HVP contribution to the muon anomalous magnetic moment from lattice QCD

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HPQCD collaboration

Seattle
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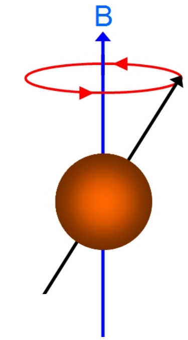
Work with: Bipasha Chakraborty, Gordon Donald, Rachel Dowdall, Jonna Koponen, Peter Lepage

Using the Darwin (9600 core) Sandybridge/infiniband cluster at Cambridge, part of STFC's DiRAC HPC facility

Muon anomalous magnetic moment

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad a_\mu = \frac{g-2}{2}$$



Measure using polarised muons circulating in E and B fields. At a momentum where $\beta \times E$ terms cancel, difference between precession and cyclotron frequencies:

$$\omega_a = -\frac{e}{m}a_\mu B$$

BNL result:

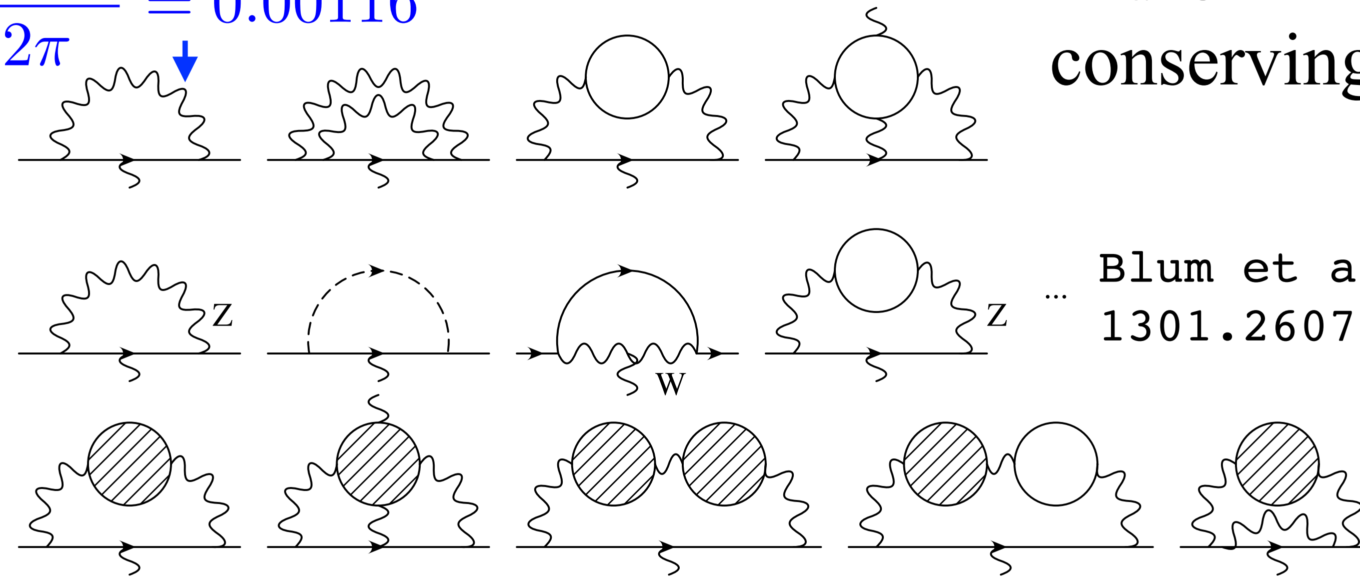
$$a_\mu^{expt} = 11659208.9(6.3) \times 10^{-10}$$

E989 (FNAL) will reduce exptl uncty to 1.6, starting 2017

Standard Model theory expectations

Contributions from QED, EW and QCD interactions. QED dominates. QCD contribs start at α_{QED}^2

$\frac{\alpha_{QED}}{2\pi} = 0.00116$



…

Blum et al, 1301.2607

flavour and CP conserving

LO Hadronic vacuum polarisation (HVP)

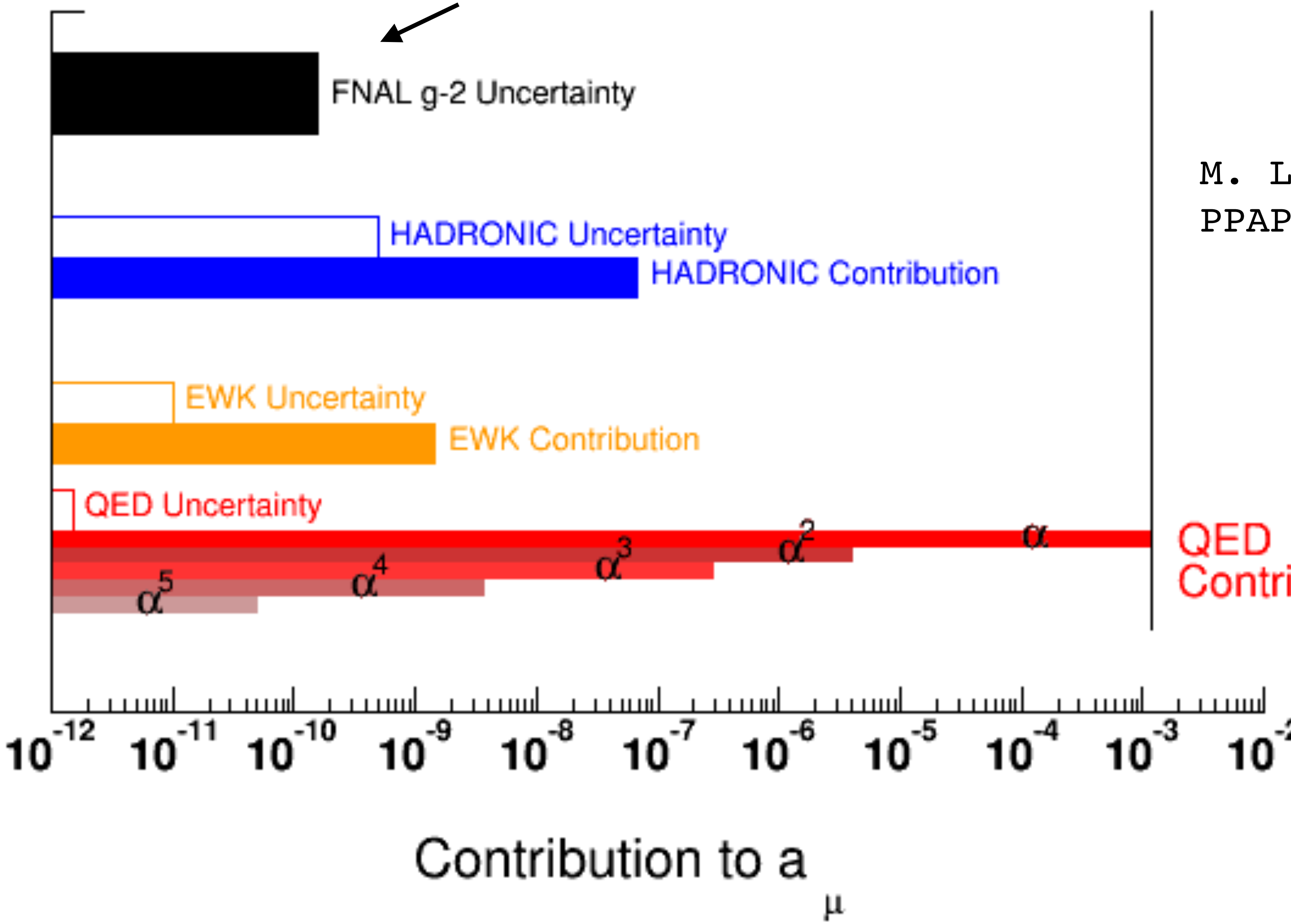
$$a_\mu^{QED} = 11658471.885(4) \times 10^{-10}$$

$$a_\mu^{EW} = 15.4(2) \times 10^{-10}$$

$$a_\mu^{E821} = 11659208.9(6.3) \times 10^{-10}$$

Uncertainty dominated by that from hadronic contribns

planned



M. Lancaster
PPAP2015

Hadronic contributions

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation, so assume:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$

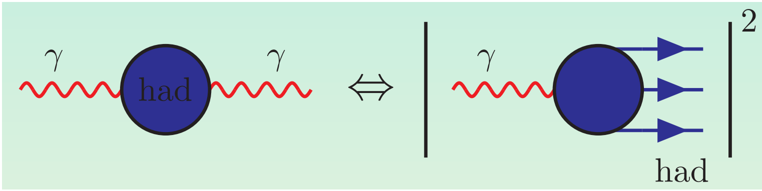
NLO+NNLO

Kurz et al, 1403.6400

$$a_\mu^{HVP, no\,new\,physics} = 719.8(6.8) \times 10^{-10}$$

Best method to date for HVP uses exptl e+e- cross-section

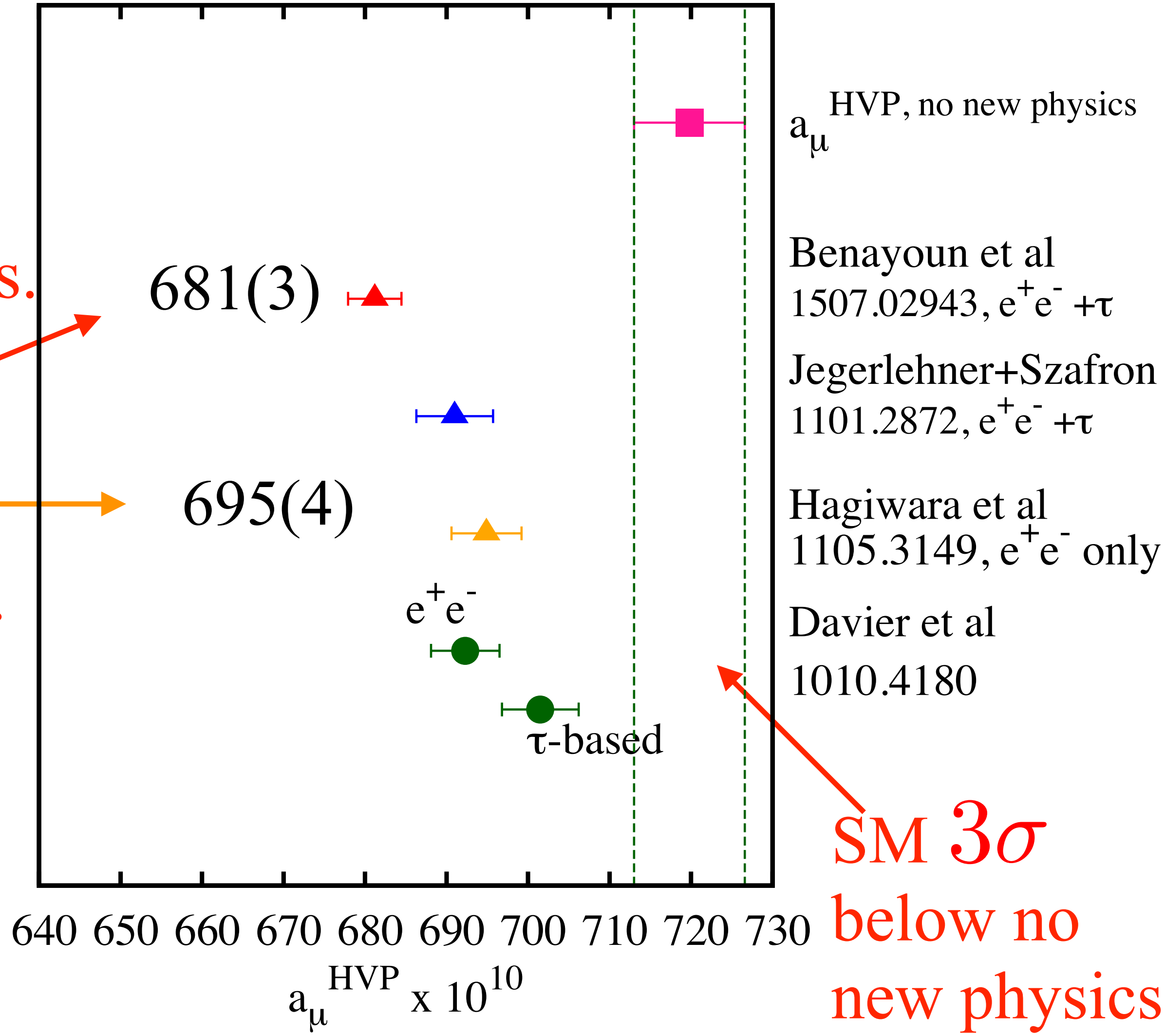
$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\sigma^0_{had}(s)K(s)$$



$e^+e^- \to \gamma^* \to hadrons$

"bare" cross-section
but inc. final-state radiation

some "tension" between results.
Difference is
use of BaBar radiative
return data.
BES III data appearing now …



SM 3σ
below no
new physics

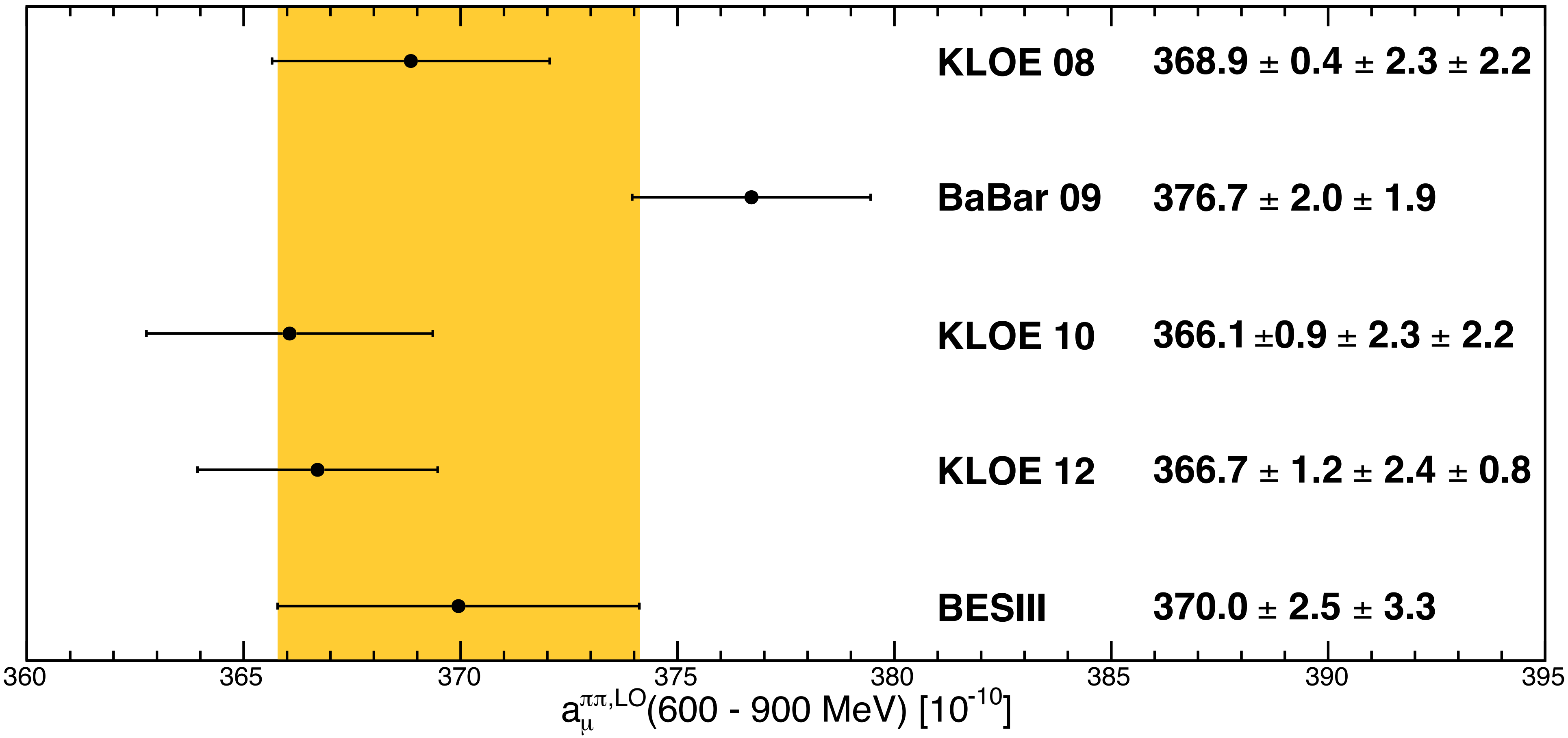
Hagiwara et al:

$$a_\mu^{HVP} = 694.9(4.3) \times 10^{-10}$$

SM discrepancy:

$$24.9(8.0) \times 10^{-10}$$

$\pi^+\pi^-$: new data from BESIII; arXiv:1507.08188v2



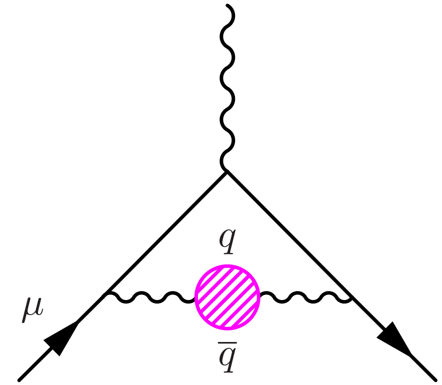
Full analysis inc. BES data still to be done

T. Teubner, talk at Benasque, 2015

Lattice calculation of HVP

Analytically continue to Euclidean q^2.

$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$



Blum, hep-lat/0212018

connected contribution for flavour i

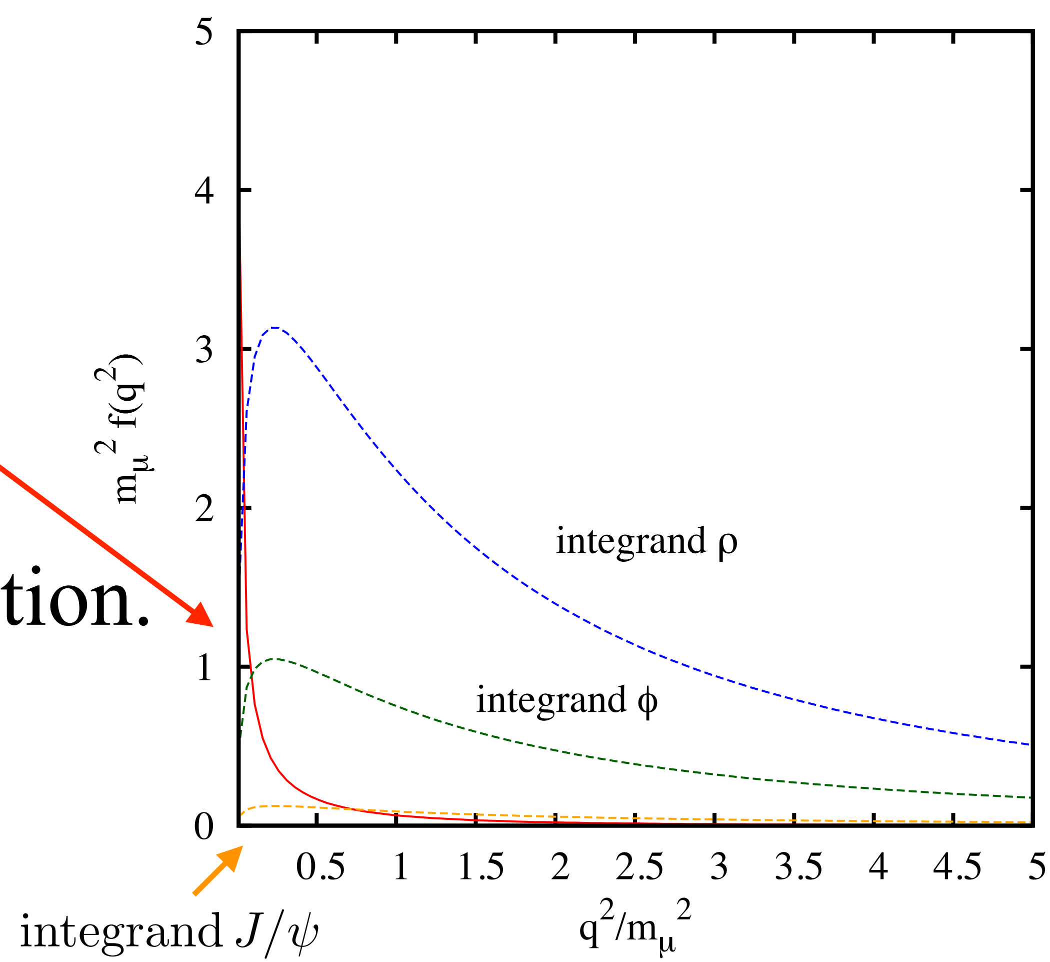
$f(q^2)$ is divergent function with scale set by m_μ

$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

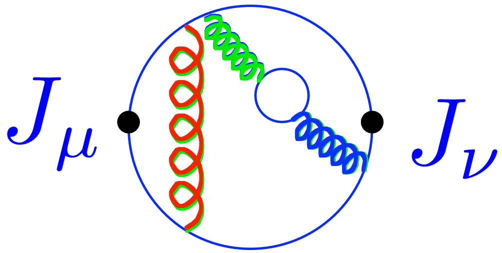
is vacuum polarisation function.

Test with mesons:

$$\hat{\Pi}(q^2) \propto \frac{1}{m_V^2} - \frac{1}{q^2 + m_V^2}$$



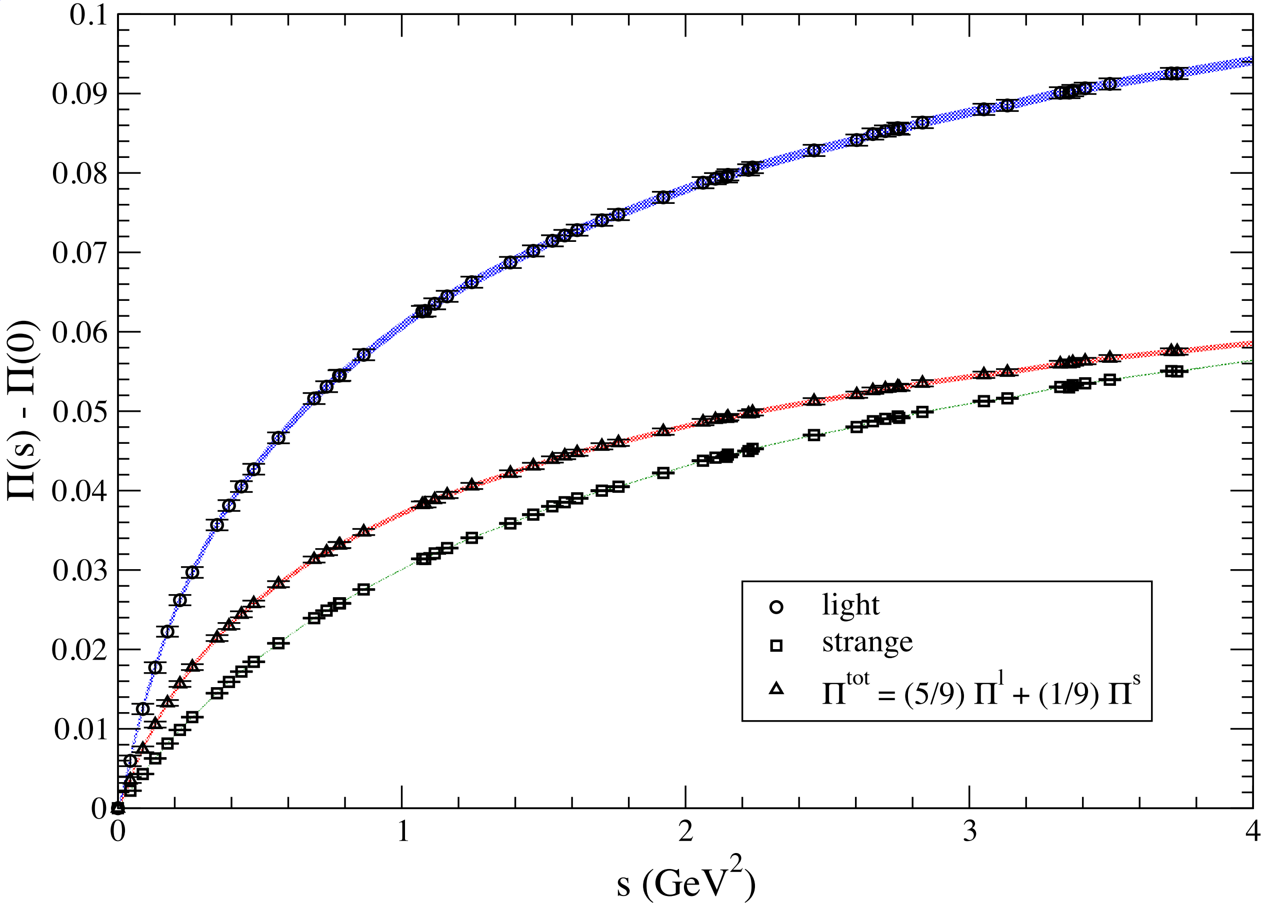
Calculation with quarks



$$= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Calculation required is correlation function of quark and antiquark propagators, created and destroyed by vector (photon) current

Fourier transform
and
plot out as a function of q^2



E. Gregory, BMW, LAT15.
Smeared clover action

Simpler method

HPQCD 1403.1778

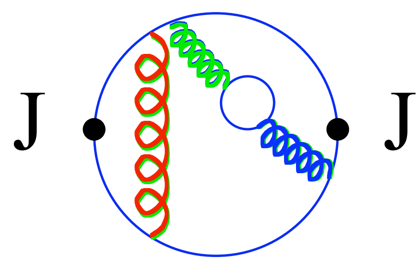
For spatial vector currents at zero spatial momentum

$$\Pi^{jj}(q^2) = q^2\Pi(q^2) = a^4 \sum_t e^{iqt} \sum_{\vec{x}} \langle j^j(\vec{x},t) j^j(0) \rangle$$

Time-moments of lattice current-current correlators

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^j(\vec{x},t) j^j(0) \rangle$$

$$= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2) \bigg|_{q^2=0}$$

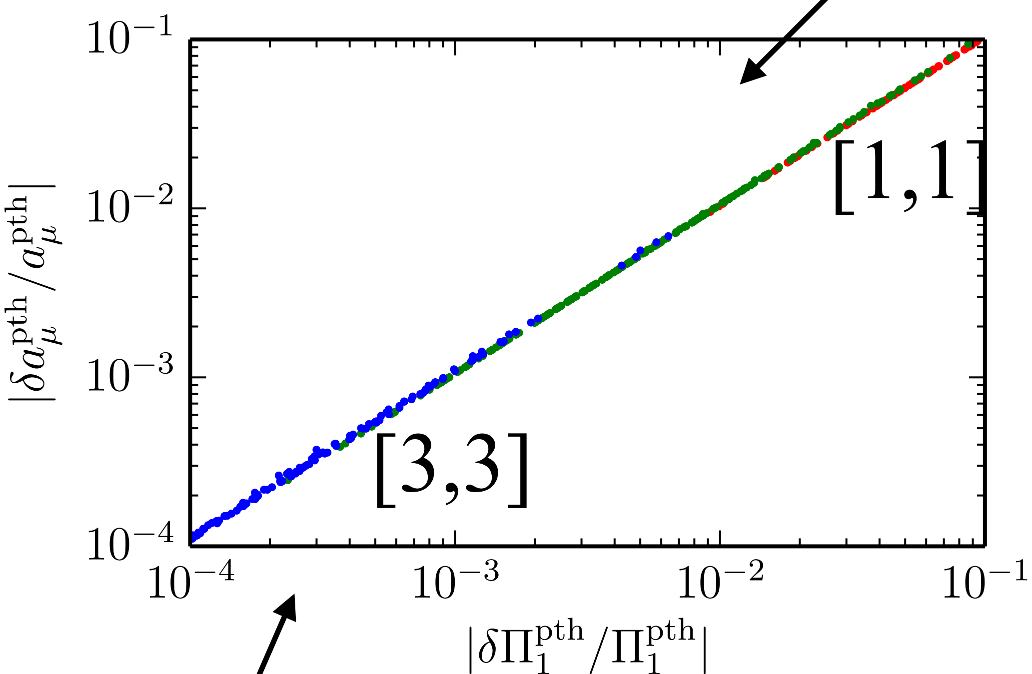


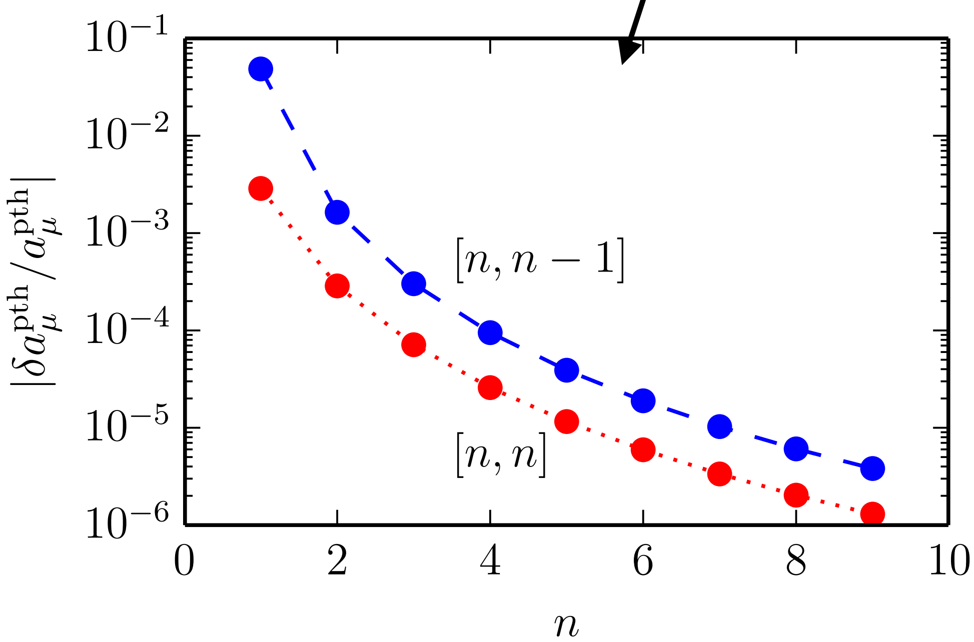
$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j \quad \text{with} \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Allows us to reconstruct $\hat{\Pi}(q^2)$ and integrate

Use Pade approximants (ratio of m/n polynomials) rather than Taylor expansion for better large q^2 behaviour.

Test Pade approximants in similar scenarios (1-loop quark vacuum polarisation, with noise added)





Improved precision allows higher order Pade - we use [2,2]

CHARM contribution

HPQCD 1004.4285, 1208.2855

Part of the set of calculations that gave

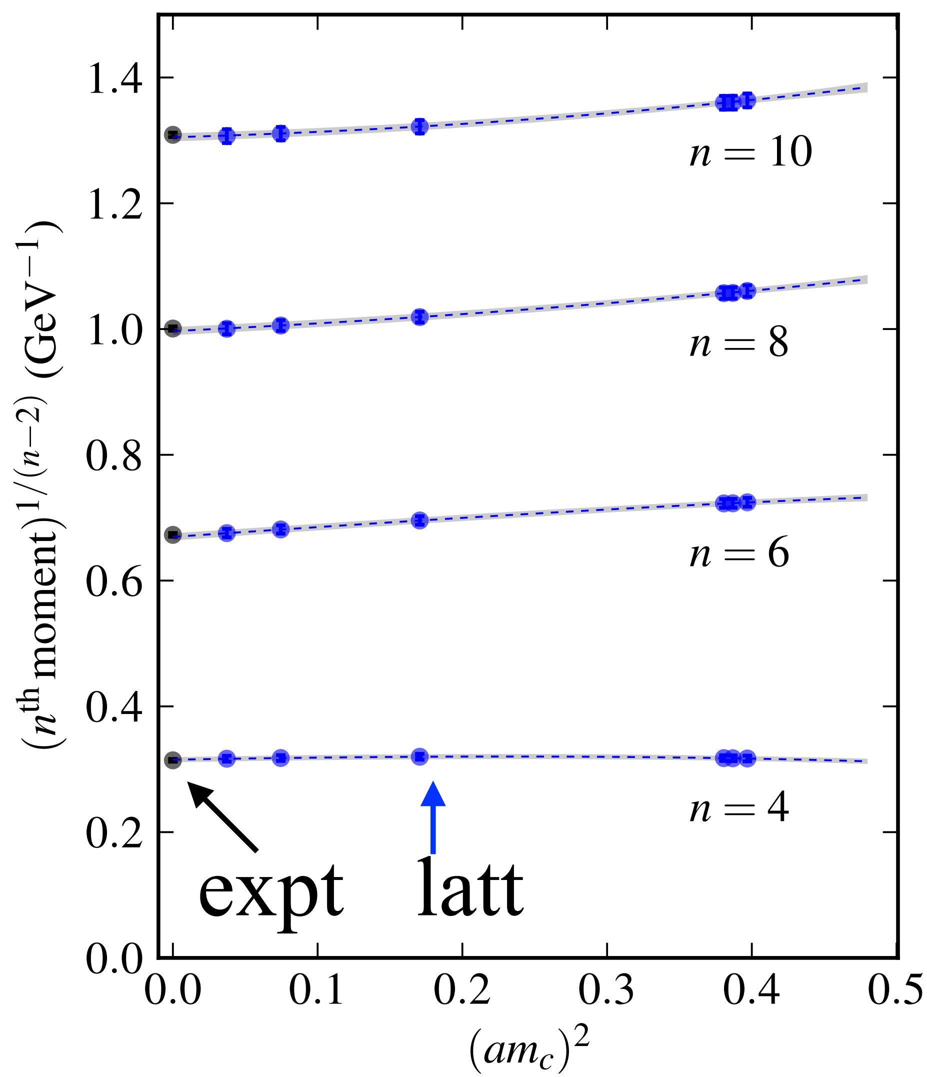
$$m_c, M(J/\psi) - M(\eta_c), \Gamma(J/\Psi \to e^+e^-), \Gamma(J/\psi \to \eta_c\gamma)$$

Used HISQ valence quarks on MILC 2+1 asqtad configs. Z_V from contnm QCD pert. th.

Extrapolation to physical point allows us to compare directly to moments from e+e- expt. in charm region

$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

HPQCD 1403.1778



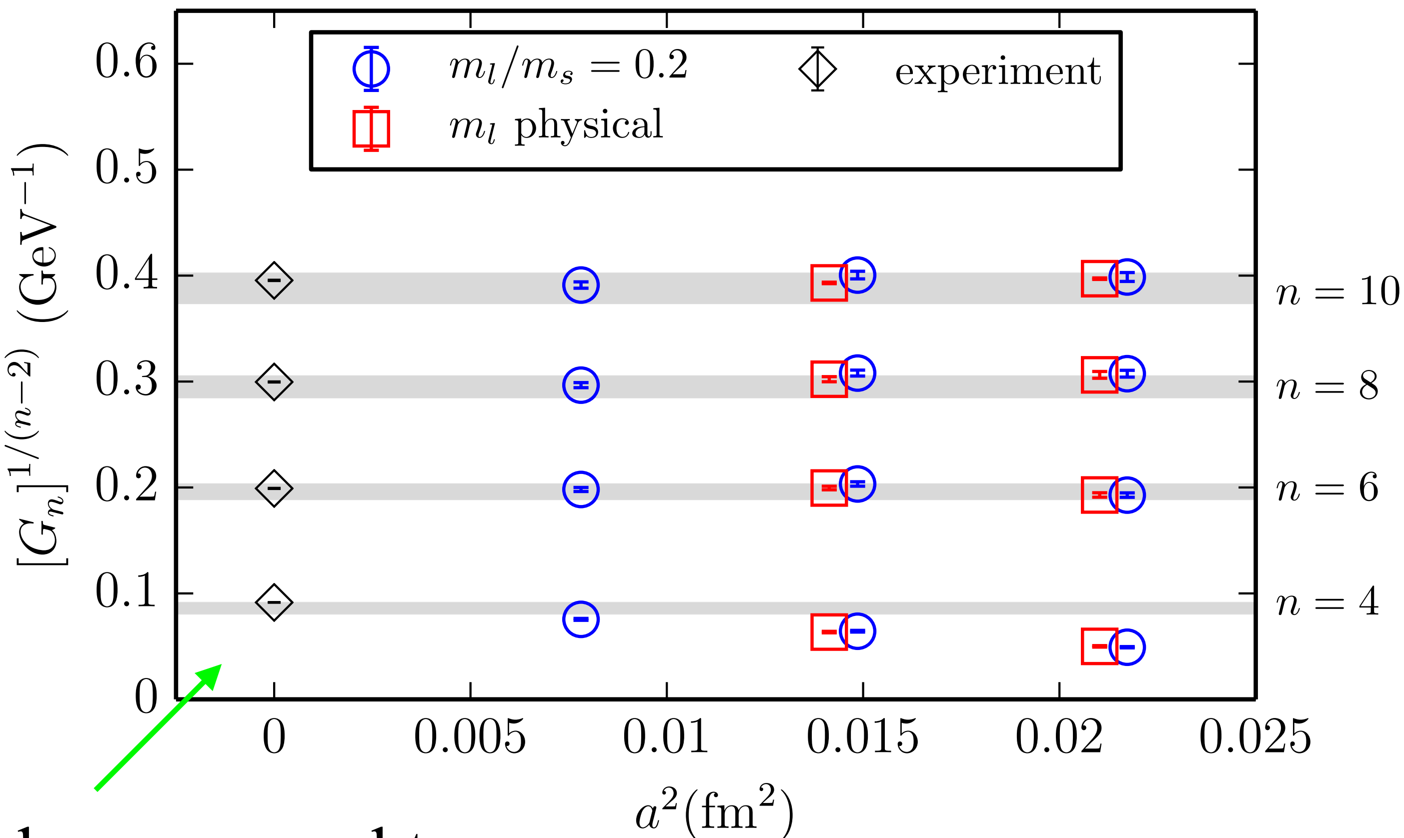
BOTTOM contribution

HPQCD 1110.6887, 1309.5797, 1408.5768

Part of the set of calculations that gave

$$m_b, M(\Upsilon) - M(\eta_b), M(\Upsilon') - M(\eta_b'), \Gamma(\Upsilon \to e^+e^-), \Gamma(\Upsilon' \to e^+e^-)$$

Used NRQCD valence quarks on MILC 2+1+1 HISQ configs. Z_V from contnm QCD pert. th.



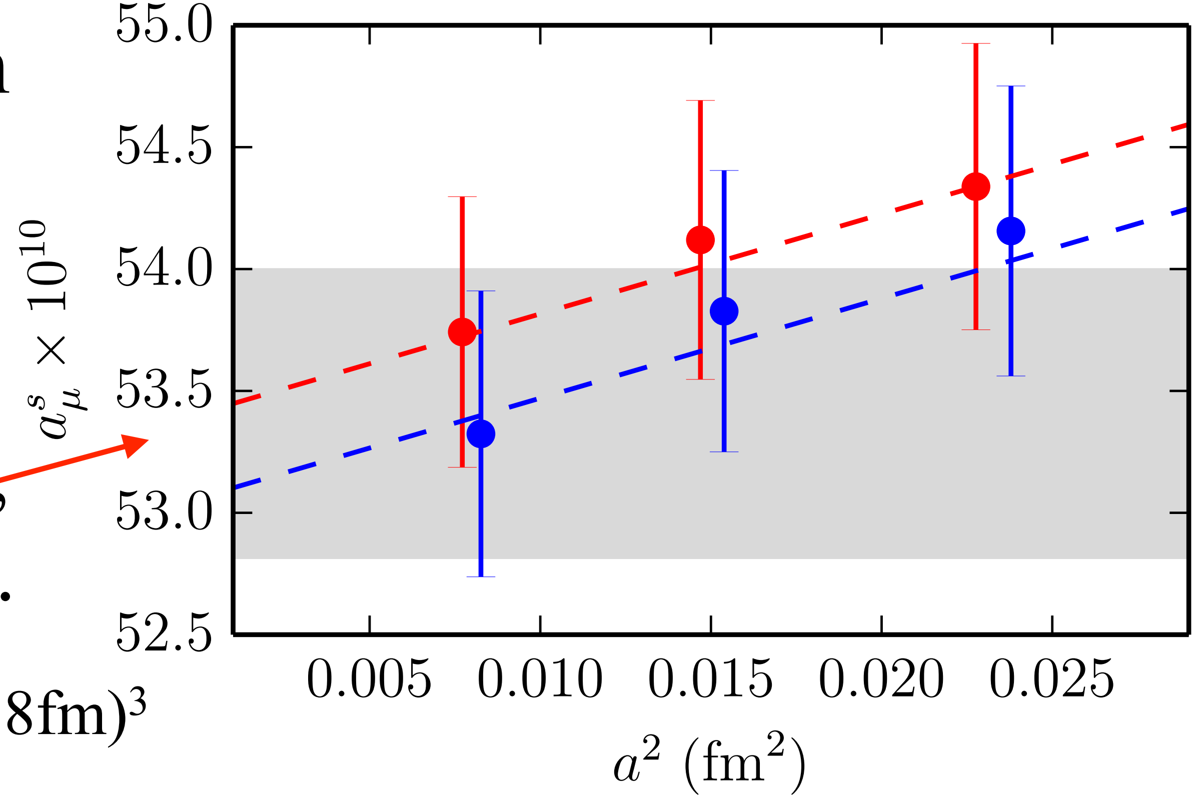
Again, moments can be compared to those extracted from expt.

$$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$$

STRANGE contribution

HISQ valence quarks on MILC 2+1+1 HISQ configs. Local J_V - nonpert. Z_V.
multiple a (fixed by w_0), m_l (inc. phys.), volumes.
Tune s from η_s

up to $(5.8\text{fm})^3$

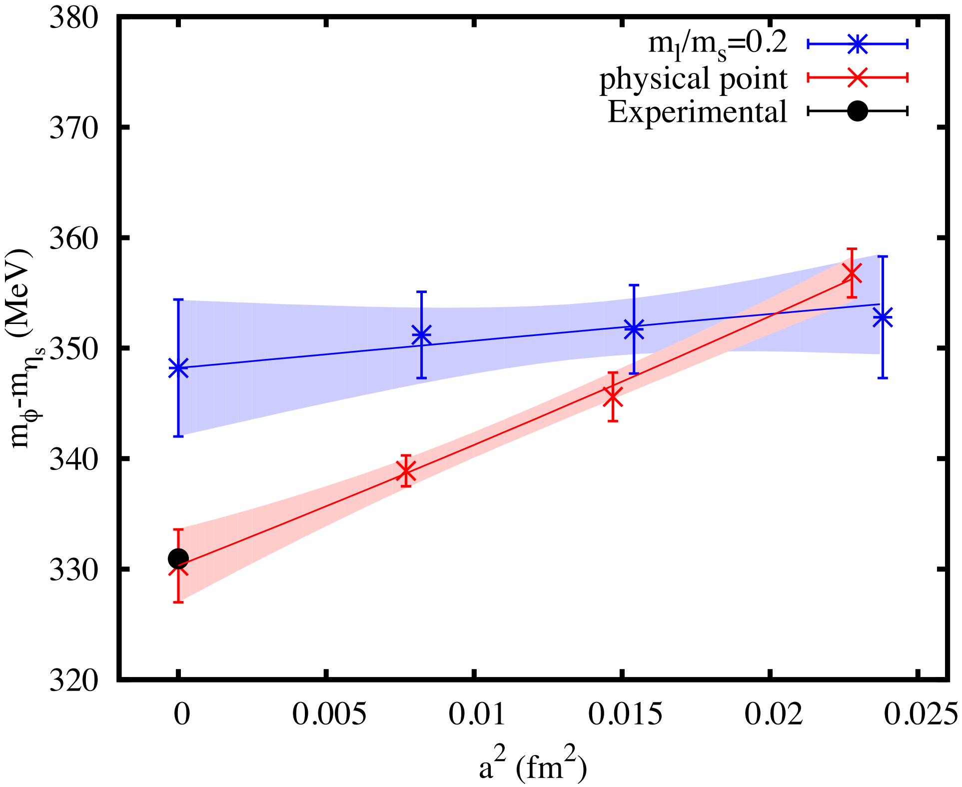


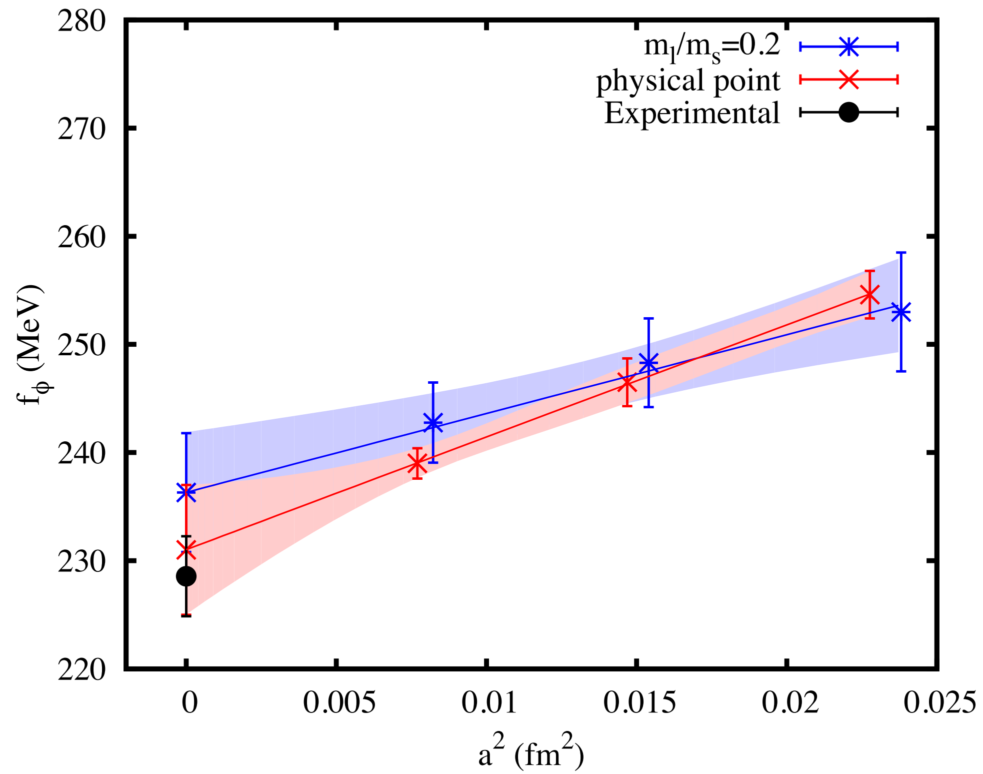
	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	1.1%

$$a^s_{\mu,\text{lat}} = a^s_\mu \times (1 + c_{a^2}(a\Lambda_{\text{QCD}}/\pi)^2 + c_{\text{sea}}\delta x_{\text{sea}} + c_{\text{val}}\delta x_{\text{val}})$$

$$a_\mu^{HVP,s} = 53.41(59) \times 10^{-10}$$

Check mass and decay constant of ϕ from these correlators against expt

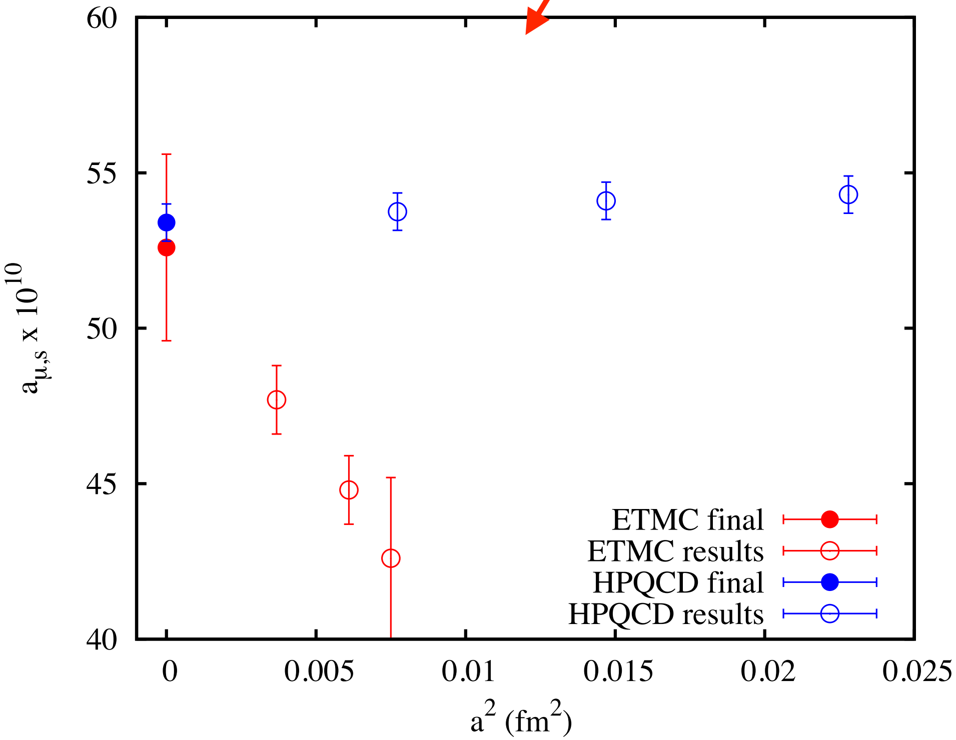




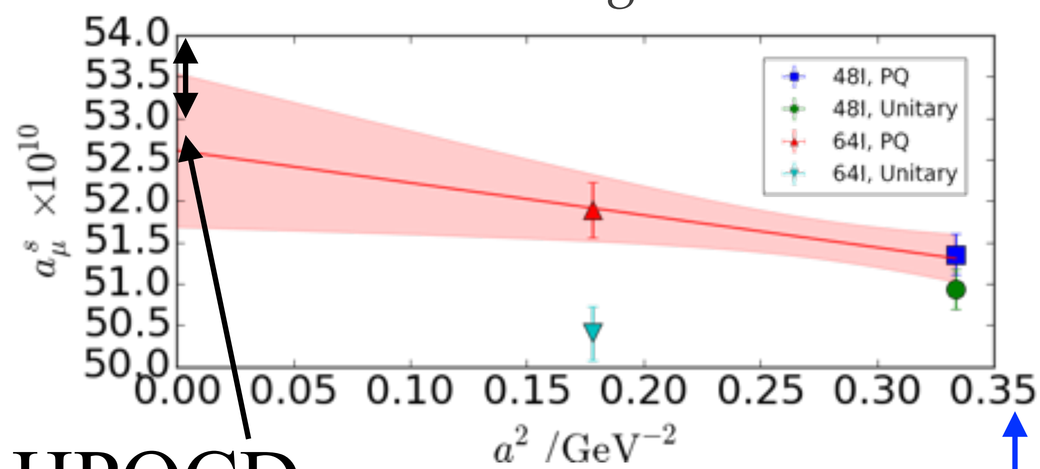
Results from ETMC, RBC/UKQCD + BMW underway

twisted mass

domain wall

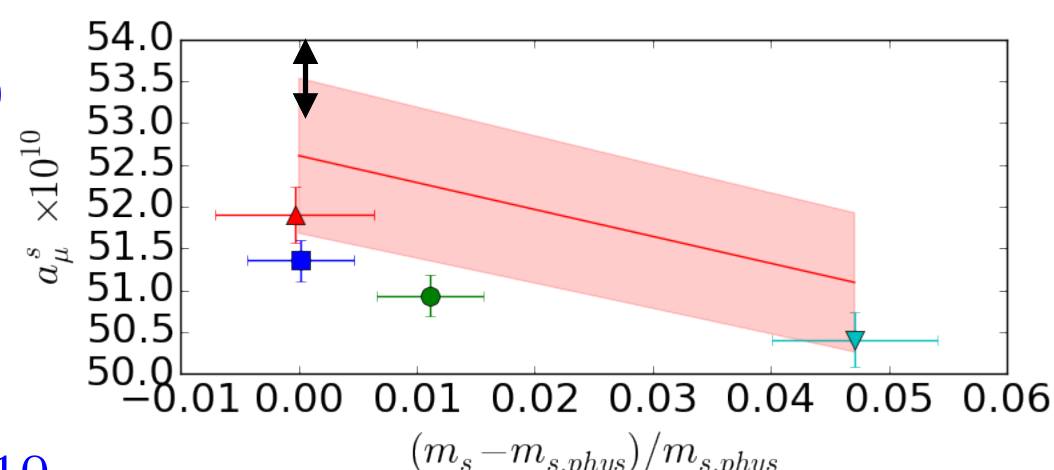


continuum limit for strange contribution to a_μ



0.014fm^2

HPQCD



$$a_\mu^{HVP,s,ETMC} = 53(3) \times 10^{-10}$$

Continuum estimate/
upper limit:

$$a_\mu^{HVP,s,cont} = 55.3(8) \times 10^{-10}$$

1403.1778

LIGHT contribution $m_u = m_d$

HISQ valence quarks on MILC 2+1+1 HISQ configs. Use Z_V from s calc.

Multiple a (use w_0), m_l (inc. phys.), volumes (at ml/ms=0.1).

New ingredient since correlators much noisier. Use:

$$G(t) = \begin{cases} G_{\mathrm{data}}(t) & \text{for } t \leq t^* \\ G_{\mathrm{fit}}(t) & \text{for } t > t^* \end{cases}$$

← from Monte Carlo

← from multi-exponential fit

$t^* = 1.5\mathrm{fm} = 6/m_\rho$ so 70% of result from G_{data}

- 80% of result comes from ρ meson pole, so need to understand ρ on lattice, inc. finite-volume from $\pi\pi$.
- 10% from $\pi\pi$, sensitive to finite-volume and m_π (so taste-issues for staggered quarks).

One approach is to correct Taylor coefficients

$$\hat{\Pi}_j^{latt} \rightarrow (\hat{\Pi}_j^{latt} - \hat{\Pi}_j^{latt}(\pi\pi)) \left[\frac{m_\rho^{2j,latt}}{m_\rho^{2j,expt}} \right] + \hat{\Pi}_j^{cont}(\pi\pi)$$

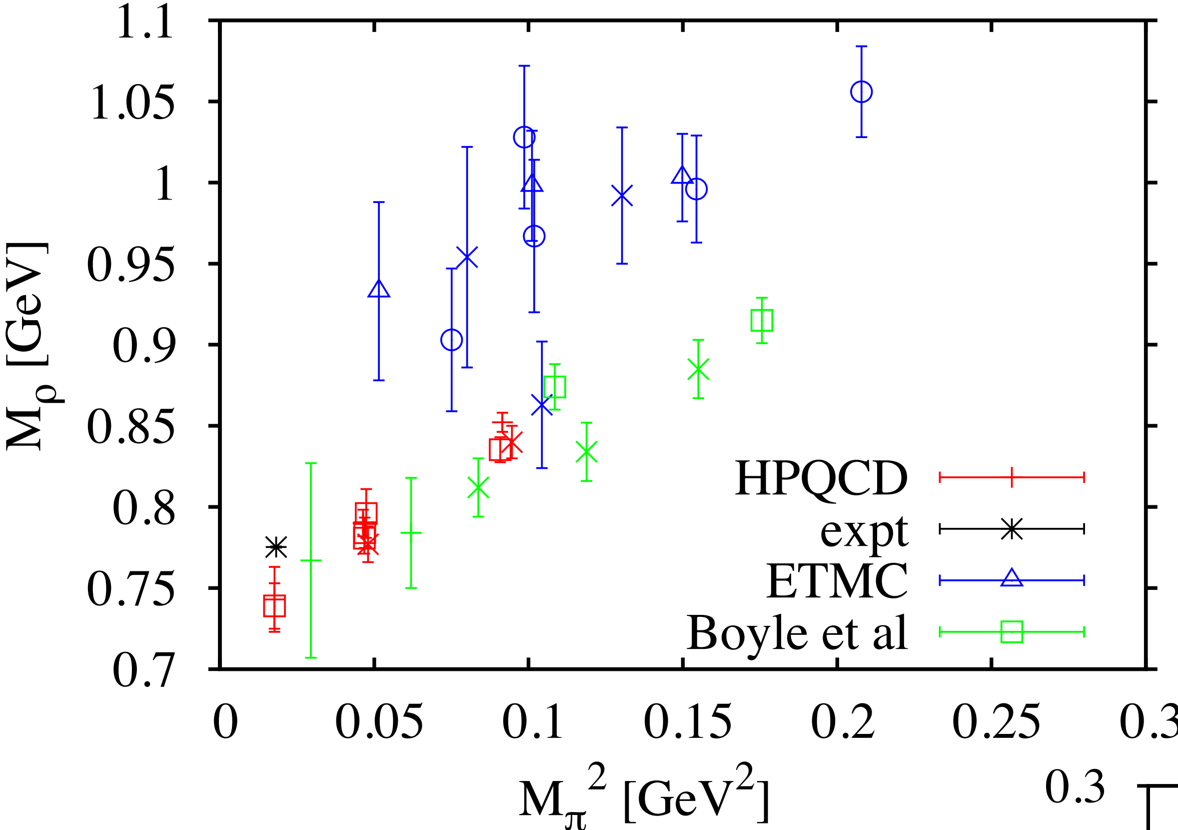
Remove lattice $\pi\pi$ using effective theory of ρ, π, γ inc. staggered quark effects and finite vol.

Jegerlehner +Szafron, 1101.2872

Rescale using exptl m_ρ elaborating on ETMC : 1308.4327. Reduces lattice systematics from light quark mass effects

Restore $\pi\pi$ from continuum effective theory

$\pi\pi$ contribution distorted at physical point using staggered quarks on these coarse lattices. Important to inc. other masses. But note: need 7fm lattice to reduce finite vol effects below 1% for contnm $\pi\pi$



Analysis of ρ parameters

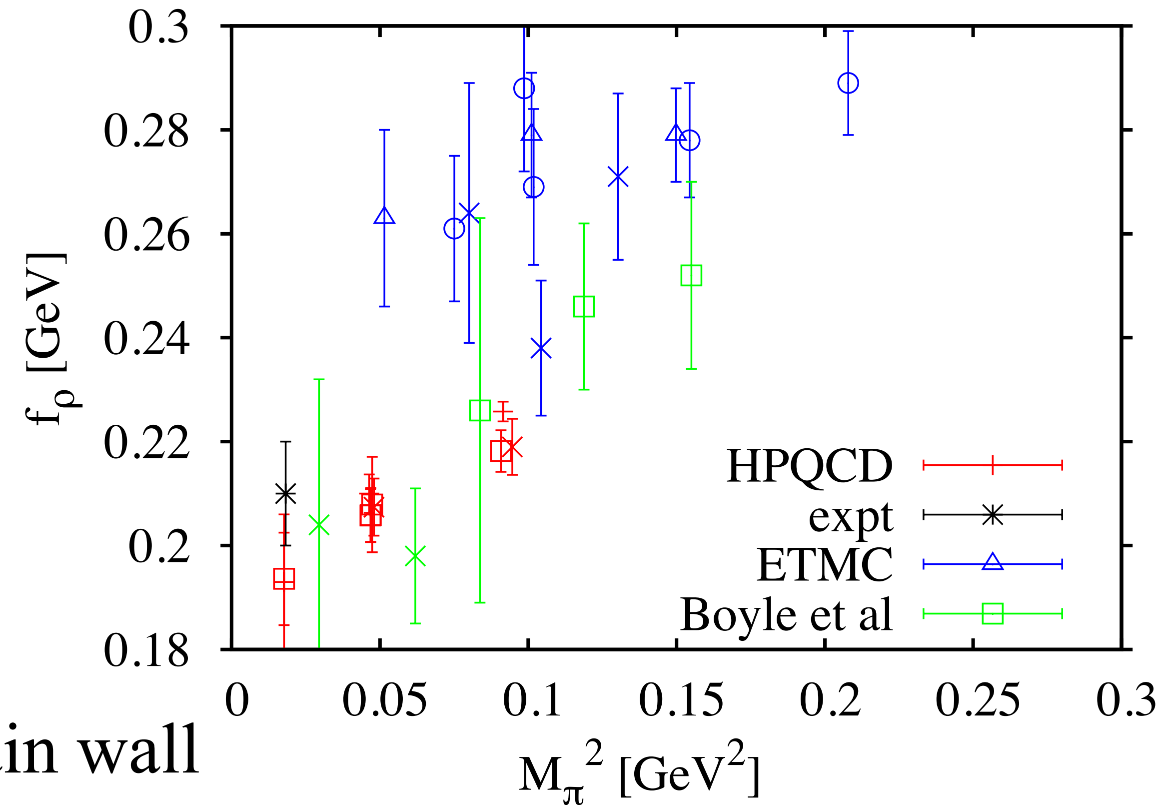
Direct comparison with ETMC (1308.4327) and Boyle et al (1107.1497) possible

ETMC a 0.06-0.08fm
L 2.5- 2.9fm

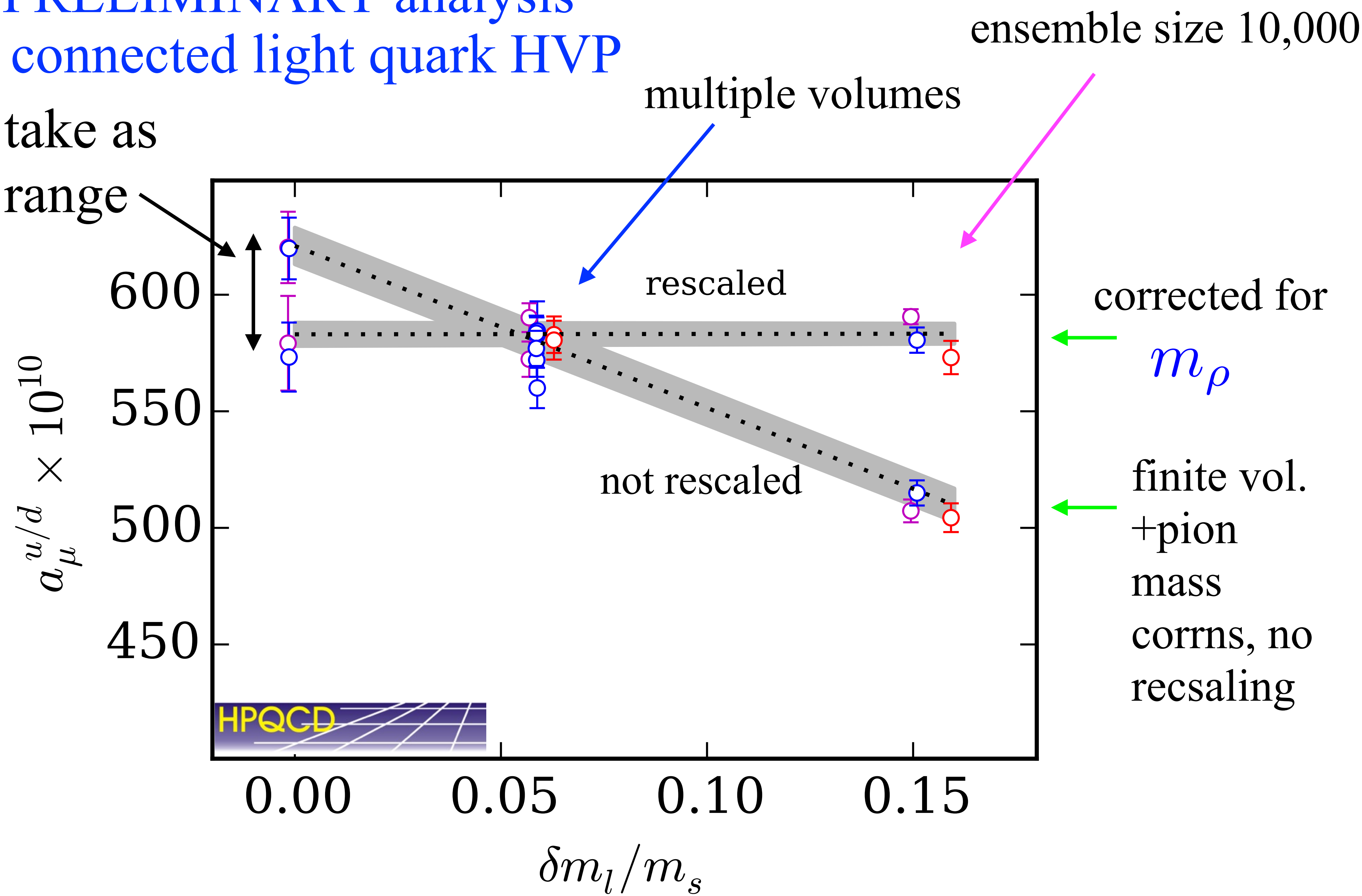
HPQCD a 0.09-0.15fm
L 2.5-5.8fm

Boyle et al a 0.09-0.14fm
L 2.7-4.6fm

domain wall

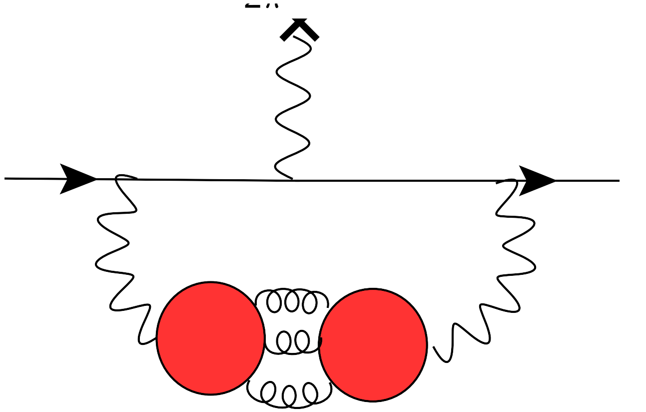


PRELIMINARY analysis
connected light quark HVP



Future: improve statistics at physical point, finer lattices

DISCONNECTED contribution to HVP

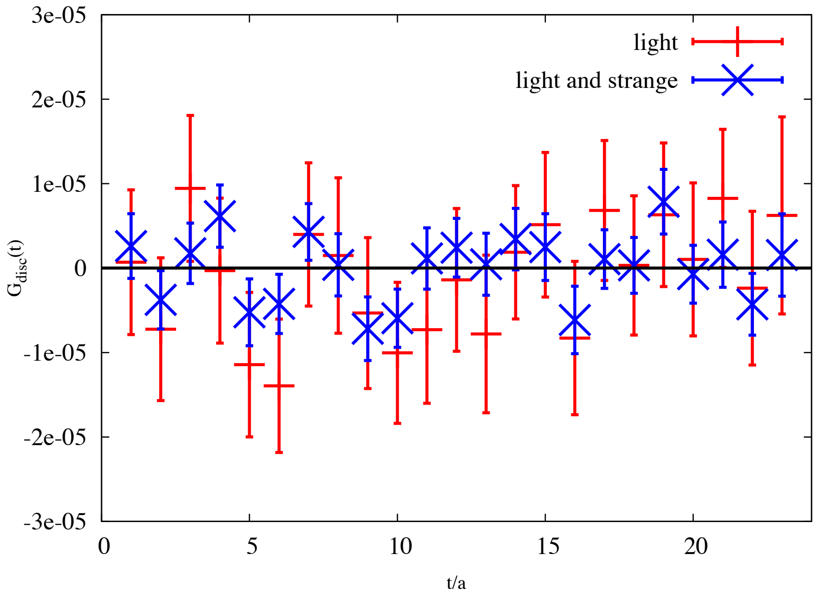


Vanishes if $m_u = m_d = m_s$

since $\sum_i e_i = 0$ Blum, hep-lat/0212018

For real masses, result is disconn. correlator for (l-s) current with charge 1/3 (so e^2 factor is 1/5 of connected)

Focus has been on stochastic methods. Using same source for l and s helps Guelpers, Mainz,LAT14



Chakraborty,HPQCD,LAT15

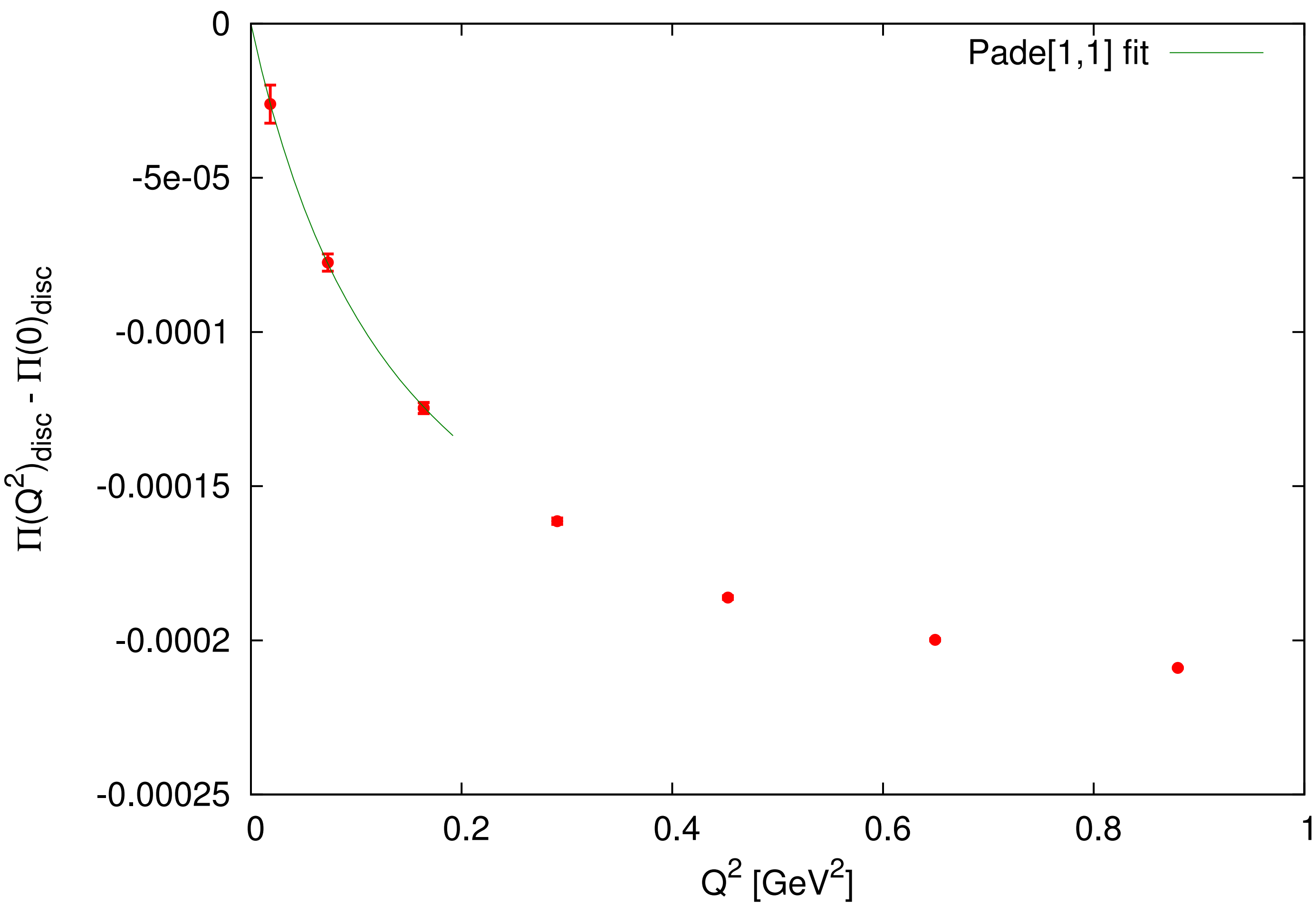
no signal from 50+50 sources (all-to-all) per config. One-link J

BMWc collaboration - new method

Toth,LAT15

Use loop expansion of inverse + improved algorithm for inversion to reduce uncertainty in calc. of disc. pieces

$a = 0.095\,\text{fm}$, physical quark masses, $64^3 \times 96$



staggered quarks

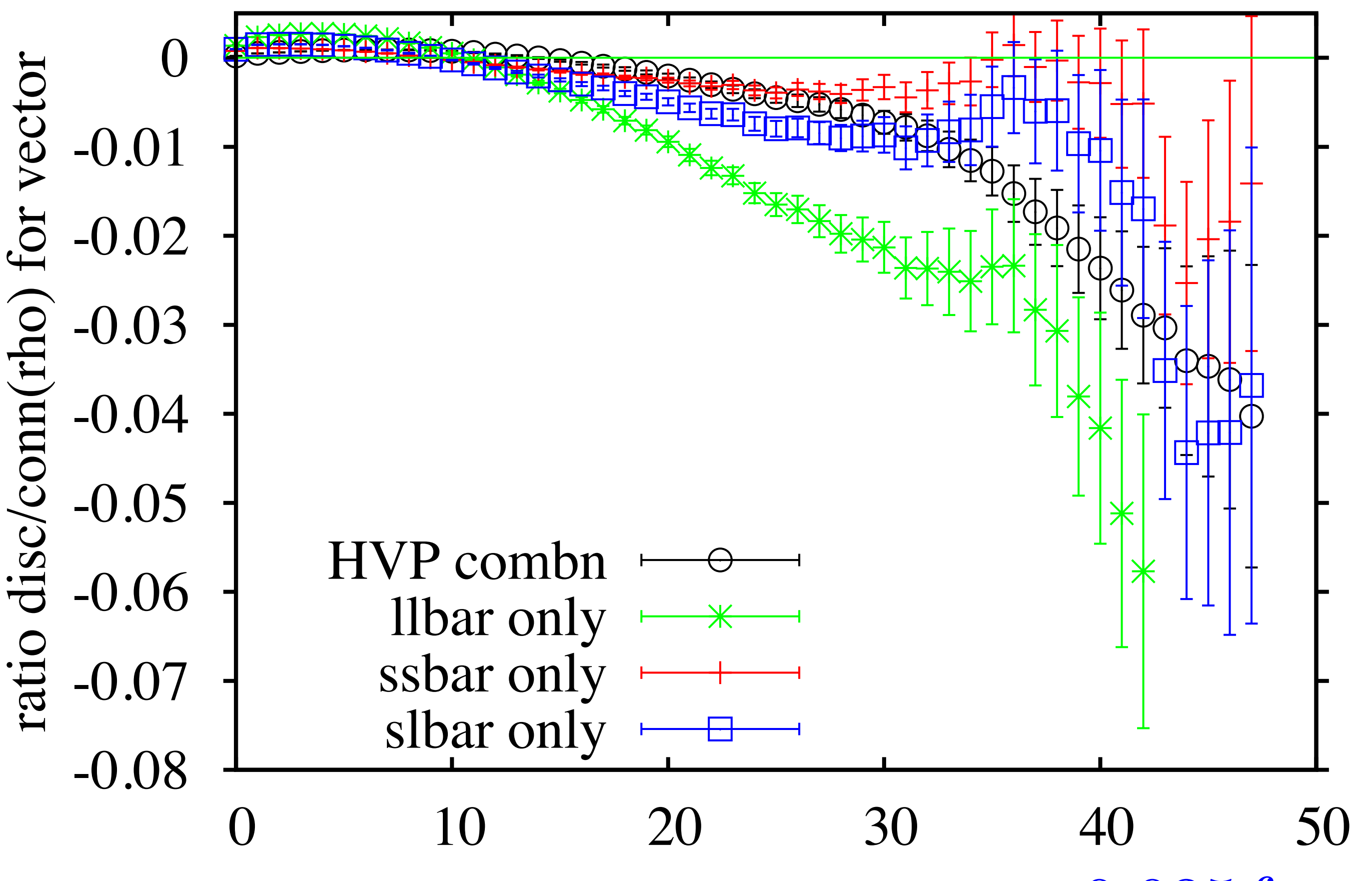
good signal - work ongoing

contrbn negative and very small

HadSpec results

e.g. Hadspec,1309.2608

Use instead many (~150) source vectors (eigenvectors of gauge-covariant Laplacian) for both conn. and disc. correlators to obtain good signal.



$a_t = 0.035$ fm

$m_\pi = 390$ MeV

PRELIMINARY

Fitting and normalising to connected light, gives HVP disc. contribn of ~ -0.2%

Hadspec+HPQCD,in prep.

anisotropic clover action

Simple (but conservative) argument on size of disc. pieces

l-l disc.pieces provide key difference between ω and ρ

$$2D_{ll} = -\frac{f_\rho^2 m_\rho}{2} e^{-m_\rho t} + \frac{f_\omega^2 m_\omega}{2} e^{-m_\omega t}$$

$$\frac{\hat{\Pi}_{j,disc}}{\hat{\Pi}_{j,conn}} = \frac{1}{2}\left[\frac{m_\rho^{2j+2} f_\omega^2}{m_\omega^{2j+2} f_\rho^2} - 1\right]$$

We do not have accurate information on decay constants because of width of ρ , mixing of ω etc

Taking $f_\rho = 0.21(1)\,\mathrm{GeV}, f_\omega = 0.20(1)\,\mathrm{GeV}$

Disc. contribn reduced by factor of 5 from electric charge

⟶ HVP : disc-ll/conn-ll = -1.5(1.5) %

Adding contributions to (g-2)/2

ETMC
1308.4327

567(11)stat

674(28)

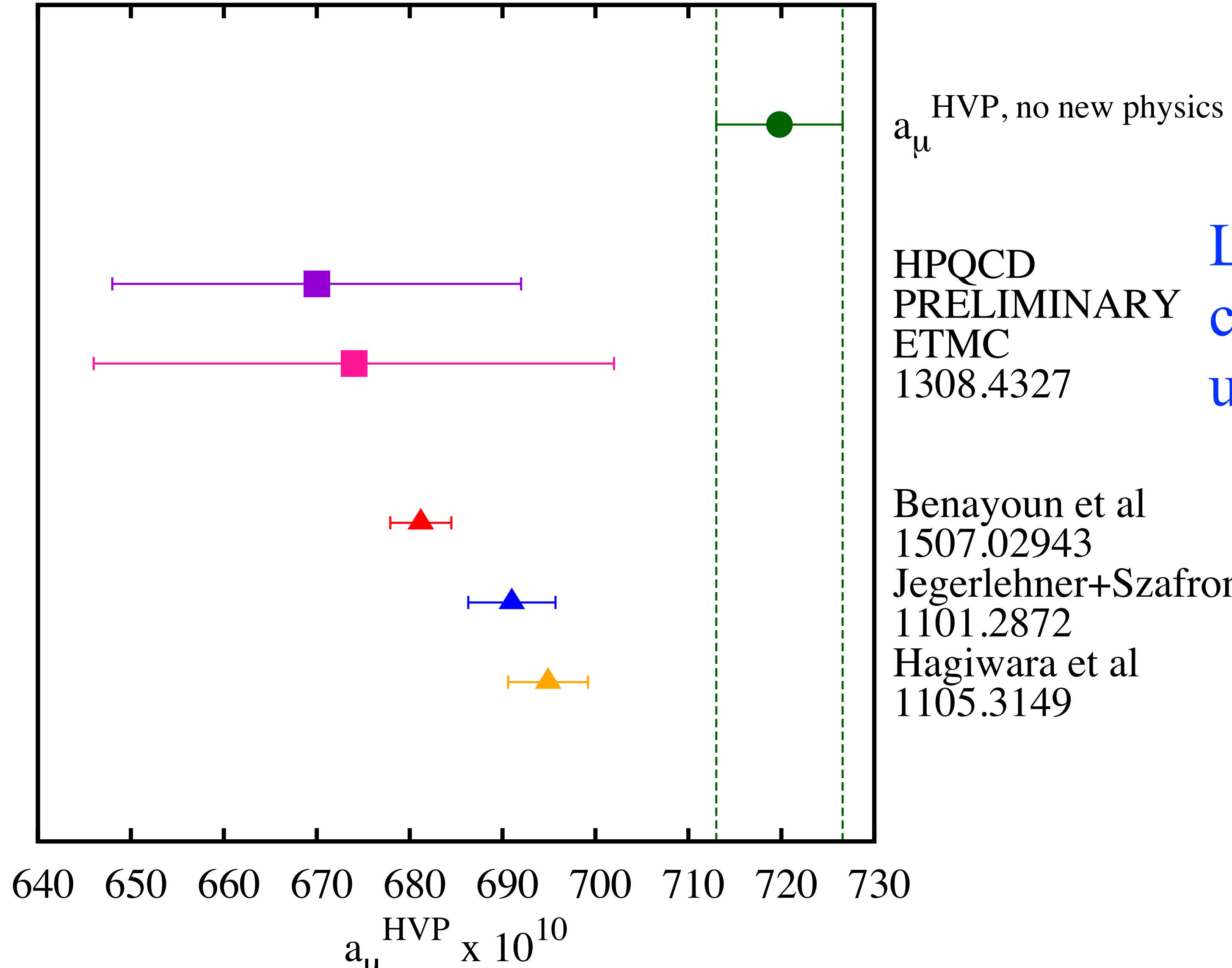
HPQCD: $\times 10^{-10}$

light, connected	602(20)	preliminary. inc. 1% QED and 1% isospin uncty
strange connected	53.4(6)	1403.1778
charm connected	14.4(4)	1403.1778, 1208.2855
bottom connected	0.27(4)	1408.5768
disconn. (estimate)	0(9)	Take 1.5% as uncty. Contrbn negative
TOTAL	670(22)	

CONCLUSIONS: Lattice - continuum comparison

2.3σ from no new physics

lattice/non-lattice agreement



Lattice calcs inc. u,d,s,c

A lot of progress but lattice uncty (all from u/d) still too big. Need to calc. QED, m_u/m_d effects (~1% and positive?) and disc. (negative). More calculations underway (Mainz, BMW, RBC/UKQCD …)

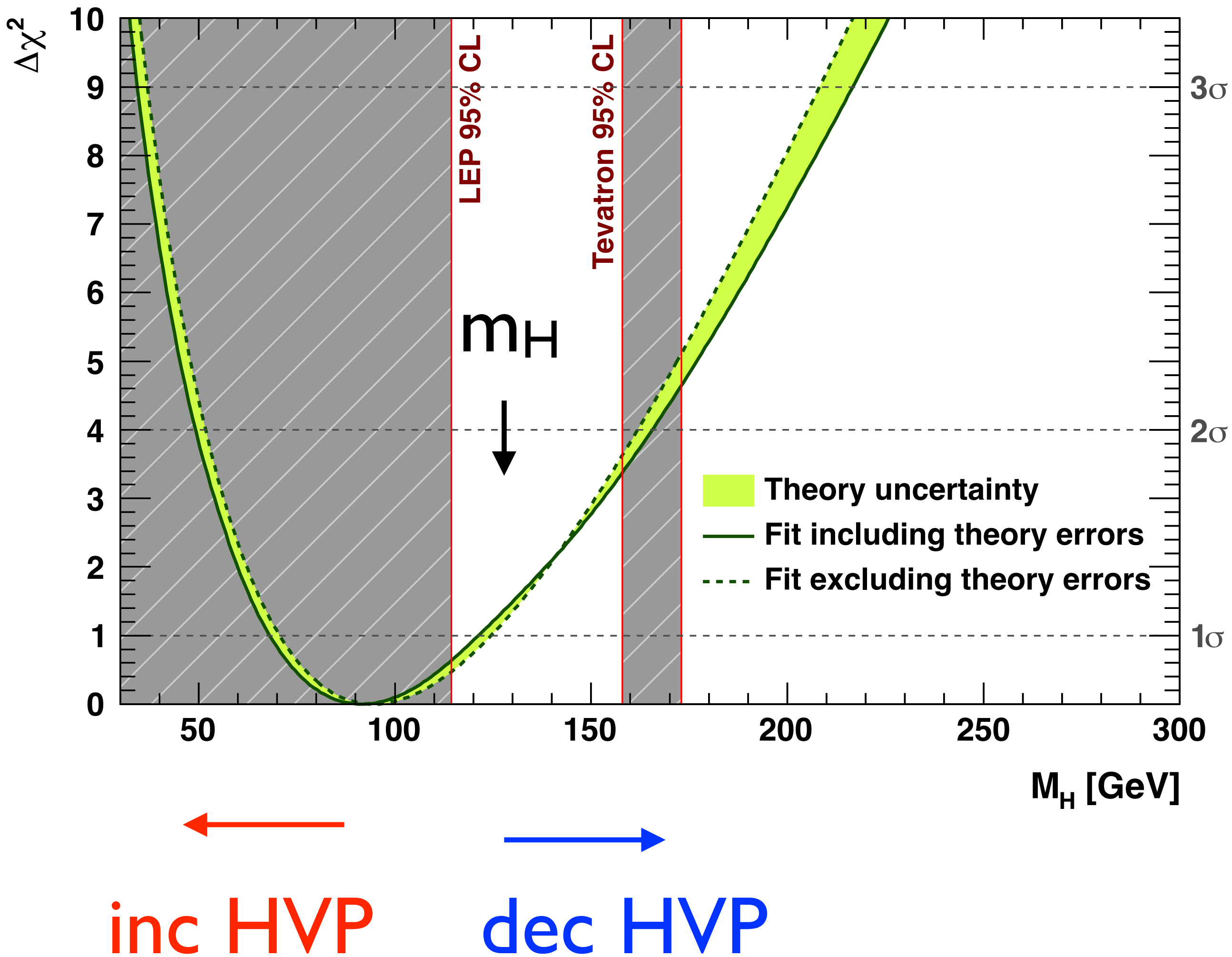
Backup Slides

Precision electroweak Higgs bounds

sees HVP through α_{QED} but sensitivity to range of exptl data is different

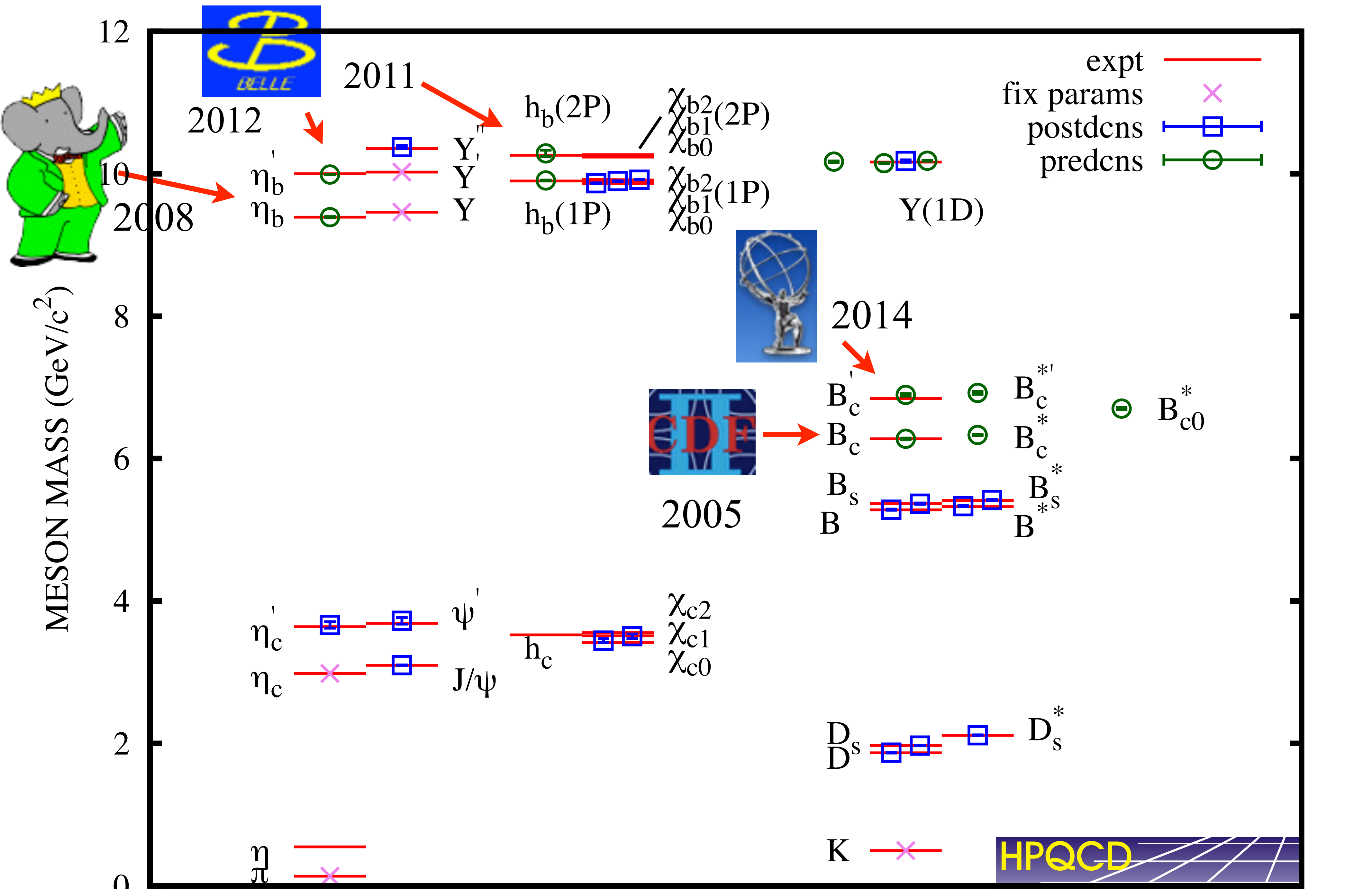
Hagiwara et al, 1105.3149

Gfitter,1107.0975



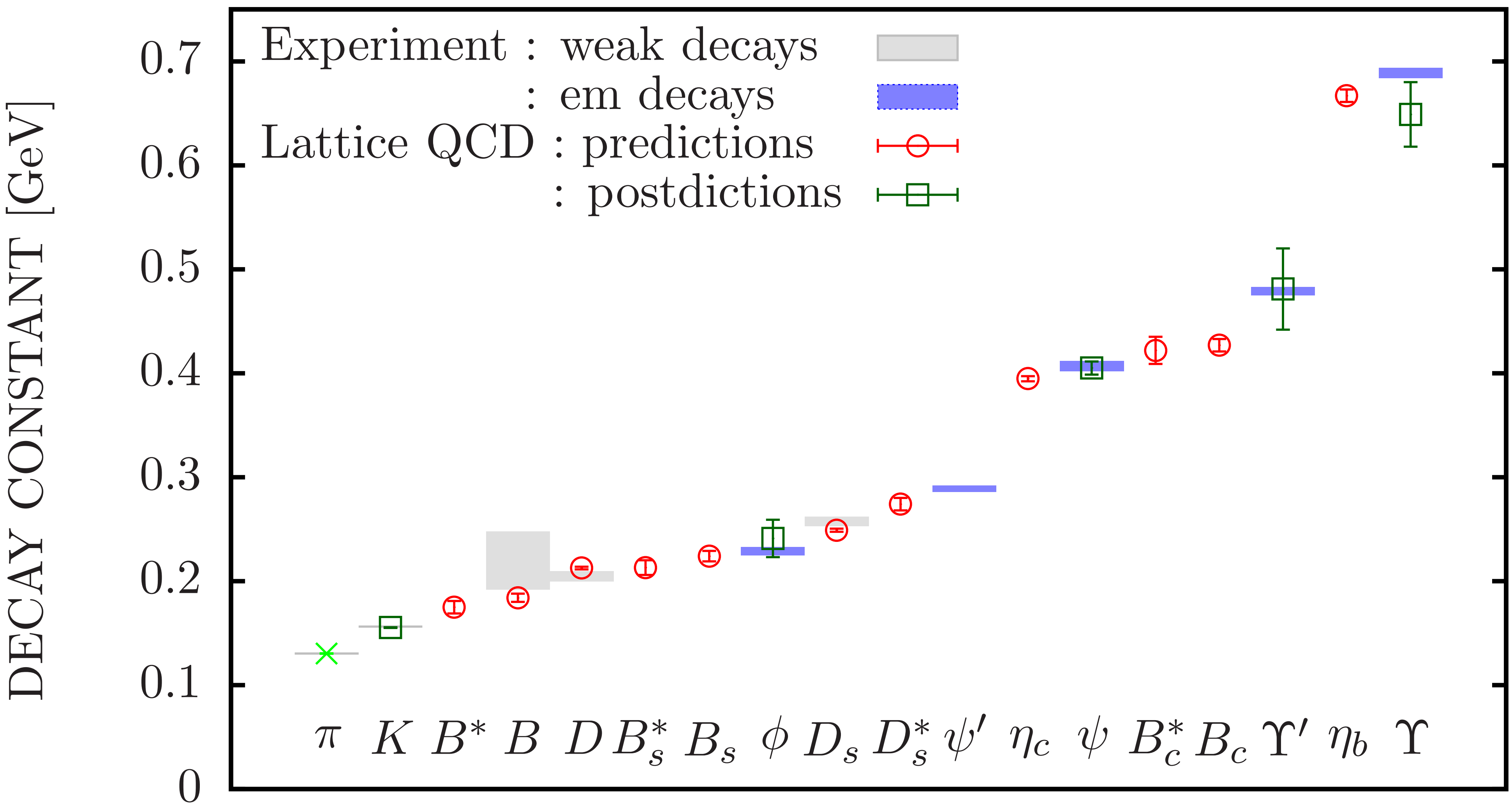
Lattice QCD calcs of $\Delta\alpha^{had}_{QED}$ also underway (ETMC, Mainz)

Keep an eye on the 'big' picture whilst doing this …..



few MeV uncertainties in many cases

Keep an eye on the 'big'picture whilst doing this …..



HPQCD 1503.05762