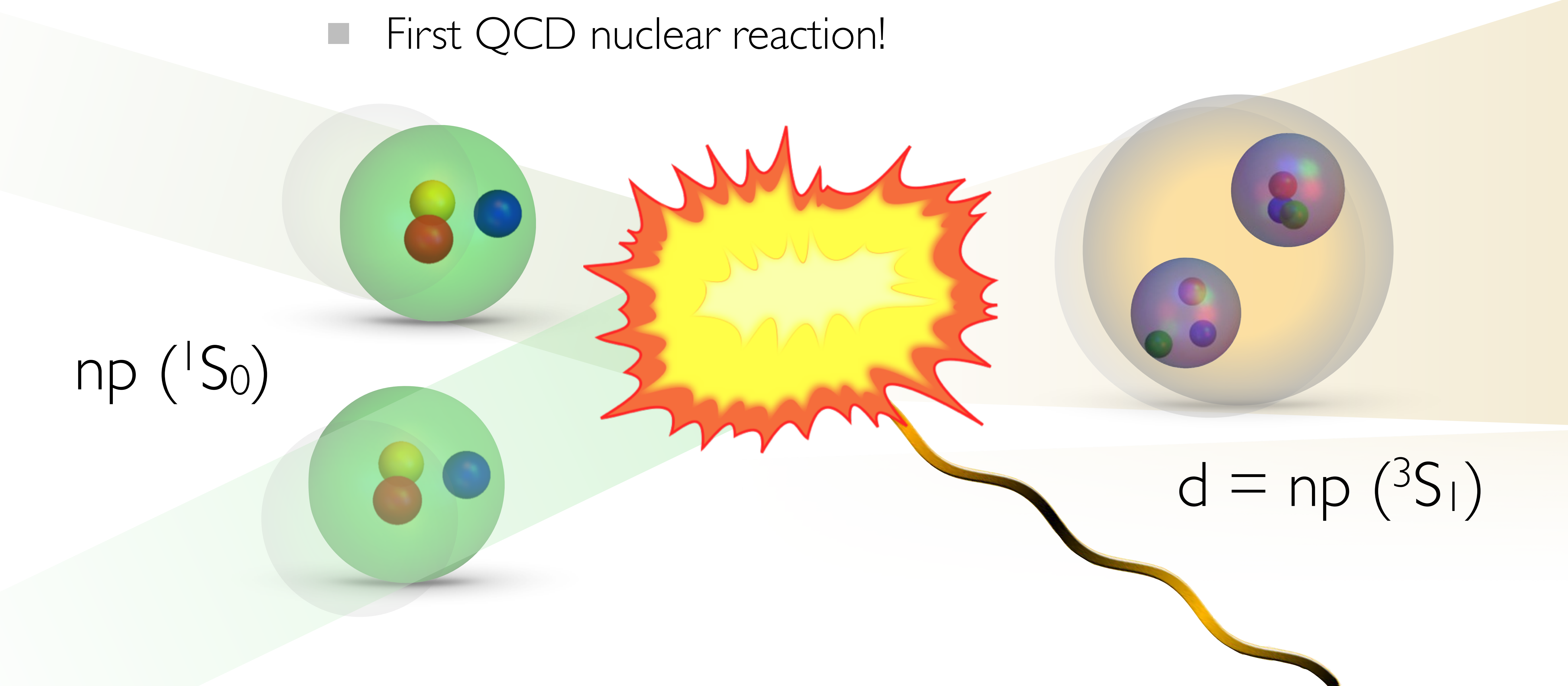




Thermal Neutron Capture Cross-Section

[NPLQCD PRL 115, 132001 (2015)]

- Thermal neutron capture cross-section: $np \rightarrow d\gamma$
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: 2-body contributions ~10%
  - First QCD nuclear reaction!

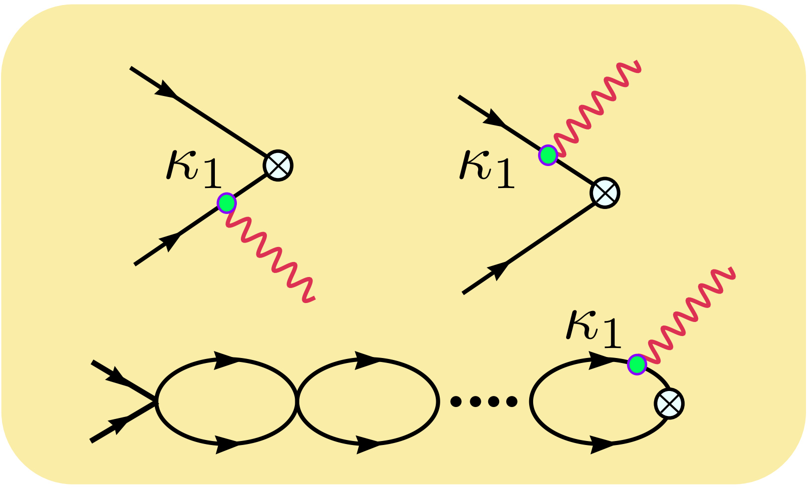


np→dγ in pionless EFT

$$Z_d = 1/\sqrt{1-\gamma_0 r_3}$$

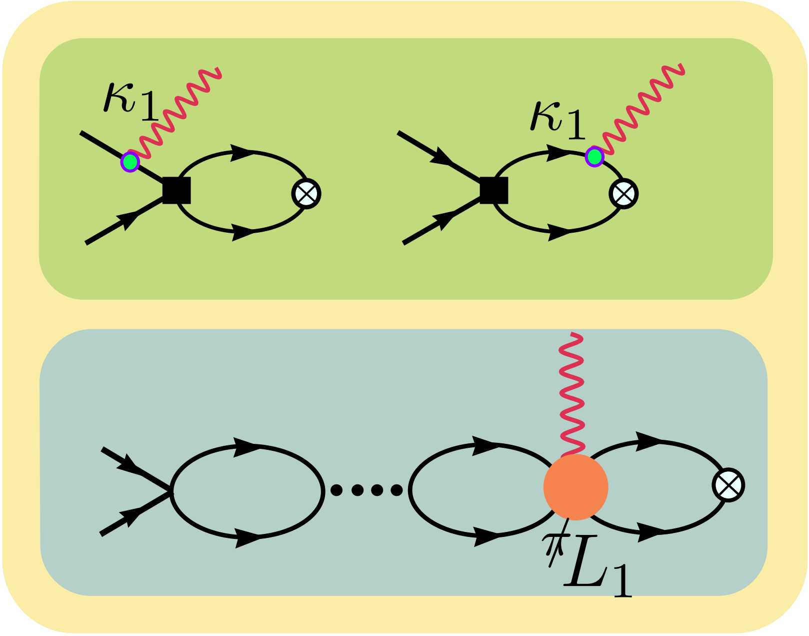
- Cross-section at threshold calculated in pionless EFT

$$\sigma(np \to d\gamma) = \frac{e^2(\gamma_0^2 + |\mathbf{p}|^2)^3}{M^4 \gamma_0^3 |\mathbf{p}|} |\tilde{X}_{M1}|^2 + \ldots$$



- EFT expansion at LO given by mag. moments NLO contributions from short-distance two nucleon operators

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 - i|\mathbf{p}|} \times \left[\frac{\kappa_1 \gamma_0^2}{\gamma_0^2 + |\mathbf{p}|^2} \left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2} r_1 |\mathbf{p}|^2 \right) + \frac{\gamma_0^2}{2} l_1 \right]$$



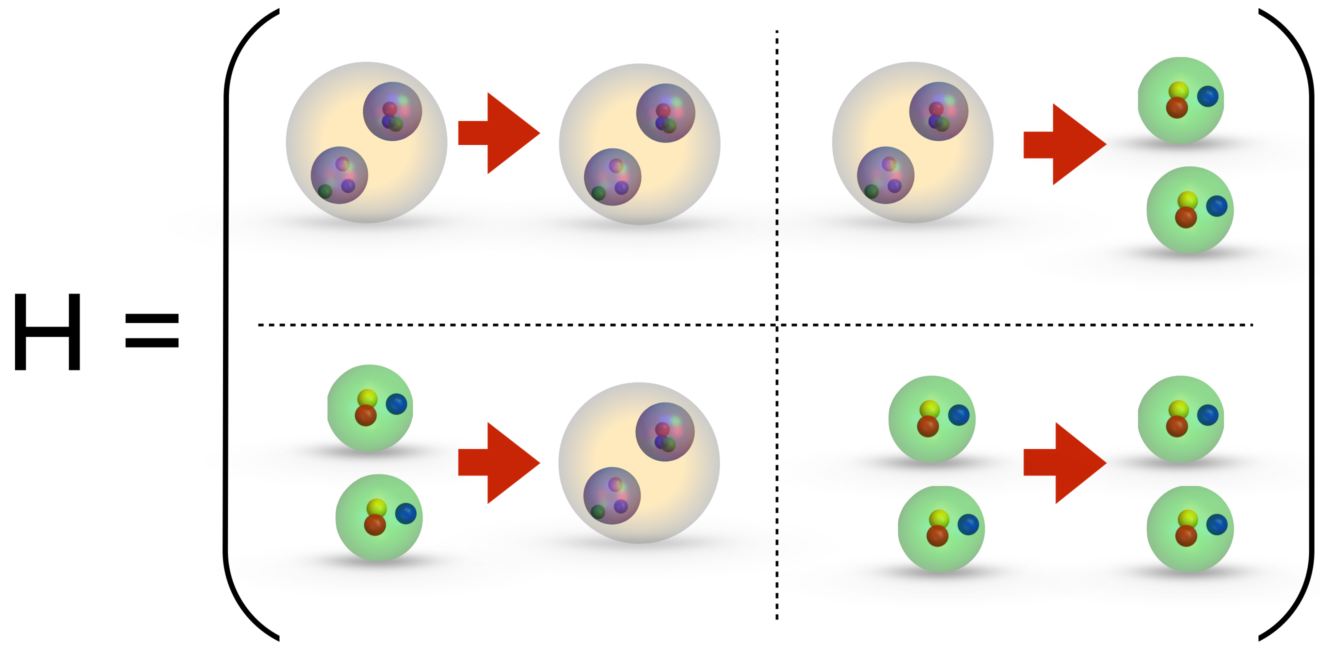
- Phenomenological description with 1% accuracy for E< 1MeV
  - Short distance (MEC) contributes ~10%

MECs: Riska, Phys.Lett. B38 (1972) 193
Hokert et al, Nucl.Phys. A217 (1973) 14
EFT: Chen et al.,Nucl.Phys. A653 (1999) 386
Chen et al, Phys.Lett. B464 (1999) 1
Rupak Nucl.Phys. A678 (2000) 405

$np \rightarrow d\gamma$

[NPLQCD PRL 115, 132001 (2015)]

- Presence of magnetic field mixes $I_z=J_z=0$ $^3S_1$ and $^1S_0$ *np* systems



- Wigner SU(4) super-multiplet (spin-flavour) symmetry relates $^3S_1$ and $^1S_0$ states (diagonal elements approximately equal)
  - Shift of eigenvalues determined by transition amplitude

$$\Delta E_{^3S_1,^1S_0} = \mp\left(\kappa_1 + \overline{L}_1\right)\frac{eB}{M} + \ldots$$

- More generally eigenvalues depend on transition amplitude
[WD, & M Savage 2004, H Meyer 2012]

[NPLQCD PRL 115, 132001 (2015)]

- Iz=Jz=0 correlation matrix

Lattice correlator with $^3S_1$ source and $^1S_0$ sink

$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C_{^3S_1,^3S_1}(t;\mathbf{B}) & C_{^3S_1,^1S_0}(t;\mathbf{B}) \\ C_{^1S_0,^3S_1}(t;\mathbf{B}) & C_{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$

- Generalised eigenvalue problem

$$[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}\mathbf{C}(t;\mathbf{B})[\mathbf{C}(t_0;\mathbf{B})]^{-1/2}v = \lambda(t;\mathbf{B})v$$

- Ratio of correlator ratios to extract 2-body

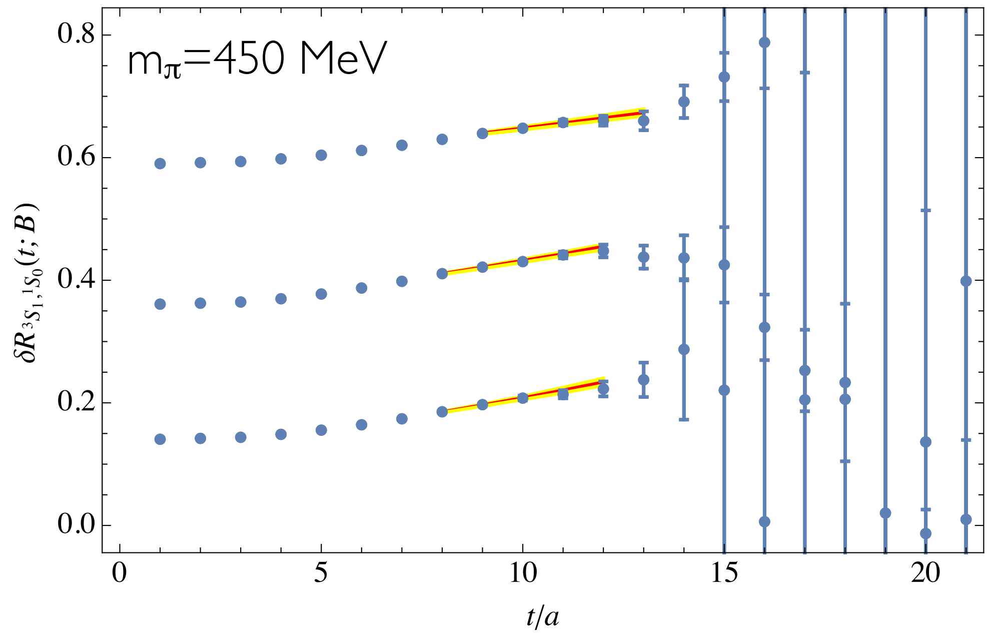
$$R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{\lambda_+(t;\mathbf{B})}{\lambda_-(t;\mathbf{B})} \overset{t\to\infty}{\longrightarrow} \hat{Z}\exp\left[2\ \Delta E_{^3S_1,^1S_0} t\right]$$

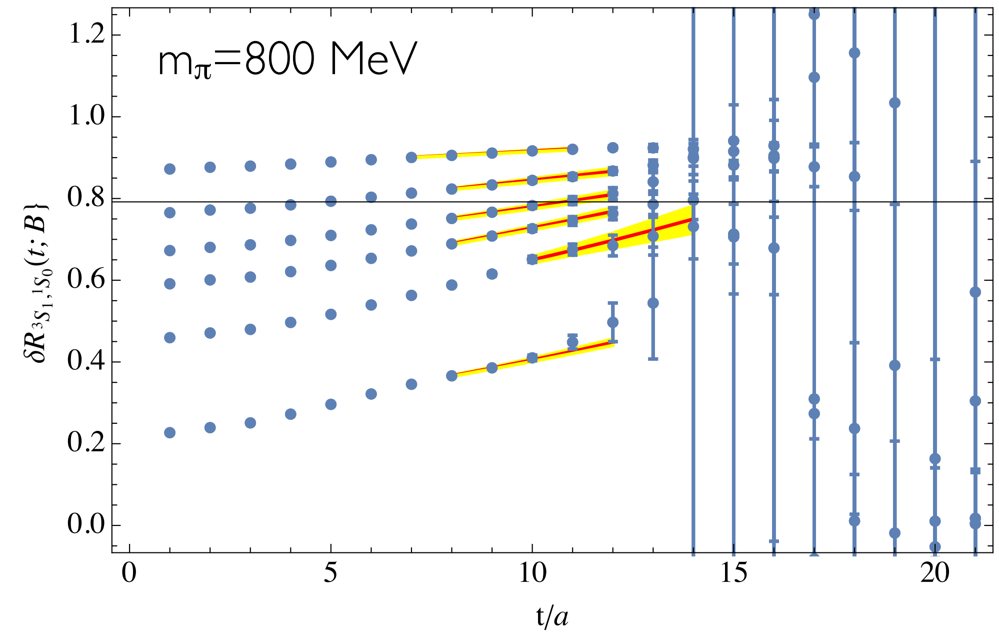
$$\delta R_{^3S_1,^1S_0}(t;\mathbf{B}) = \frac{R_{^3S_1,^1S_0}(t;\mathbf{B})}{\Delta R_p(t;\mathbf{B})/\Delta R_n(t;\mathbf{B})} \to A\ e^{-\delta E_{^3S_1,^1S_0}(\mathbf{B})t}$$

$$\begin{aligned} \delta E_{^3S_1,^1S_0} &\equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \\ &\to 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2) \end{aligned}$$

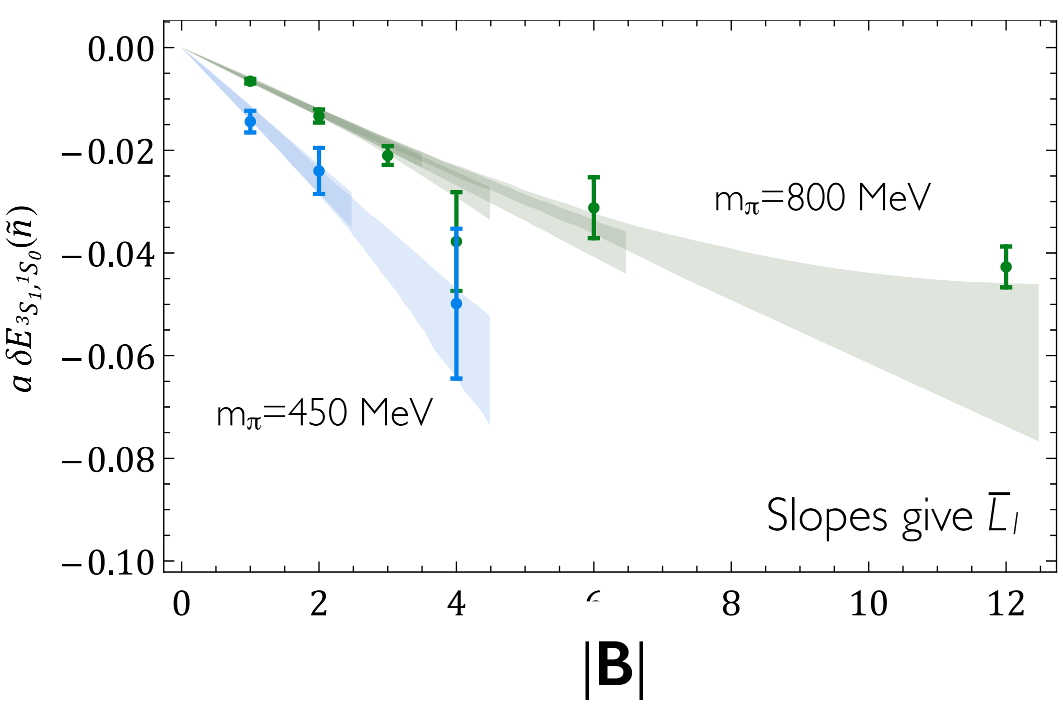
[NPLQCD PRL 115, 132001 (2015)]

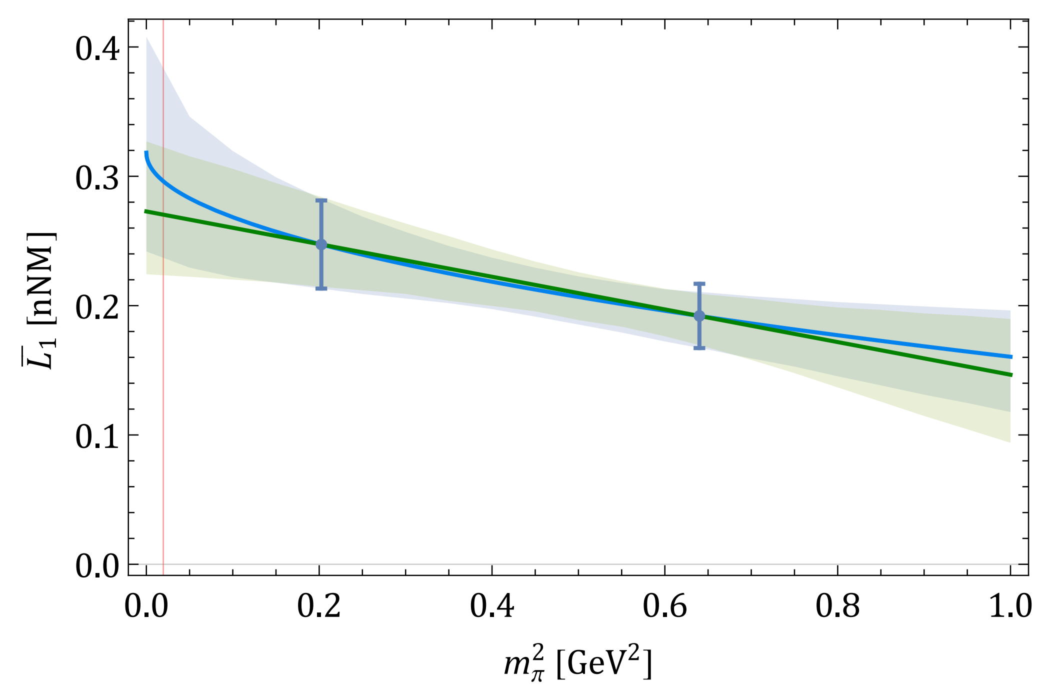
- Correlator ratios for different field strengths





- Field strength & mass dependence





np→dγ

[NPLQCD PRL 115, 132001 (2015)]

- Extracted short-distance contribution at physical mass

$$\overline{L}_1^{\text{lqcd}} = 0.285(\,^{+63}_{-60}\,)\ \text{nNM} \qquad l_1^{\text{lqcd}} = -4.48(\,^{+16}_{-15}\,)\ \text{fm}$$

- Combine with phenomenological nucleon magnetic moment, scattering parameters at incident neutron velocity v=2,200 m/s

$$\sigma^{\text{lqcd}}(np \to d\gamma) = 307.8(1 + 0.273\ \overline{L}_1^{\text{lqcd}})\ \text{mb}$$

$$\sigma^{\text{lqcd}}(np \to d\gamma) = 332.4(\,^{+5.4}_{-4.7}\,)\ \text{mb}$$

c.f. phenomenological value

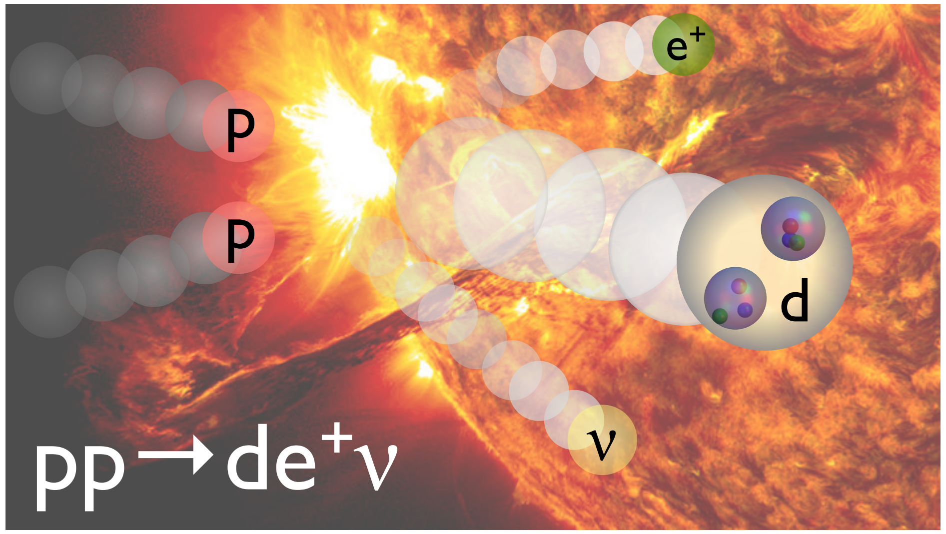
$$\sigma^{\text{expt}}(np \to d\gamma) = 334.2(0.5)\ \text{mb}$$

- NB: at m_π=800 MeV, use LQCD for all inputs (ab initio)

$$\sigma^{800\ \text{MeV}}(np \to d\gamma) \sim 10\ \text{mb}$$

Further matrix elements

- Background field approach to other cases
- Axial coupling to NN system
  - pp fusion: "Calibrate the sun"
  - Muon capture: MuSun @ PSI
  - $d\nu \rightarrow n n e^{+}$: SNO



- Quadrupole moments: requires non-constant fields [Z Davoudi, WD 1507.01908]
- Axial form factors
- Scalar, … matrix elements for dark matter
- Twist-2 operators: EMC effect $\langle N, Z|\bar{q}\gamma_{\{\mu_1}D_{\mu_2}\ldots D_{\mu_n\}}q|N, Z\rangle$

QCD for nuclei

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei
  - Structure: magnetic moments and polarisabilities
  - Electroweak interactions: thermal capture cross-section
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Critical role in current and upcoming intensity frontier experimental program
  - Learn many interesting things about nuclear physics along the way

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