Review on nucleon charges from lattice QCD

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QCD for New Physics at the Precision Frontier

September 28th, 2015

University of Cyprus

The Cyprus Institute

OUTLINE

A Motivation

B Nucleon on the Lattice

- Introduction
- Systematics

C Nucleon Charges

- Axial Charge
- Scalar & Tensor Charges

D Spin Structure of the Nucleon

- Quark contributions
- Gluon contributions

E Conclusions & Perspectives

slide: 3

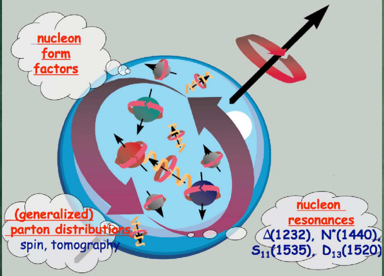
A MOTIVATION

slide: 4

Lattice QCD meets Nature

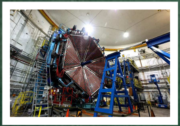
Why Lattice QCD ?

- ★ Well-established non-perturbative approach to QCD
- ★ Makes contact with well determined experimental measurements
- ★ Provides input for quantities not easily accessible in expriments
- ★ Interpretation of experimental data
- ★ Tests of SM and New Physics searches

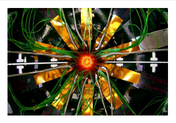


Rich experimental activities in major facilities

JLAB



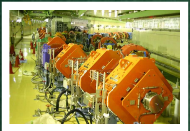
ALICE



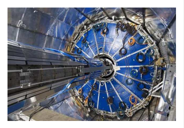
COMPASS

- Investigation of Baryon and meson structure
- Origin of mass and spin
- New physics searches:
  scalar/tensor interactions, $(g-2)_\mu$, dark photon, EDMs
- proton radius puzzle
- the list is long...

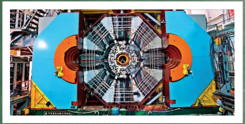
JPARC



RHIC

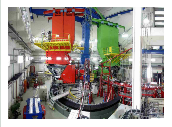


BES III



PSI

MAMI



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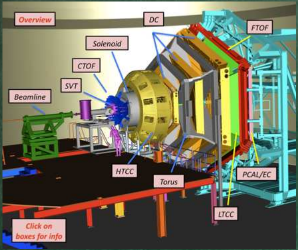
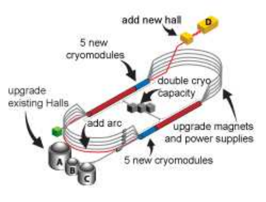
12GeV Upgrade at JLab





Continuous Electron Beam Accelerator Facility

"...to employ new methods for studying the basic properties of the building blocks of the universe, how they are formed, how they interact and the forces that mediate these interactions." ... *"expanding our knowledge of nuclear and particle physics well beyond its current level. "*

Physics Program for CLAS12 (Selected Experiments)

- Spin/Flavor Structure of the Nucleon
- Nucleon Resonance Studies with CLAS12
- Origins of quark confinement
- High Precision Measurement of the Proton Charge Radius
- Scalar and Tensor interactions
- The Transverse Structure of the Hadrons

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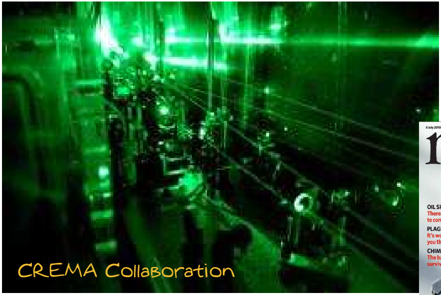
Proton Radius Puzzle

$< r_p^2 >$ from muonic hydrogen μp 7.1σ smaller than hydrogen spectroscopy



[R. Pohl et al. Nature 466, 213 (2010)]

Talk by C. Carlson
Wed @ 09: 45

MAMI in Mainz

- measured energy difference between the 2P and 2S states of muonic hydrogen
- μp: 10 times more accurate than other measurements
- very sensitive to the proton size
- no obvious way to connect with other measurements (4% diff)

[I. Lorenz et al., (2014), arXiv:**1405.6582**]
[J. Bernauer et al. (2010), arXiv:**1007.5076**]

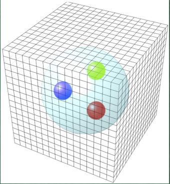
B

NUCLEONS ON THE LATTICE

Lattice formulation of QCD

K. Wilson

formulation (1974)



M. Creutz

1st numerical computation

- Space-time discretization on a finite-sized 4-D lattice

  L: lattice size, a: lattice spacing

  - $\Psi(x)$, $\bar{\Psi}(x)$: Quark fields on lattice points
  - $U_\mu(x)$: Gauge fields (gluons) on links (Wilson lines)

- Finite degrees of freedom

- Construction of an action $\quad S = S_{\text{fermions}} + S_{\text{gluons}}$

  (with correct continuum limit)

- Numerical simulations and perturbative lattice calculations

Improved fermion action

- Clover improved Wilson
  - ✓ computationally fast
  - ✗ broken chiral symmetry & requires operator improvement
  - ★ Employed by: ALPHA, BMW, CLS, LHPC, NPQCD, PACS-CS, QCDSF, RQCD
- Twisted Mass
  - ✓ computationally fast & automatic improvement
  - ✗ broken chiral symmetry & violation of isospin
  - ★ Employed by: ETMC
- Staggered
  - ✓ computationally fast &
  - ✗ 4 doublers & difficult contractions
  - ★ Employed by: MILC, LHPC
- Overlap
  - ✓ exact chiral symmetry
  - ✗ computationally expensive
  - ★ Employed by: JLQCD
- Domain Wall
  - ✓ improved chiral symmetry
  - ✗ computationally demanding & requires tuning
  - ★ Employed by: RBC-UKQCD

★ Fermion actions: $\mathcal{O}(a)$-improved
★ Gluon actions: $\mathcal{O}(a^2)$-improved

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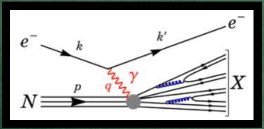
Probing Nucleon Structure



- **Generalized Parton Distributions (GPDs)**

  Introduced late '90s

  Deep inelastic scattering (DIS)

  Comprehensive description of hadron structure from first principles

- **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

- $q = p' - p$, $\bar{P} = (p' + p)/2$
- n: light-cone vector ($\bar{P}.n = 1$)
- $\xi = -n \cdot \Delta/2$

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- Choices of operators in LQCD:
  towers of local twist-2 operators

- Rely on OPE to extract moments

$$f^n = \int_{-1}^{1} dx\, x^{n-1} f(x)$$

Contain information of:

★ Form factors and parton distributions

★ Quark orbital angular momentum

★ spin structure of the nucleon

A **Unpolarized**

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar q\, \gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

DIS, Drell-Yan, W-asymmetry, γ^+ jet, ...

B **Helicity (polarized)**

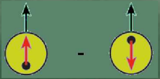
$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar q\, \gamma_5 \gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

polarized DIS, SIDIS, pp collisions,photo/electro production, ...

C **Transversity**

$$\mathcal{O}^{\mu_1\cdots\mu_{n-1}} = \bar q\, \sigma^{\mu\{\nu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

single-spin asymmetry in SIDIS, ...



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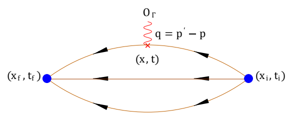
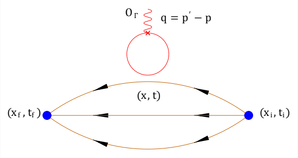
Nucleon on the Lattice in a nutshell

1. Diagrams:





Connected

Disconnected

2. Two-pt and three-pt functions:

$$\text{2pt}: \quad G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}}\, \Gamma^0_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \overline{J}_\beta(0) \rangle$$

$$\text{3pt}: \quad G_{\mathcal{O}}(\Gamma^\kappa, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}}\, e^{-i\vec{x}_f \cdot \vec{p}'}\, \Gamma^\kappa_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \overline{J}_\beta(0) \rangle$$

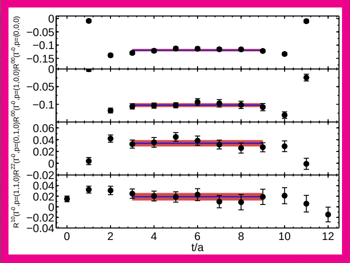
$$\Gamma^0 \equiv \frac{1}{4}(1 + \gamma_0)$$

$$\Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i$$

and other variations

slide: 14

3. Optimized ratio:



$$R_{\mathcal{O}}(\Gamma,\vec{q},t) = \frac{G_{\mathcal{O}}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)} \times \sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}}$$

$$\xrightarrow[t-t_i\to\infty]{t_f-t\to\infty} \Pi(\Gamma,\vec{q}) \quad \text{(Plateau Method)}$$

4. Renormalization:

connection to experiments

$$\Pi^R(\Gamma,\vec{q}) = Z_{\mathcal{O}}\, \Pi(\Gamma,\vec{q})$$

5. Extraction of form factors

e.g. Axial current:

$$A^3_\mu \equiv \bar{\psi}\,\gamma_\mu\,\gamma_5\,\frac{\tau^3}{2}\,\psi \Rightarrow \bar{u}_N(p')\left[G_A(q^2)\,\gamma_\mu\,\gamma_5 + G_p(q^2)\,\frac{q_\mu\,\gamma_5}{2\,m_N}\right]u_N(p)$$

Isovector Combination

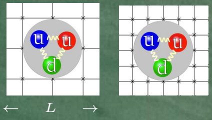
- disconnected contributions cancel out
- Simpler renormalization

Isoscalar Combination

- disconnected contributions
- operator mixing

Systematic uncertainties: Challenges & Progress

1 Cut-off Effects: finite lattice spacing



- Continuum limit $a \to 0$
- Simulations with fine lattices ($a < 0.1$ fm)
- Improve actions, algorithmic improvements

2 Finite Volume Effects

- Infinite volume limit $L \to \infty$
- Simulating hadrons in large volumes (Rule of thumb: $L\, m_\pi > 3.5$)

3 Contamination from other hadron states

- Various methods for extracting information from lattice data

4 Not simulating the physical world

- Chiral extrapolation
- Simulations at physical parameters are now feasible

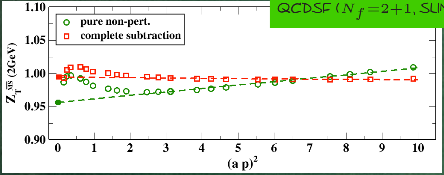
5 Renormalization and mixing

- Subtraction of lattice artifacts, utilize perturbation theory

RENORMALIZATION: Lattice artifacts: important!

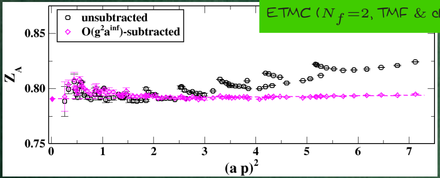
Synergy of perturbative and non-perturbative results

QCDSF ($N_f = 2+1$, SLINC)



[M. Constantinou et al. (QCDSF), arXiv:**1408.6047**]

ETMC ($N_f = 2$, TMF & clover)



[M. Constantinou et al. (ETMC), arXiv:**1509.00213**]

- Lattice artifacts computed perturbatively
- Subtraction from non-perturbative estimates

★ Usage of momentum-source method :

- Dirac equation solved with momentum source
- # of inversion depends on # of momenta considered
- Application of any operator
- High statistical accuracy

Control of lattice artifacts (Lorentz non-invariant):

$$\frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} < 0.4 \quad \text{(empirically)}$$

slide: 17

C

NUCLEON CHARGES

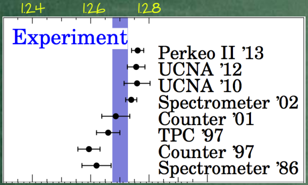
AXIAL CHARGE

Nucleon Axial current: $A^3_\mu \equiv \bar\psi\, \gamma_\mu\, \gamma_5\, \frac{\tau^3}{2}\, \psi$

$$\langle N(\vec{p}\,')\mathcal{O}^a_A N(\vec{p})\rangle = \bar{u}_N(p')\left[G_A(q^2)\,\gamma_\mu\,\gamma_5 + G_p(q^2)\,\frac{q_\mu\,\gamma_5}{2\,m_N}\right]u_N(p)$$

$$g_A \equiv \langle N(\vec{p}\,')\mathcal{O}^a_A N(\vec{p})\rangle\Big|_{q^2=0} = G_A(0)$$

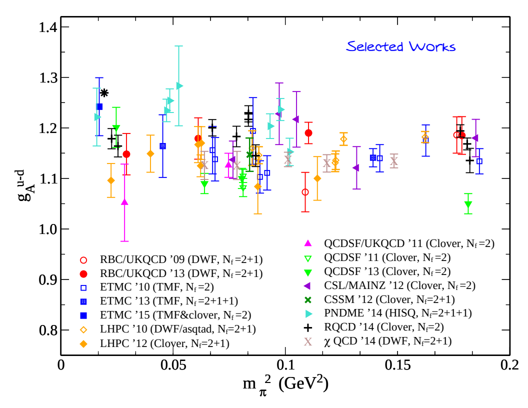
- governs the rate of β-decay (Well-determined!)



[T. Bhattacharya et al. (PNDME), arXiv1306.5435]

- related to the fraction of the nucleon spin carried by the quarks
- On the lattice: requires the lowest moment and zero momentum
- determined directly from lattice data (no fit necessary)

AXIAL CHARGE

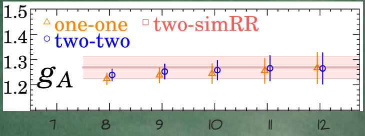


- $g_A^{exp} = 1.2701(25)$ [PDG'12]
- $m_\pi > 200$MeV: lattice results below exp.: ~10-15%

Systematic uncertainties

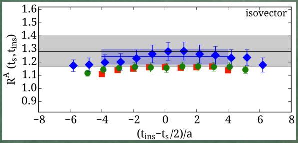
⟿ Excited States

m_π=310 MeV



[T. Bhattacharya et al. (PNDME), arXiv:1306.5435]

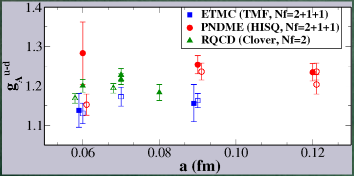
m_π=135 MeV



[A.Abdel-Rehim et al. (ETMC), arXiv:**1507.04936**]

- ★ Lattice data from 'plateau' methods
- ★ Latest achievement: lattice results at physical m_π
- ★ No necessity of chiral extrapolation
- ★ Different strategies for addressing systematic uncertainties

Cut-off effects



Continuum extrapolation requires 3 lattice spacings

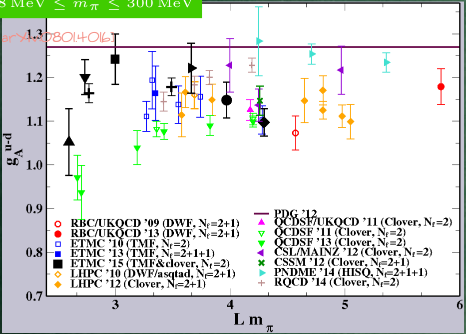
[C. Alexandrou et al. (ETMC), arXiv:1012.0857]
[G. Bali et al. (RQCD), 2014]
[R. Gupta et al. (PNDME), 2014]

$a < 0.1$ fm is sufficient

Finite Volume Effects

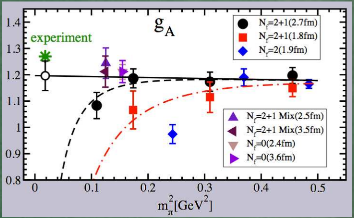
128 MeV $\leq m_\pi \leq 300$ MeV

arXiv:0801.4016



Lattice data for plateau method
No volume corrections

[T. Yamazaki et al. (RBC/UKQCD)]



Volume effects not fully understood

SCALAR & TENSOR CHARGES

- ★ Non $V-A$ structure of weak interaction
- ★ Small contributions of scalar/tensor interactions in SM (10^{-3})
- ★ ϵ_S, ϵ_T: low-energy couplings

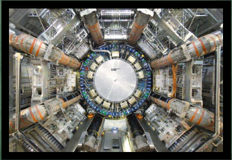
$$H_{eff} = G_F \left(J^l_{V_A} \times J^q_{V_A} + \sum_i \epsilon_i \mathcal{O}^l_i \times \mathcal{O}^q_i \right)$$

  - related to masses of new TeV-scale particles
  - require knowledge of g_S: $\langle p|\bar{u}d|n\rangle$, g_T: $\langle p|\bar{u}\sigma^{\mu\nu}d|n\rangle$

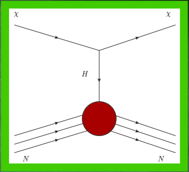
- ★ scalar interactions: $0^+ \to 0^+$ nucleon decays
- ★ tensor interactions: radiative pion decay $\pi \to e\nu\gamma$
- ★ Upcoming experiments (TeV scale) that probe small signals: UCNB & UCNb at LANL, Nab at ORNL, ATLAS at LHC

UCN @ LANL

ATLAS @ LHC



SCALAR CHARGE



- sensitivity of m_N to m_q: $\langle N|\bar{q}q|N\rangle = \frac{\partial m_N}{\partial m_q}$
- no direct experimental measurements
- Indirect measurements: meson-nucleon scattering amplitudes (large system.)
- related to nucleon σ - terms:

$$\sigma_l = \frac{1}{2}(m_u+m_d)\langle N|\bar{u}u+\bar{d}d|N\rangle \qquad \sigma_s = m_s\langle N|\bar{s}s|N\rangle$$

nucleon mass generated by the quarks via spontaneous chiral symmetry breaking

Strange quark content of nucleon:

$$y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} = 1 - \frac{\sigma_0}{\sigma_l}$$

- important for direct search of dark matter
  large coupling of strange quarks to candidate dark matter [J. Ellis et al., arXiv:0801.3656]

Lattice calculations

Direct Method
$\langle N|\bar{q}q|N\rangle$ (3pt CI & DI)
discussed in this talk

Spectrum Method
Feynman-Hellmann on $\frac{\partial m_N}{\partial m_q}$
R. Young, [arXiv:1301.1765]

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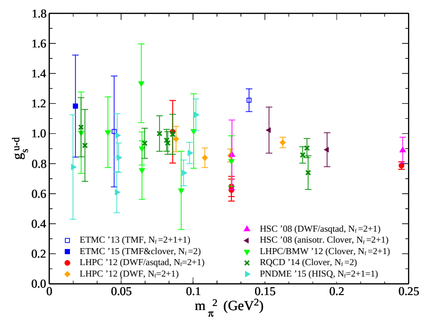
SCALAR CHARGE: The Squiggly One

$$g_S \equiv \langle N|\bar{u}u - \bar{d}d|N\rangle|_{Q^2=0}$$



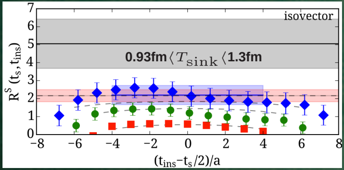
Challenging calculation:

- smallest signal-to-noise ratio
- systematics are not well-controlled
- disconnected contributions not negligible
- requires vacuum subtraction

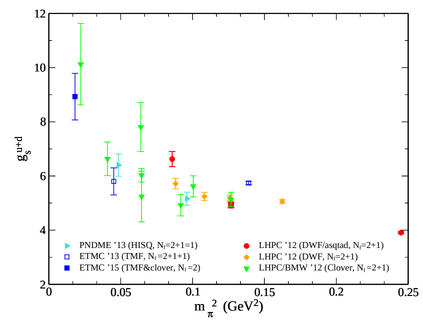
★ Severe contamination of excited states

TMF&Clover: $N_f=2$ $m_\pi=135$MeV

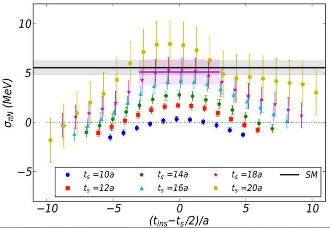
- Increasing trend for plateau value for large T_{sink}

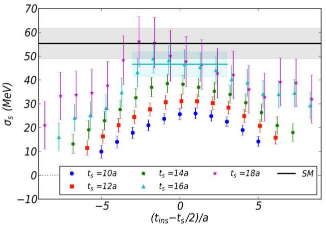


[A.Abdel-Rehim et al. (ETMC), arXiv:1507.04936]



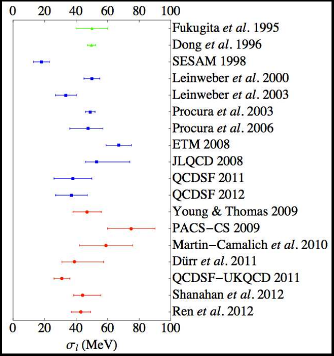
σ - terms



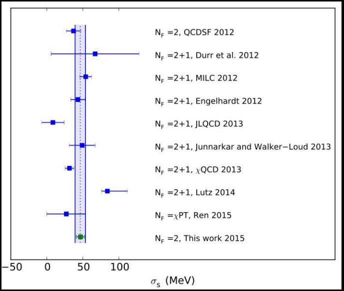


$m_\pi = 135 MeV$

[A.Abdel-Rehim et al. (ETMC), preliminary]



[R. Young, Lattice 2012]



[A. Vaquero et al., Lattice 2015]

Phenomenology:

$\sigma_l = 45 \pm 8$ MeV

[J. Gasser et al., **PLB 253(1991) 252**]

$\sigma_l = 64 \pm 7$ MeV

[M. Pavan et al., **hep-ph/0111066**]

$\sigma_l = 59 \pm 7$ MeV

[J. Alarcon et al., arXiv:**1110.3797**]

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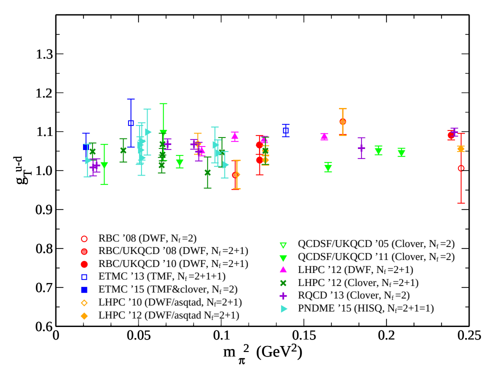
TENSOR CHARGE

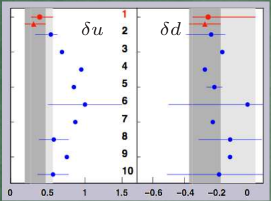
$$g_T \equiv \langle N|\sigma^{\mu\nu}|N\rangle|_{Q^2=0}$$



SIDIS results (HERMES, COMPASS) and BELLE e+ e- analysis

- $g_T^{\exp}(0.8\mathrm{GeV}^2)=0.77^{+0.13}_{-0.27}$

[M.Anselmino et al., arXiv:0812.4366]



[M.Anselmino et al., arXiv:1303.3822]

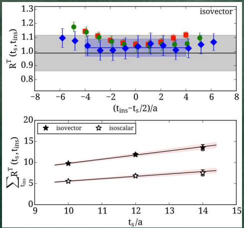
★ Agreement among most lattice points
★ Mild m_π dependence

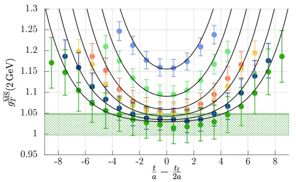
g_T^{IV} input in analysis of neutron β-decay

★ strong scale-dependence

slide: 26

Investigation of Systemetics





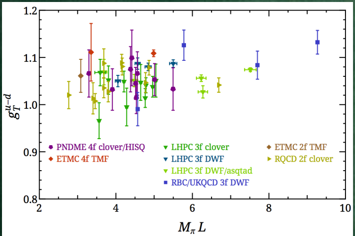
[C. Alexandrou et al. (ETMC), arXiv:**1507.04936**]

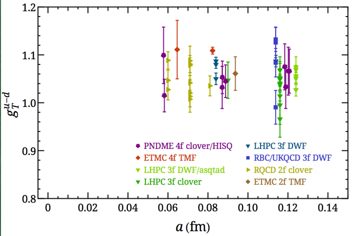
m_π=135MeV, a=0.093fm, $T_{\rm sink}$:0.93−1.31fm

[G. Bali et al. (RQCD), arXiv:**1412.7336**]

m_π=290MeV, a=0.071fm, $T_{\rm sink}$:0.5−1.2fm

$T_{\rm sink} > 1$ fm is safe





[T. Bhattacharya et al. (PNDME), arXiv:**1506.06411**]

Little sensitivity to m_π, a, $T_{\rm sink}$

Implication of g_T to New Physics Searches

- New interaction at TeV scale :
  - source of CP violation
  - may give rise to nEDM (quark-photon coupling)

- g_T related to the quark contributions to the nEDM:

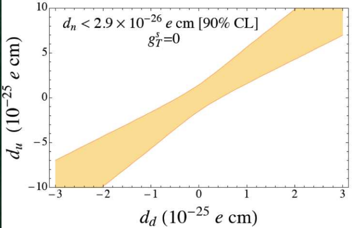
$$d_n = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

  - LQCD may constrain the low-energy effective couplings d_u, d_d, d_s
  - individual quark contributions $\Rightarrow$ disconnected contributions
  - current best exp. upper limit:

$$|d_n| < 2.9 \times 10^{-26}\, e\, cm \text{ (90 \% C.L.)}$$

(ILL Grenoble)

90 % confidence interval bounds of d_u, d_n (Assumption: $g_T^s = 0$)



χPT to 135MeV, $a \to 0$, $L \to \infty$

$g_T^u = +0.774(66)$

$g_T^d = -0.233(28)$

Include DI

- assume Peccei-Quinn mechanism
- ignore θ-term contribution to nEDM
  contributions of higher dim. operators negligible

Talks by G. Schierholz & T. Bhattacharya on Thu

[T. Bhattacharya et al. (PNDME), arXiv:**1506.06411**]

slide: 28

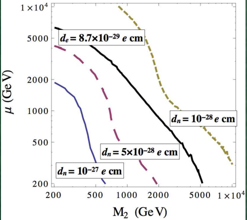
Consequenses in split SUSY models

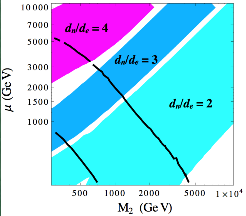
[J. Wells, arXiv:hep-ph/0306127]
[G. Giudice et al., arXiv:hep-ph/0406088]
[N. Arkani-Hamed et al., arXiv:hep-ph/0409232]

- all scalars much heavier than electroweak scale (except one Higgs doublet)
- preserves gauge coupling unification
- there is a dark matter candidate
- avoids constraints related to flavor & CP problem
- QEDMs leading contributions
- gaugino (M_2) and Higgsino μ masses, their relative phase ϕ and Higgs vacuum expectation value $\tan(\beta)$

Talks by V. Cirigliano & T. Bhattacharya on Thursday

$d_e = 8.7 \times 10^{-29}$ $e\,cm$ (90% C.L.) (ACME)





[T. Bhattacharya et al. (PNDME), arXiv:1506.04196]

Not overlaping bands due to precision of lattice results!

$d_n < 4 \times 10^{-28}$ $e\,cm$ for split-SUSY to hold

Thanks to V. Cirigliano for material

D

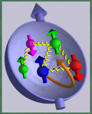
SPIN STRUCTURE

SPIN STRUCTURE OF NUCLEON

Understanding of nucleon spin has evolved:

$\Rightarrow$

$\Rightarrow$



Simple parton model
$\frac{1}{2}(\Delta u_v + \Delta d_v) = \frac{1}{2}$

Sea quarks & Gluons are polarized
$\frac{1}{2}(\Delta q + \bar{\Delta} q) + \Delta G = \frac{1}{2}$

Parton orbital angular momentum
$\frac{1}{2}(\Delta q + \bar{\Delta} q) + \Delta G + L_z = \frac{1}{2}$

Where does the nucleon spin come from? Exper. Status

- Quark Contributions
  - Spin 20% - 30% (DIS)
  - Orbital angular momentum (Upcoming experiments of GPDs and TMDs)
- Gluon Contributions
  - Spin 40% (STAR, PHENIX, COMPASS)
  - Orbital angular momentum zero

[S. Brodsky et al., hep-ph/0608219]

There is a need to find the missing contributions to the spin !

slide: 31

NUCLEON SPIN

Input from Lattice QCD

Spin Sum Rule:

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

Quark orbital angular momentum

Quark Spin

Gluon part

Extraction from LQCD:

$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right) , \quad L^q = J^q - \Sigma^q , \quad \Sigma^q = g_A^q$$

Status of Lattice Calculations

- Quark Contributions
  - Quark Spin (Connected) $\sim$ 40% - 50%
  - Light Quark Spin (Disconnected) $\sim$ 5% - 7%
  - Strange Quark Spin (Disconnected) $\sim$ 3%
  - Orbital angular momentum $L^{u+d} \sim 0$ ($L^u \sim -L^d$)
  - Total Spin carried almost exclusively by the up quark ($J^d \sim 0$)

slide: 32

Gluon unpolarized disctribution

⟿ Lattice Calculations

Disconnected diagram is required

- Direct lattice computation of gluon moment $\langle x \rangle_g$:

Gluon Operator

$$\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho} G_{\nu\rho}\right]$$

$$\langle N(p)|\mathcal{O}_{44} - \frac{1}{3}\sum_{j=1}^{3} \mathcal{O}_{jj}|N(p)\rangle = \left(m_N + \frac{2}{3\,E_N}\vec{p}^{\,2}\right)\langle x \rangle_g$$

- Decomposition of Energy-momentum Tensor

$$J^i_{q,g} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x \,\left(\mathcal{T}^{0k}_{q,g}\, x^j - \mathcal{T}^{0j}_{q,g}\, x^k\right)$$

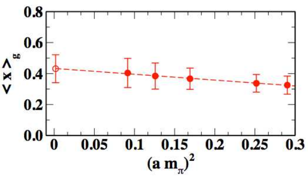
$$\mathcal{T}^{(E)}_{\{4i\}q} = -\frac{i}{4}\sum_f \overline{\psi}_f \left[\gamma_4 \overrightarrow{D}_i + \gamma_i \overrightarrow{D}_4 - \gamma_4 \overleftarrow{D}_i - \gamma_i \overleftarrow{D}_4\right]\psi_f$$

$$\mathcal{T}^{(E)}_{\{4i\}g} = -\frac{i}{2}\sum_{k=1}^{3} 2\mathrm{Tr}^c\left[G_{4k}G_{ki} + G_{ik}G_{k4}\right]$$

Lattice Results

Quenched

Feynman-Hellmann

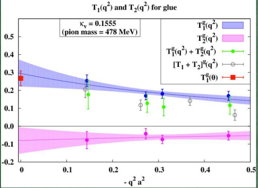


[R. Horsley et al. (QCDSF), 2012, arXiv:1205.6410]

$N_f = 0$ Clover, $m_\pi = 314 - 555$ MeV

$\langle x \rangle_g = 0.43(7)(5)$

Energy-Momentum tensor

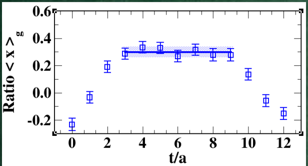


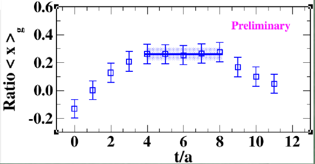
[M. Deka et al. (χQCD), 2013, arXiv:1312.4816]

$N_f = 0$ Wilson, $m_\pi = 478 - 650$ MeV

$\langle x \rangle_g = 0.313(56)$

Dynamical





[C. Alexandrou et al. (ETMC), 2015]

$N_f = 2{+}1{+}1$ ETM, $m_\pi = 375$ MeV

$\langle x \rangle_g = 0.309(25)$

$N_f = 2$ TMF & Clover, $m_\pi = 135$ MeV

$\langle x \rangle_g = 0.283(41)$

Smearing: improves signal

Challenges with Renormalization of Gluon operator

- Mixing with Quark singlet operator: $\mathcal{O}^q \equiv \bar{\psi}\gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}}\psi$

Unavoidable

- Mixing with other Operators:
  - Gauge invariant
  - BRS-variations
  - vanish by the e.o.m.

Vanish in physical matrix elements

2 x 2 mixing matrix

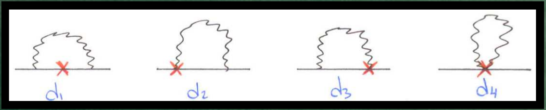
$$\begin{pmatrix} \langle x \rangle_g^{\overline{\text{MS}}}(\mu) \\ \sum_q \langle x \rangle_q^{\overline{\text{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{gg}^{\overline{\text{MS}}}(\mu) & Z_{gq}^{\overline{\text{MS}}}(\mu) \\ Z_{qg}^{\overline{\text{MS}}}(\mu) & Z_{qq}^{\overline{\text{MS}}}(\mu) \end{pmatrix} \begin{pmatrix} \langle x \rangle_g \\ \sum_q \langle x \rangle_q \end{pmatrix}$$

$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g^B + Z_{gq} \sum_q \langle x \rangle_q^B$ and $\sum_q \langle x \rangle_q^R = Z_{qq} \sum_q \langle x \rangle_q^B + Z_{qg} \langle x \rangle_g^B$

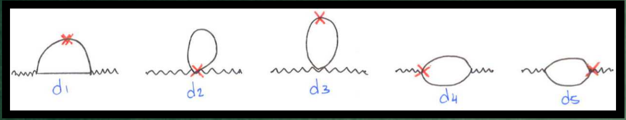
★ Quenched case: $Z_{qg} = 1 - Z_{gg}$, $Z_{gq} = 1 - Z_{qq}$

Perturbative computation

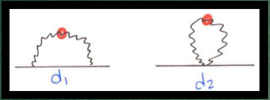
× Z_{qq} : $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$



× Z_{qg} : $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$

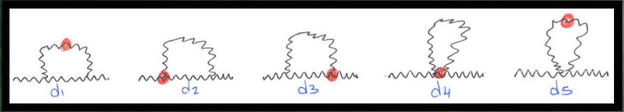


• Z_{gq} : $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$



Many millions of terms...

• Z_{gg} : $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$



- Multiplicative renormalization
- Identification of mixing
- General action parameters
- (wide applications)
- Stout smearing (action & operator)

Example

Clover fermions, Iwasaki gluons, 2 stout smearing steps for $\mathcal{O}_g$

$$Z_{gg} = 1 + \frac{g^2}{16\pi^2}\left(1.0574\, N_f + \frac{-13.5627}{N_c} - \frac{2\, N_f}{3}\log(a^2\bar{\mu}^2)\right)$$

$$Z_{gq} = 0 + \frac{g^2\, C_f}{16\pi^2}\left(0.8114 + 0.4434\, c_{SW} - 0.2074\, c_{SW}^2 + \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

$$Z_{qq} = 1 + \frac{g^2}{16\pi^2}\left(-1.8557 + 2.9582\, c_{SW} + 0.3984\, c_{SW}^2 - \frac{8}{3}\log(a^2\bar{\mu}^2)\right)$$

$$Z_{qg} = 0 + \frac{g^2\, N_f}{16\pi^2}\left(0.2164 + 0.4511\, c_{SW} + 1.4917\, c_{SW}^2 - \frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

[M. Constantinou et al. (Cyprus Group), 2015]

Application for $N_f=2$ TMF & clover, $m_\pi=135$ MeV:

$\langle x\rangle_{u+d}^R = 0.587(18)$ $\quad\langle x\rangle_g^R = 0.283(41)$ $\quad\Rightarrow\quad$ $\langle x\rangle_{u+d}^R + \langle x\rangle_{u+d}^R = 0.870(43)$

Missing quark disconnected contributions

E

CONCLUSIONS

SUMMARY & CHALLENGES

- Simulating the physical world
- New physics BSM
  - Lattice QCD provides predictions
- Dedication of human force and computational resources on:
  - Control of statistical uncertainties ⇒ noise reduction techniques crucial
  - comprehensive study of systematic uncertainties
  - study of DI at lower masses (Target: physical m_π!)
    - challenging task
    - exploid techniques: AMA, hierarchical probing, others
    - usage of GPUs
    - current computations of DI provide bounds
- Nucleon spin: dynamical simulations for gluon angular momentum
  - Becoming feasible
  - Overcoming difficulties with renormalization and mixing
  - Rely on perturbation theory

Stay Tuned!

THANK YOU

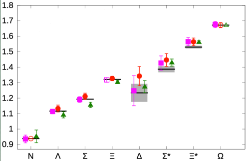
BACKUP SLIDES

slide: 41

Computation of Observables

Configuration Generation

Physics!



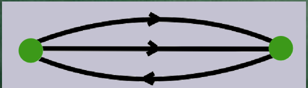
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int_U \mathcal{O}(D^{-1}, U)\, det(D[U]^{N_f})\, e^{-S[U]}$$

Quark Propagators

(CPUs, GPUs)

Data analysis

Contraction of propagators



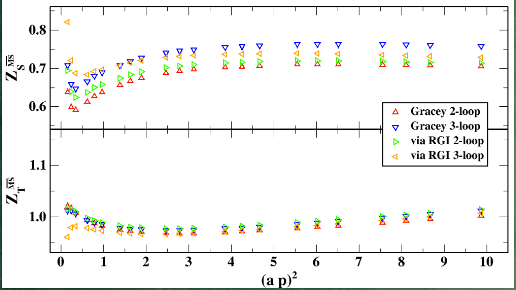
RENORMALIZATION

RI' scheme:

$$Z_q = \frac{1}{12}\mathrm{Tr}[\left(S^L(p)\right)^{-1} \; S^{\mathrm{Born}}(p)]\Big|_{p^2=\bar{\mu}^2}$$

$$Z_q^{-1}\, Z_{\mathcal{O}}\, \tfrac{1}{12}\mathrm{Tr}[\Gamma^L_{\mathcal{O}}(p)\, \left(\Gamma^{\mathrm{Born}}_{\mathcal{O}}(p)\right)^{-1}]\Big|_{p^2=\bar{\mu}^2} = 1$$

★ Conversion to $\overline{\mathrm{MS}}(\mu = 2\mathrm{GeV})$:



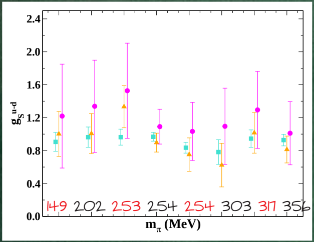
[M. Constantinou et al. (QCDSF), arXiv:1408.6047]

- Systematics due to conversion to $\overline{\mathrm{MS}}$ under control
- Scalar Operator: 3-loop expressions necessary

LHPC: $m_\pi = 149 - 356$MeV

[J.R.Green et al. (LHPC), arXiv:**1206.4527**]

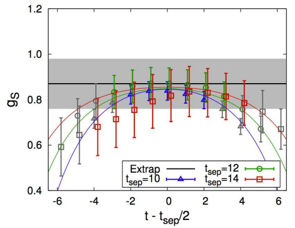
★ $m_\pi = 149$MeV: 0.93 fm $\langle T_{\text{sink}} \langle$ 1.39 fm



PNDME: $m_\pi = 220$MeV

[T. Bhattacharya et al. (PNDME), arXiv:**1501.07639**]

★ $m_\pi = 149$MeV: 0.9 fm $\langle T_{\text{sink}} \langle$ 1.26 fm

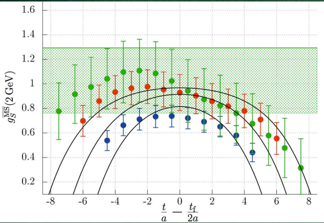


Increasing trend for the plateau value for larger values of T_{sink}

RQCD: $m_\pi = 150$MeV

[G. Bali et al. (RQCD), arXiv:**1412.7336**]

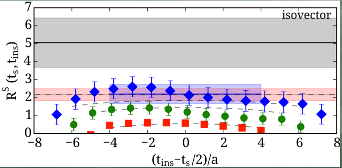
★ 0.64 fm $\langle T_{\text{sink}} \langle$ 1.07 fm



ETMC: $N_f = 2\&\ c_{SW}$, $m_\pi = 135$MeV

[A.Abdel-Rehim et al. (ETMC), arXiv:**1507.04936**]

- 0.93 fm $\langle T_{\text{sink}} \langle$ 1.3 fm
- $T_{\text{sink}} \geq$1.5 fm: agreement with SM



slide: 44

NUCLEON SPIN

Input from Lattice QCD

Spin Sum Rule:

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

Quark orbital angular momentum

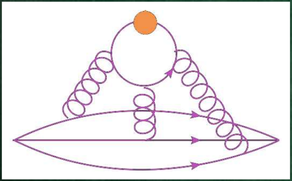
Quark Spin

Gluon part

Extraction from LQCD:

$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right) \ , \quad L^q = J^q - \Sigma^q \ , \quad \Sigma^q = g_A^q$$

★ Individual quark contributions: disconnected insertion contributes
★ Status of proper $Z_{\mathcal{O}}^{singlet}$: Perturbatively, Feynman-Hellmann



$$\mathrm{Tr}\left[\mathcal{O}G(x;x)\right]$$

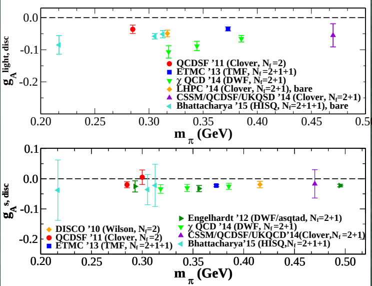
★ We need to compute all-to-all propagator
★ extremely difficult to compute
★ very noisy and very expensive conputationally
★ We've come far in development of techniques:

- Truncated Solver Method
- One-end-trick
- All-Mode-Averaging
- Hierarchical probing

slide: 45

NUCLEON SPIN

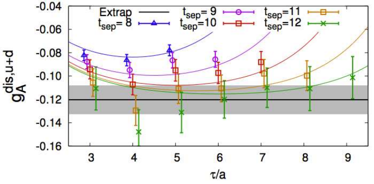
Disconnected Contributions



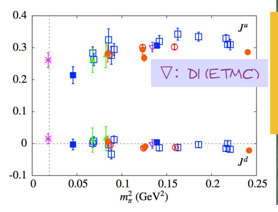
★ Agreement of various results

★ DI for g_A lower the total value

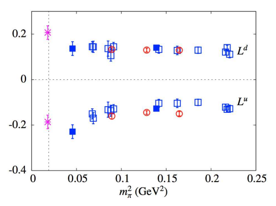
[T. Bhattacharya et al., arXiv:1503.05975]



Collected results

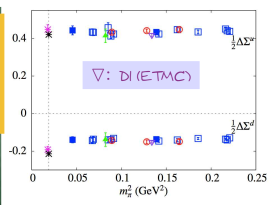


○ LHPC (DWF/asqtad, $N_f=2+1$)
● LHPC (DWF, $N_f=2+1$)
▲ QCDSF (Clover, $N_f=2$)
□ ETMC (TMF, $N_f=2$)
■ ETMC (TMF, $N_f=2+1+1$)
★ ETMC (TMF&c_{sw}, $N_f=2$)



Most results: only CI
TMF: include $Z_A^s - Z_A^{ns}$
m_π=131 MeV:
- $J^u \sim 0.25$, $J^d \sim 0$
- $\Delta\Sigma^u$, $\Delta\Sigma^d$ agrees with exp.

$L^{u+d} \sim 0$



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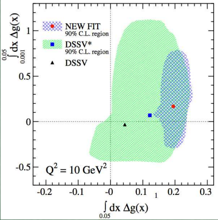
Gluon unpolarized disctribution

Experimental Status

- Spin 40% (STAR, PHENIX, COMPASS)
- Orbital angular momentum zero
- Glue helicity 0.2 (STAR, COMPASS)



Lattice Calculations (disconnected diagram)

- Direct lattice computation of gluon moment $\langle x \rangle_g$:

Gluon Operator

$$\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\,[G_{\mu\rho} G_{\nu\rho}]$$

$$\langle N(p)|\mathcal{O}_{44} - \tfrac{1}{3}\textstyle\sum_{j=1}^{3} \mathcal{O}_{jj}|N(p)\rangle = \left(m_N + \frac{2}{3\,E_N}\vec{p}^{\,2}\right)\langle x \rangle_g$$

- Decomposition of Energy-momentum Tensor

$$J^i_{q,g} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x \left(\mathcal{T}^{0k}_{q,g}\, x^j - \mathcal{T}^{0j}_{q,g}\, x^k\right)$$

$$\mathcal{T}^{(E)}_{\{4i\}q} = -\frac{i}{4}\sum_f \overline{\psi}_f \left[\gamma_4 \overrightarrow{D}_i + \gamma_i \overrightarrow{D}_4 - \gamma_4 \overleftarrow{D}_i - \gamma_i \overleftarrow{D}_4\right]\psi_f$$

$$\mathcal{T}^{(E)}_{\{4i\}g} = -\frac{i}{2}\sum_{k=1}^{3} 2\mathrm{Tr}^c\,[G_{4k}G_{ki} + G_{ik}G_{k4}]$$