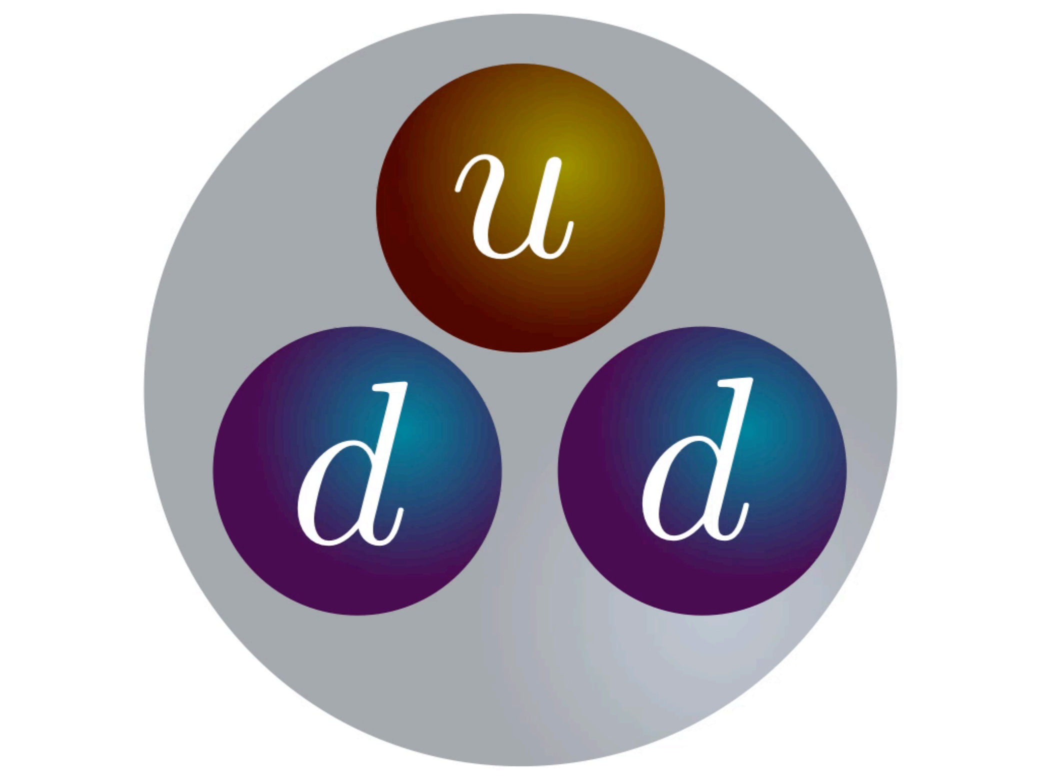
NEUTRON-ANTINEUTRON TRANSITIONS:

CONTROLLED CONNECTIONS TO NEW PHYSICS FROM FIRST-PRINCIPLES



Michael I. Buchoff
Lawrence Livermore National Laboratory

In collaboration with Sergey Syritsyn, Michael Wagman, Chris Schroeder, and Joe Wasem

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.
LLNL-PRES-677388

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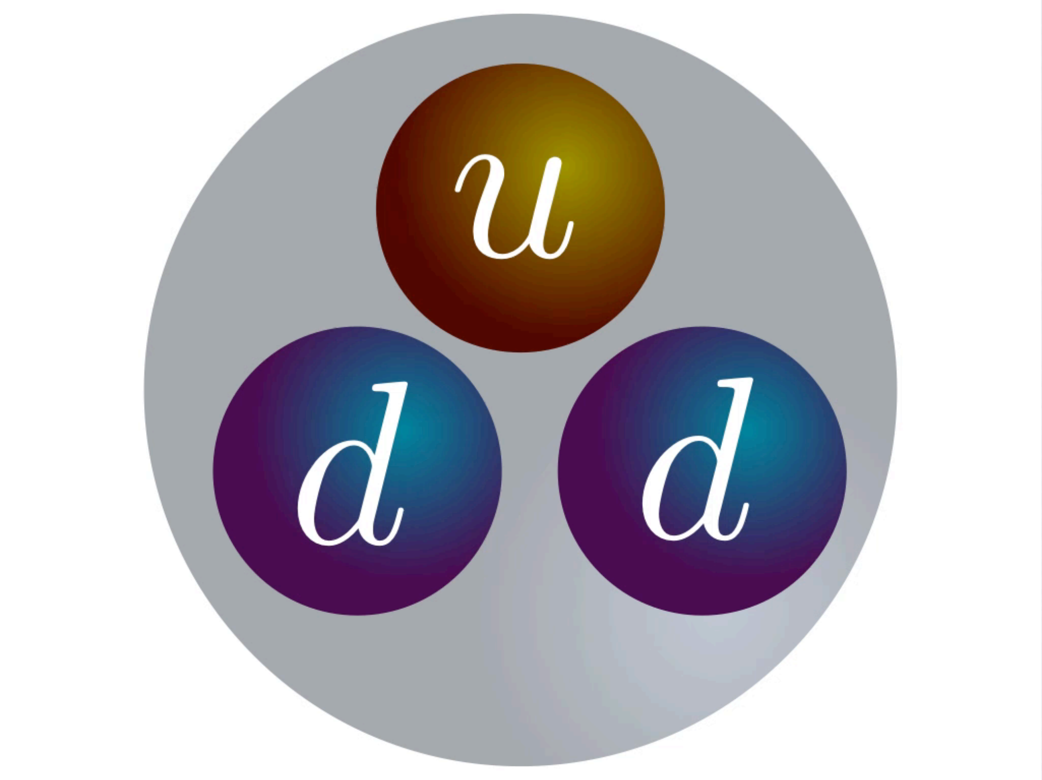

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What we know

1) There exists a baryon-antibaryon asymmetry

2) The Standard Model alone cannot reproduce

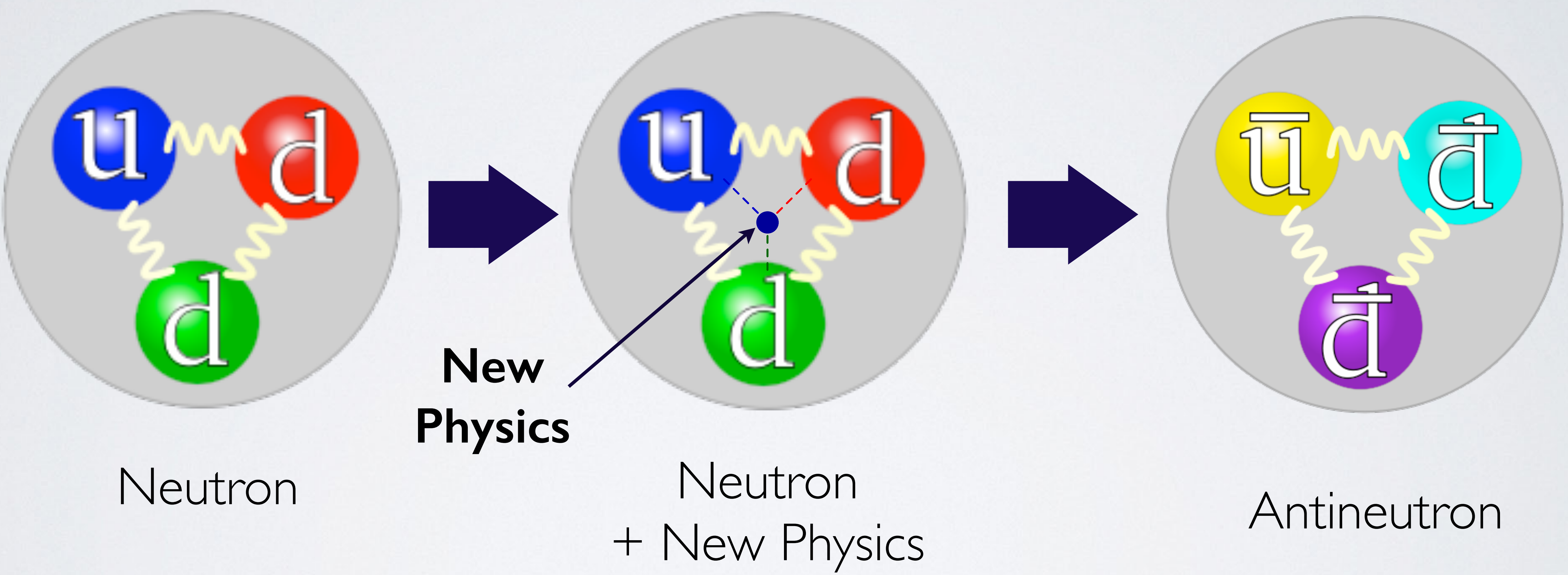
- **Too little CP violation**
- **Too little baryon number (B) violation**

3) Can be encoded in low-energy "hints"

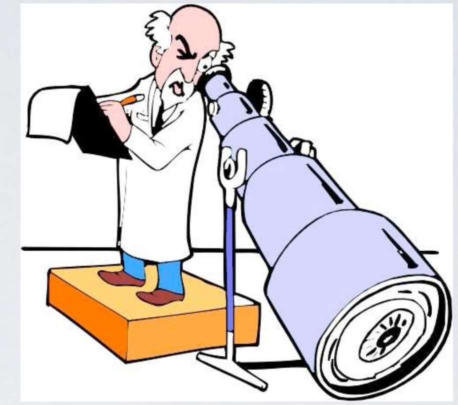
- **Neutron EDM (CP violation)**
- **Proton decay (B violation)**
- **Neutron-antineutron transition (B violation)**

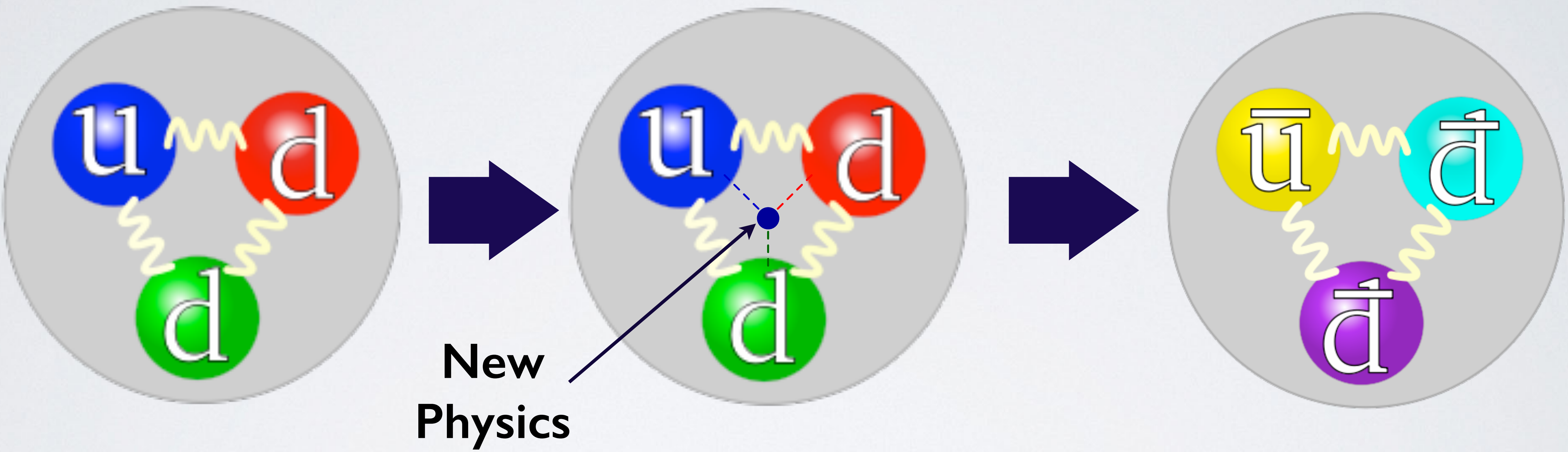
"Hints" can strongly constrain how the Universe evolved

Neutron-antineutron schematic



Neutron-antineutron schematic





Neutron

Neutron
+ New Physics

Antineutron

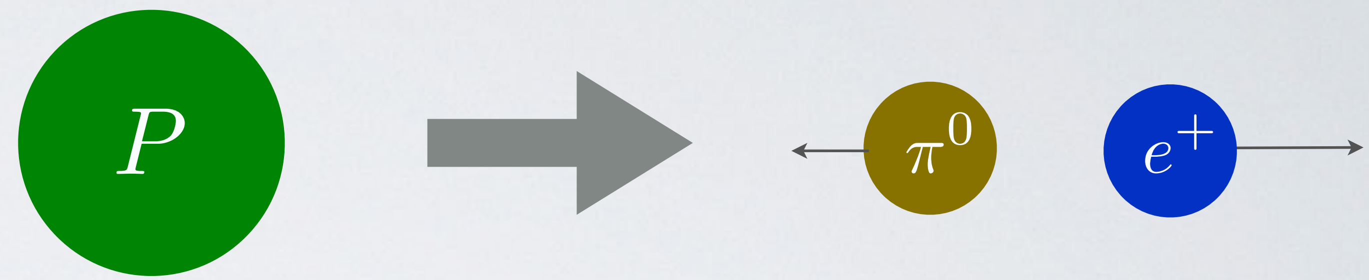
Can learn about new physics by measuring transition rate

Two search channels in nuclear physics

PROTON DECAY

$|\Delta B| = 1$ $\quad$ $|\Delta L| = 1$

$M_{NP} \sim 10^{12} - 10^{13}$ TeV *



Leading Effective interaction:

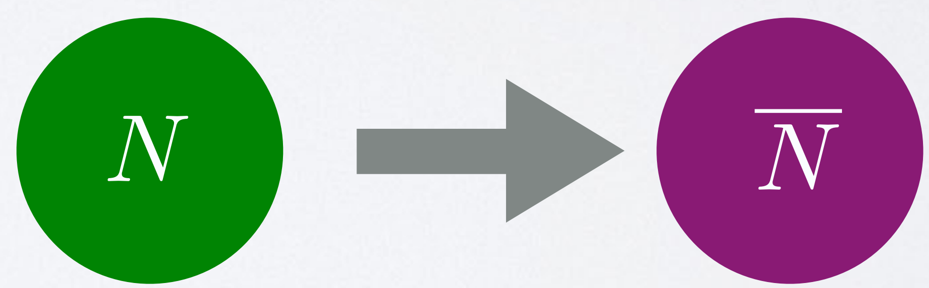
$B - L = 0$

$$\mathcal{L}_{\not B} \sim \frac{1}{M_{NP}^2} QQQL$$

NEUTRON-ANTINEUTRON TRANSITIONS

$|\Delta B| = 2$ $\quad$ $|\Delta L| = 0$

$M_{NP} \sim 1 - 1000$ TeV *



Leading Effective interaction:

$B - L \neq 0$

$$\mathcal{L}_{n\bar{n}} \sim \frac{1}{M_{NP}^5} QQQQQQ$$

* Can be altered significantly by details of new physics

Enter neutron-antineutron transitions!

In 1937, Majorana conjectured neutrons and antineutrons could be states of the same particle

" ... this method ... allows not only to cast the electron-positron theory into a symmetric form, but also to construct an essentially new theory for particles not endowed with an electric charge (neutrons and the hypothetical neutrinos)."

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In 1937, Majorana conjectured neutrons and antineutrons could be states of the same particle

“ ... this method ... allows not only to cast the electron-positron theory into a symmetric form, but also to construct an essentially new theory for particles not endowed with an electric charge (neutrons and the hypothetical neutrinos).”

N-NBar: $|\Delta B| = 2$ $|\Delta L| = 0$ $|B - L| = 2$

Sensitive to different possible B-violating processes

PROTON DECAY

- Insensitive to process where $|\Delta B| > 1$
- Insensitive to processes independent of L $|\Delta L| = 0$

Basic idea

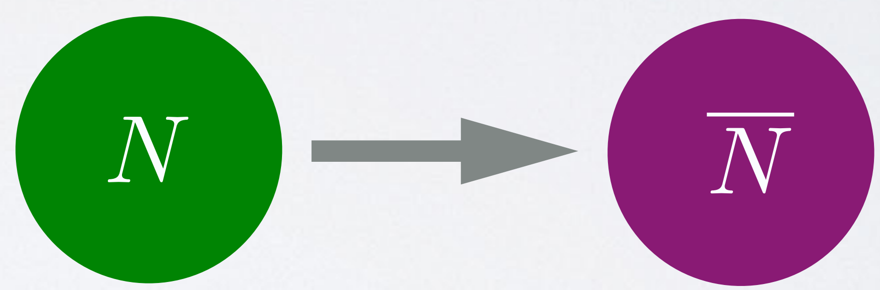
- New physics leads to neutron-antineutron mixing

δm (From new physics) $\qquad \tau_n = \frac{1}{\lambda} \approx 14.7 \text{ min}$

For no external interactions:

$$H = \begin{pmatrix} \langle n| & \langle \bar{n}| \end{pmatrix} \begin{pmatrix} m_n - i\lambda/2 & \delta m \\ \delta m & m_n - i\lambda/2 \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

- Transition Probability



$$P_{n\to\bar{n}}(t) = \sin^2(\delta m\ t)\, e^{-\lambda t}$$

$$\tau_{n\bar{n}} = \frac{1}{\delta m}$$

Model Estimates

Examples: $\tau_{n\bar{n}}$

TeV-scale seesaw mechanism for neutrino masses in $SU(2)_L \times SU(2)_R \times SU(4)_c$	$300 - 3000$ years	Babu, Bhupal Dev, Mohapatra (2009)
SO(10) seesaw mechanism with adequate baryogenesis	$30 - 30,000$ years	Babu, Mohapatra (2012)
Certain extra-dimensional particles	> 3 years	Nussinov, Shrock (2002); Ng, Winslow (2010)
500 - 1000 TeV Scalar extensions to SM without proton decay	$10 - 1000$ years	Arnold, Fornal, Wise (2012)

Others:

- R-parity Violation — hep-ph/0406039
- MFV SUSY — Csaki, Grossman, Heidenreich arXiv:1111.1239
- Low Scale Gravity — Dvali, Gabadadze PLB 1999
- ....

- **Estimates** for confirming/ruling out large classes of models

$$\tau_{n\bar{n}} > 300 - 3000 \text{ years}$$

Model Estimates

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	$\tau_{n\bar{n}}$	
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Dvali, Gabadadze
PLB 1999

....

Uncontrolled NDA estimates for QCD matrix element

- **Estimates** for confirming/ruling out large classes of models

$$\tau_{n\bar{n}} > 300 - 3000 \text{ years}$$

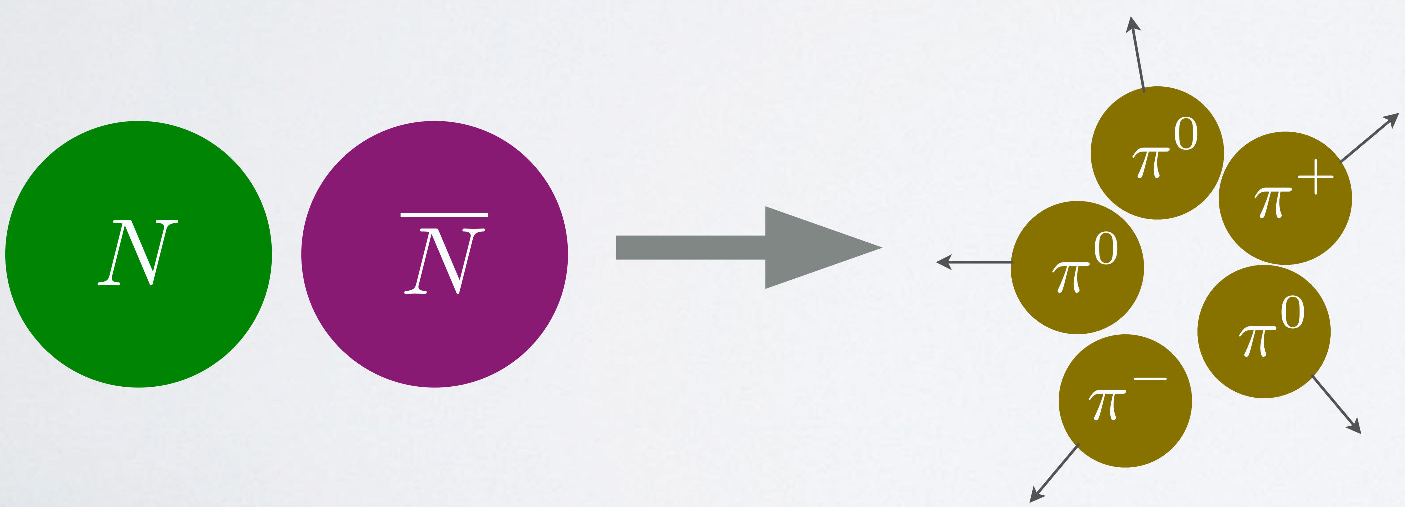
Experimental Basics

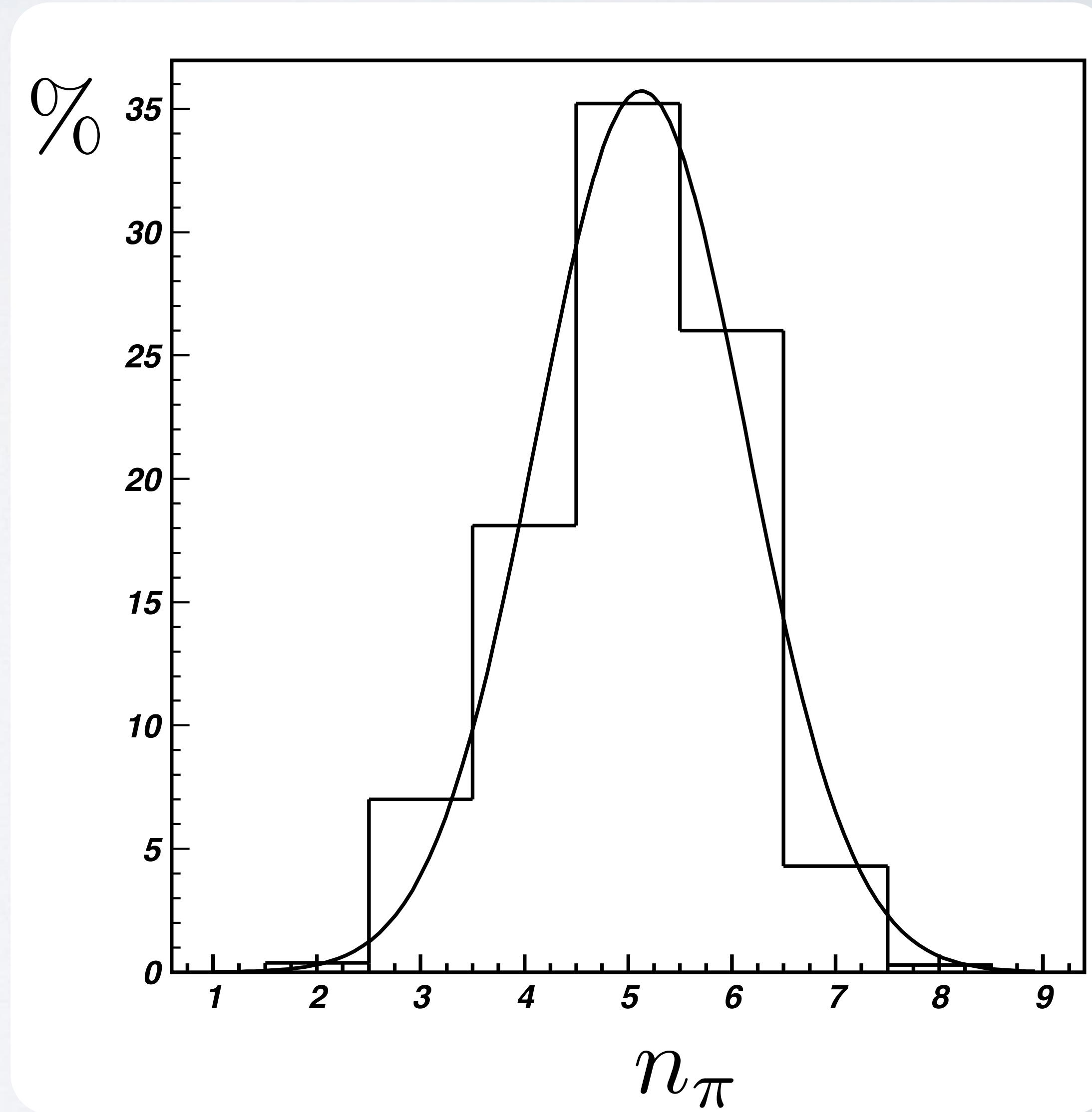
Many neutrons → Many neutrons + antineutron → Annihilation

- Neutron-antineutron annihilation signals

Primary channel $n\bar{n} \rightarrow 5\pi$

Crystal Barrel experiment

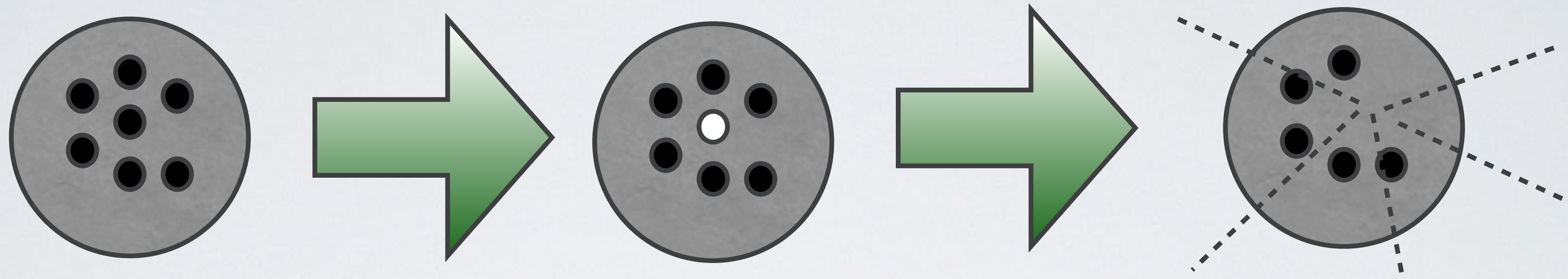




- Two types of experiments

Experimental Progress

1. Neutron-antineutron annihilation in nuclei



A human contains roughly 2×10^{28} neutrons

Straight-forward question:
Why have we not annihilated yet?

Why?
$\tau_{n\bar{n}} \ll \tau_{\text{nuclei}}$

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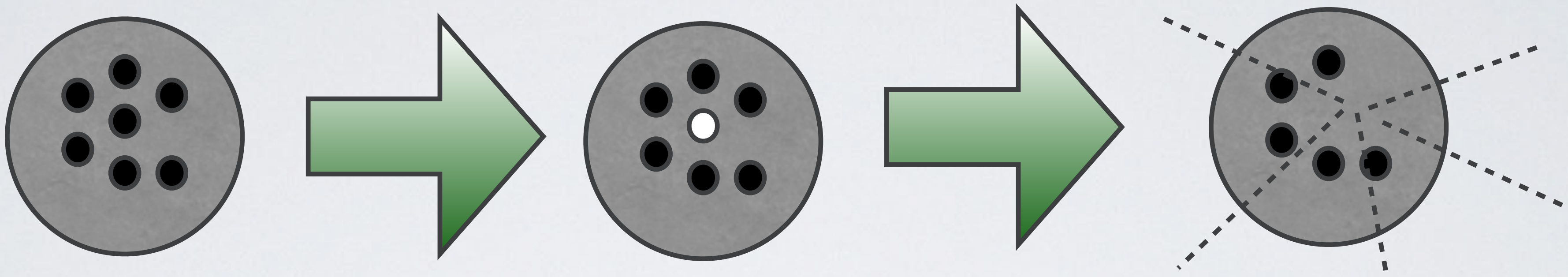

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Precise answer is a difficult nuclear structure question

Can show from QM: $\tau_{\rm nuclei} \sim \tau_{n\bar{n}}^2$

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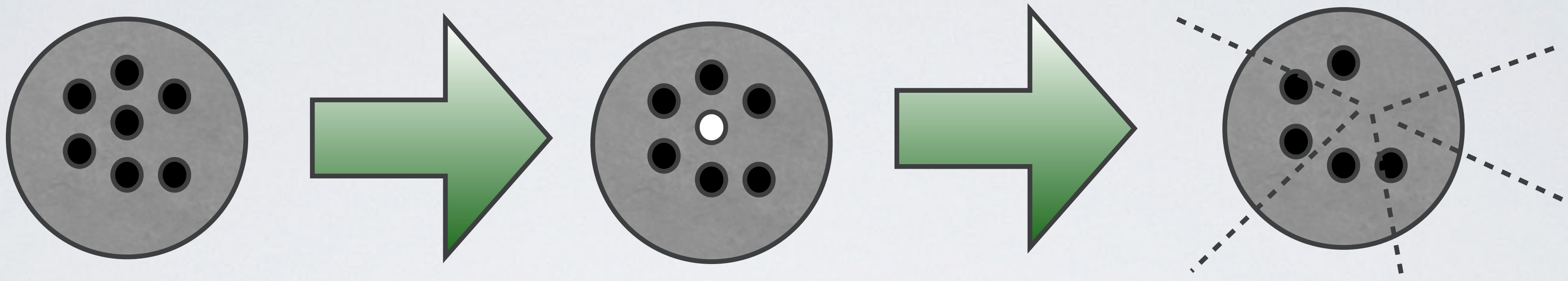

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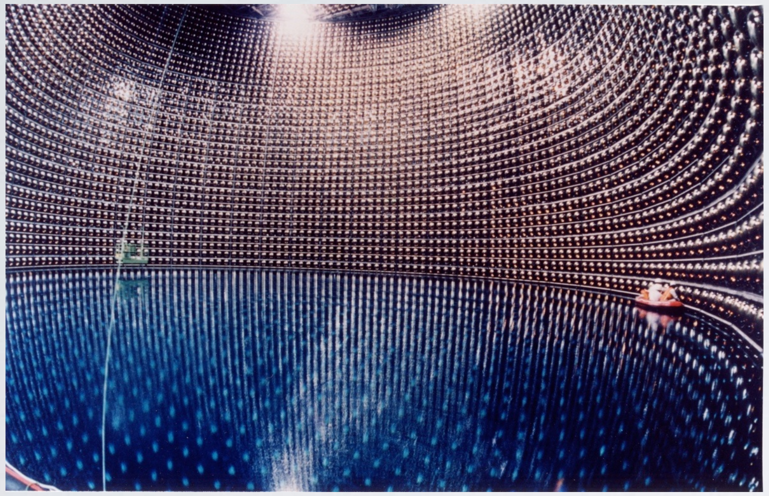
Can show from QM: $\tau_{\text{nuclei}} \sim \tau_{n\bar{n}}^2$

Crude estimate: Primary dimensionful scale binding energy E_B

$\tau_{\text{nuclei}} = R\,\tau_{n\bar{n}}^2$ ➡ $[R] = \dfrac{1}{\text{time}} = \text{Energy}$ ➡ $R \approx E_B \approx 8\ \text{MeV} \approx \dfrac{4 \times 10^{29}}{1\ \text{year}}$

Experimental Progress

1. Neutron-antineutron annihilation in nuclei



H_2O

Super-K bounds (2011)

$\tau_{n\bar{n}} > 11$ years

$$R = \frac{1.6 \times 10^{30}}{\text{year}}$$

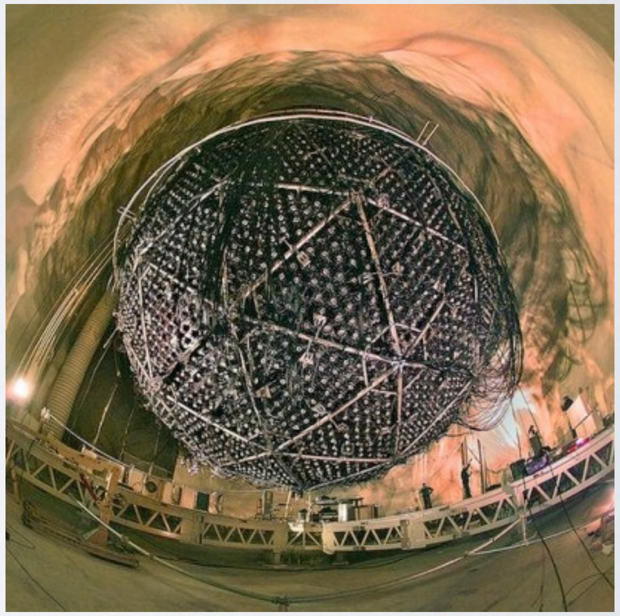
Friedman, Gal 2008

SNO Laboratory

1,100 tons of heavy water

Focus on deuterium → D_2O ← Not on oxygen

$\tau_{n\bar{n}} > 5.7$ years (Preliminary)



$$R = \frac{(3.7 - 9.3) \times 10^{19}}{\text{year}}$$

Experimental Progress

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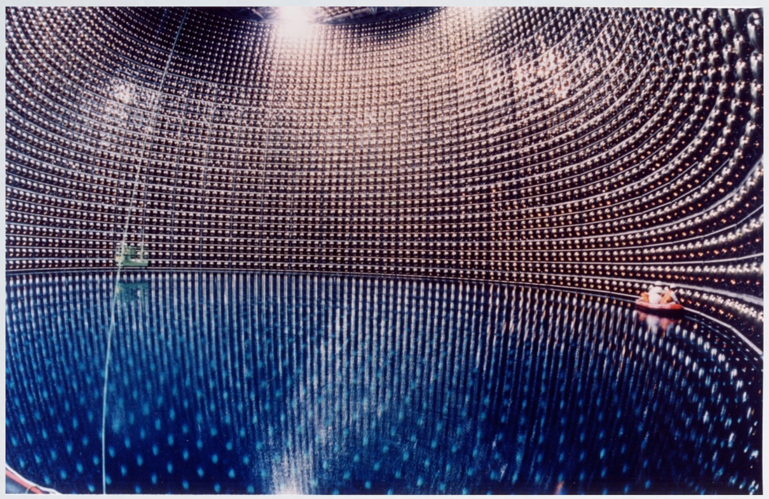

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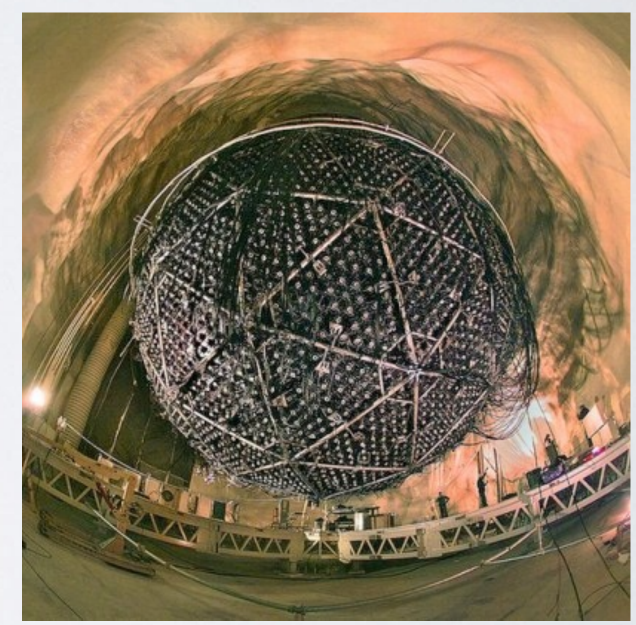
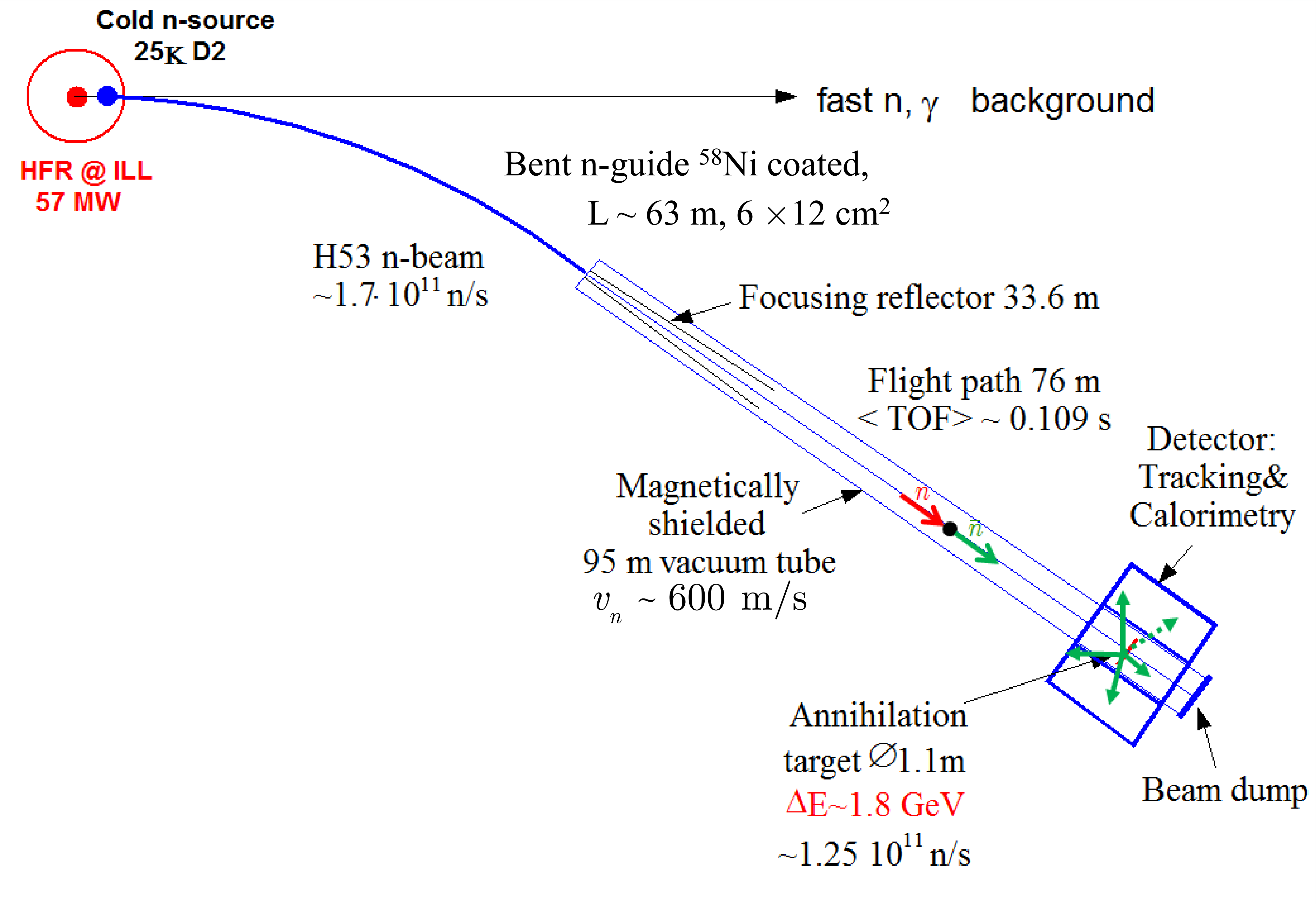


More theoretical controlled nuclear suppression factor
(but still has factor of 2 discrepancies between models)

$$R = \frac{(3.7 - 9.3) \times 10^{19}}{\text{year}}$$

Experimental Progress

2. Free, Cold neutron annihilation with target



ILL/Grenoble (1993)

Experimental Progress

2. Free, Cold neutron annihilation with target

Designed to:

1. Maximize number of neutrons
2. Minimize energy of neutrons
3. Maximize time of flight
4. Minimize External Magnetic Field

Minimize external potential → Most controlled measurement

ILL bound (1993)

$\tau_{n\bar{n}} > 2.7$ years

Experimental Progress

2. Free, Cold neutron annihilation with target

Designed to:

1. Maximize number of neutrons
2. Minimize energy of neutrons
3. Maximize time of flight
4. Minimize External Magnetic Field

Lots of recent discussion:
Gardner, Jafari (2014)
Babu, Mohapatra (2015)
Berezhiani, Vainshtein (2015)

See Susan and Arkady's talks last week

Minimize external potential → Most controlled measurement

ILL bound (1993)

$$\tau_{n\bar{n}} > 2.7 \text{ years}$$

Experimental Prospects

- Project X meeting summary (arXiv:1306.5009)

NNBarX:

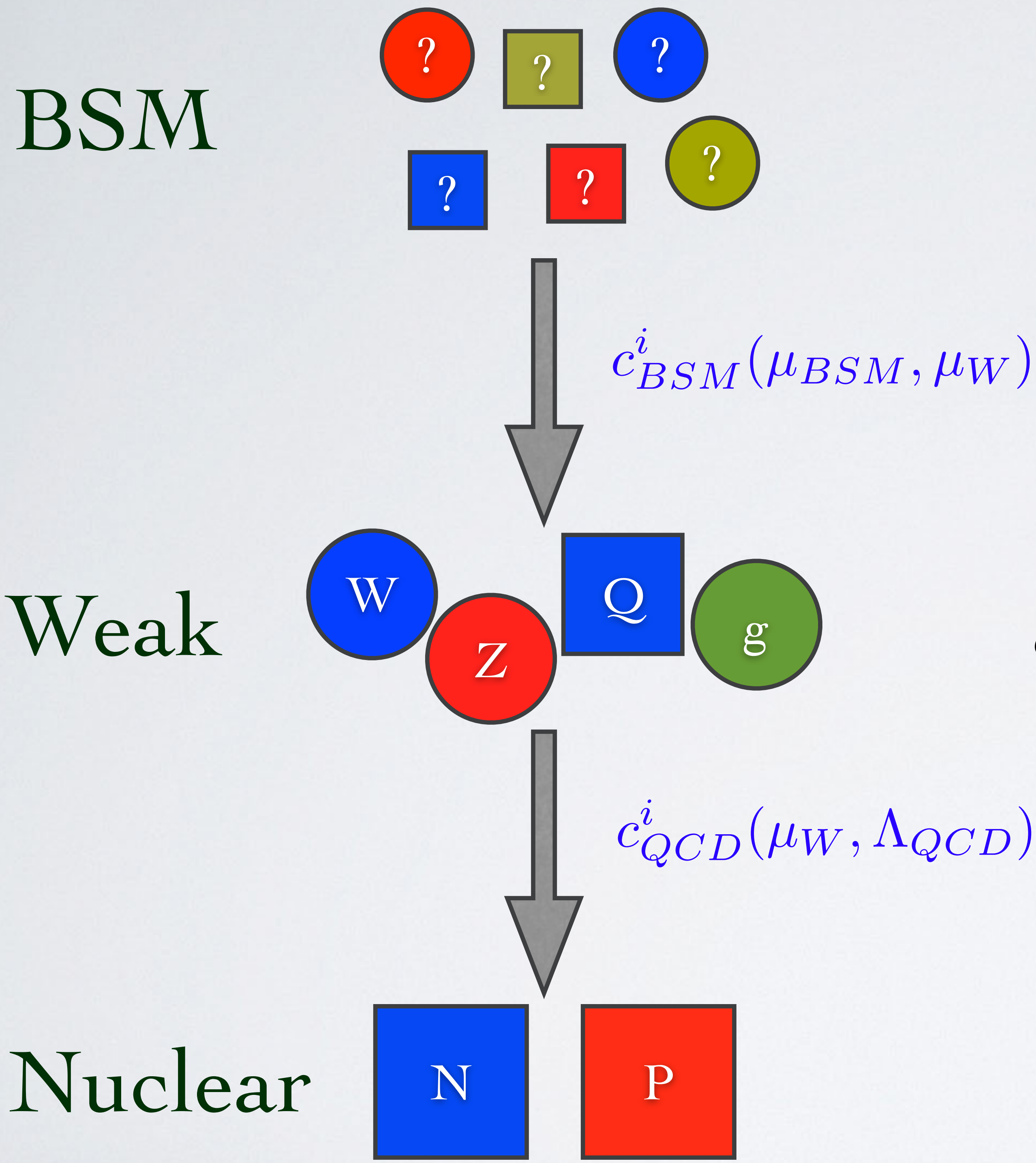
(Fermilab)

First Stage: $\tau_{n\bar{n}} > 80$ years
(2-3 years)

Second Stage: $\tau_{n\bar{n}} > 8000$ years

Discussions ongoing for other new high-flux, low energy neutron experiment… (see Yuri's talk last week)

Origin of Oscillations



- Running of BSM interaction to nuclear scale

$$\mathcal{L}_{n\bar{n}} \sim \frac{1}{M_{NP}^5} QQQQQQ$$

$$c^i = c^i_{BSM}(\mu_{BSM}, \mu_W) c^i_{QCD}(\mu_W, \Lambda_{QCD})$$

$$\frac{1}{\tau_{n\bar{n}}} = \delta m = \frac{1}{M_\chi^5} \sum_i c^i \langle \bar{n} | \mathcal{O}^i | n \rangle$$

$$M_\chi \sim \left(\frac{\Lambda^6}{\delta m} \right)^{\frac{1}{5}} \gtrsim 500 \text{ TeV}$$

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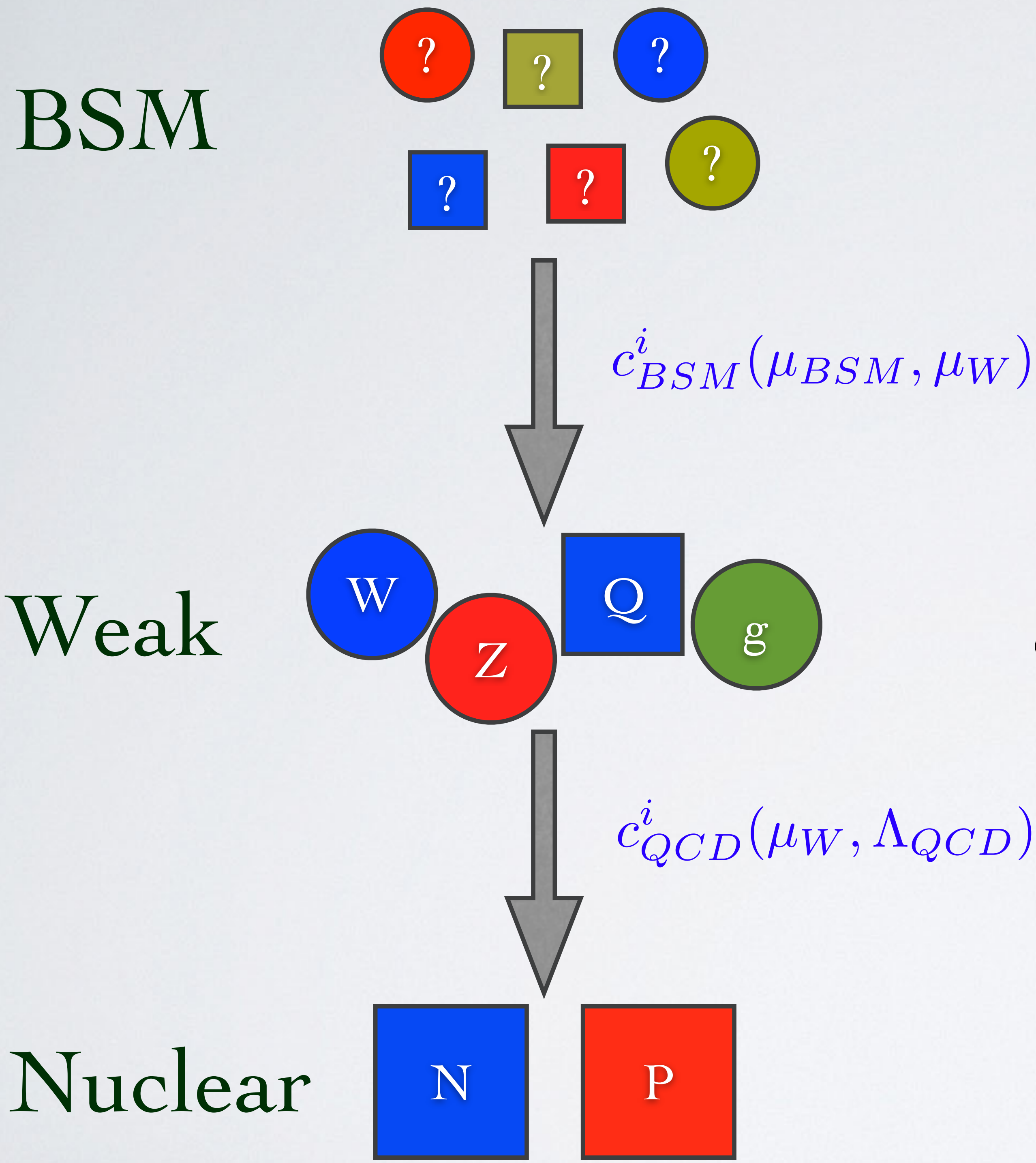

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$$\frac{1}{\tau_{n\bar{n}}} = \delta m = \frac{1}{M^5_\chi} \sum_i c^i \boxed{\langle \bar{n} | \mathcal{O}^i | n \rangle}$$

Non-perturbative

$$M_\chi \sim \left(\frac{\Lambda^6}{\delta m} \right)^{\frac{1}{5}} \gtrsim 500 \text{ TeV}$$

What can we say without supercomputer?

Answer: Dimensional estimate not reliable

Strong Dynamics Scale: rho meson mass $\Lambda_{QCD} \sim m_\rho$

Strong Physics	How much can we trust?
Baryon & Meson Masses $\sim m_\rho$	Within factor of 2, at best
Proton Decay Matrix Element $\sim m_\rho^2$	Within factor of 4, at best
Neutron-Antineutron Matrix Element $\sim m_\rho^6$	Within factor of 64, at best

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Cannot guarantee better without first-principles calculation

Six-quark Operators

Rao, Shrock (1982)

Three pairs of quarks: **2 u's** **4 d's**

I. Flavor: $u^T C u$ or $u^T C d$ or $d^T C d$

Six-quark Operators

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2. Spin: $q_L^T C q_L$ or $q_R^T C q_R$

3. Color: $q^{iT} C q^j$ → $\bar{3}_c \oplus 6_c$

$$T_{\{ij\}\{kl\}\{mn\}} = \epsilon_{mik}\epsilon_{njl} + \epsilon_{nik}\epsilon_{mjl} + \epsilon_{mjk}\epsilon_{nil} + \epsilon_{mil}\epsilon_{njk} \qquad 1_c \subset 6_c \otimes 6_c \otimes 6_c$$

$$T_{[ij][kl]\{mn\}} = \epsilon_{mij}\epsilon_{nkl} + \epsilon_{nij}\epsilon_{mkl} \qquad 1_c \subset \bar{3}_c \otimes \bar{3}_c \otimes 6_c$$

Six-quark Operators

Rao, Shrock (1982)

$\chi_i = L, R$

1. $\mathcal{O}^1_{\chi_1\chi_2\chi_3} = (u_i^T C u_j)_{\chi_1} (d_k^T C d_l)_{\chi_2} (d_m^T C d_n)_{\chi_3} T_{\{ij\}\{kl\}\{mn\}}$

2. $\mathcal{O}^2_{\chi_1\chi_2\chi_3} = (u_i^T C d_j)_{\chi_1} (u_k^T C d_l)_{\chi_2} (d_m^T C d_n)_{\chi_3} T_{\{ij\}\{kl\}\{mn\}}$

3. $\mathcal{O}^3_{\chi_1\chi_2\chi_3} = (u_i^T C d_j)_{\chi_1} (u_k^T C d_l)_{\chi_2} (d_m^T C d_n)_{\chi_3} T_{[ij][kl]\{mn\}}$

# of operators: 24

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# of operators: ~~24~~ 18

$\mathcal{O}^1_{\chi_1 LR} = \mathcal{O}^1_{\chi_1 RL}$

$\mathcal{O}^{2,3}_{LR\chi_3} = \mathcal{O}^{2,3}_{RL\chi_3}$

Six-quark Operators

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# of operators: ~~24~~ ~~18~~ 14

$\mathcal{O}^1_{\chi_1 LR} = \mathcal{O}^1_{\chi_1 RL}$

$\mathcal{O}^{2,3}_{LR\chi_3} = \mathcal{O}^{2,3}_{RL\chi_3}$

$\mathcal{O}^2_{\sigma\sigma\rho} - \mathcal{O}^1_{\sigma\sigma\rho} = 3\mathcal{O}^3_{\sigma\sigma\rho}$

Caswell, Milutinovic, Sejanovic (1983)

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# of operators: 7

Transition flips parity

$$L \leftrightarrow R$$

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# of operators: ~~7~~ 4

Transition flips parity

$L \leftrightarrow R$

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

Operator Symmetries

MIB, M.Wagman (2015)

Special thanks to B.Tiburzi

Chiral properties important for renormalization and EFT calculations

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \mathcal{D}_\chi \equiv (\psi C P_\chi i\tau^2 \psi) \qquad \mathcal{D}^A_\chi \equiv (\psi C P_\chi i\tau^2 \tau^A \psi)$$

Example:

$$Q_1 = (\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \tau^+ \psi) T^{AAS}$$

Chiral Basis	Fixed-Flavor Basis	Chiral Tensor Structure	Chiral Irrep
Q_1	$\mathcal{O}^3_{RRR}$	$\mathcal{D}_R\mathcal{D}_R\mathcal{D}^+_R T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_2	$\mathcal{O}^3_{LRR}$	$\mathcal{D}_L\mathcal{D}_R\mathcal{D}^+_R T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_3	$\mathcal{O}^3_{LLR}$	$\mathcal{D}_L\mathcal{D}_L\mathcal{D}^+_R T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_4	$4/5\ \mathcal{O}^2_{RRR} + 1/5\ \mathcal{O}^1_{RRR}$	$\mathcal{D}^{33+}_R T^{SSS}$	$(\mathbf{1}_L, \mathbf{7}_R)$
Q_5	$\mathcal{O}^1_{RLL}$	$\mathcal{D}^-_R\mathcal{D}^{++}_L T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_6	$\mathcal{O}^2_{RLL}$	$\mathcal{D}^3_R\mathcal{D}^{3+}_L T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_7	$2/3\ \mathcal{O}^2_{LLR} + 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}^+_R\mathcal{D}^{33}_L T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
$\widetilde{Q}_1$	$1/3\ \mathcal{O}^2_{RRR} - 1/3\ \mathcal{O}^1_{RRR}$	$\mathcal{D}_R\mathcal{D}_R\mathcal{D}^+_R T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
$\widetilde{Q}_3$	$1/3\ \mathcal{O}^2_{LLR} - 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}_L\mathcal{D}_L\mathcal{D}^+_R T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$

Operator Symmetries

MIB, M.Wagman (2015)   *Special thanks to B.Tiburzi*

Chiral properties important for renormalization and EFT calculations

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \mathcal{D}_\chi \equiv (\psi C P_\chi i\tau^2 \psi) \qquad \mathcal{D}_\chi^A \equiv (\psi C P_\chi i\tau^2 \tau^A \psi)$$

Isospin-2

$$\mathcal{D}_\chi^{AB} \equiv \mathcal{D}_\chi^{\{A}\mathcal{D}_\chi^{B\}} - \frac{1}{3}\delta^{AB}\mathcal{D}_\chi^C\mathcal{D}_\chi^C$$

Example:

$$Q_1 = (\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \tau^+ \psi) T^{AAS}$$

Chiral Basis	Fixed-Flavor Basis	Chiral Tensor Structure	Chiral Irrep
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Q_2	$\mathcal{O}^3_{LRR}$	$\mathcal{D}_L\mathcal{D}_R\mathcal{D}_R^+ T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
Q_3	$\mathcal{O}^3_{LLR}$	$\mathcal{D}_L\mathcal{D}_L\mathcal{D}_R^+ T^{AAS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
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Q_5	$\mathcal{O}^1_{RLL}$	$\mathcal{D}_R^-\mathcal{D}_L^{++} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_6	$\mathcal{O}^2_{RLL}$	$\mathcal{D}_R^3\mathcal{D}_L^{3+} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
Q_7	$2/3\ \mathcal{O}^2_{LLR} + 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}_R^+\mathcal{D}_L^{33} T^{SSS}$	$(\mathbf{5}_L, \mathbf{3}_R)$
$\tilde{Q}_1$	$1/3\ \mathcal{O}^2_{RRR} - 1/3\ \mathcal{O}^1_{RRR}$	$\mathcal{D}_R\mathcal{D}_R\mathcal{D}_R^+ T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$
$\tilde{Q}_3$	$1/3\ \mathcal{O}^2_{LLR} - 1/3\ \mathcal{O}^1_{LLR}$	$\mathcal{D}_L\mathcal{D}_L\mathcal{D}_R^+ T^{SSS}$	$(\mathbf{1}_L, \mathbf{3}_R)$

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Isospin-3

$$\mathcal{D}_\chi^{ABC} \equiv \mathcal{D}_\chi^{\{A}\mathcal{D}_\chi^{B}\mathcal{D}_\chi^{C\}} - \frac{1}{5}\left[\delta^{AB}\mathcal{D}_\chi^{\{C}\mathcal{D}_\chi^{D}\mathcal{D}_\chi^{D\}} + \delta^{AC}\mathcal{D}_\chi^{\{B}\mathcal{D}_\chi^{D}\mathcal{D}_\chi^{D\}} + \delta^{BC}\mathcal{D}_\chi^{\{A}\mathcal{D}_\chi^{D}\mathcal{D}_\chi^{D\}}\right]$$

Example:

$$Q_1 = (\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \psi)(\psi C P_R i\tau^2 \tau^+ \psi) T^{AAS}$$

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Isospin-3 → Q_4

Not SM gauge inv. → Q_5, Q_6, Q_7

Redundant in D=4 → $\tilde{Q}_1$, $\tilde{Q}_3$

And so our quest began...

GOAL:

To calculate neturon-antineutron matrix elements crucial for connecting theory & experiment

MIB, C. Schroeder, S. Syritsyn, J. Wasem, M. Wagman

Initial Lattice QCD calculation:

Pion Mass:	390 MeV	(Note: Physical value ~139 MeV)
Lattice Spacing:	0.125 fm	
Lattice Extent:	2.5 fm	(Number of sites: 20^3 x 256)
Pion Mass x Lattice Extent:	4.875	(Note: Typically > 4)
Measurements:	57,500	(Note: 1150 cfg, sep by 5 tu)
Anisotropic Clover-Wilson lattices with anisotropy factor of 3.5		(Note: Discretization information)

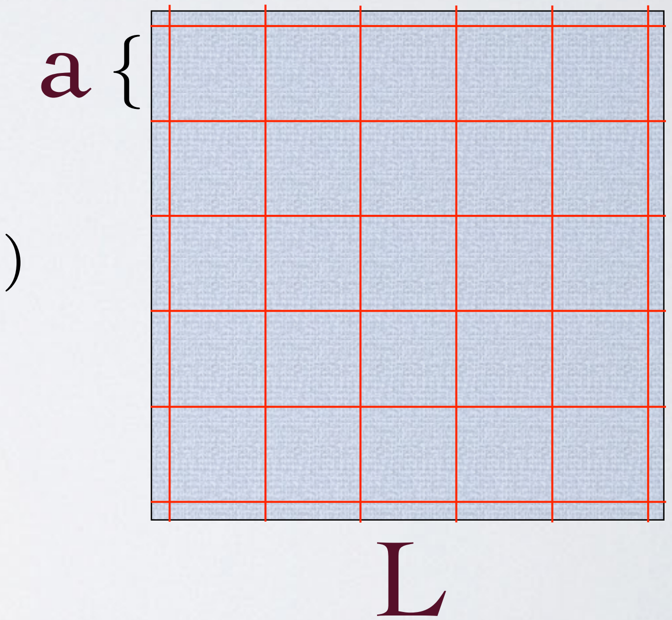
Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = Z^{-1} \int d[U]d[\overline{\psi}]d[\psi] \; \mathcal{O} \; e^{i(S_F(\overline{\psi},\psi,U)+S_G(U))}$$

$$= Z^{-1} \int d[U] \; \mathcal{O} \; \det(D_F(U))e^{iS_G(U)}$$

Approximate continuum with discrete lattice:



$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = Z^{-1} \int d[U] \; \mathcal{O} \; \det(D_{F,lat}(U))e^{iS_{G,lat}(U)}$$

Stochastically estimate integral via importance sampling

Euclidean: $$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = Z^{-1} \sum_{U} \mathcal{O} \det(D_{F,lat}(U))e^{-S_{G,lat}(U)}$$

Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U] \; \mathcal{O} \; \det(D_{lat}(U)) e^{-S_G(U)}$$

Parity +
Spin +

$$C_N^{(+)}(t) = \langle N^{(+)}(t) \overline{N}^{(+)}(0) \rangle \stackrel{t \to \infty}{\longrightarrow} (Z_N)^2 e^{-m_n t}$$

Parity -
Spin -

$$C_N^{(-)}(-t) = \langle \overline{N}^{(-)}(-t) N^{(-)}(0) \rangle \stackrel{t \to \infty}{\longrightarrow} (Z_{\bar{N}})^2 e^{-m_n t}$$

Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U]\ \mathcal{O}\ \det(D_{lat}(U))e^{-S_G(U)}$$

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Spin -

$$C_N^{(-)}(-t) = \langle \overline{N}^{(-)}(-t)N^{(-)}(0)\rangle \overset{t\to\infty}{\longrightarrow} (Z_{\bar{N}})^2 e^{-m_n t}$$

$$C_{N\mathcal{O}\bar{N}}(-t_1, t_2) = \langle N^{(+)}(t_2)\mathcal{O}(0)N^{(-)}(-t_1)\rangle \overset{t_1,t_2\to\infty}{\longrightarrow} (Z_{\bar{N}} Z_N)\langle n|\mathcal{O}|\bar{n}\rangle e^{-m_n(t_1+t_2)}$$

Parity +
Spin +

Parity-
Spin -

Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U]\ \mathcal{O}\ \det(D_{lat}(U))e^{-S_G(U)}$$

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Spin -

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Parity +
Spin +

Parity-
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Lattice Calculation

Correlation Functions via path integral:

$$C_{\mathcal{O}} = \langle \mathcal{O} \rangle = \int d[U] \; \mathcal{O} \; \det(D_{lat}(U)) e^{-S_G(U)}$$

Parity +
Spin +

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$$C_{N\mathcal{O}\bar{N}}(-t_1, t_2) = \langle N^{(+)}(t_2) \mathcal{O}(0) N^{(-)}(-t_1) \rangle \xrightarrow{t_1, t_2 \to \infty} (Z_{\bar{N}} Z_N) \langle n | \mathcal{O} | \bar{n} \rangle e^{-m_n (t_1 + t_2)}$$

Parity +
Spin +

Parity-
Spin -

$$\mathcal{R} = \frac{C_{N\mathcal{O}\bar{N}}(-t_1, t_2)}{C_{\bar{N}}^{(-)}(-t_1) C_N^{(+)}(t_2)} \to \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\bar{N}}} + \mathcal{O}(e^{-E_\Delta t_1}) + \mathcal{O}(e^{-E_\Delta t_2})$$

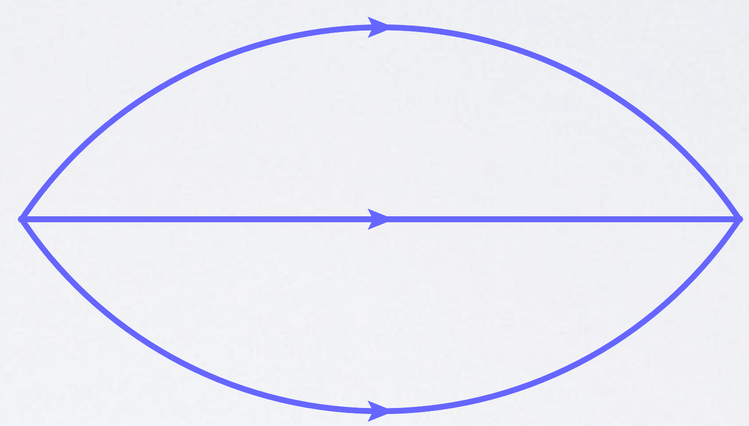
$$E_\Delta \approx m_{n^*} - m_n$$

Lattice Contractions

Propagator Contractions:

$$\overline{q}_{i'}^{\alpha'}(y)\; q_i^{\alpha}(x) = S_{i'i}^{\alpha'\alpha}(y,x) \qquad\qquad S^\dagger = \gamma_5 S \gamma_5$$

$C_{NN}(t)$



$\tau = 0$ $\qquad\qquad\qquad$ $\tau = t$

Lattice Contractions

Propagator Contractions:

$$\overline{q}_{i'}^{\alpha'}(y)\; q_i^{\alpha}(x) = S_{i'i}^{\alpha'\alpha}(y,x) \qquad\qquad S^{\dagger} = \gamma_5 S \gamma_5$$

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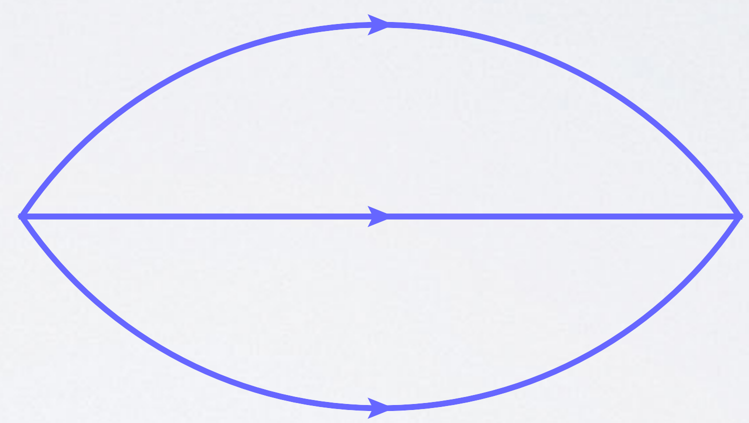

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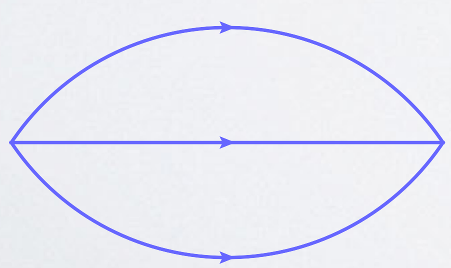
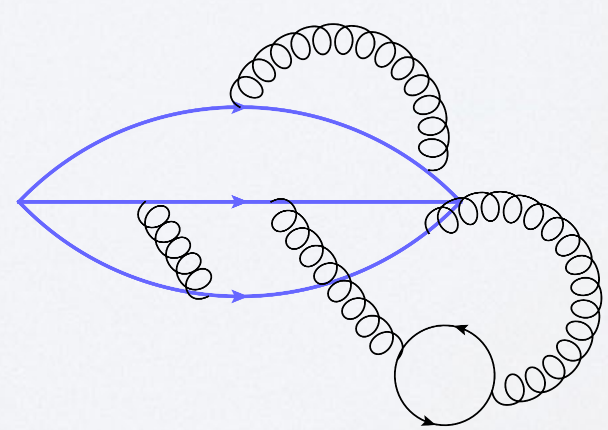


$=$

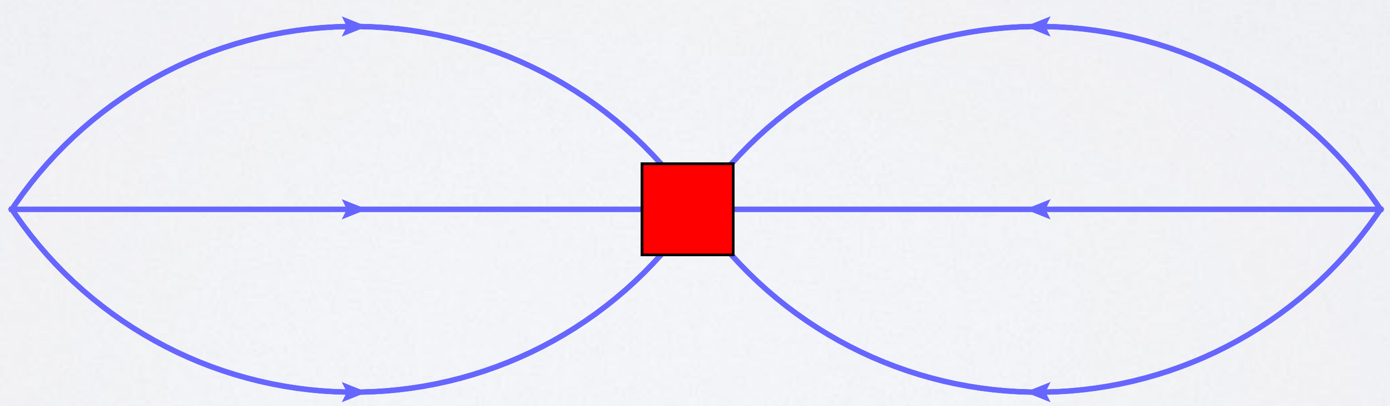


$+ \quad \cdots$

Lattice Contractions

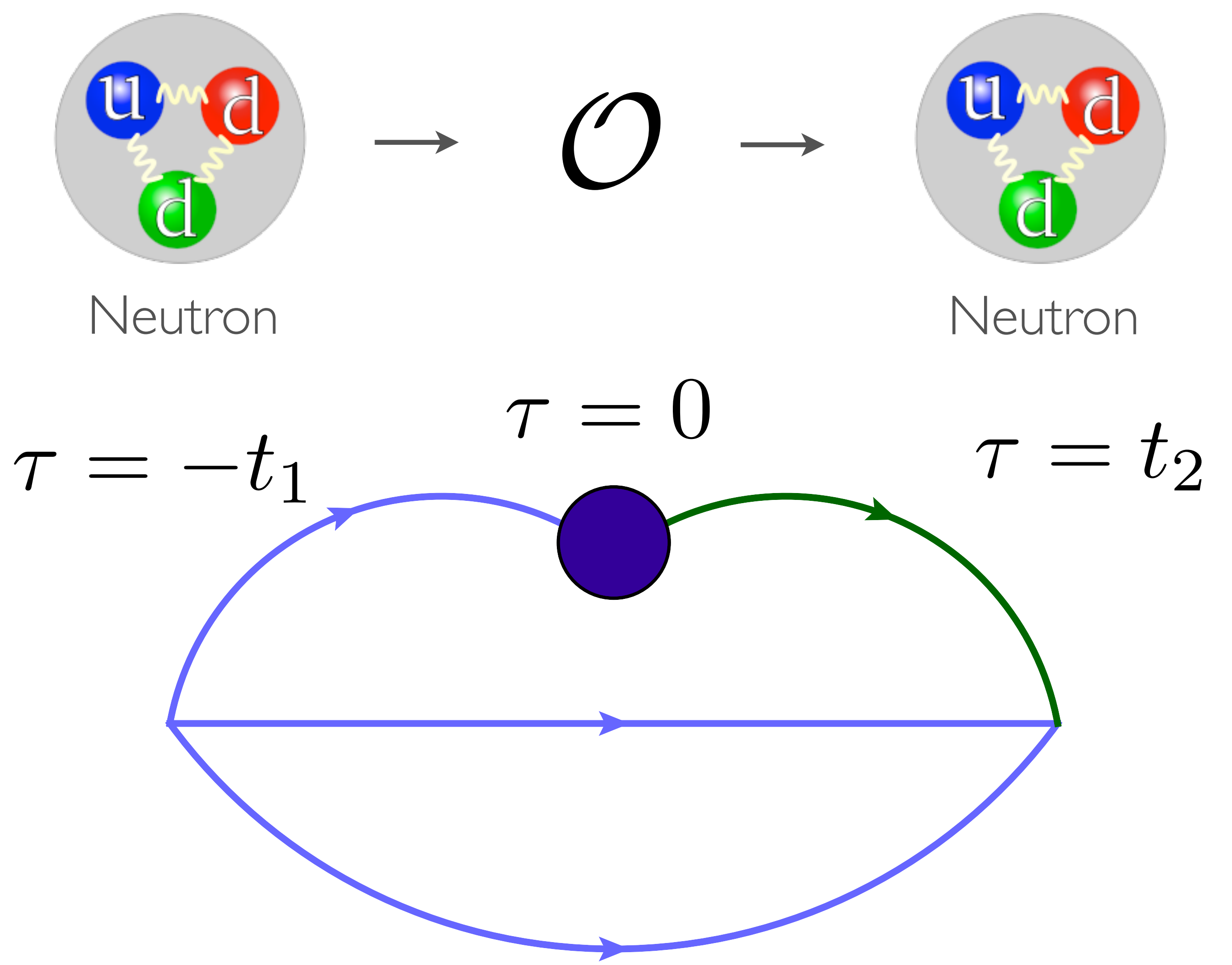
$\tau = -t_1$ $\tau = 0$ $\tau = t_2$

$C_{N\mathcal{O}\bar{N}}(-t_1, t_2)$



1 Propagator → Two measurement ALL time insertion

Lattice Contractions



Typical
3-point

$C_{N\mathcal{O}}$

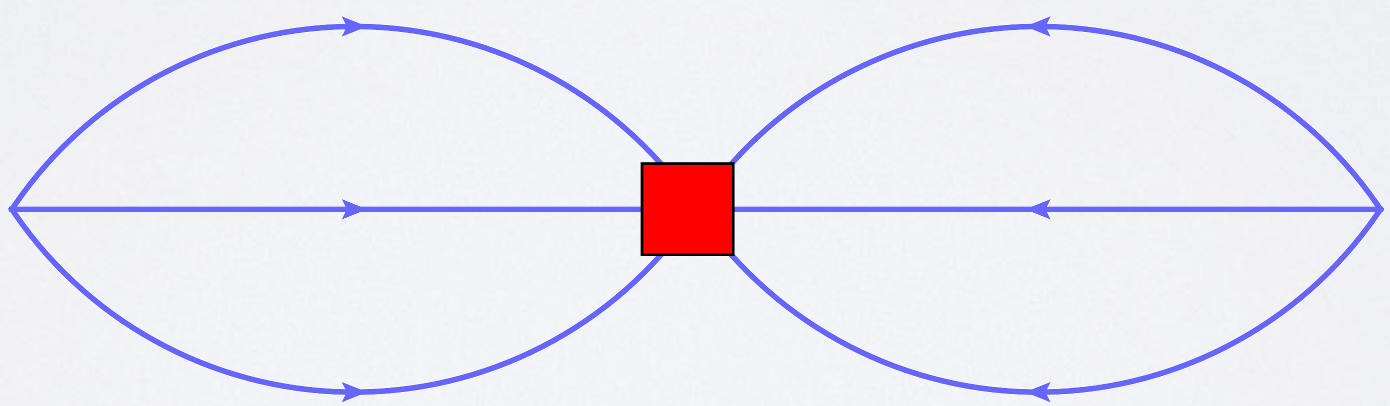
2 Propagators → One measurements
One time insertion

ALL time insertion

Lattice Contractions

$\tau = -t_1$ $\tau = 0$ $\tau = t_2$

$C_{N\mathcal{O}\bar{N}}(-t_1, t_2)$

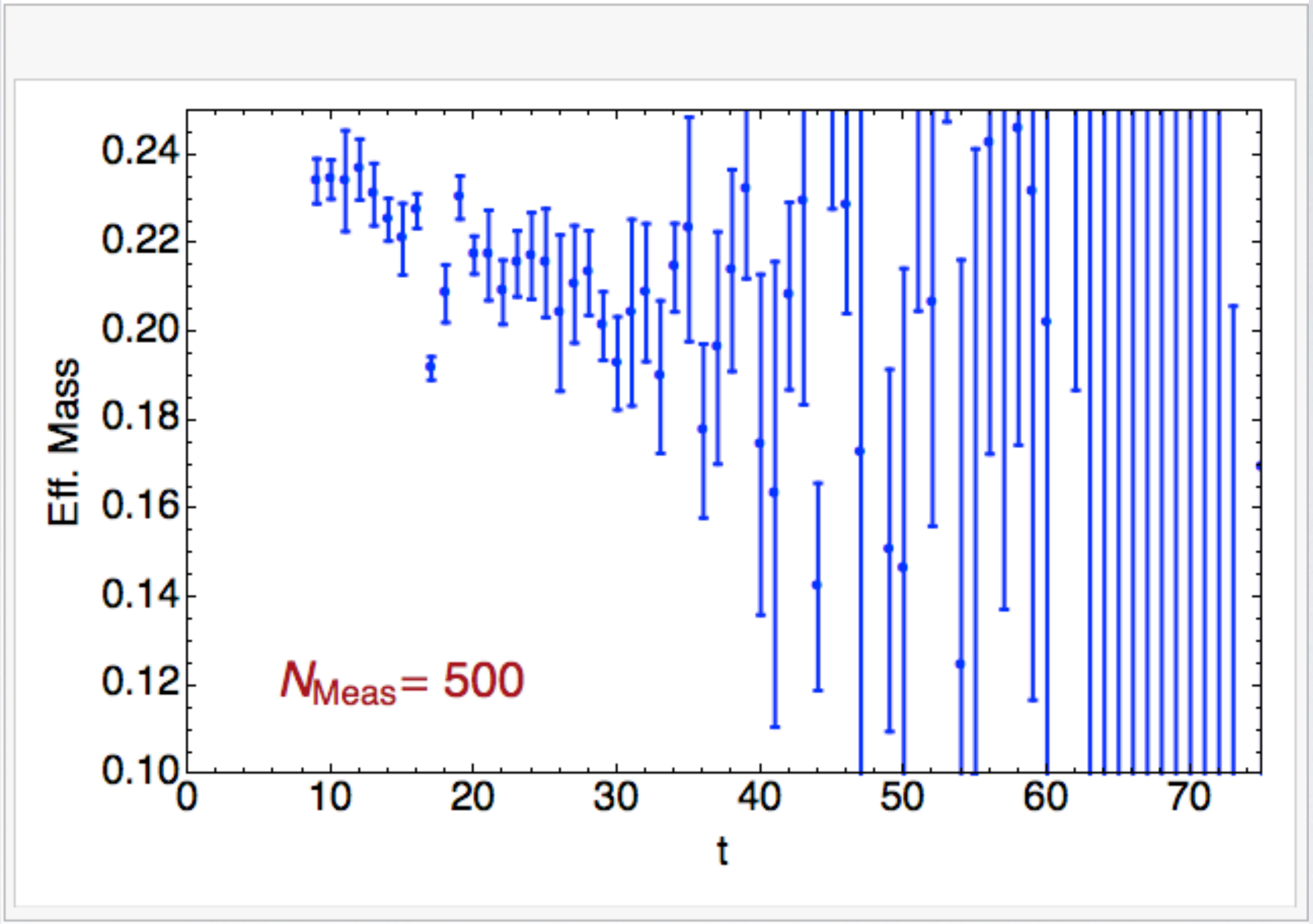


1 Propagator → Two measurement ALL time insertion

Neutron Mass

$$\text{Eff. Mass} = \ln \frac{C_N(t+1)}{C_N(t)} \xrightarrow{t\to\infty} m_n$$

Neutron Mass



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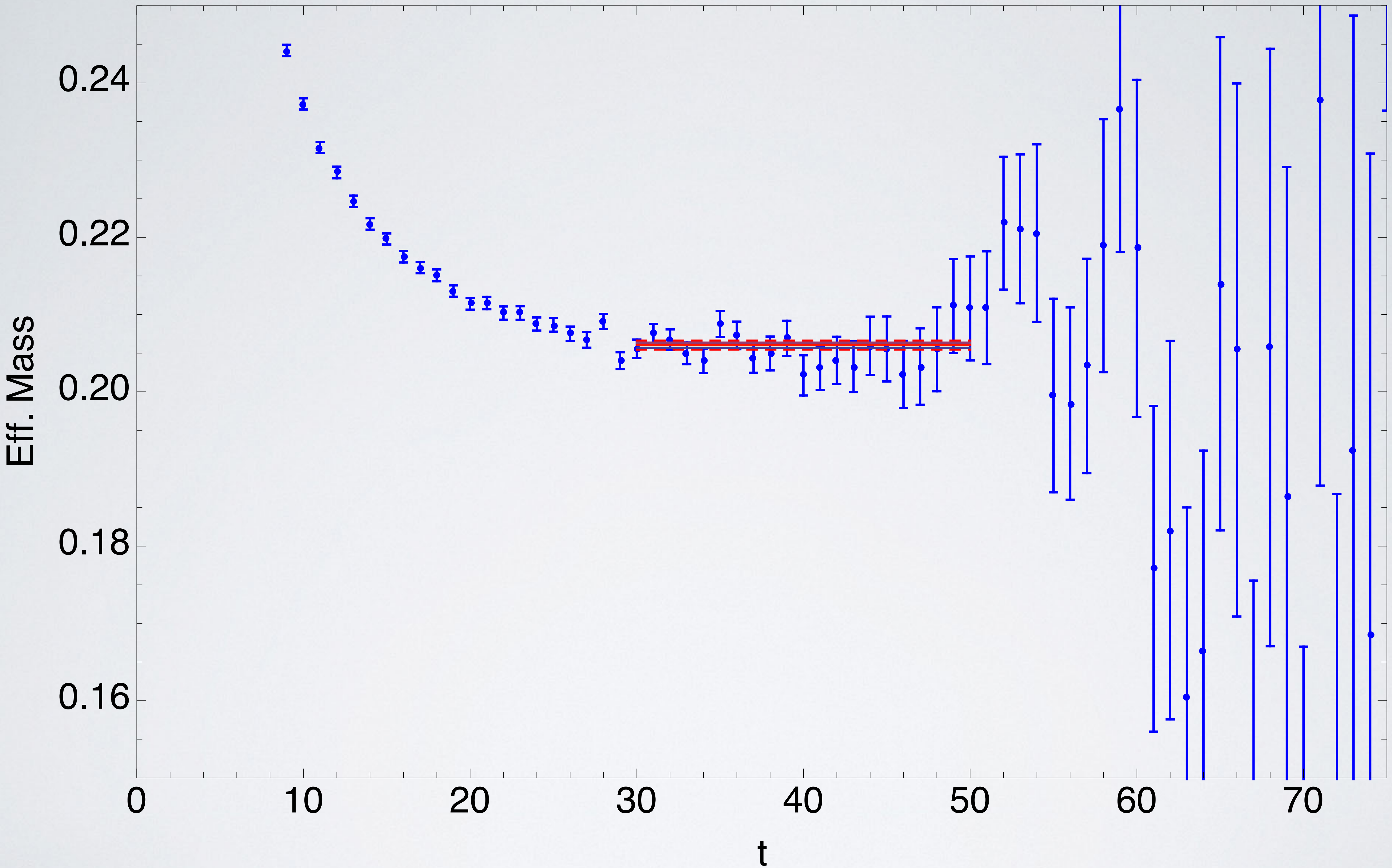
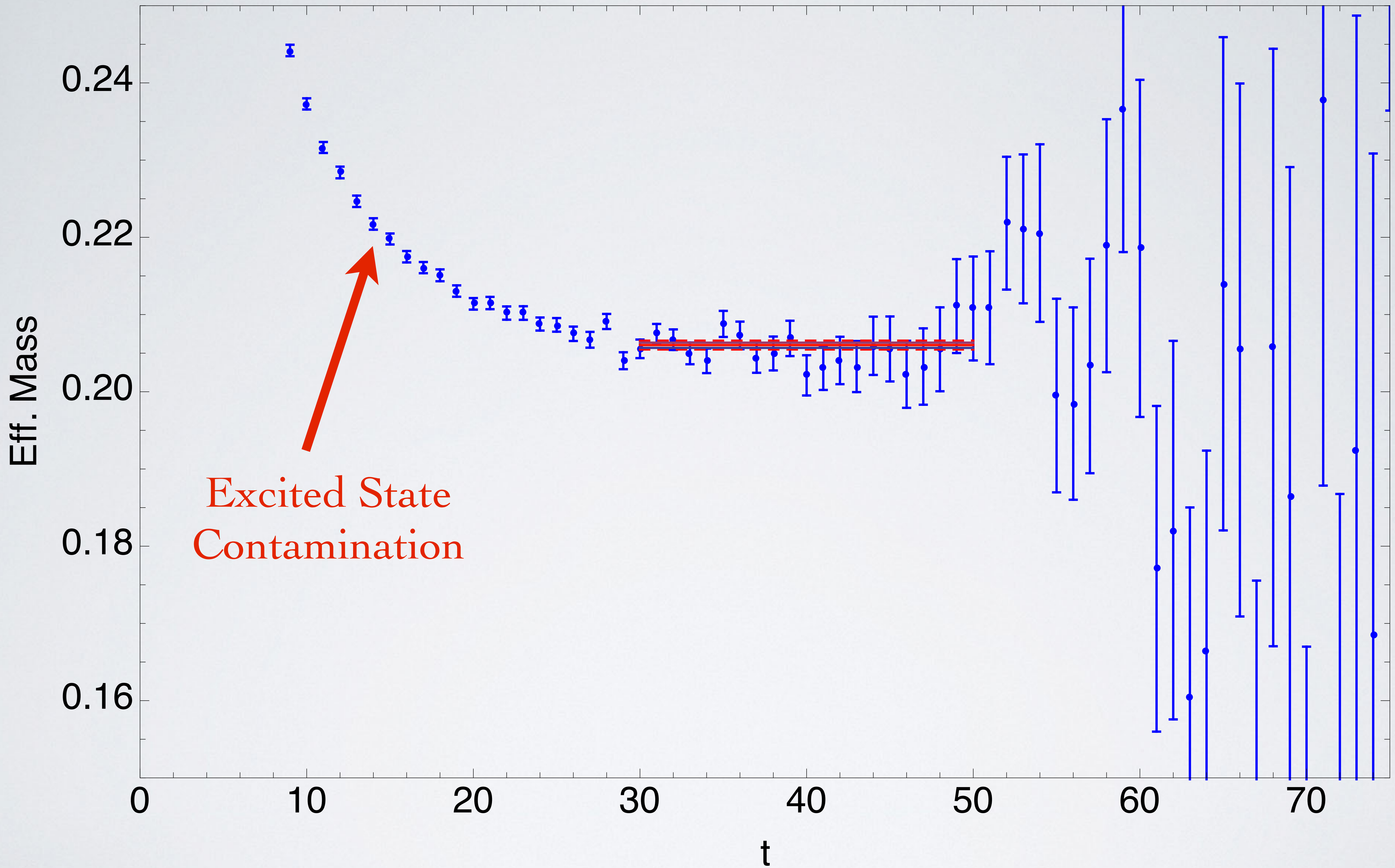
Neutron Mass



$$M_N = 1.1539 \pm 0.0032^{+0.0017}_{-0.0020}\ \text{GeV}$$

Neutron Mass



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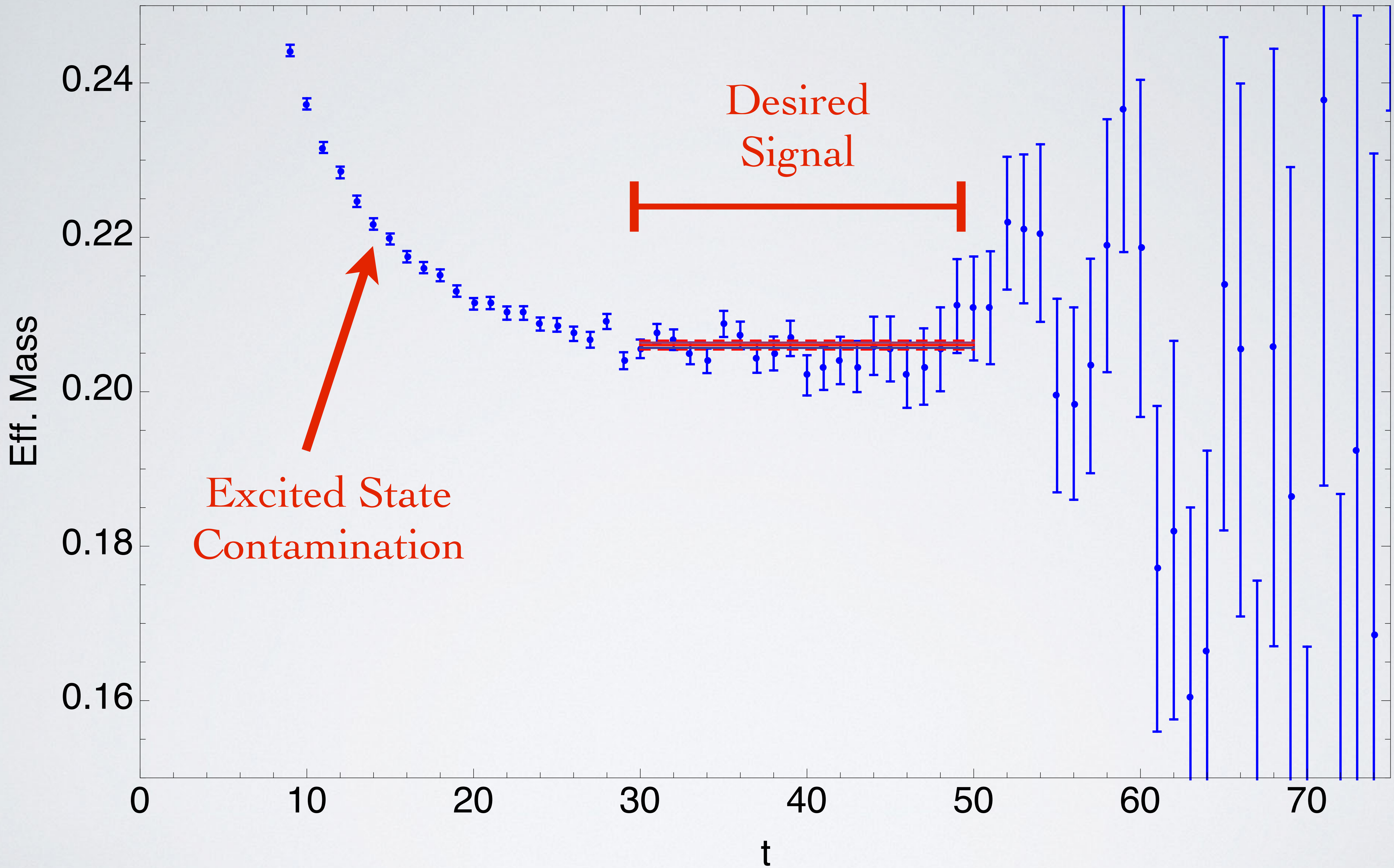

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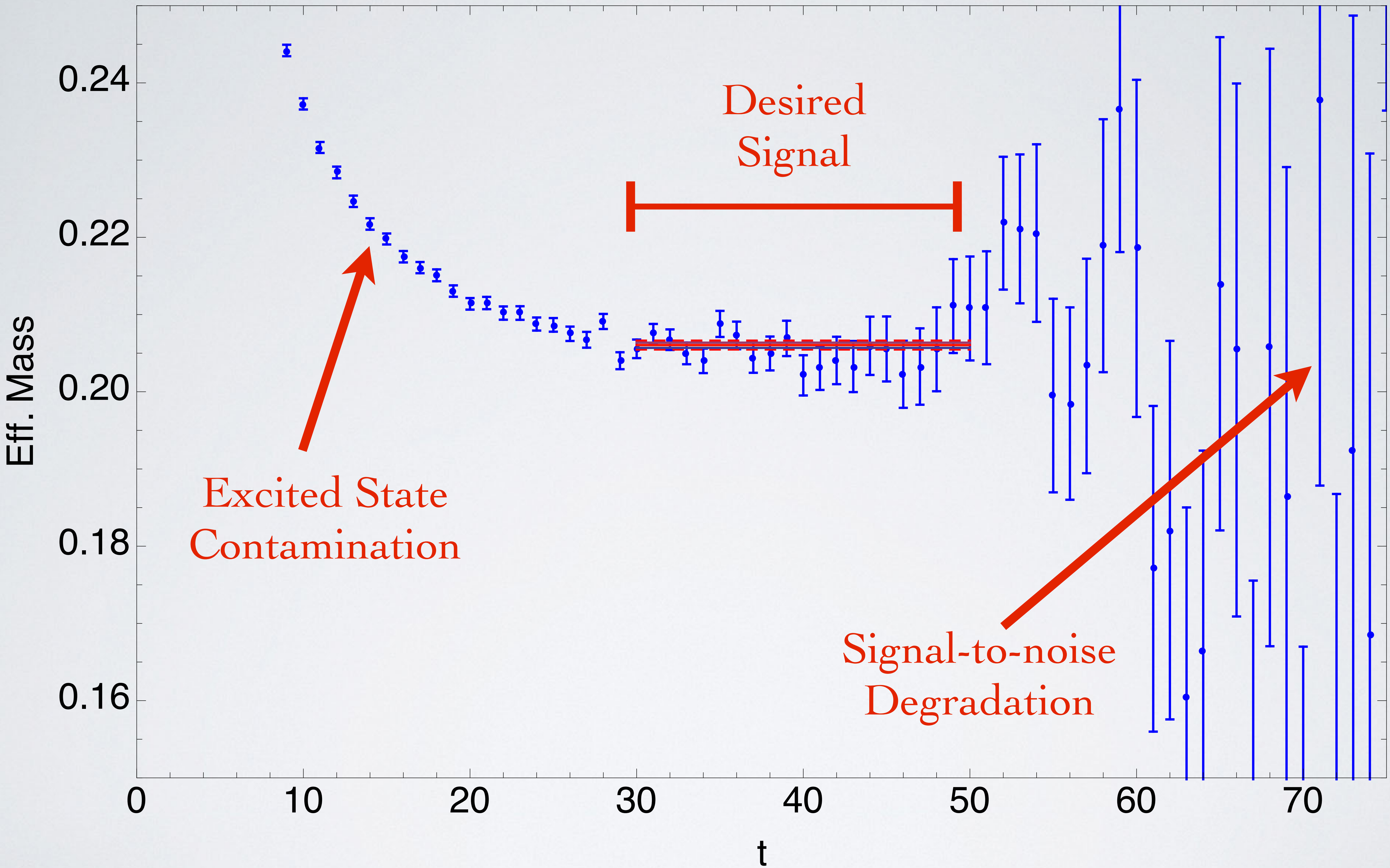
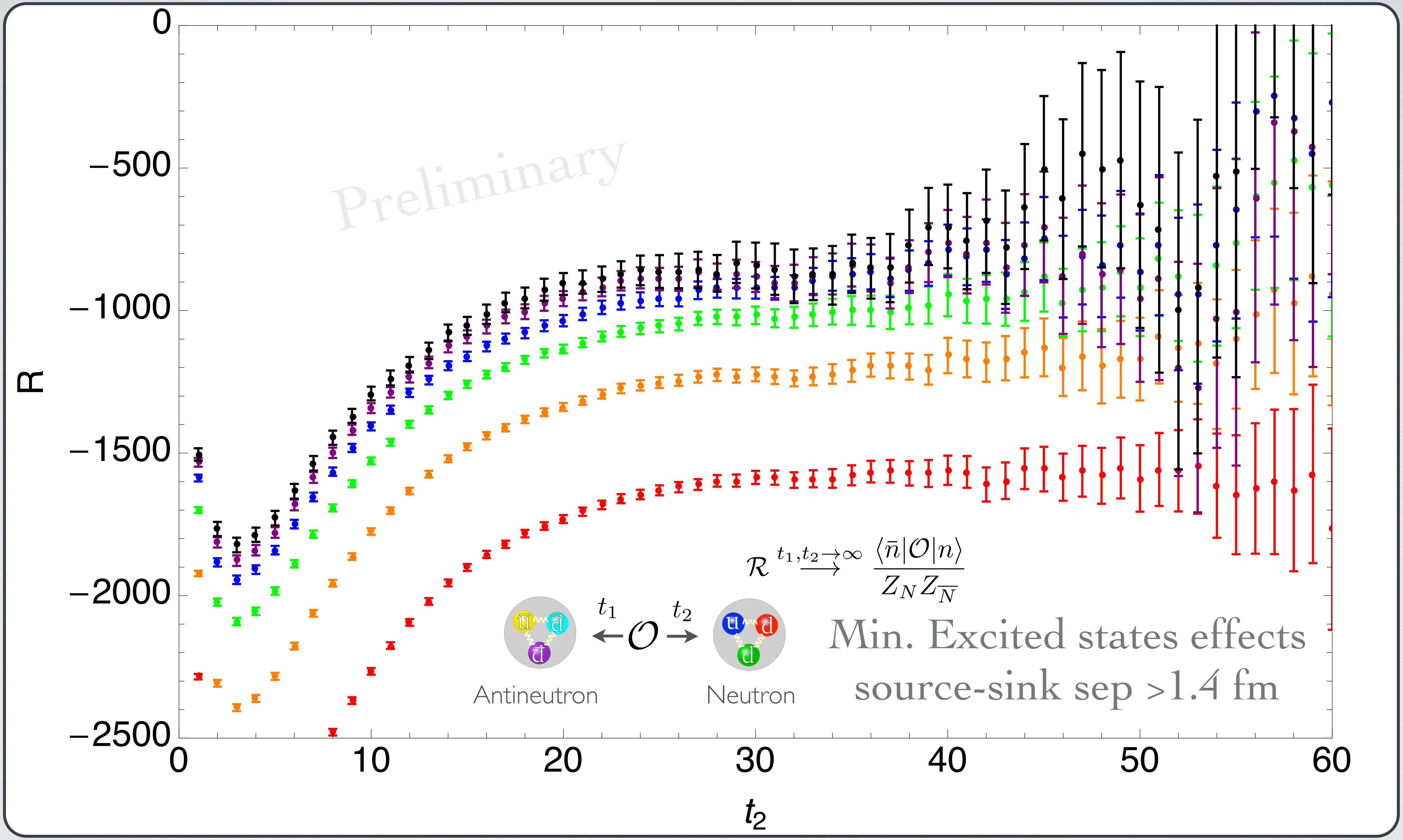
Neutron Mass



$$M_N = 1.1539 \pm 0.0032^{+0.0017}_{-0.0020}\ \mathrm{GeV}$$

N-NBar Matrix Element

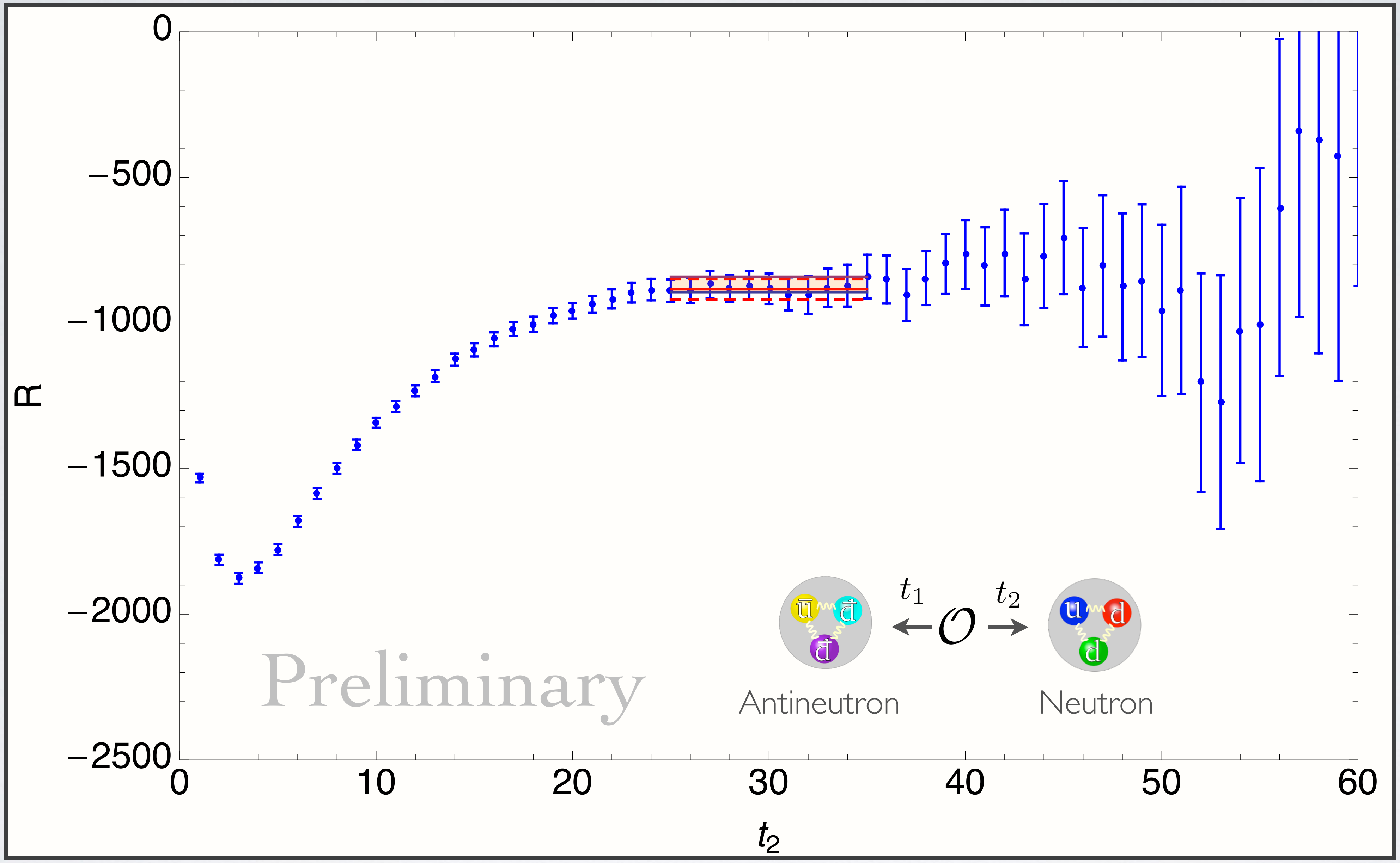


$t_1 = 5$ $t_1 = 10$ $t_1 = 15$ $t_1 = 20$ $t_1 = 25$

A multitude of fits

$\mathcal{R} \xrightarrow{t_1, t_2 \to \infty} \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\overline{N}}}$

$t_1 = 25$



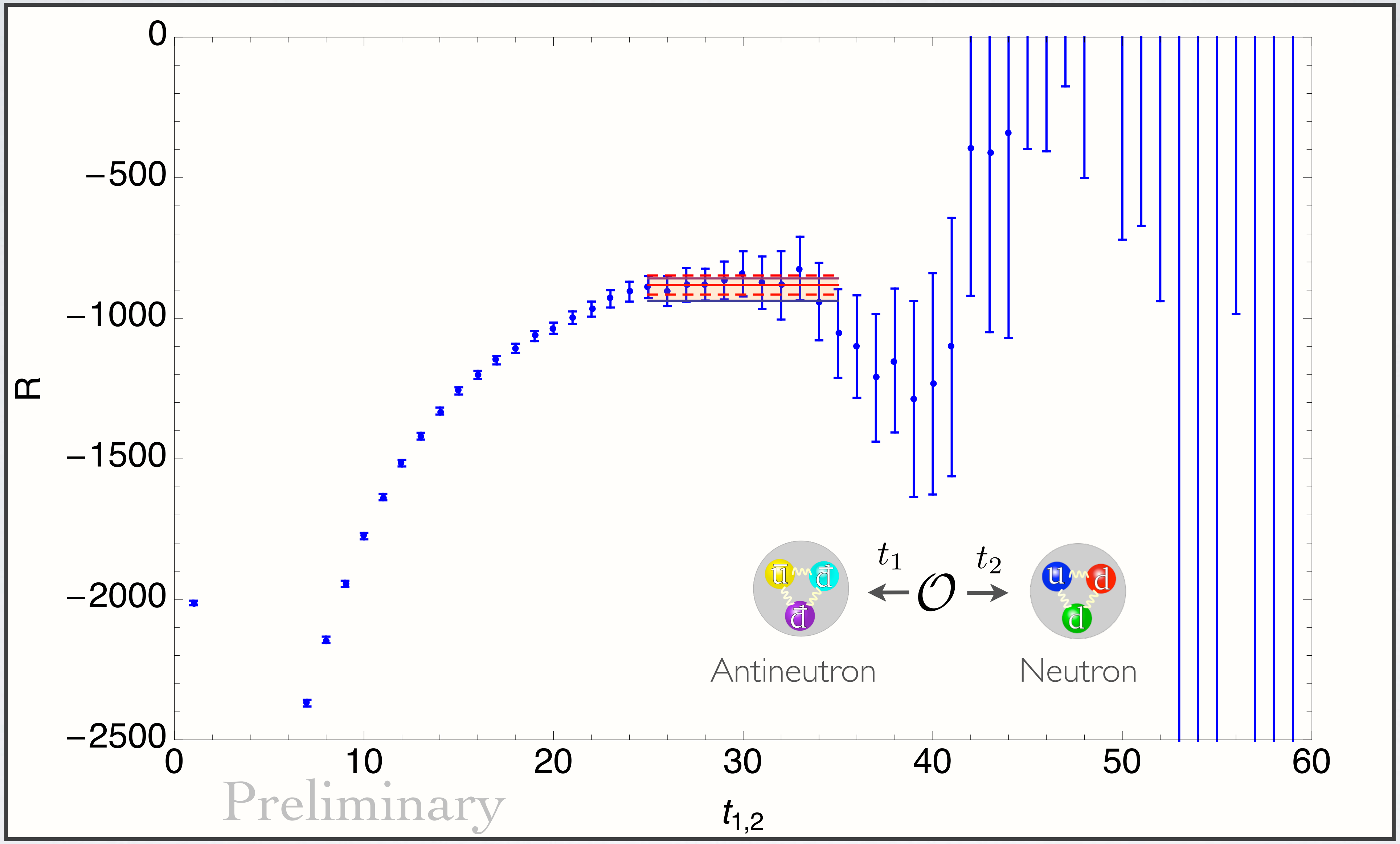
$$a_s^6 \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\bar{N}}} = -884 \pm 35^{+43}_{-10}$$

$\chi^2/\mathrm{dof} = 1.20$

A multitude of fits

$$\mathcal{R} \xrightarrow{t_1,t_2\to\infty} \frac{\langle \bar{n}|\mathcal{O}|n\rangle}{Z_N Z_{\overline{N}}}$$

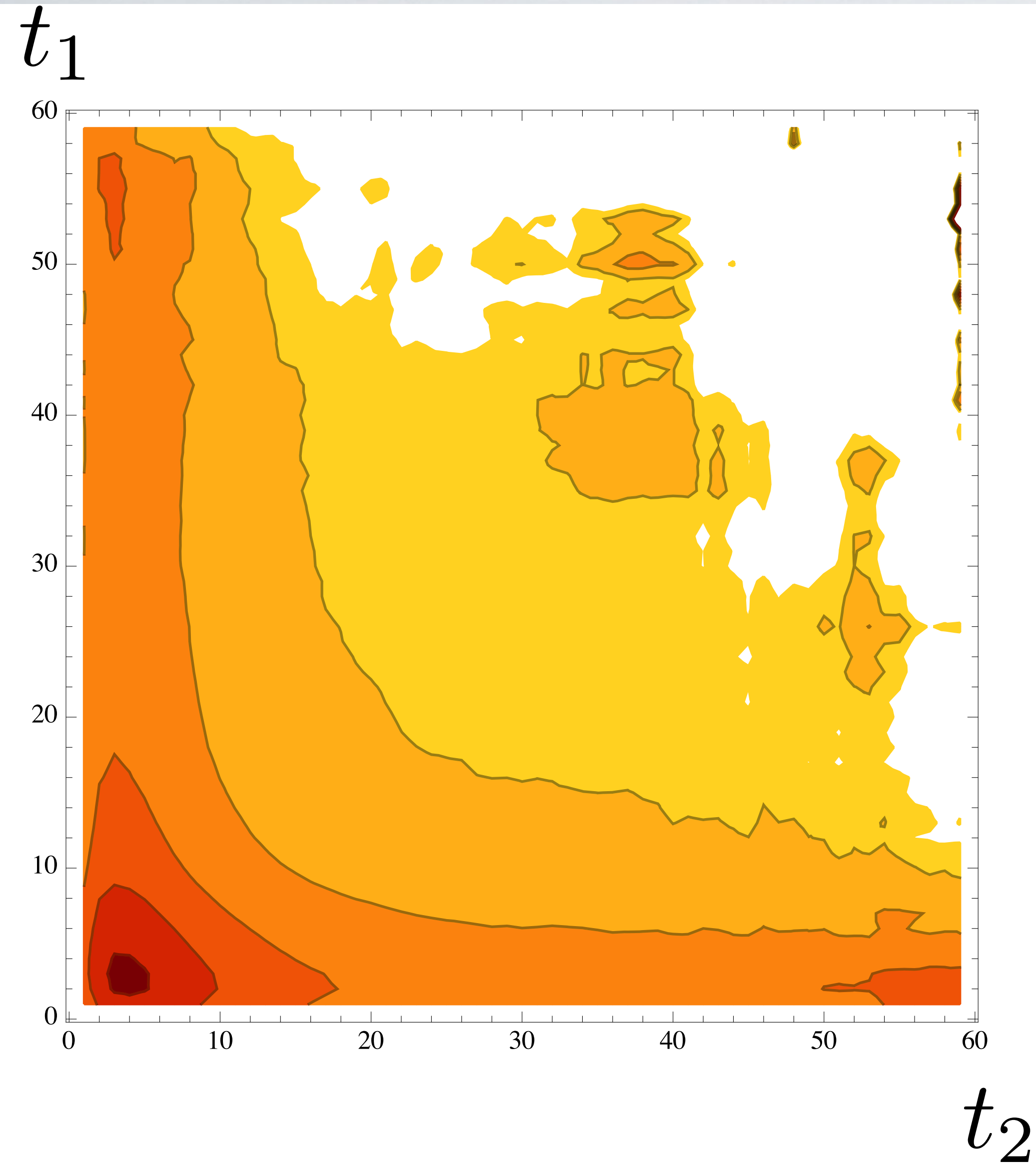
$$t_{1,2} = t_1 = t_2$$

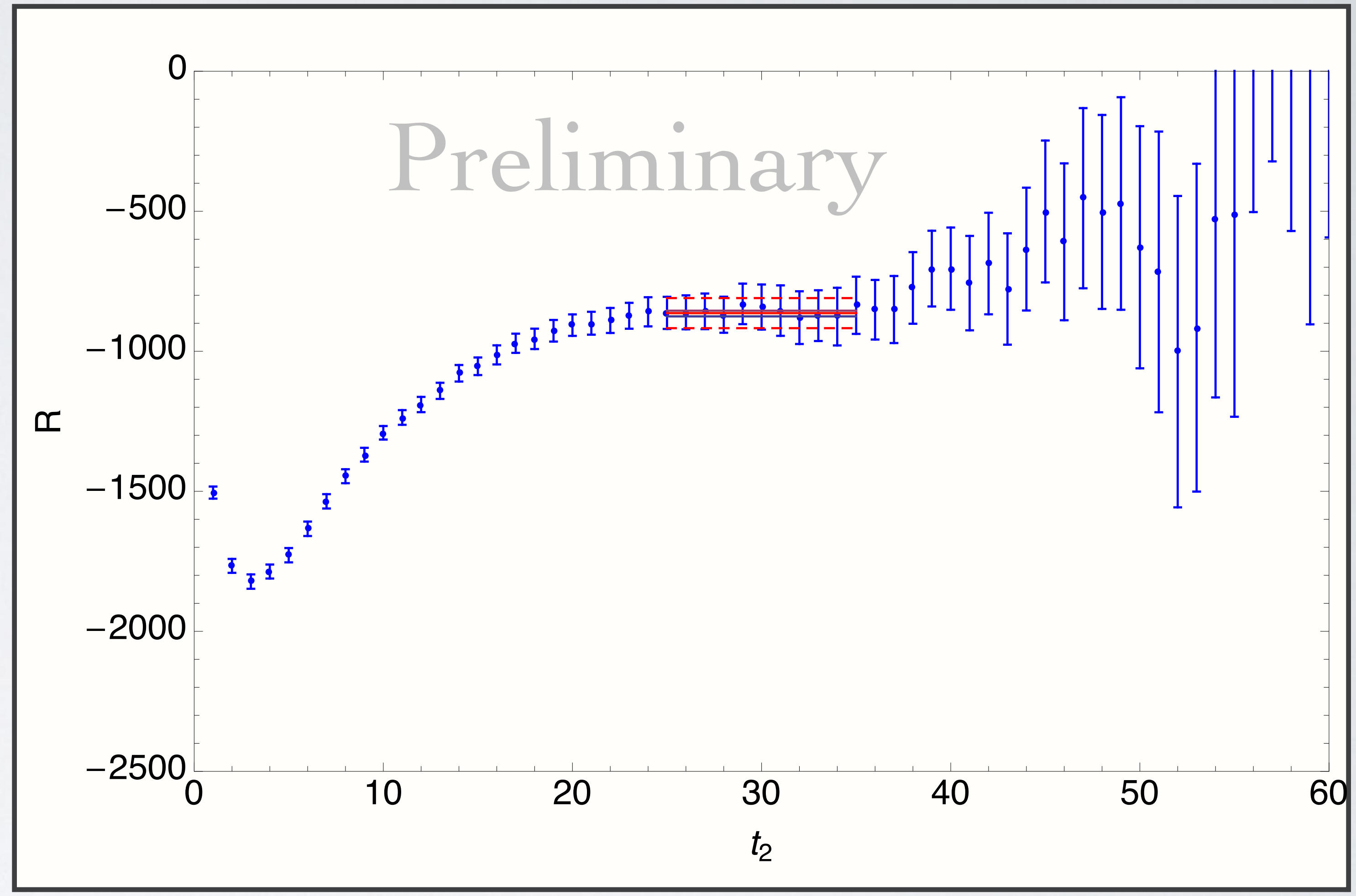


$$a_s^6 \frac{\langle \bar{n}|\mathcal{O}|n\rangle}{Z_N Z_{\bar{N}}} = -881 \pm 34^{+23}_{-56}$$

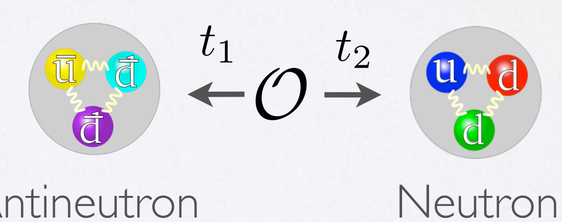
$$\chi^2/\text{dof} = 1.25$$

A multitude of fits





$$25 \leq t_1 \leq 35$$
$$25 \leq t_2 \leq 35$$

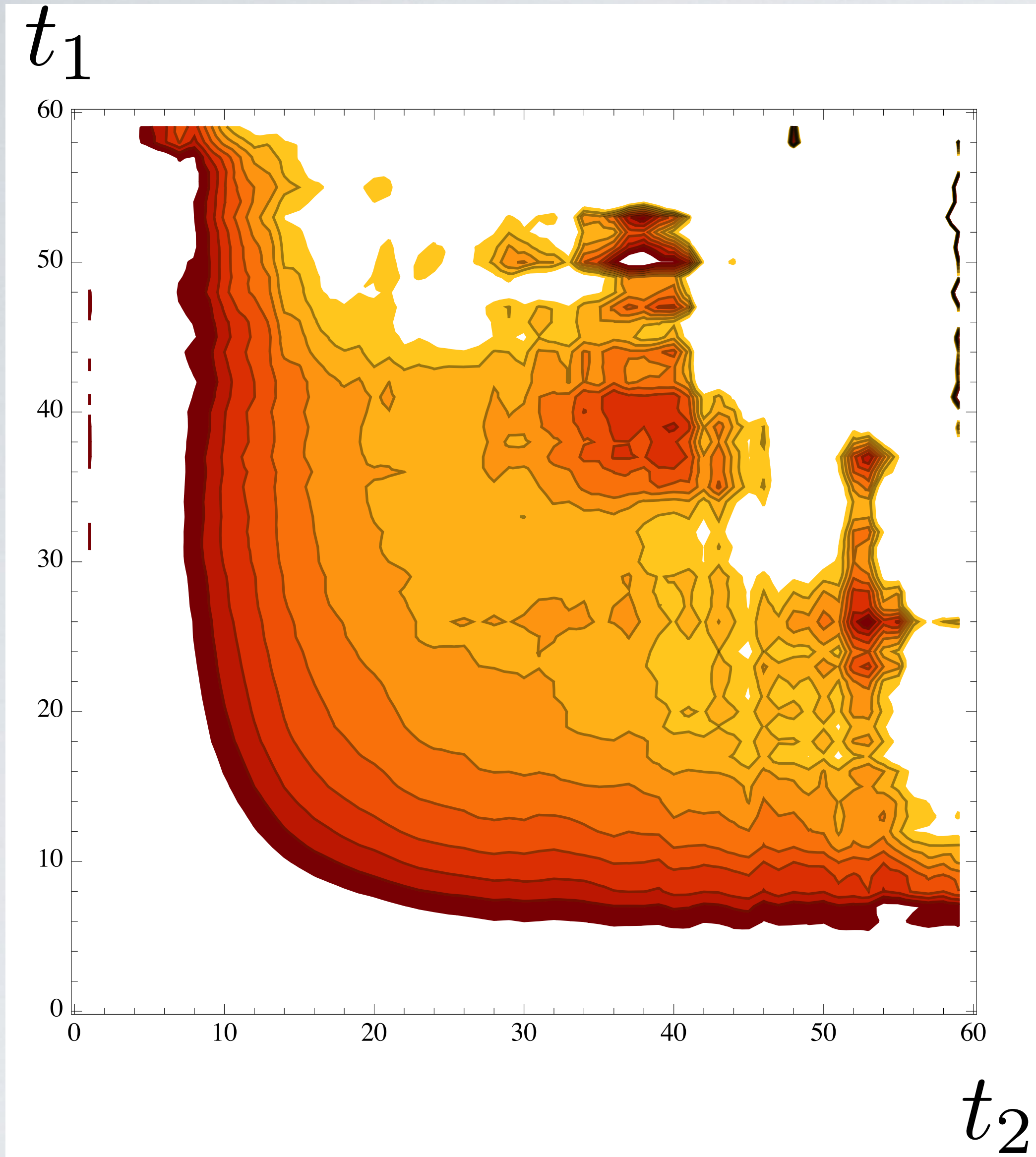


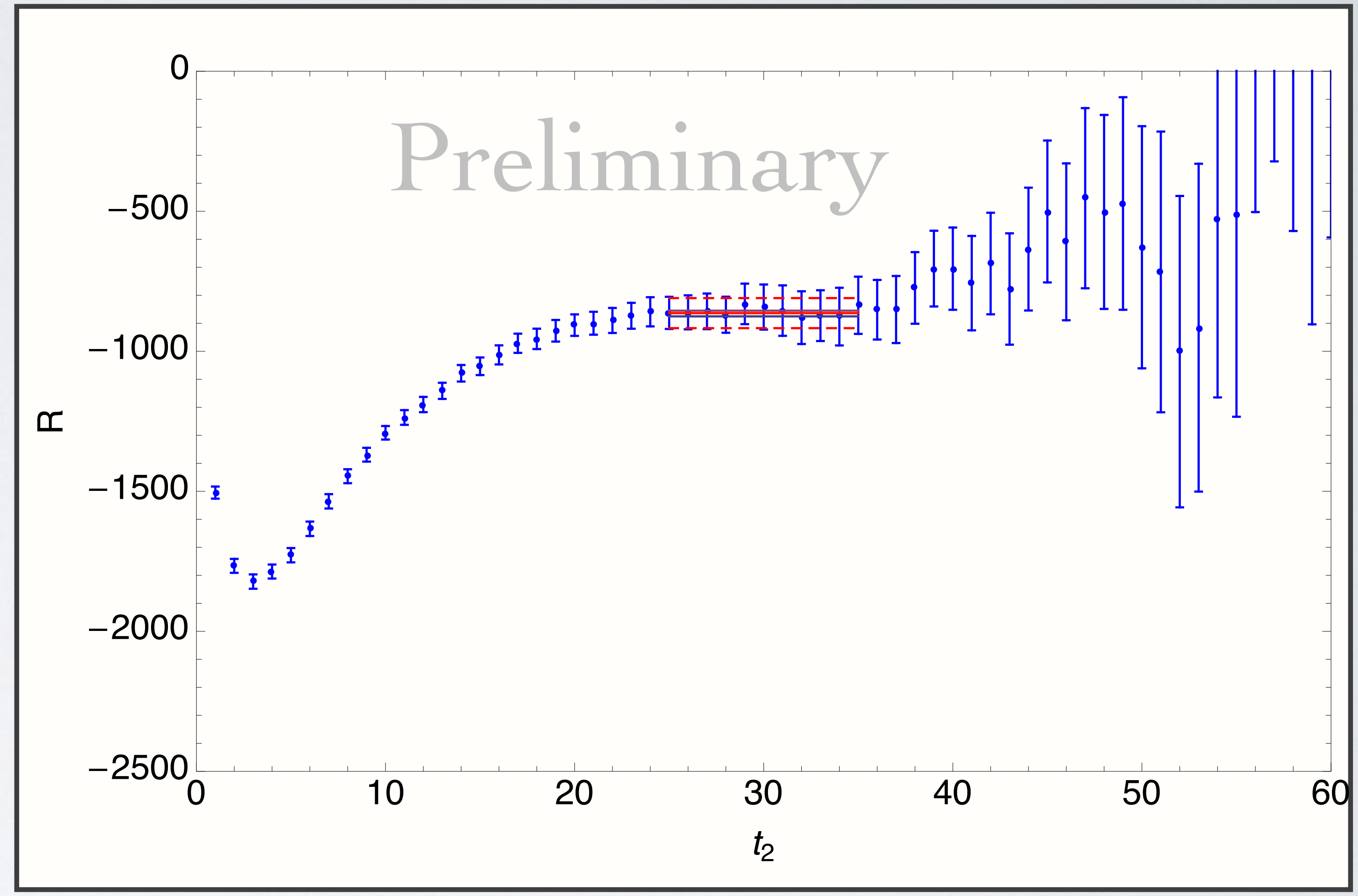
$$\mathcal{R} \xrightarrow{t_1, t_2 \to \infty} \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\overline{N}}}$$

$$a_s^6 \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\bar{N}}} = -863 \pm 54^{+9}_{-11}$$

$$\chi^2/\mathrm{dof} = 0.40$$

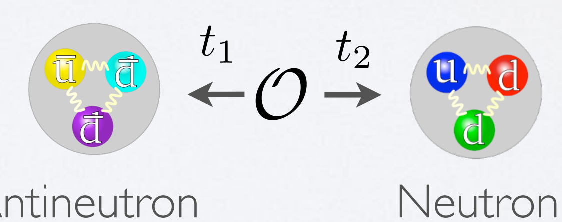
A multitude of fits





$$25 \leq t_1 \leq 35$$

$$25 \leq t_2 \leq 35$$

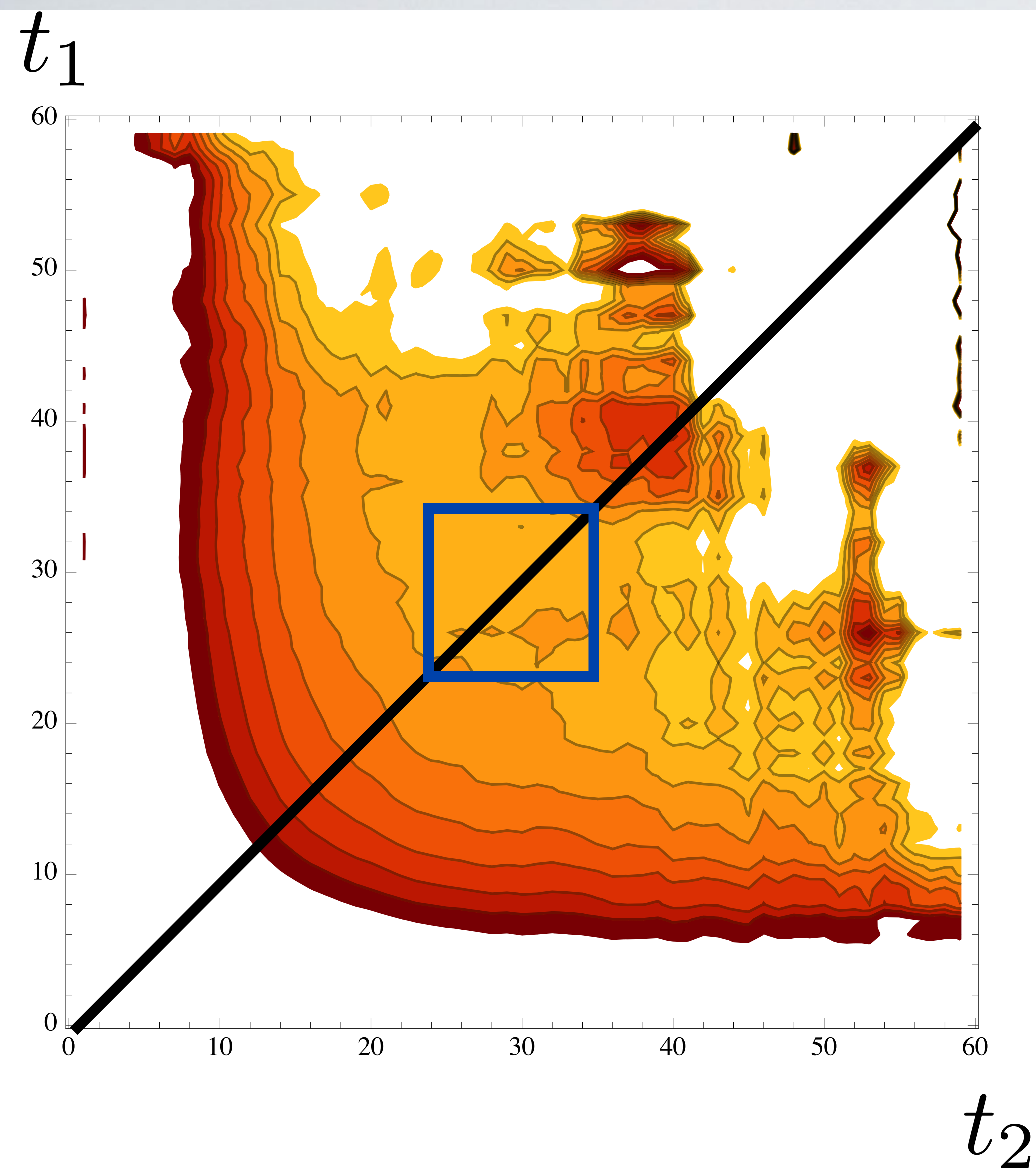


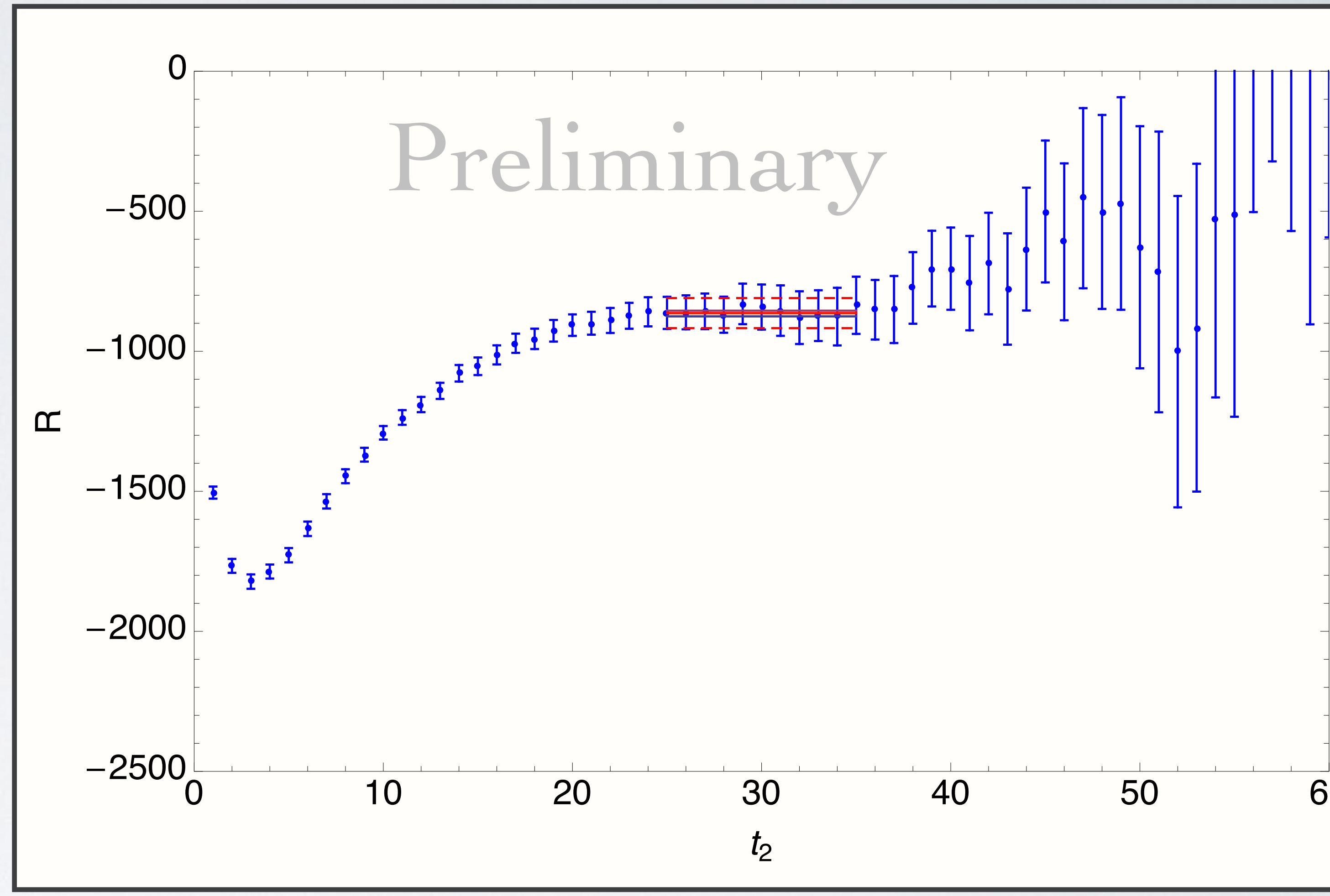
$$\mathcal{R} \xrightarrow{t_1, t_2 \to \infty} \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\overline{N}}}$$

$$a_s^6 \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\bar{N}}} = -863 \pm 54^{+9}_{-11}$$

$$\chi^2/\text{dof} = 0.40$$

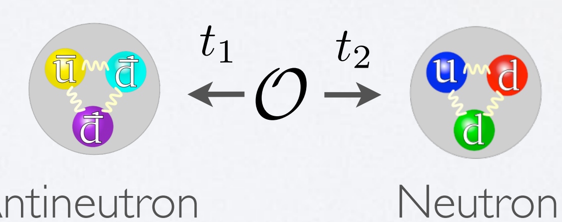
A multitude of fits





$$25 \le t_1 \le 35$$

$$25 \le t_2 \le 35$$

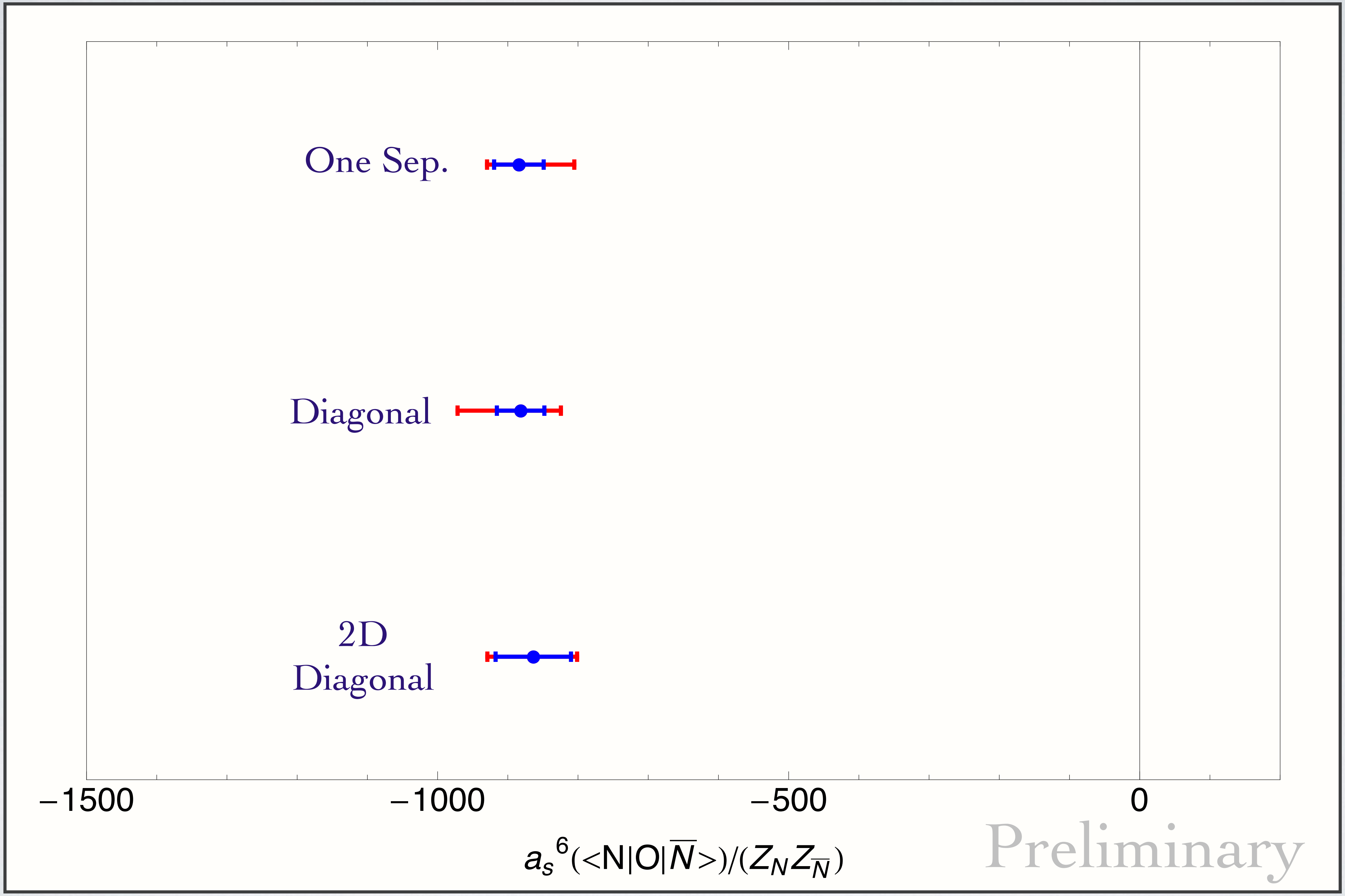


$$\mathcal{R} \xrightarrow{t_1, t_2 \to \infty} \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\overline{N}}}$$

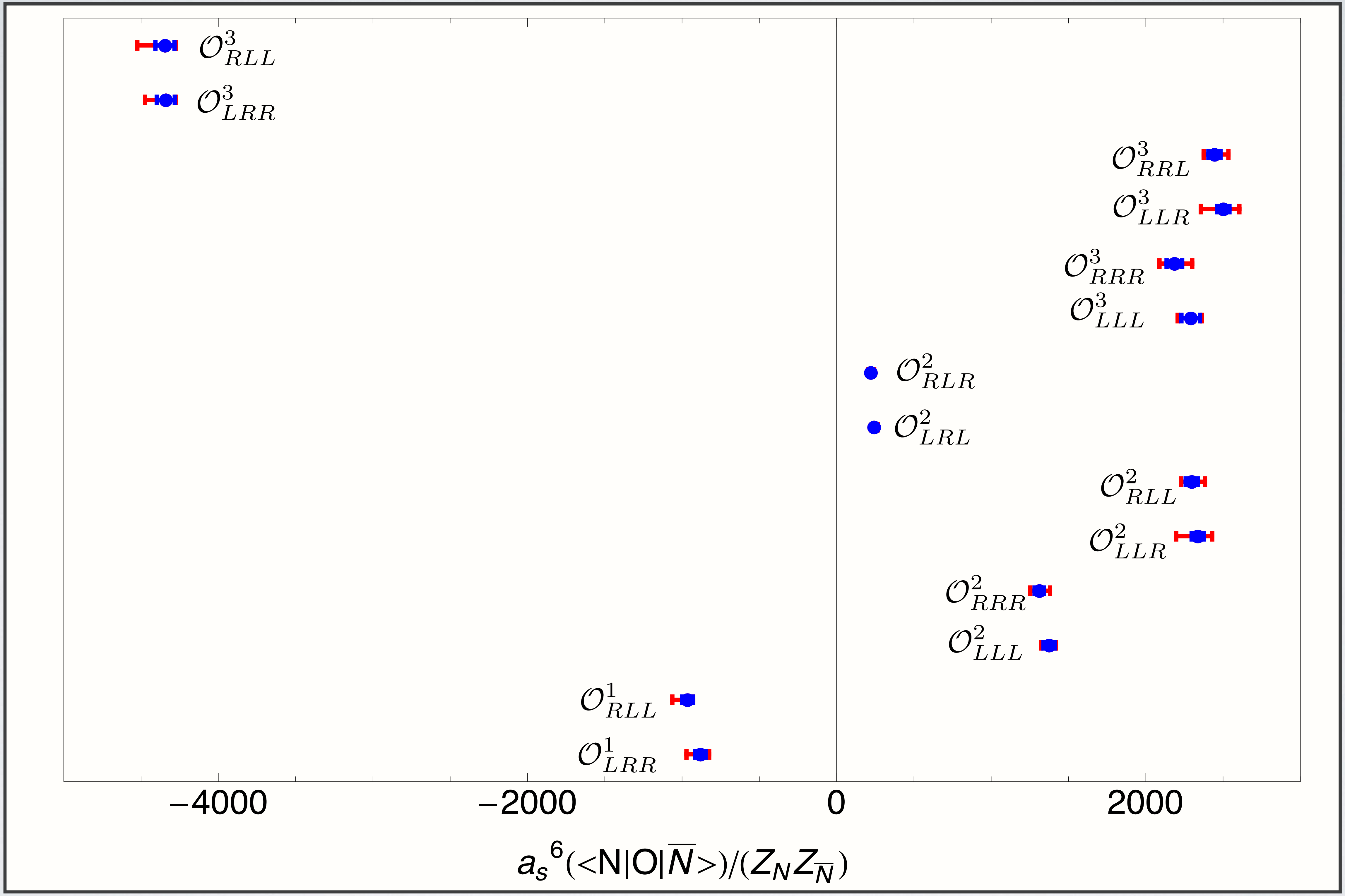
$$a_s^6 \frac{\langle \bar{n} | \mathcal{O} | n \rangle}{Z_N Z_{\bar{N}}} = -863 \pm 54^{+9}_{-11}$$

$$\chi^2/\mathrm{dof} = 0.40$$

Different fits agree



Preliminary (Bare) Results



Back to the big question...

Namely, what is the overall scale?

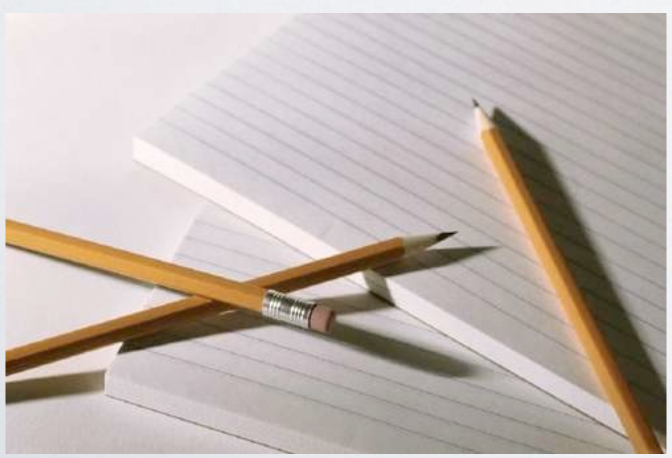
Reminder: $\langle \bar{n} | \mathcal{O} | n \rangle \sim \Lambda_{\mathrm{QCD}}^6$

Unfortunately, requires much additional work to extract reliably

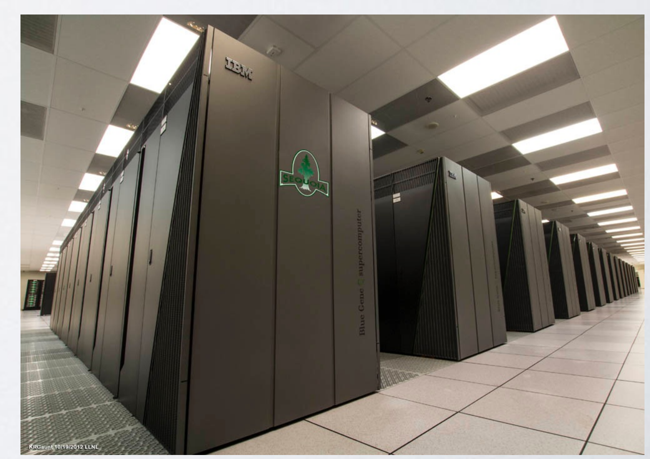
Analytically - Two loop QCD renormalization & one loop matching

Numerically - Full non-perturbative renormalization

$$\mathcal{O}^{\overline{MS}}(\mu) = U^{\overline{MS}}(\mu, p_0) \frac{Z^{\overline{MS}}(p_0)}{Z_{\mathrm{cont}}^{MOM}(p_0)} Z_{\mathrm{latt}}^{MOM}(p_0) \mathcal{O}_{\mathrm{latt}}^{\mathrm{bare}}$$



$$\Lambda_{QCD} < p_0 < \frac{1}{a}$$



Corrections: $\mathcal{O}(ap_0)\,,\,\mathcal{O}\big(g(p_0)^2\big)$

Perturbative Renormalization

BSM

Lattice

$$\mathcal{H}_{eff}^{n\bar{n}} = \sum_I C_I(\mu) Q_I(\mu) \quad \Rightarrow \quad \mathcal{H}_{eff}^{n\bar{n}} = \sum_I C_I^{\bar{\mathrm{MS}}}(\mu) U_I(\mu, p_0) Q_I^{\mathrm{RI}}(p_0)$$

Perturbative Renormalization

BSM Lattice

$$\mathcal{H}^{n\bar{n}}_{eff} = \sum_I C_I(\mu) Q_I(\mu) \quad \Rightarrow \quad \mathcal{H}^{n\bar{n}}_{eff} = \sum_I C_I^{\bar{\mathrm{MS}}}(\mu) U_I(\mu, p_0) Q_I^{\mathrm{RI}}(p_0)$$

$$U_I(\mu, p_0) = \begin{cases} U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, m_b) U_I^{N_f=4}(m_b, p_0) & \text{for} \quad m_c < p_0 < m_b \\ U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, p_0) & \text{for} \quad m_b < p_0 < m_t \end{cases}$$

Perturbative Renormalization

BSM Lattice

$$\mathcal{H}^{n\bar{n}}_{eff} = \sum_I C_I(\mu) Q_I(\mu) \quad \Longrightarrow \quad \mathcal{H}^{n\bar{n}}_{eff} = \sum_I C_I^{\overline{\rm MS}}(\mu) U_I(\mu, p_0) Q_I^{\rm RI}(p_0)$$

$$U_I(\mu, p_0) = \begin{cases} U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, m_b) U_I^{N_f=4}(m_b, p_0) & \text{for} \quad m_c < p_0 < m_b \\ U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, p_0) & \text{for} \quad m_b < p_0 < m_t \end{cases}$$

$$U_I^{N_f}(\mu_1, \mu_2) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)} \right)^{-\gamma_I^{(0)}/2\beta_0} \left[1 - \delta_{\mu_2, p_0} r_I^{(0)} \frac{\alpha_s(p_0)}{4\pi} + \left(\frac{\beta_1 \gamma_I^{(0)}}{2\beta_0^2} - \frac{\gamma_I^{(1)}}{2\beta_0} \right) \frac{\alpha_s(\mu_2) - \alpha_s(\mu_1)}{4\pi} + O(\alpha_s^2) \right]$$

One loop divergent — *W. Caswell, J. Milutinovic, G. Senjanovic (1983)*

One loop finite
Two loop divergent — *MIB, M. Wagman (2015)*

Perturbative Renormalization

$$\mathcal{H}_{eff}^{n\bar{n}} = \sum_I C_I(\mu) Q_I(\mu) \quad \Rightarrow \quad \mathcal{H}_{eff}^{n\bar{n}} = \sum_I \underbrace{C_I^{\overline{\text{MS}}}(\mu)}_{\text{BSM}} U_I(\mu, p_0) \underbrace{Q_I^{\text{RI}}(p_0)}_{\text{Lattice}}$$

$$U_I(\mu, p_0) = \begin{cases} U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, m_b) U_I^{N_f=4}(m_b, p_0) & \text{for} \quad m_c < p_0 < m_b \\ U_I^{N_f=6}(\mu, m_t) U_I^{N_f=5}(m_t, p_0) & \text{for} \quad m_b < p_0 < m_t \end{cases}$$

$$U_I^{N_f}(\mu_1, \mu_2) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\gamma_I^{(0)}/2\beta_0} \left[1 - \delta_{\mu_2, p_0} r_I^{(0)} \frac{\alpha_s(p_0)}{4\pi} + \left(\frac{\beta_1 \gamma_I^{(0)}}{2\beta_0^2} - \frac{\gamma_I^{(1)}}{2\beta_0}\right) \frac{\alpha_s(\mu_2) - \alpha_s(\mu_1)}{4\pi} + O(\alpha_s^2)\right]$$

One loop divergent *W. Caswell, J. Milutinovic, G. Senjanovic (1983)*

One loop finite
Two loop divergent *MIB, M. Wagman (2015)*

RI-MOM:

$$\begin{aligned} [\Lambda_I]_{ijklmn}^{\alpha\beta\gamma\delta\eta\zeta}(p) = & \frac{1}{5} \langle Q_I(0)\, \bar{u}_i^\alpha(p) \bar{u}_j^\beta(p) \bar{d}_k^\gamma(p) \bar{d}_l^\delta(-p) \bar{d}_m^\eta(-p) \bar{d}_n^\zeta(-p) \rangle \Big|_{amp} \\ & + \frac{3}{5} \langle Q_I(0)\, \bar{u}_i^\alpha(p) \bar{u}_j^\beta(-p) \bar{d}_k^\gamma(p) \bar{d}_l^\delta(p) \bar{d}_m^\eta(-p) \bar{d}_n^\zeta(-p) \rangle \Big|_{amp} \\ & + \frac{1}{5} \langle Q_I(0)\, \bar{u}_i^\alpha(-p) \bar{u}_j^\beta(-p) \bar{d}_k^\gamma(p) \bar{d}_l^\delta(p) \bar{d}_m^\eta(p) \bar{d}_n^\zeta(-p) \rangle \Big|_{amp}, \end{aligned}$$

$$\text{tr}\left[\mathcal{P}_I \Lambda_J^{(0)}\right] = \delta_{IJ}$$

Perturbative Renormalization

BSM

Lattice

$$(\mathcal{P}_1)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{92160}\left(-T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_R)^{\alpha\beta}(CP_R)^{\gamma\delta}(CP_R)^{\eta\zeta} + 2T^{AAS}_{[ij][kl]\{mn\}}(CP_R)^{\alpha\delta}(CP_R)^{\gamma\beta}(CP_R)^{\eta\zeta}\right),$$

$$(\mathcal{P}_2)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{18432}\left(-T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_L)^{\alpha\delta}(CP_R)^{\gamma\beta}(CP_R)^{\eta\zeta} + 2T^{AAS}_{[ij][kl]\{mn\}}(CP_L)^{\alpha\delta}(CP_R)^{\gamma\zeta}(CP_R)^{\eta\beta}\right),$$

$$(\mathcal{P}_3)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{36864}\left(-T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_L)^{\alpha\beta}(CP_L)^{\gamma\delta}(CP_R)^{\eta\zeta} + 2T^{AAS}_{[ij][kl]\{mn\}}(CP_L)^{\alpha\delta}(CP_L)^{\gamma\beta}(CP_R)^{\eta\zeta}\right),$$

$$(\mathcal{P}_4)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{221184}\left(T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_R)^{\alpha\beta}(CP_R)^{\gamma\delta}(CP_R)^{\eta\zeta} + 3T^{AAS}_{[ij][kl]\{mn\}}(CP_R)^{\alpha\delta}(CP_R)^{\gamma\beta}(CP_R)^{\eta\zeta}\right),$$

$$(\mathcal{P}_5)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{221184}\left(T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_R)^{\alpha\beta}(CP_L)^{\gamma\delta}(CP_L)^{\eta\zeta}\right),$$

$$(\mathcal{P}_6)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{55296}\left(T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_R)^{\alpha\delta}(CP_L)^{\gamma\beta}(CP_L)^{\eta\zeta} + 6T^{AAS}_{[ij][kl]\{mn\}}(CP_R)^{\alpha\delta}(CP_L)^{\gamma\zeta}(CP_L)^{\eta\beta}\right),$$

$$(\mathcal{P}_7)^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn} = -\frac{1}{73728}\left(T^{SSS}_{\{ij\}\{kl\}\{mn\}}(CP_L)^{\alpha\beta}(CP_L)^{\gamma\delta}(CP_R)^{\eta\zeta} + 2T^{AAS}_{[ij][kl]\{mn\}}(CP_L)^{\alpha\delta}(CP_L)^{\gamma\beta}(CP_R)^{\eta\zeta}\right)$$

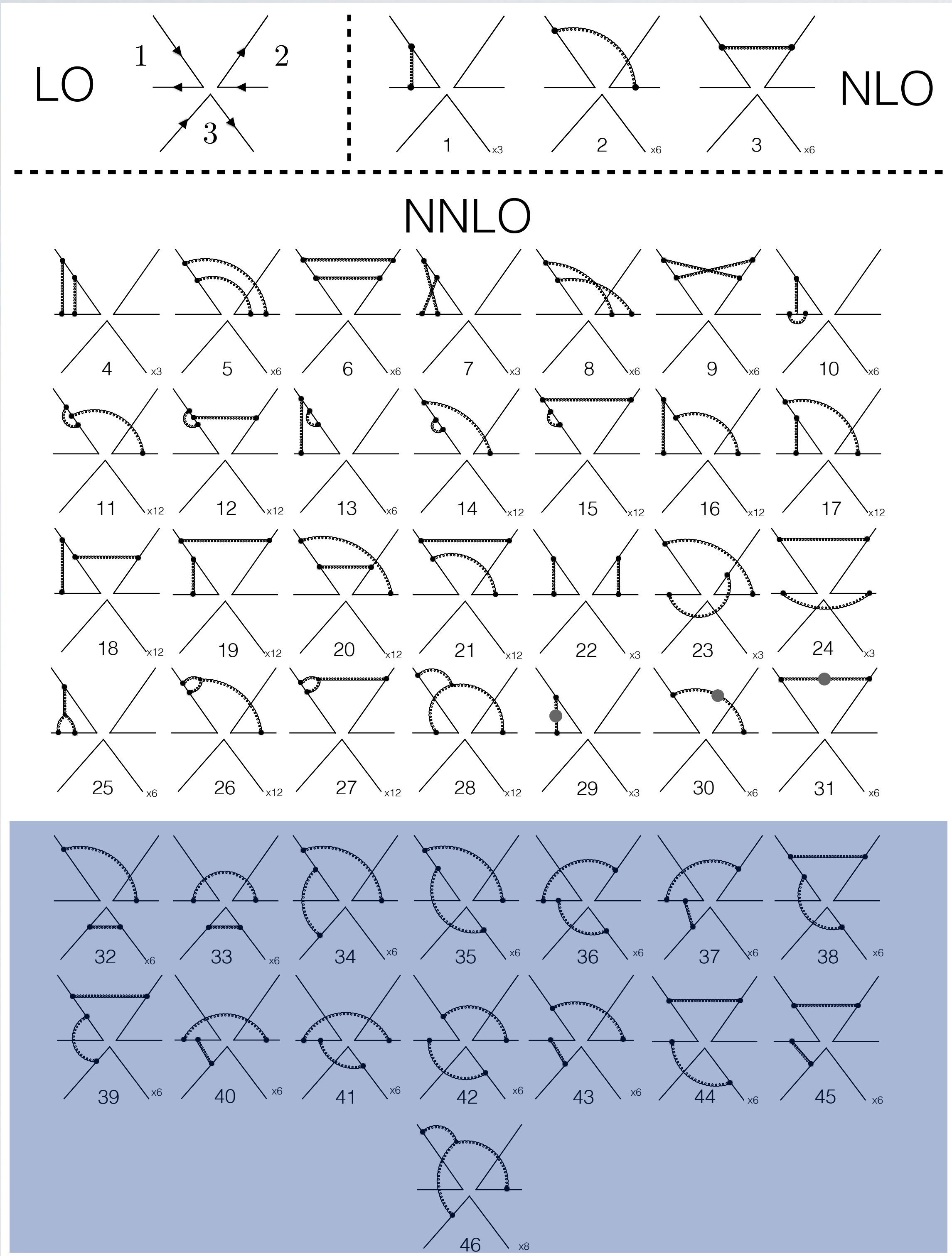
RI-MOM:

$$\begin{aligned}[\Lambda_I]^{\alpha\beta\gamma\delta\eta\zeta}_{ijklmn}(p) = &\frac{1}{5}\langle Q_I(0)\,\bar u^\alpha_i(p)\bar u^\beta_j(p)\bar d^\gamma_k(p)\bar d^\delta_l(-p)\bar d^\eta_m(-p)\bar d^\zeta_n(-p)\rangle\Big|_{amp} \\ &+\frac{3}{5}\langle Q_I(0)\,\bar u^\alpha_i(p)\bar u^\beta_j(-p)\bar d^\gamma_k(p)\bar d^\delta_l(p)\bar d^\eta_m(-p)\bar d^\zeta_n(-p)\rangle\Big|_{amp} \\ &+\frac{1}{5}\langle Q_I(0)\,\bar u^\alpha_i(-p)\bar u^\beta_j(-p)\bar d^\gamma_k(p)\bar d^\delta_l(p)\bar d^\eta_m(p)\bar d^\zeta_n(-p)\rangle\Big|_{amp},\end{aligned}$$

$$\mathrm{tr}\left[\mathcal{P}_I\Lambda^{(0)}_J\right] = \delta_{IJ}$$

Perturbative Renormalization

MIB, M. Wagman (2015)
(See Mike's talk last week)



One Loop:

3 diagram classes, 15 diagrams

Two Loop:

43 diagram classes, 320 diagrams

First Calculation of three diquarks

Evanescent operators real complication!!

$D \neq 4$ → Fierz identities no longer hold

Two-loop anomalous dimension not unique!
(depends on generalization of Fierz to D dimensions)

Effectively, treat all operators independently: Q_I $\tilde{Q}_I$ $Q_I - \tilde{Q}_I$

Perturbative Renormalization

MIB, M. Wagman (2015)

Chiral Basis	Flavor Basis	$\gamma_I^{(0)}$	$\gamma_I^{(1)}$	$r_I^{(0)}$
Q_1	$\mathcal{O}^3_{RRR},\ \mathcal{O}^3_{LLL}$	4	$335/3 - 34N_f/9$	$101/30 + 8/15\ln 2$
Q_2	$\mathcal{O}^3_{LRR},\ \mathcal{O}^3_{RLR},\ \mathcal{O}^3_{RLL},\ \mathcal{O}^3_{LRL}$	-4	$91/3 - 26N_f/9$	$-31/6 + 88/15\ln 2$
Q_3	$\mathcal{O}^3_{LLR}, \mathcal{O}^3_{RRL}$	0	$64 - 10N_f/3$	$-9/10 + 16/5\ln 2$
Q_4	$\left(4/5\ \mathcal{O}^2_{RRR} + 1/5\ \mathcal{O}^1_{RRR}\right)$, $\left(4/5\ \mathcal{O}^2_{LLL} + 1/5\ \mathcal{O}^1_{LLL}\right)$	24	$229 - 46N_f/3$	$177/10 - 64/5\ln 2$
Q_5	$\mathcal{O}^1_{RLL},\ \mathcal{O}^1_{LRR},\ \mathcal{O}^2_{RLL},$ $\mathcal{O}^2_{LRL},\ \mathcal{O}^2_{LRR},\ \mathcal{O}^2_{RLR},$ $(2/3\ \mathcal{O}^2_{LLR} + 1/3\ O^1_{LLR}),$ $(2/3\ \mathcal{O}^2_{LLR} + 1/3\ O^1_{LRL}),$ $(2/3\ \mathcal{O}^2_{RRL} + 1/3\ O^1_{RRL}),$ $(2/3\ \mathcal{O}^2_{RRL} + 1/3\ O^1_{RLR})$	12	$238 - 14N_f$	$49/10 - 24/5\ln 2$
$\widetilde{Q}_1$	$(1/3\ \mathcal{O}^2_{RRR} - 1/3\ \mathcal{O}^1_{RRR}),$ $(1/3\ \mathcal{O}^2_{LLL} - 1/3\ \mathcal{O}^1_{LLL})$	4	$797/3 - 118N_f/9$	$-109/30 + 8/15\ln 2$
$\widetilde{Q}_3$	$(1/3\ \mathcal{O}^2_{LLR} - 1/3\ \mathcal{O}^1_{LLR}),$ $(1/3\ \mathcal{O}^2_{RRL} - 1/3\ \mathcal{O}^1_{RRL})$	0	$218 - 38N_f/3$	$-79/10 + 16/5\ln 2$

$$U_I^{N_f}(\mu_1,\mu_2) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-\gamma_I^{(0)}/2\beta_0} \left[1 - \delta_{\mu_2,p_0} r_I^{(0)} \frac{\alpha_s(p_0)}{4\pi} + \left(\frac{\beta_1\gamma_I^{(0)}}{2\beta_0^2} - \frac{\gamma_I^{(1)}}{2\beta_0}\right)\frac{\alpha_s(\mu_2)-\alpha_s(\mu_1)}{4\pi} + O(\alpha_s^2)\right]$$

Back to the big question...

Namely, what is the overall scale?

Reminder: $\langle \bar{n} | \mathcal{O} | n \rangle \sim \Lambda^6_{\mathrm{QCD}}$

Unfortunately, requires additional work to extract reliably

Analytically - Two loop QCD renormalization, EFT calculations

- One loop divergent ✓
- Projection Operators ✓
- One loop matching ✓
- Two loop running ✓

Numerically - Full non-perturbative renormalization

14 operators with delicate chiral-structure → Extremely difficult with anisotropic Wilson fermions

Last leg of the race…

GOAL:
To calculate neturon-antineutron matrix elements crucial for connecting theory & experiment

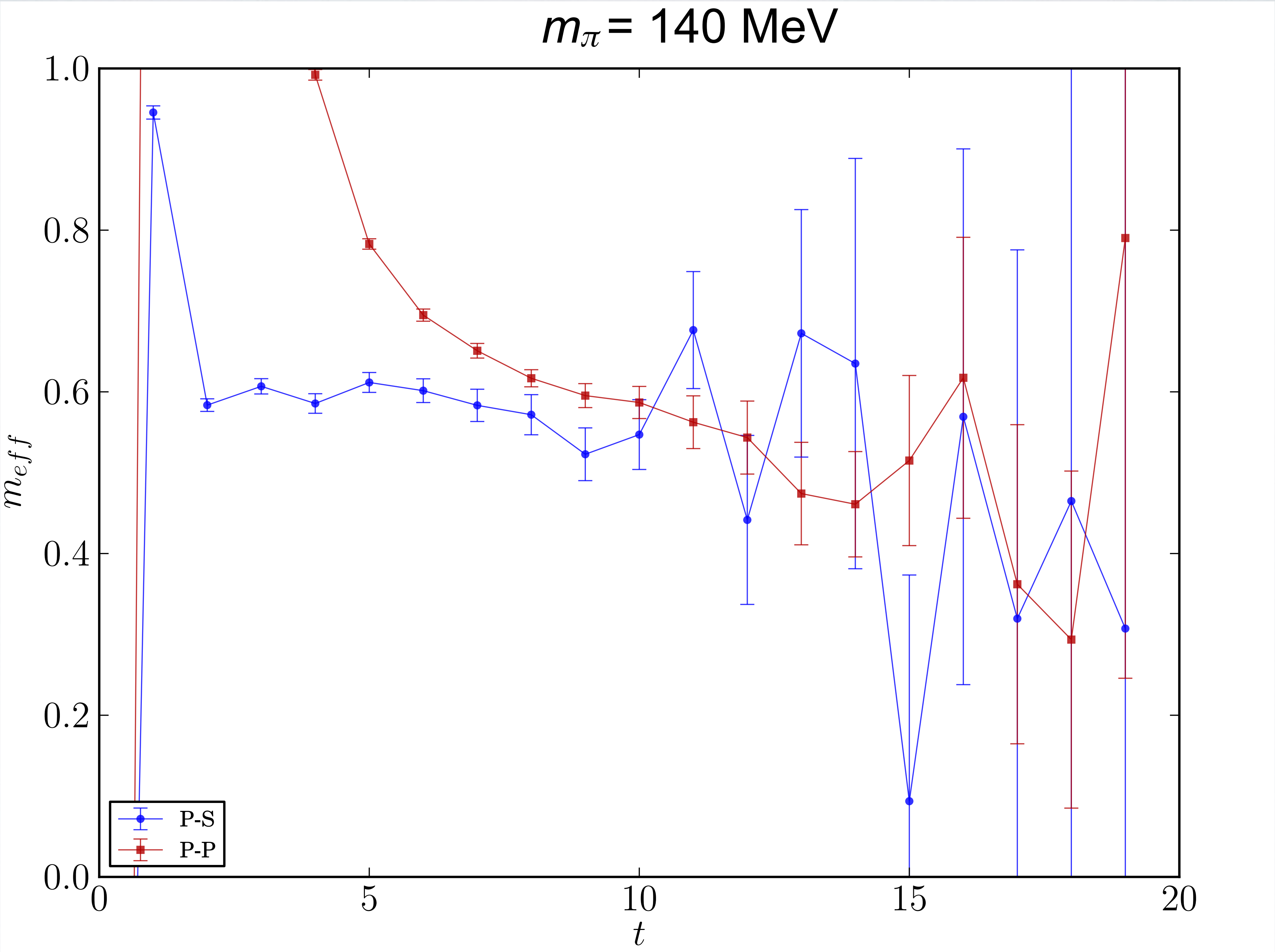
MIB, C. Schroeder, S. Syritsyn, J. Wasem, M. Wagman

Physical DWF Lattice QCD calculation:

Pion Mass:	140 MeV	(Note: Physical value ~139 MeV)
Lattice Spacing:	0.123 fm	
Lattice Extent:	5.5 fm	(Number of sites: 48^3 x 96)
Pion Mass x Lattice Extent:	3.9	(Note: Typically > 4)
Measurements:	2268	(Note: 81 cfg, sep by 25 tu, 28 AMA meas per cfg)

Neutron Mass

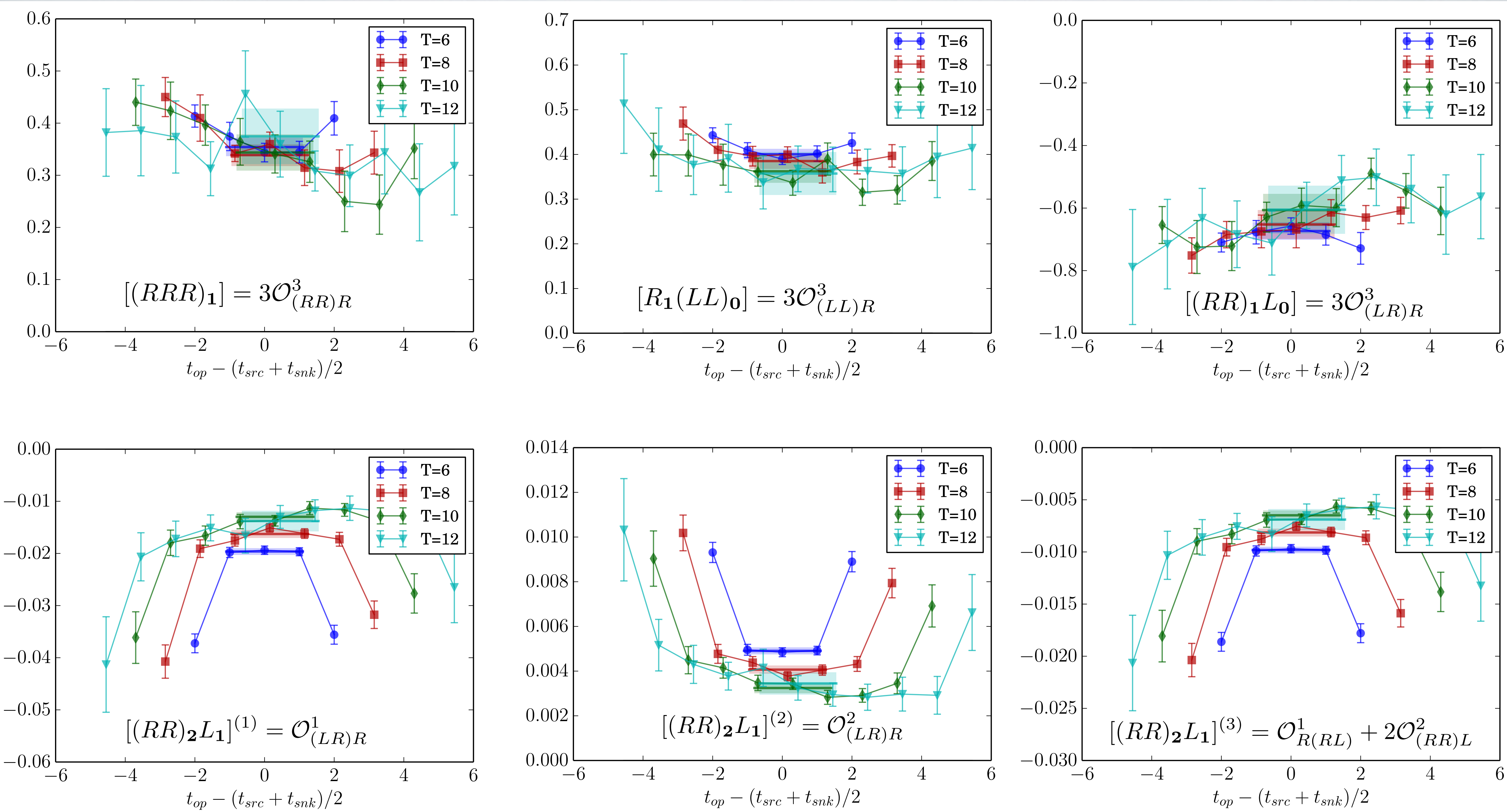
Credit: Sergey Syritsyn



$$am_{eff}(t) = \log \frac{\langle N(t)N(0)\rangle}{\langle N(t+1)N(0)\rangle} \xrightarrow{t\to\infty} aM_N$$

Neutron-Antineutron Matrix elements

Credit: Sergey Syritsyn



$$C^{2\text{pt}}_{PP} = Z_P \; e^{-m_n t}$$

$$C^{2\text{pt}}_{PS} = \sqrt{Z_P Z_S} \; e^{-m_n t}$$

$$C^{3\text{pt}}_{SS} = Z_S \; e^{-m_n(t_1+t_2)} \langle \bar{n} | \mathcal{O} | n \rangle$$

$$C^{3\text{pt}}_{PP} = Z_P \; e^{-m_n(t_1+t_2)} \langle \bar{n} | \mathcal{O} | n \rangle$$

Scaled $\times\ 10^6$, kinematic factors not divided out

T=10: ~10% stat uncertainty, consistant with T=12

Non-perturbative Renormalization

Non-perturbative Renormalization

$$S_{i'i}^{\alpha'\alpha}(p) = \sum_y e^{ip\cdot y}\,\overline{q}_{i'}^{\alpha'}(x)\,q_i^{\alpha}(0)$$

MOM: $p_0^2 = p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = p_6^2$

Non-perturbative Renormalization

$$S^{\alpha'\alpha}_{i'i}(p) = \sum_y e^{ip\cdot y}\overline{q}^{\alpha'}_{i'}(x)q^{\alpha}_i(0)$$

MOM: $p_0^2 = p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = p_6^2$

$a = 1 \cdots 14$

$${}^a\Lambda^{\alpha\beta\cdots}_{ij\cdots}\left[S^{c'c}_{s's}\right] =$$

$S^{\beta\beta'}_{jj'}(-p_0) \rightarrow$, $S^{\delta\delta'}_{ll'}(-p_0) \rightarrow$, $S^{\eta\eta'}_{nn'}(-p_0) \rightarrow$ ${}^a\mathcal{O}^{\alpha'\beta'\cdots}_{i'j'\cdots}$ $\leftarrow S^{\alpha'\alpha}_{i'i}(p_0)$, $\leftarrow S^{\gamma'\gamma}_{k'k}(p_0)$, $\leftarrow S^{\epsilon'\epsilon}_{k'k}(p_0)$

$${}^a\mathcal{P}^{\alpha\beta\cdots}_{ij\cdots}\left({}^b\Lambda^{\alpha\beta\cdots}_{ij\cdots}\left[\delta^{c'c}_{s's}\right]\right) = \delta^{ab}$$

Non-perturbative Renormalization

$$S^{\alpha'\alpha}_{i'i}(p) = \sum_y e^{ip\cdot y} \overline{q}^{\alpha'}_{i'}(x) q^{\alpha}_{i}(0)$$

MOM: $p_0^2 = p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_5^2 = p_6^2$

$a = 1 \cdots 14$

$${}^a\Lambda^{\alpha\beta\cdots}_{ij\cdots}\left[S^{c'c}_{s's}\right] = \left\{ S^{\beta\beta'}_{jj'}(-p_0),\ S^{\delta\delta'}_{ll'}(-p_0),\ S^{\eta\eta'}_{nn'}(-p_0) \right\} \rightarrow {}^a\mathcal{O}^{\alpha'\beta'\cdots}_{i'j'\cdots} \leftarrow \left\{ S^{\alpha'\alpha}_{i'i}(p_0),\ S^{\gamma'\gamma}_{k'k}(p_0),\ S^{\epsilon'\epsilon}_{k'k}(p_0) \right\}$$

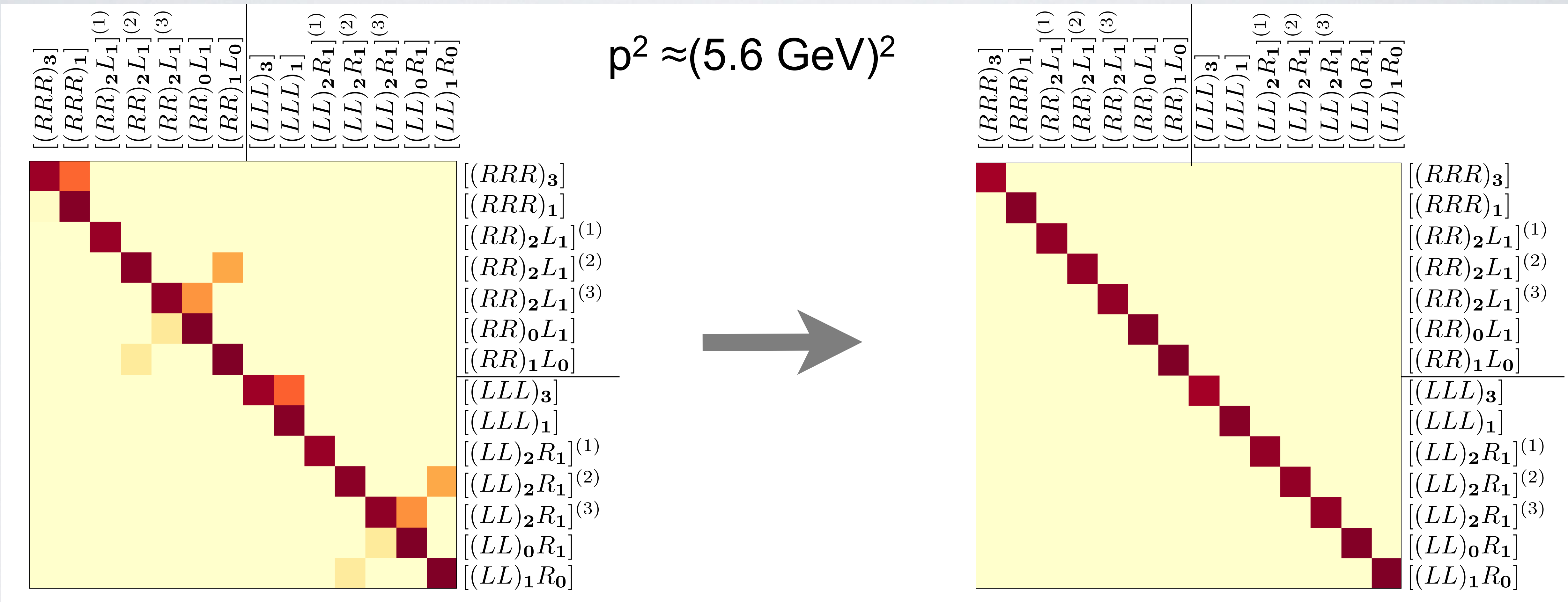
$${}^a\mathcal{P}^{\alpha\beta\cdots}_{ij\cdots}\left({}^b\Lambda^{\alpha\beta\cdots}_{ij\cdots}\left[\delta^{c'c}_{s's}\right]\right) = \delta^{ab}$$

$${}^a\mathcal{P}^{\alpha\beta\cdots}_{ij\cdots}(Z_q^{-3})({}^{bc}Z^{MOM}_{\rm latt})\ {}^c\Lambda^{\alpha\beta\cdots}_{ij\cdots} = \delta^{ab}$$

Non-perturbative Renormalization

Credit: Sergey Syritsyn

Issue: $\begin{pmatrix} u(p) \\ d(-p) \end{pmatrix}$ Not SU(2) doublet (ruins chiral symmetry)

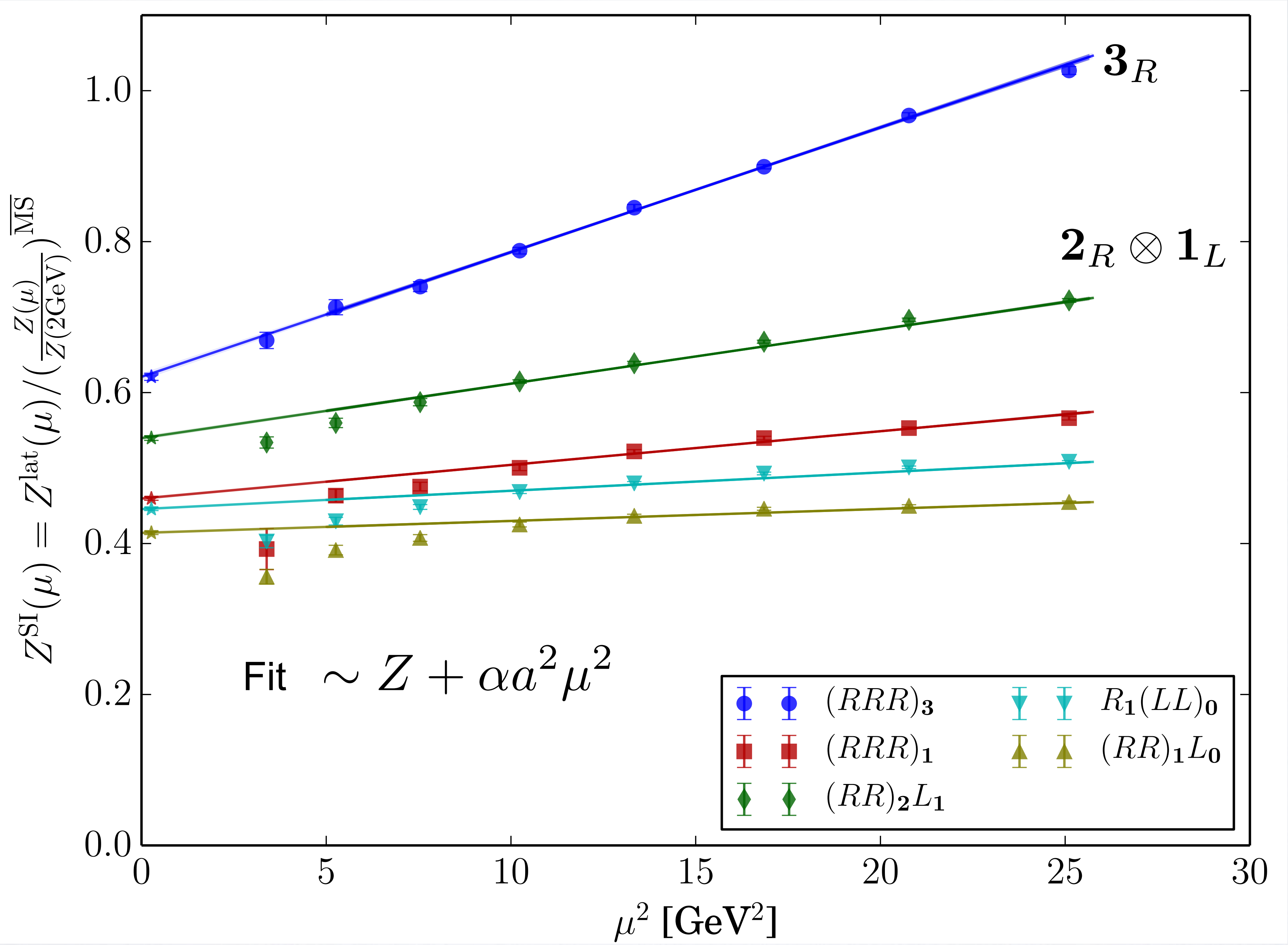


Trick:
Symmetrize over external momenta

$$\frac{1}{5}\langle \mathcal{O}^{6q}\, \bar{u}(+p)\bar{u}(+p)\bar{d}(+p)\bar{d}(-p)\bar{d}(-p)\bar{d}(-p)\rangle$$
$$+\frac{3}{5}\langle \mathcal{O}^{6q}\, \bar{u}(+p)\bar{u}(-p)\bar{d}(+p)\bar{d}(+p)\bar{d}(-p)\bar{d}(-p)\rangle$$
$$+\frac{1}{5}\langle \mathcal{O}^{6q}\, \bar{u}(-p)\bar{u}(-p)\bar{d}(+p)\bar{d}(+p)\bar{d}(+p)\bar{d}(-p)\rangle$$

Non-perturbative Renormalization

Credit: Sergey Syritsyn



Props from RBC/UKQCD HQ BBar Project

Systematic error estimate:

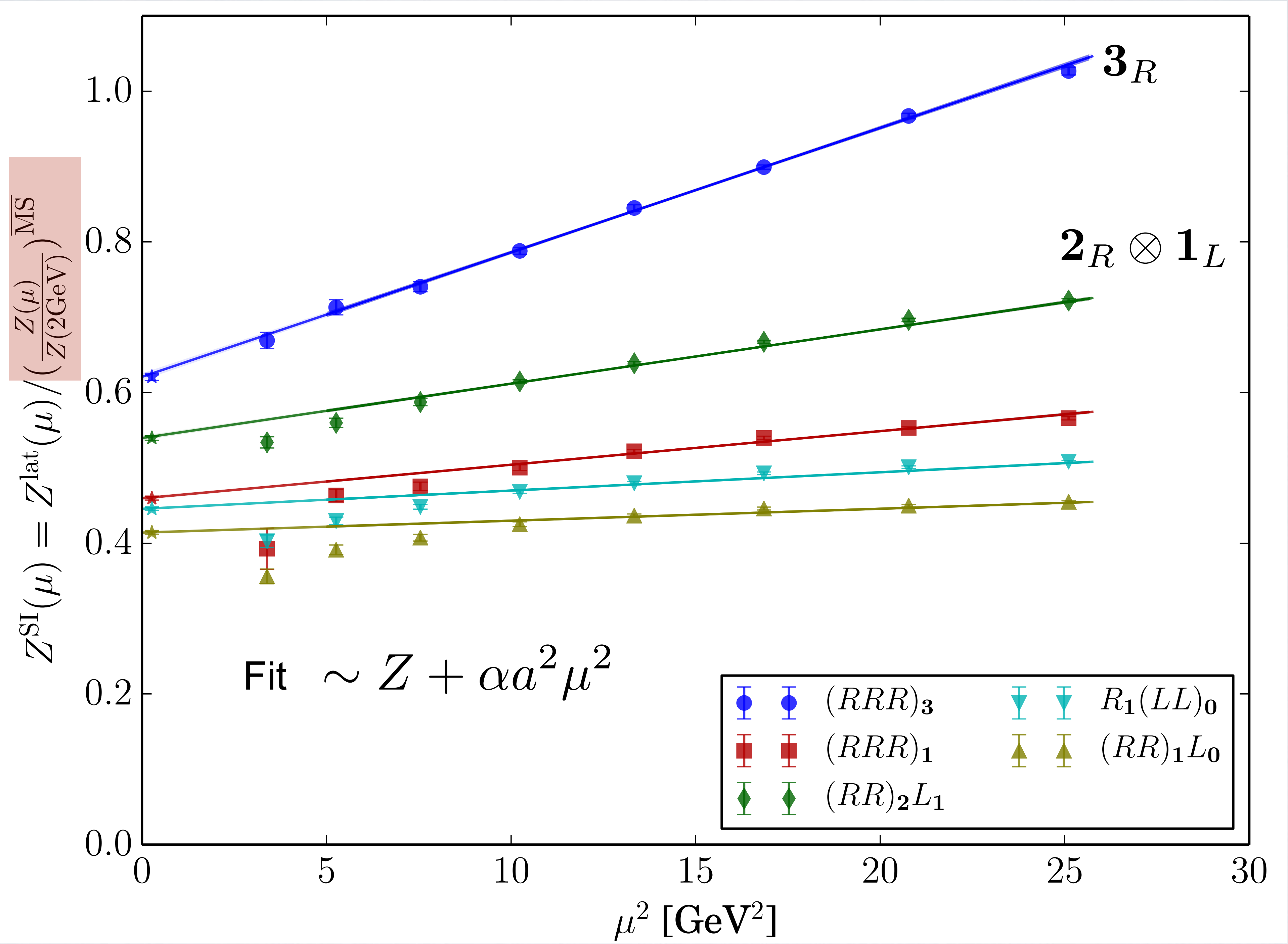
Variance between 2 - 4 GeV

Variance between 4 - 6 GeV

Non-perturbative Renormalization

Credit: Sergey Syritsyn

Perturbative 1-loop



Props from RBC/UKQCD HQ BBar Project

Systematic error estimate:

Variance between 2 - 4 GeV

Variance between 4 - 6 GeV

Results

Credit: Sergey Syritsyn

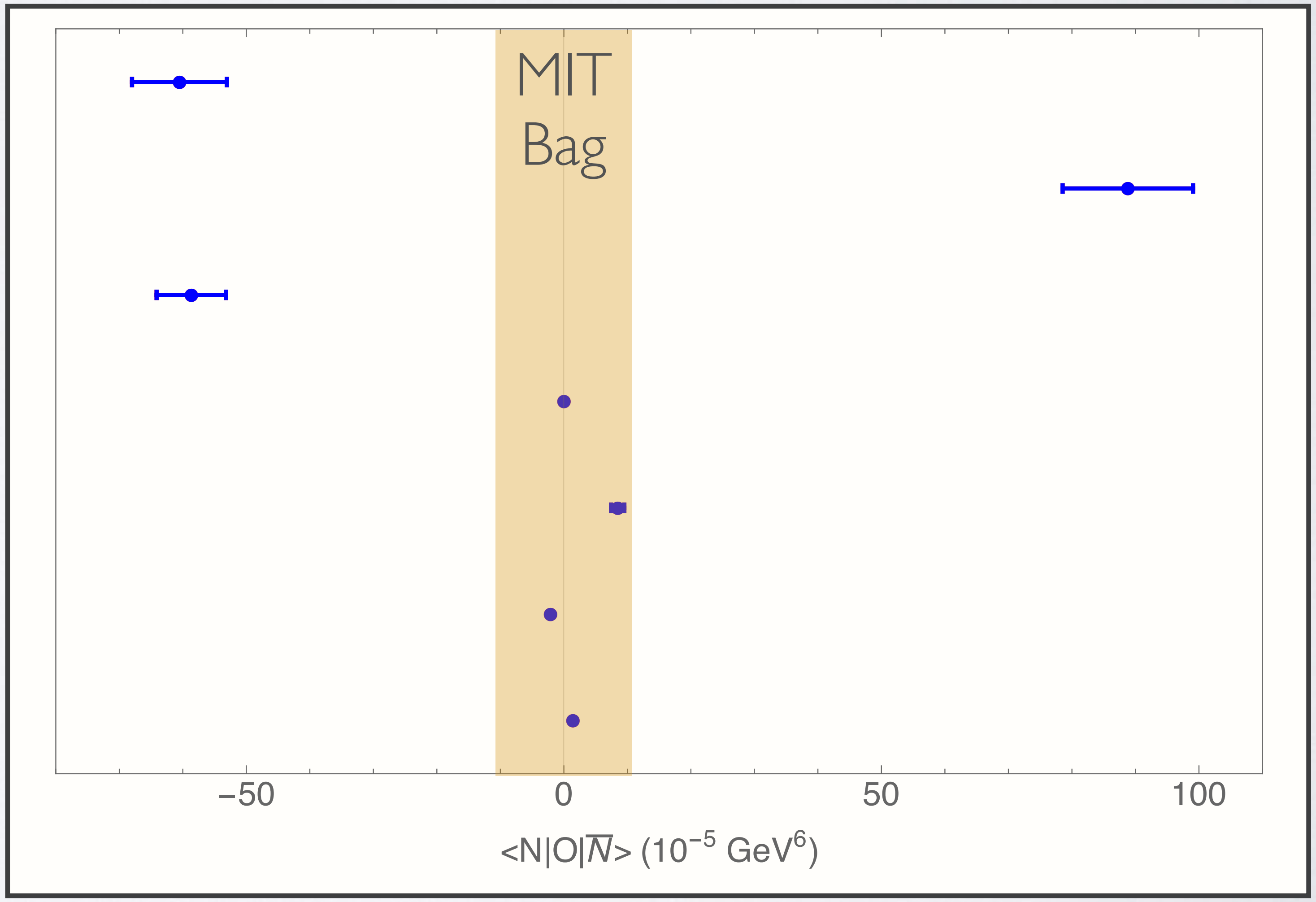
Chiral	Fixed-Flavor (equiv. $L \leftrightarrow R$)	Lattice ($\mu = 2$ GeV) $\times 10^{-5}$ GeV	Bag Model 1 $\times 10^{-5}$ GeV	Bag Model 2 $\times 10^{-5}$ GeV	$\frac{\text{LQCD}}{\text{Bag 2}}$
Q_1	$\mathcal{O}^3_{RRR}$	-60.5(7.5)	-10.92	-8.88	6.8
Q_2	$\mathcal{O}^3_{LRR}$	88.8 (10.2)	12.72	10.88	8.1
Q_3	$\mathcal{O}^3_{LLR}$	-58.7(5.4)	-9.64	-8.12	7.2
Q_4	$4/5\ \mathcal{O}^2_{RRR} + 1/5\ \mathcal{O}^1_{RRR}$	0	-	-	-
Q_5	$\mathcal{O}^1_{RLL}$	8.48 (1.04)	5.04	2.664	3.2
Q_6	$\mathcal{O}^2_{RLL}$	-2.12 (0.26)	1.256	-0.668	3.2
Q_7	$2/3\ \mathcal{O}^2_{LLR} + 1/3\ \mathcal{O}^1_{LLR}$	1.41 (0.17)	-0.84	0.44	3.2
$\widetilde{Q}_1$	$1/3\ \mathcal{O}^2_{RRR} - 1/3\ \mathcal{O}^1_{RRR}$	-60.5(7.5)	-10.92	-8.88	6.8
$\widetilde{Q}_3$	$1/3\ \mathcal{O}^2_{LLR} - 1/3\ \mathcal{O}^1_{LLR}$	-58.7(5.4)	12.72	-8.12	8.1

Obey SM Gauge

No Contribution

Break SM Gauge

Differ in Pert. running



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Phenomenological Example

$$\frac{1}{\tau_{n\bar{n}}} = \delta m = \langle \bar{n} | \mathcal{H}^{n\bar{n}}_{eff} n | \rangle$$

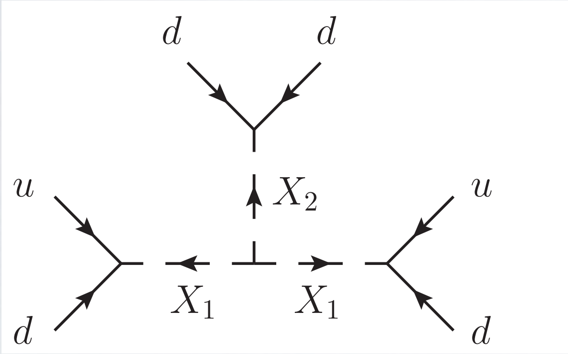
Arnold, Fornal, and Wise (2012):
(see Bartosz's talk last week)

Scalars

$$X_1 \in (\bar{6}, 1, -1/3)$$
$$X_2 \in (\bar{6}, 1, 2/3)$$

Interactions

$$g_1 X_1 Q_L Q_L$$
$$g_2 X_2 d_R d_R$$
$$g_1' X_1 u_R d_R$$
$$\lambda X_1 X_1 X_2$$



$$\mathcal{H}^{n\bar{n}}_{eff} = -\frac{(g_1'^{11})^2 g_2^{11} \lambda}{4M_1^4 M_2^2} \mathcal{O}^2_{RRR} = \frac{(g_1'^{11})^2 g_2^{11} \lambda}{16 M_1^4 M_2^2} \left[Q_4 + \frac{3}{5} \tilde{Q}_1 \right]$$

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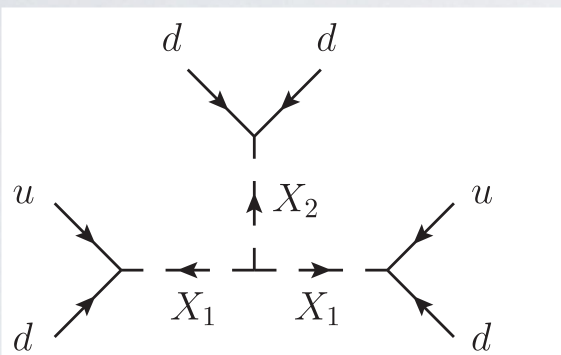

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T^{SSS} T^{AAS} T^{ASA} T^{SAA} T^{AAA}

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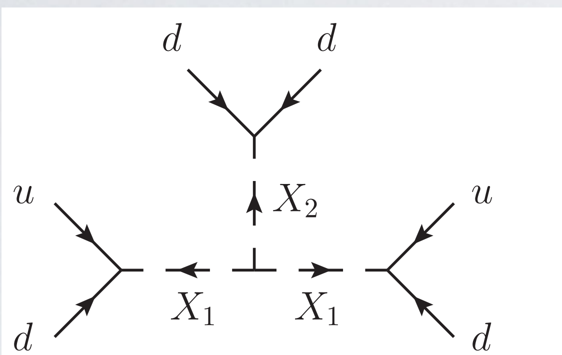

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Phenomenological Example

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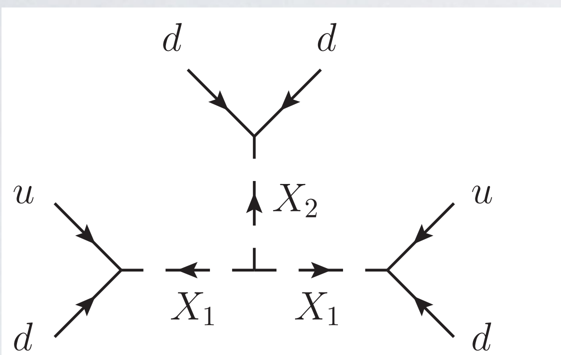

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$$|\delta m| = (\underbrace{7.26}_{\text{LO}} \quad \underbrace{-2.63}_{\text{NLO}} \quad \underbrace{-0.35}_{\text{NNLO matching}} \quad \underbrace{-0.92}_{\text{NNLO running}} \quad \underbrace{\pm 0.41}_{\text{Lattice Statistical}}) \times 10^{-34} \text{ GeV}$$
$$= (3.37 \pm 0.41) \times 10^{-34} \text{ GeV}$$

Wait until paper before quoting! Final round of checks are currently underway!

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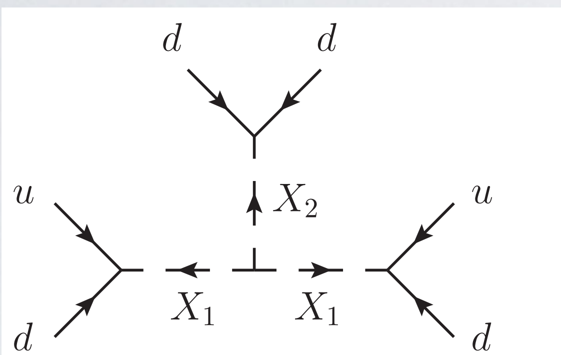

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$$|\delta m| < 2 \times 10^{-33}\ \mathrm{GeV}$$

$$M > (\underbrace{408}_{\rm LO} \ \underbrace{-34}_{\rm NLO} \ \underbrace{-6}_{\rm NNLO\ matching} \ \underbrace{-17}_{\rm NNLO\ running} \ \underbrace{\pm 9}_{\rm Lattice\ Statistical}) \ \mathrm{TeV},$$

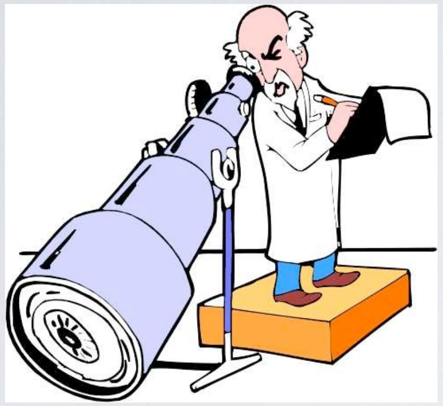
$$M > 350 \pm 9\ \mathrm{TeV}$$

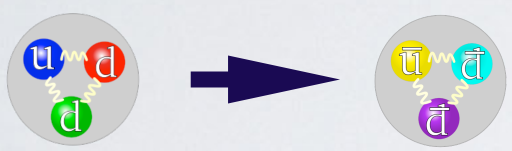
Wait until paper before quoting! Final round of checks are currently underway!

Final Roundup

Final Roundup

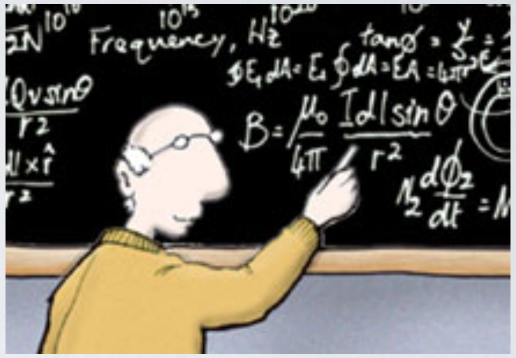
Experiment

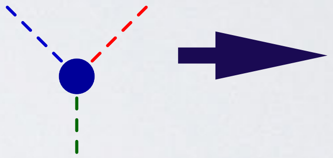




Neutron Antineutron

Theory

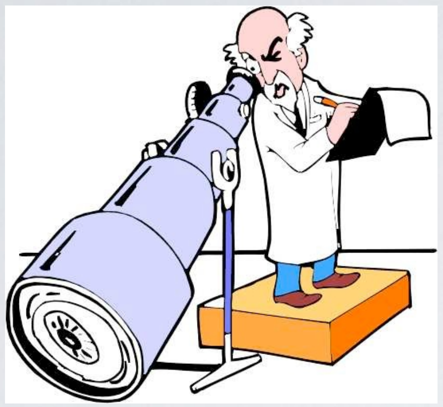




New Physics Universe

Final Roundup

Experiment



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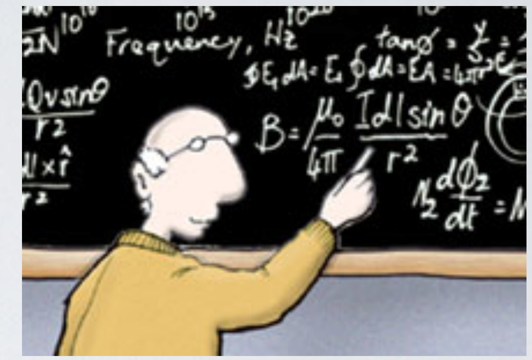
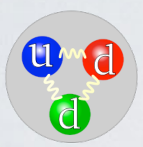
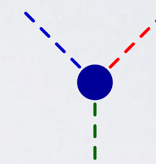




Neutron

Antineutron

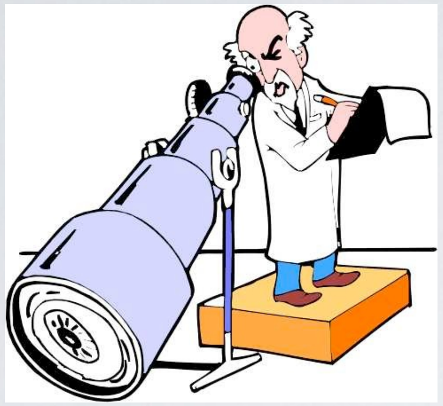


New Physics

Universe

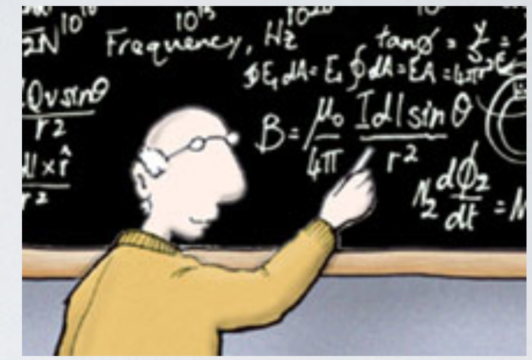
Final Roundup

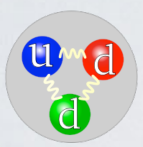
Experiment



Possible factor of 100 ambiguity

Theory





Neutron

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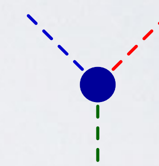
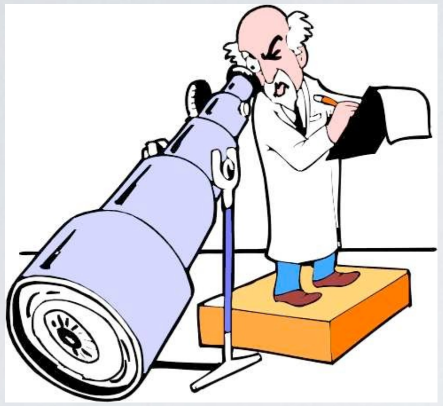


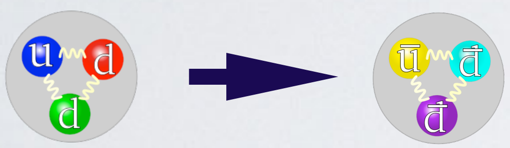
New Physics

Universe

Final Roundup

Experiment





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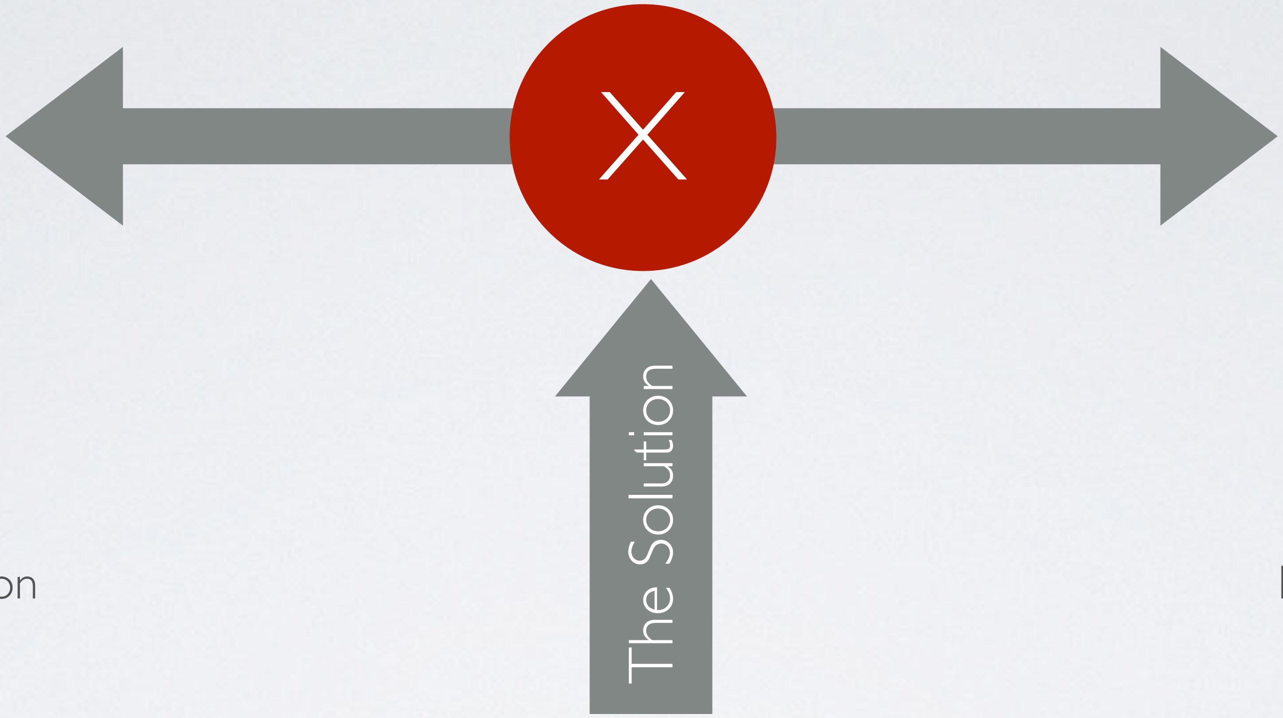
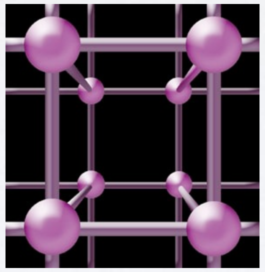
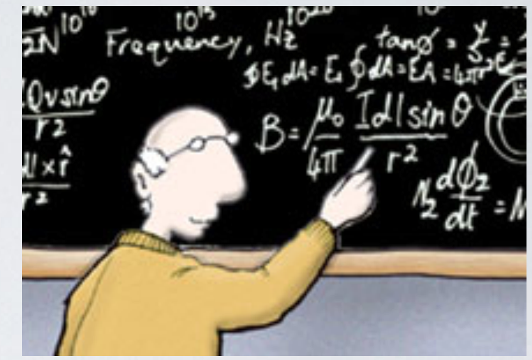
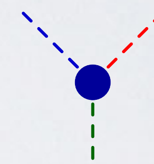




Lattice

Theory

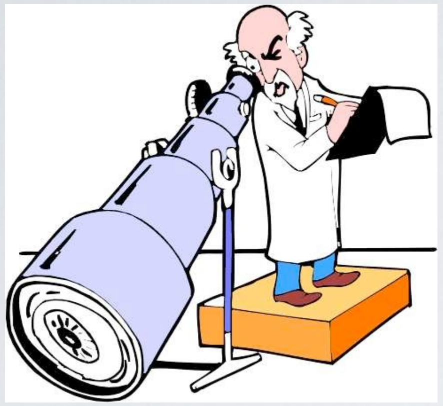




New Physics Universe

Final Roundup

Experiment



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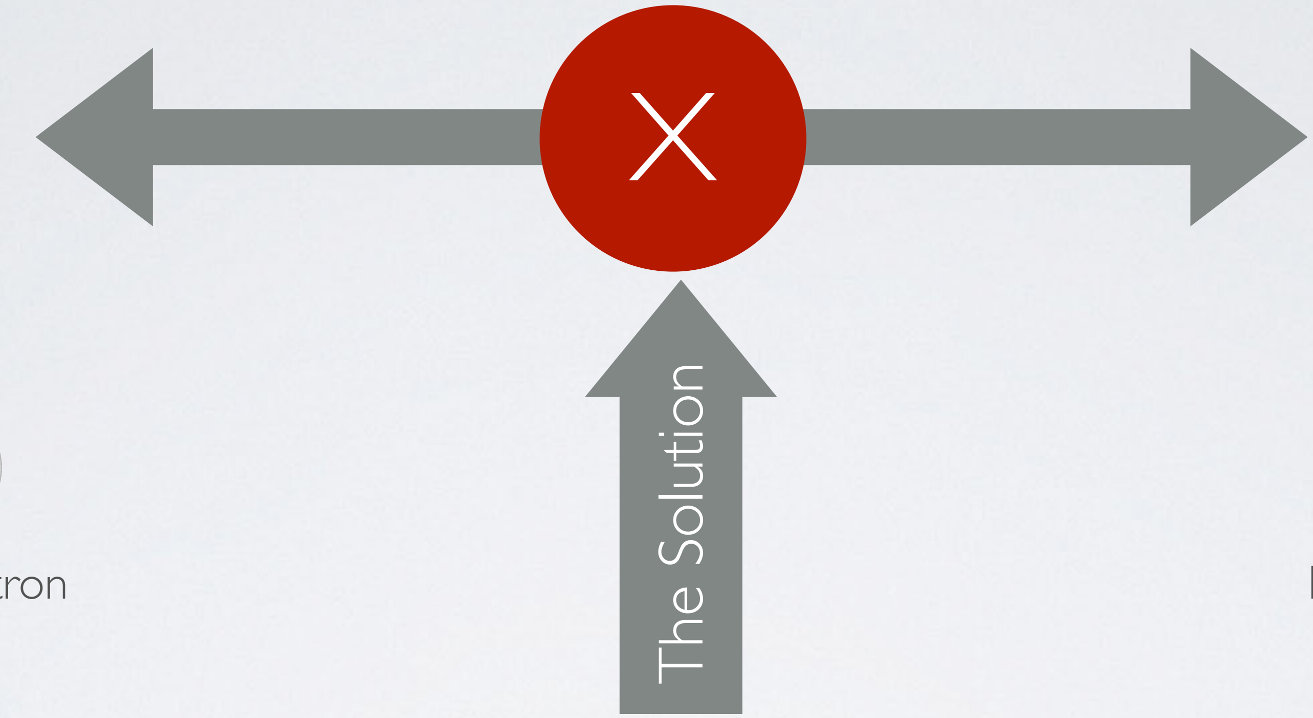

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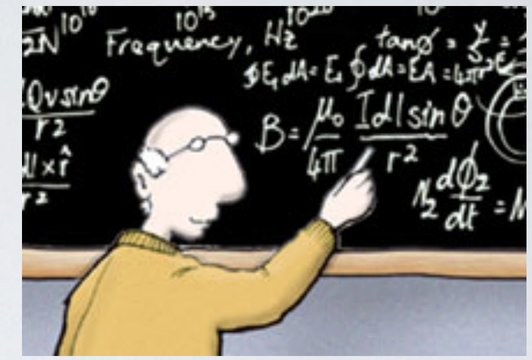

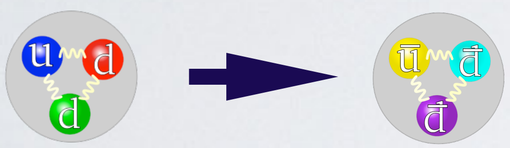

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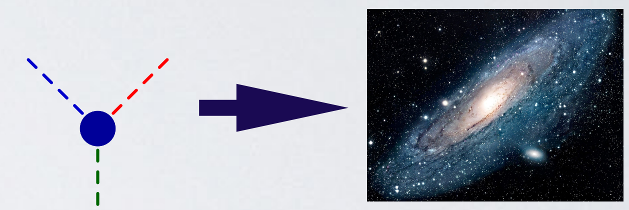
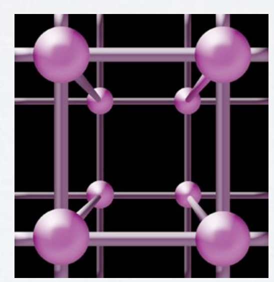


New Physics

Universe



Lattice

Results: (Lattice vs. Models)

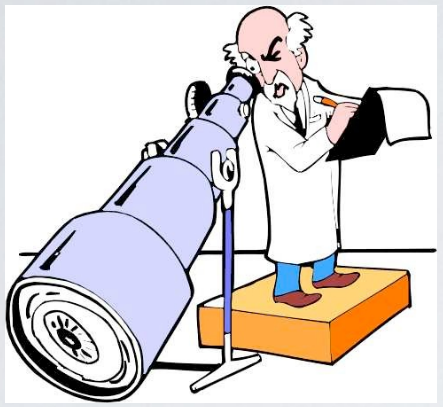
Final round of checks are currently underway!

Total Uncertainty within 15%

Operators 2 - 8 times larger than MIT bag model predictions

Final Roundup

Experiment



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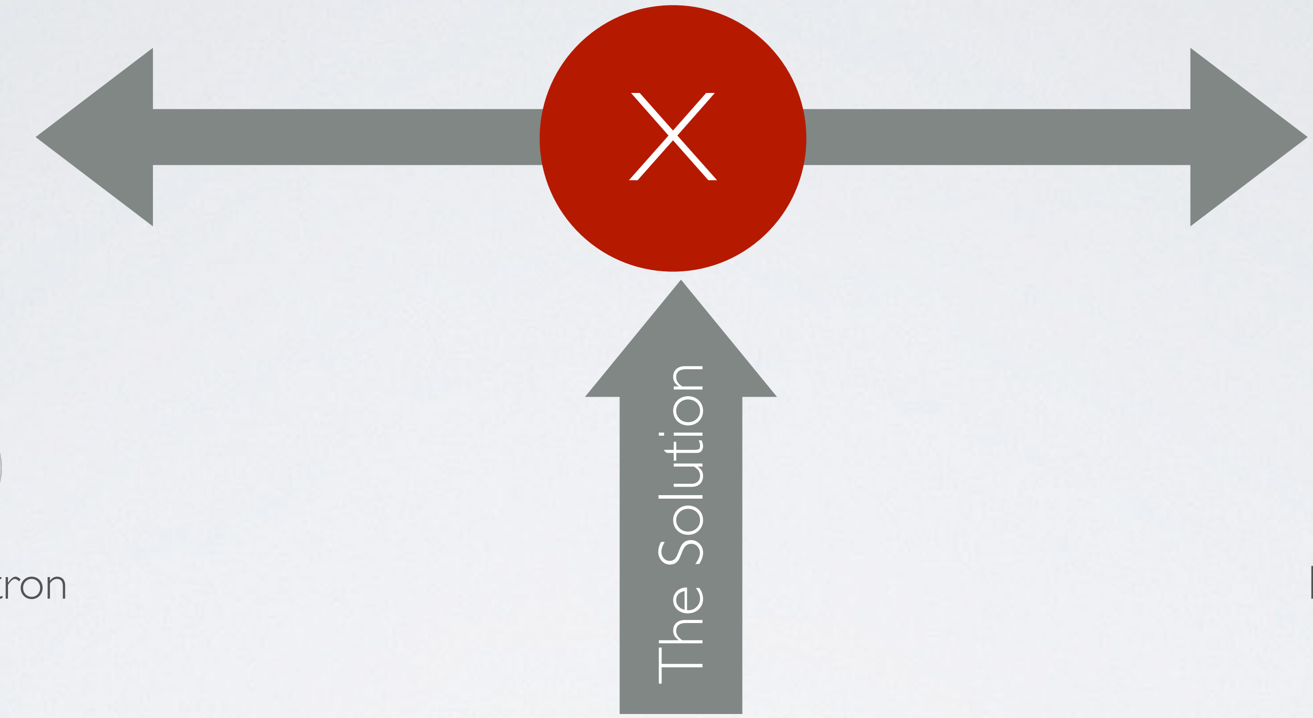

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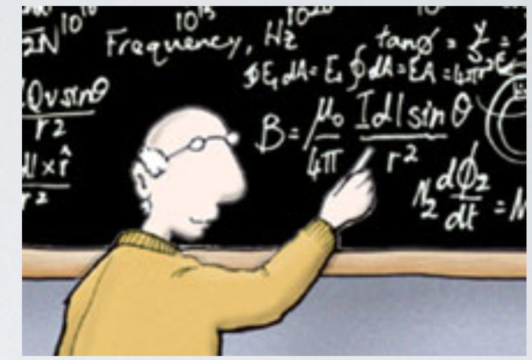

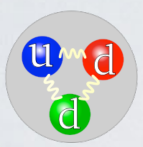

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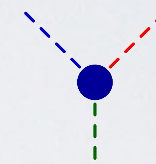

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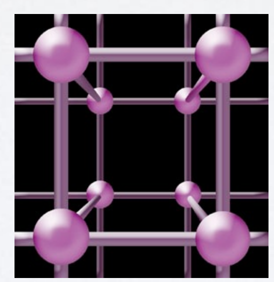


Lattice

LONG TERM:

More lattices and statistics

Volume & lattice spacing effects

Backup

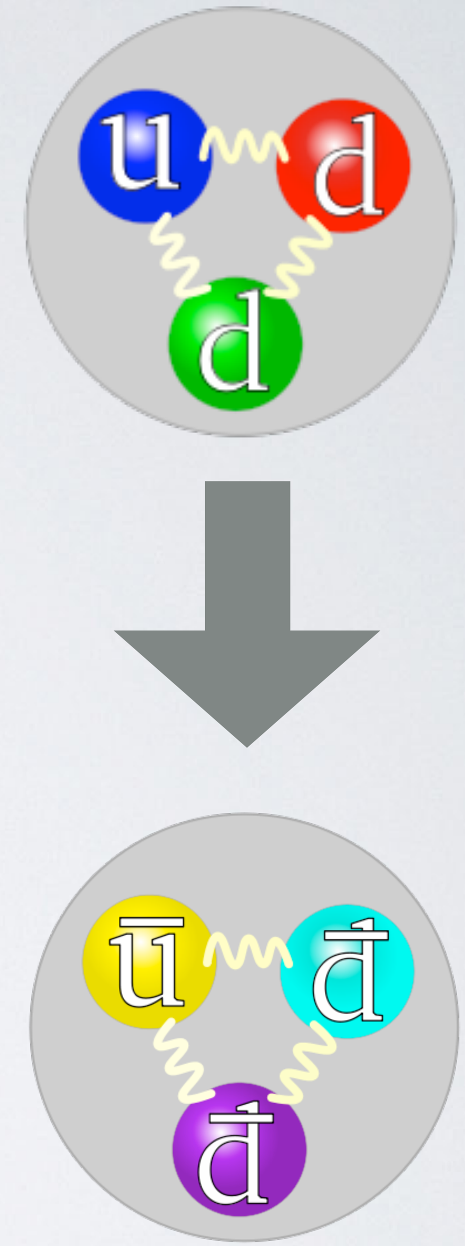
Would you believe...

...That a neutron could switch to an antineutron at any time?

...That this process is predicted for various classes of new physics?

...That by observing this process (or bounding it), we could address questions about how the matter filled universe exists?

Neutron



Antineutron

What is needed for baryon asymmetry?

Sakharov conditions for baryon creation:

1. Interactions that violate Baryon Number
2. Interactions that violate charge conj. and charge conj. x parity symmetries
3. Interactions outside thermal equilibrium

$$L = 0 \quad B = +1 \quad qqq$$

P N

$$L = 0 \quad B = -1 \quad \bar{q}\bar{q}\bar{q}$$

$\overline{P}$ $\overline{N}$

$$L = 0 \quad B = 0 \quad \bar{q}q$$

π K η

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3. Interactions outside thermal equilibrium

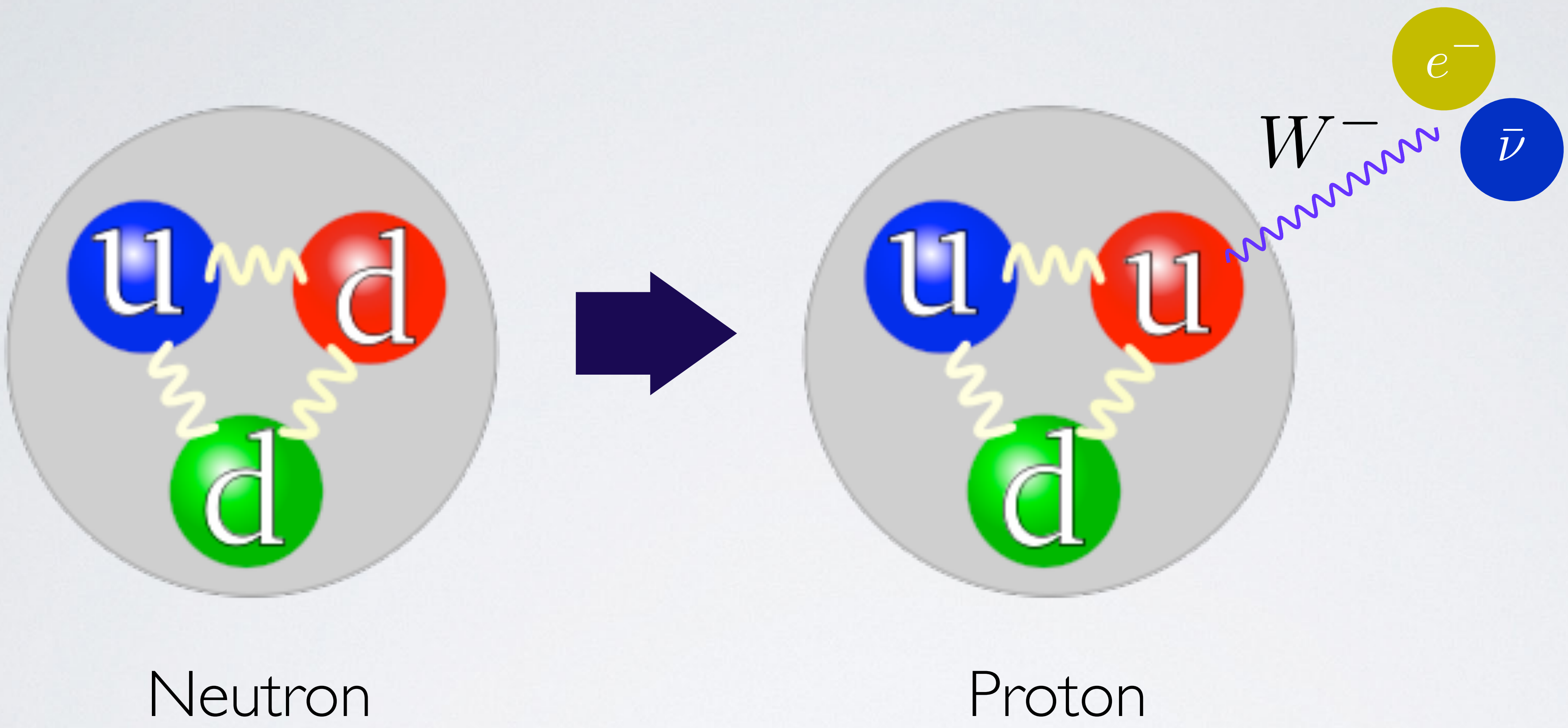
$$L = +1 \qquad\qquad L = -1$$
$$B = 0 \qquad\qquad B = 0$$

e^- μ^- τ^- $\qquad$ e^+ μ^+ τ^+

ν $\qquad$ $\bar{\nu}$

Start with a more familiar picture

We know unbound neutrons decay (beta decay)



Roughly 15 minute lifetime

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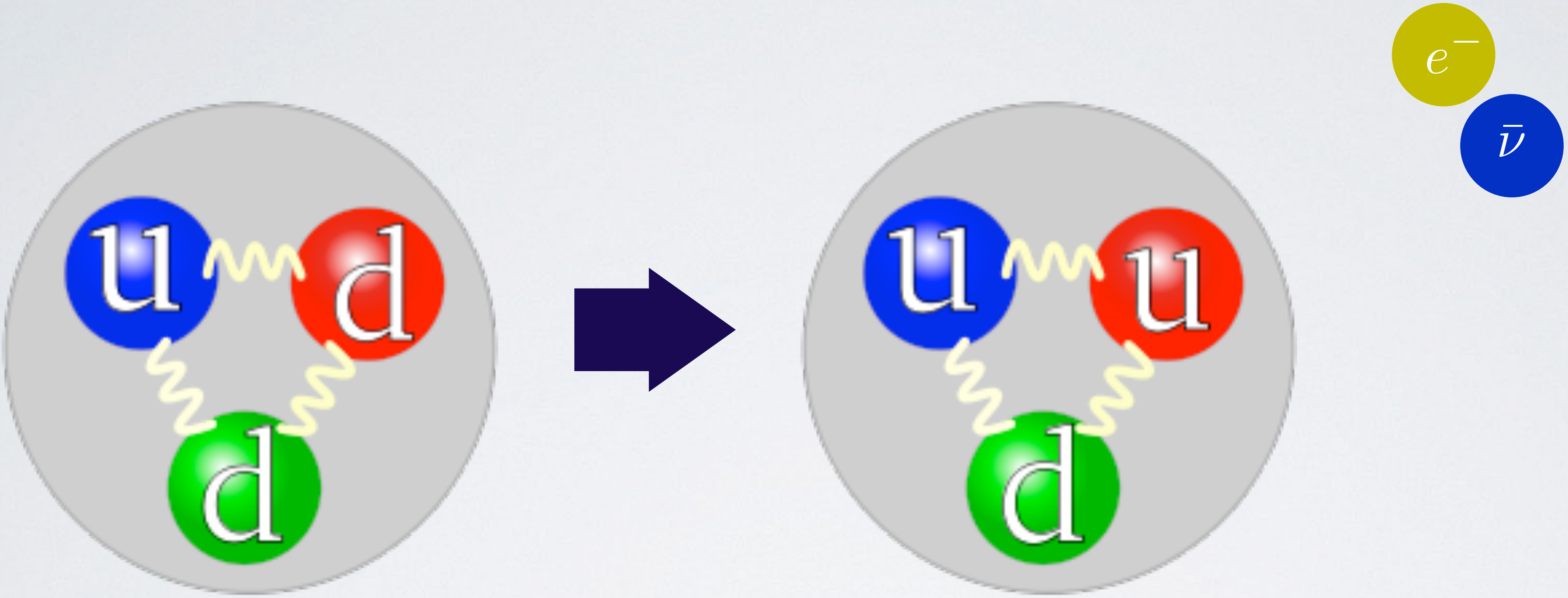

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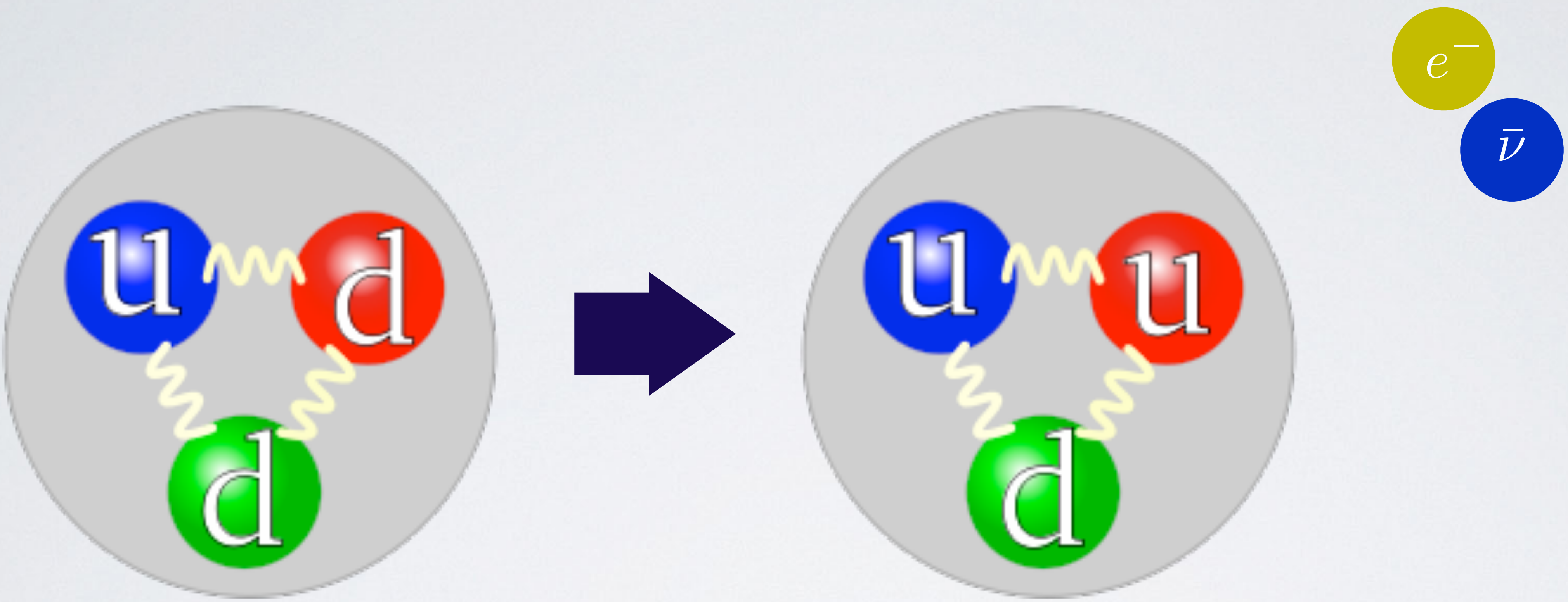


Neutron

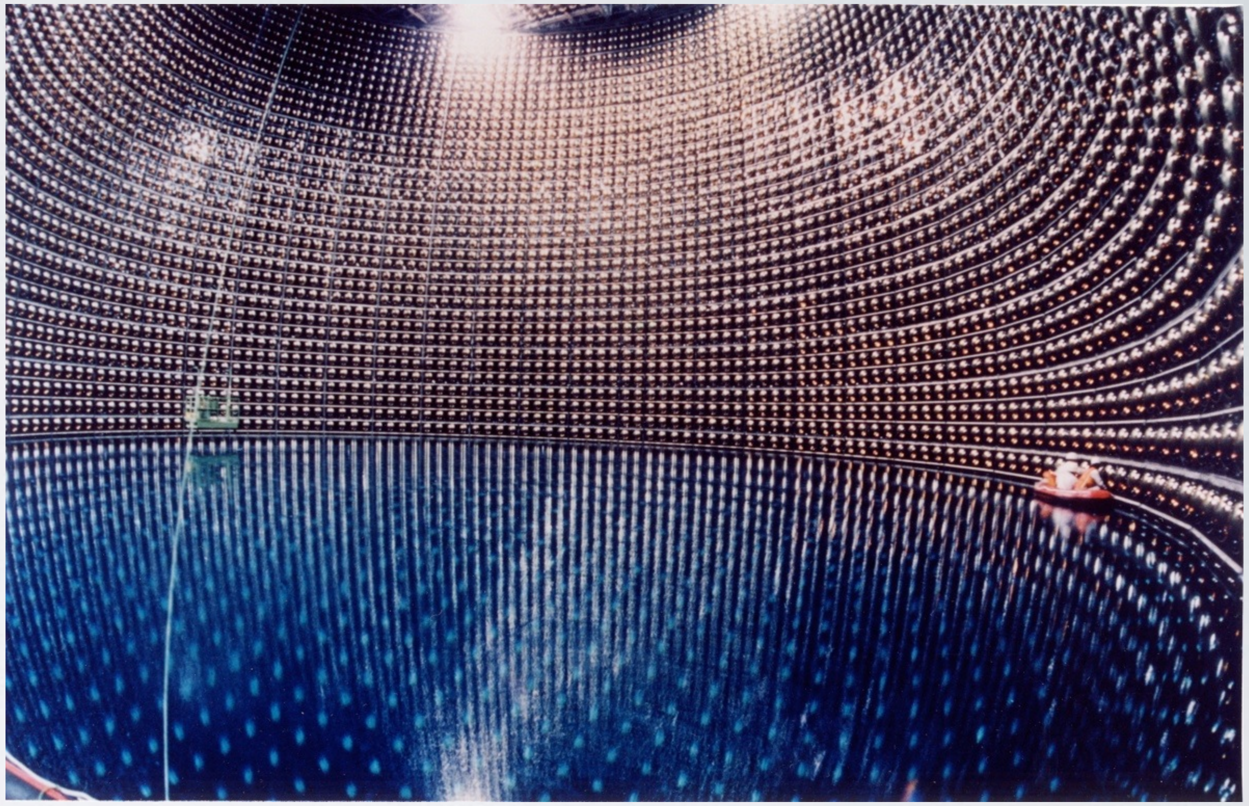
Proton

Tells us about weak physics

Roughly 15 minute lifetime

Background on proton decay

Latest bounds from water Cherenkov detectors



Super-K: $\tau > 8.3 \times 10^{33}$ years

50,000 tons of ultra-pure water

-Probes B-violation from new physics

$$|\Delta B| = 1$$

$$M_{NP} \sim 10^{12} - 10^{13} \text{ TeV}$$

-In particular, probes B-L conserving processes at these scales

Effective interaction
(new physics at low energy)

$$\mathcal{L}_{\not{B}} \sim \frac{1}{M_{NP}^2} QQQL$$

Back to the big question...

Namely, what is the overall scale?

Reminder: $\langle \bar{n} | \mathcal{O} | n \rangle \sim \frac{1}{r_p^6}$

Unfortunately, requires additional work to extract reliably

Analytically - Two loop QCD renormalization, EFT calculations

Numerically - Full non-perturbative renormalization

Renormalization: Crude Estimate

$$\mathcal{O}^{\overline{MS}}(\mu) = U^{\overline{MS}}(\mu, p_0)\frac{Z^{\overline{MS}}(p_0)}{Z^{MOM}_{\rm cont}(p_0)} Z^{MOM}_{\rm latt}(p_0)\mathcal{O}^{\rm bare}_{\rm latt}$$

TREE LEVEL:

$$Z^{MOM}_{\rm latt}(p_0) = 1 \qquad \frac{Z^{\overline{MS}}(p_0)}{Z^{MOM}_{\rm cont}(p_0)} = 1 \qquad U^{\overline{MS}}(\mu, p_0) = \left[\frac{\alpha_s(\mu)}{\alpha_s(p_0)}\right]^{\gamma_0/2\beta_0}$$

TADPOLE-IMPROVED TREE LEVEL:

$$Z^{MOM}_{\rm latt}(p_0) = Z_q^{-3} = u_0^3$$

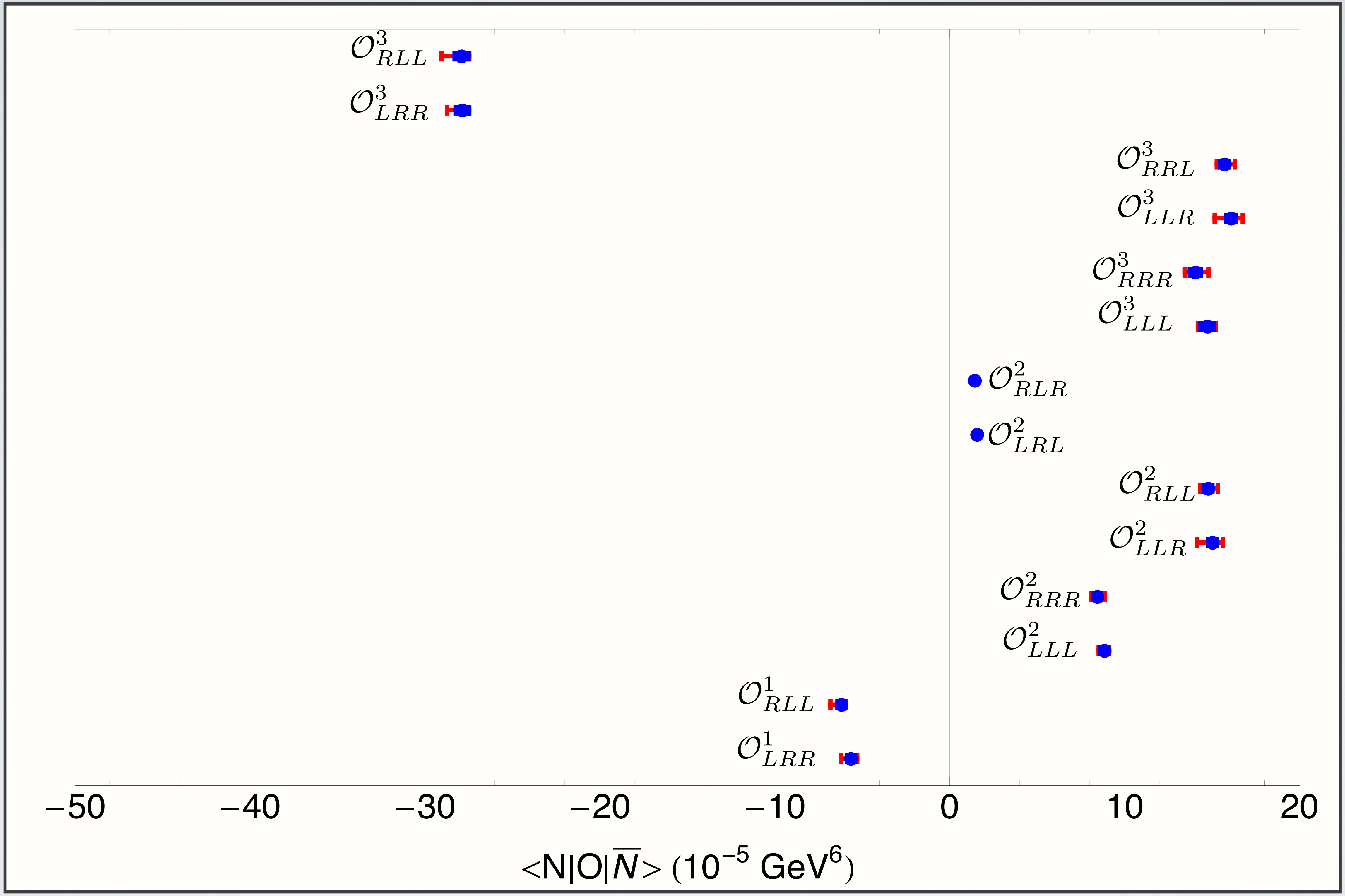
Lepage, Mackenzie (1992)

$$u_0 = \left[\frac{1}{3}\mathrm{Tr}\, U_{\rm Plaq}\right]^{1/4}$$

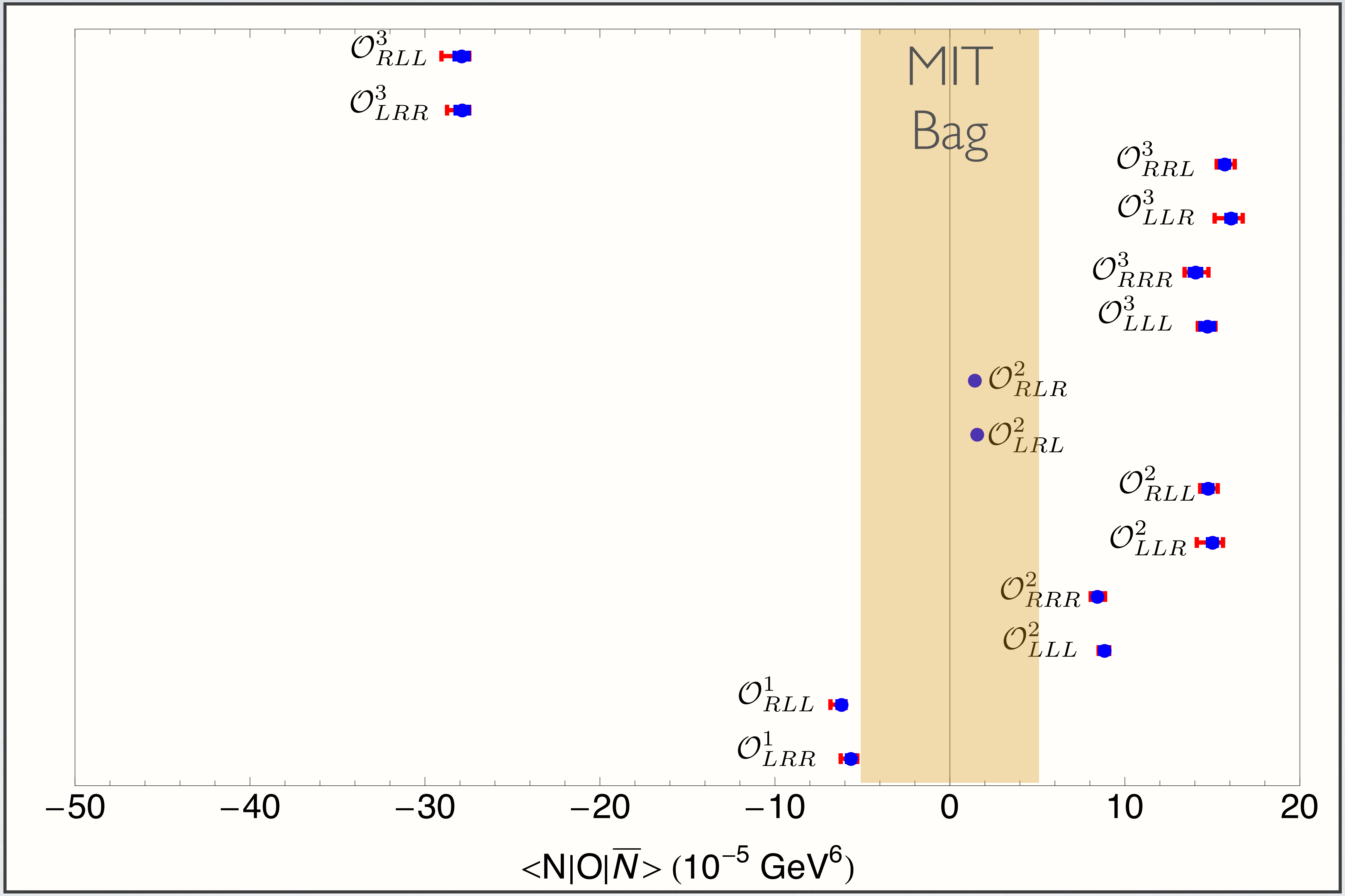
Closer to physical Expansion:

$$U_\mu(x) = e^{iaA_\mu(x)} \quad \longrightarrow \quad U'_\mu(x) = u_0^{-1} e^{iaA_\mu(x)}$$

Preliminary Results

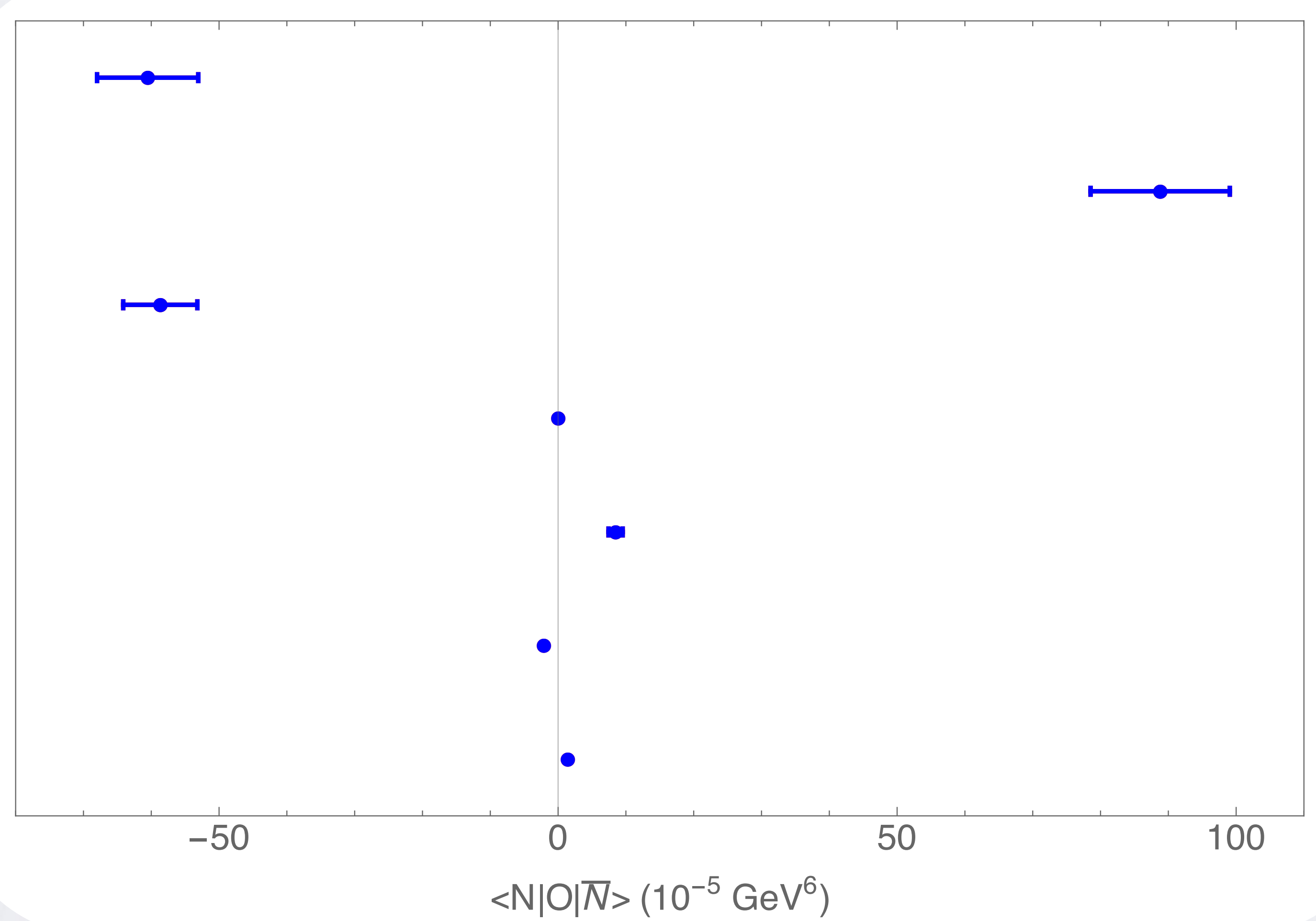


Preliminary Results



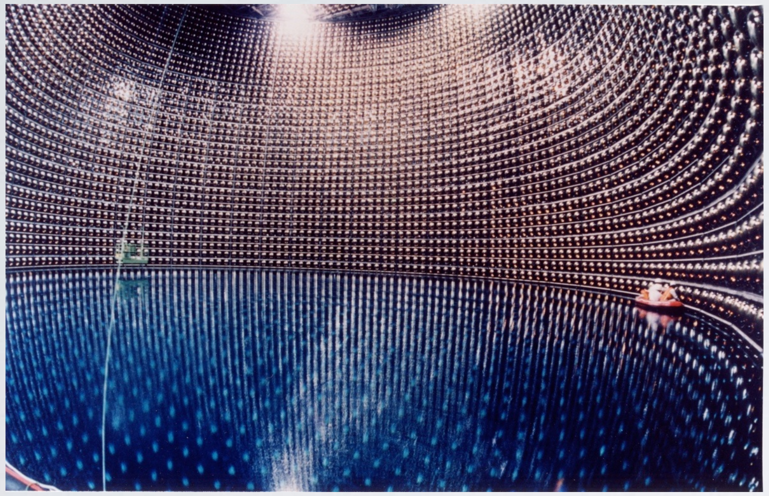
Final Results

	$Z(\text{lat} \to \overline{MS})$	$\mathcal{O}^{\overline{MS}}(2\text{ GeV})$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$
$[(RRR)_{\mathbf{3}}]$	0.62(12)	0	0	–	0	–
$[(RRR)_{\mathbf{1}}]$	0.454(33)	45.4(5.6)	8.190	5.5	6.660	6.8
$[R_{\mathbf{1}}(LL)_{\mathbf{0}}]$	0.435(26)	44.0(4.1)	7.230	6.1	6.090	7.2
$[(RR)_{\mathbf{1}}L_{\mathbf{0}}]$	0.396(31)	-66.6(7.7)	-9.540	7.0	-8.160	8.1
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(1)}$	0.537(52)	-2.12(26)	1.260	-1.7	-0.666	3.2
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(2)}$	0.537(52)	0.531(64)	-0.314	-1.7	0.167	3.2
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(3)}$	0.537(52)	-1.06(13)	0.630	-1.7	-0.330	3.2



Experimental Progress

1. Neutron-antineutron annihilation in nuclei



H_2O

Super-K bounds (2011)

$$\tau_{n\bar{n}} > 11 \text{ years}$$

$$R = \frac{1.6 \times 10^{30}}{\text{year}}$$

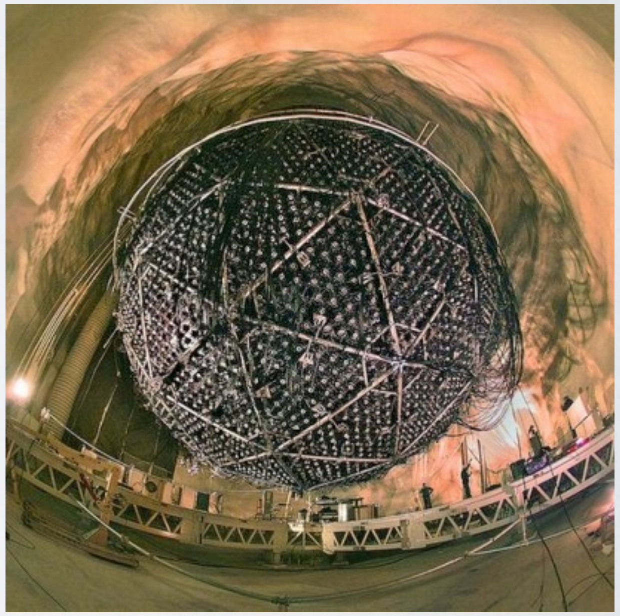
Friedman, Gal 2008

SNO Laboratory

1,100 tons of heavy water

Focus on deuterium → D_2O ← Not on oxygen

$$\tau_{n\bar{n}} > 5.7 \text{ years}$$ (Preliminary)



$R = (3.8 - 6.3) \times 10^{29}$ year^{-1} L. Kondratyuk (1996)

$R = (8.5 - 8.7) \times 10^{29}$ year^{-1} C. Dover, A. Gal, J. Richard (1982)

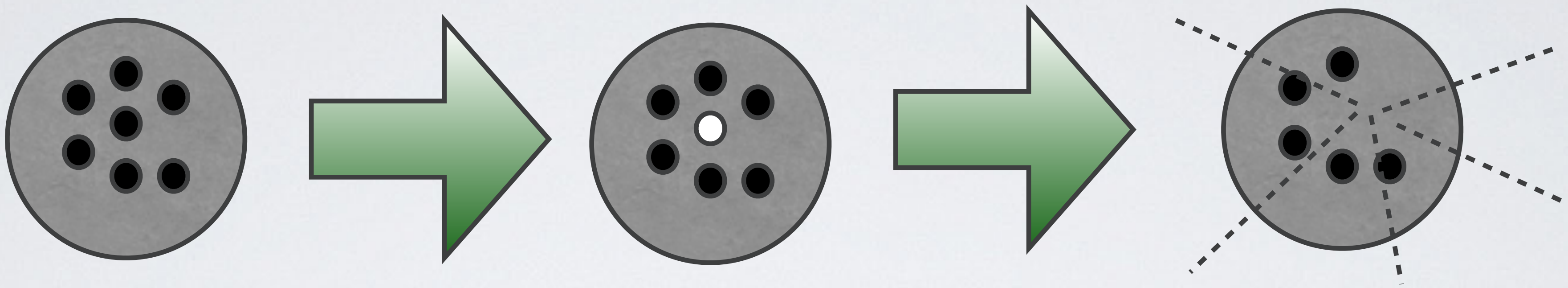
$R = 9.27 \times 10^{29}$ year^{-1} V. Kopeliovich and I. Potashnikova (2011)

Bingwei Long (Ph. D Thesis, 2008):

$R = (3.75 \pm 0.64 \pm 0.38) \times 10^{29}$ year^{-1}

Nuclear Suppression

1. Neutron-antineutron annihilation in nuclei



Straight-forward question:
Why have we not annihilated yet?

$$H = \begin{pmatrix} E_n & \delta m \\ \delta m & E_{\bar{n}} \end{pmatrix} = \begin{pmatrix} m_n + V_{nR} & \delta m \\ \delta m & m_n + V_{\bar{n}R} - iV_{\bar{n}I} \end{pmatrix}$$

$V \sim \mathcal{O}(100\ \mathrm{MeV})$

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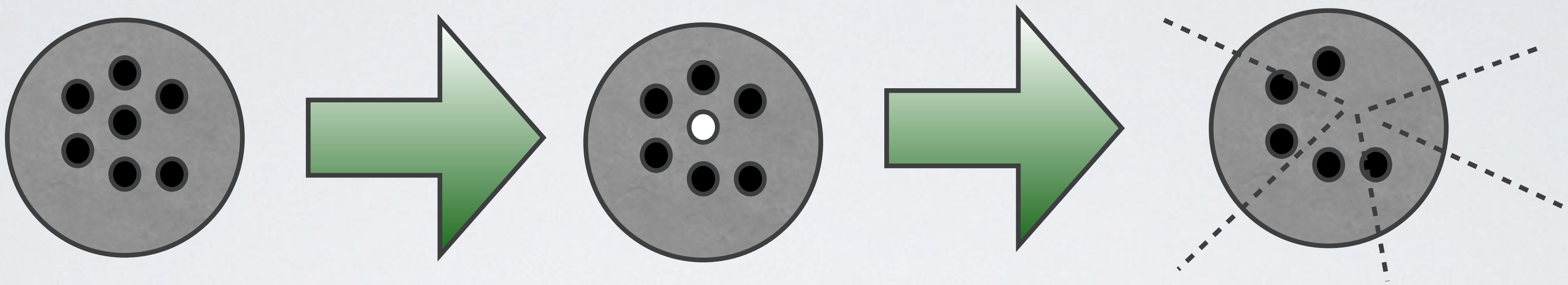

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$$P_{n \to \bar{n}}(t) \sim \sin^2[t/\tau_{Nucl}] \qquad \tau_{Nucl} \approx \frac{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}{2|V_{\bar{n}I}|(\delta m)^2}$$

$$\tau_{Nucl} \approx R\, \tau_{\bar{n}n}^2$$

Nuclear Suppression

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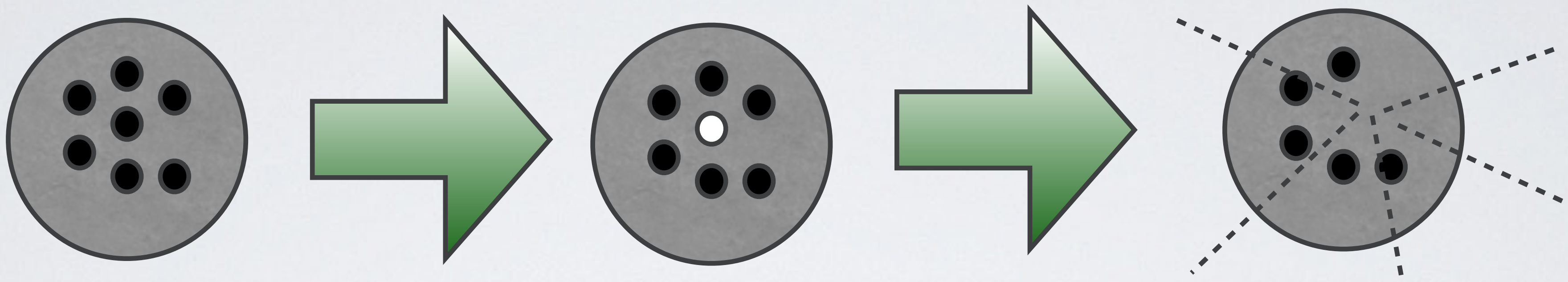

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What is this?

Magnetic Field Limit

2. Free, Cold neutron annihilation with target

- Hamiltonian: $$H = \begin{pmatrix} m_n - \vec{\mu} \cdot \vec{B} - i\lambda/2 & \delta m \\ \delta m & m_n + \vec{\mu} \cdot \vec{B} - i\lambda/2 \end{pmatrix}$$

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- Diagonalizing: $|n_1\rangle = \cos\theta|n\rangle + \sin\theta|\bar{n}\rangle \qquad |n_2\rangle = -\sin\theta|n\rangle + \cos\theta|\bar{n}\rangle$

$$\tan(2\theta) = -\frac{\delta m}{\vec{\mu}\cdot\vec{B}}$$

$$m_{1,2} = m_n \pm \sqrt{(\vec{\mu}\cdot\vec{B})^2 + (\delta m)^2} - i\lambda/2$$

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- When: $|\delta m|t \ll |\vec{\mu}\cdot\vec{B}|t \ll \lambda t \ll 1$ $\qquad t \sim 0.1\ \text{sec}$

$$P_{n\to\bar{n}}(t) \approx (2\theta)^2\left(\frac{(m_1 - m_2)t}{2}\right)^2 \approx \left(\frac{\delta m}{\vec{\mu}\cdot\vec{B}}\right)^2(\vec{\mu}\cdot\vec{B}\ t)^2 = (\delta m\ t)^2$$

Preliminary Taste

Rao, Shrock (1982)

	Lattice (bare)	MIT Bag (1)	MIT Bag (2)
$\frac{\langle \bar{n} \vert \mathcal{O}^3_{LRR} \vert n \rangle}{\langle \bar{n} \vert \mathcal{O}^3_{LRR} \vert n \rangle}$	1	1	1
$\frac{\langle \bar{n} \vert \mathcal{O}^3_{LLR} \vert n \rangle}{\langle \bar{n} \vert \mathcal{O}^3_{LRR} \vert n \rangle}$	$-0.576 \pm 0.012^{+0.014}_{-0.026}$	-0.758	-0.746
$\frac{\langle \bar{n} \vert \mathcal{O}^1_{RRL} \vert n \rangle}{\langle \bar{n} \vert \mathcal{O}^3_{LRR} \vert n \rangle}$	$0.222 \pm 0.009^{+0.001}_{-0.015}$	-0.858	0.245
$\frac{\langle \bar{n} \vert \mathcal{O}^2_{RRR} \vert n \rangle}{\langle \bar{n} \vert \mathcal{O}^3_{LRR} \vert n \rangle}$	$-0.302 \pm 0.008^{+0.009}_{-0.008}$	-0.516	-0.489

MIT Bag Model - Substitute QCD with quarks in sphere