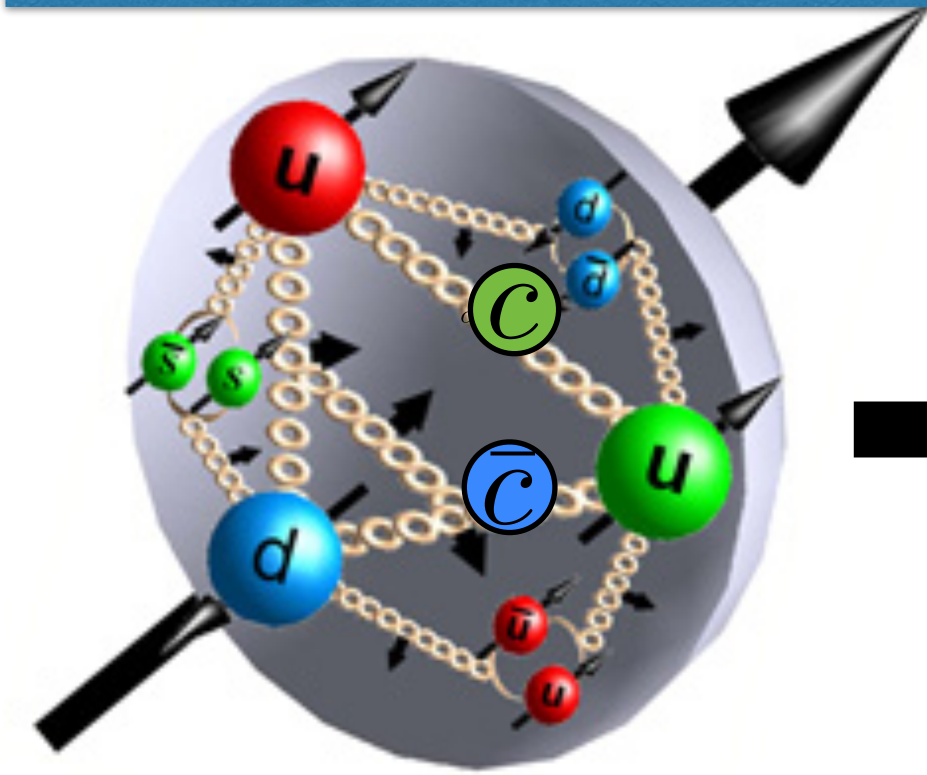
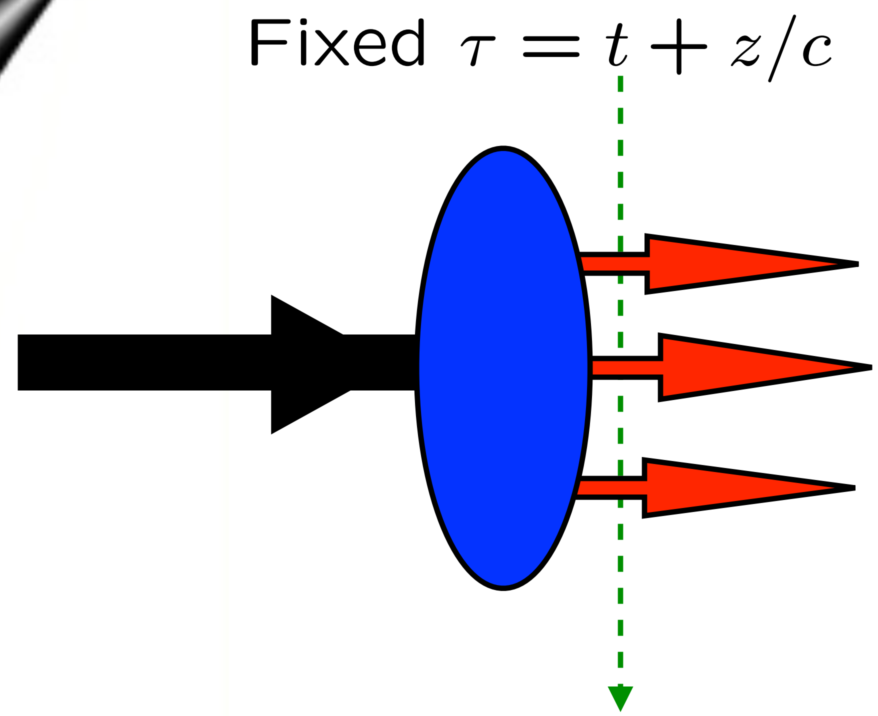
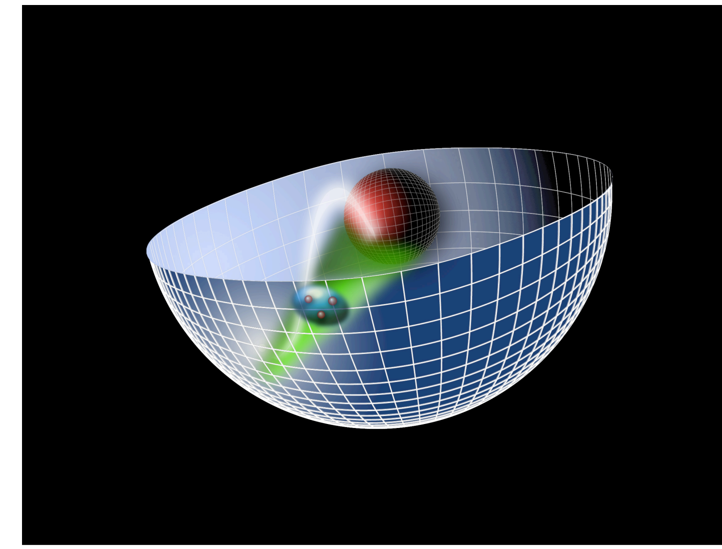
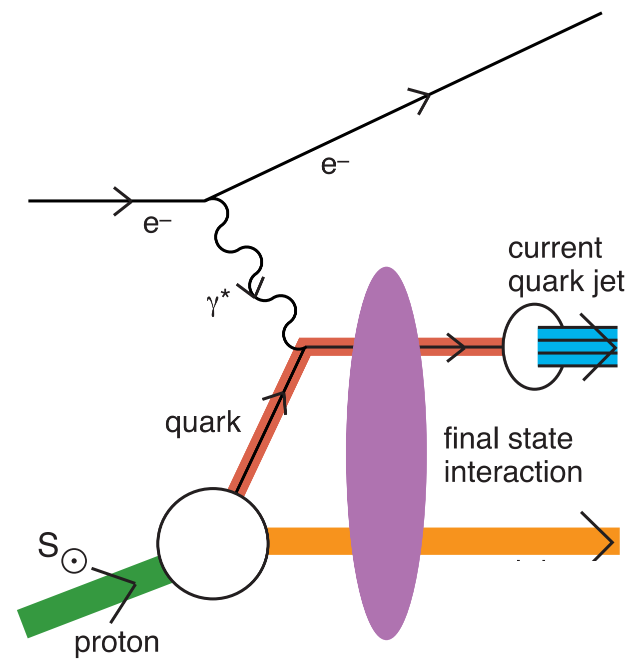
Breakdown of Factorization, Sum Rules, and Insights for QCD from Light-Front Holography









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INSTITUTE FOR NUCLEAR THEORY

Intersections of BSM Phenomenology and QCD for New Physics Searches (INT-15-3)

October 15, 2015, INT, University of Washington

QCD Myths

- ISI and FSI are higher twist effects - only a phase
- Momentum and Spin Sum Rules valid for nuclei - in fact not proven!
- Anti-Shadowing is Universal -
  In fact, anti-shadowing is Flavor Dependent!
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks arise only from gluon splitting — Intrinsic Strange, Charm, and Bottom
- Renormalization scale cannot be fixed — PMC
- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant

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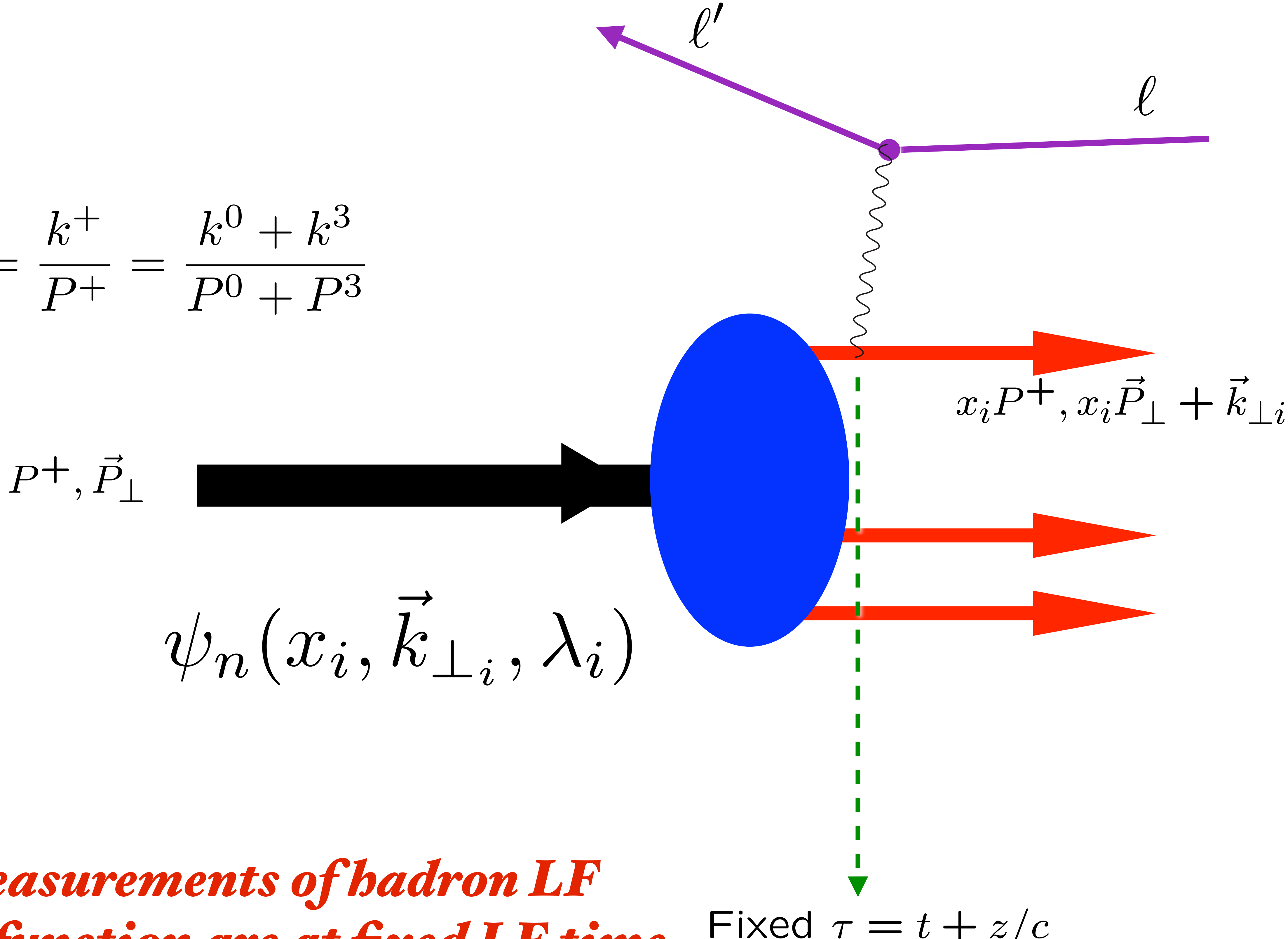
$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

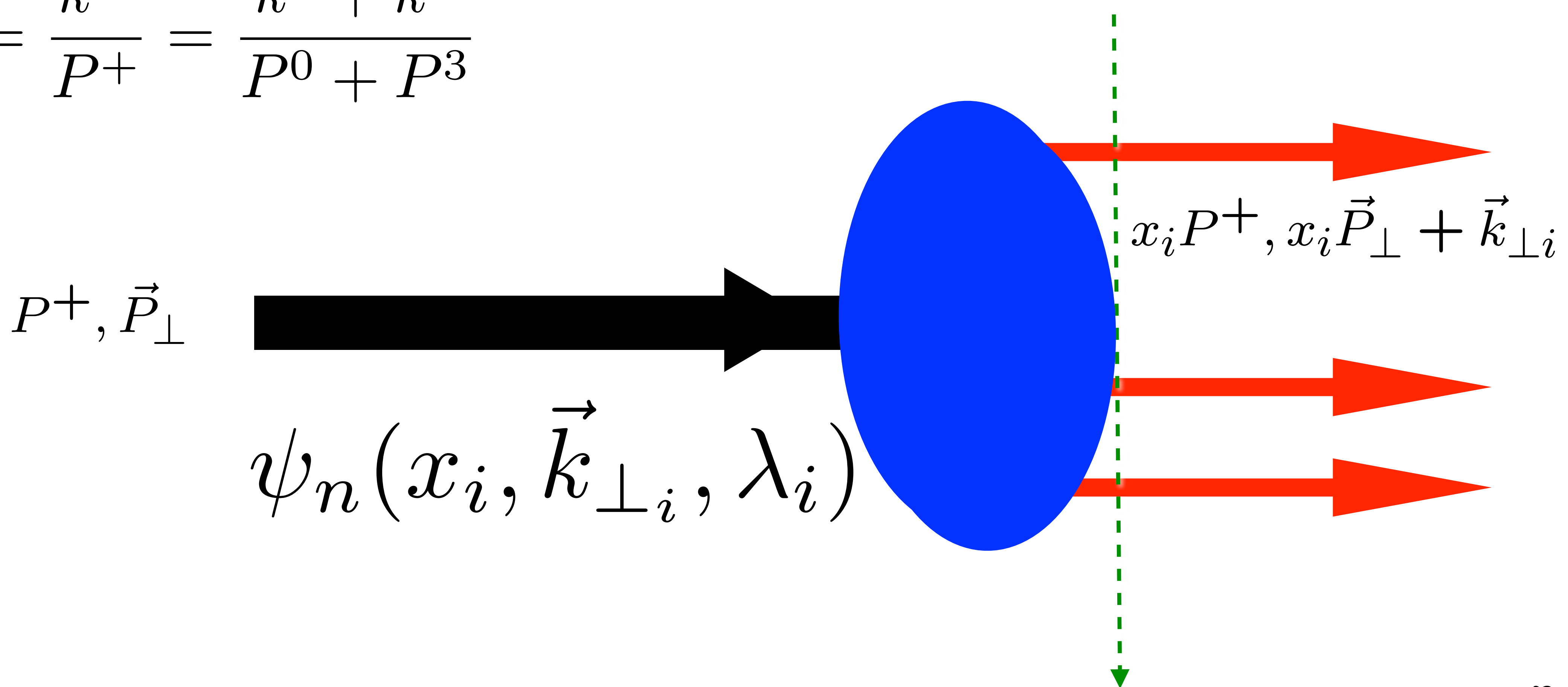
H E L E N   B R A D L E Y   -   P H O T O G R A P H Y

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

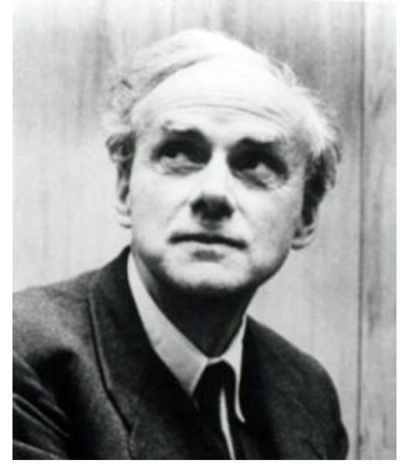
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Dirac's Amazing Idea:
The "Front Form"

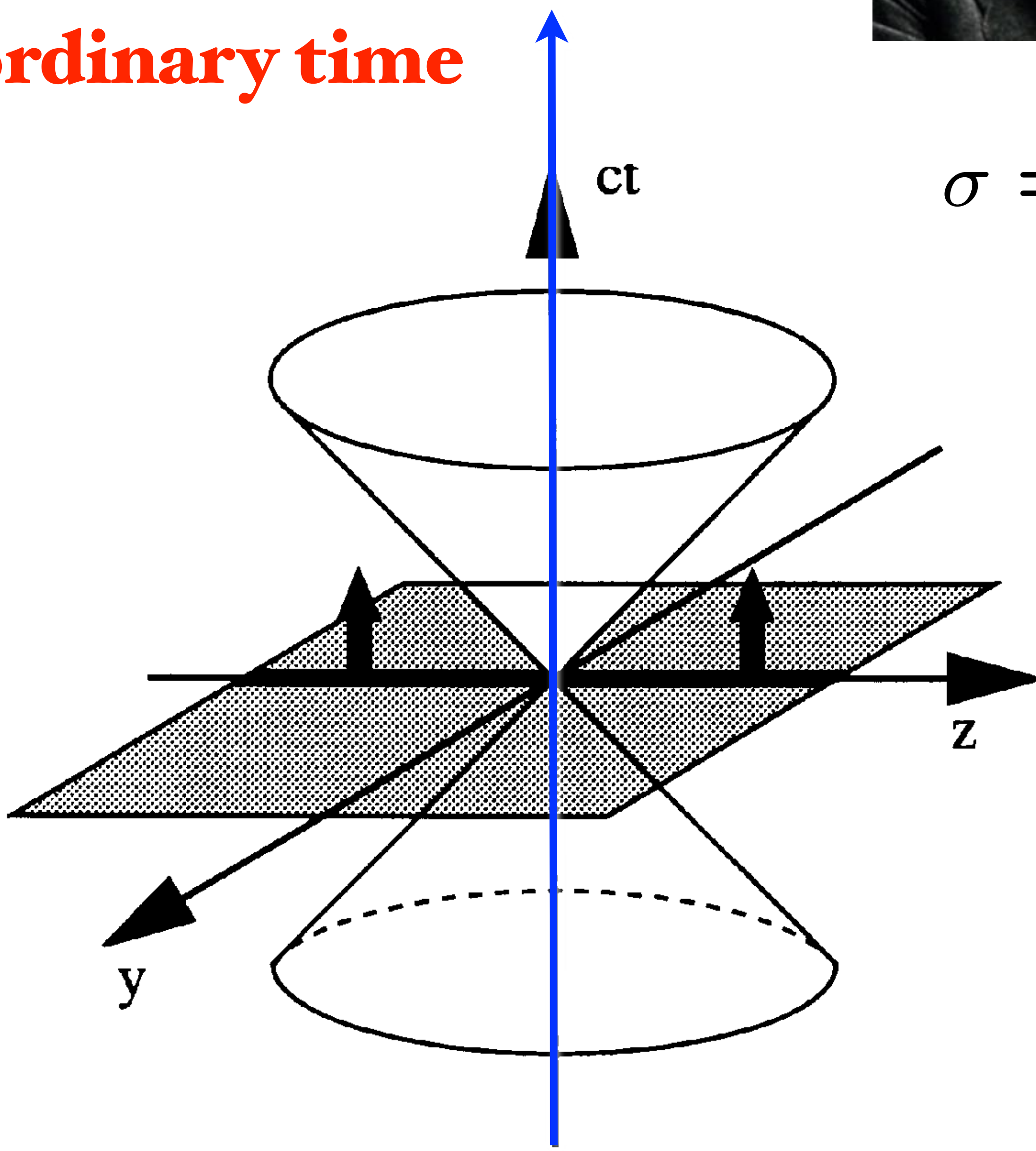


P.A.M Dirac,
Rev. Mod. Phys. 21, 392 (1949)

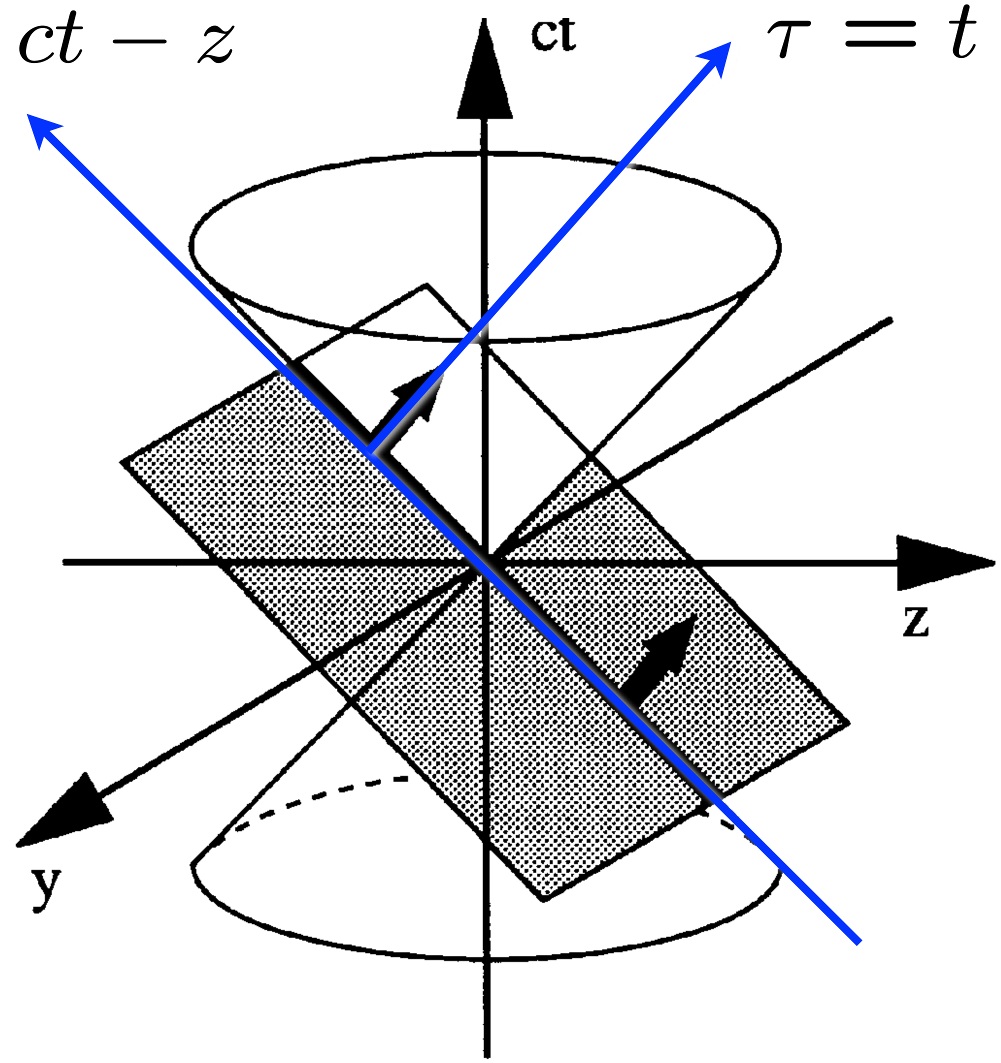
Evolve in ordinary time

Evolve in light-front time!

$\sigma = ct - z$ $\tau = t + z/c$



Instant Form



Front Form

Boost Invariant!


Love what you do
Fire the buggers
So what?
Too big to feel?
What's the scale?
COLOR
Smile!
puzzled
Yikes!
Who?
Oh my!
DON'T GIVE UP
Do what you love

J. D. Bjorken

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Sum Rules are valid
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function in e p collider and p rest frame
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

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Angular Momentum on the Light-Front

LC gauge $A^+=0$

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Sum Rules, Orbital Angular Momenta: Properties of LFWFS

Nonzero Anomalous Moment requires
Nonzero quark orbital angular momentum!

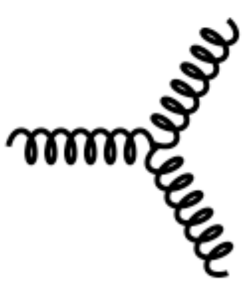
pQED: Ma, Hwang, Schmidt, sjb

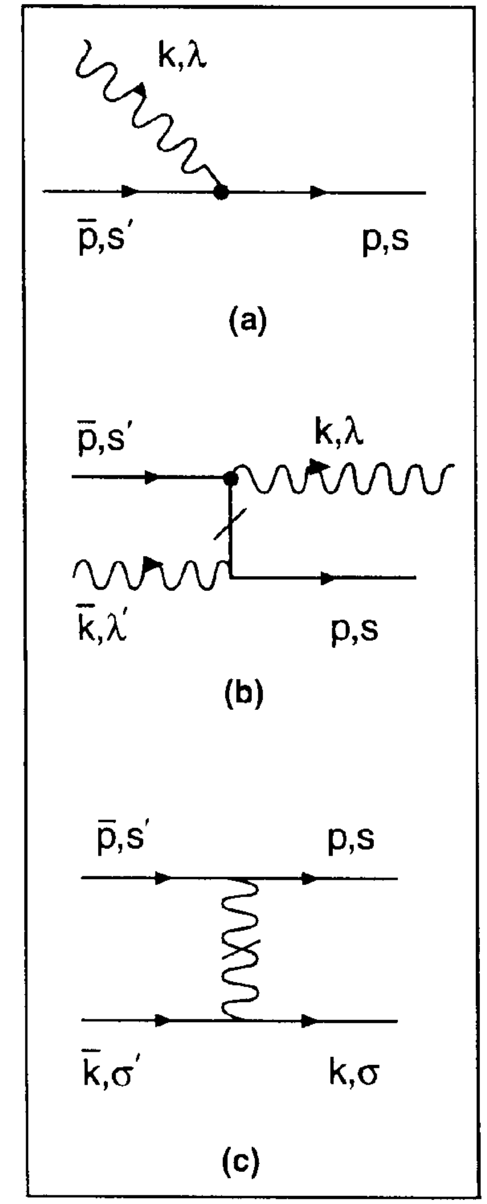
Hadron Spin Dynamics from LF Holography

- LFWF: Rigorous Definition of Angular Momentum
- Sum rules valid LF Fock state by Fock State
- Proton in AdS: Quark + Scalar-Diquark
- Equal probability for $|L^z| = 0, 1$; Proton Spin carried by L^z_q
- Anomalous proton moment requires nonzero quark L^z_q
- Sivers Effect requires nonzero quark L^z_q
- Counting Rules at large x: $G_{q/p}(x) \propto (1-x)^{2n_s - 1 + 2\Delta|S_z|}$
- Shadowing Destroys Sum Rules for Nuclear PDFs

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned}
H^{LF}_{QCD} = & \frac{1}{2}\int d^3x \bar{\tilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\tilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
& - \frac{1}{2} g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\
& + \frac{1}{2} g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\
& - g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\
& + g^2 \int d^3x \mathbf{Tr}\left(\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\
& + \frac{1}{2} g^2 \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\frac{\gamma^+}{i\partial^+}\tilde{\not{A}}\tilde{\psi} \\
& + g \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\tilde{\psi} \\
& + 2g \int d^3x \mathbf{Tr}\left(i\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right)
\end{aligned}$$





Physical gauge: $A^+ = 0$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

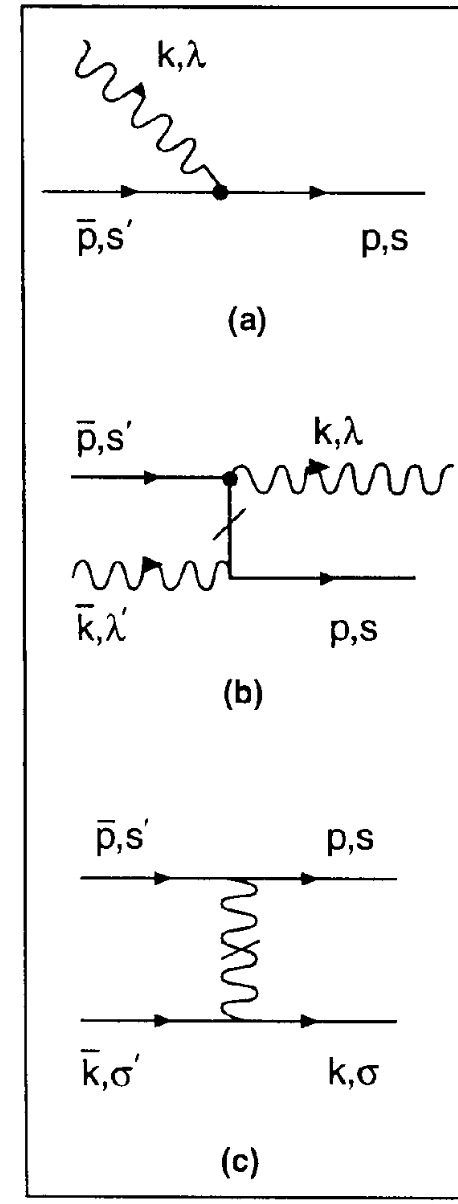
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

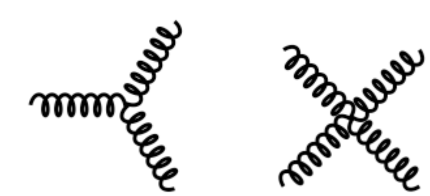
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h > = \mathcal{M}^2_h |\Psi_h >$$

$$|p, J_z > = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i >$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



LFWFs: Off-shell in P- and invariant mass

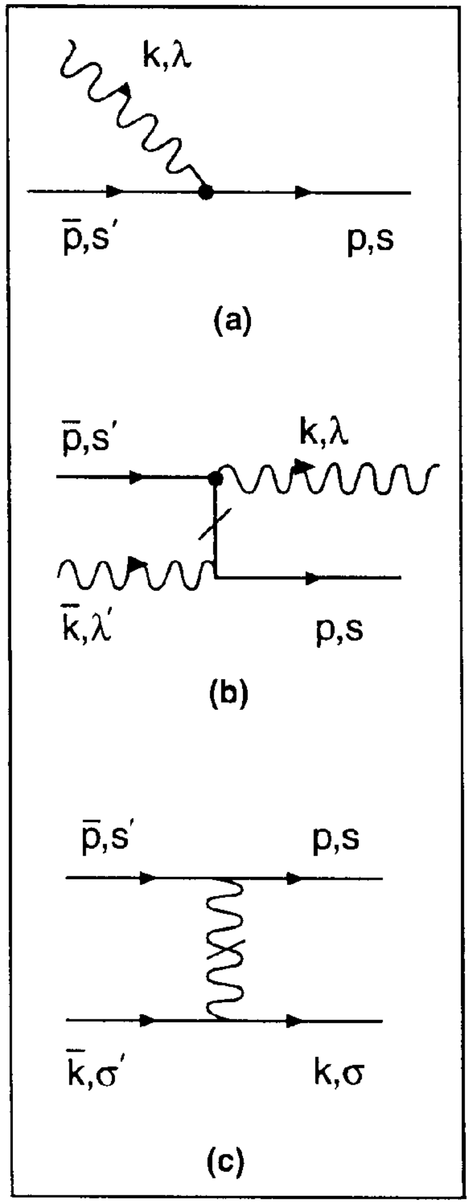
$$H^{int}_{LF}$$

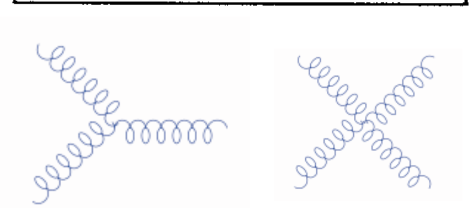
Light-Front QCD Heisenberg Equation

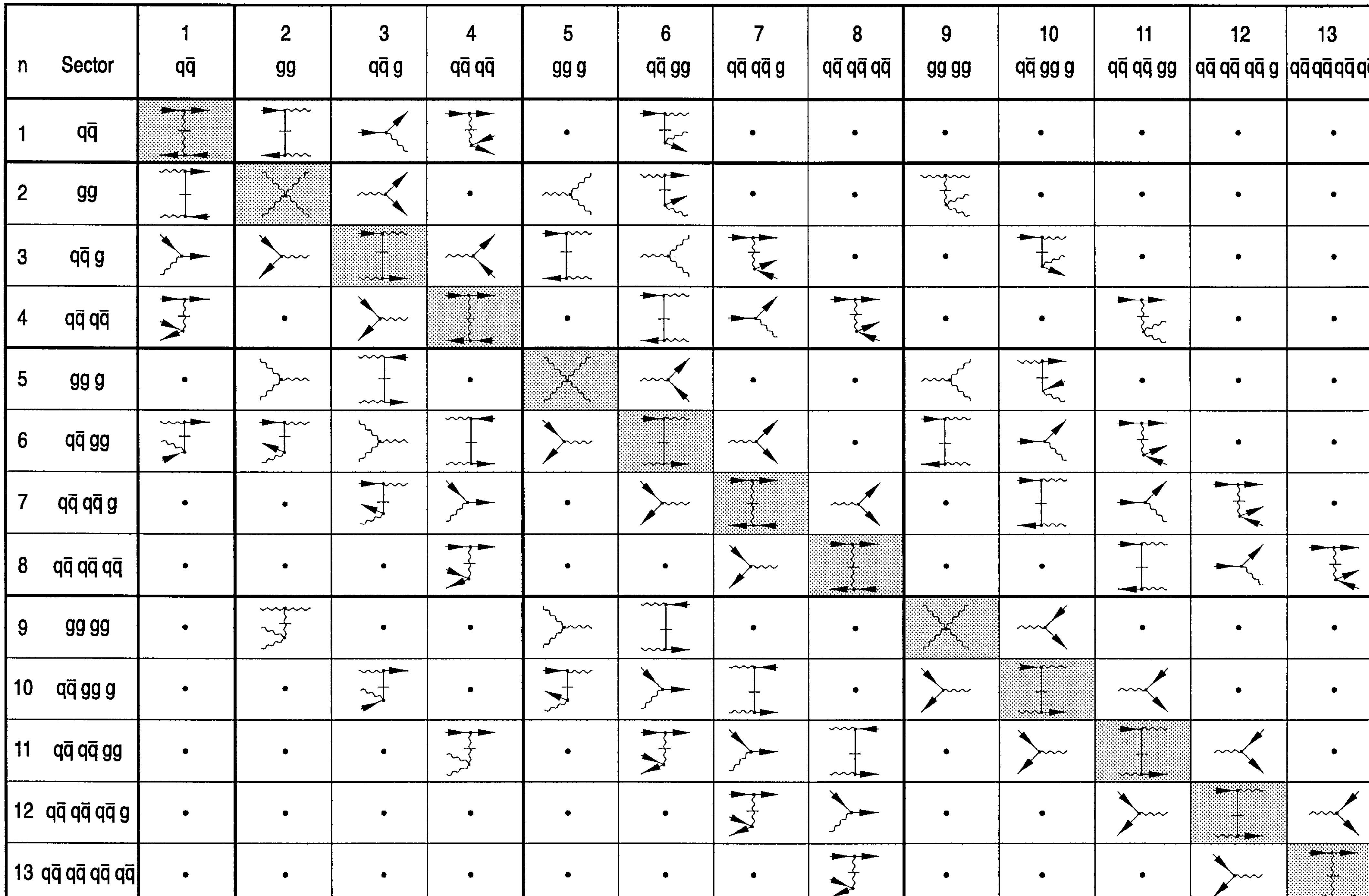
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

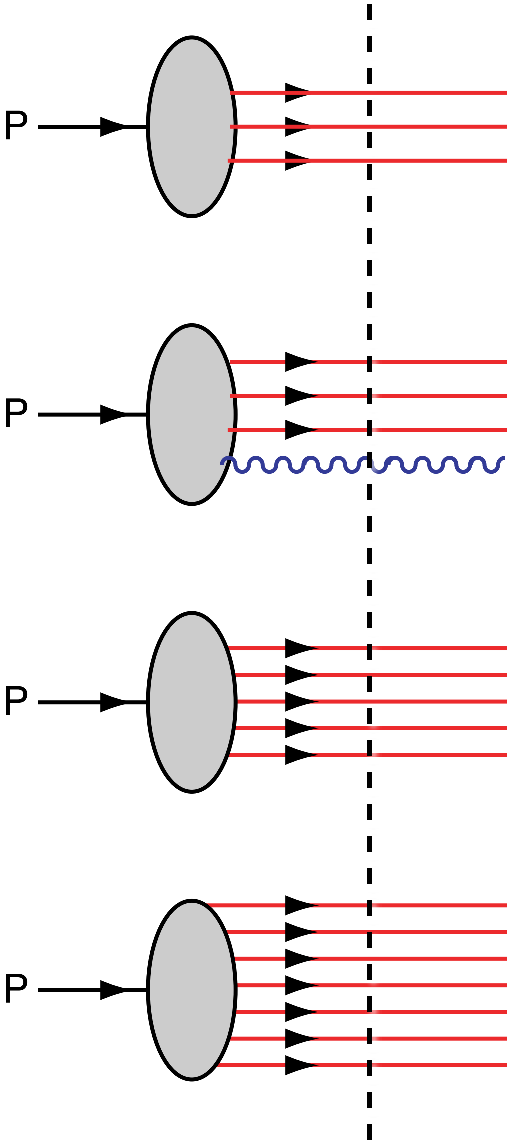
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Important work on strange quark distributions by W. Chen and J. C. Peng

$$\left|\psi_p(P^+,\vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3}\,16\pi^3\delta\left(1-\sum_{i=1}^{n}x_i\right)\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{\perp i}\right)$$

$$\times\,\psi_n(x_i,\vec{k}_{\perp i},\lambda_i)\left|n;\,x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp i},\lambda_i\right\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x,\Lambda) = \sum_{n,q_a}\int\prod_{j=1}^{n}\mathrm{d}x_j\,\mathrm{d}^2\vec{k}_{\perp j}\sum_{\lambda_i}\left|\psi^{(\Lambda)}_{n/H}(x_i,\vec{k}_{\perp i},\lambda_i)\right|^2$$

$$\times\,\delta\left(1-\sum_{i}^{n}x_i\right)\delta^{(2)}\left(\sum_{i}^{n}\vec{k}_{\perp i}\right)\delta(x-x_q)\delta_{\lambda_a\lambda_q}\Theta\left(\Lambda^2-\mathcal{M}_n^2\right),$$

Obeys DGLAP Evolution *Defines quark distributions*

Connection to Bethe-Salpeter:

$$\int dk^-\Psi_{BS}(k,P) \rightarrow \psi_{LF}(x,\vec{k}_\perp) \qquad \Psi_{BS}(x,P)|_{x^+=0}$$

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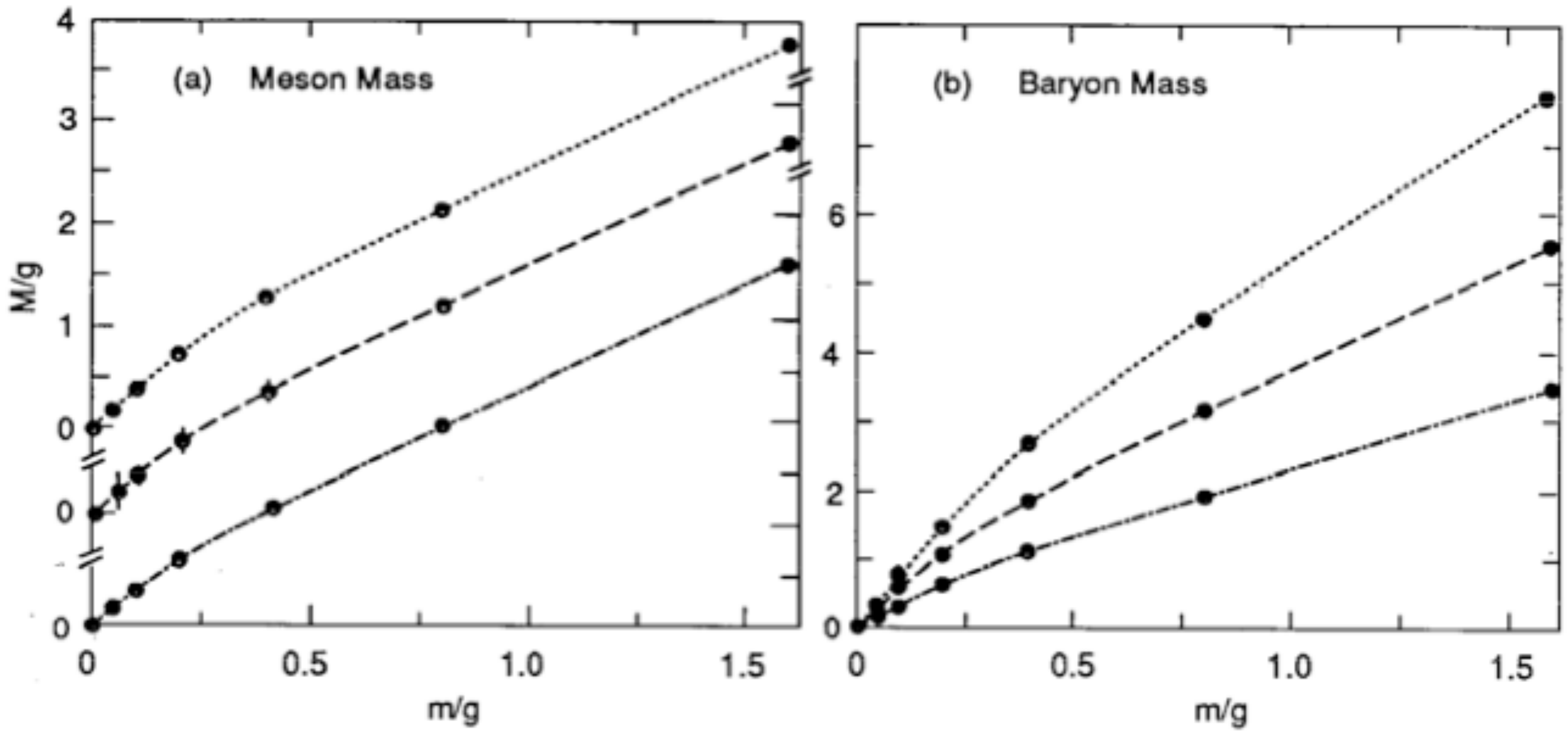
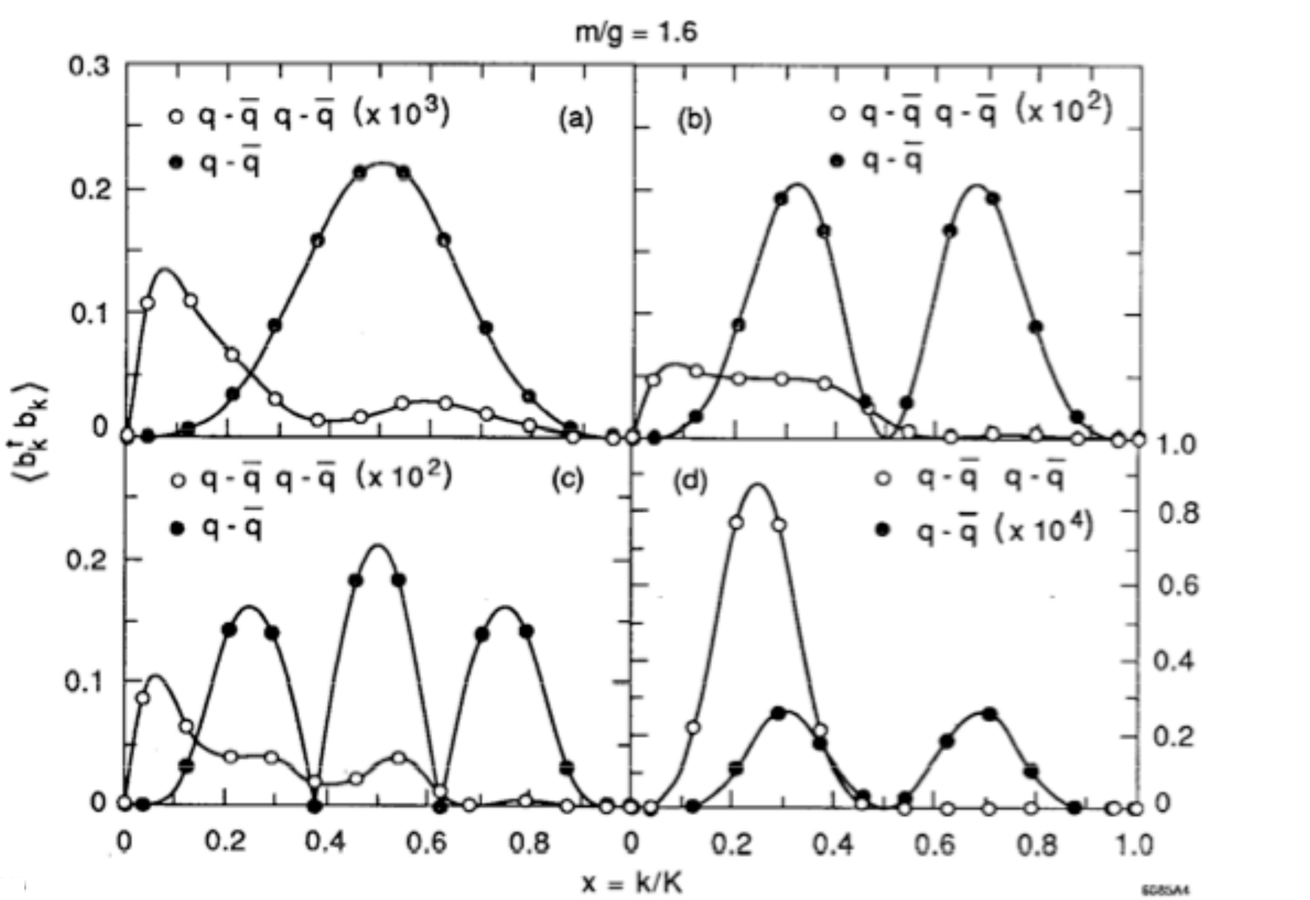
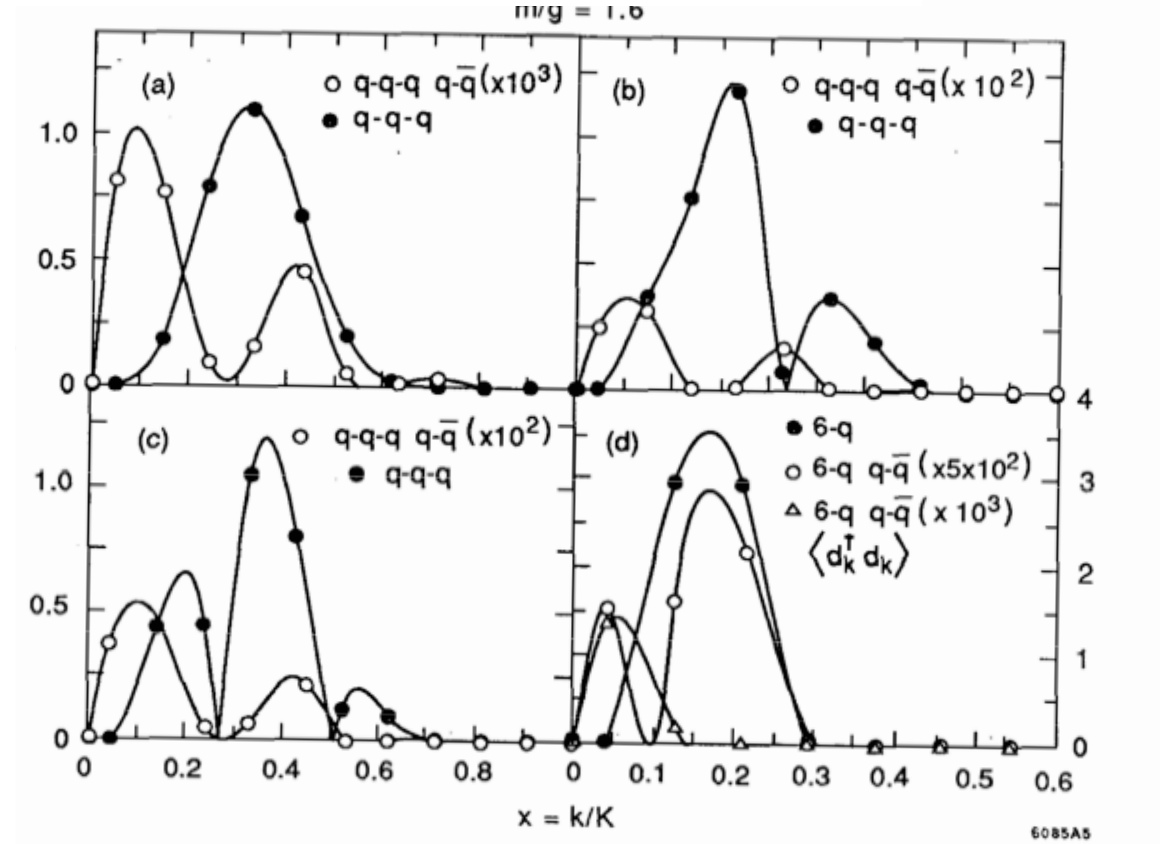
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

Light Front Theory

- Frame-Independent, causal, Minkowski space,
- DLCQ, BLFQ: No fermion doubling
- Equivalent to Bethe-Salpeter $\int dk^{-}\psi_{BS} = \psi_{LF}$
- Hadronization at the Amplitude Level
- Holographically Dual to AdS_5

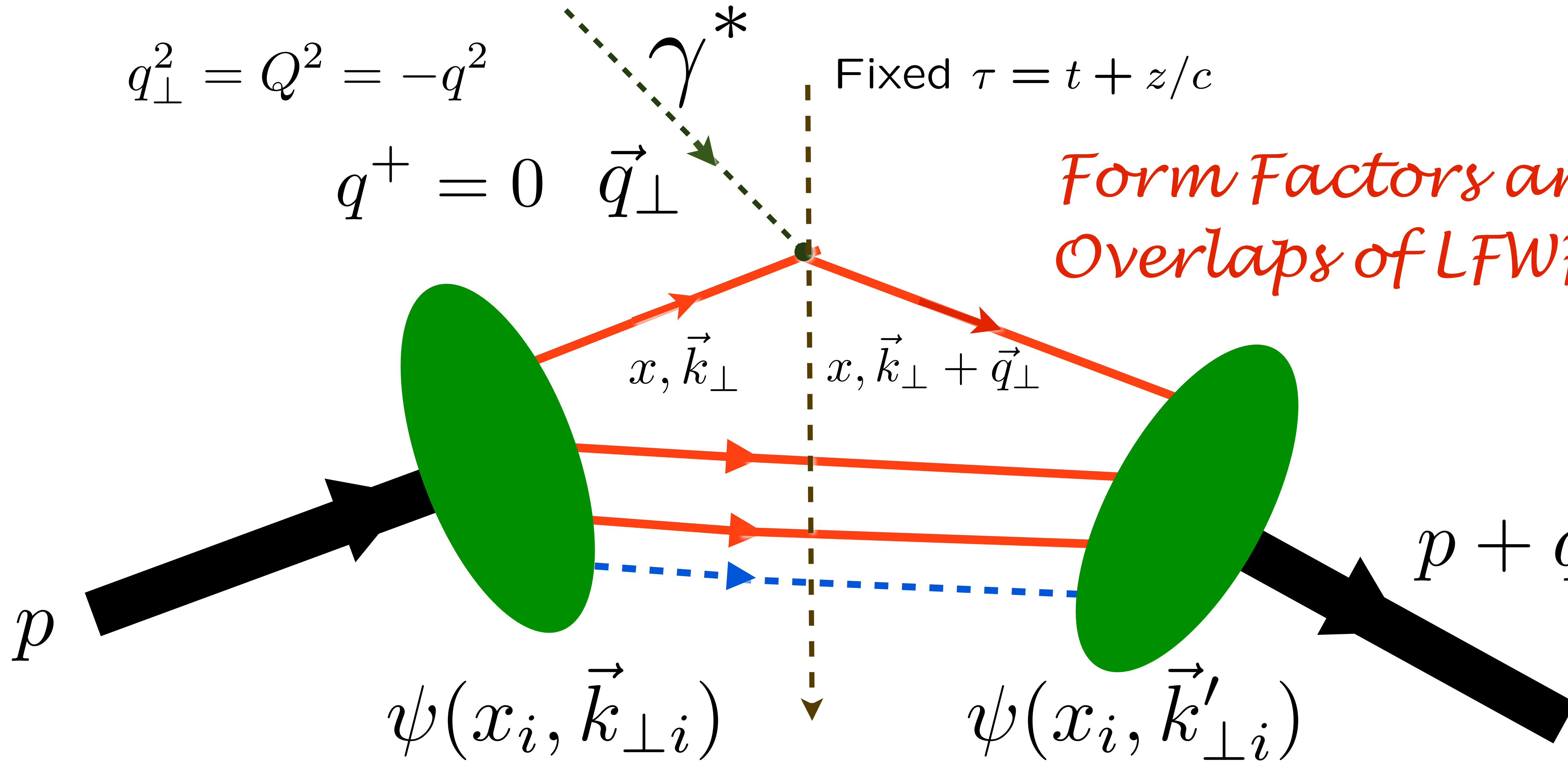
$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

$$q_\perp^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_\perp$$

Form Factors are Overlaps of LFWFs

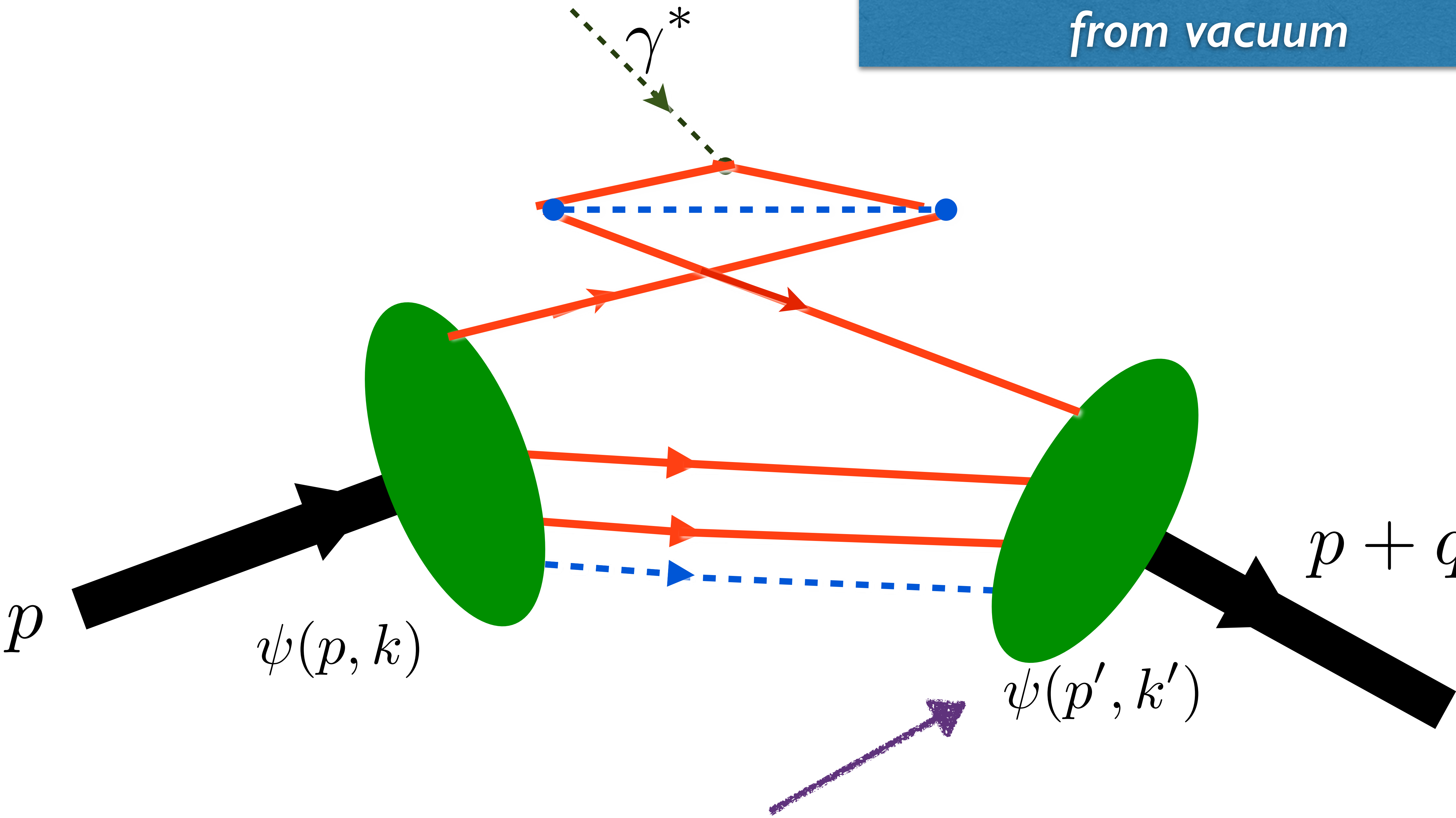


struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1-x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Pair creation and annihilation from vacuum



Need boosted instant-form wavefunction

Calculation of current matrix elements not possible in Instant Form
Must include vacuum-induced currents!

Exact LF Formula for Pauli Form Factor

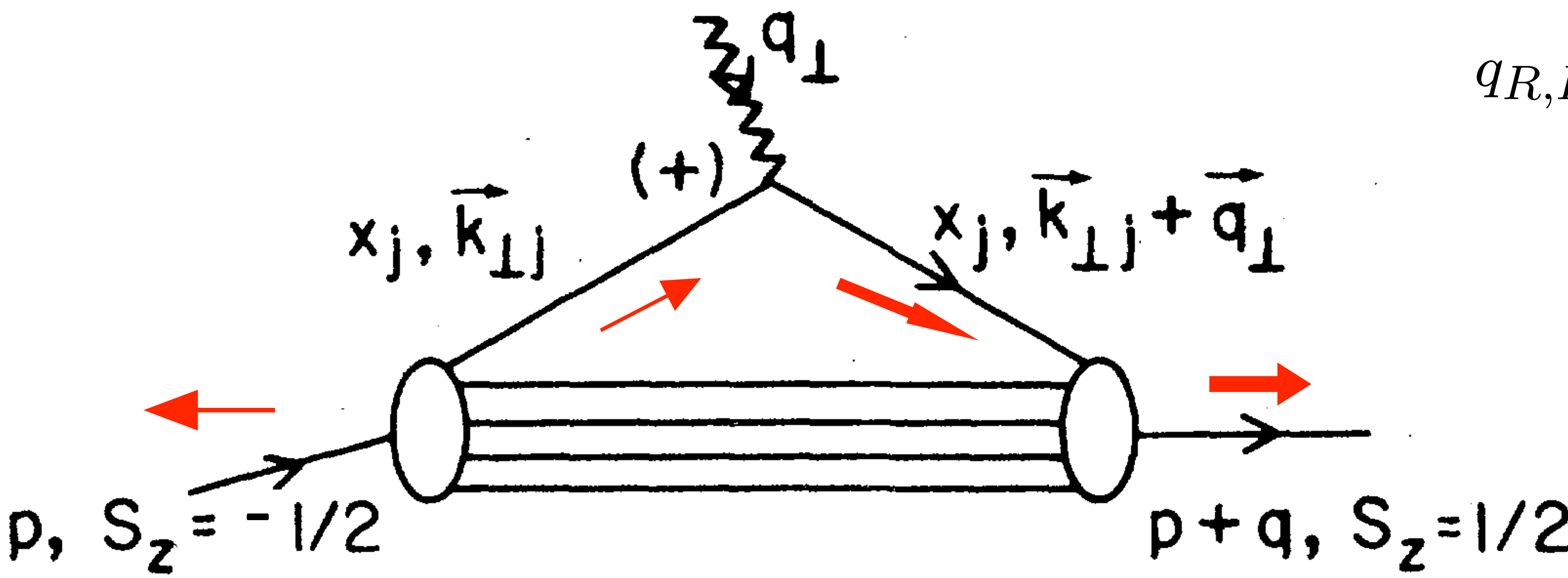
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta\ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P) ,$$

where $q^\mu = (P'-P)^\mu$, $\overline{P}^\mu = \frac{1}{2}(P'+P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

$$\left\langle P+q,\uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P,\uparrow \right\rangle = A(q^2) ,$$

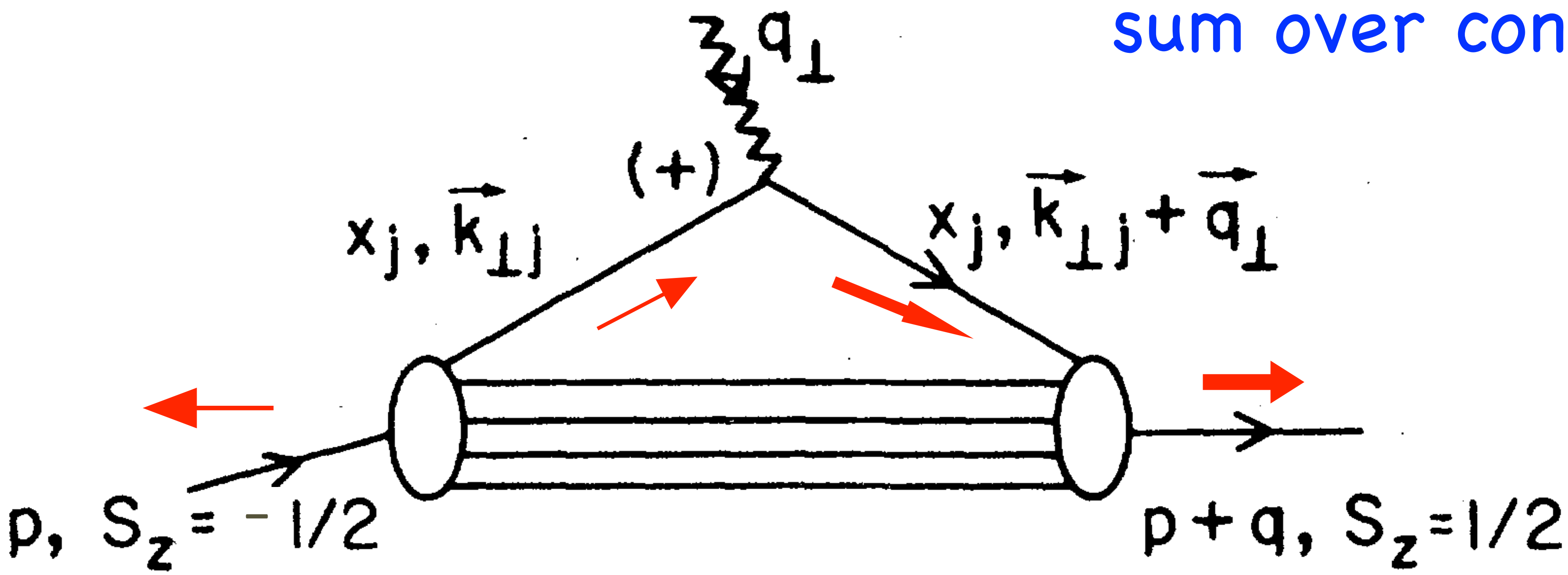
$$\left\langle P+q,\uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P,\downarrow \right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M} .$$

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: B(0) Must vanish because of Equivalence Theorem

graviton

sum over constituents

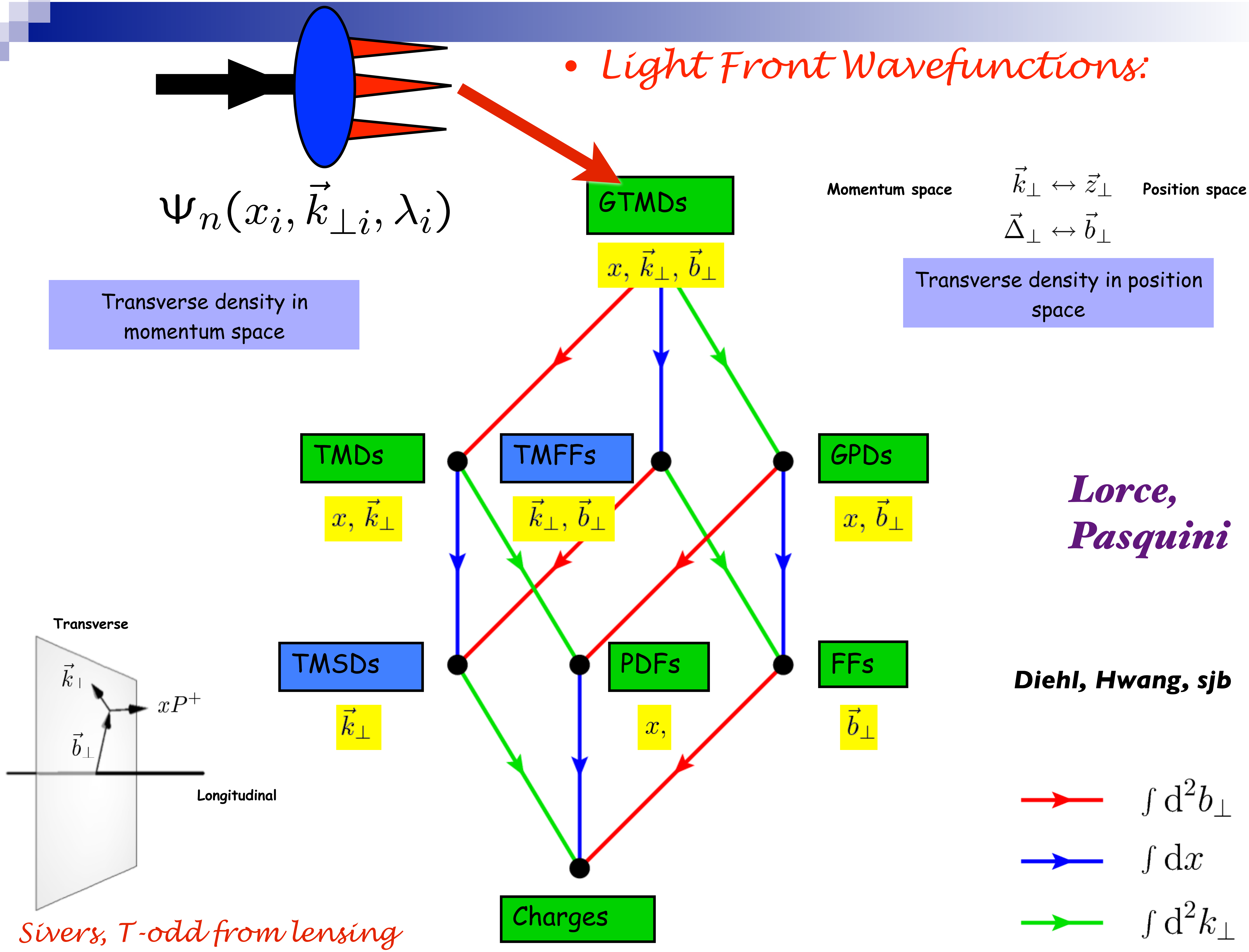


Hwang, Schmidt, sjb; Holstein et al

$$B(0) = 0$$

Each Fock State

- *Light Front Wavefunctions:*



Single-spin asymmetries

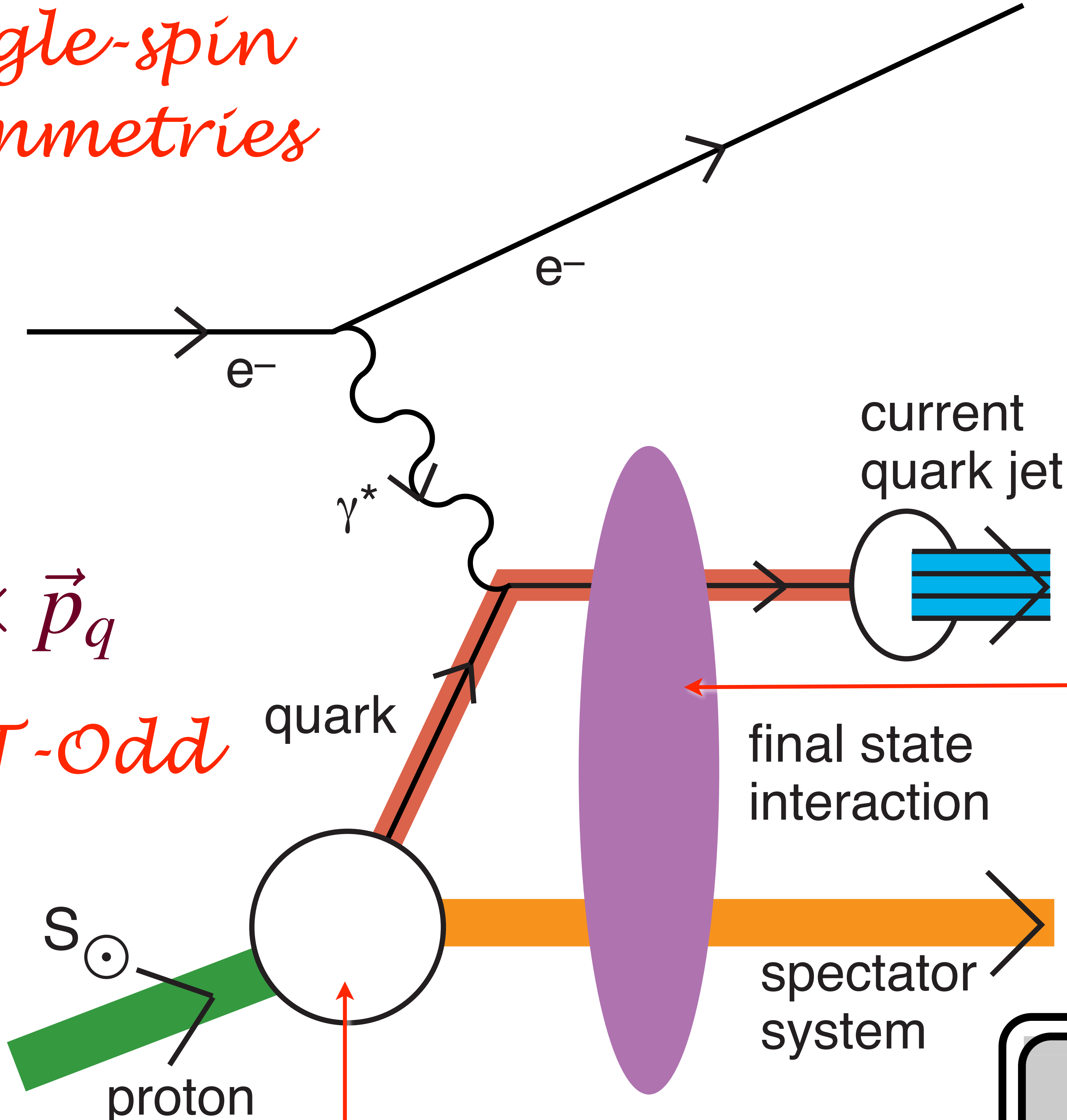
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

QED: Lensing involves soft scales

Light-Front Wavefunction S and P- Waves!

Sign reversal in DY!

Collins
Hwang, Schmidt, sjb

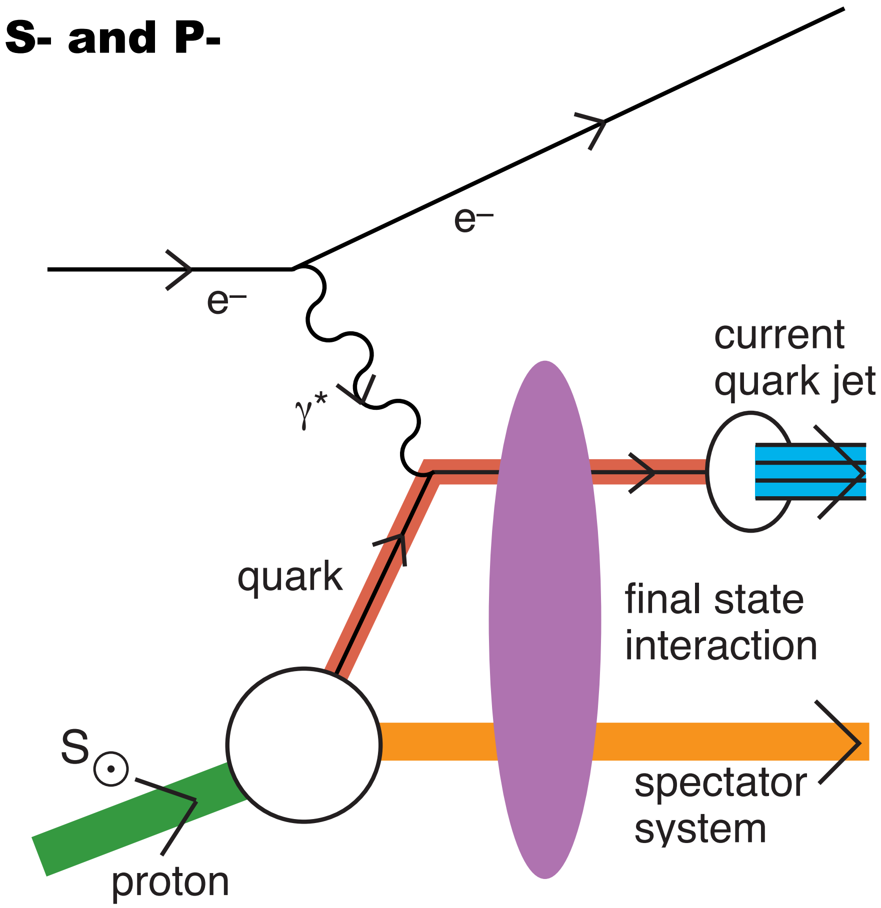
Leading-Twist Rescattering Violates pQCD Factorization!

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$i\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;
- Wilson line effect -- lc gauge prescription
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!
- Alternate: Retarded and Advanced Gauge: Augmented LFWFs



Pasquini, Xiao, Yuan, sjb
Mulders, Boer   Qiu, Sterman

Dae Sung Hwang, Yuri V. Kovchegov,
Ivan Schmidt, Matthew D. Sievert, sjb

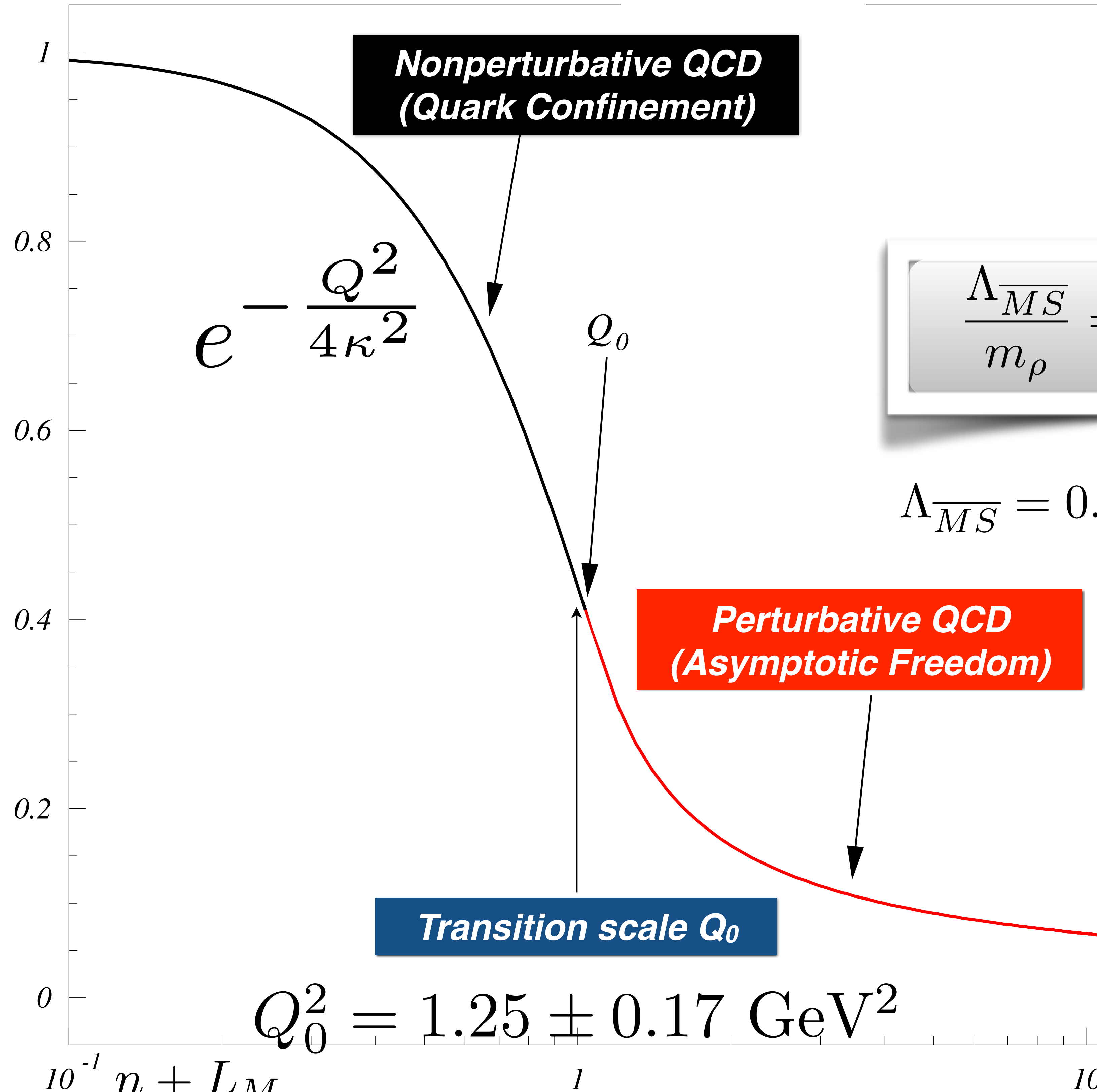
$$m_\rho = \sqrt{2}\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

Prediction from AdS/QCD:

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$$e^{-\frac{Q^2}{4\kappa^2}}$$

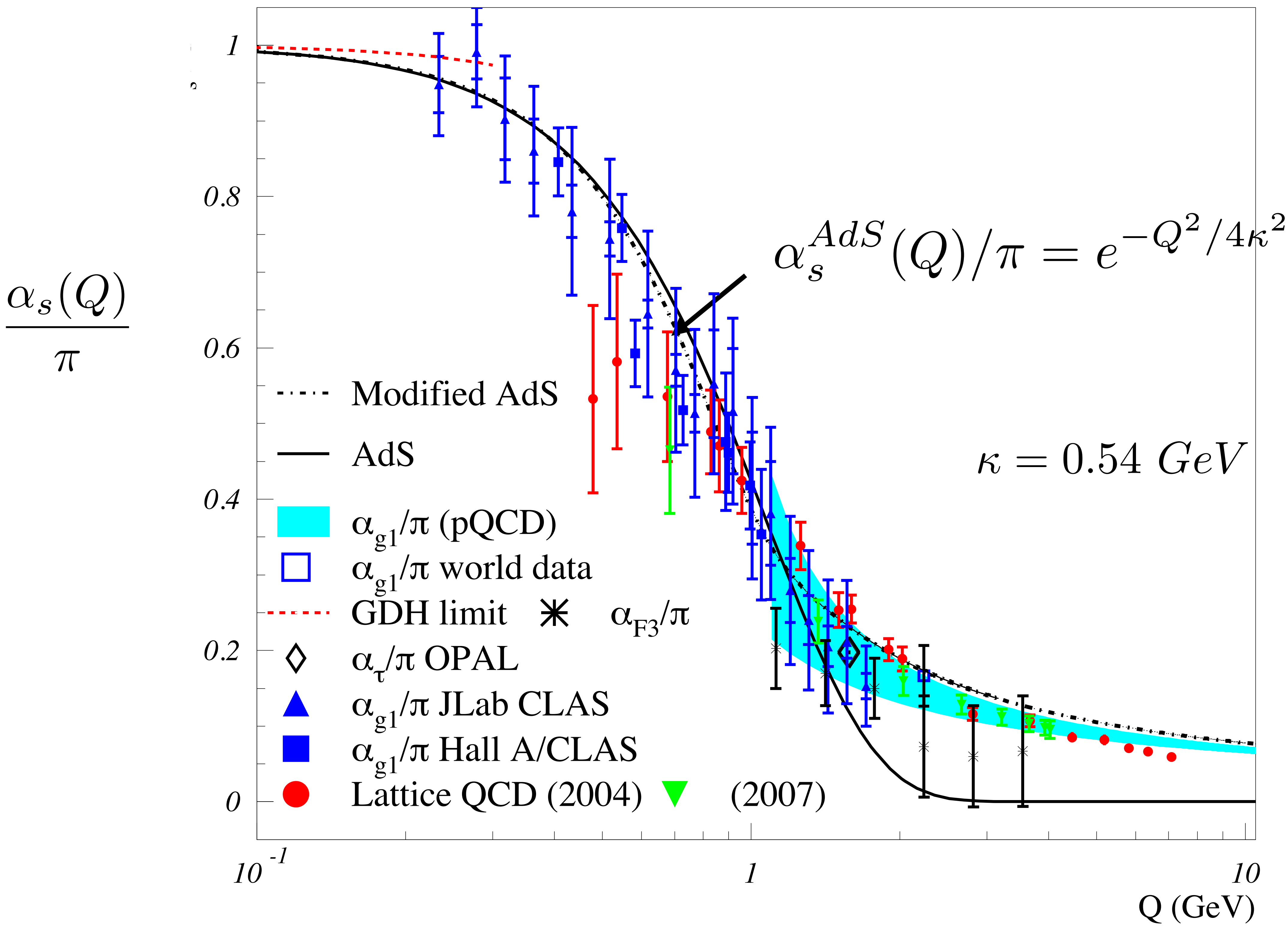
$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

$$\Lambda_{\overline{MS}} = 0.351 \pm 0.024 \ GeV$$

$$Q_0^2 = 1.25 \pm 0.17 \text{ GeV}^2$$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Analytic, defined at all scales, IR Fixed Point

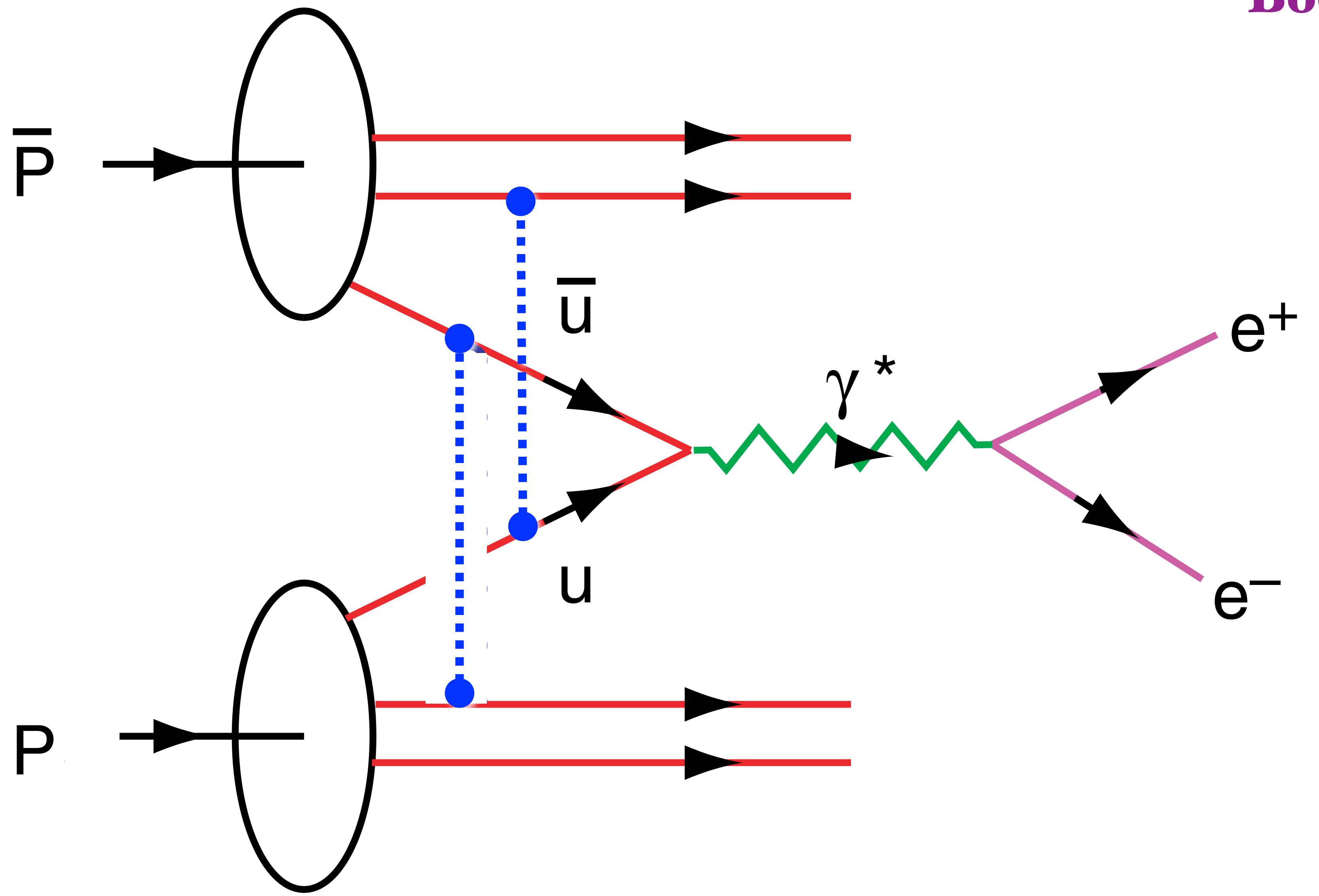


AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Boer, Hwang, sjb



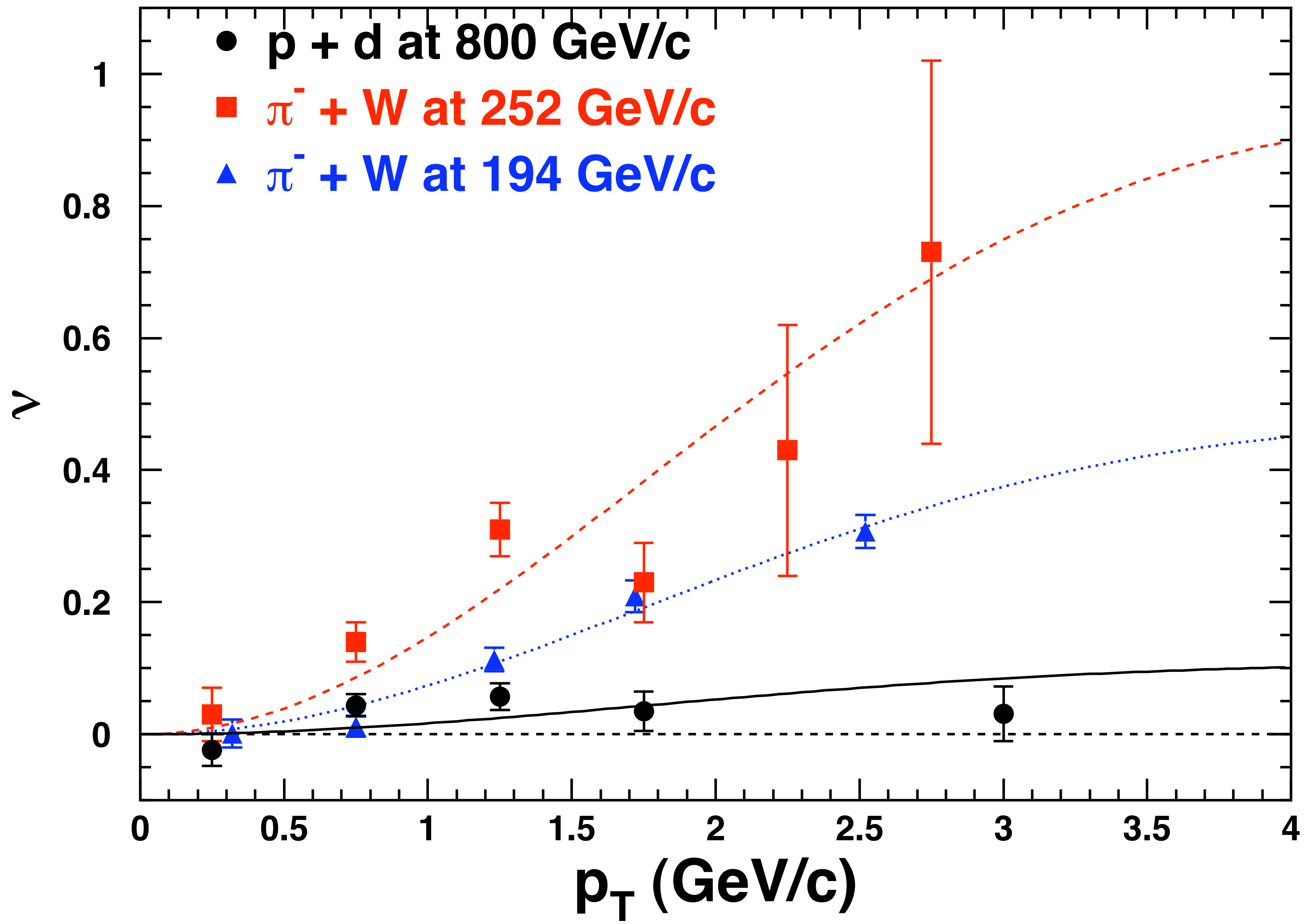
DY $\cos 2\phi$ **correlation at leading twist from double ISI**

Product of Boer - Mulders Functions

$$h_1^{\perp}(x_1, \boldsymbol{p}_{\perp}^2) \times \bar{h}_1^{\perp}(x_2, \boldsymbol{k}_{\perp}^2)$$

Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)



Huge Effect in $\pi W \to \mu^+\mu^- X$

Negligible Effect $pd \to \mu^+\mu^- X$

Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

PHYSICAL REVIEW D **75**, 114014 (2007)

k_T factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

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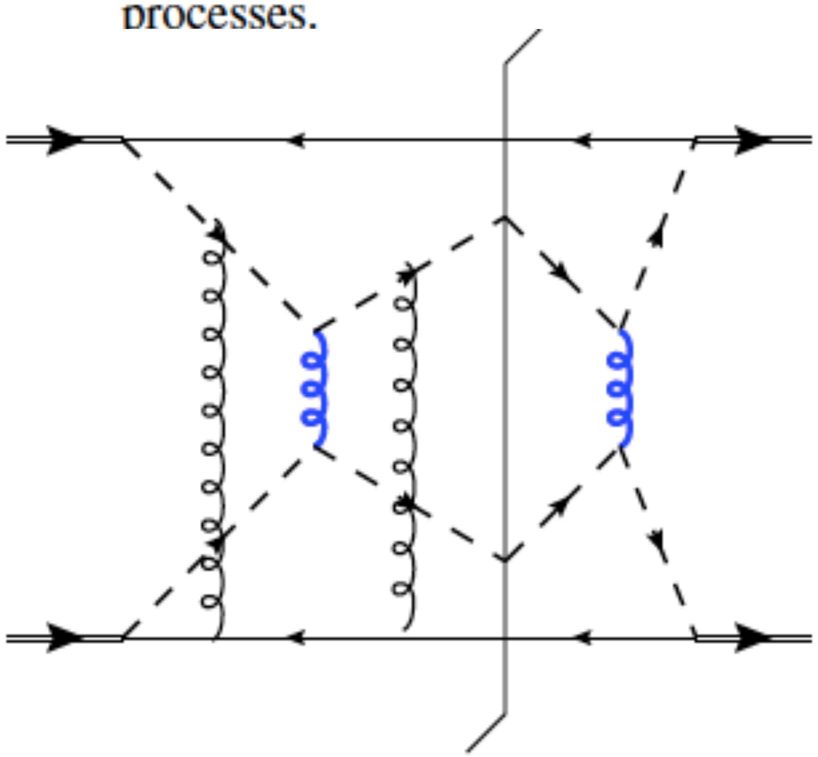
(Received 15 May 2007; published 28 June 2007)

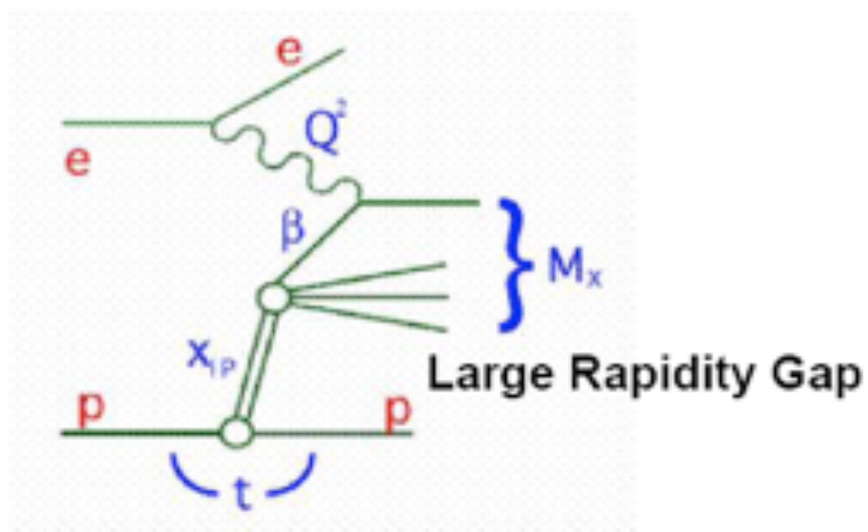
We show that hard-scattering factorization is violated in the production of high-p_T hadrons in hadron-hadron collisions, in the case that the hadrons are back-to-back, so that k_T factorization is to be used. The explicit counterexample that we construct is for the single-spin asymmetry with one beam transversely polarized. The Sivers function needed here has particular sensitivity to the Wilson lines in the parton densities. We use a greatly simplified model theory to make the breakdown of factorization easy to check explicitly. But the counterexample implies that standard arguments for factorization fail not just for the single-spin asymmetry but for the unpolarized cross section for back-to-back hadron production in QCD in hadron-hadron collisions. This is unlike corresponding cases in e^+e^- annihilation, Drell-Yan, and deeply inelastic scattering. Moreover, the result endangers factorization for more general hadroproduction processes.

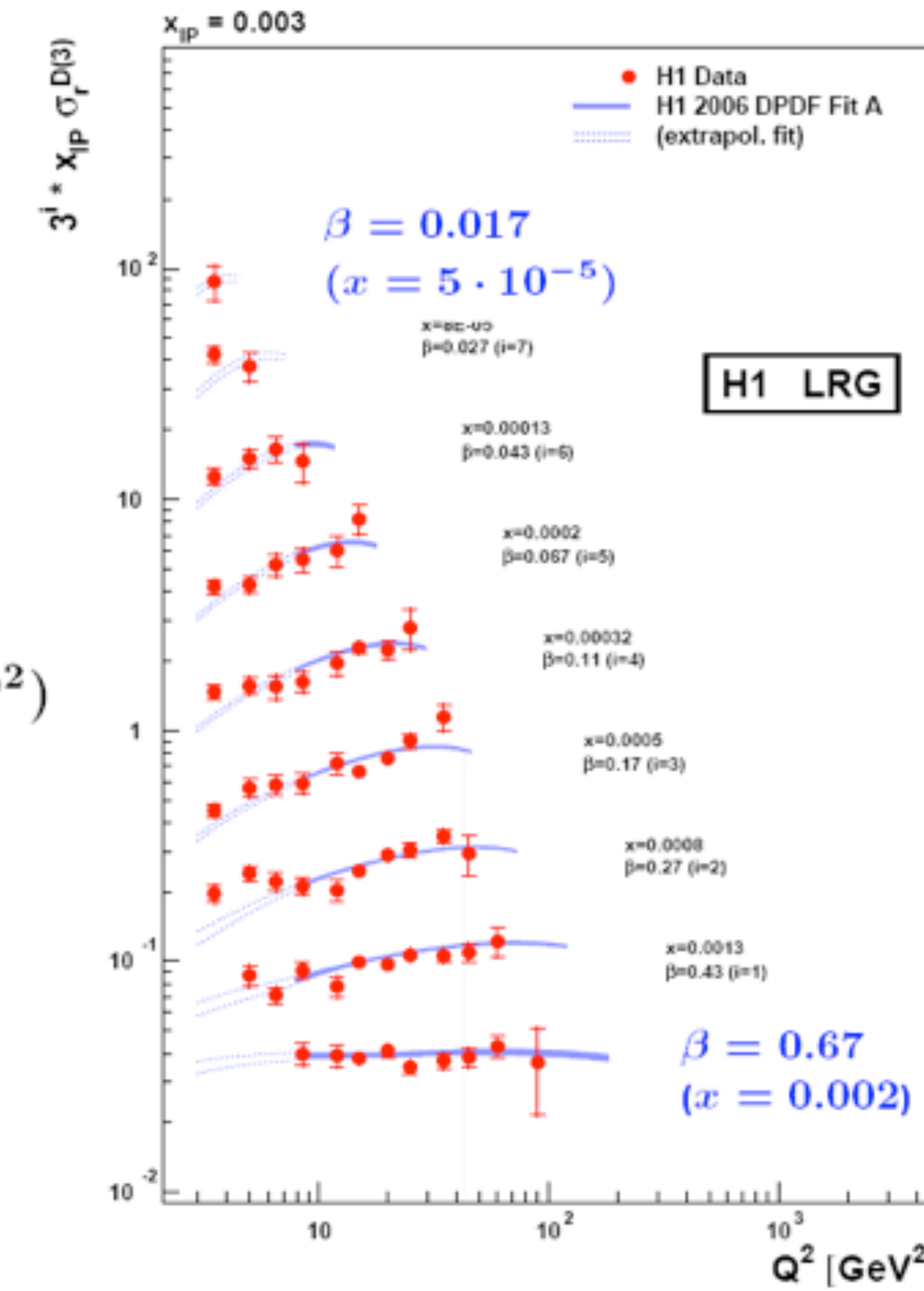


Exchange of two extra gluons —- non-factorization of inclusive unpolarized cross section

Example: $pp \to c\bar{c}X$

Diffractive Structure Function F_2^D





Diffractive inclusive cross section

$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\, d\beta\, dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$$

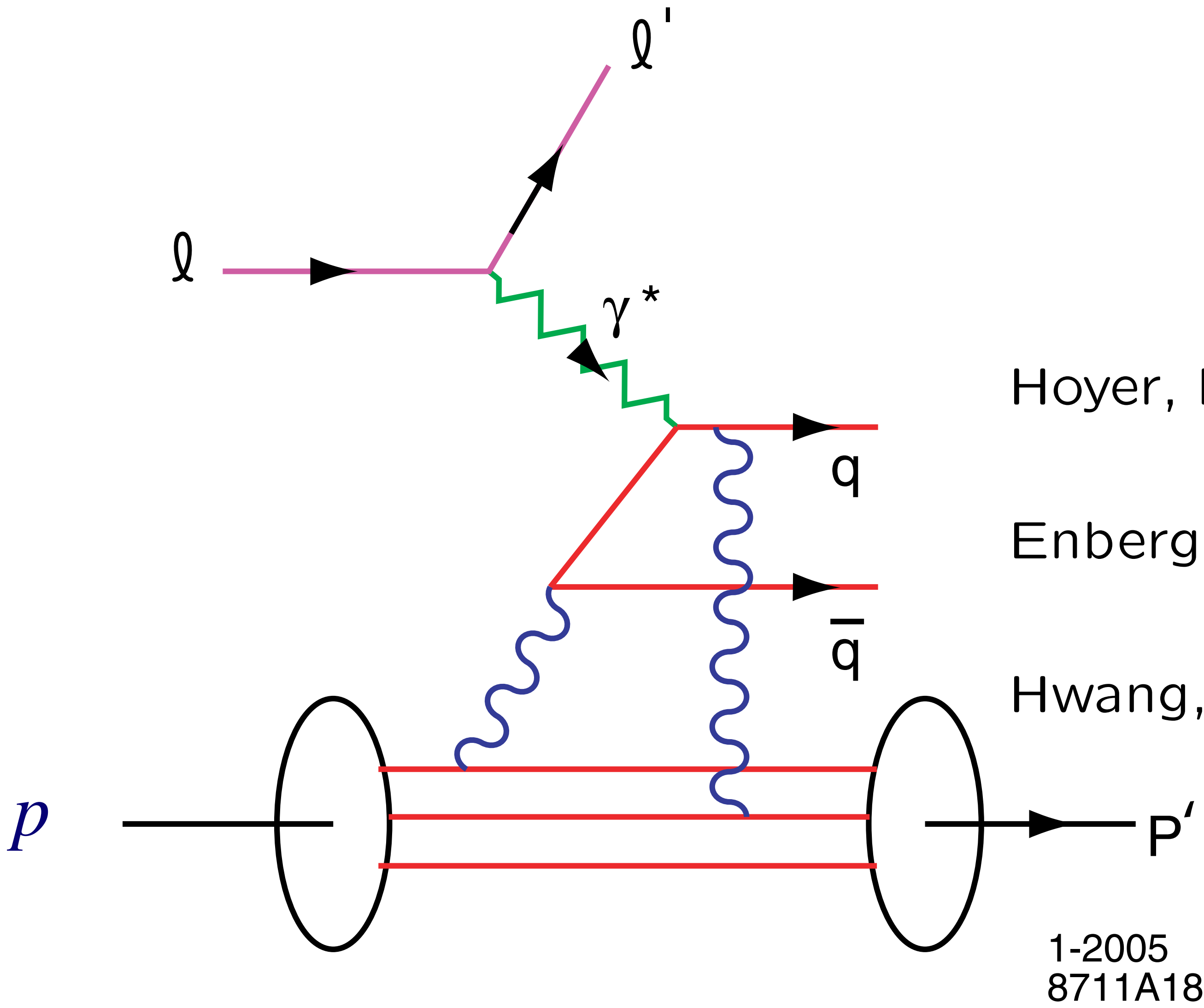
$$F_2^D(x_{I\!P}, \beta, Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

Precise measurements *sys 5%, stat 5–20%*

Quark Rescattering



Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

Enberg, Hoyer, Ingelman, SJB

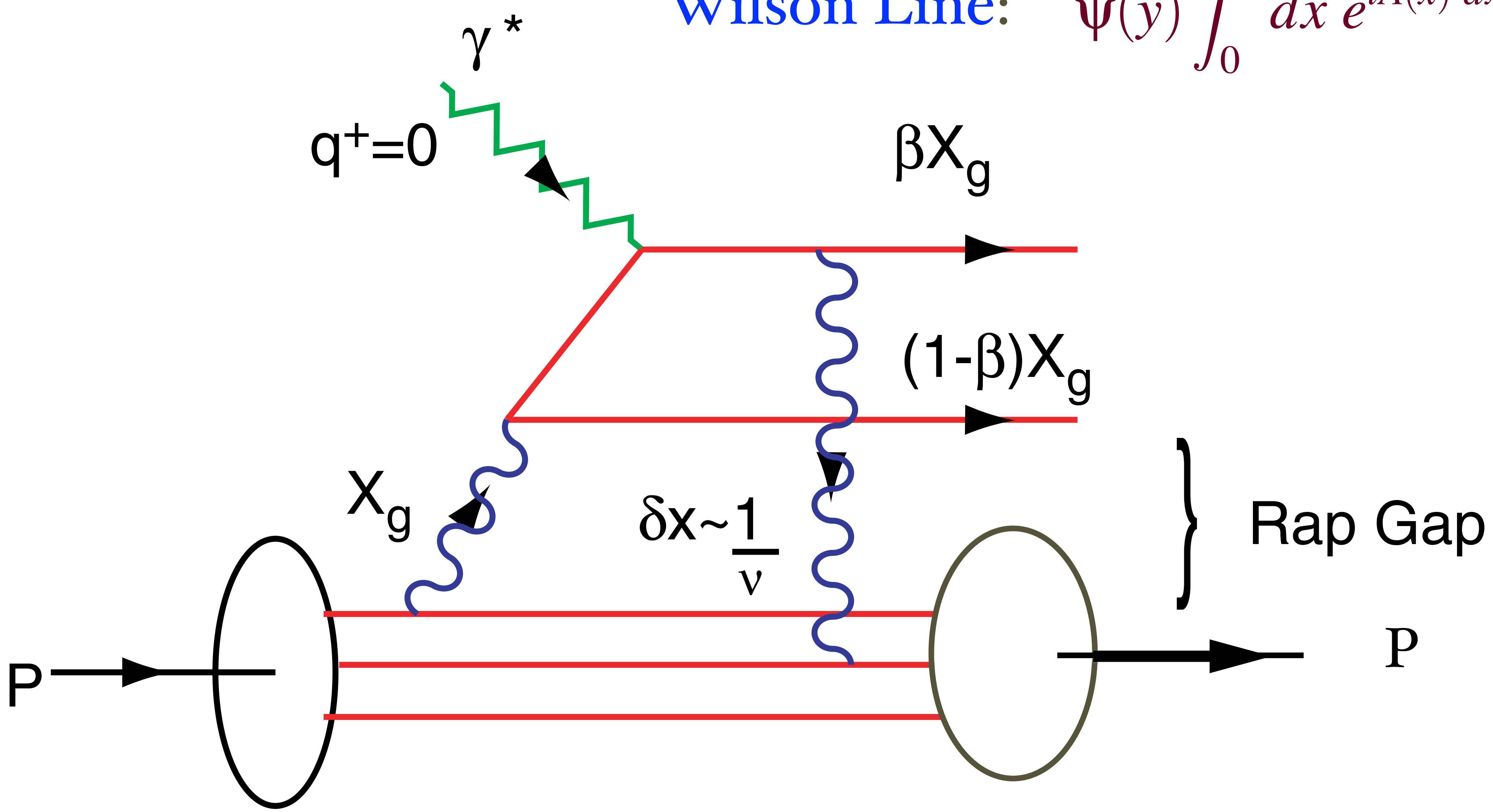
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

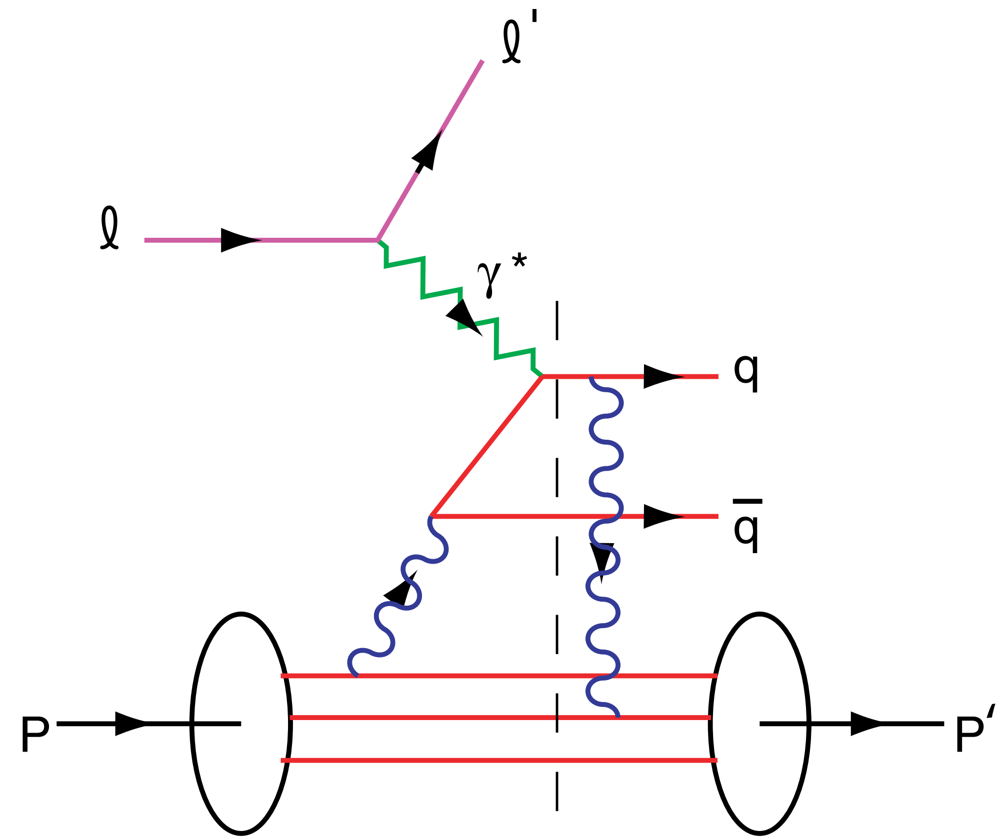
Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



Reproduces lab-frame color dipole approach



Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron and DDIS

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target!

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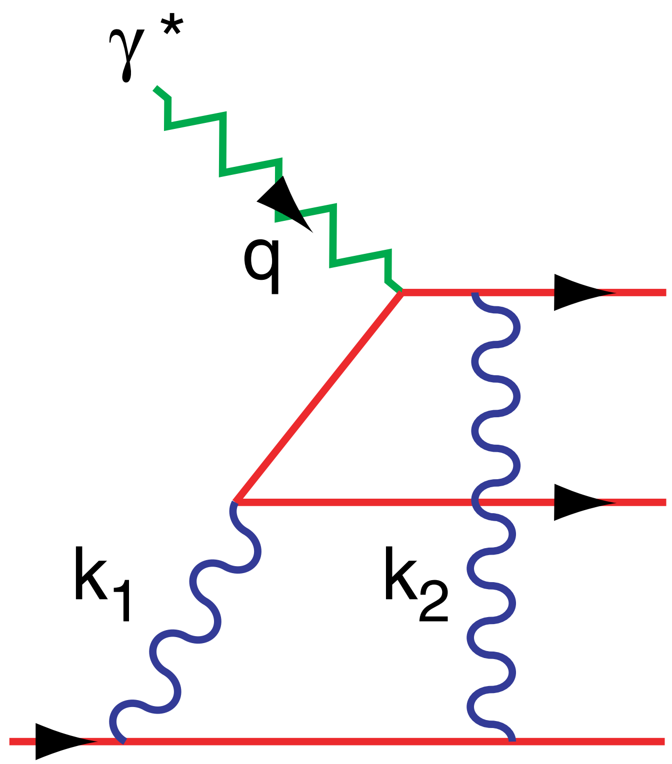
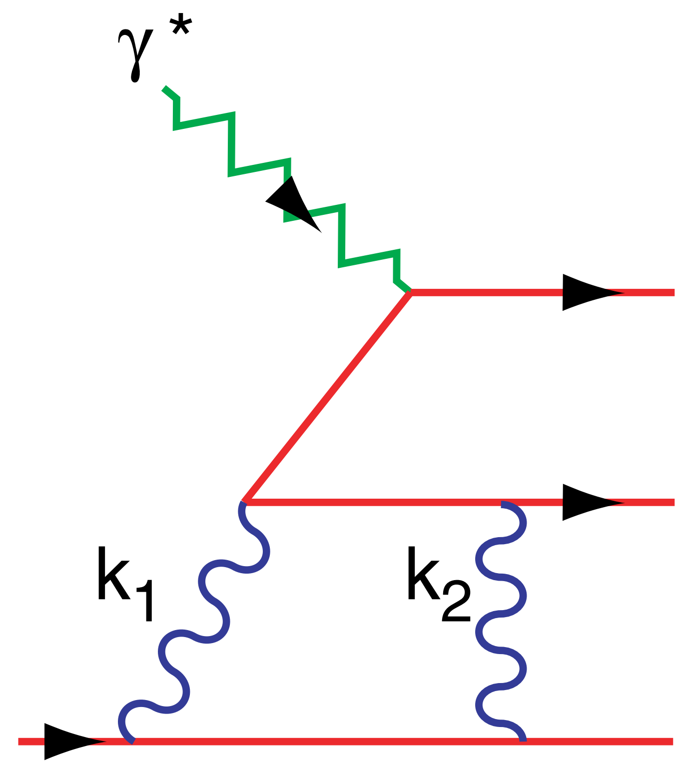
Final State Interactions in QCD



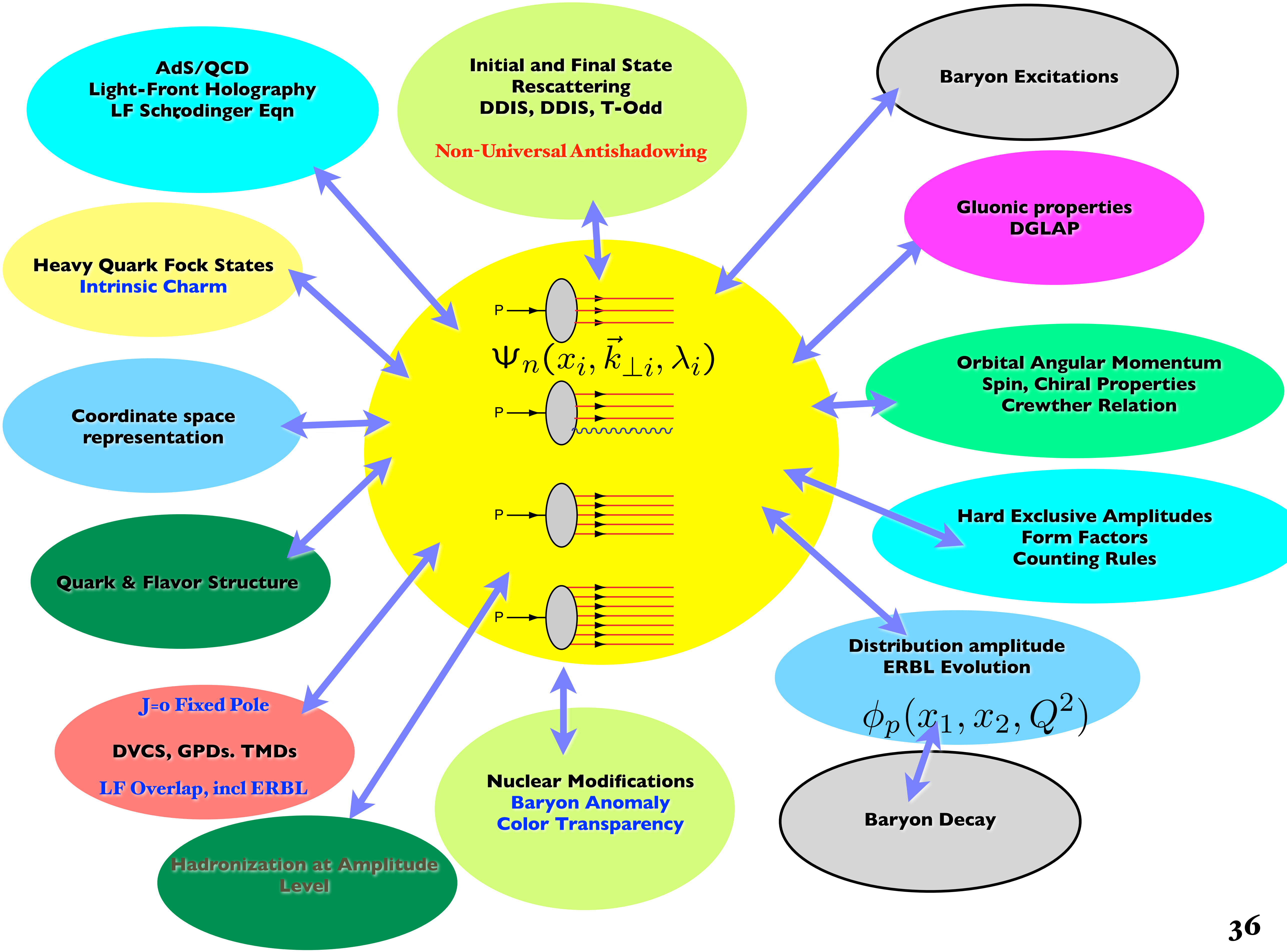
Feynman Gauge



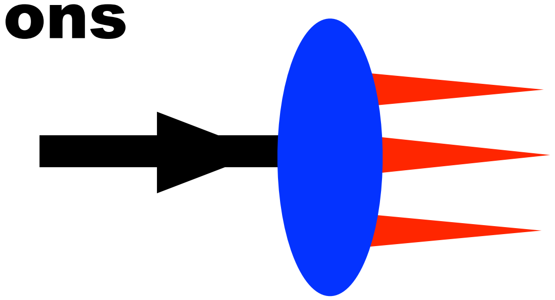
Light-Cone Gauge

Result is Gauge Independent

QCD and the LF Hadron Wavefunctions



- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

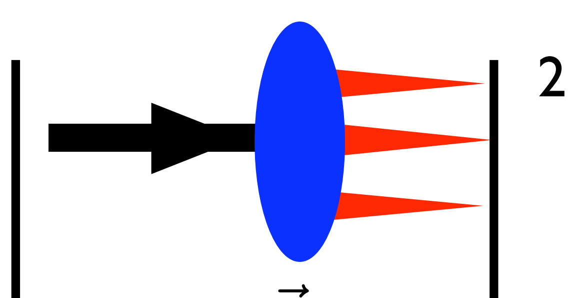
Hwang, Schmidt, sjb,

Mulders, Boer

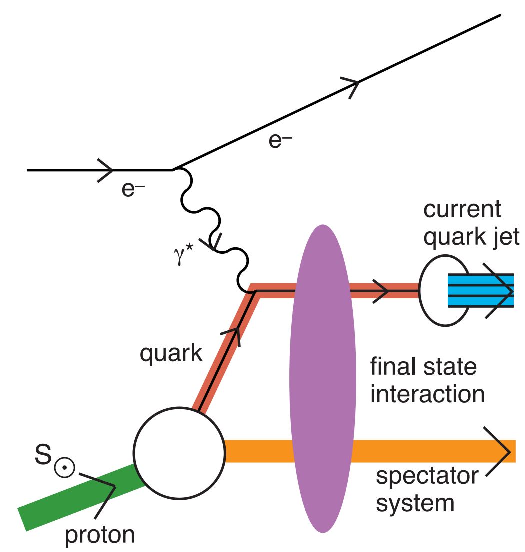
Qiu, Sterman

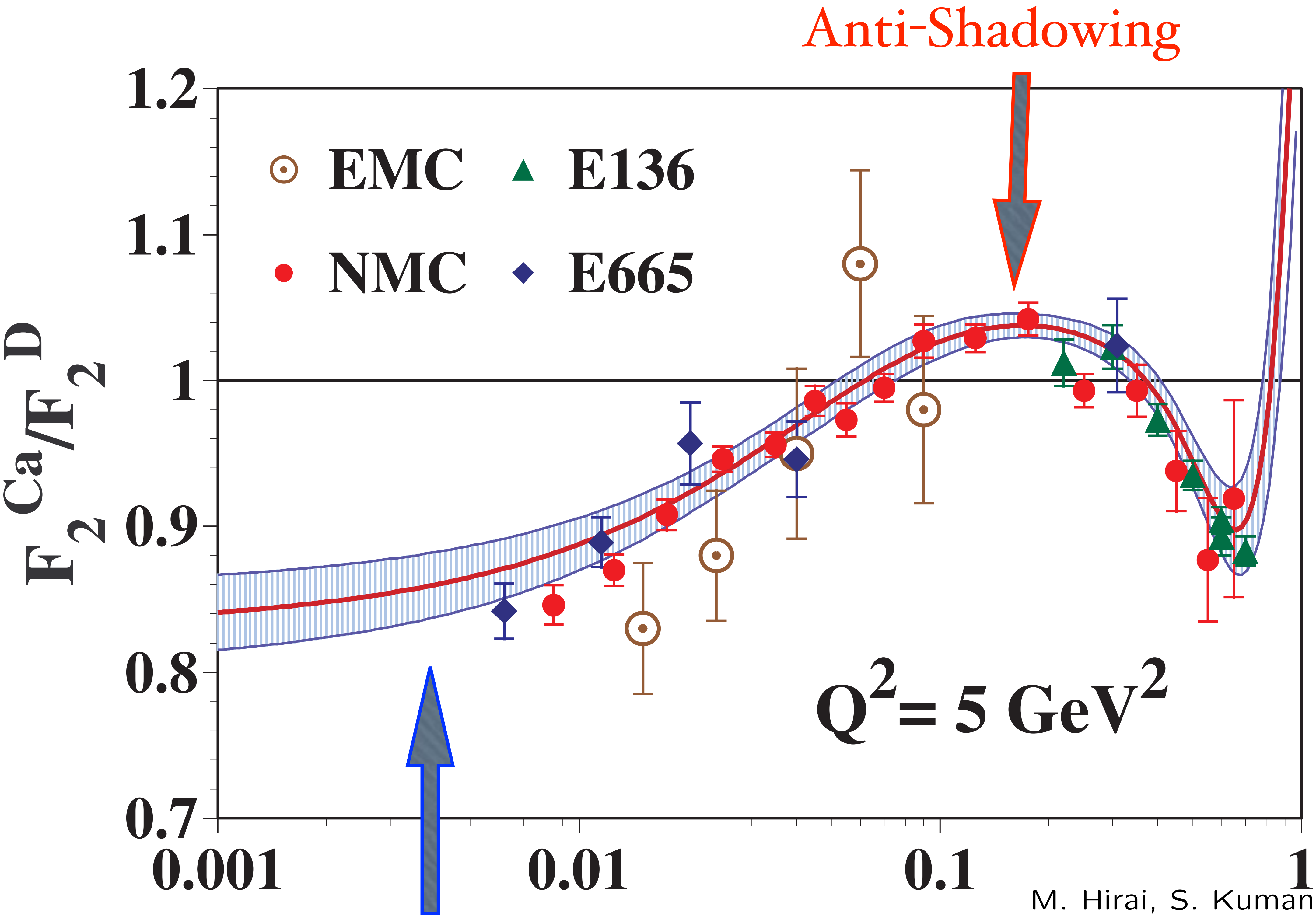
Collins, Qiu

Pasquini, Lorce, Xi Yuan, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$





M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

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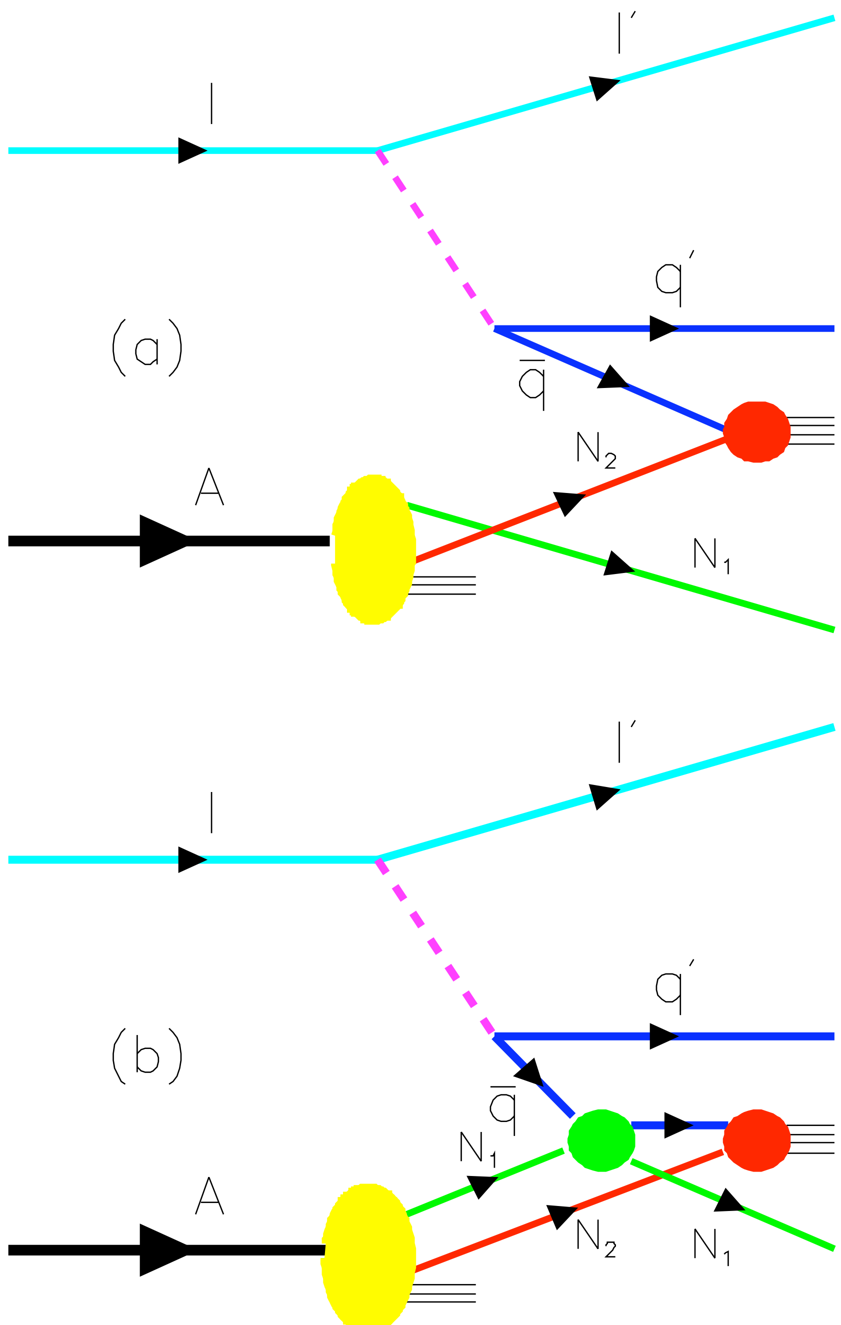
Factorization Issues and Light-Front Holographic QCD



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

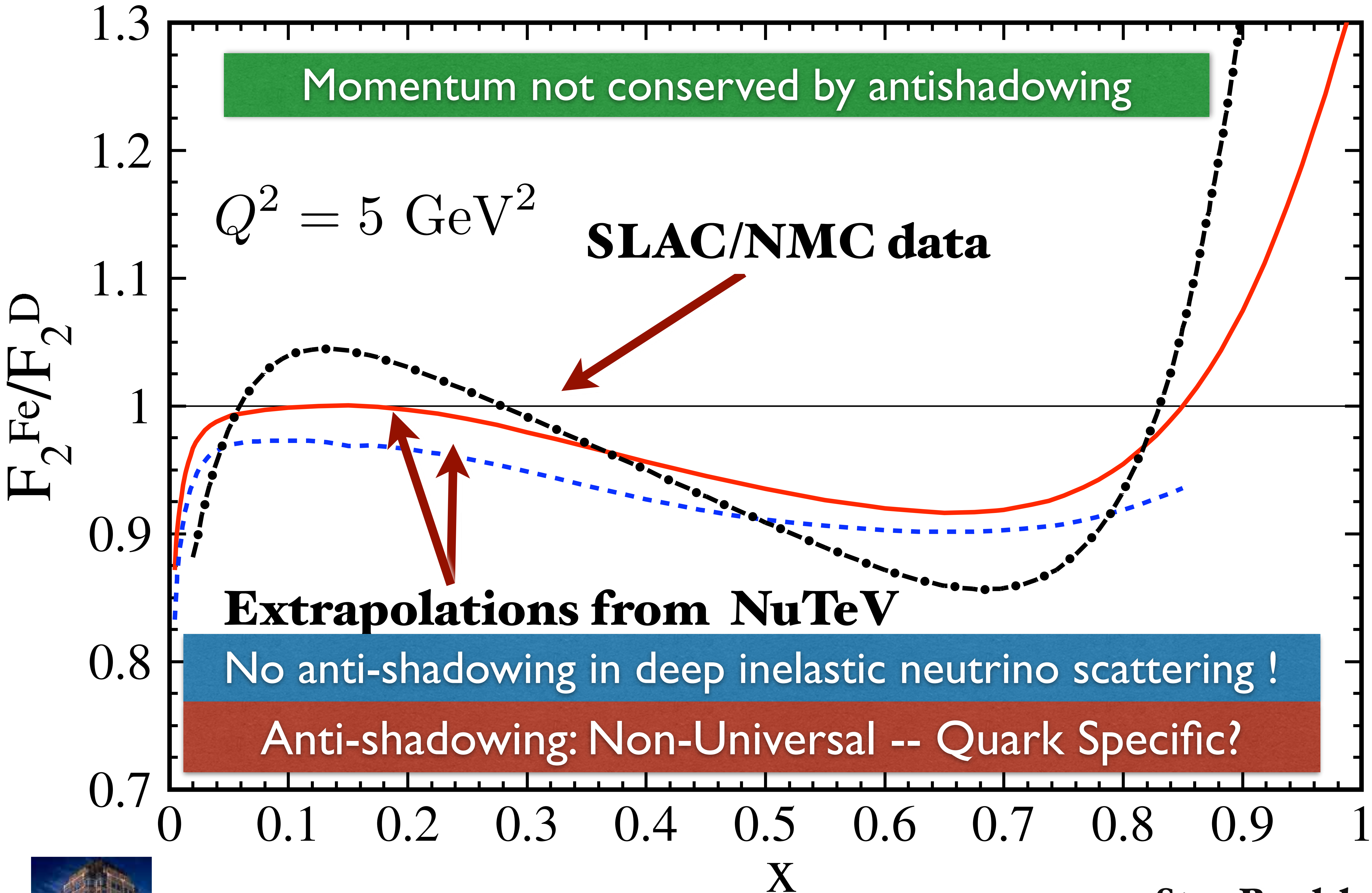
If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

$\rightarrow$ Shadowing of the DIS nuclear structure functions.

Diffraction via Pomeron Exchange gives destructive interference

Shadowing

Scheinbein, Yu, Keppel, Morfin, Olness, Owens

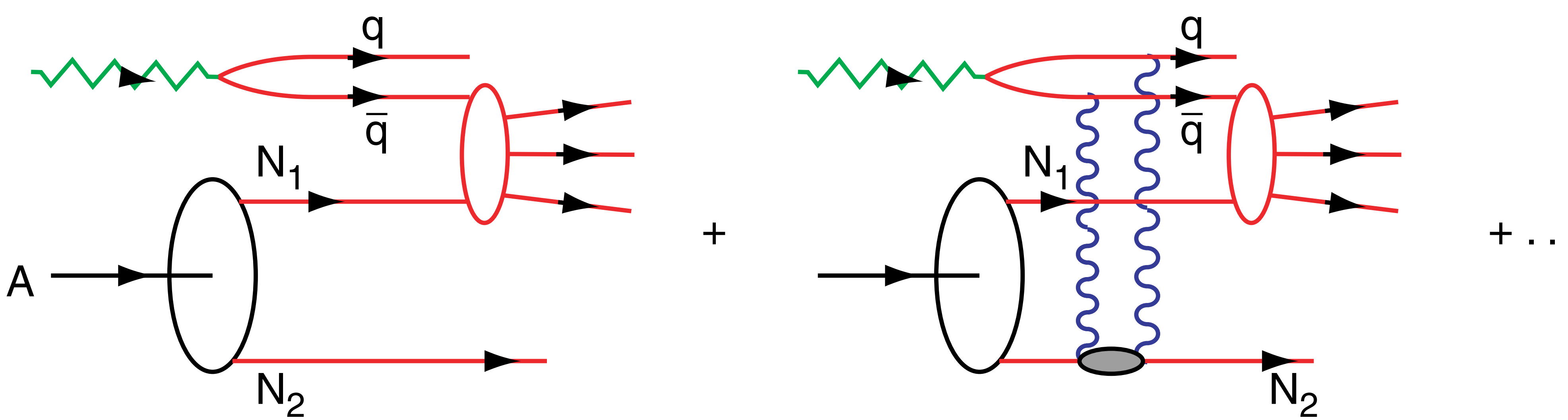


Are Momentum, Flavor, and Spin Sum Rules Valid for Nuclear PDFs?

- Conversations with Simonetta Liuti and Paul Hoyer

Stodolsky
Pumplin, sjb
Gribov

Nuclear Shadowing in QCD



Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus

Diffraction via Reggeon gives constructive interference!

Anti-shadowing not universal

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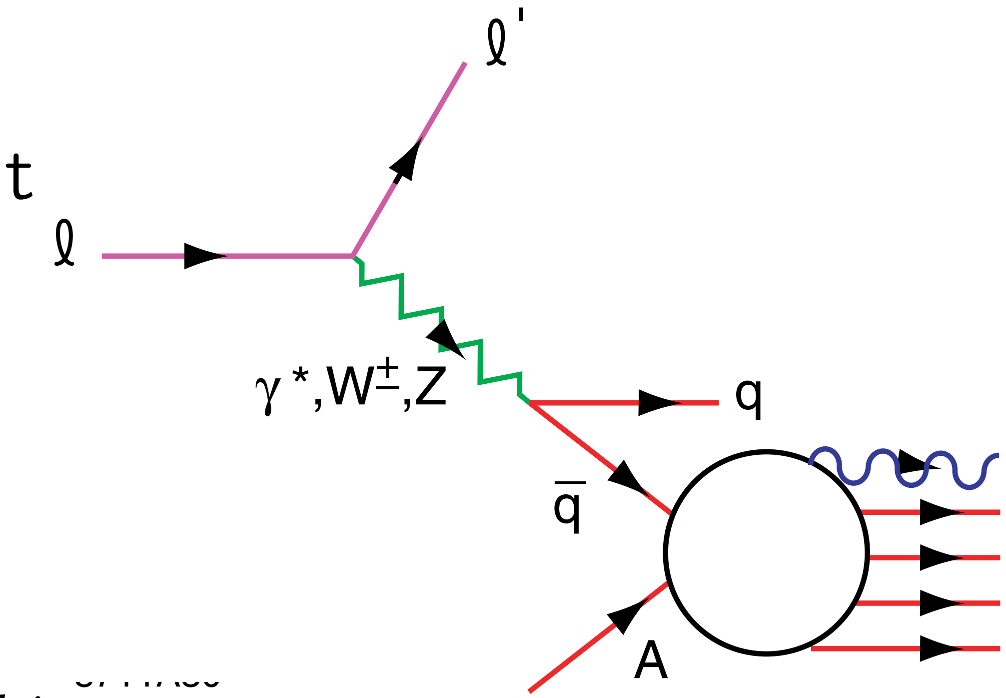
Origin of Regge Behavior of Deep Inelastic Structure Functions

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$

Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$



Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

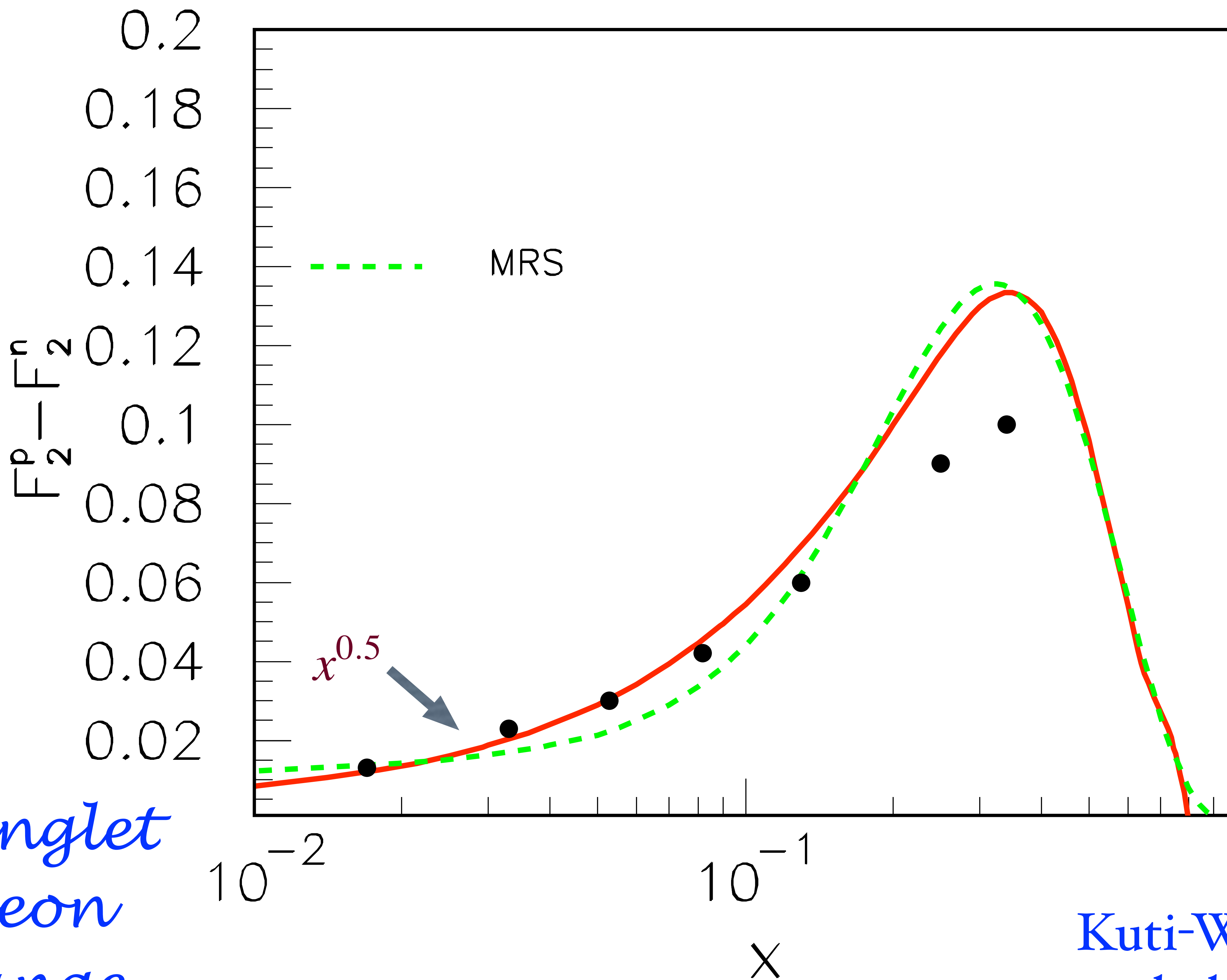
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$ at small x_{bj}.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Schmidt, Yang, Lu, sjb



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i)\times i = \frac{1}{\sqrt{2}}(i+1)$$

Constructive Interference

Depends on quark flavor!

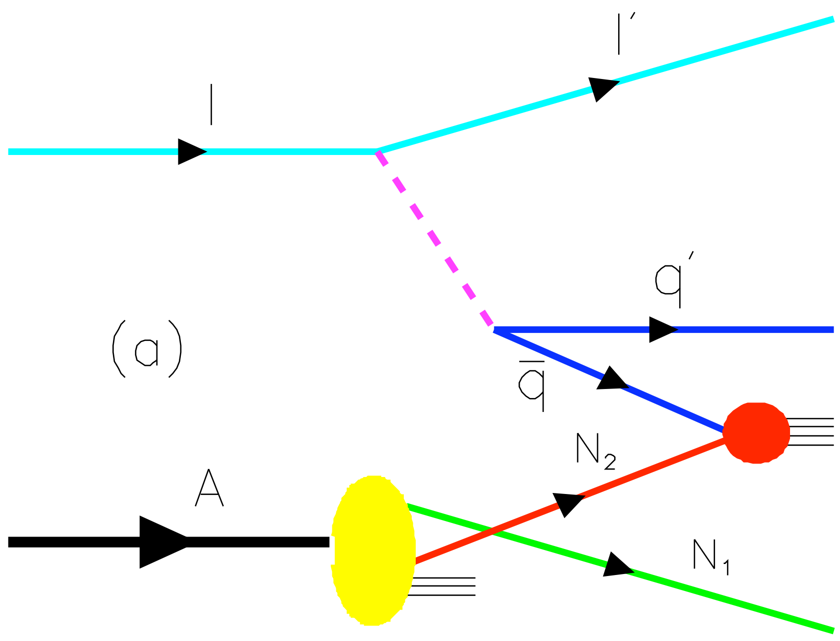
Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

test: Tagged Drell-Yan

test: scaling of charge exchange DDIS $\gamma^* p \to V^* n$

H. J. Lu, sjb



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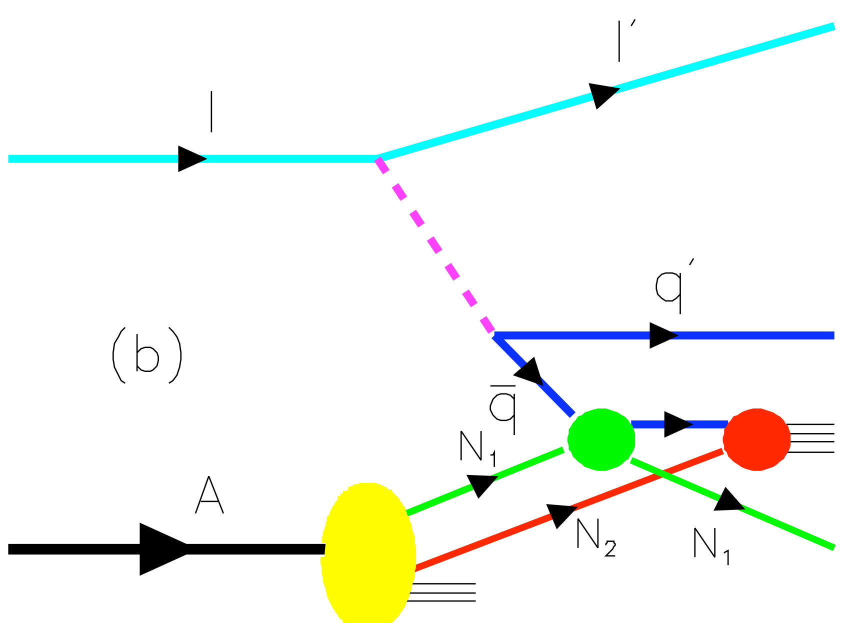


Regge

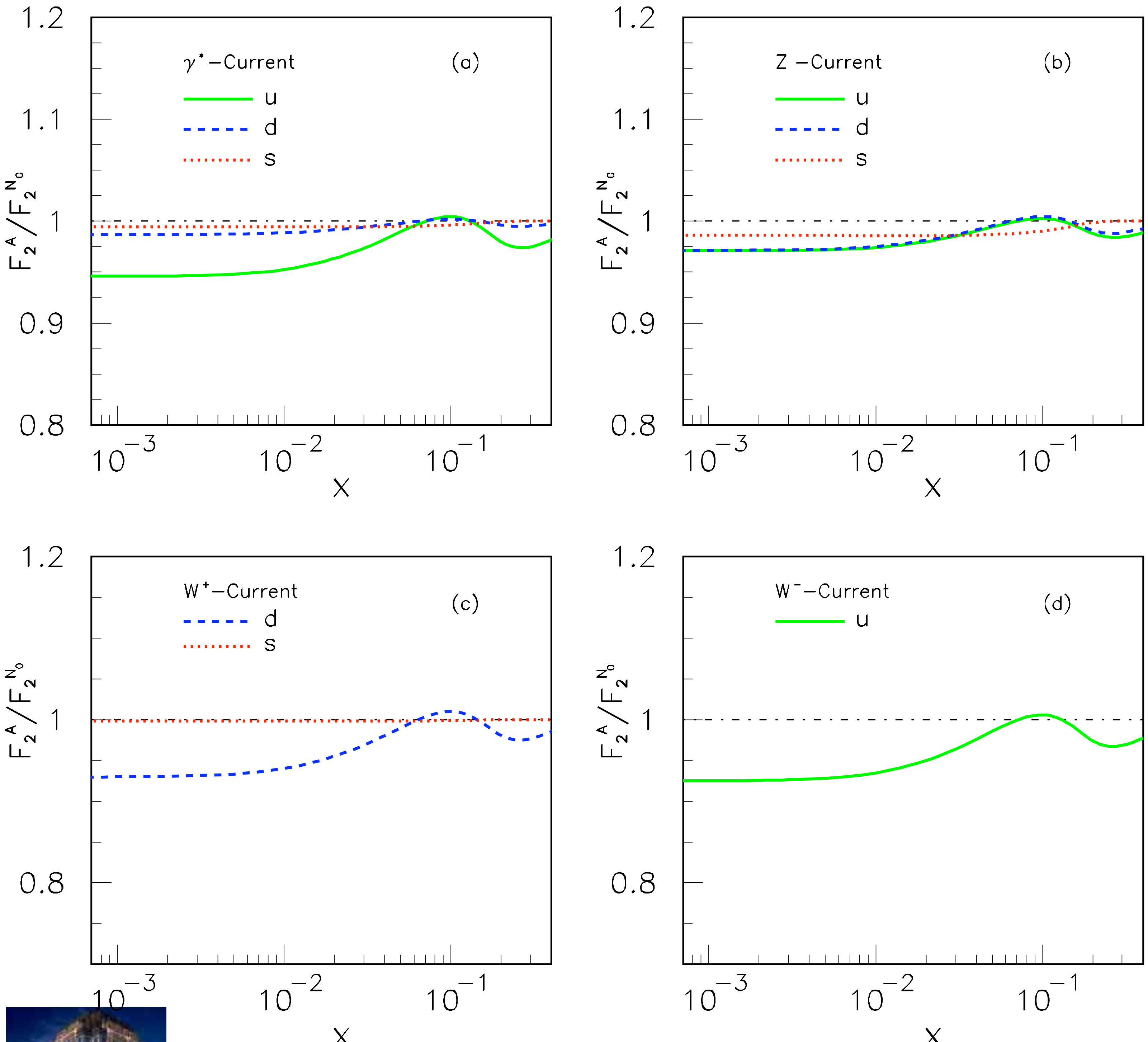
If the scattering on nucleon N_1 is via ~~pomeron~~ exchange, the one-step and two-step amplitudes are ~~opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.~~

constructive in phase

thus increasing the flux reaching N_2

Reggeon DDIS produces nuclear flavor-dependent anti-shadowing

Shadowing and Antishadowing of DIS Structure Functions

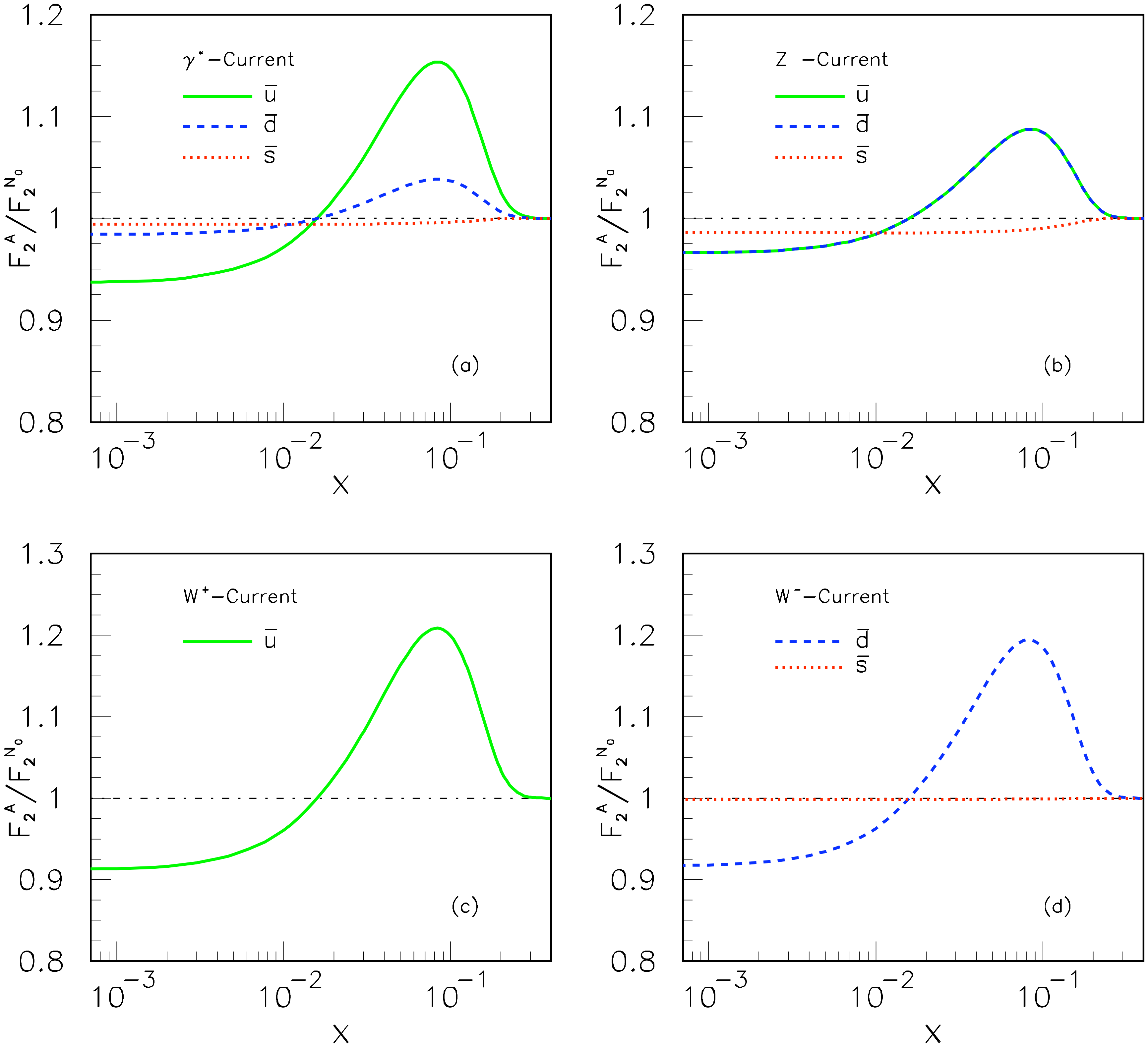


S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of

$$\sin^2\theta_W$$

Test in flavor-tagged lepton-nucleus collisions



Schmidt, Yang; sjb

Modifies NuTeV extraction of

$$\sin^2 \theta_W$$

Test in flavor-tagged DIS at the EIC

Nuclear Antishadowing not universal !

Factorization Issues and Light-Front Holographic QCD

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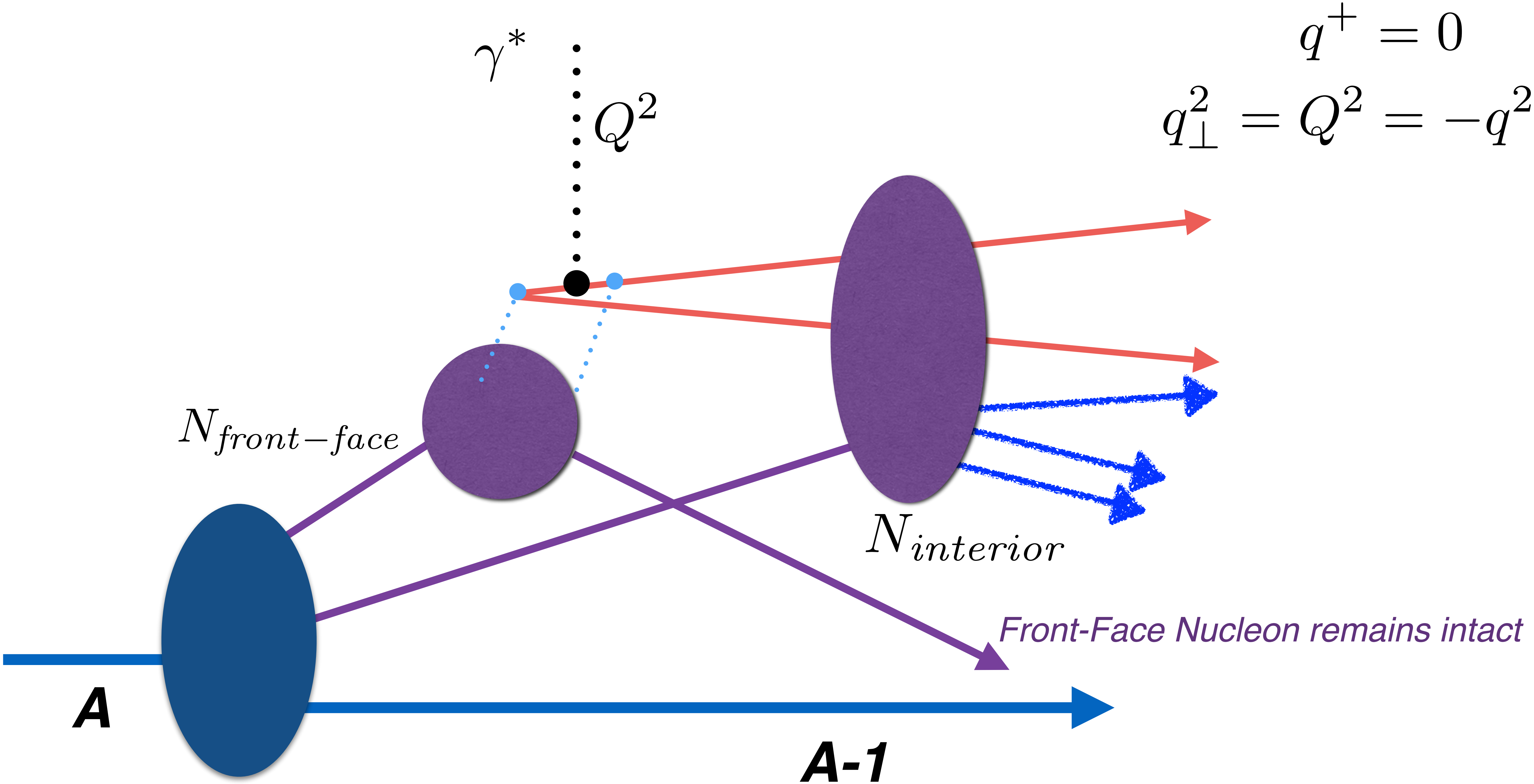
I=0 Pomeron exchange

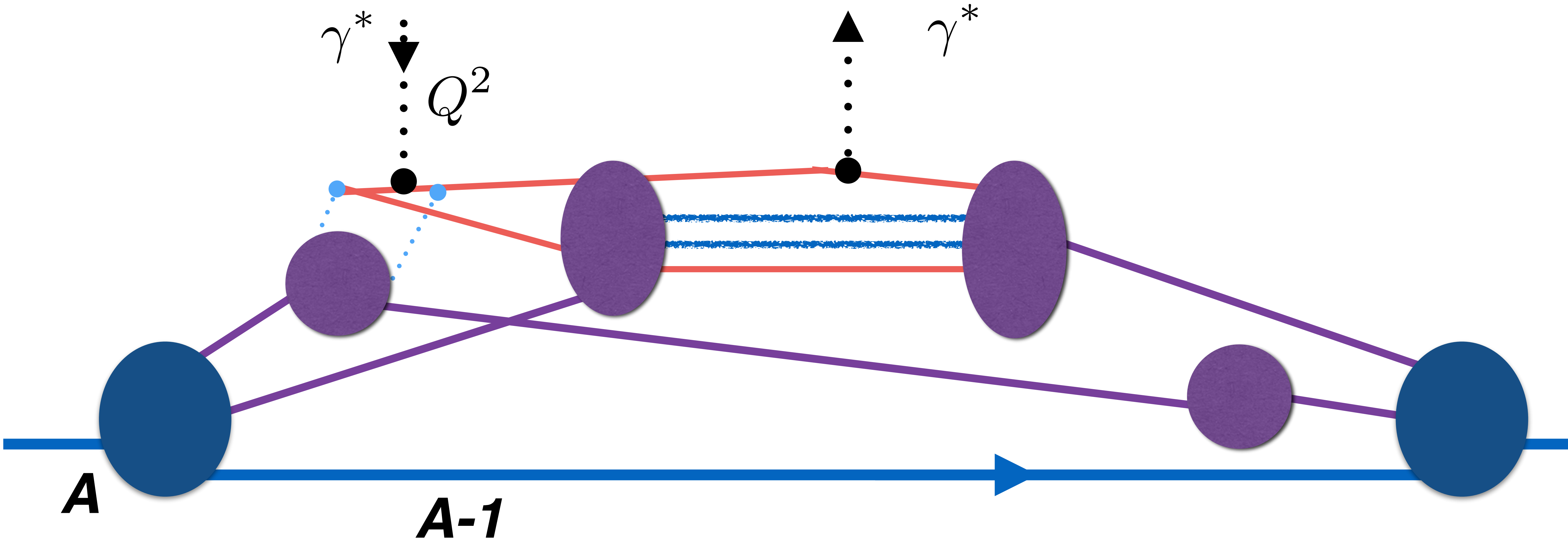


Two-Step Process in the q^+=0 Parton Model Frame

Illustrates the LF time sequence

Illustrates the LF time sequence

$$q^+ = 0 \quad q_\perp^2 = Q^2 = -q^2$$



Front-Face Nucleon struck

Front-Face Nucleon not struck

One-Step / Two-Step Interference

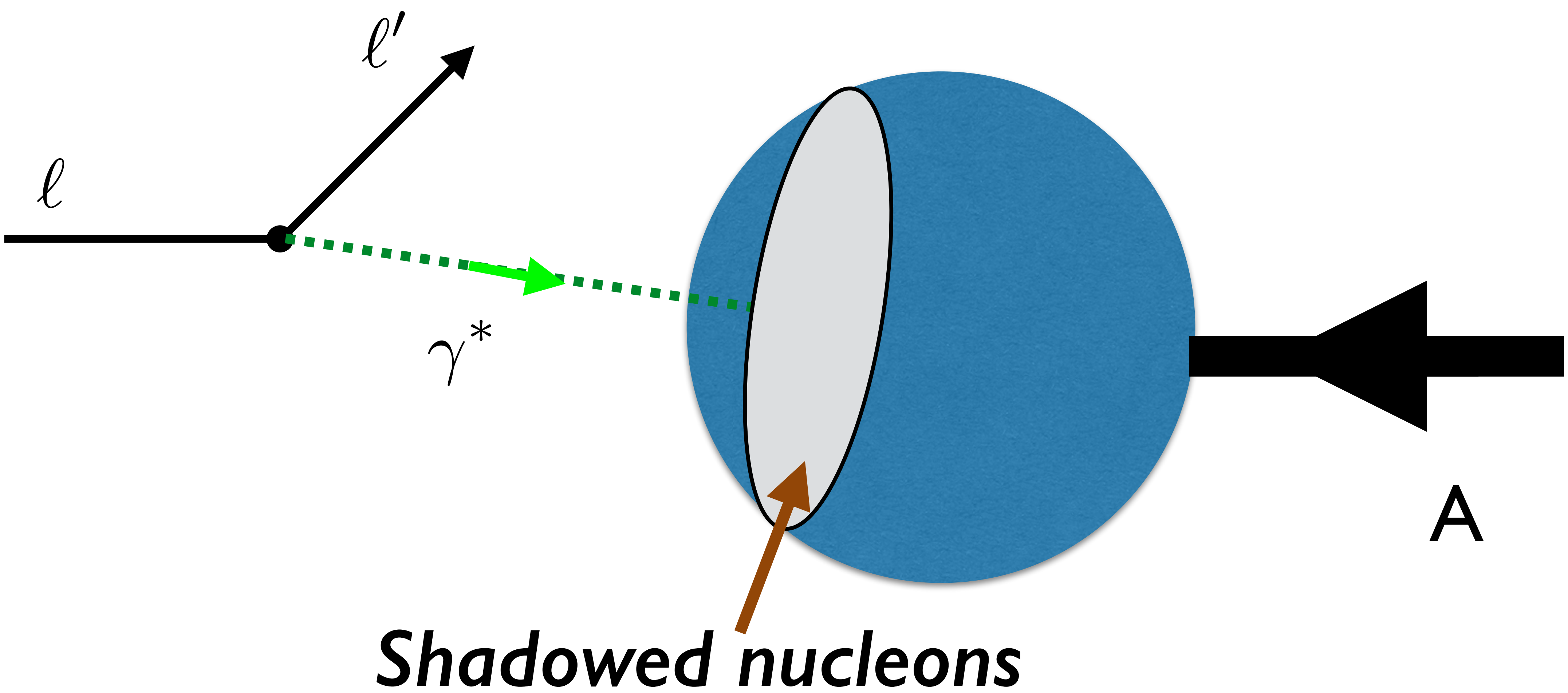
Study Double Virtual Compton Scattering $\gamma^* A \rightarrow \gamma^* A$

Cannot reduce to matrix element of local operator

Momentum and Spin Sum Rules not proven

Shadowed nucleons not exposed to photon beam

Shadowing domain is geometrically oriented toward photon beam



Light-Front Wavefunction (QCD Eigensolution) independent of beam direction!

The GPD's are non-forward matrix elements of the PDF operator:

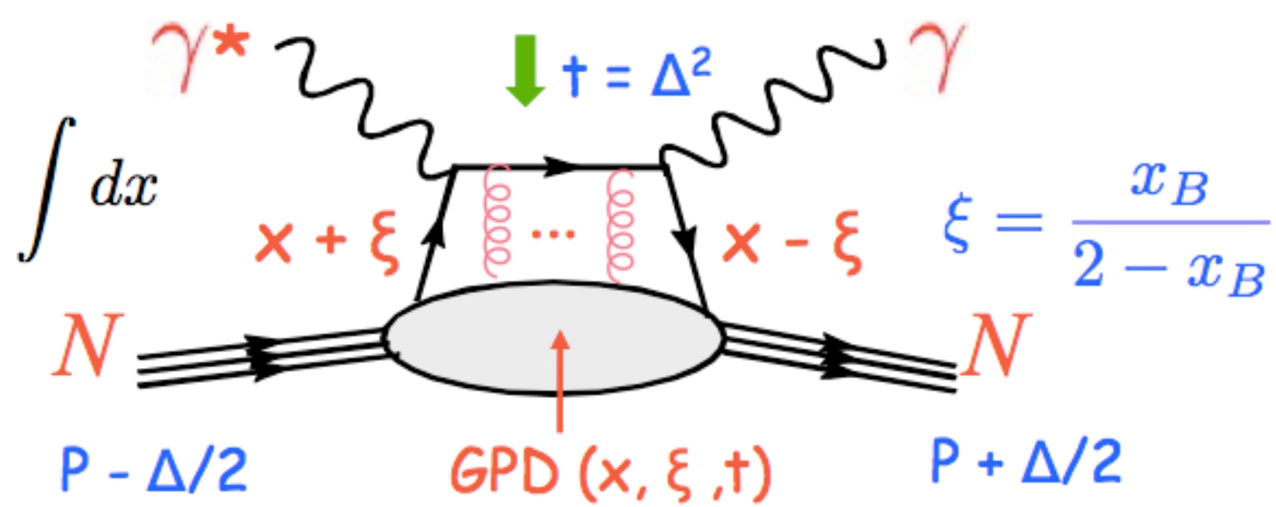
$$\frac{1}{8\pi}\int dr^- e^{imxr^-/2}\langle P+\tfrac{1}{2}\Delta|\bar{q}(-\tfrac{1}{2}r)\gamma^+ W[\tfrac{1}{2}r^-,-\tfrac{1}{2}r^-]q(\tfrac{1}{2}r)|P-\tfrac{1}{2}\Delta\rangle_{r^+=r_\perp=0}$$

$$=\frac{1}{2P^+}\bar{u}(P+\tfrac{1}{2}\Delta)\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)i\sigma^{+\nu}\frac{\Delta_\nu}{2m}\right]u(P-\tfrac{1}{2}\Delta)$$

The GPD amplitudes can be accessed experimentally through the Deeply Virtual Compton Scattering cross section at leading twist: $Q^2 \to \infty$.

DVCS: $e\,N \to e' + \gamma + N$

Through $\Delta_\perp$, the GPD's contain information about the parton distributions in transverse space.



Handbag modified by leading-twist lensing

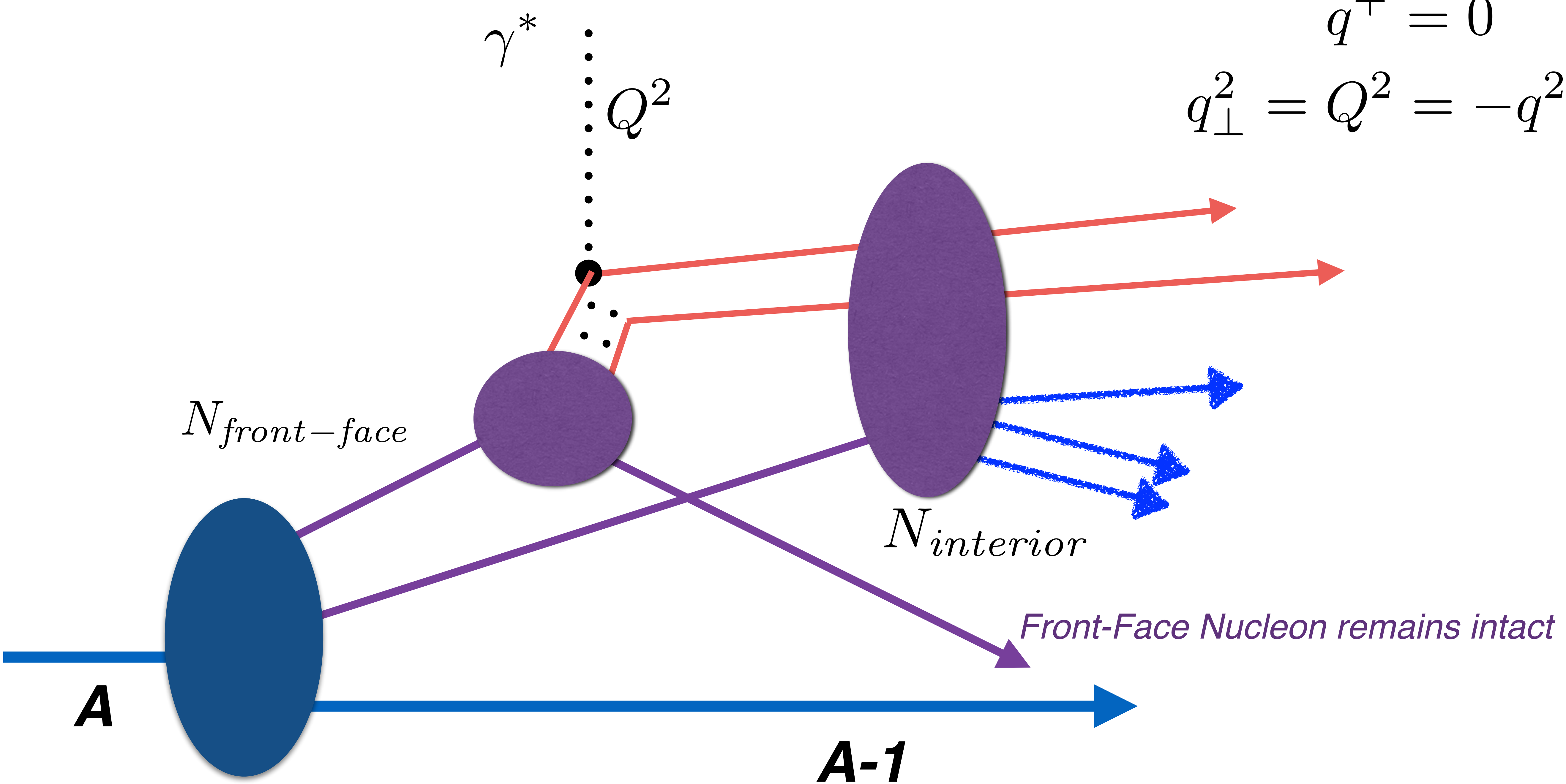
However, Shadowing involves multiple nucleons — no OPE

Momentum and Spin Sum Rules not proven!

Sum Rules are Properties of LFWFs (QCD Eigensolutions of H_{LF})

- Nuclear PDFs modified by Shadowing and Antishadowing — Physical Effects not in LFWF
- Shadowed Nucleons are Geometrically Oriented Relative to Beam — no knowledge in LFWF
- Antishadowing is Flavor-Specific — cannot balance flavor-symmetric shadowing
- Sum Rules evidently not valid for nuclear PDFs!
- Measure Nuclear DVCS and Interference with Bethe-Heitler $\quad \gamma^* A \to \gamma A'$

I=1 Reggeon exchange



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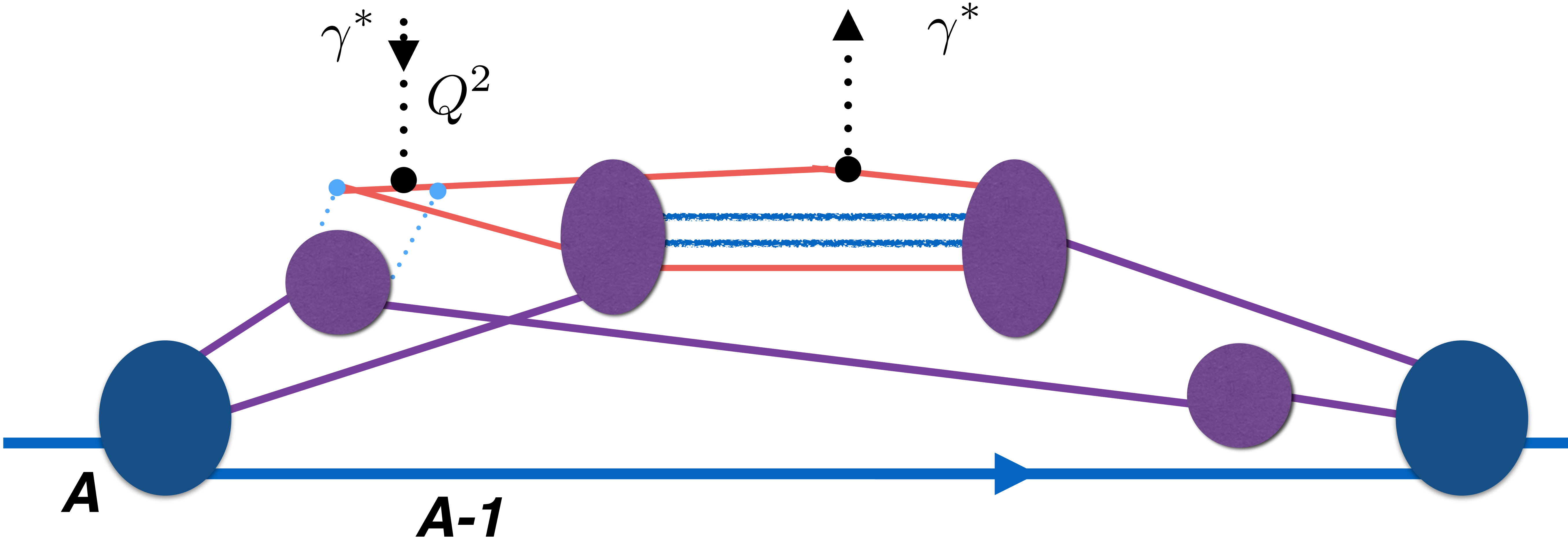

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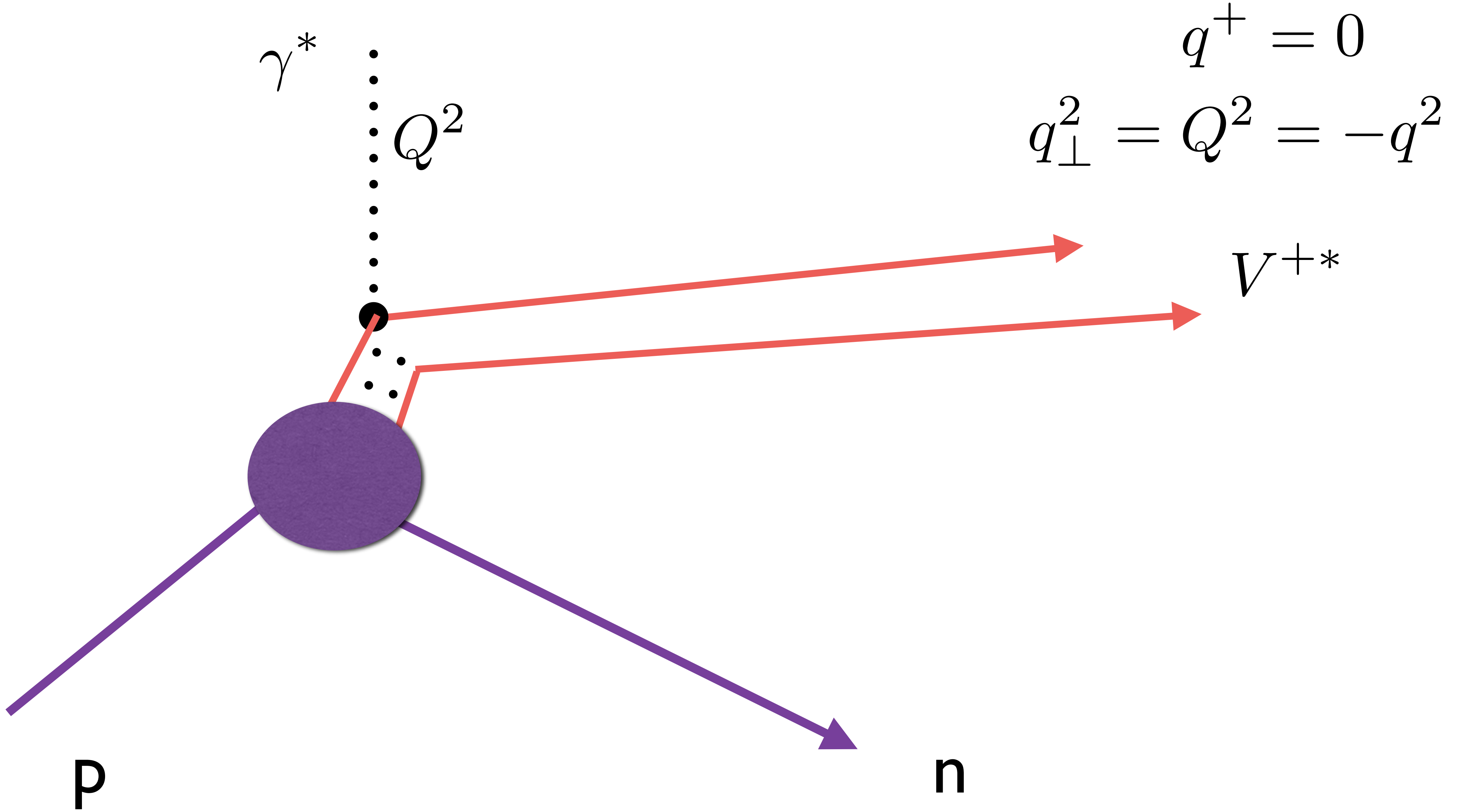
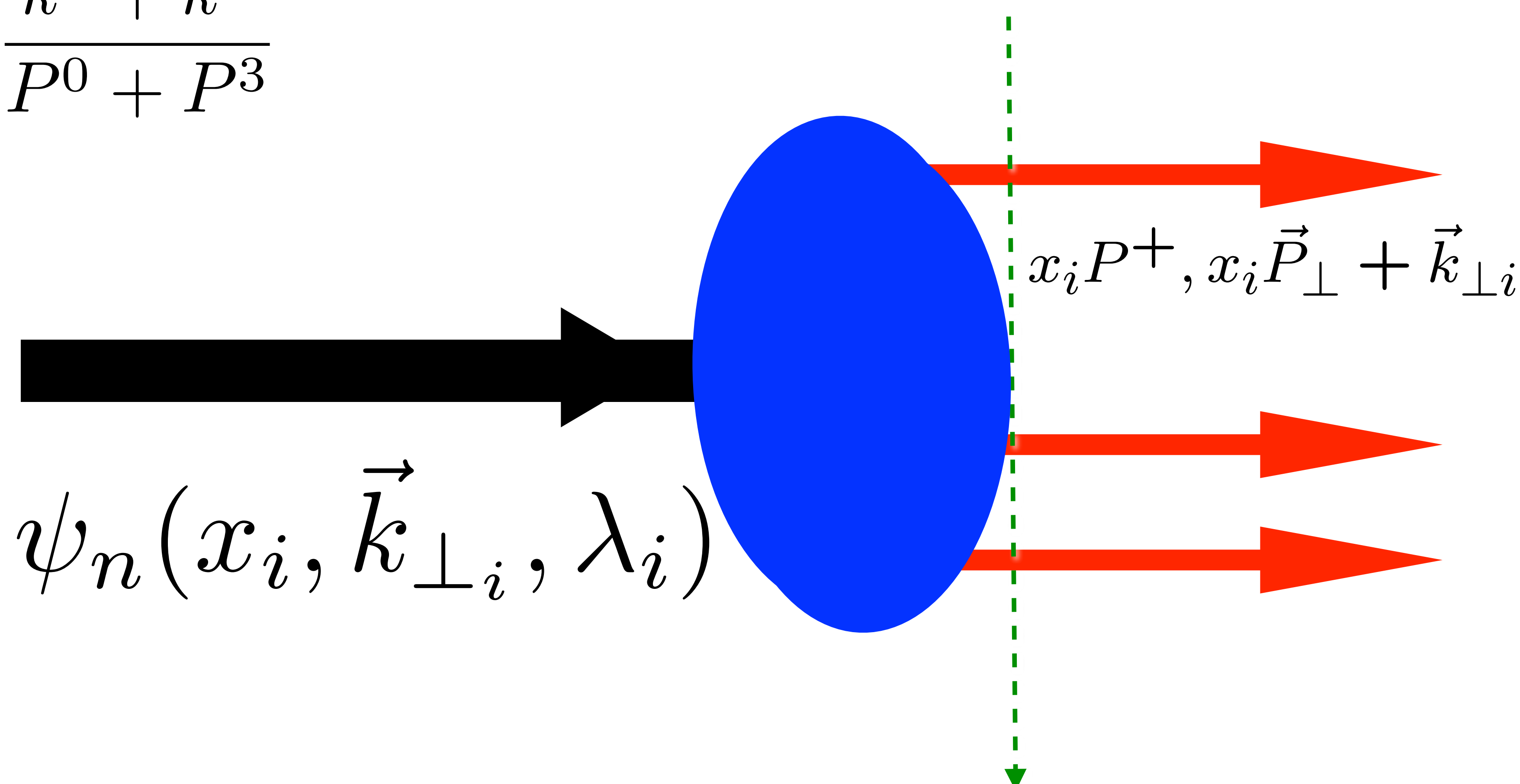


Test Bj scaling of Charge-Exchange DDIS $\gamma^* p \to V^{+*} n$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$|p, J_z >= \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

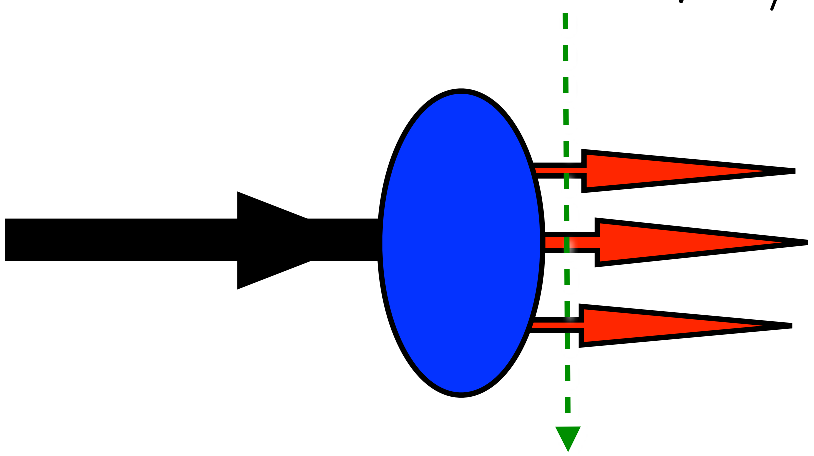
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

What do we know about hadronic LFWFs?

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

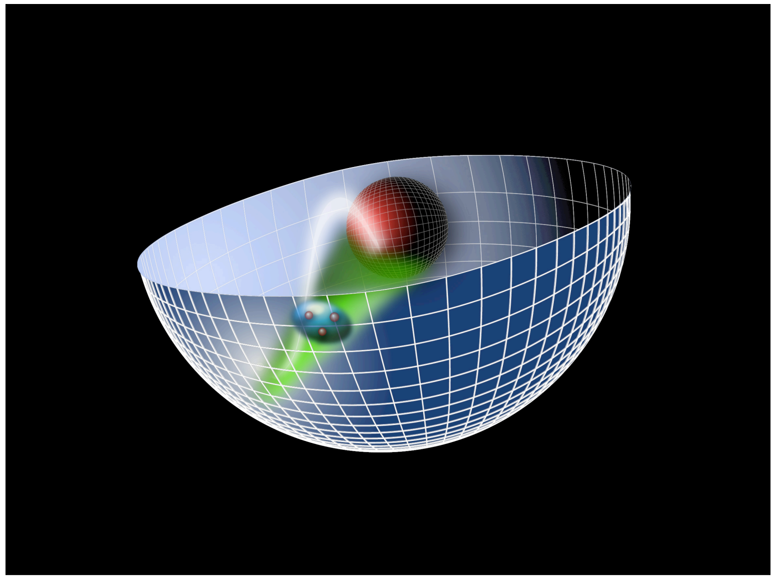
$$H^{QCD}_{LF} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$\phi(z)$

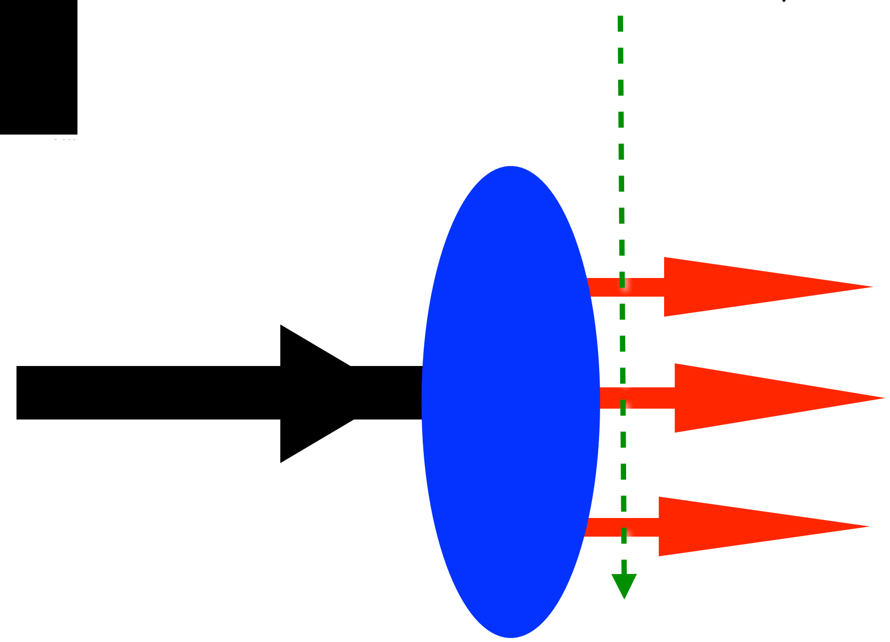


AdS5: Conformal Template for QCD

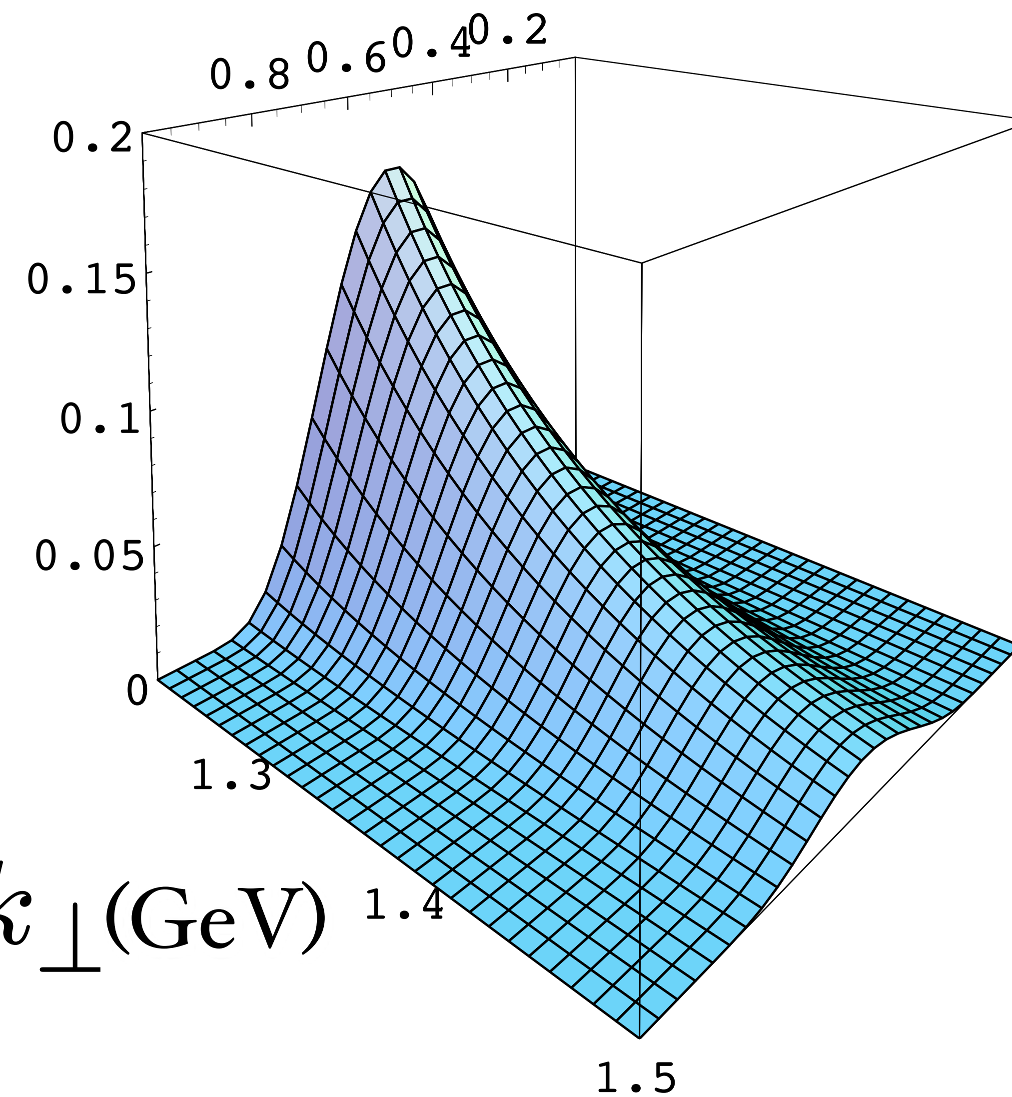
- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



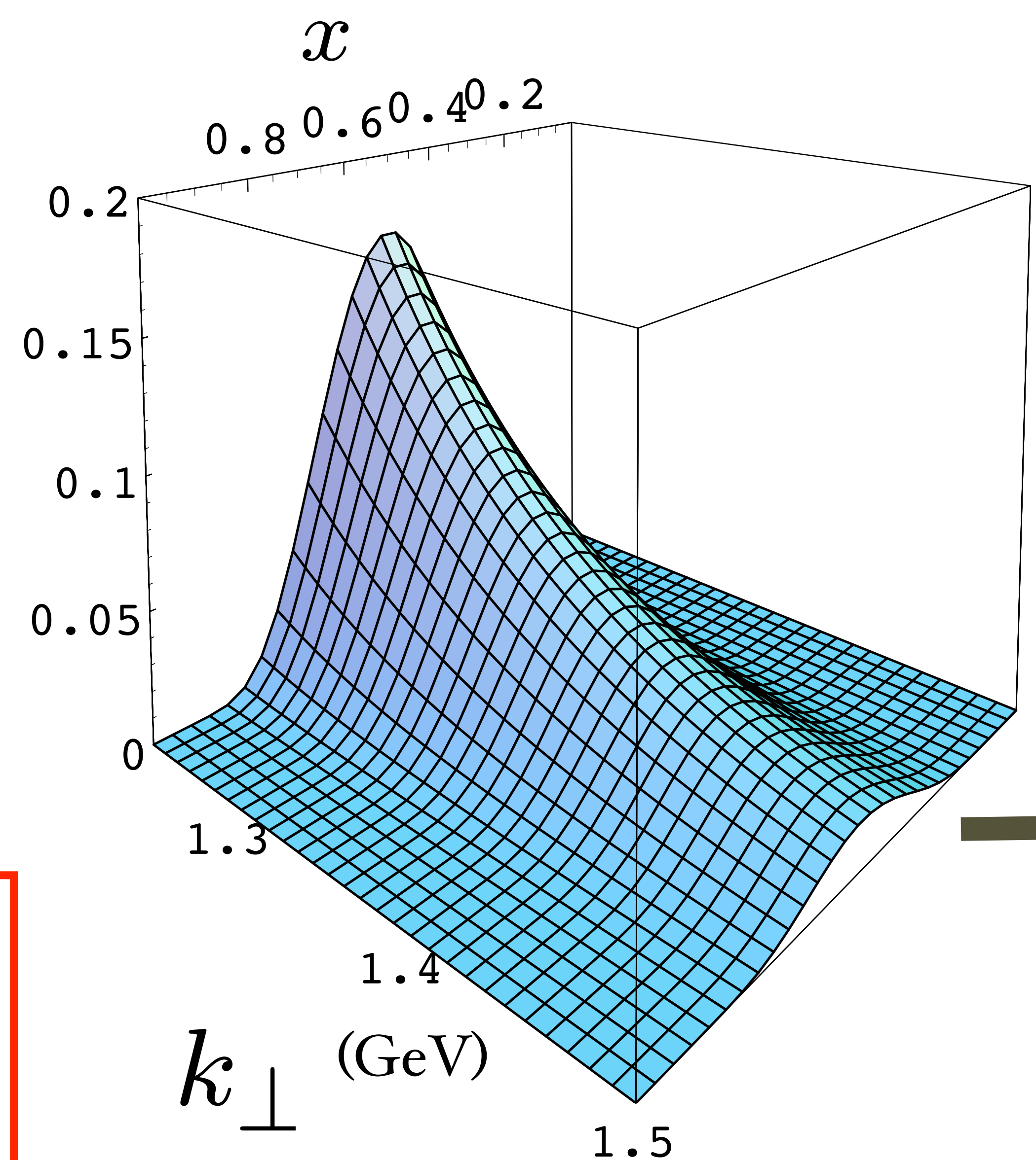
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

Prediction from AdS/QCD: Meson LFWF

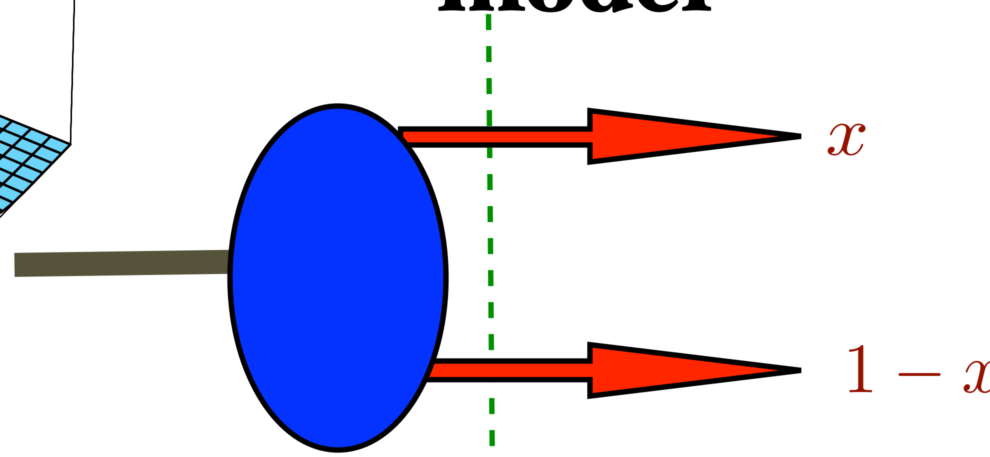
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



Note coupling

$$k_\perp^2, x$$

massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE!

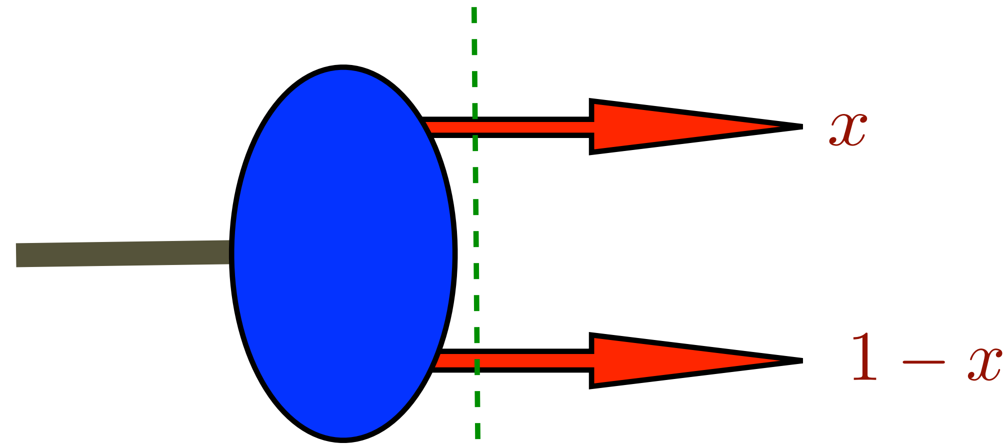
Provides Connection of Confinement to Hadron Structure

Hadron Distribution Amplitudes

$A^+ = 0$

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

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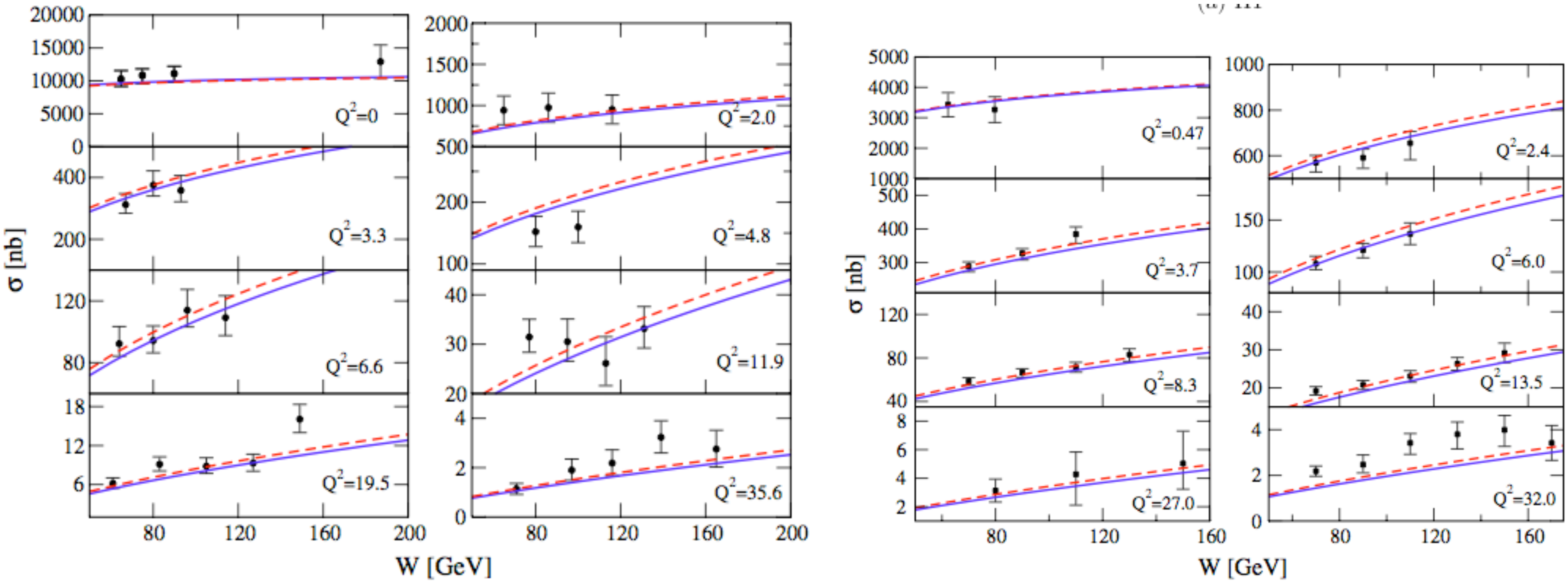
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

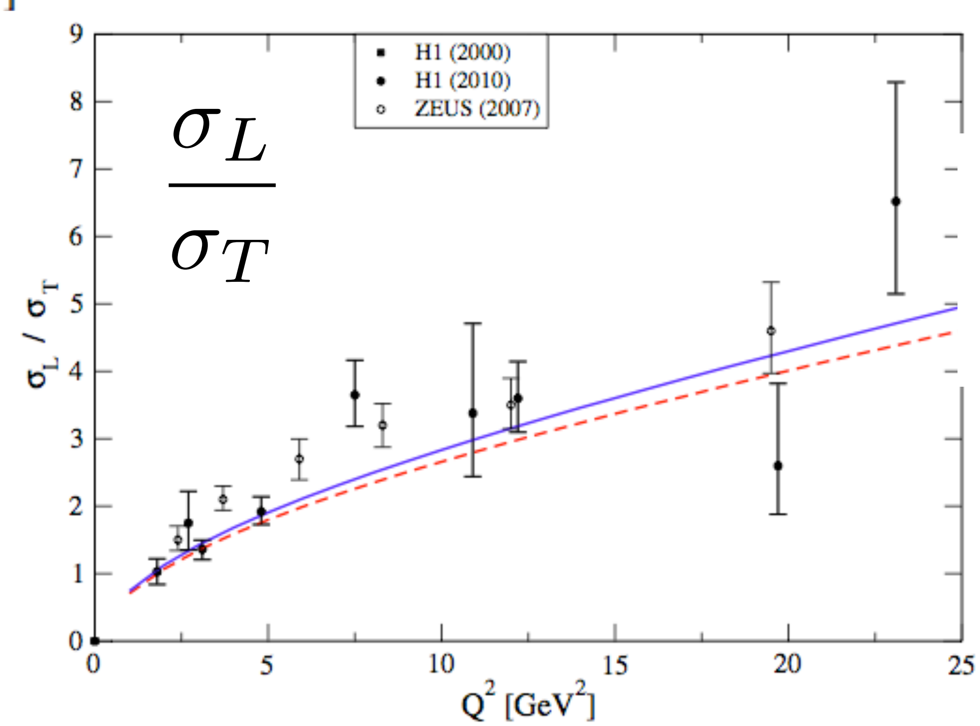
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H (b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\frac{\sigma_L}{\sigma_T}$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Dynamics + Spectroscopy!

Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) \; |\Psi >= E \; |\Psi >$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

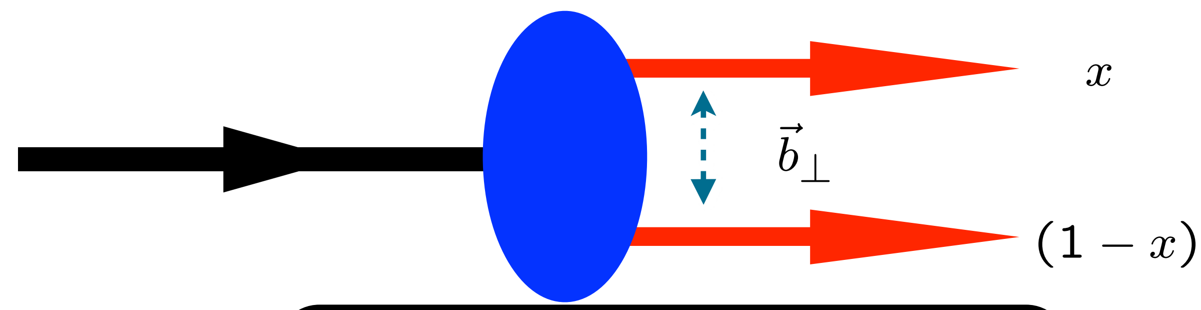
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \ \psi_{LF}(\zeta)$$

Azimuthal Basis

ζ, ϕ

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

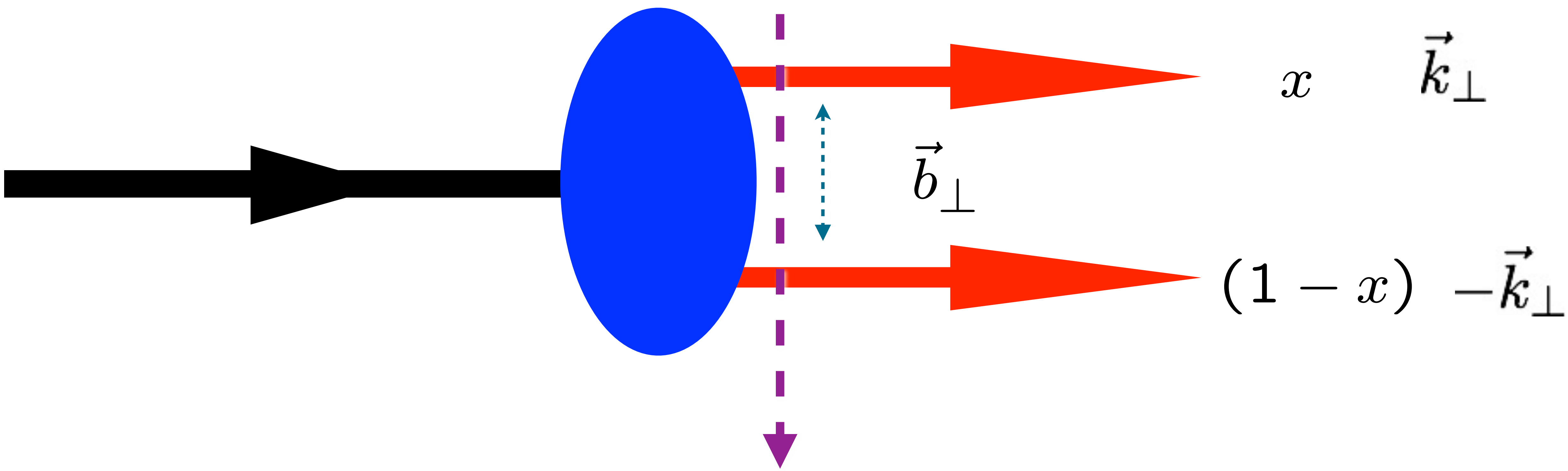
Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

de Tèramond, Dosch, sjb

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \to \psi_{LF}(x, \vec{k}_\perp)$$

Light-Front Schrödinger Equation

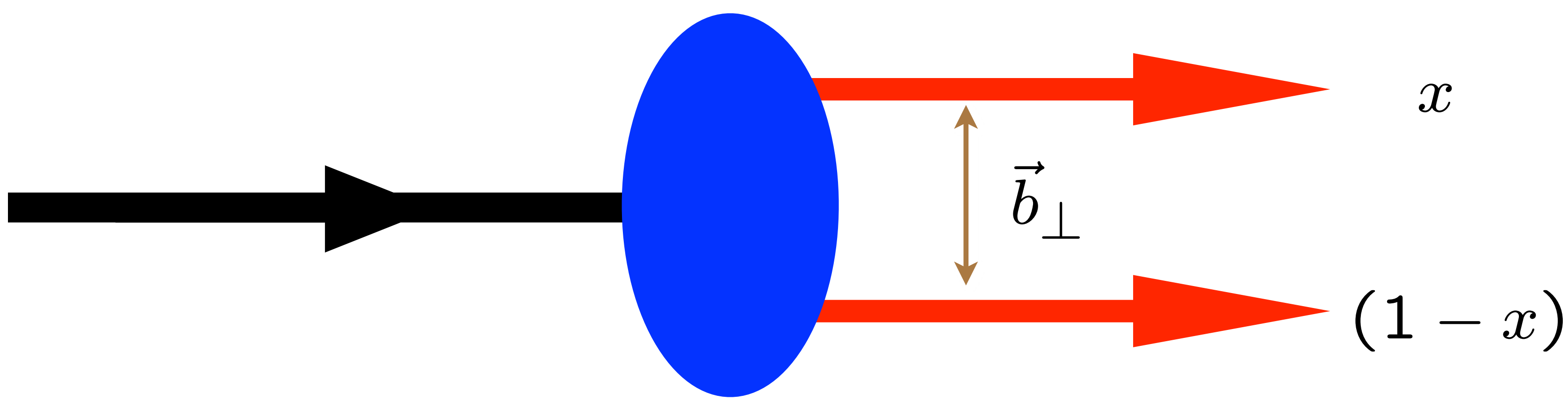
G. de Teramond, sjb

Relativistic LF radial equation for QCD & QED

Frame Independent!

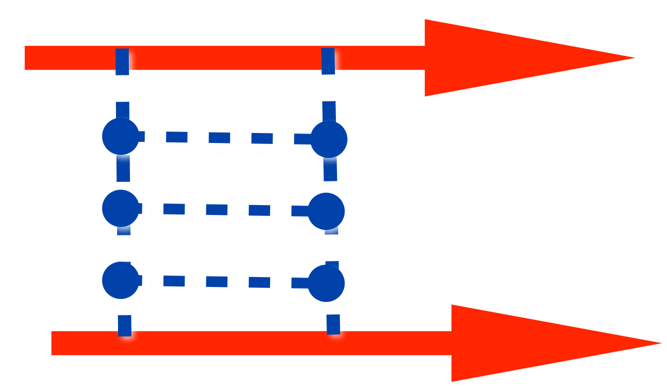
$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate

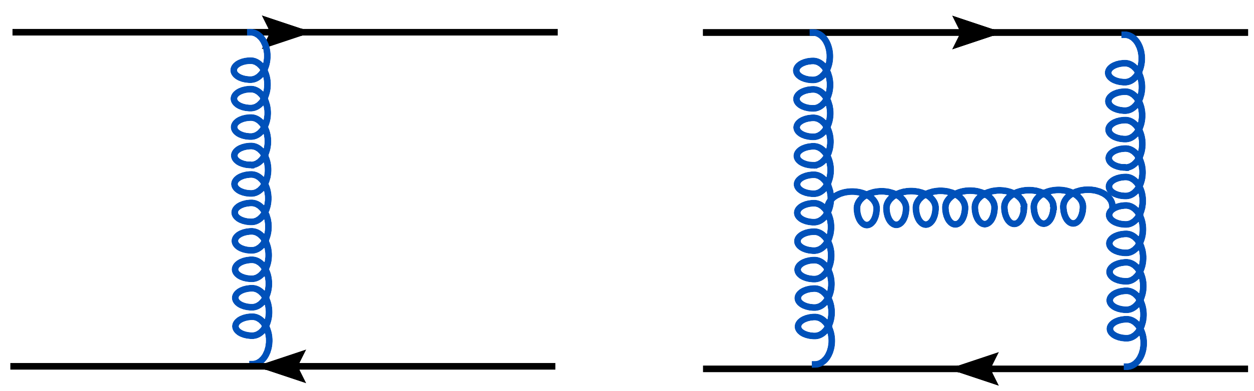
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$



Heavy Quark Potential is IR Divergent in pQCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$
$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

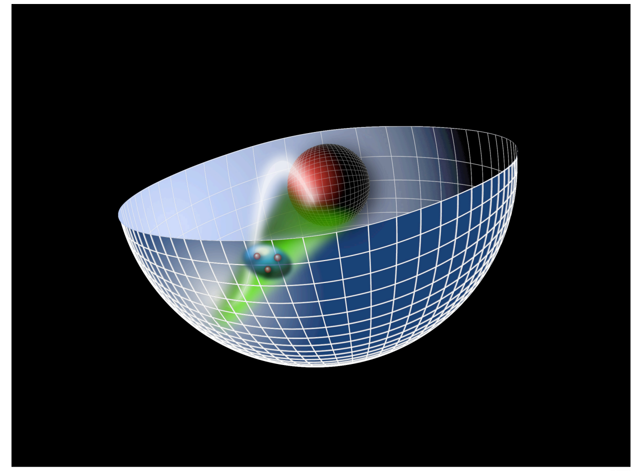
Confinement eliminates IR divergences

Self-consistent mass scale κ

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.5 \ \mathrm{GeV}$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta \, \phi^2(z)^2 = 1$

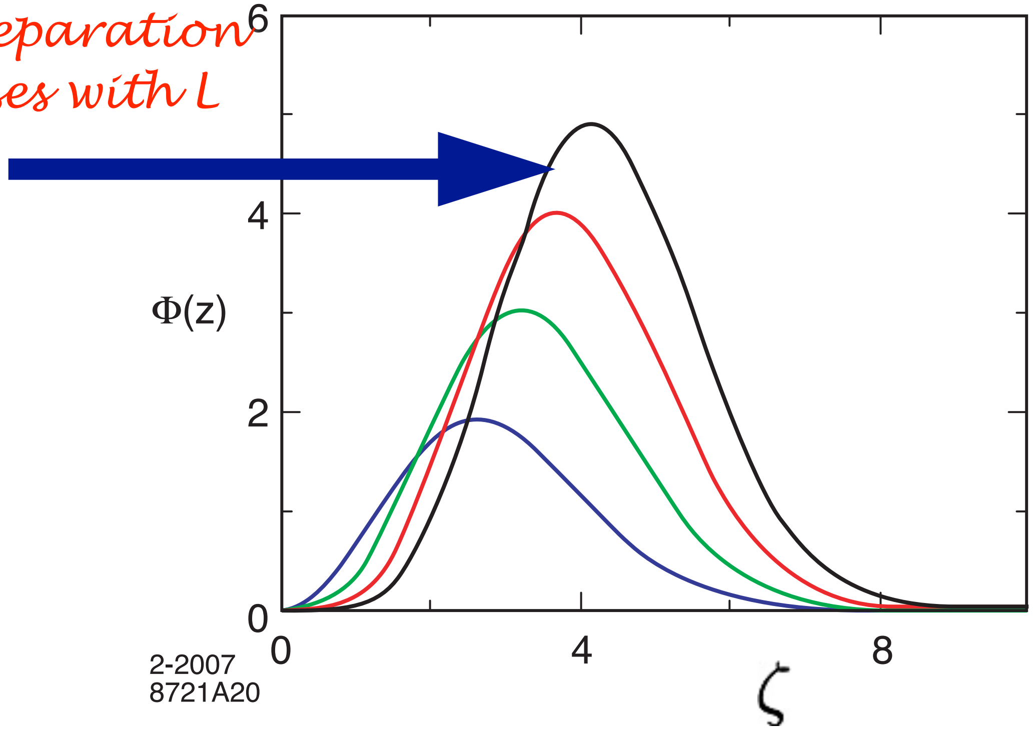
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L



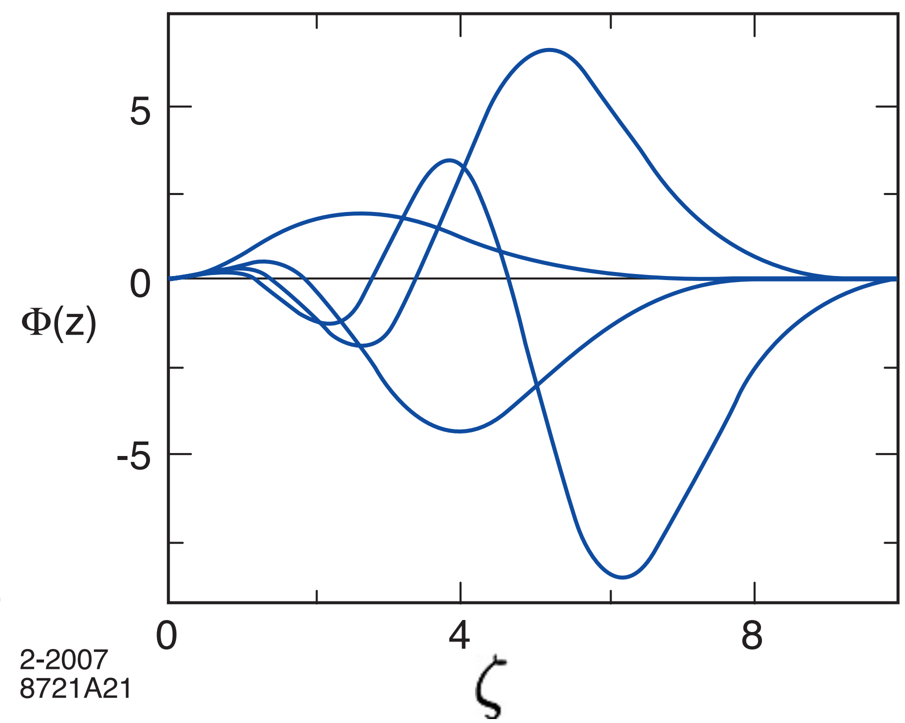
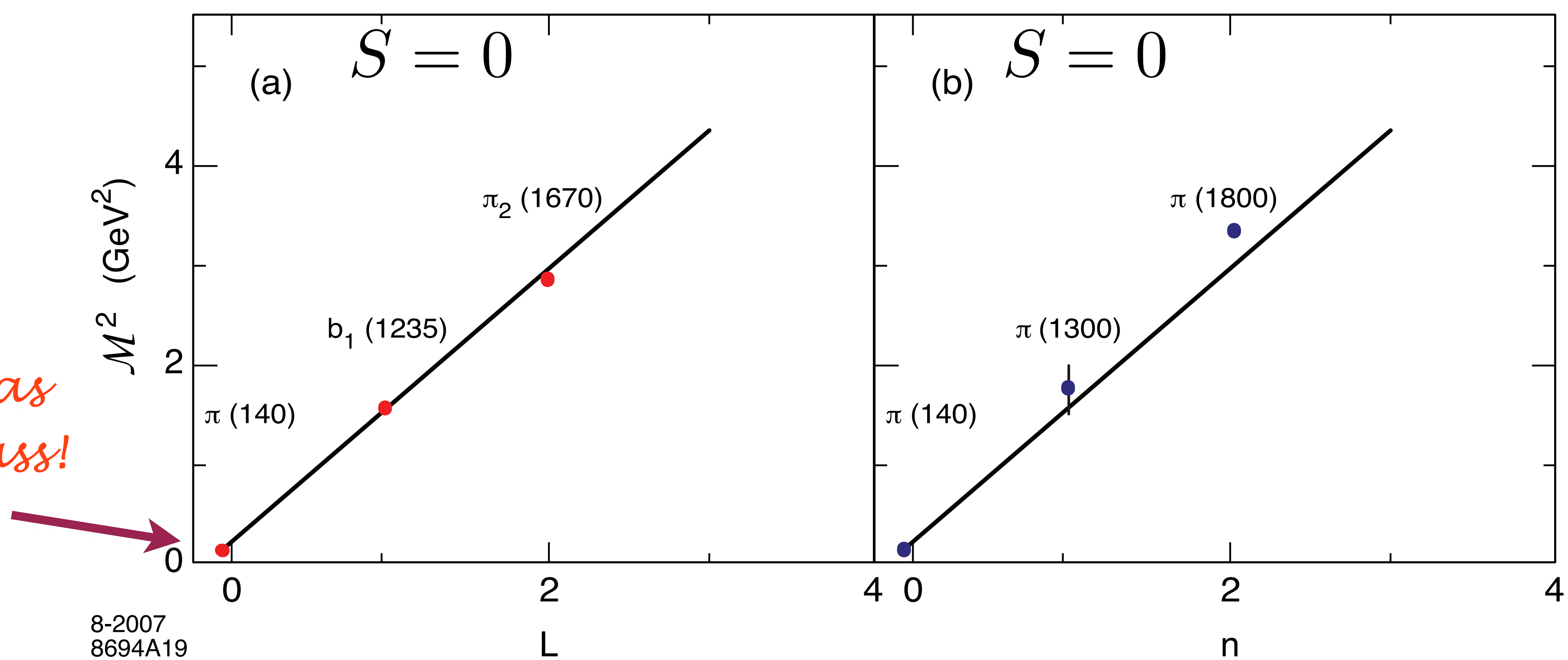


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!

Correct twist

Pion has zero mass!



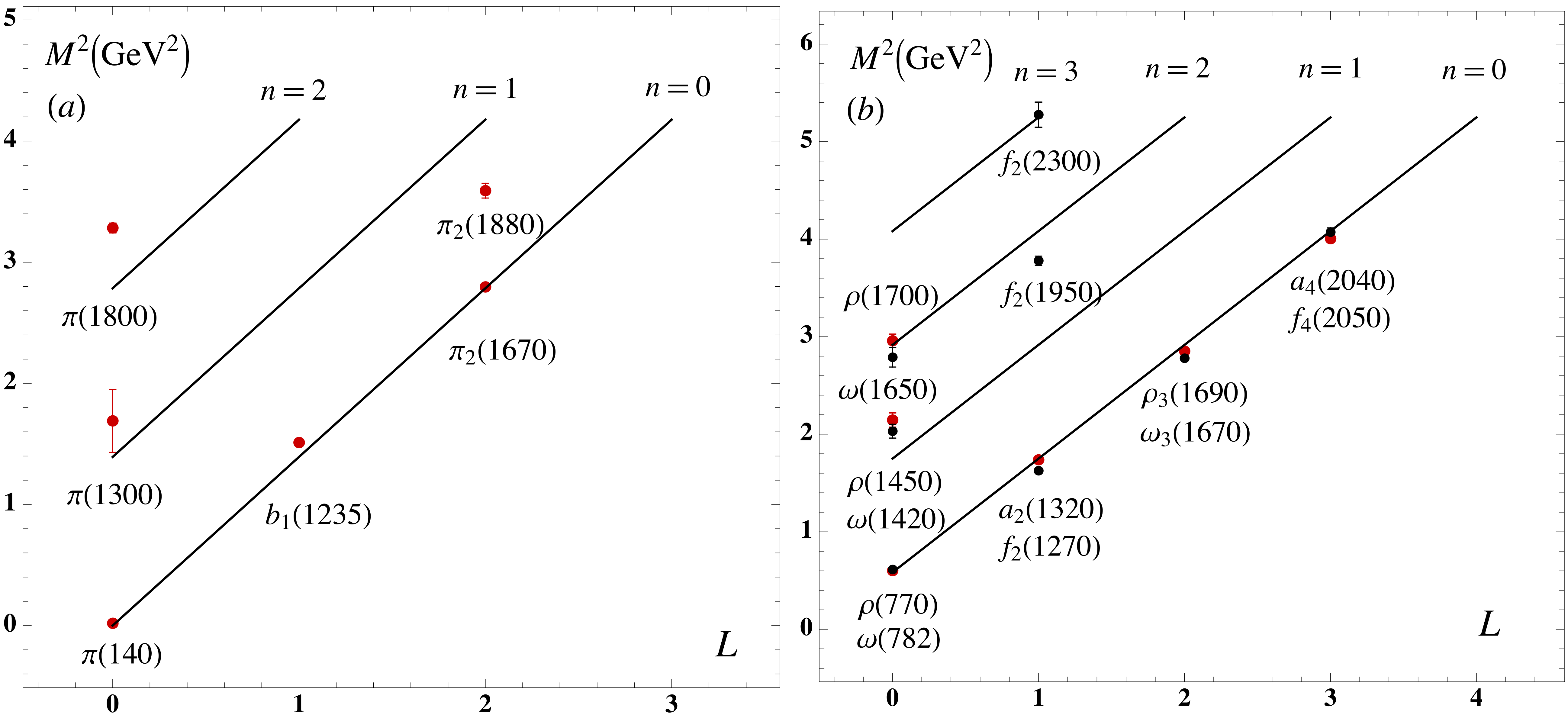
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

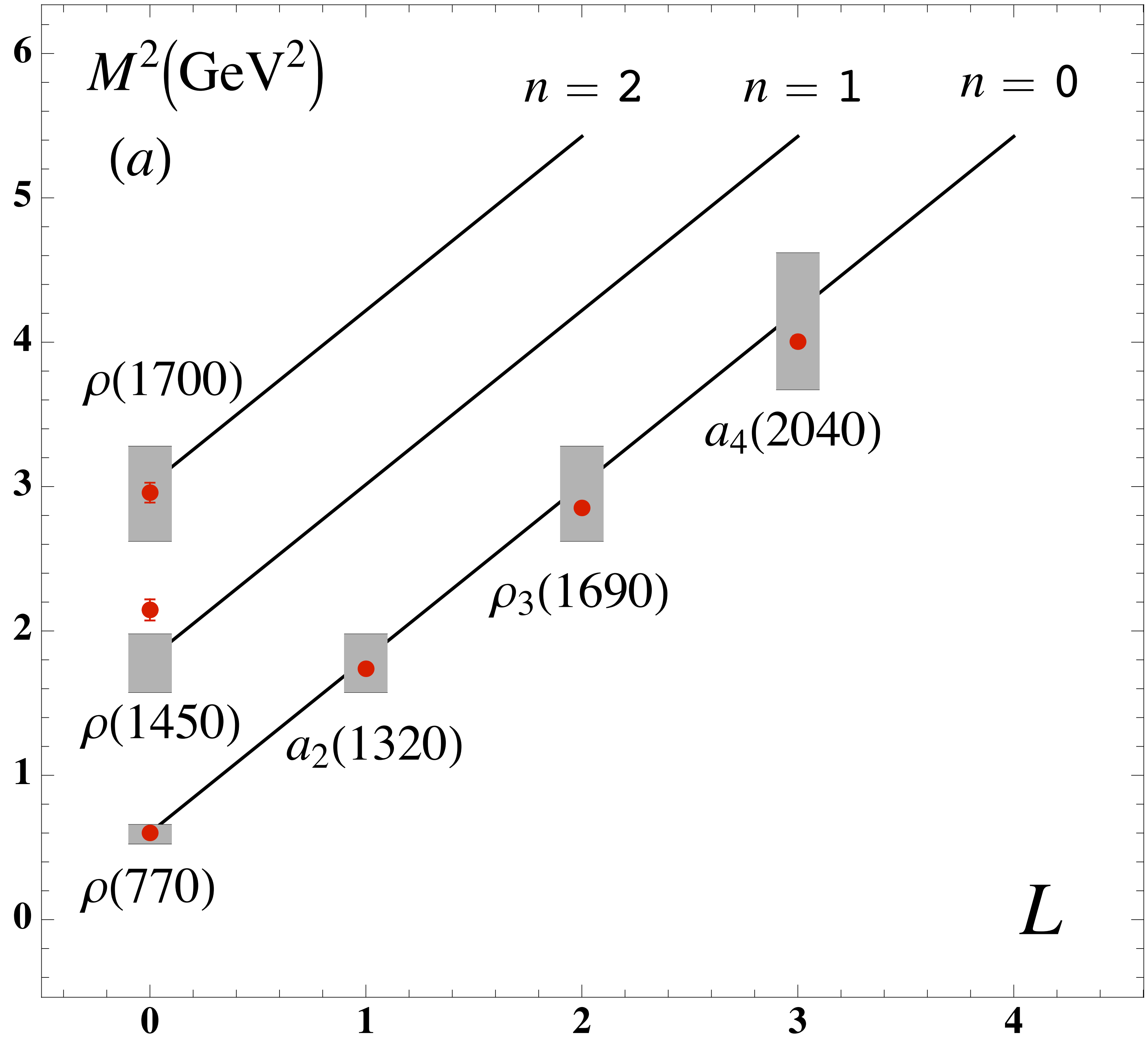
$$m_q = 0$$

$m_u = m_d = 0$

de Tèramond, Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

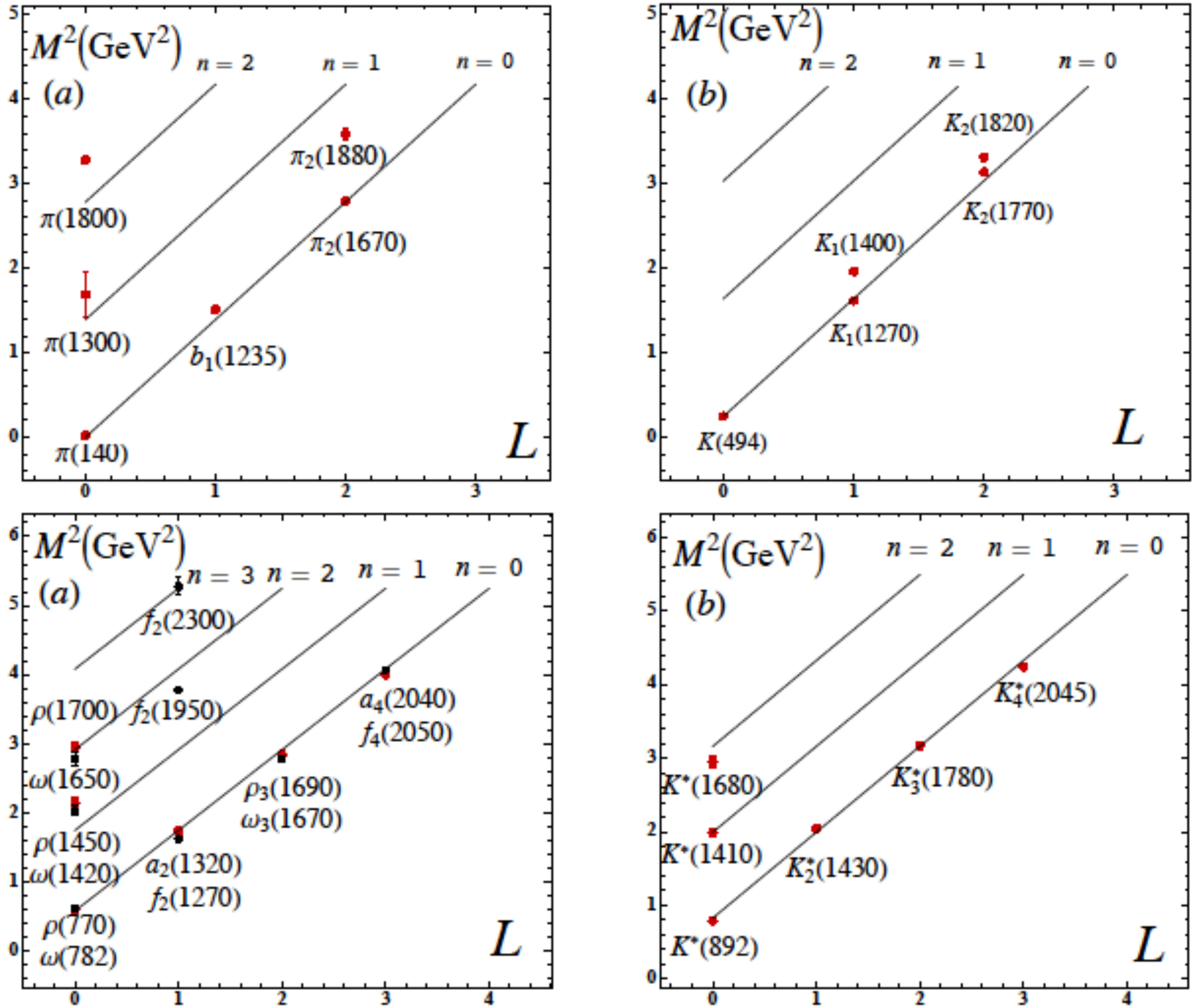


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

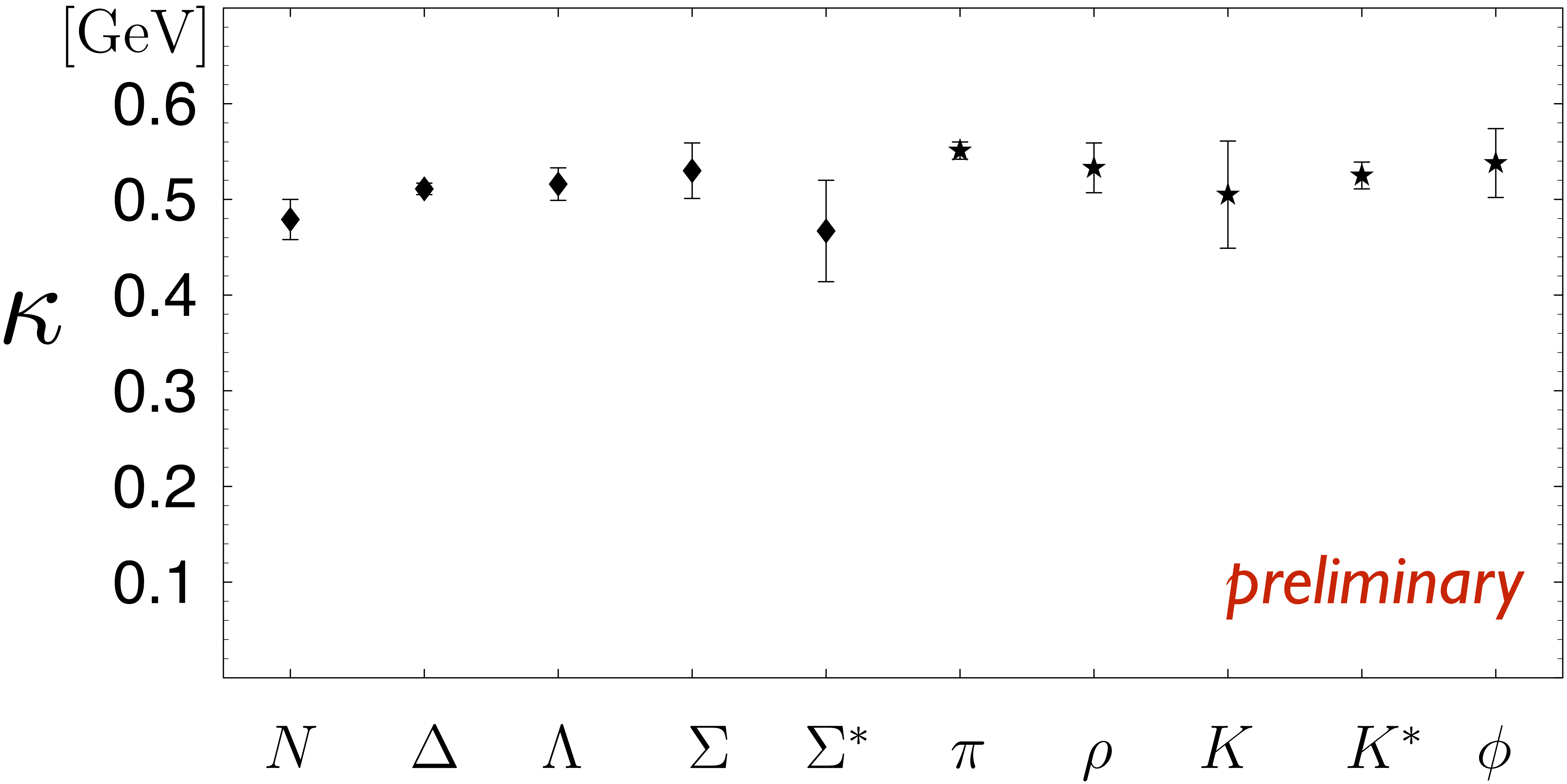
De Tèramond, Dosch, sjb

$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$



$m_u = m_d = 46$ MeV, $m_s = 357$ MeV



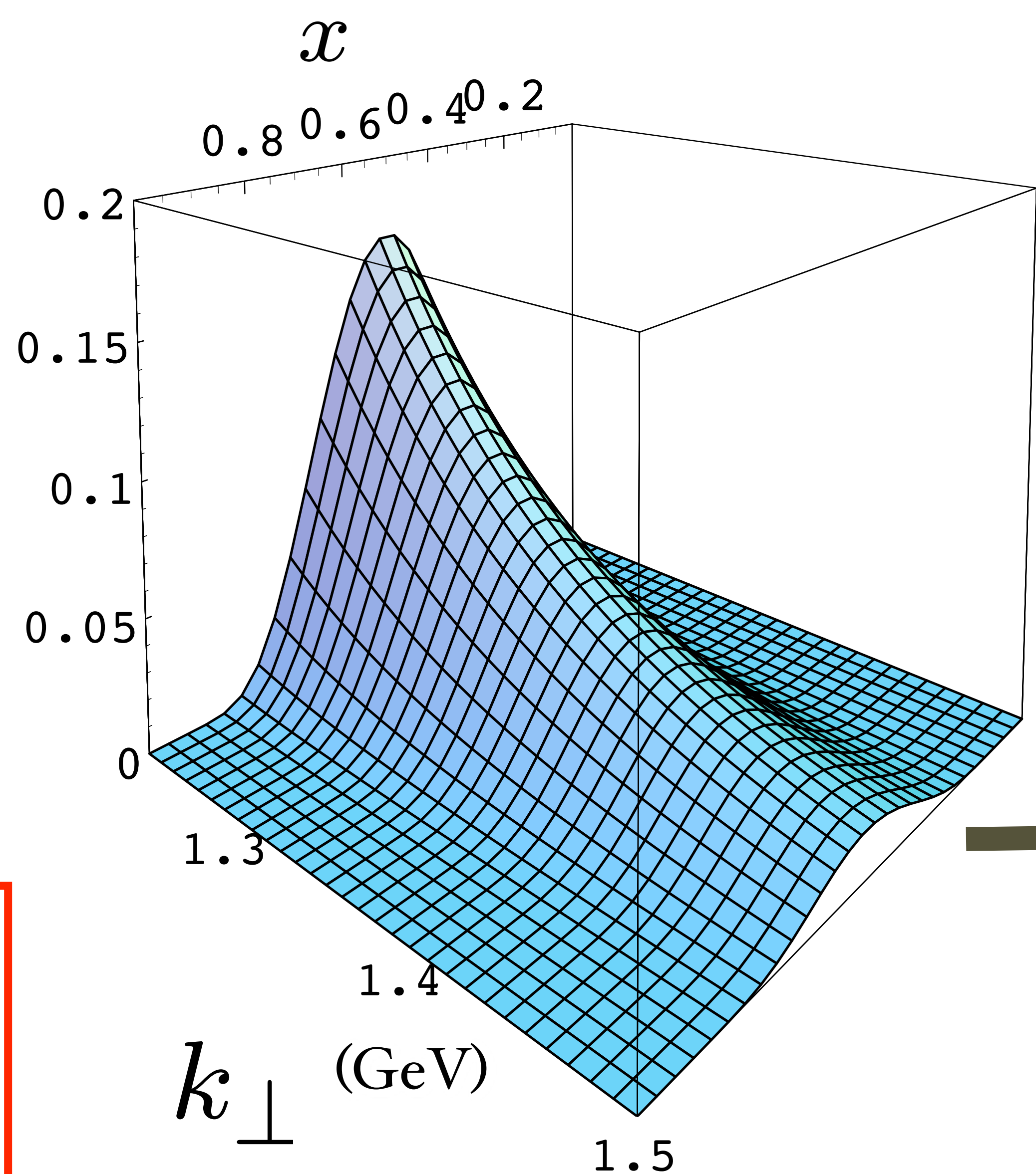
Fit to the slope of Regge trajectories, including radial excitations

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY

Prediction from AdS/QCD: Meson LFWF

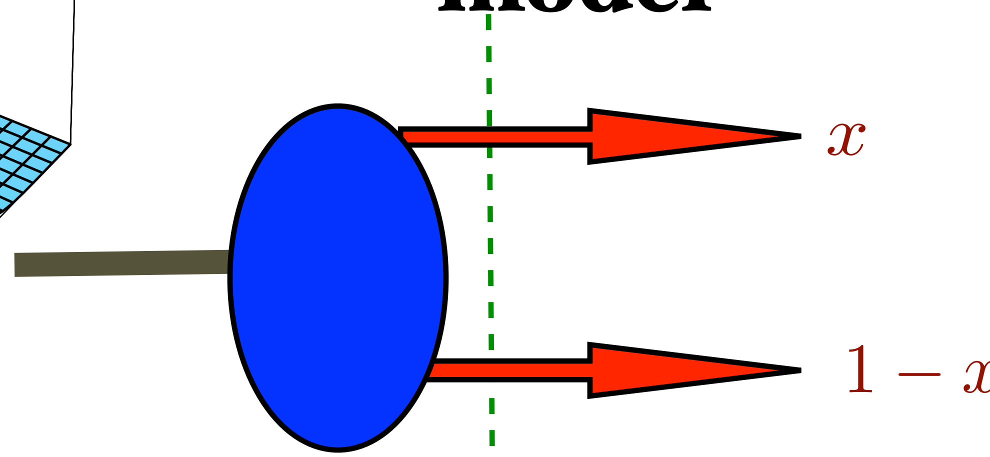
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

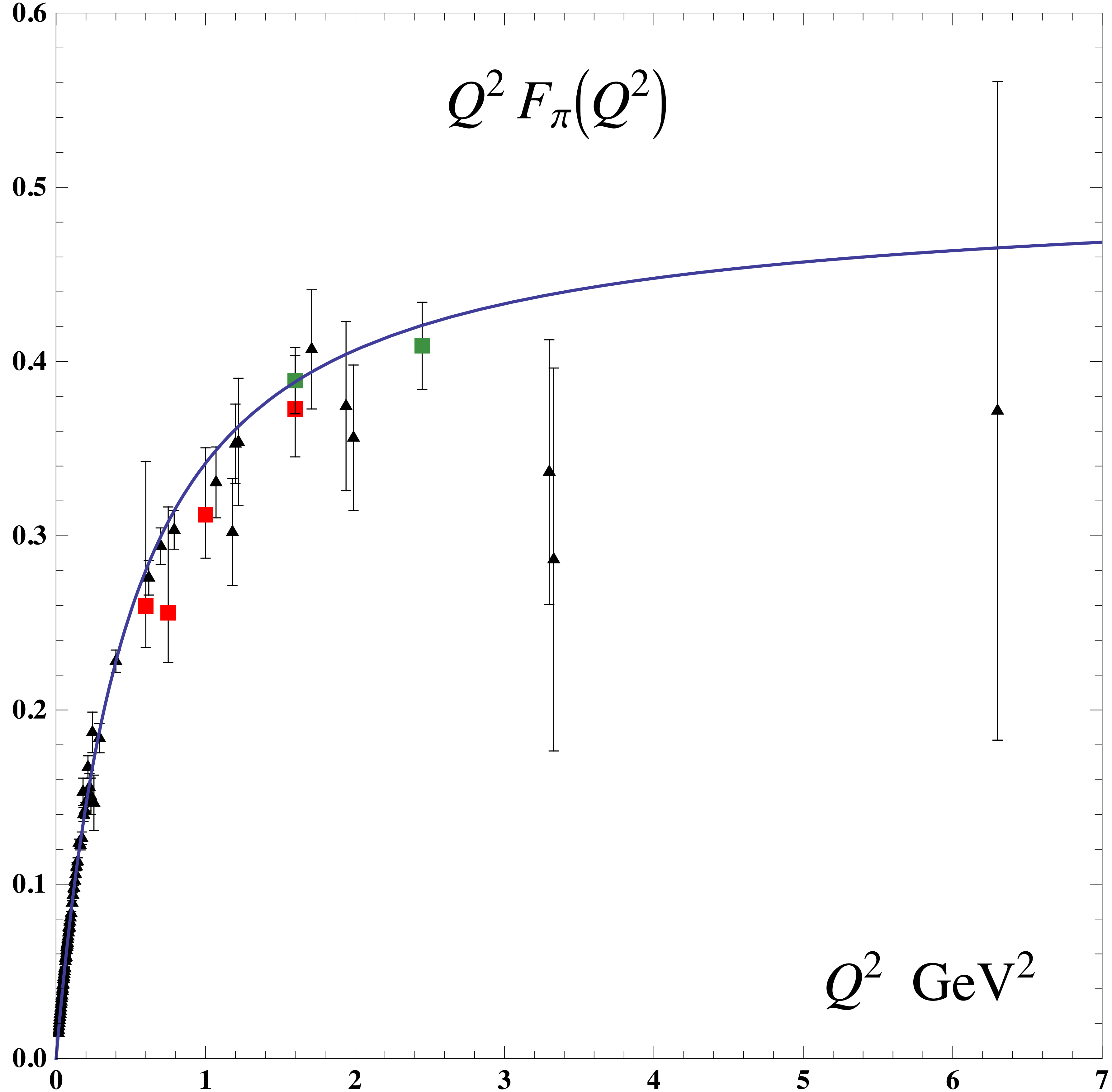
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

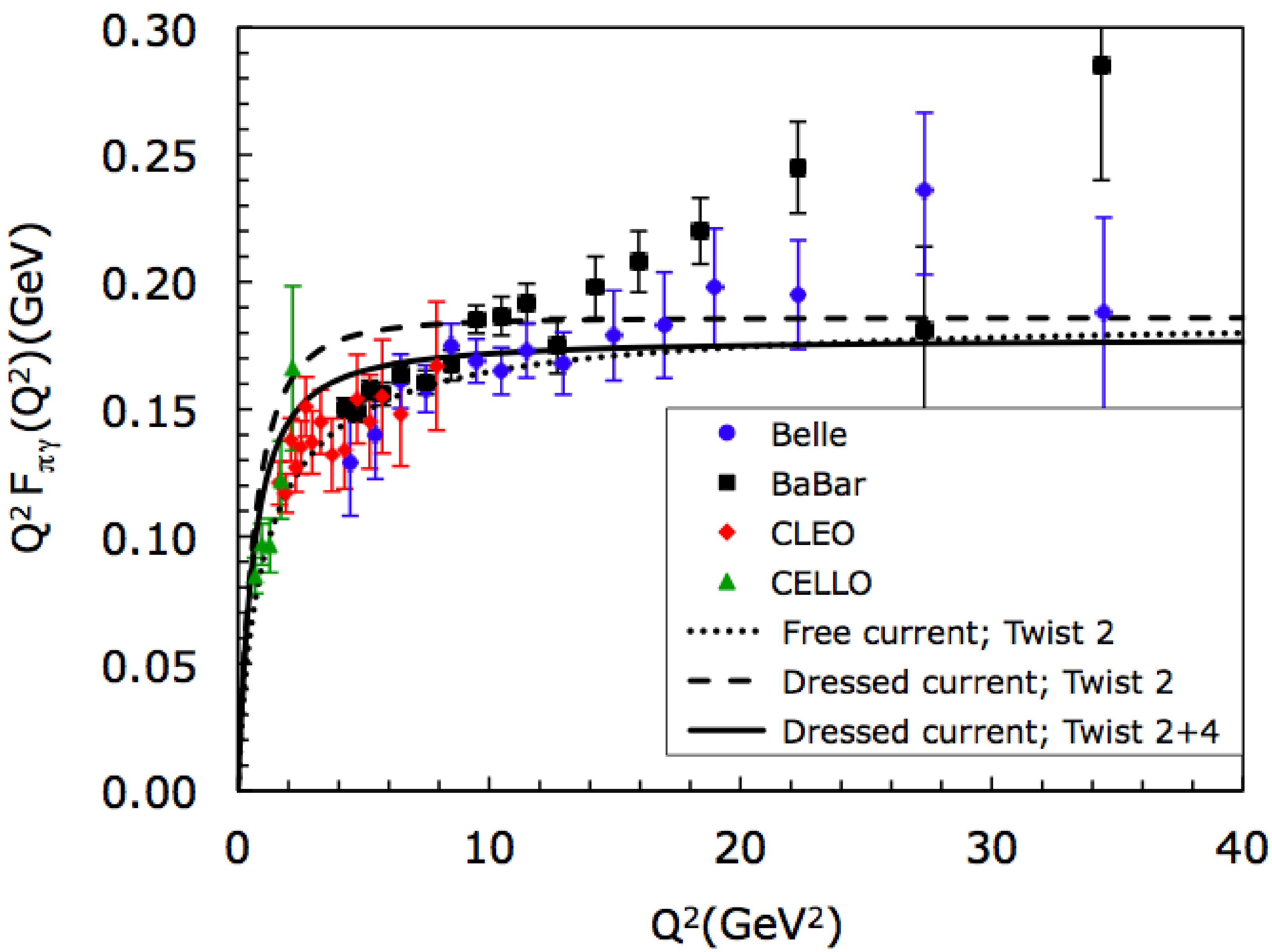
Same as DSE!

Provides Connection of Confinement to Hadron Structure



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Pion-gamma transition form factor



$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

de Teramond, Cao, sjb

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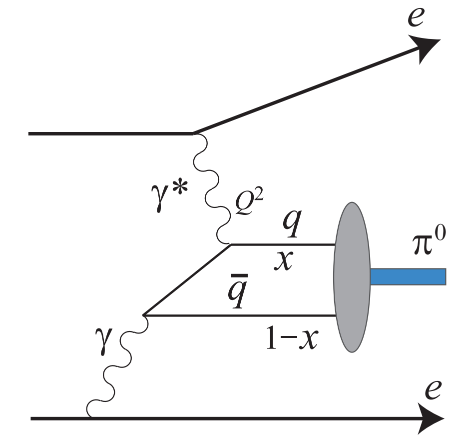
Pion Transition Form-Factor

Cao, de Teramond, sjb



- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \to \gamma$ vertex in the amplitude $e\pi \to e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2)\epsilon_{\mu\nu\rho\sigma}(p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad k^2 = 0$$

- Asymptotic value of pion TFF is determined by first principles in QCD:
  $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$ [Lepage and Brodsky (1980)]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$
$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Find for $A_z \propto \Phi_\pi(z)/z$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z} \, \Phi_\pi(z) V\left(Q^2, z\right)$$

  with normalization fixed by asymptotic QCD prediction

- $V(Q^2, z)$ bulk-to-boundary propagator of γ^*

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Unique confinement potential!

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)

- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$

- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$

- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ

- Only dimensionless mass ratios (and M times R) predicted

- Mass and time units [GeV] and [sec] from physics external to QCD

- New feature: bounded frame-independent relative time between constituents

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Fubini and Rabinovici

Superconformal Algebra

de Teramond Dosch and SJB

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Fubini, Rabinovici:

Superconformal Algebra

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates the conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\big)$$

$$G_{22} = \big(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\big)$$

Identify $f - \frac{1}{2} = L_B\ , \quad w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Algebra

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

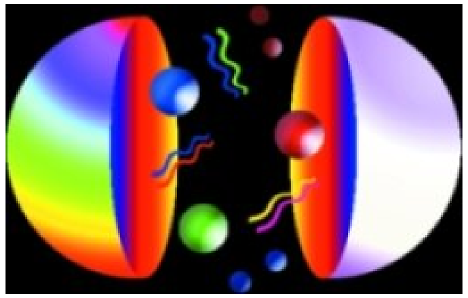
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

Fermionic Modes and Baryon Spectrum



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

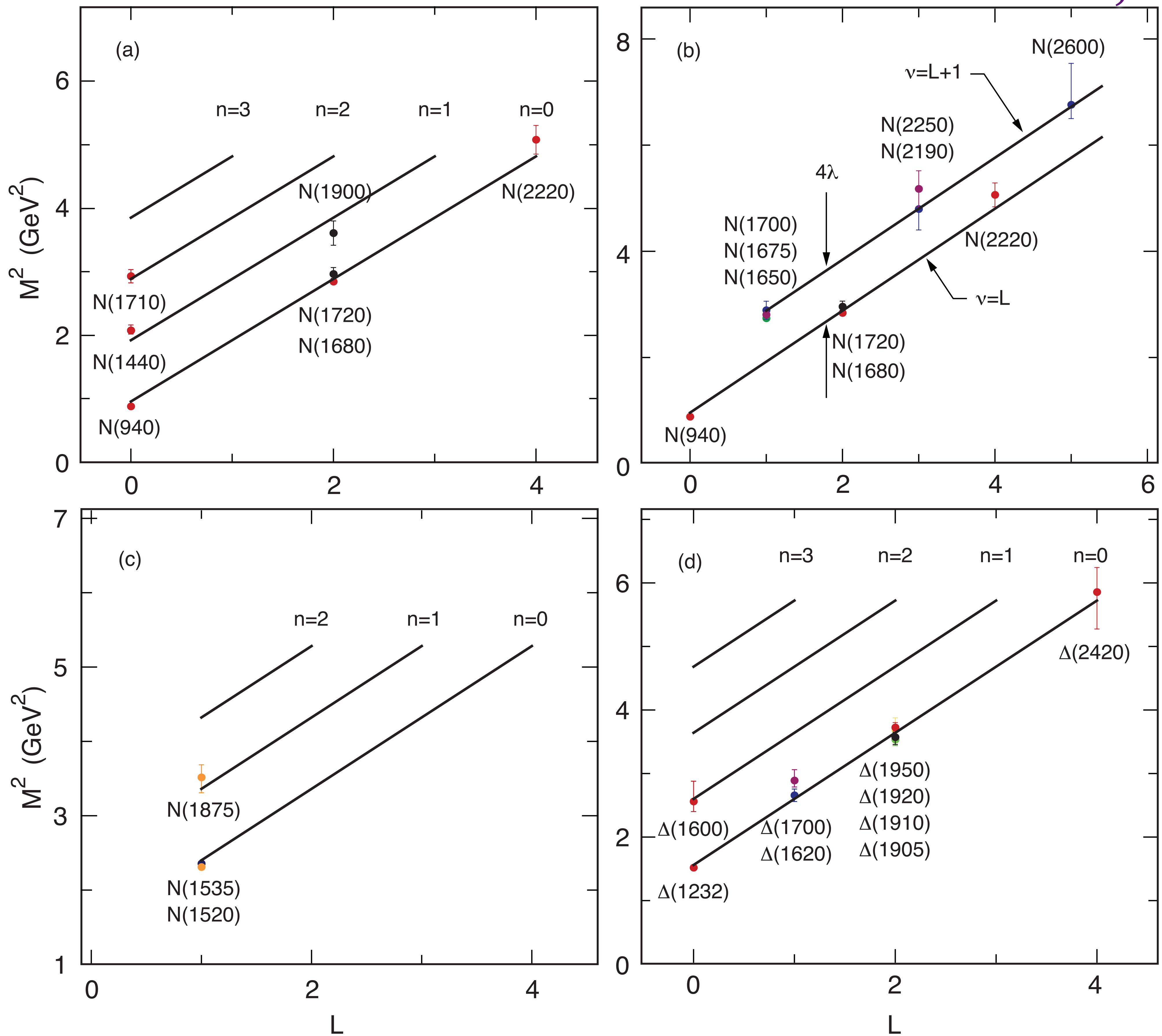
- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

de Tèramond, Dosch, sjb

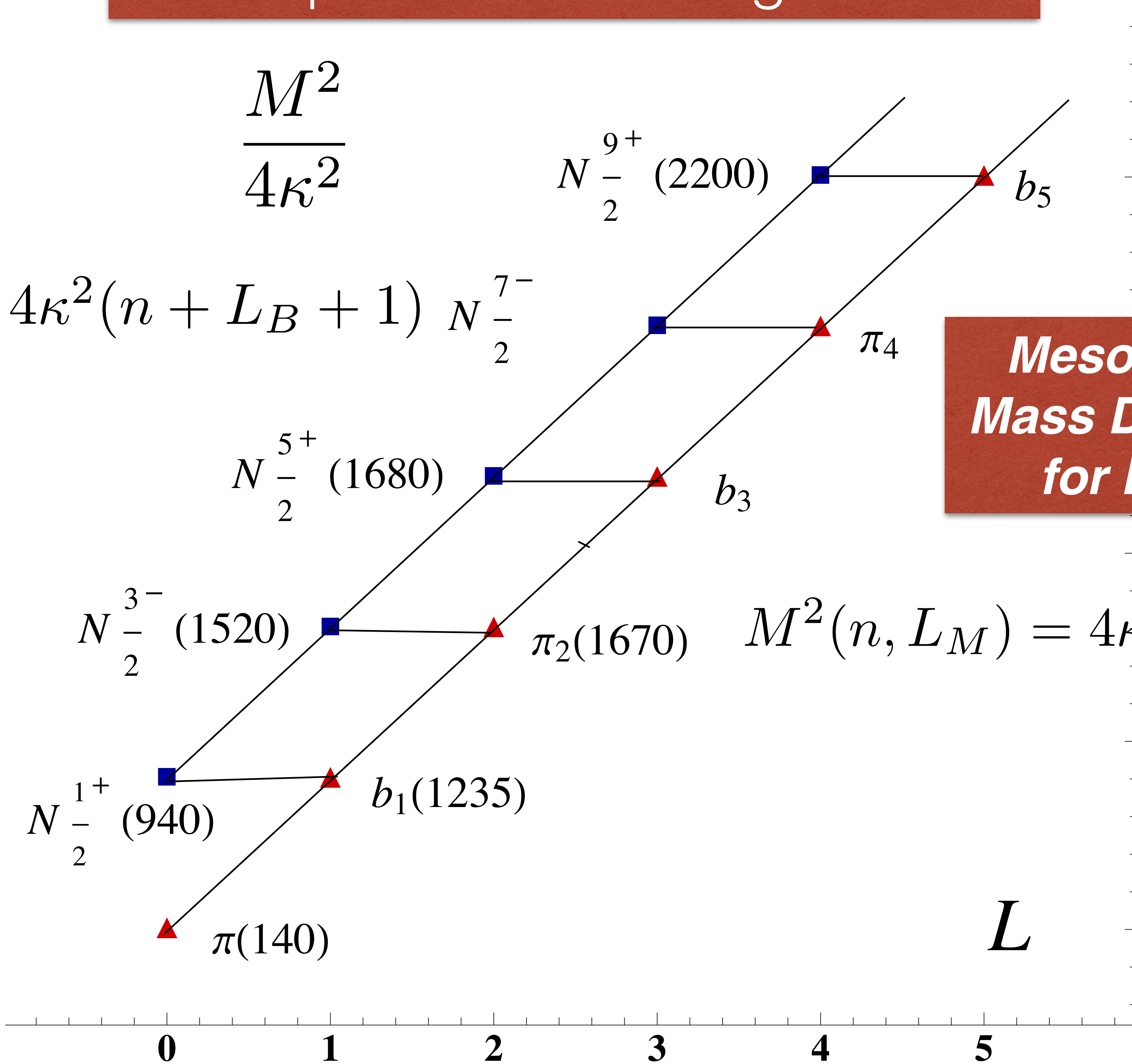


Baryon orbital and radial excitations for
$\kappa = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

Superconformal Algebra

$$\frac{M^2}{4\kappa^2}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

Same κ

$$\lambda = \kappa^2$$

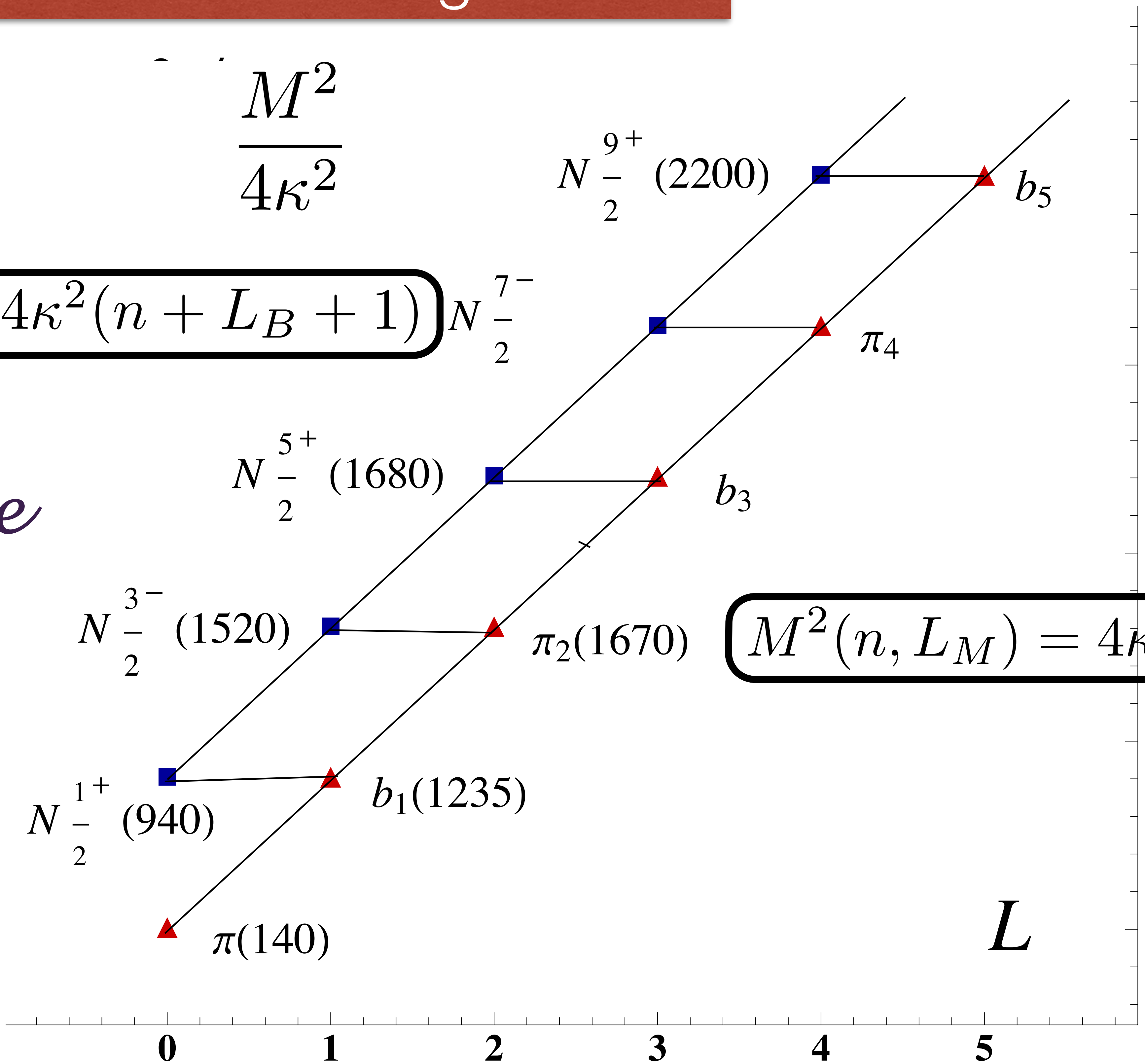
S=0, I=1 Meson is superpartner of S=1/2, I=1/2 Baryon

Superconformal Algebra

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

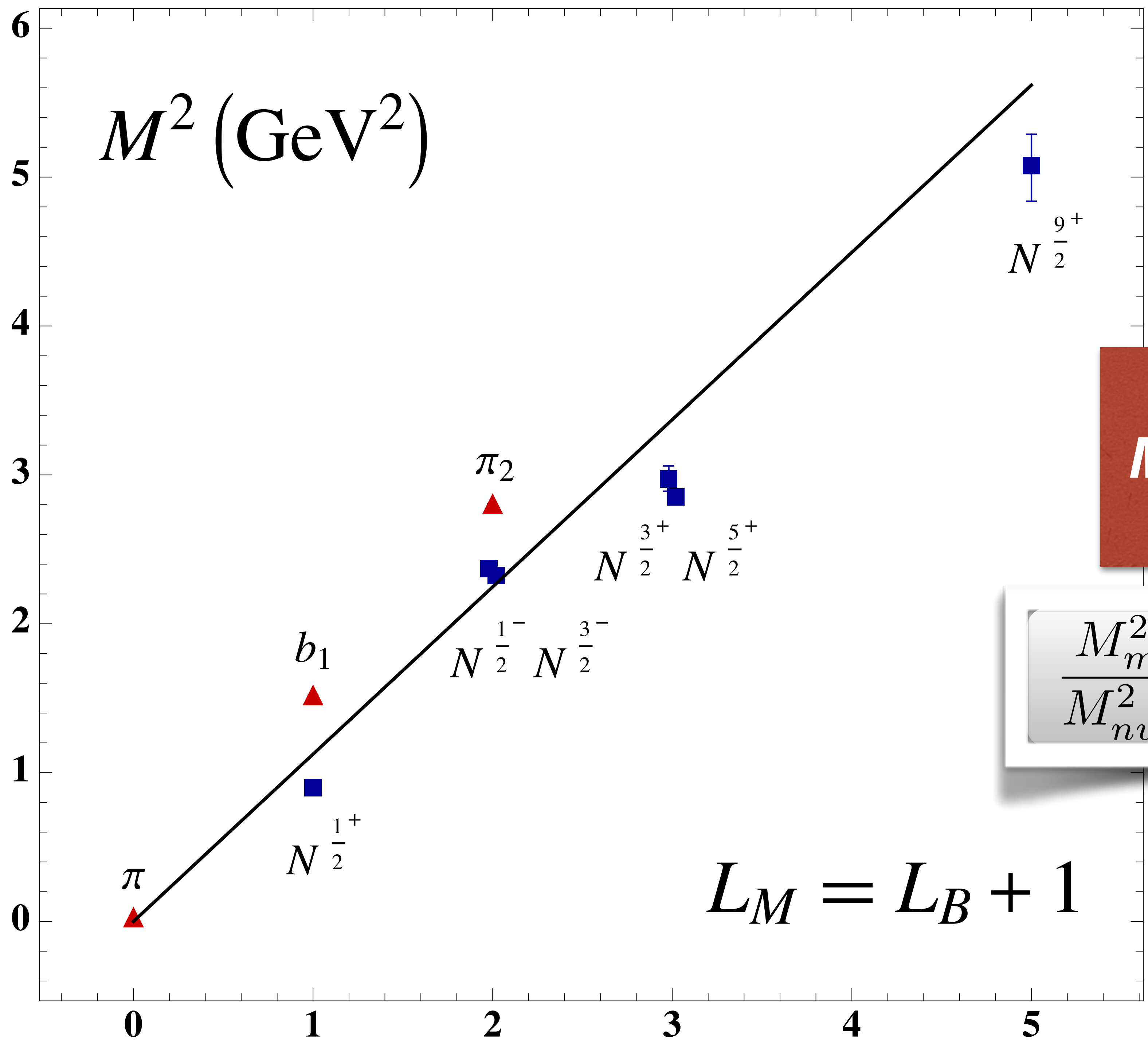


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

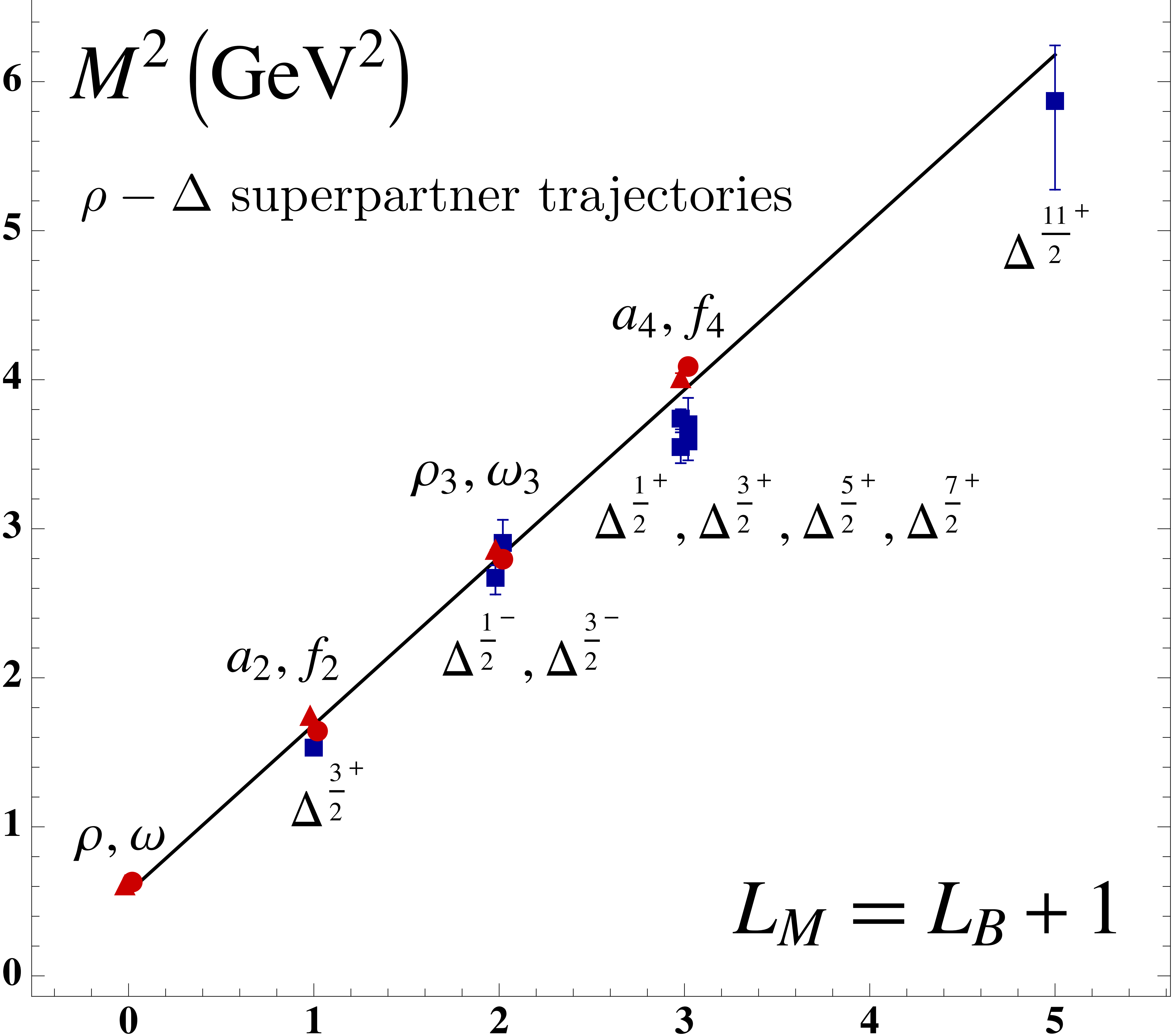
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

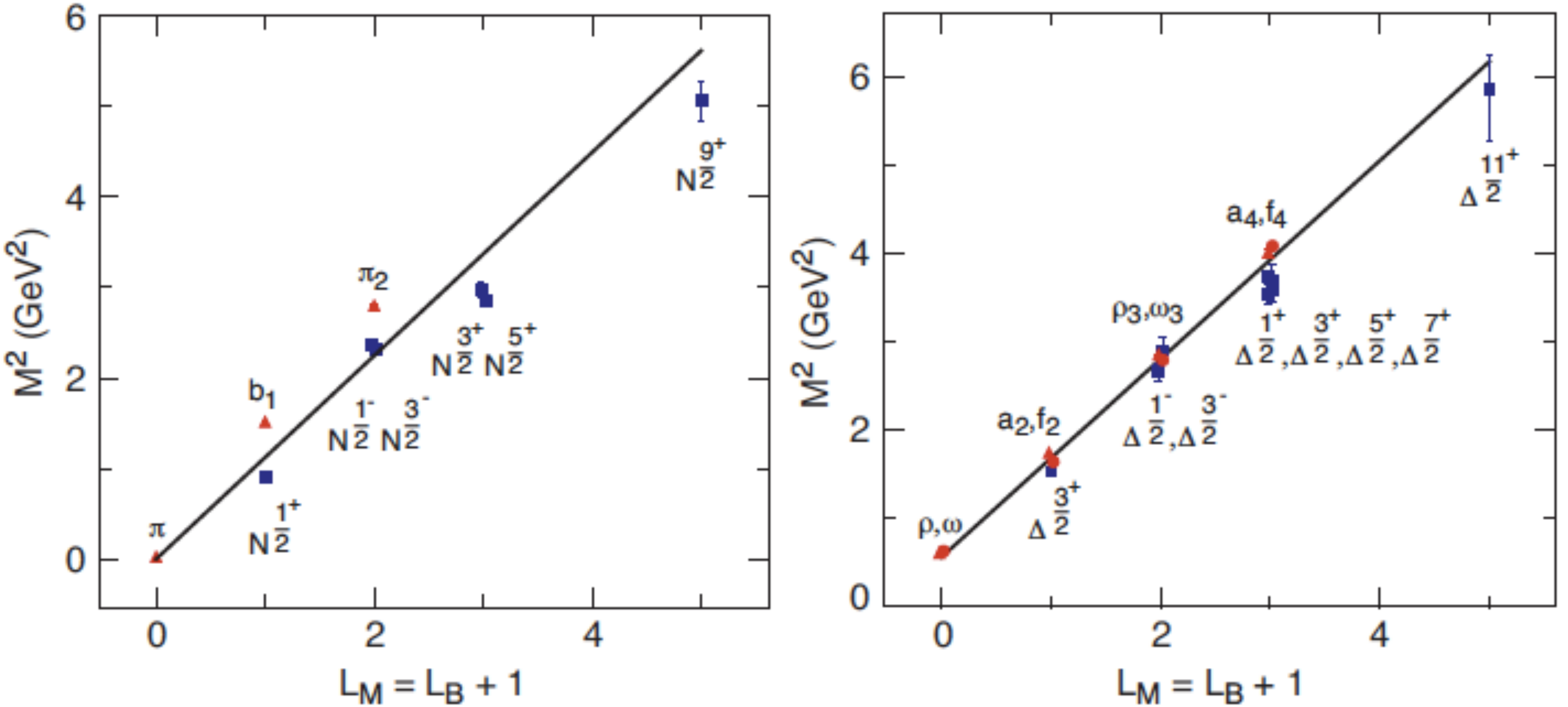
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1/2 Baryon



Dosch, de Teramond, sjb

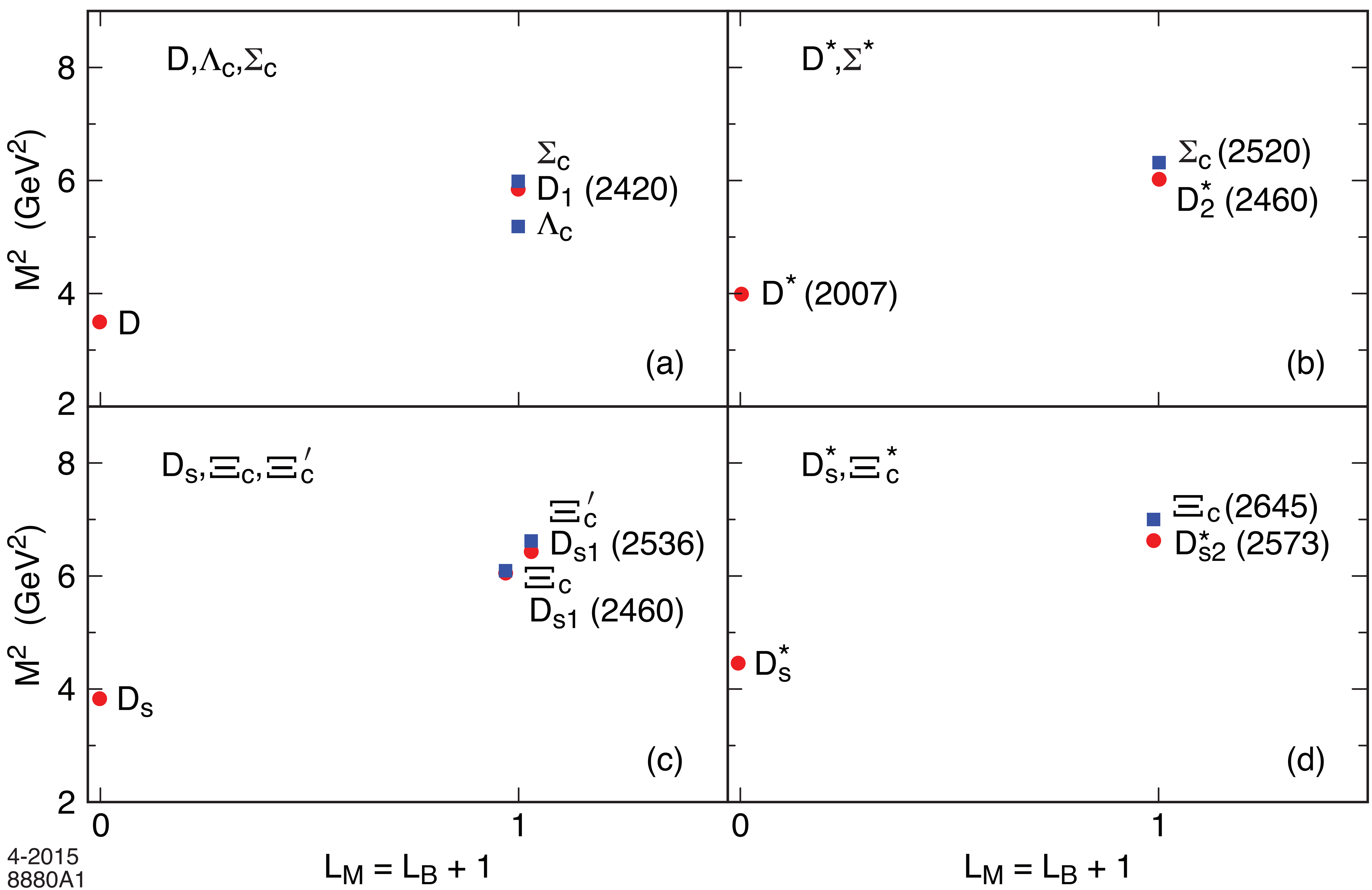
de Tèramond, Dosch, sjb



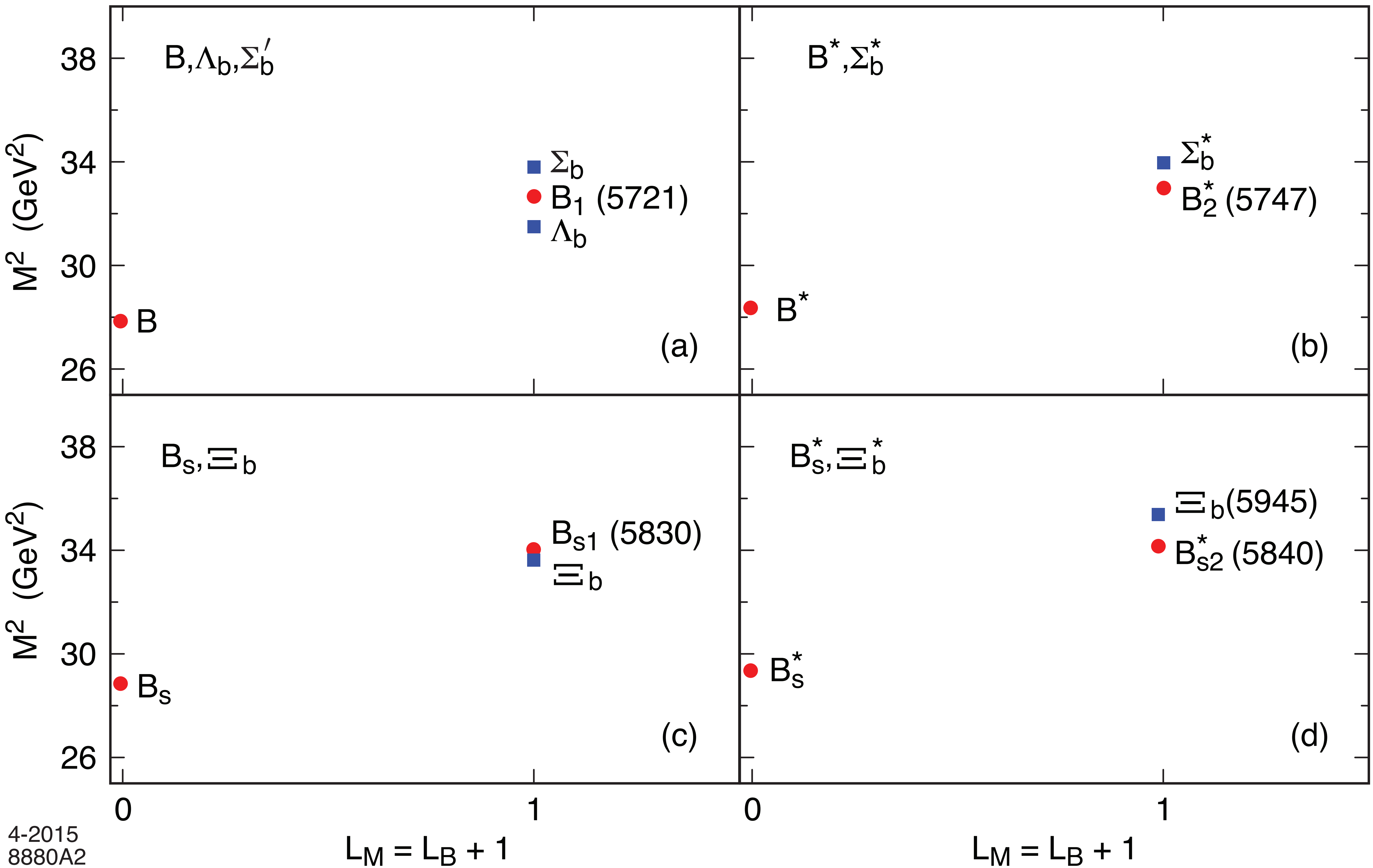
Superconformal Meson-Nucleon Partners

$$\kappa = 530 \text{ MeV}$$

Dosch, de Teramond, sjb



Dosch, de Teramond, Lorce, sjb



LF Holography

Baryon Equation

Superconformal Algebra

$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2 (L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2 (n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2 (n + L_M)$$

Same κ !

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$

- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = \langle L^z_q \rangle$

- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Meson and baryon have same κ !

Proton spin carried by quark orbital angular momentum

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons, baryons***

- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***

- ***Hadronic LFWFs***

- ***Counting Rules***

- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for L_M=L_B+1

AdS/QCD and Light-Front Holography

- Single Scale κ; Only ratios predicted
- Spectroscopy, LFWFs, and Dynamics
- LF Schrödinger Equation — Analogous to Schrödinger Equation for Atomic Physics
- QCD Running Couplings
- Matching Scale Q_0

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

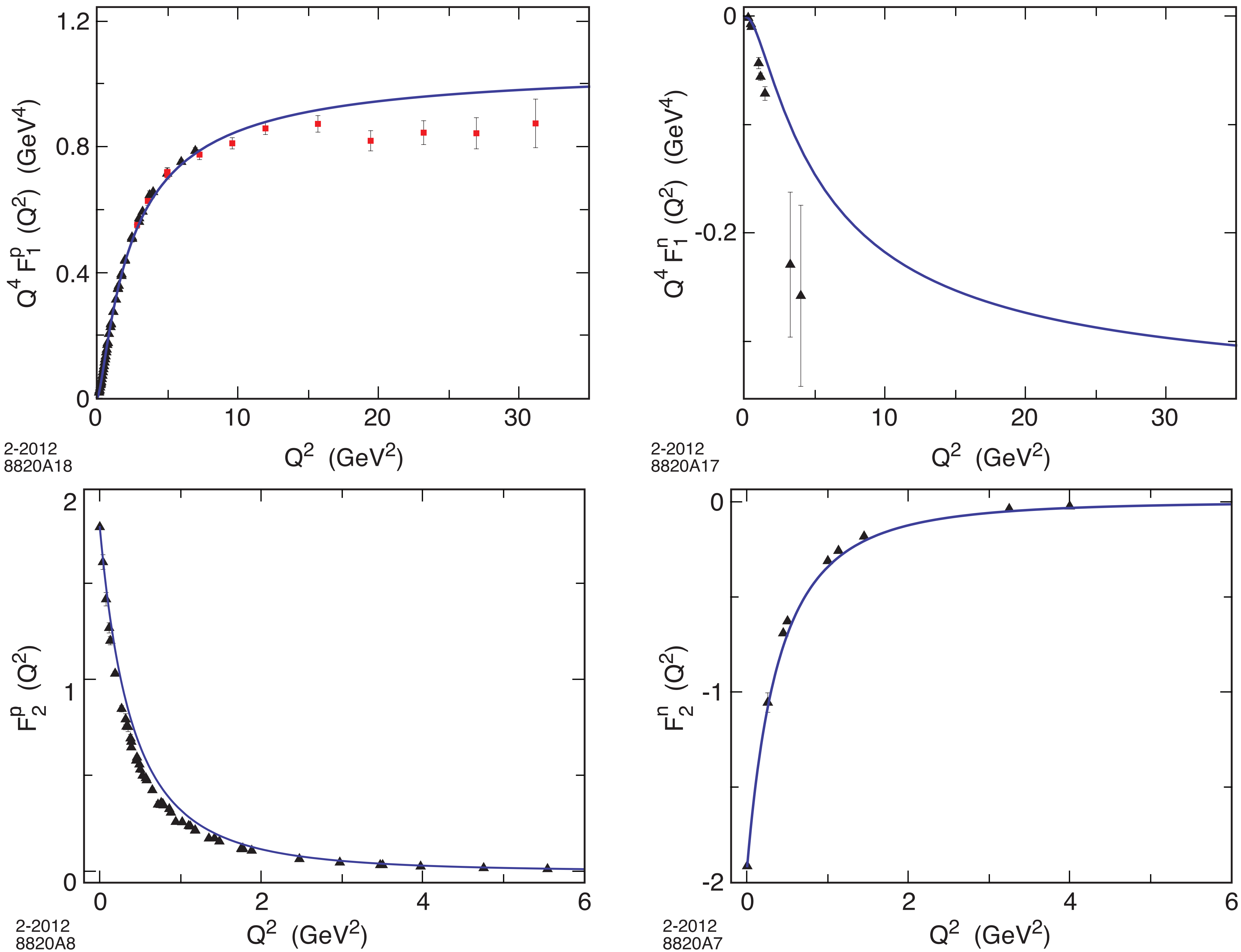
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

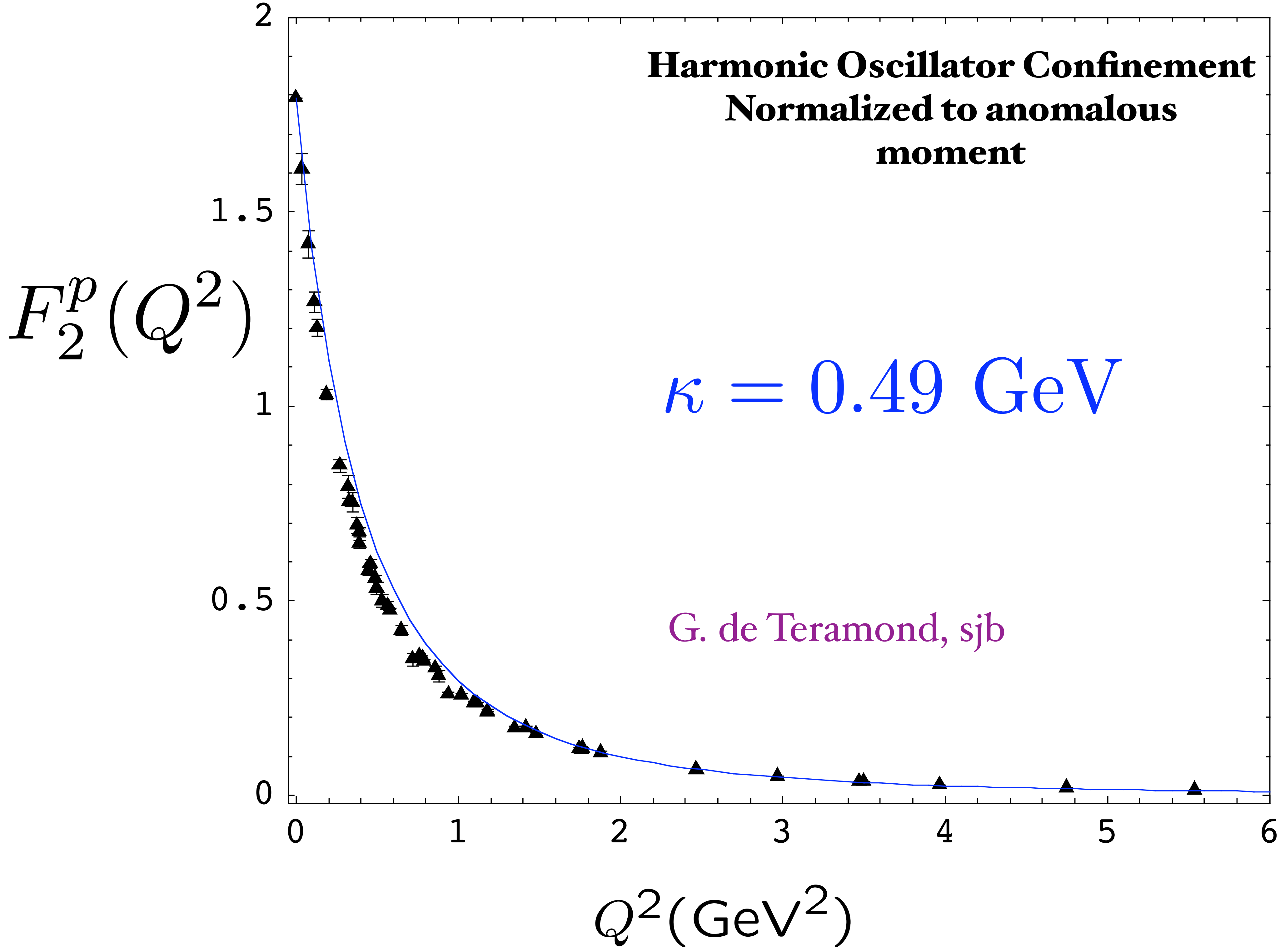
where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

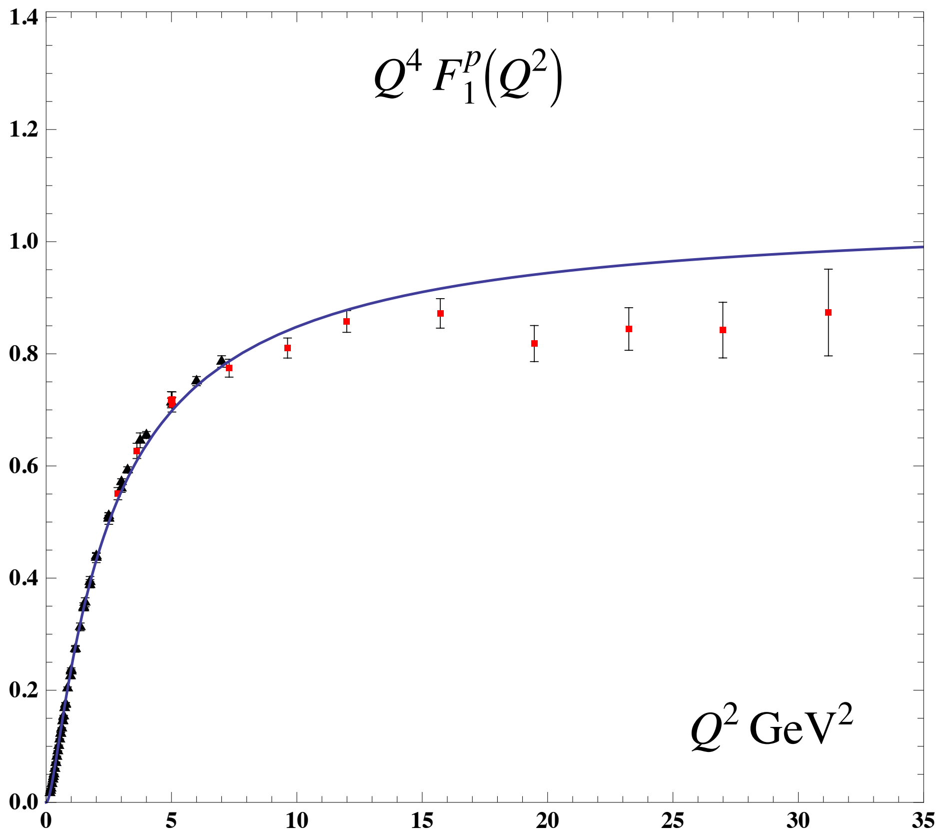
From overlap of $L = 1$ and $L = 0$ LFWFs

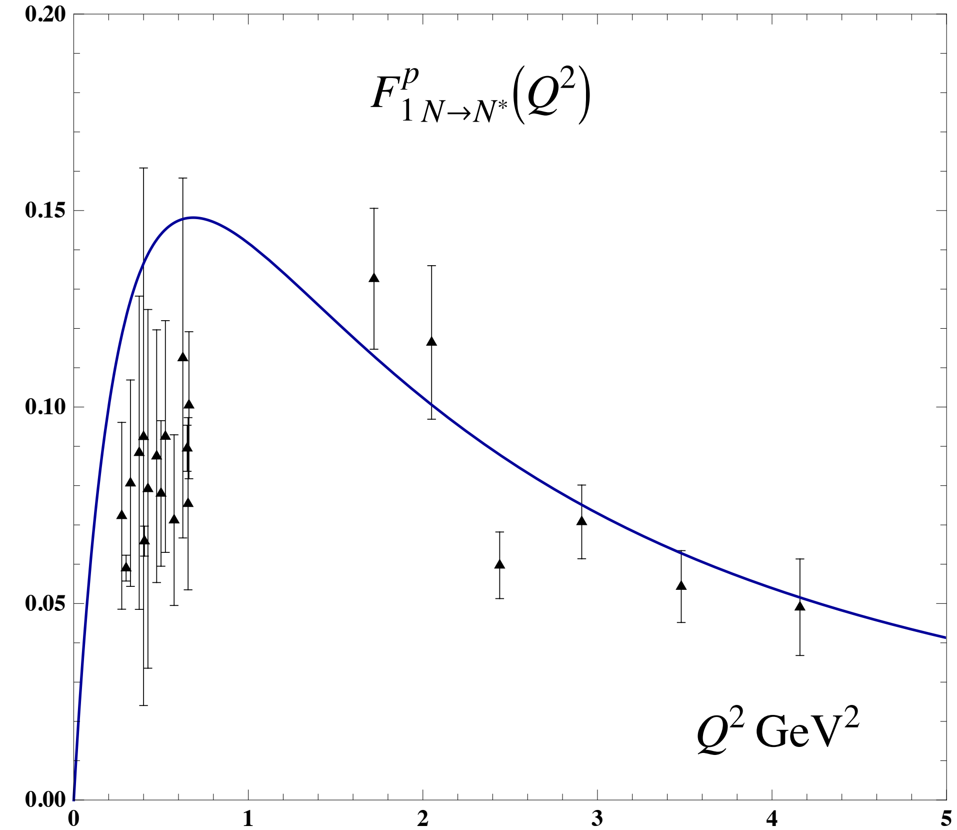


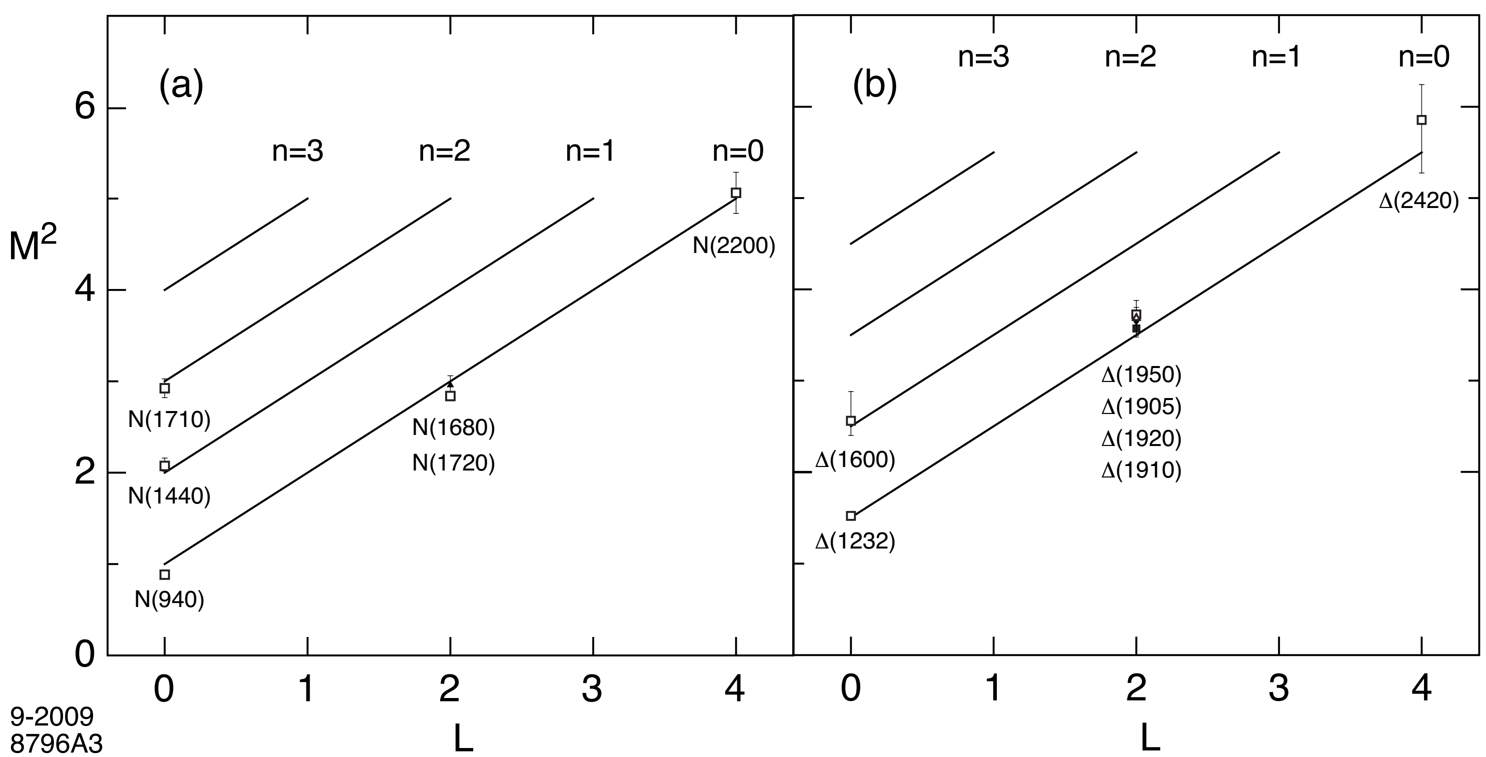
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb







Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

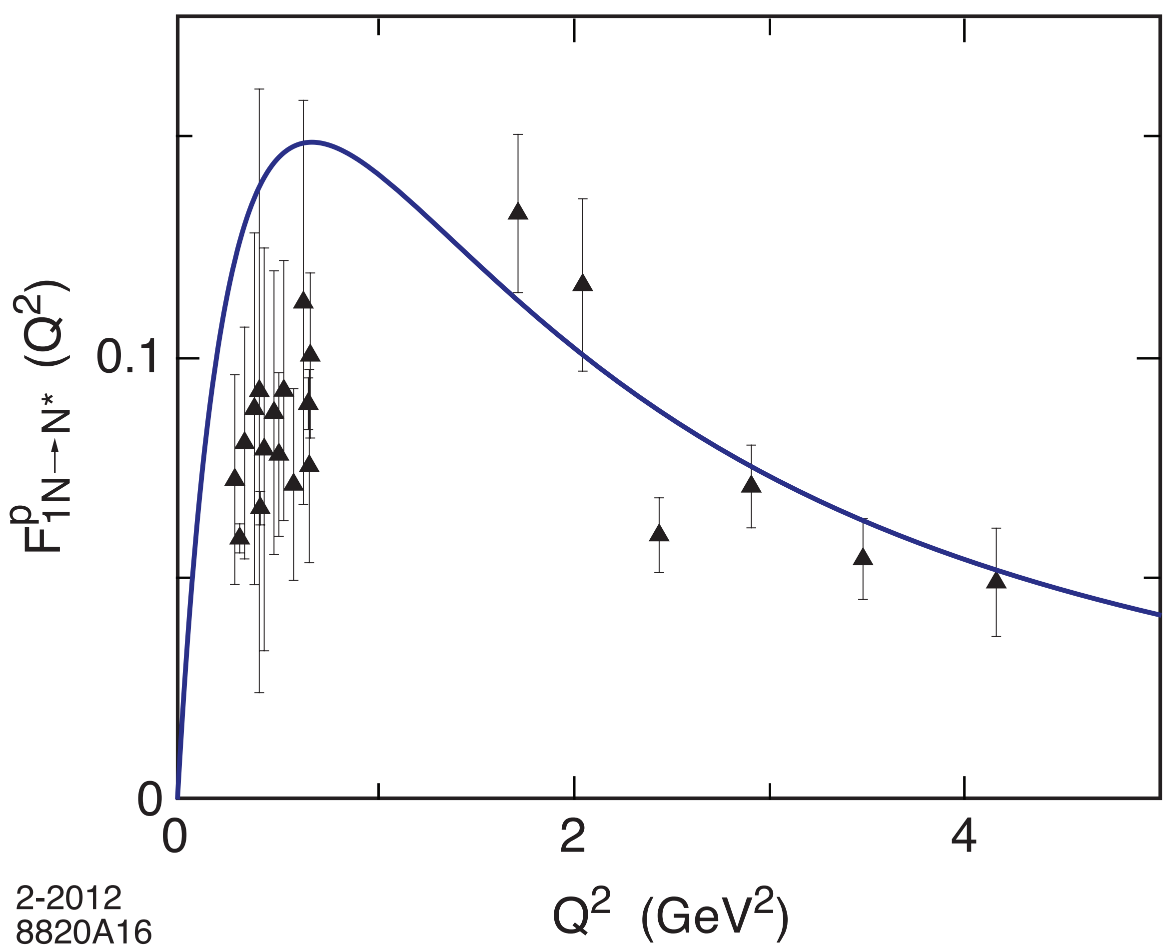
with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$

de Teramond, sjb

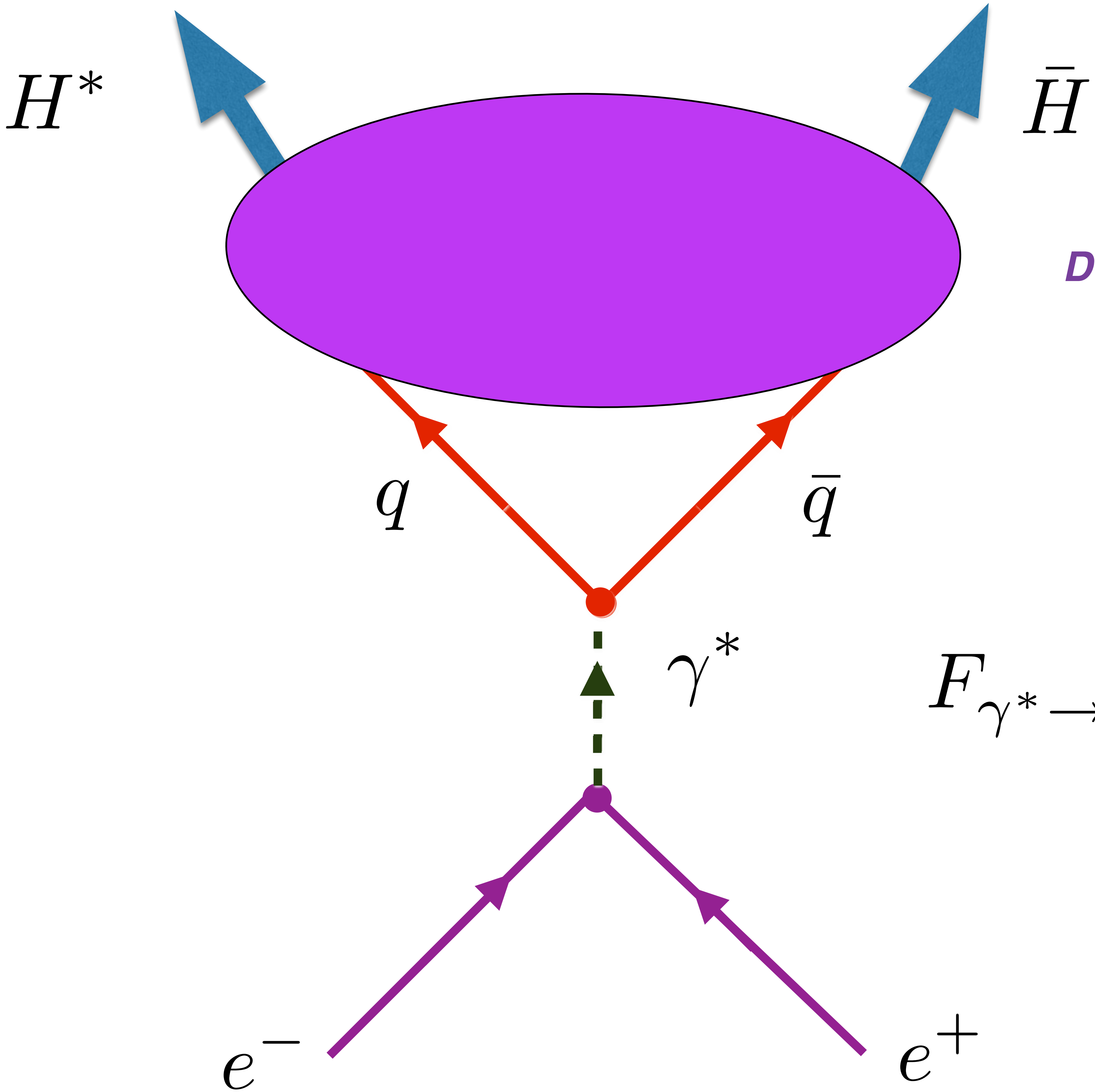
Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab



Dosch, de Teramond, sjb

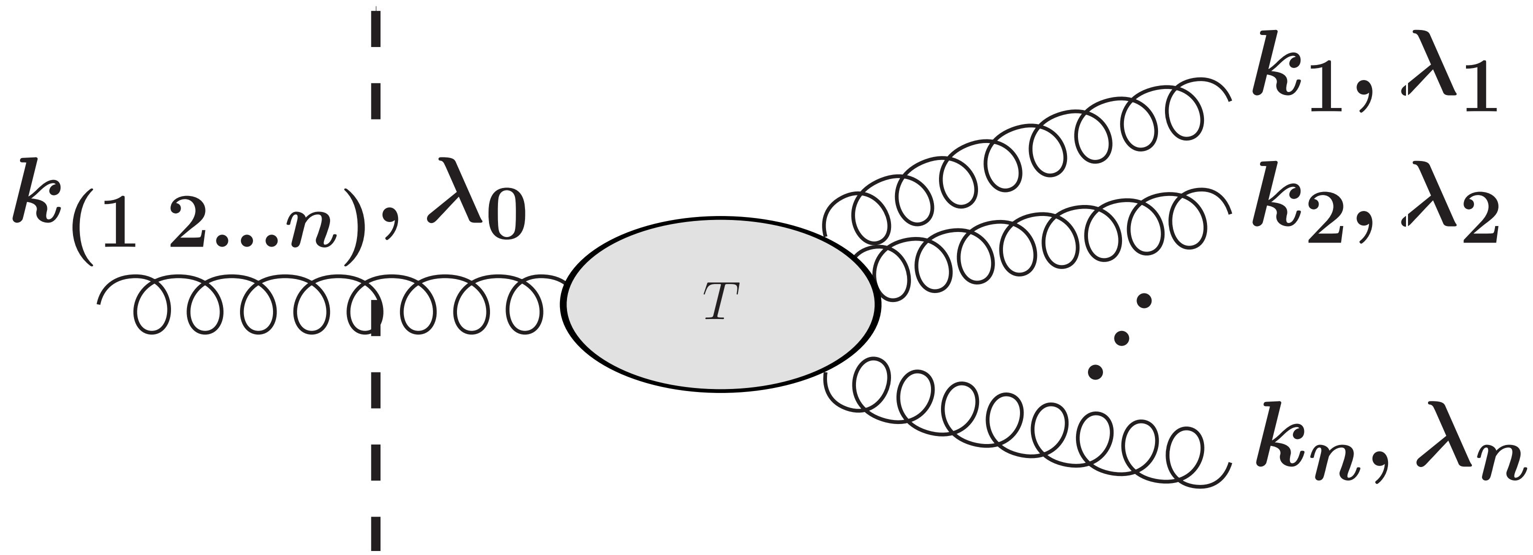
$$F_{\gamma^* \to H^* \bar{H}}(s)$$

Prediction from Super Conformal AdS/QCD:
Same Form Factors for H= M and H=B if $L_M=L_B+1$

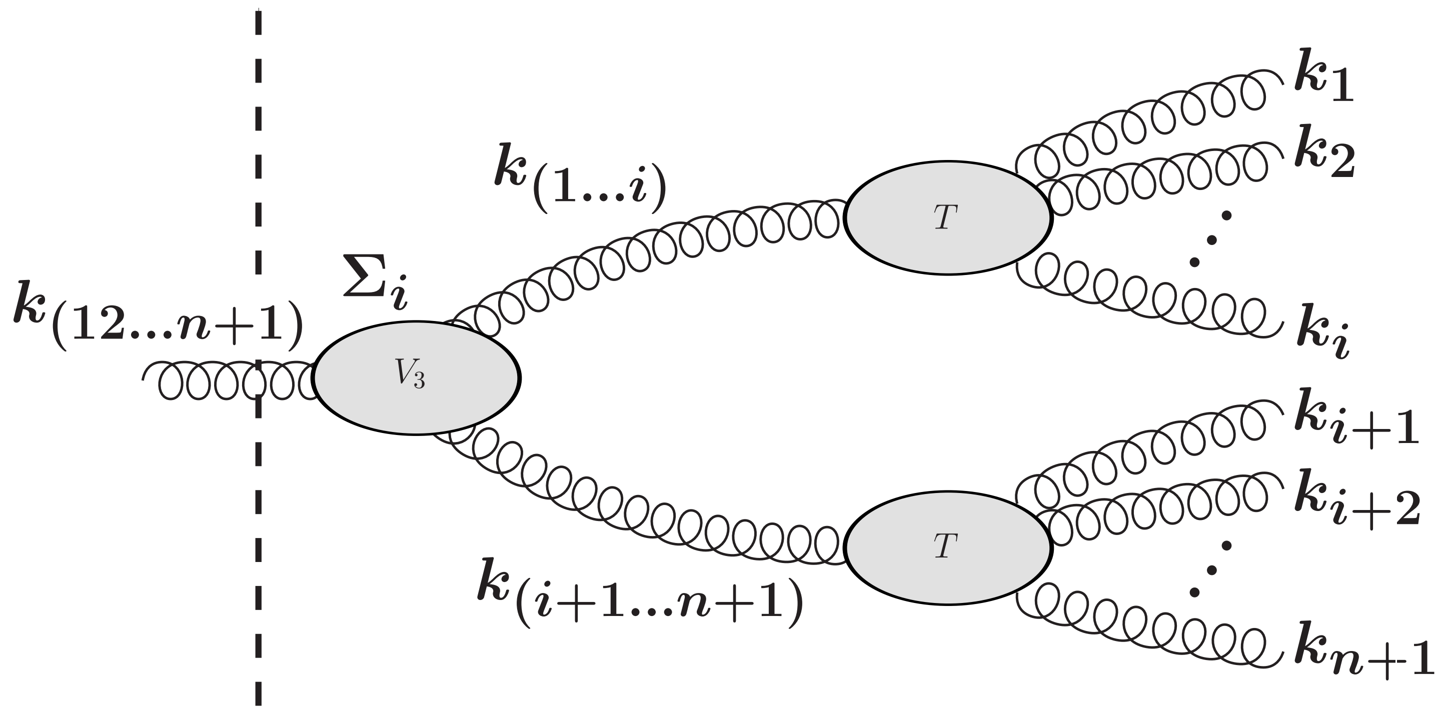
Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**



$T_{n+1} \quad =$

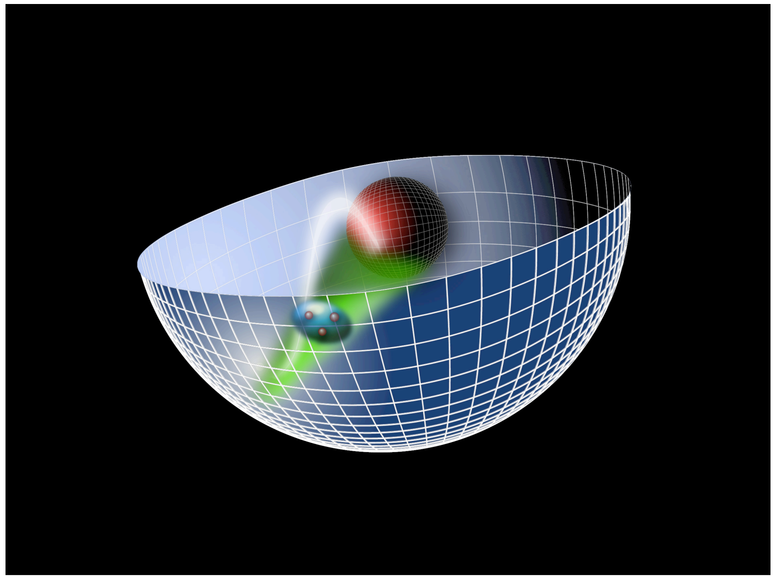


Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle i j \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) :$$

AdS5: Conformal Template for QCD

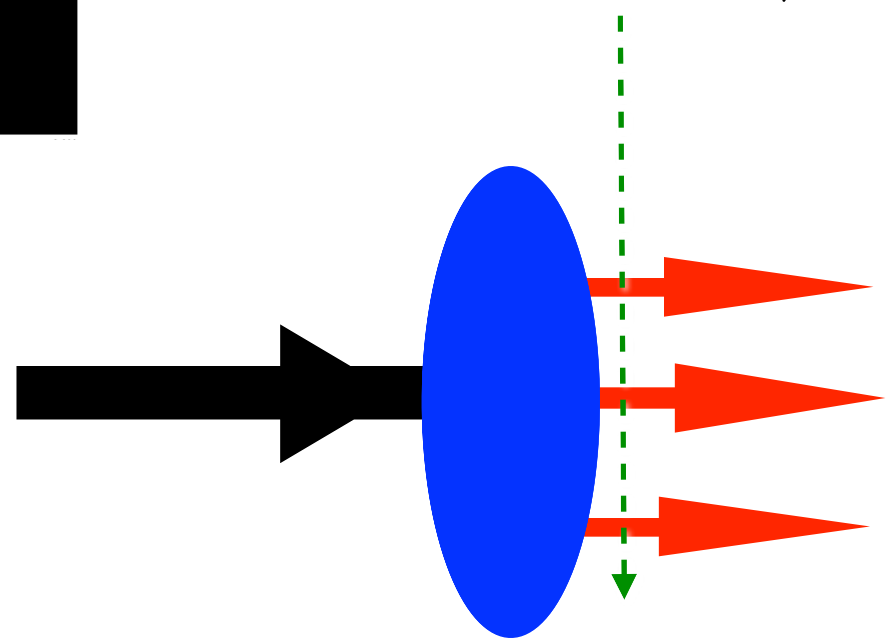
$\phi(z)$



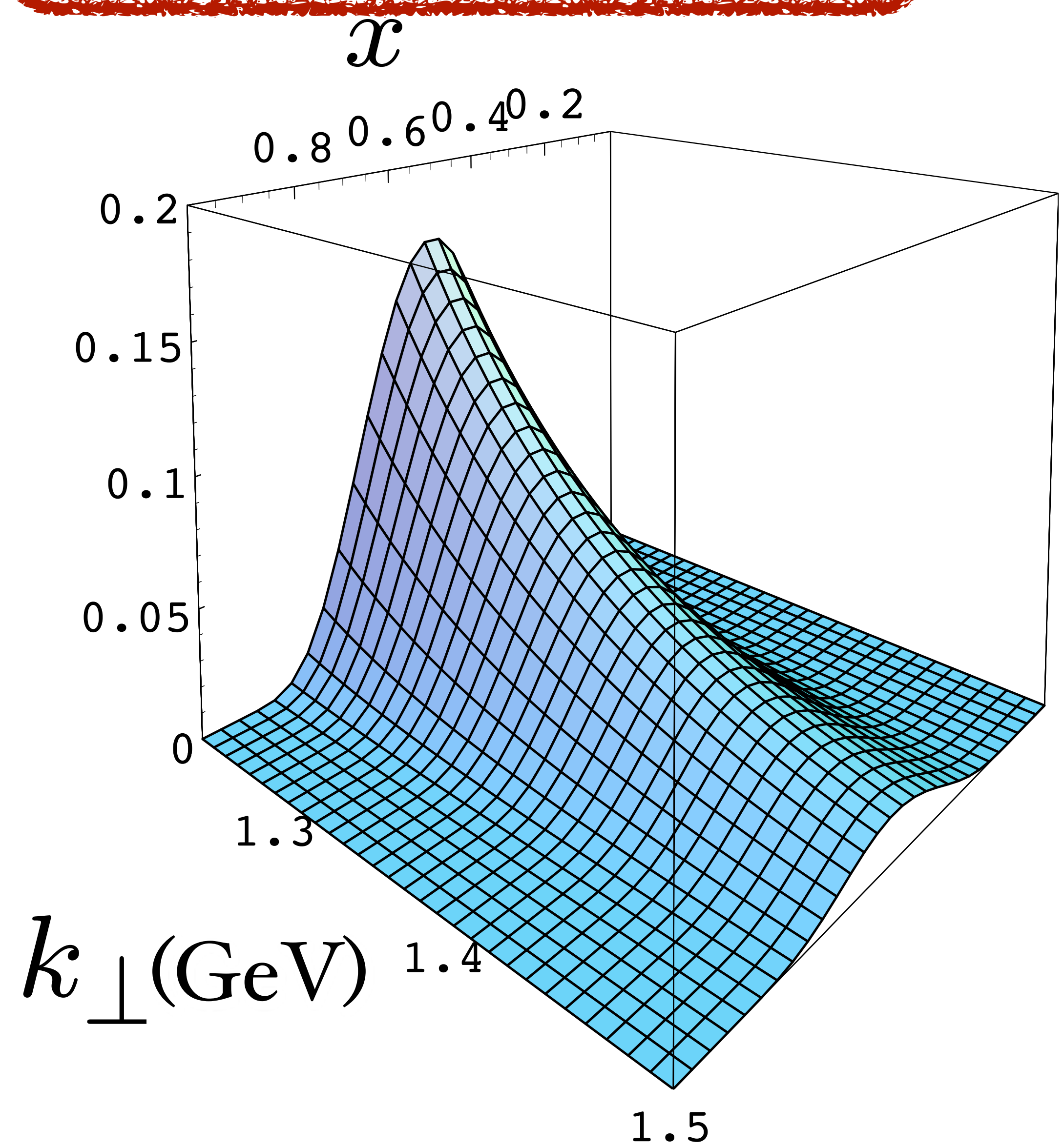
- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

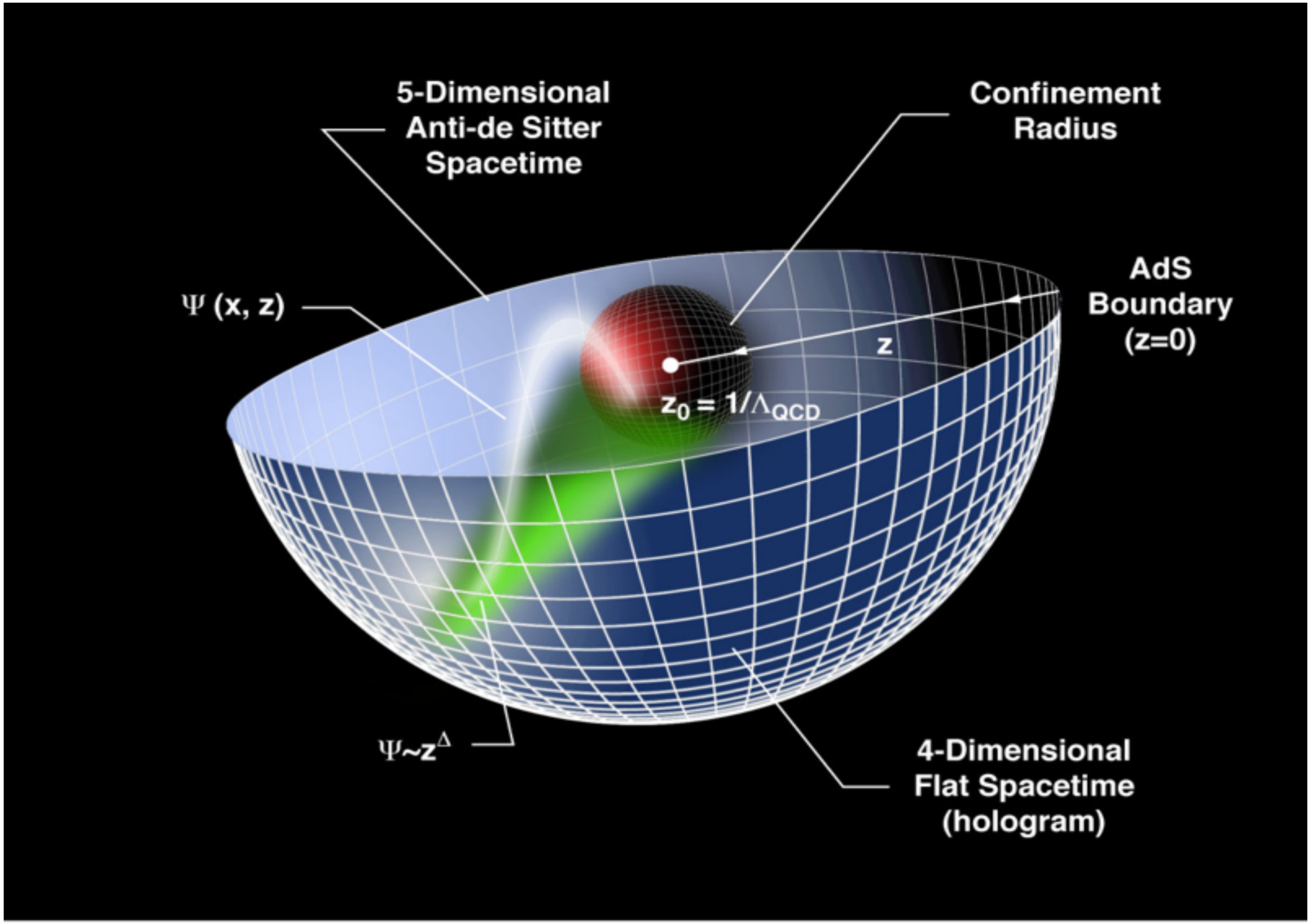


- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation

Spectroscopy and Dynamics

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

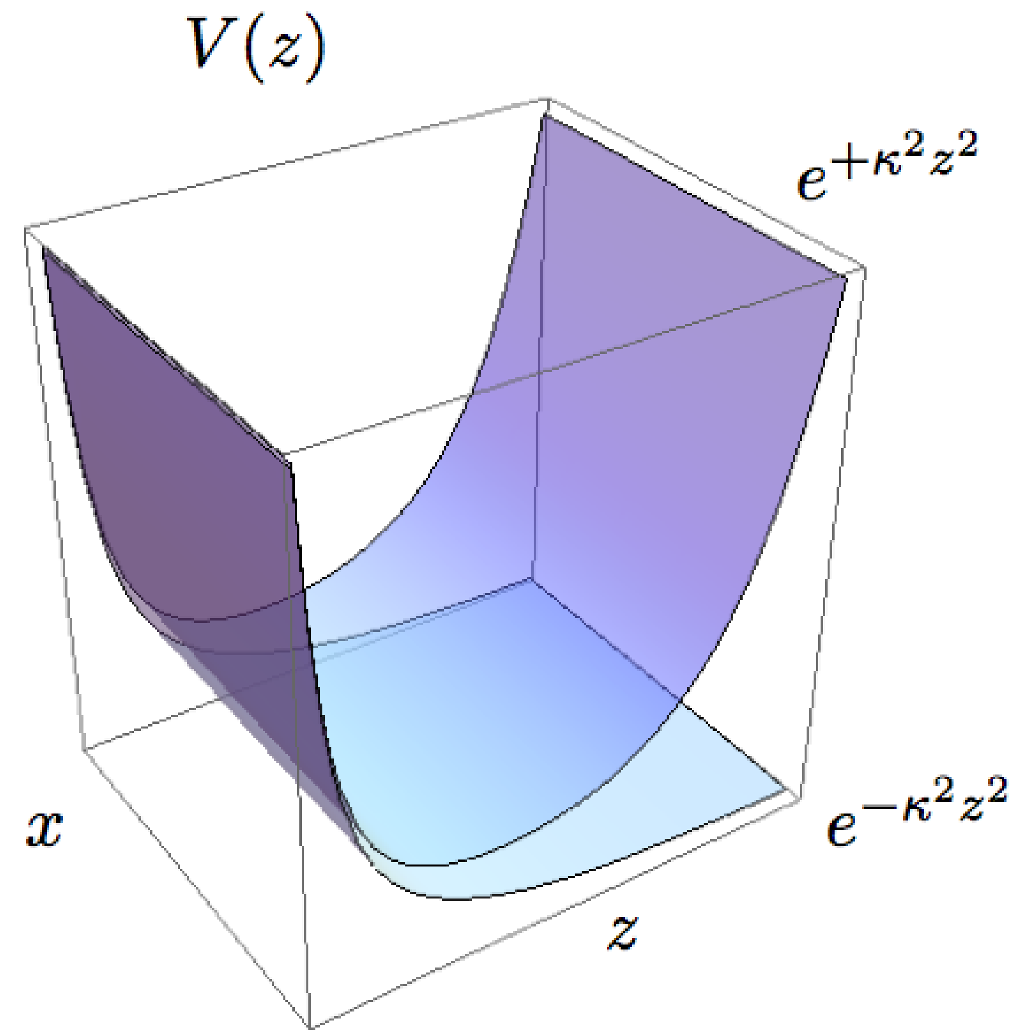
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

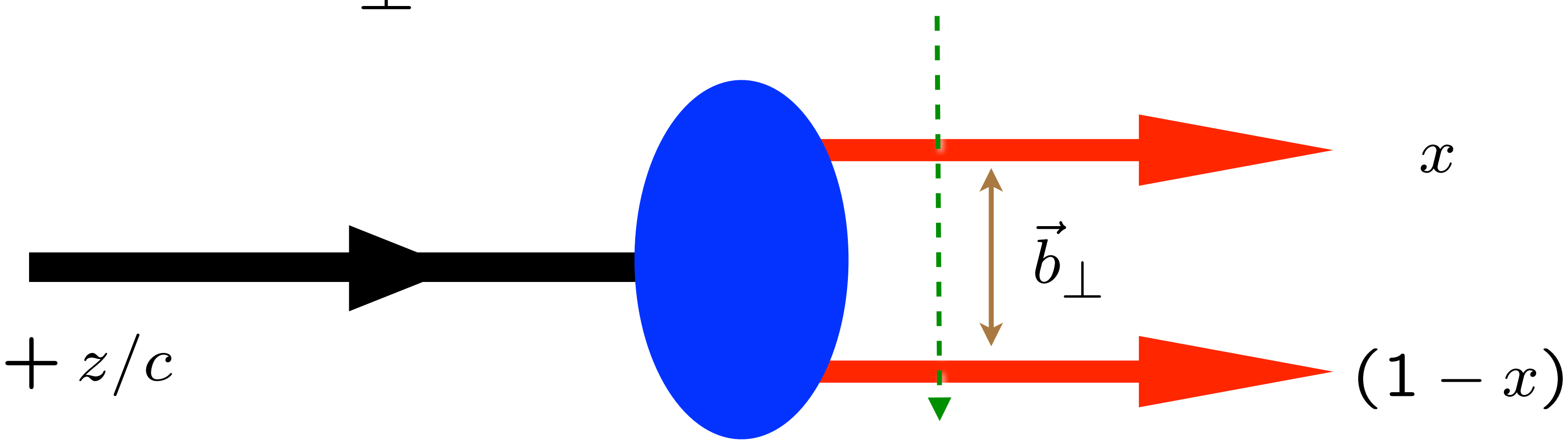
LF(3+1) ⟷ AdS_5

de Teramond, sjb

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

***Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

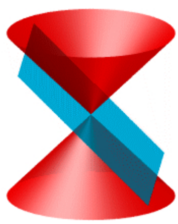
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

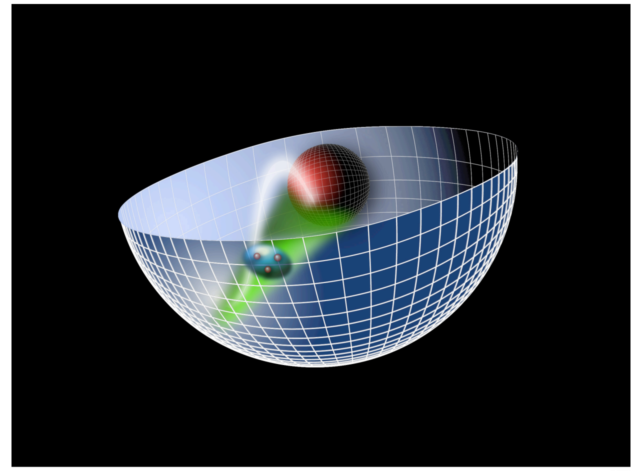


with $(\mu R)^2 = -(2-J)^2 + L^2$

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

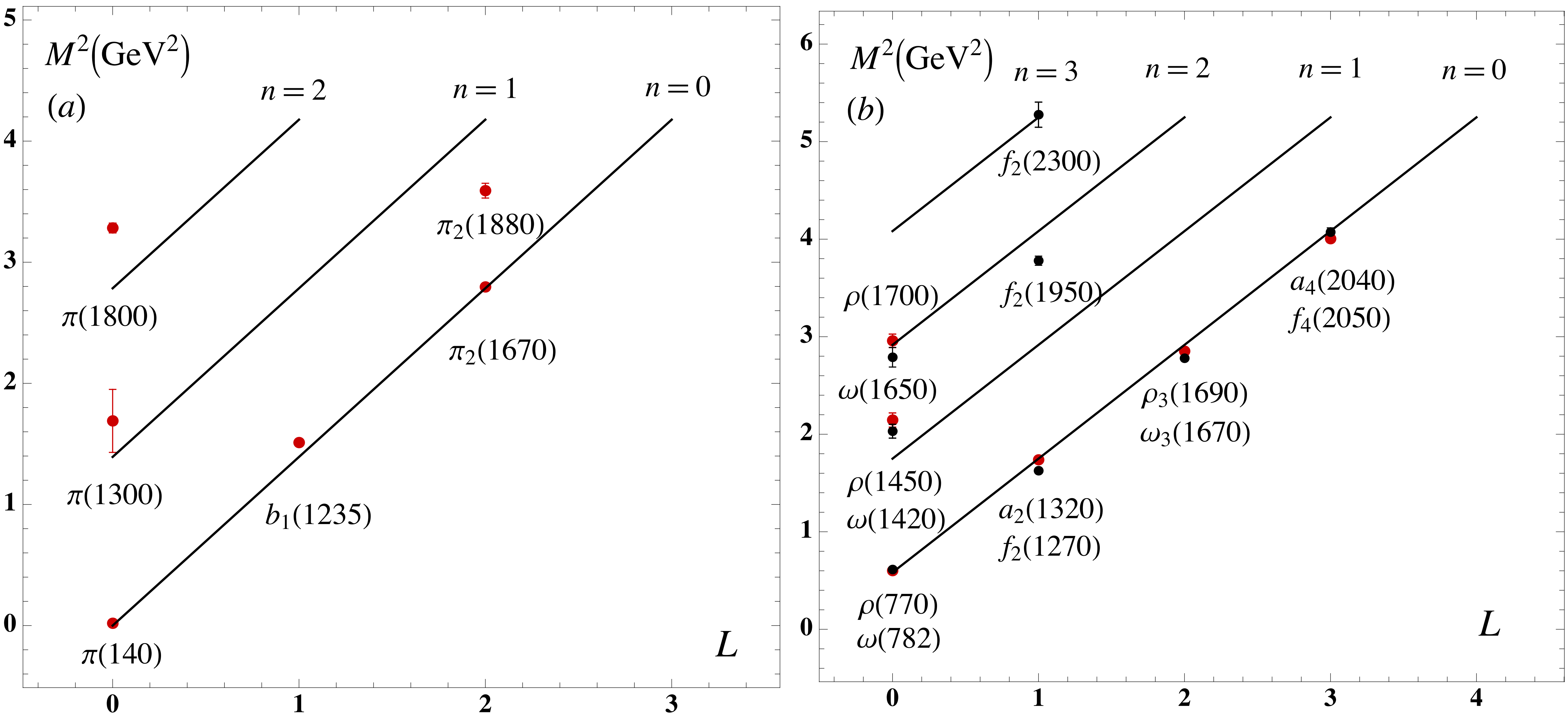
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Extended Conformal Invariance

- AdS_5: Isometries of the conformal group
- Light-Front Holography: $\mathrm{AdS}_5 \equiv H_{LF}$ $\quad z \leftrightarrow \zeta$
- dAFF: Introduce Mass Scale κ in Hamiltonian while retaining conformal symmetry of action
- Dilaton-Modified AdS_5 $S_{AdS_5} \to e^{+\kappa^2 z^2} S_{AdS_5}$
- Fubini and Rabinovici: Superconformal Algebra
- Yu-Ju Chiu, sjb: Conformal Invariance in general dimensions $\quad d \neq 4$

$m_u = m_d = 0$

de Tèramond, Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi| \sum_a m_a^2/x_a |\psi\rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda\left(n + \frac{J+L}{2}\right)$$

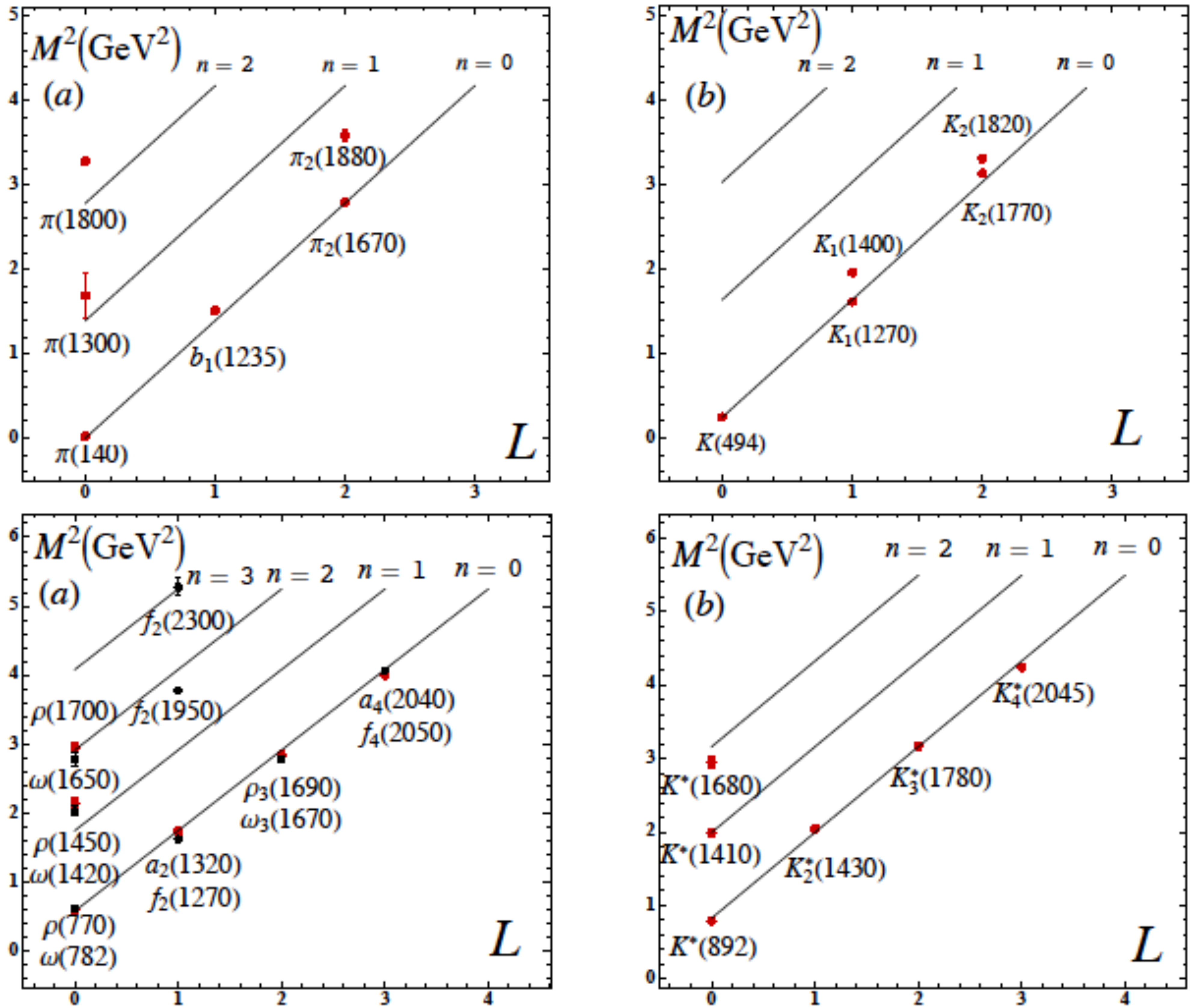
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



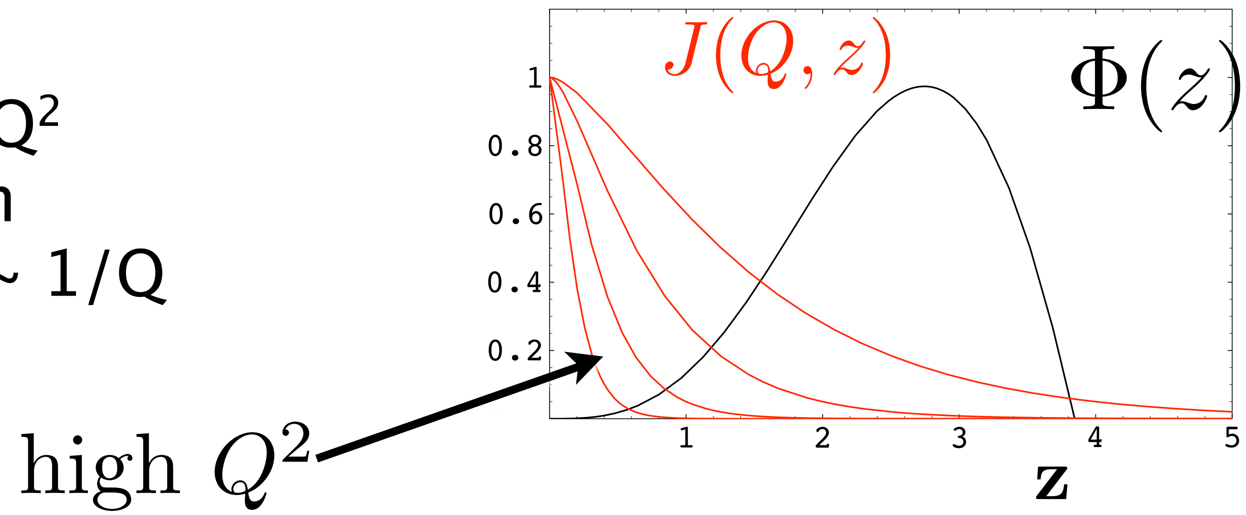
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



**Polchinski, Strassler
de Teramond, sjb**

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

**Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

$$\text{Twist } \tau = n + L$$

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

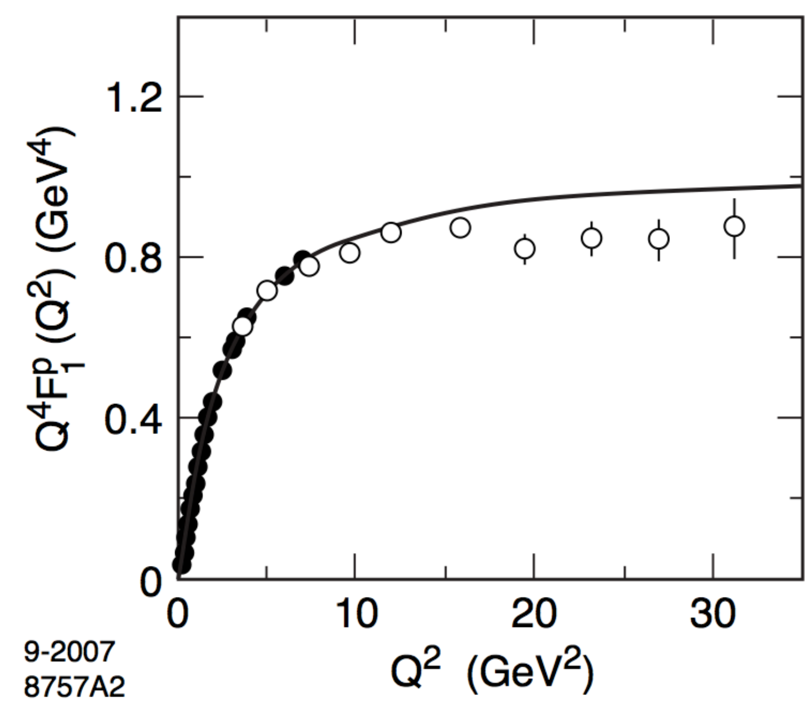
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n + 1/2)$



Uniqueness

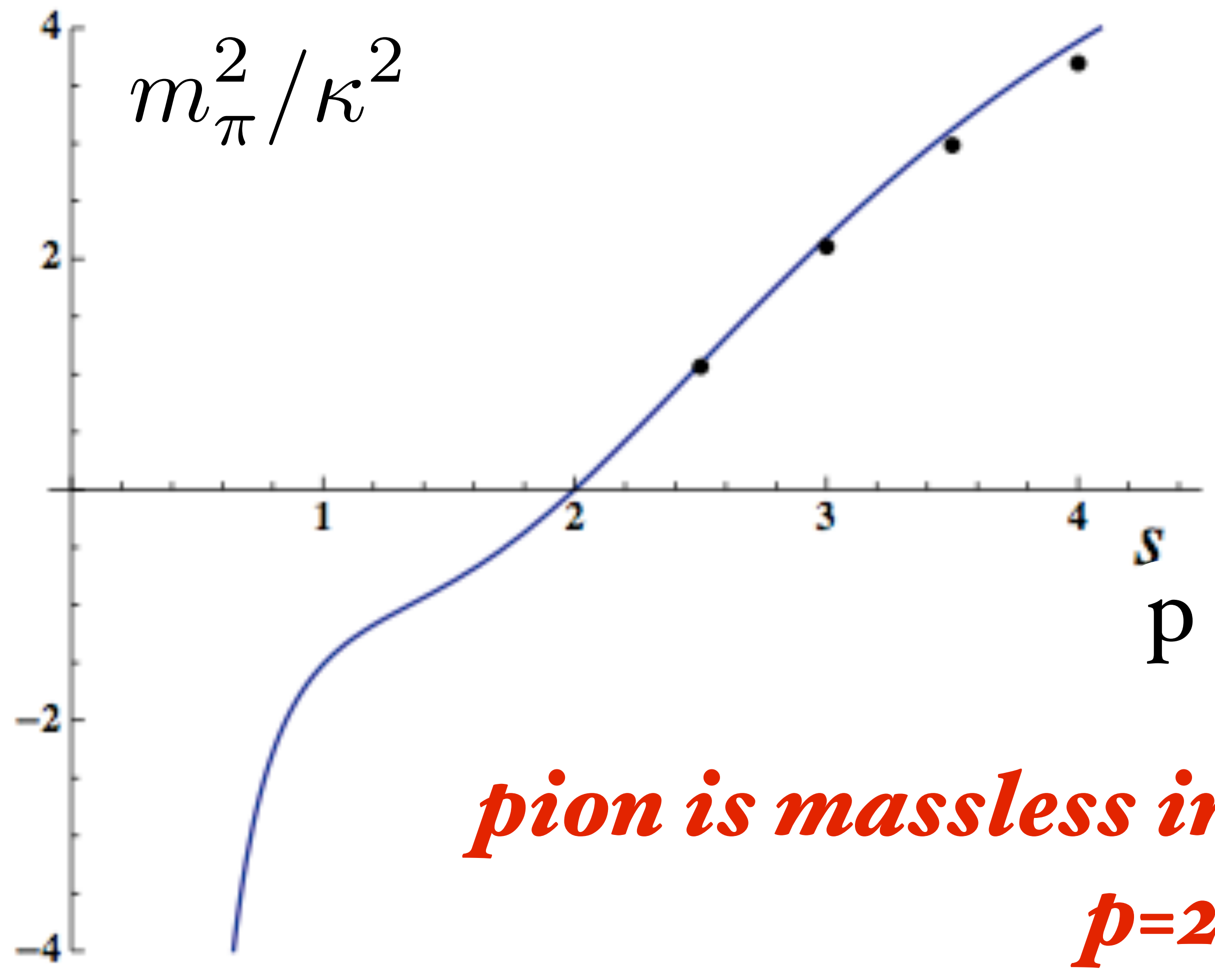
de Tèramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit! No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

Current Matrix Elements in AdS Space (SW)

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1 + t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

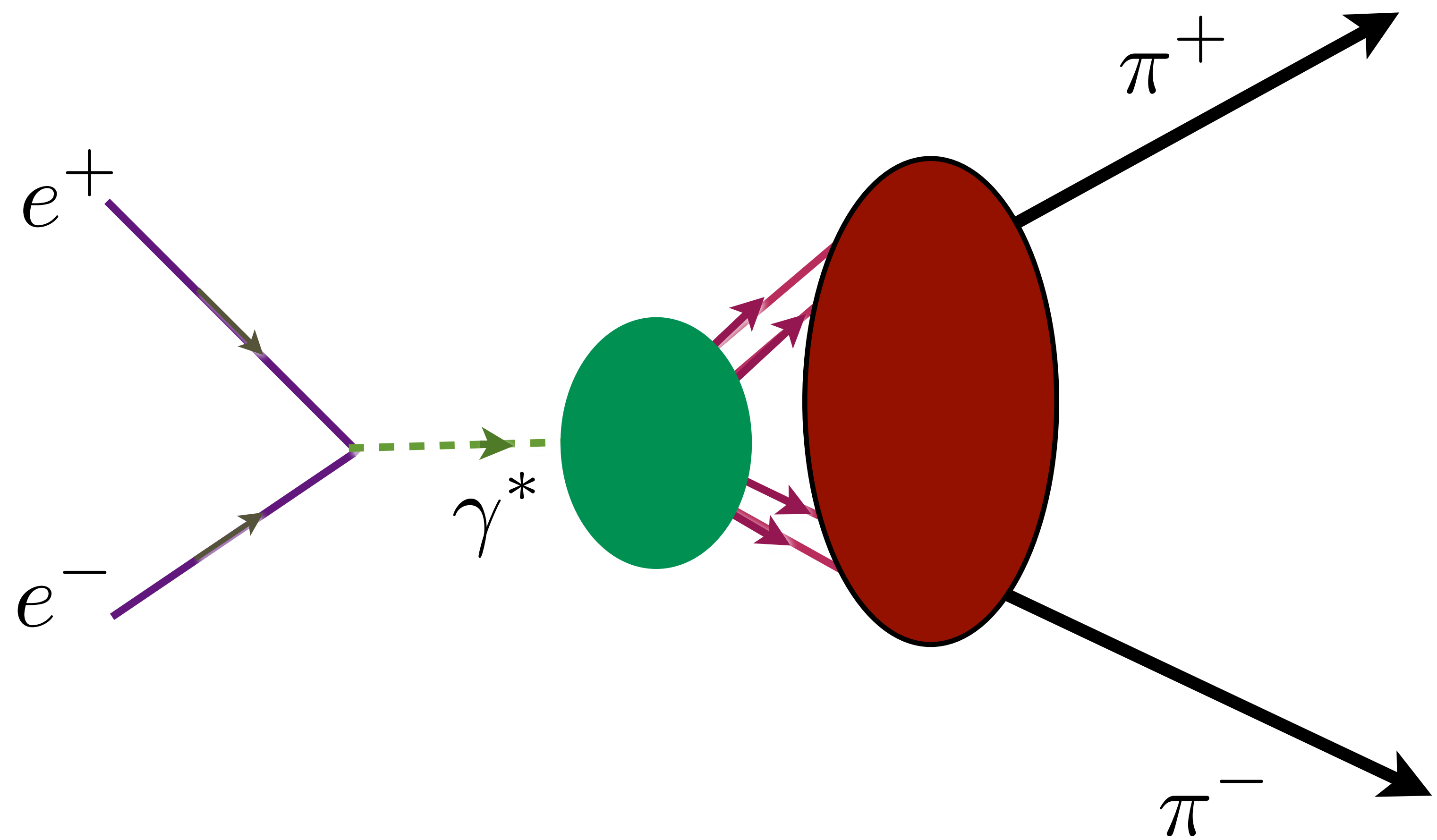
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

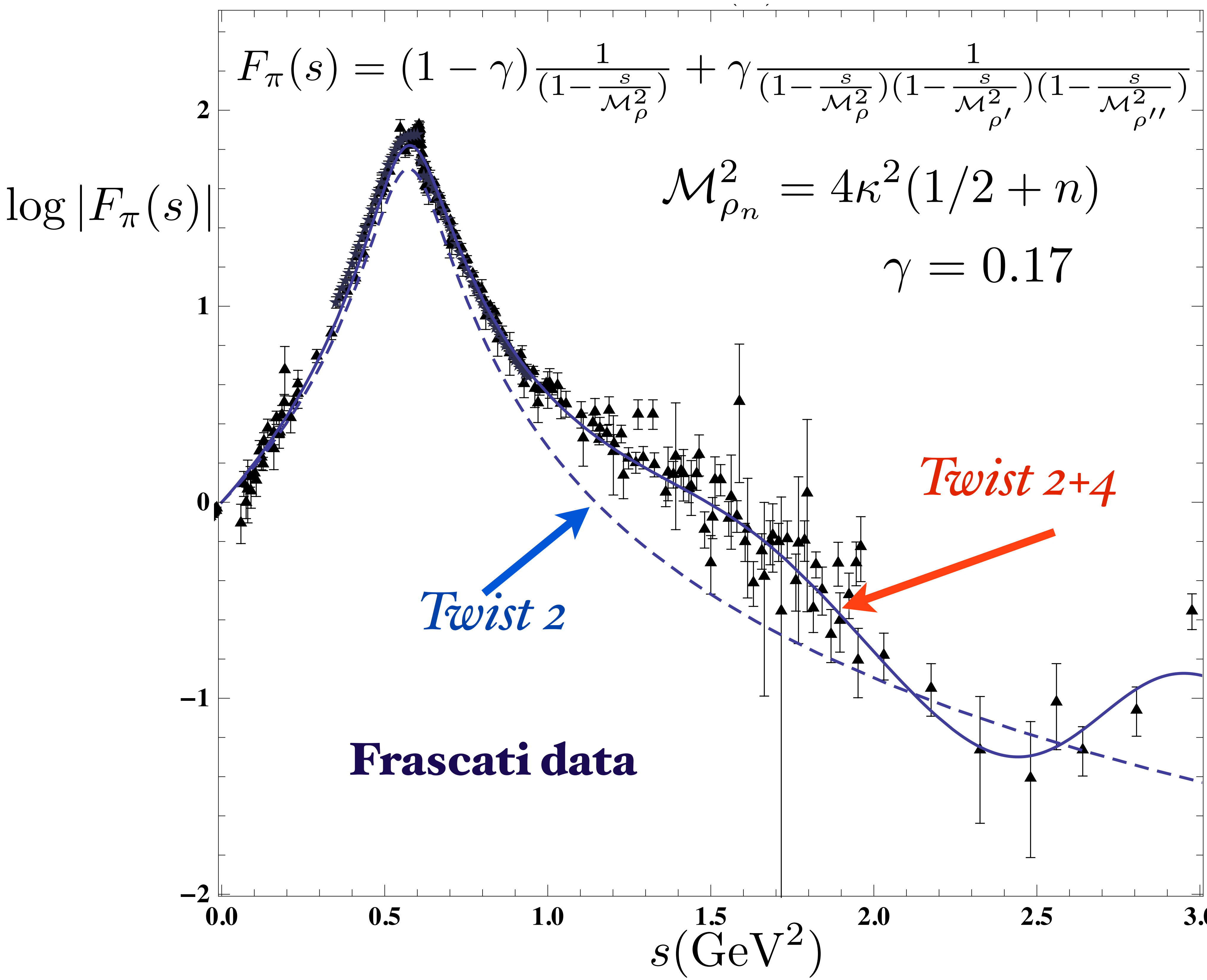
the external current decouples from the dilaton field.

de Tèramond & sjb
Grigoryan and Radyushkin

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



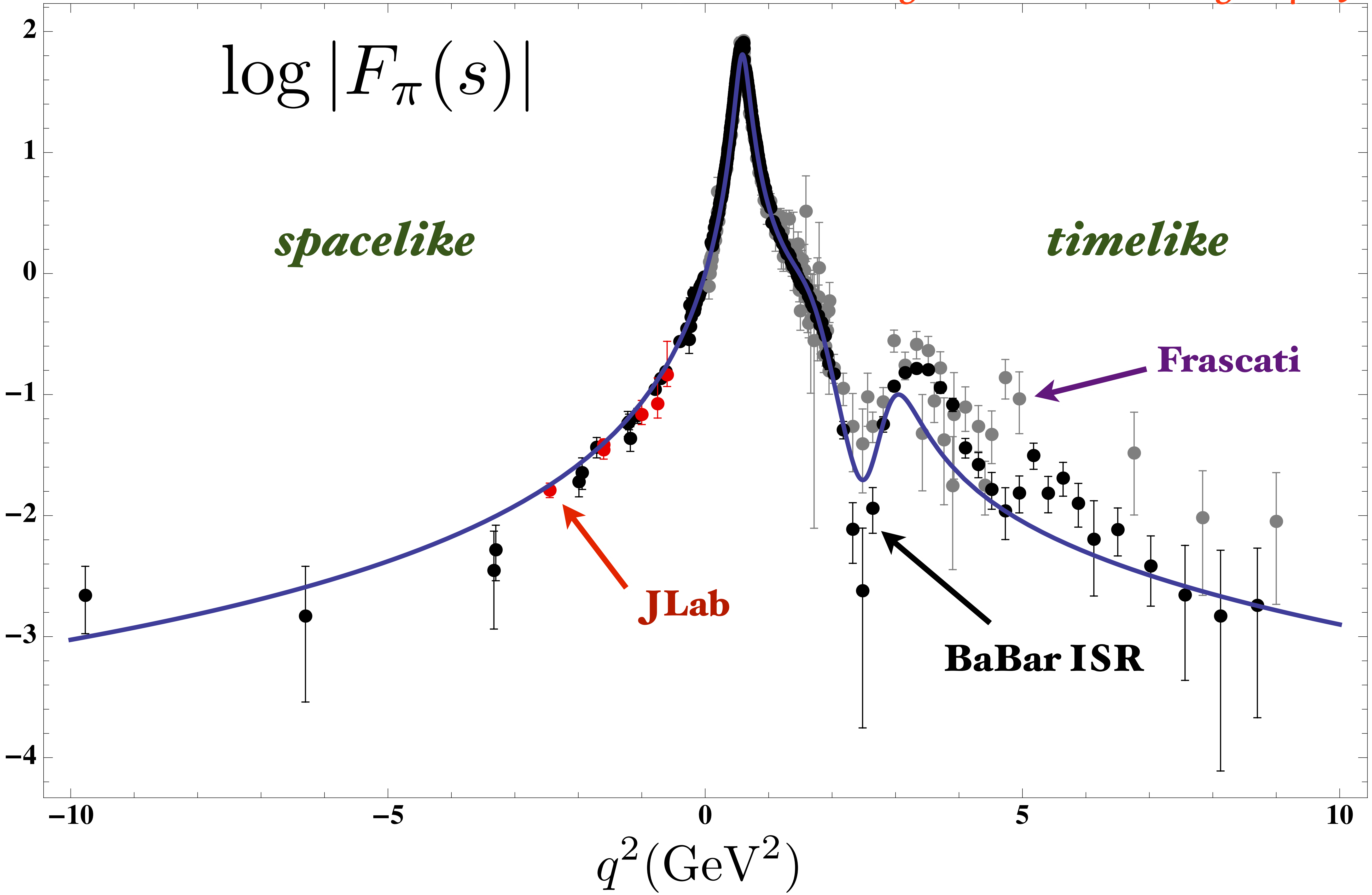
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
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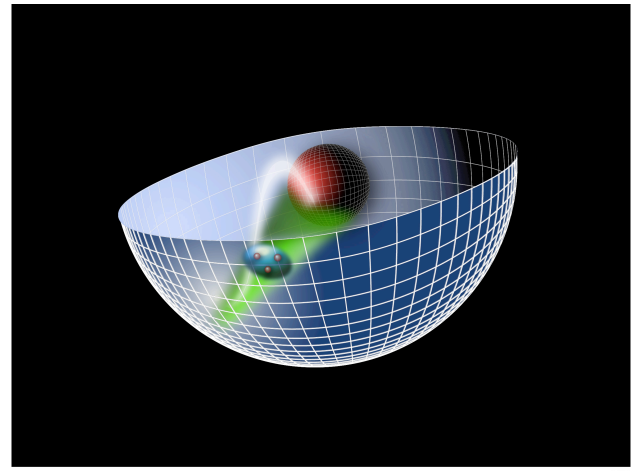
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

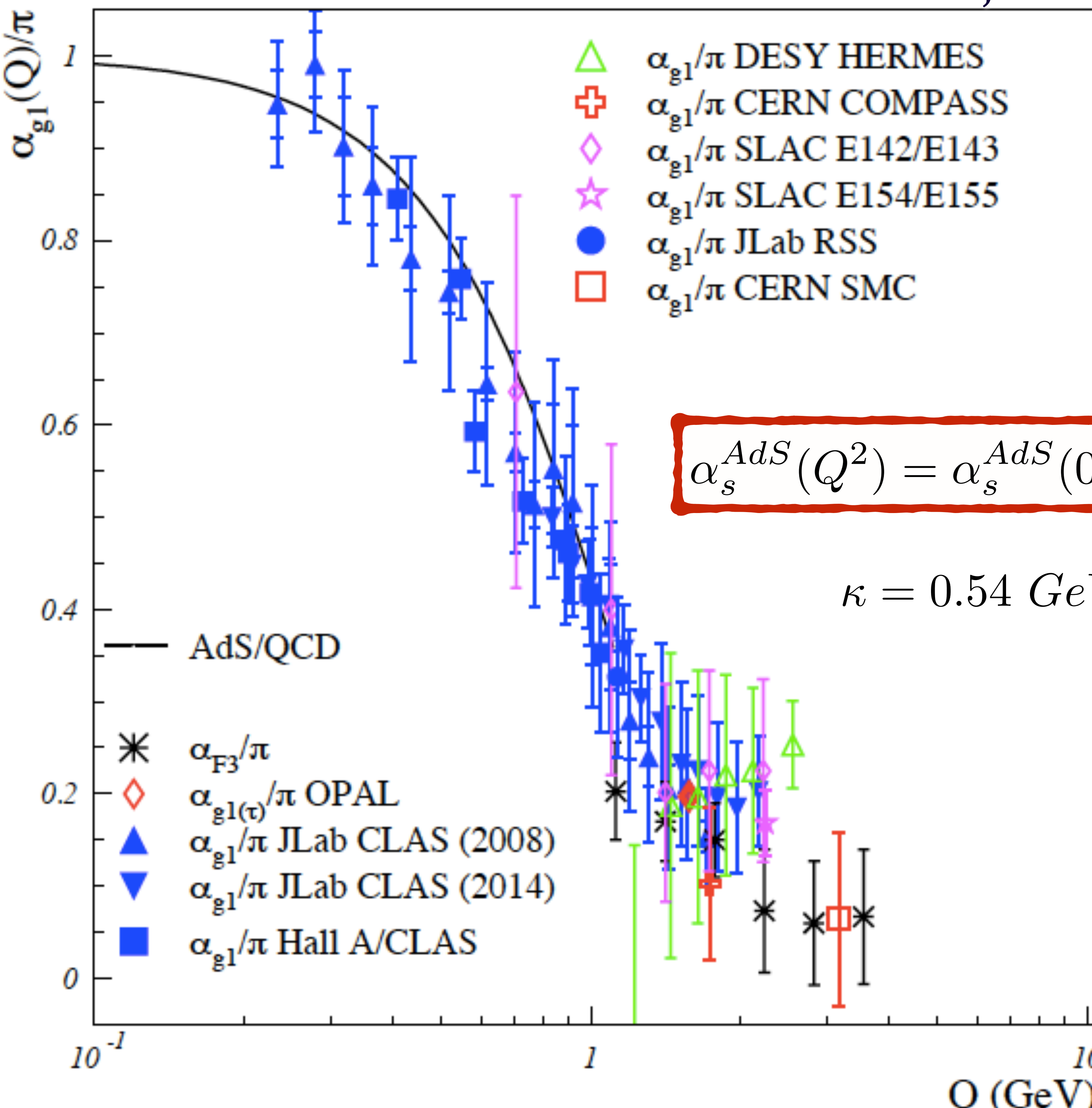
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Deur, de Teramond, sjb



$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)e^{-Q^2/4\kappa^2}$$

$$\kappa = 0.54\ GeV$$

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

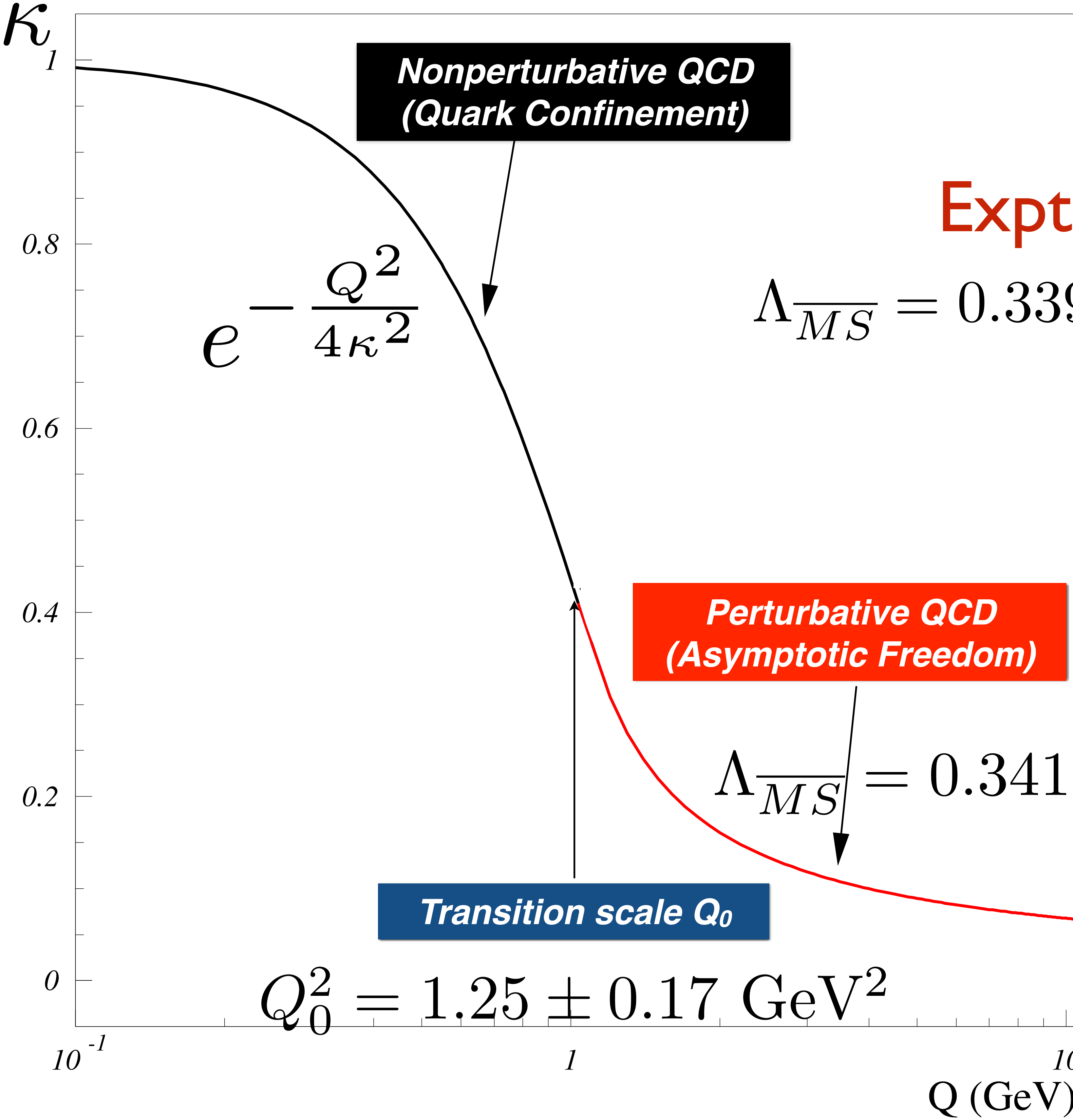
$m_\rho = \sqrt{2}\kappa$

$m_p = 2\kappa$

All-Scale QCD Coupling

Deur, de Teramond, sjb

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$$e^{-\frac{Q^2}{4\kappa^2}}$$

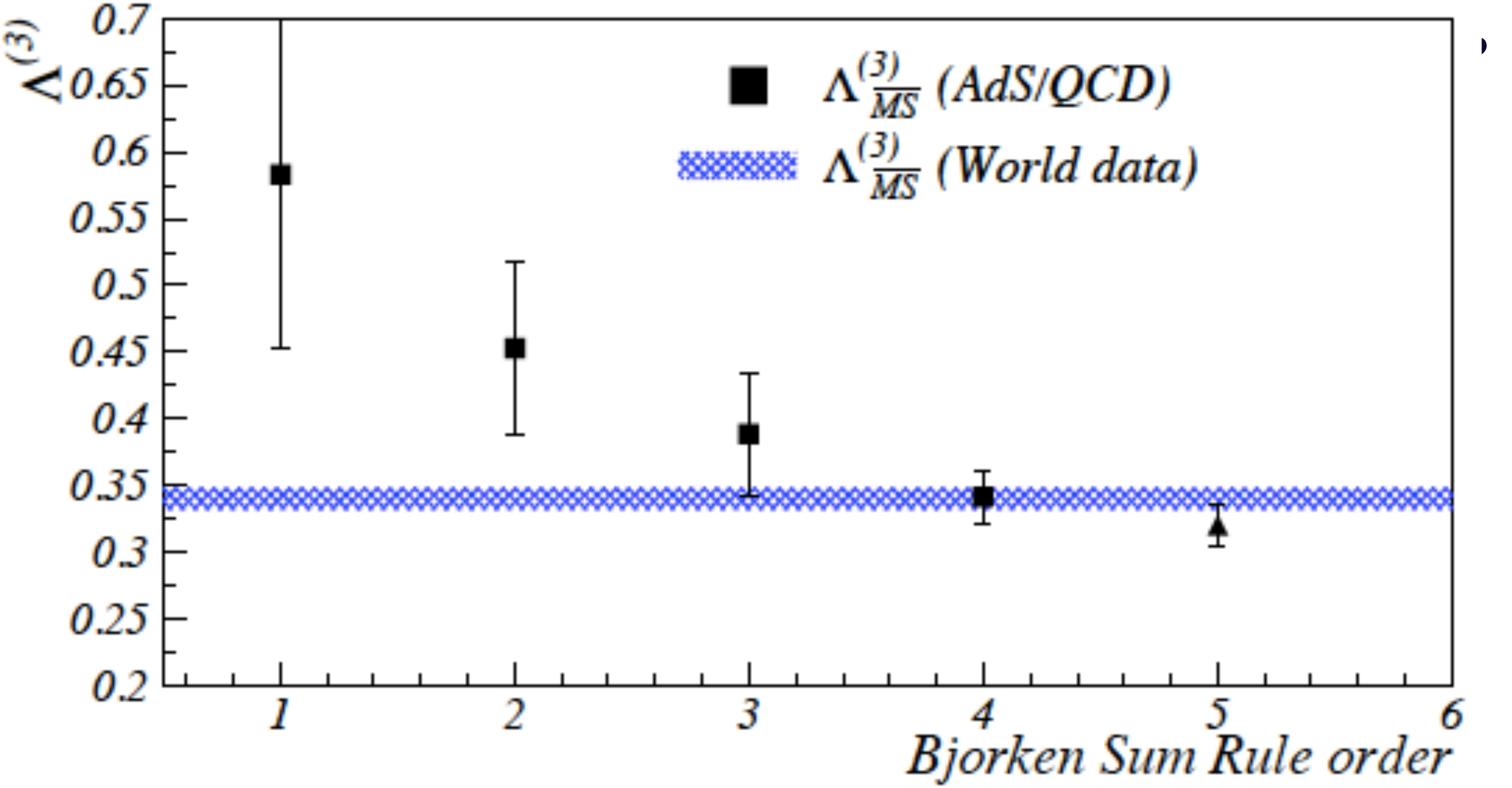
Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024\ GeV$$

$$Q_0^2 = 1.25 \pm 0.17\ \mathrm{GeV}^2$$

$\lambda \equiv \kappa^2$

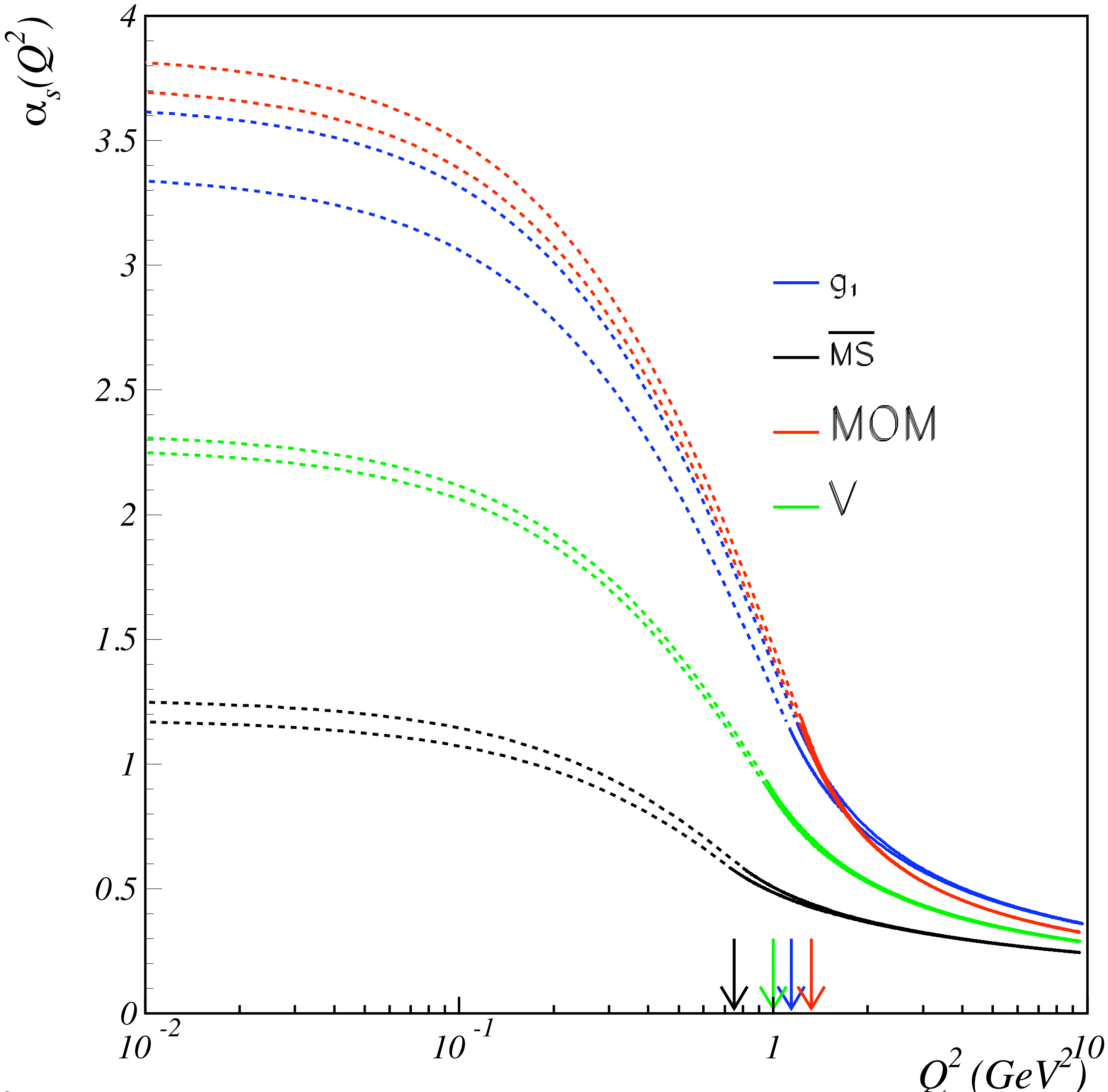


$$\Lambda_{\overline{MS}} = 0.341\ GeV = 0.440 m_\rho = 0.622\kappa$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

Experiment: $M_\rho = 0.7753 \pm 0.0003\ GeV$

Unification Predictions in Various Schemes



Preliminary

Deur, de Teramond, sjb

Unification Scale Q_0

- Matches perturbative to nonperturbative QCD
- Use for ERBL, DGLAP
- Hadronization at amplitude level
- BLFQ transition scale
- Use Principle of Maximum Conformality (PMC) to make scheme-indepedent predictions without renormalization scale ambiguity —
- PMC: Eliminates an unnecessary theory uncertainty

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21 \; \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2 \; \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ / m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$(m_q = 0)$

$m_\pi = 0$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of κ
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π
- "Zero-Parameter" Model

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Connection to the Linear Instant-Form Potential

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M^2_{q\bar{q}} = 4\, m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\bar{q}} = 0$

$$M^2_{q\bar{q}} = \frac{\mathbf{k}_\perp^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}\, V + 2\, V \sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}_\perp^2 = \frac{\mathbf{k}_\perp^2}{4x(1-x)}, \quad p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
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$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for $m_q = 0$ (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1} F(Q^2) \to \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Features of AdS/QCD

de Tèramond, Dosch, Deur, sjb

- **Color confining potential $\kappa^4\zeta^2$ and universal mass scale from dilaton**

$$e^{\phi(z)} = e^{\kappa^2 z^2} \qquad \alpha_s(Q^2) \propto \exp - Q^2/4\kappa^2$$

- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$

- **Chiral Action remains conformally invariant despite mass scale** *DAFF*

- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**

- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**

- **Massless pion for massless quark**

- **Supersymmetric meson-baryon dynamics and spectroscopy: $L_M = L_B + 1$**

- **Dynamics: LFWFs, Form Factors, GPDs**

Superconformal Algebra
Fubini and Rabinovici

de Tèramond, Dosch, Lorce, sjb

Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_o**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

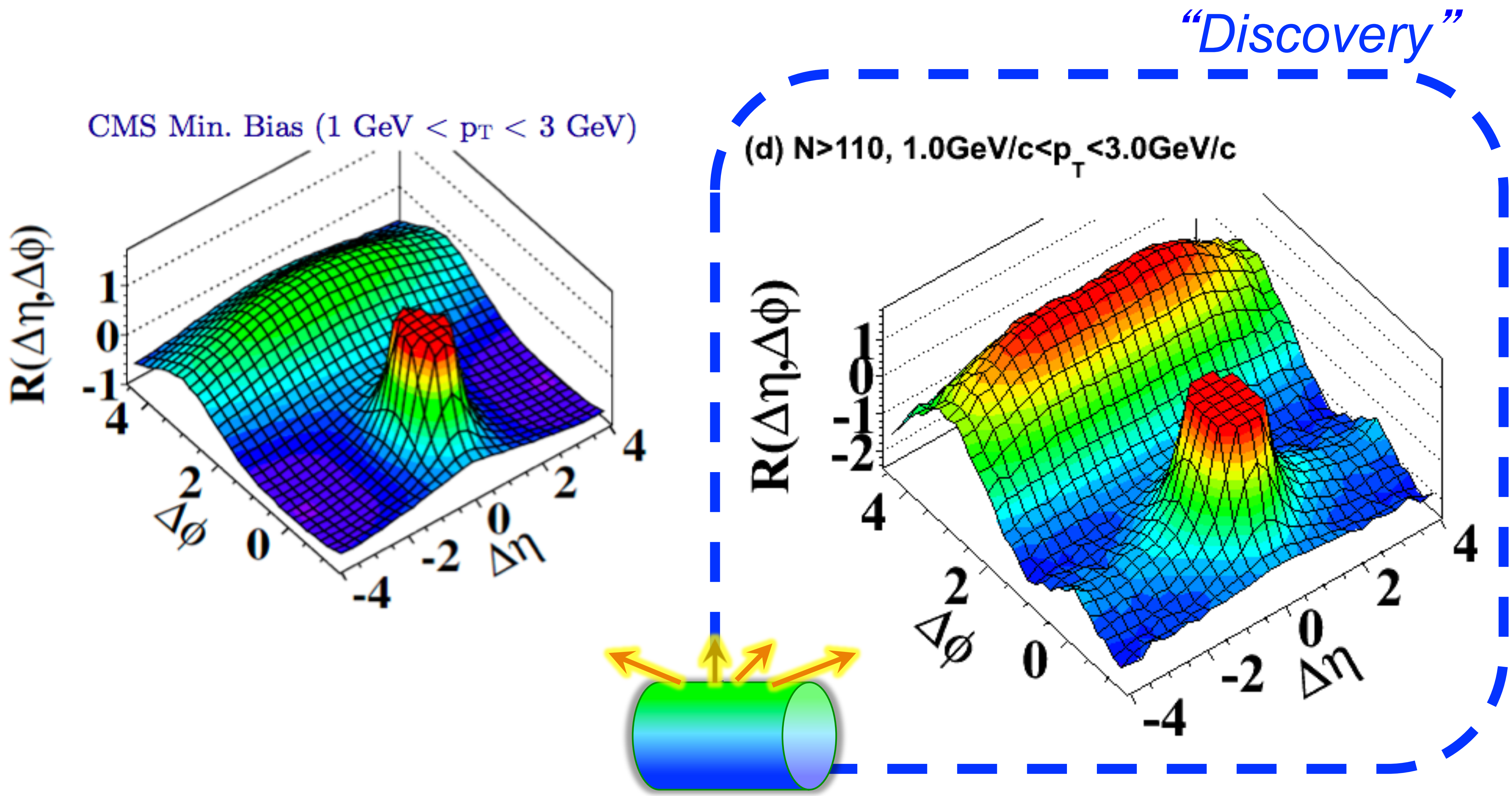
Vary, sjb, et al

Novel QCD Physics

- Collisions of Flux Tubes and the Ridge
- Factorization-Breaking Lensing Corrections
- Digluon initiated subprocesses and anomalous nuclear dependence of quarkonium production
- Higgs Production at high x_F from Intrinsic Heavy Quarks
- Direct, color-transparent hard subprocesses and the baryon anomaly
- PMC eliminates renormalization scale ambiguity order by order; increased top/anti-top asymmetry; scheme independent
- Light-Front Schrödinger Equation: New approach to confinement, origin of QCD mass scale

Ridge in high-multiplicity p p collisions

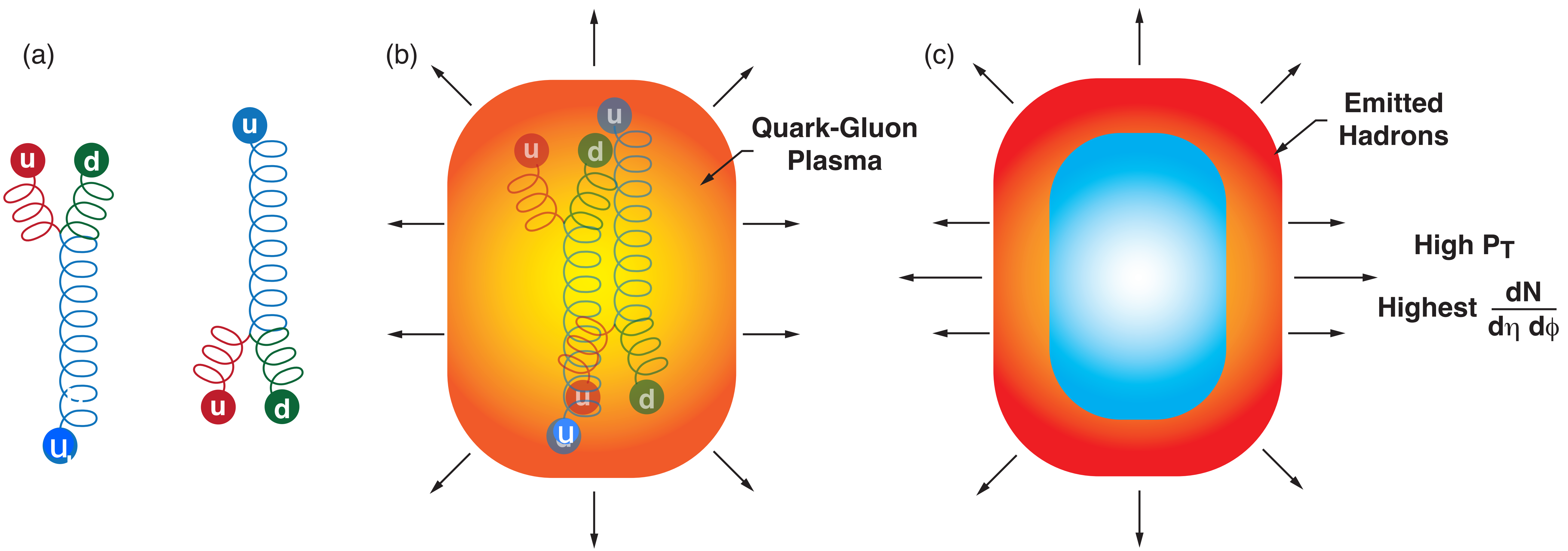
Two-particle correlations: CMS results



- Ridge: Distinct long range correlation in η collimated around ΔΦ≈ 0 for two hadrons in the intermediate 1 < p_T, q_T < 3 GeV

Raju Venugopalan

Ridge may reflect collision of aligned flux tubes



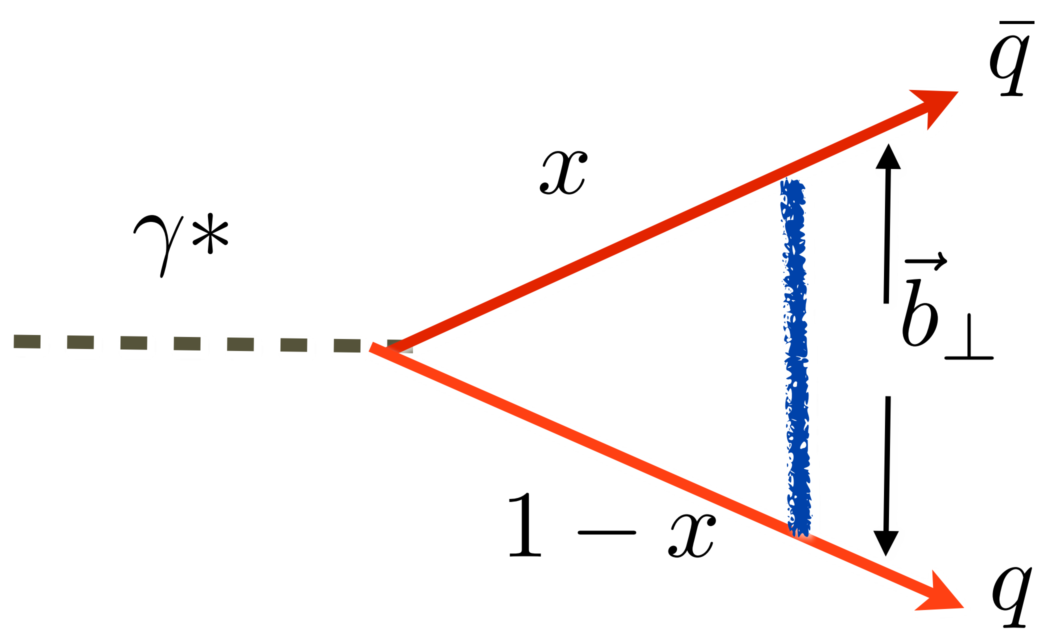
Bjorken, Goldhaber, sjb

Two-Dimensional Confinement

Interesting feature from AdS/QCD

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

$$\vec{\zeta}_\perp = \vec{b}_\perp \sqrt{x(1-x)}$$



confinement in plane of pair

Multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

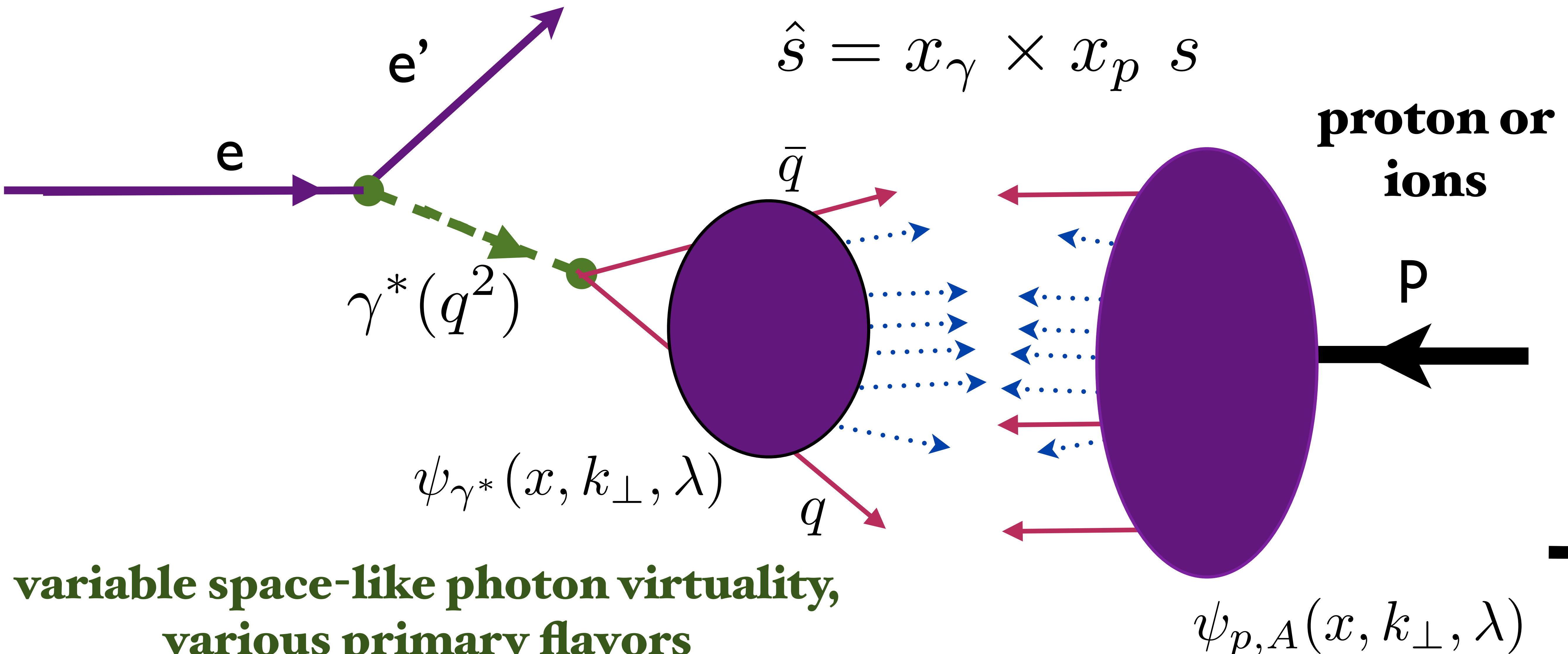
Bjorken, Goldhaber, sjb

We suggest that this "ridge"-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

The "spray" of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.

Electron-Ion Colliders: Virtual Photon-Ion Collider

Perspective from the e-p collider frame



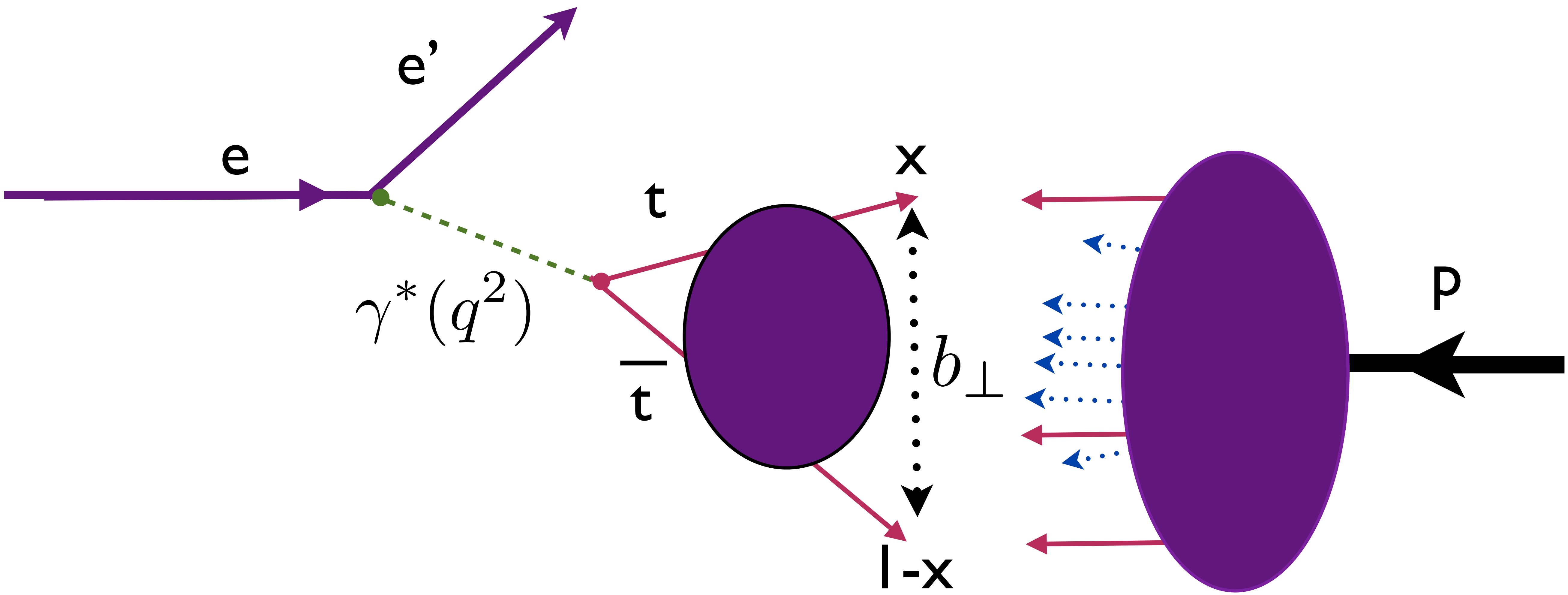
variable space-like photon virtuality, various primary flavors

$\bar{q}$ q plane aligned with lepton scattering plane ~ cos²φ

Front-surface dynamics: shadowing/antishadowing

$\bar{t}\,t$ acts as a 'drill'

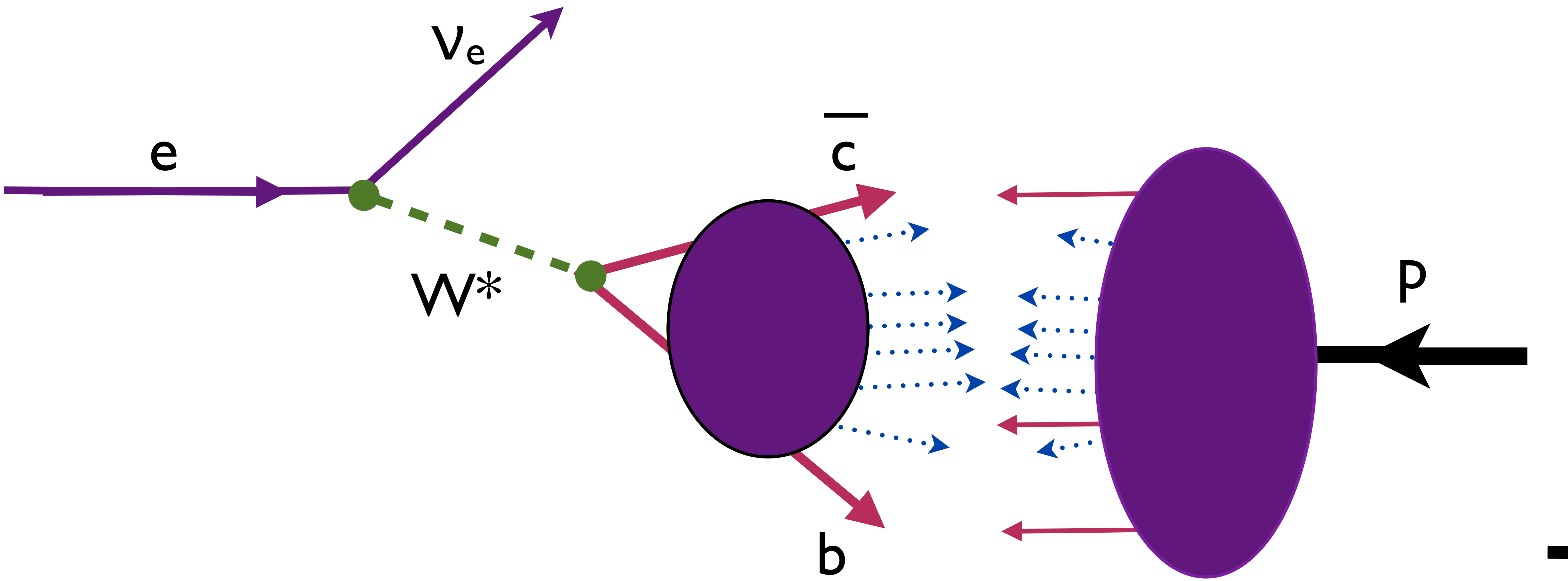
$$\langle b_\perp^2 \rangle \sim \frac{1}{Q^2 x(1-x) + M_t^2}$$



High Q^2, high M^2_Q virtual photon at LHeC acts as a precision, small bore, linearly oriented, flavor-dependent probe acting on a proton or nuclear target.

Study final-state hadron multiplicity distributions, ridges, nuclear dependence, etc.

EIC: Virtual Weak Boson-Proton Collider



variable W* virtuality,
variable flavors

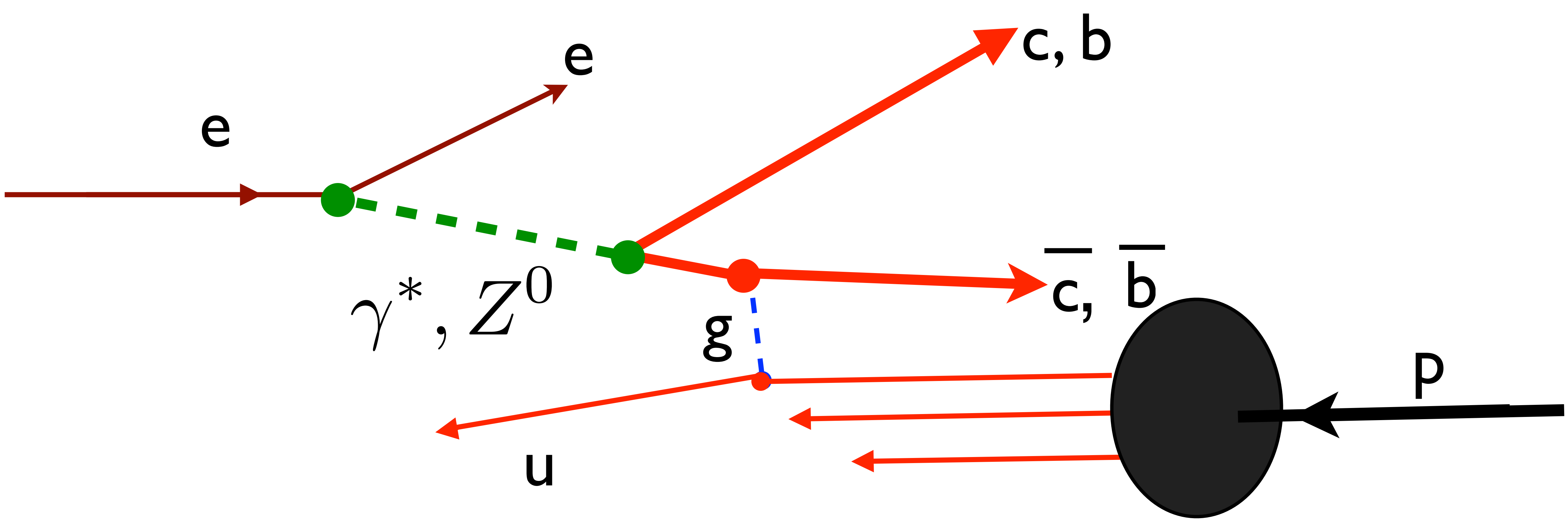
proton or ions

EIC: Virtual-Photon—Ion Collider

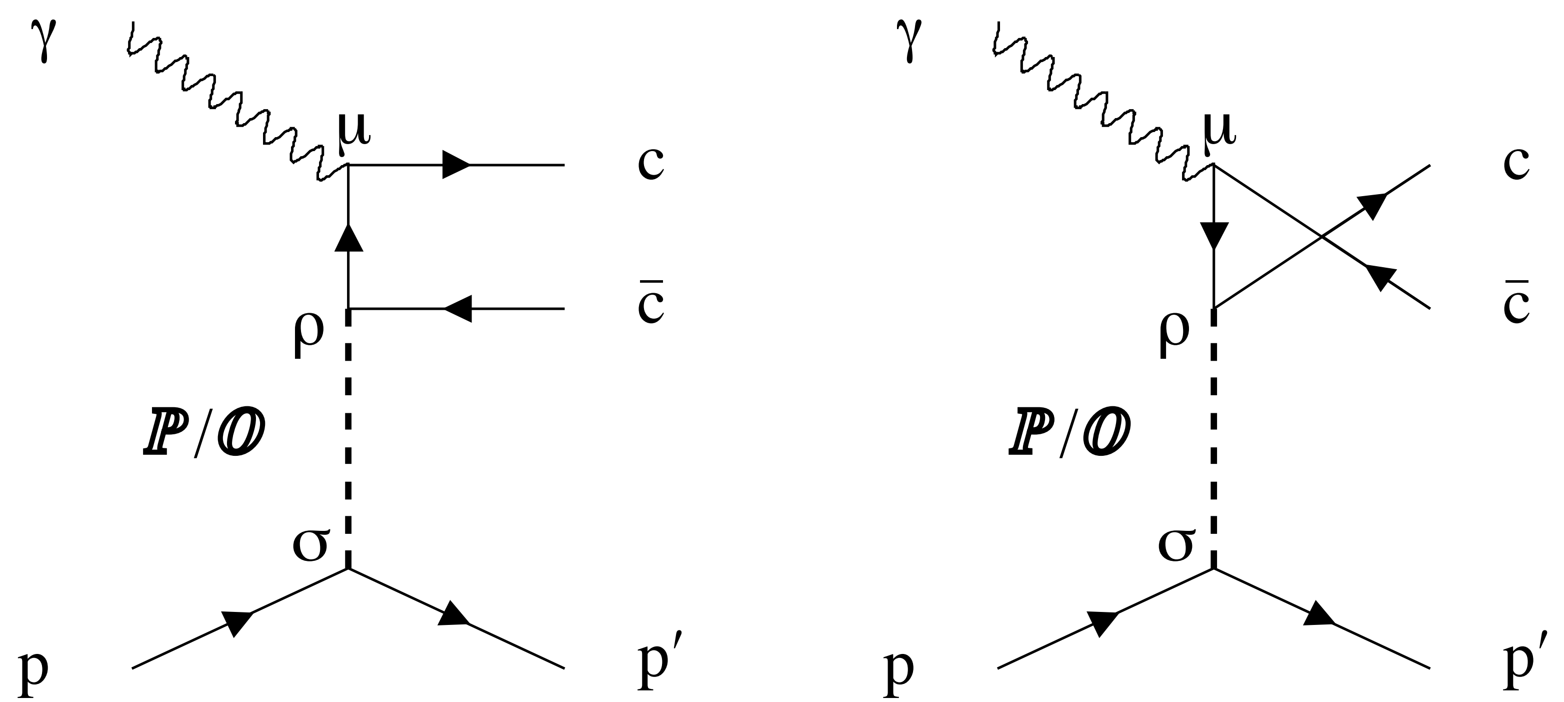
Inclusive c,b Electroproduction at the EIC

$c - \bar{c}$ asymmetry from $\gamma^* - Z^*$ or pomeron/odderon interference

Interpretation: Charm quark in photon vs. heavy sea quark in proton?

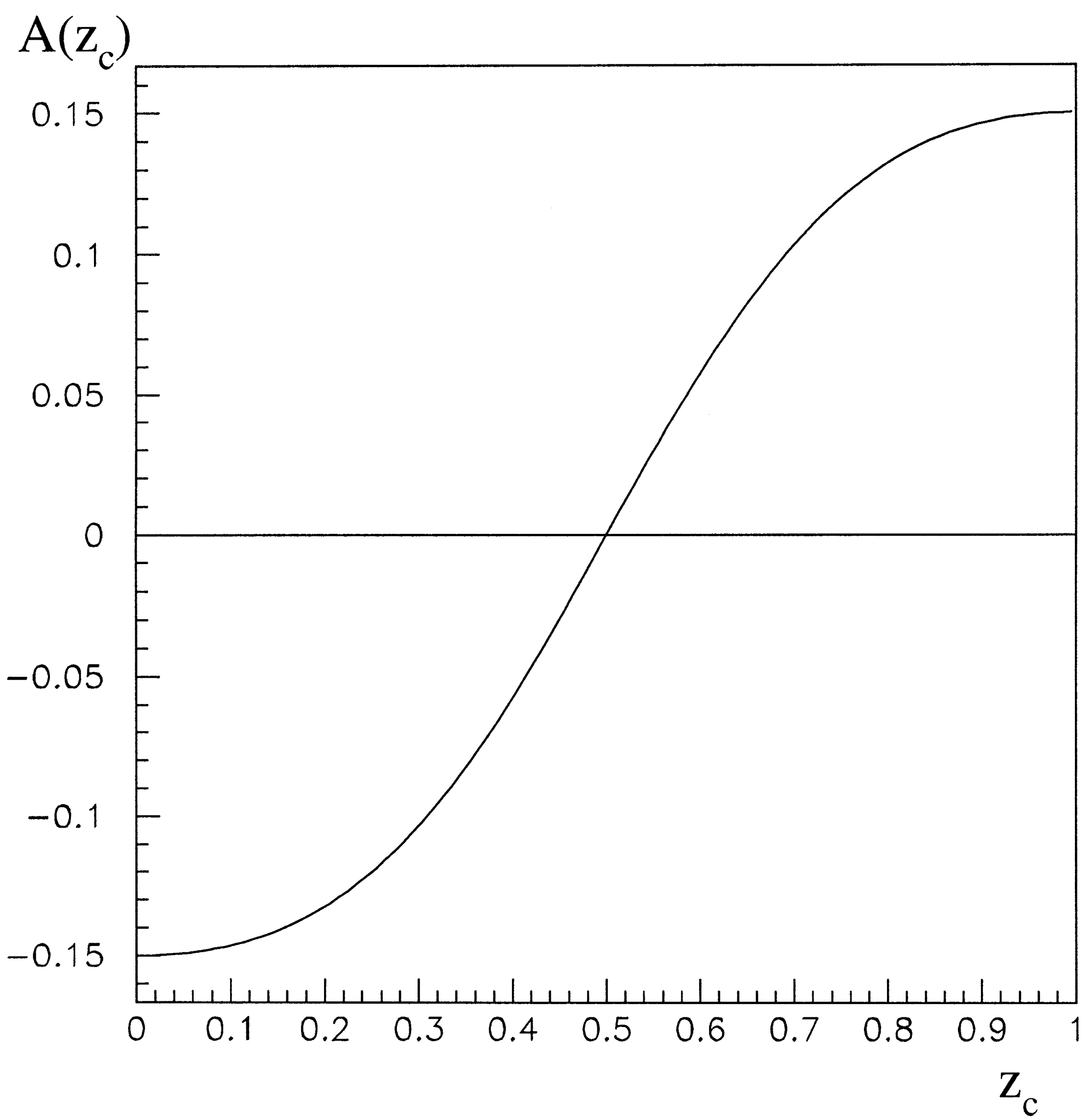


$Q\ \bar{Q}$ Plane correlated with Electron Scattering Plane



$$\gamma^* p \to c\bar{c}p$$

Odderon-Pomeron Interference!



$$\mathscr{A}(t \simeq 0, M_X^2, z_c) \simeq 0.45 \left(\frac{s_{\gamma p}}{M_X^2}\right)^{-0.25} \frac{2z_c - 1}{z_c^2 + (1 - z_c)^2}$$

Measure charm asymmetry in photon fragmentation region

Merino, Rathsman, sjb

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- Higgs Production at high x_F from Intrinsic Heavy Quarks
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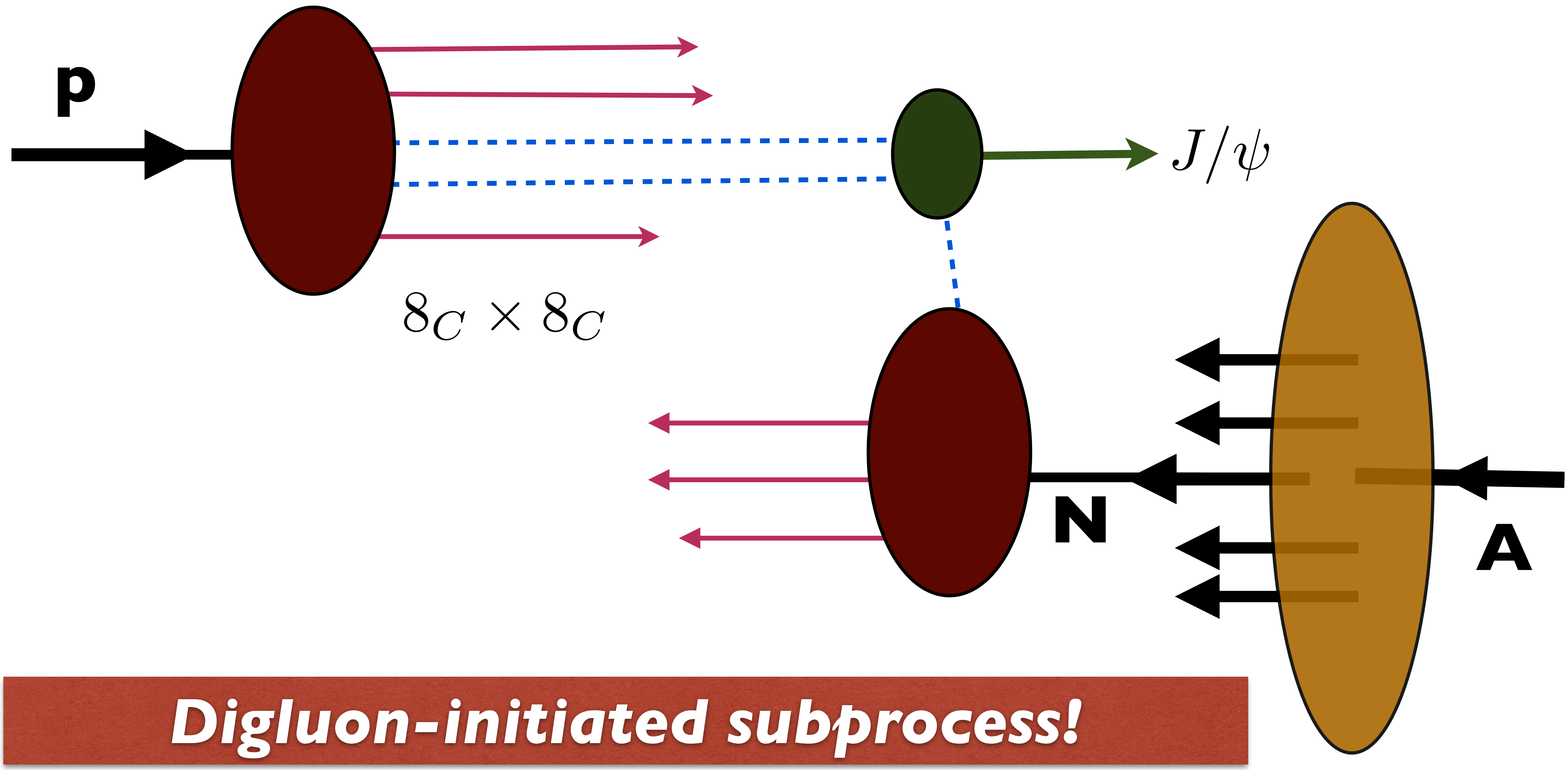
- Light-Front Schrödinger Equation: New approach to confinement, origin of QCD mass scale

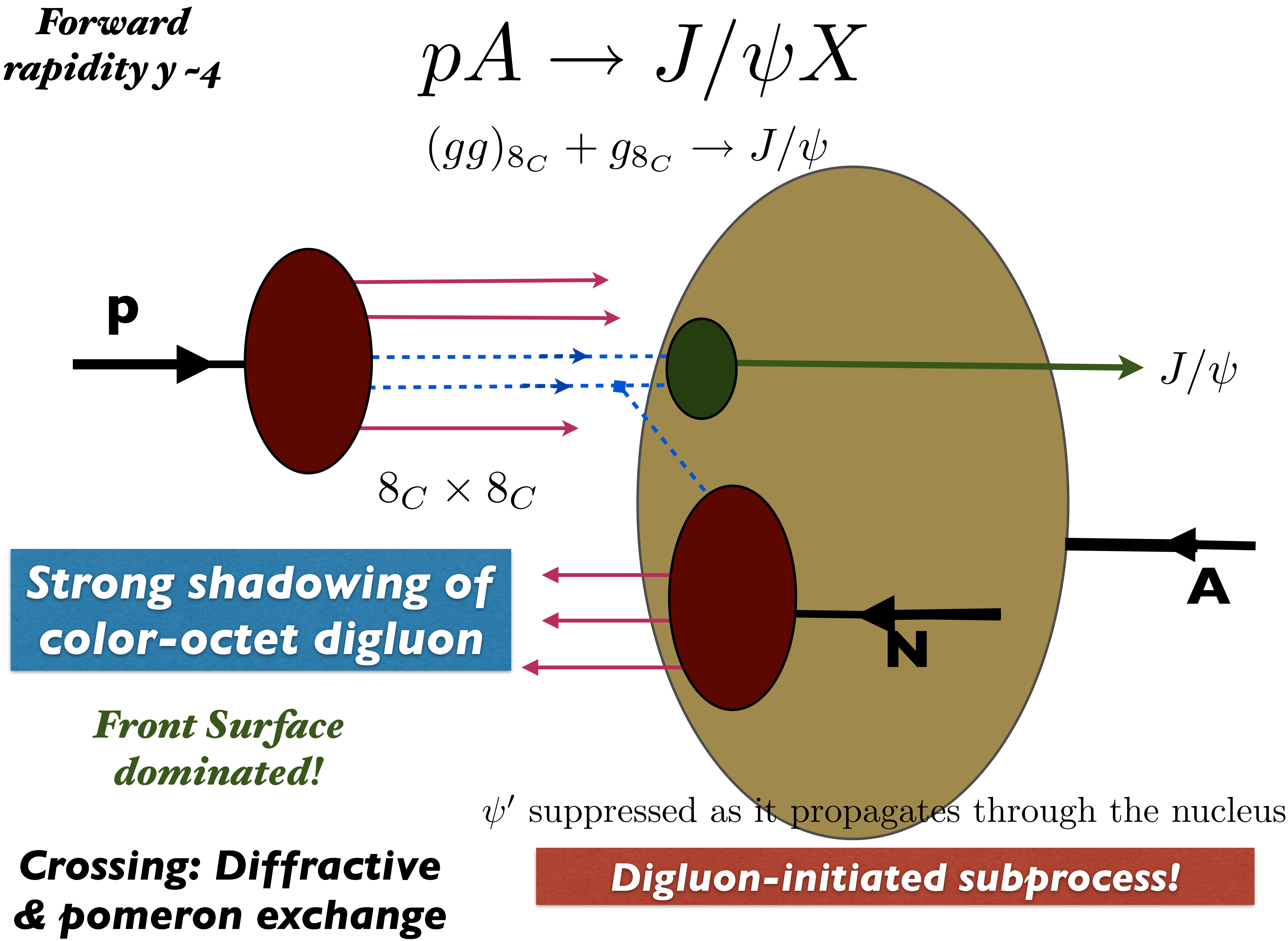
$$pA \to J/\psi X$$

$$(gg)_{8_C} + g_{8_C} \to J/\psi$$



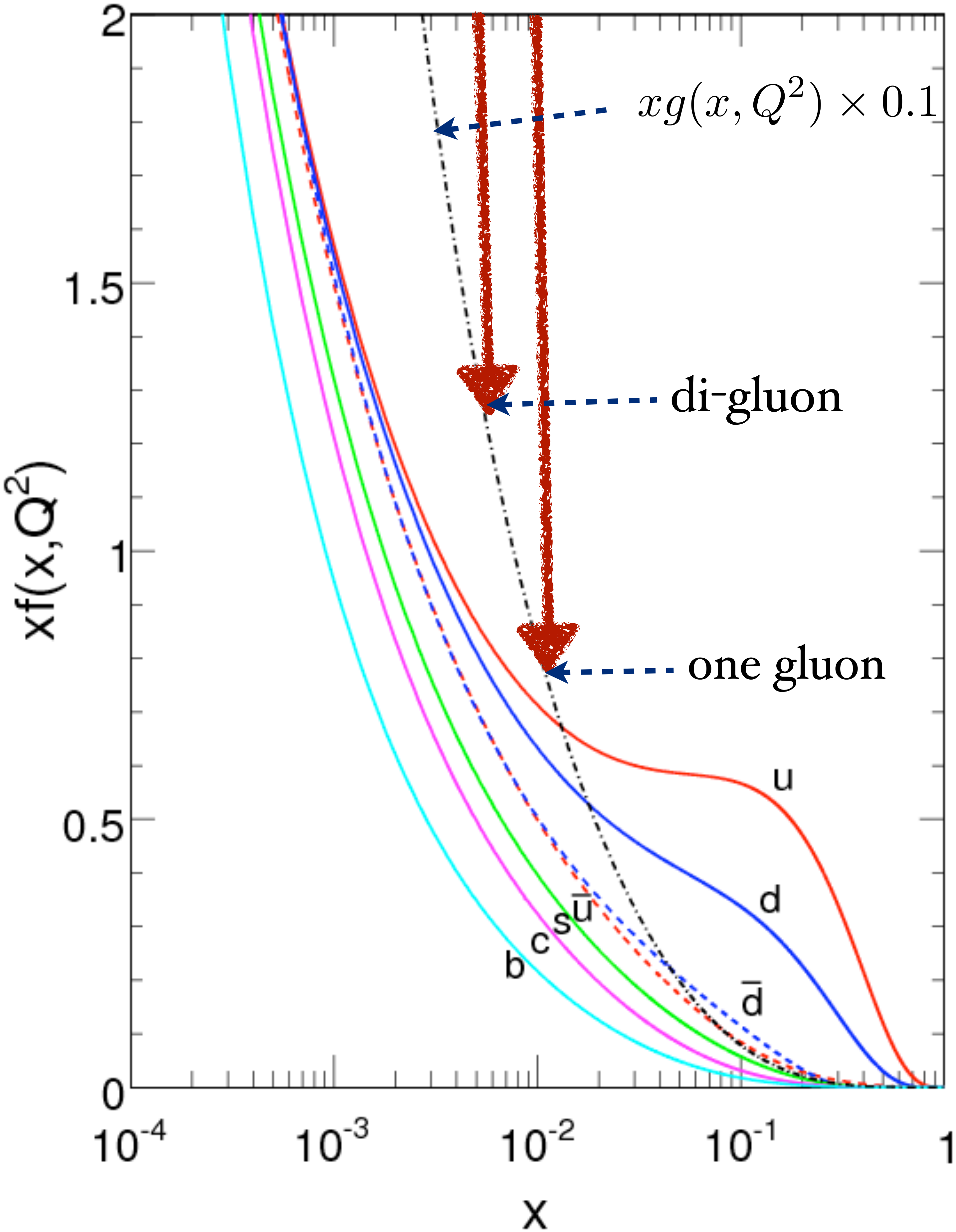
Digluon-initiated subprocess!

Higher-Twist but can dominate at forward rapidity, small p_T



Forward rapidity y ~4
$pA \to J/\psi X$
$(gg)_{8_C} + g_{8_C} \to J/\psi$
p
$8_C \times 8_C$
J/ψ
A
N
Strong shadowing of color-octet digluon
Front Surface dominated!
ψ' suppressed as it propagates through the nucleus
Crossing: Diffractive & pomeron exchange
Digluon-initiated subprocess!

Two gluons at $g(0.005) \sim \frac{13}{0.005} = 2600$ vs. one gluon at $g(0.01) \sim \frac{8}{0.01} = 800$



“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

Revised Gell Mann-Oakes-Renner Formula in QCD

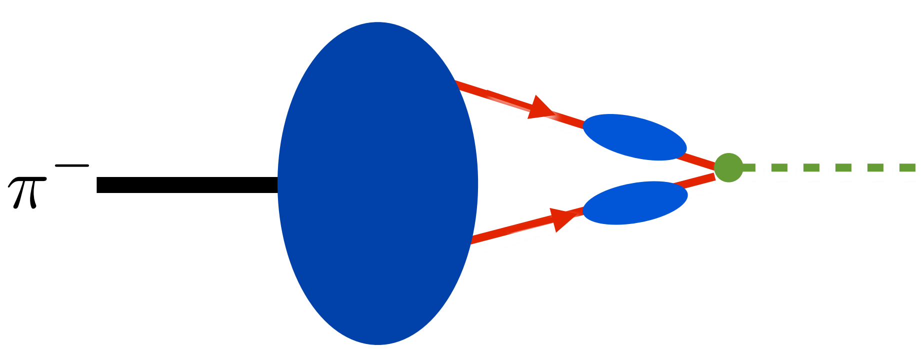
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0 >_{LF}= M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time τ = t+z/c within causal horizon

Frame-independent description of the causal physical universe!

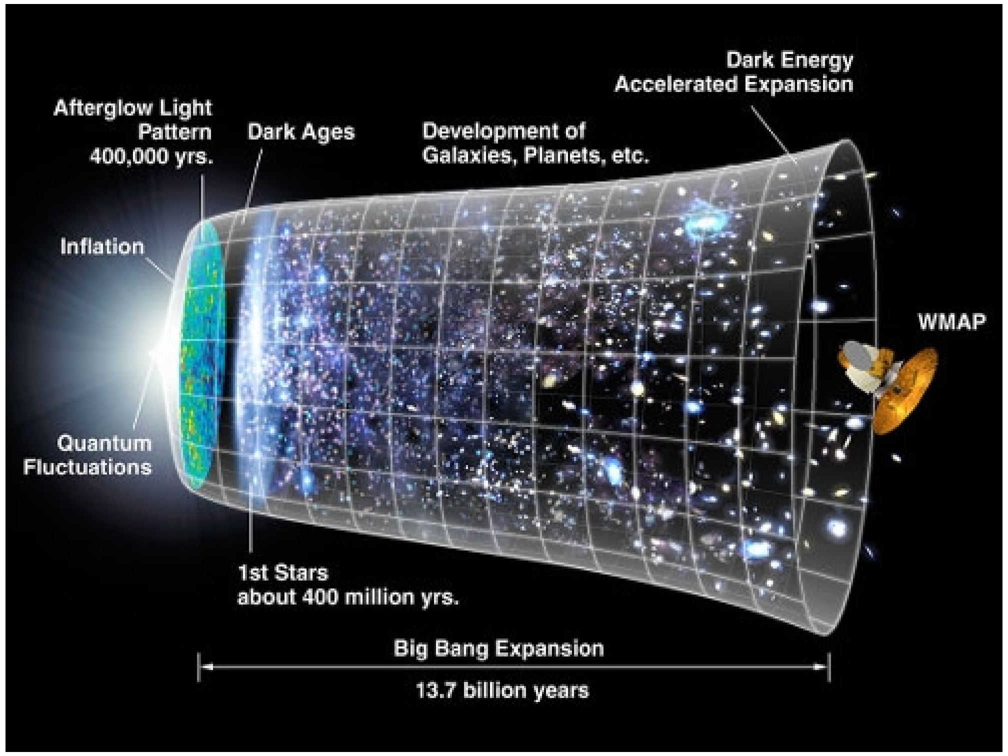
Front Form Vacuum Describes the Empty, Causal Universe

- $P^+ = \sum_i p_i^+,\ p_i^+ > 0$: LF vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$ (usual vacuum bubbles are kinematically forbidden in the front form !)
- Frame independent definition of the vacuum within the causal horizon

$$\boxed{P^2|0\rangle = 0}$$

(LF vacuum also has zero quantum numbers and $P^+ = 0$)

- LF vacuum is defined at fixed LF time $x^+ = x^0 + x^3$ over all $x^- = x^0 - x^3$ and $\mathbf{x}_\perp$, the expanse of space that can be observed within the speed of light
- Causality is maintained since LF vacuum only requires information within the causal horizon
- The front form is a natural basis for cosmology: universe observed along the front of a light wave



Light-Front vacuum can simulate empty universe

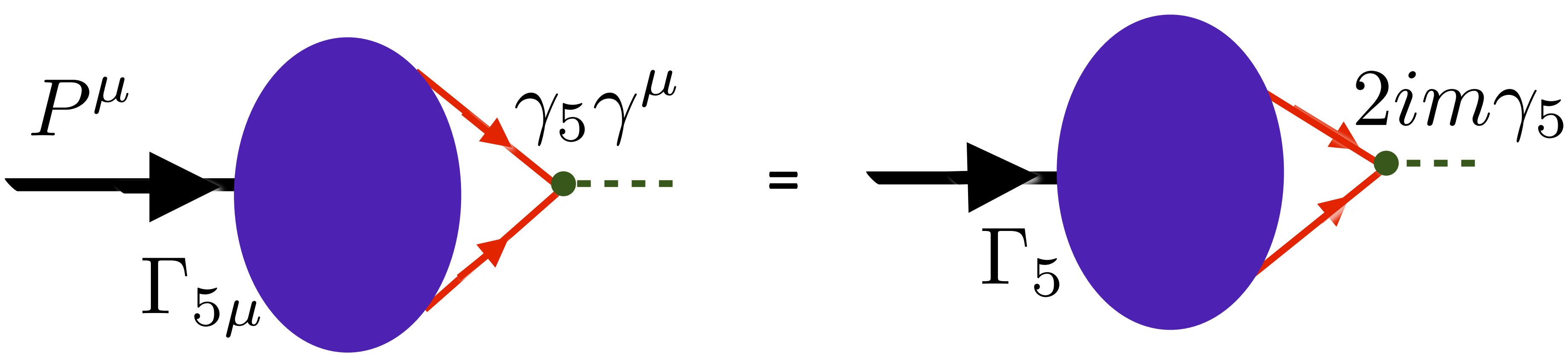
Shrock, Tandy, Roberts, sjb

- **Independent of observer frame**
- **Causal**
- **Lowest invariant mass state M= 0.**
- **Trivial up to k^+=0 zero modes-- already normal-ordering**
- **Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)**
- **QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.**
- **QED vacuum; no loops**
- **Zero cosmological constant from QED, QCD**

Ward-Takahashi Identity for axial current

$$P^{\mu}\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

Identify pion pole at $P^2 = m_{\pi}^2$

$$P^{\mu} < 0|\bar{q}\gamma_5\gamma^{\mu}q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_{\pi}m_{\pi}^2 = -(m_u + m_d)\rho_{\pi}$$

PHYSICAL REVIEW D **66**, 045019 (2002)

Light-front formulation of the standard model

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(Received 20 February 2002; published 20 August 2002)

Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+=0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

P. Srivastava, sjb

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode!***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to $T^\mu{}_\mu$; zero coupling to gravity**

P. Srivastava, sjb

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+\phi^\dagger\partial_-\phi + \partial_-\phi^\dagger\partial_+\phi - \partial_\perp\phi^\dagger\partial_\perp\phi - \mathcal{V}(\phi^\dagger\phi)$$

where $V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ with $\lambda > 0,\ \mu^2 < 0$

Constraint equation: $\int d^2x_\perp dx^- \left[\partial_\perp\partial_\perp\phi - \frac{\delta V}{\delta\phi^\dagger}\right] = 0$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$\omega(\tau, x_\perp)$ is a $k^+ = 0$ zero mode

$\omega = v/\sqrt{2}$ where $v = \sqrt{-\mu^2/\lambda}$

Thus a c-number in LF replaces conventional Higgs VEV

No coupling to gravity!

Possibility: $\partial_\perp\omega \neq 0$

Goals

- Test QCD to maximum precision at the LHC
- Maximize sensitivity to new physics
- High precision determination of fundamental parameters
- Determine renormalizations scales without ambiguity
- Eliminate scheme dependence

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

$through\ the\ PMC - BLM\ correspondence\ principle$

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\} - terms$

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

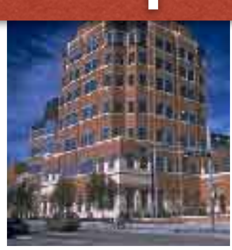
QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB



Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

Electron-Electron Scattering in QED

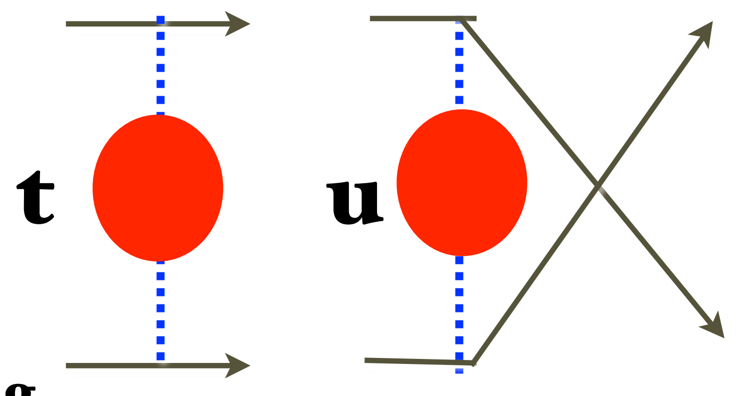
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



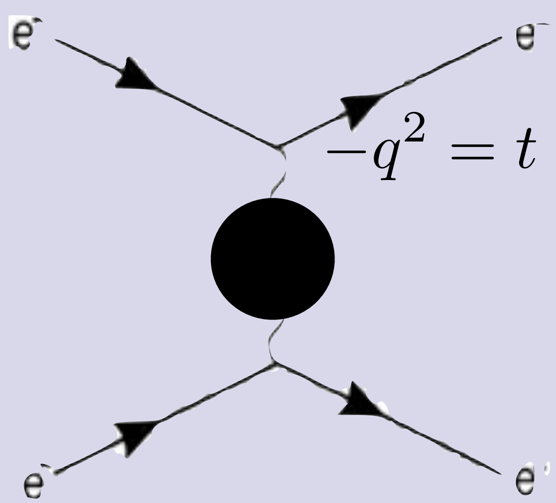
- **Two separate physical scales: t, u = photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

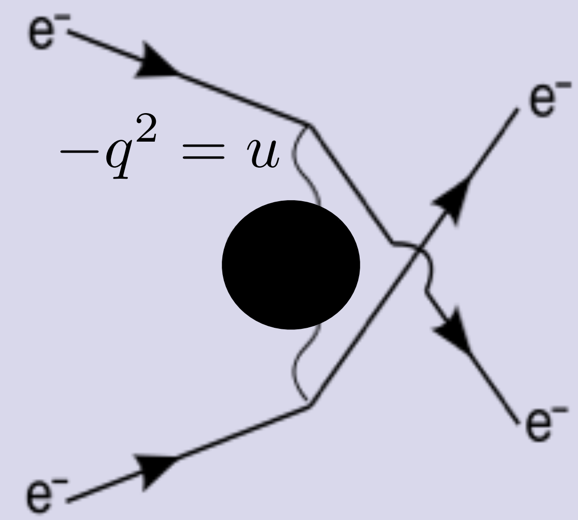
$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee\to ee} = \frac{8\pi s}{t}\alpha(t) + \frac{8\pi s}{u}\alpha(u)$$

Two separate scales;
one for each skeleton graph.



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log\frac{\mu^2_{\overline{MS}}}{m_\ell^2} = 6\int_0^1 dx\, x(1-x)\log\frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad \xrightarrow{Q^2 \gg m_\ell^2} \log\frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\alpha_{\overline{MS}}(e^{-5/3}q^2) = \alpha_{GM-L}(q^2).$$

Effective method in quest for new physics

November 14, 2013 - 06:29

ScienceNordic

CERN's Large Hadron Collider particle accelerator smashes protons together with such great force that it can give birth to hitherto unknown particles. A new method makes it easier to recognise the new particles.

Keywords: Physics

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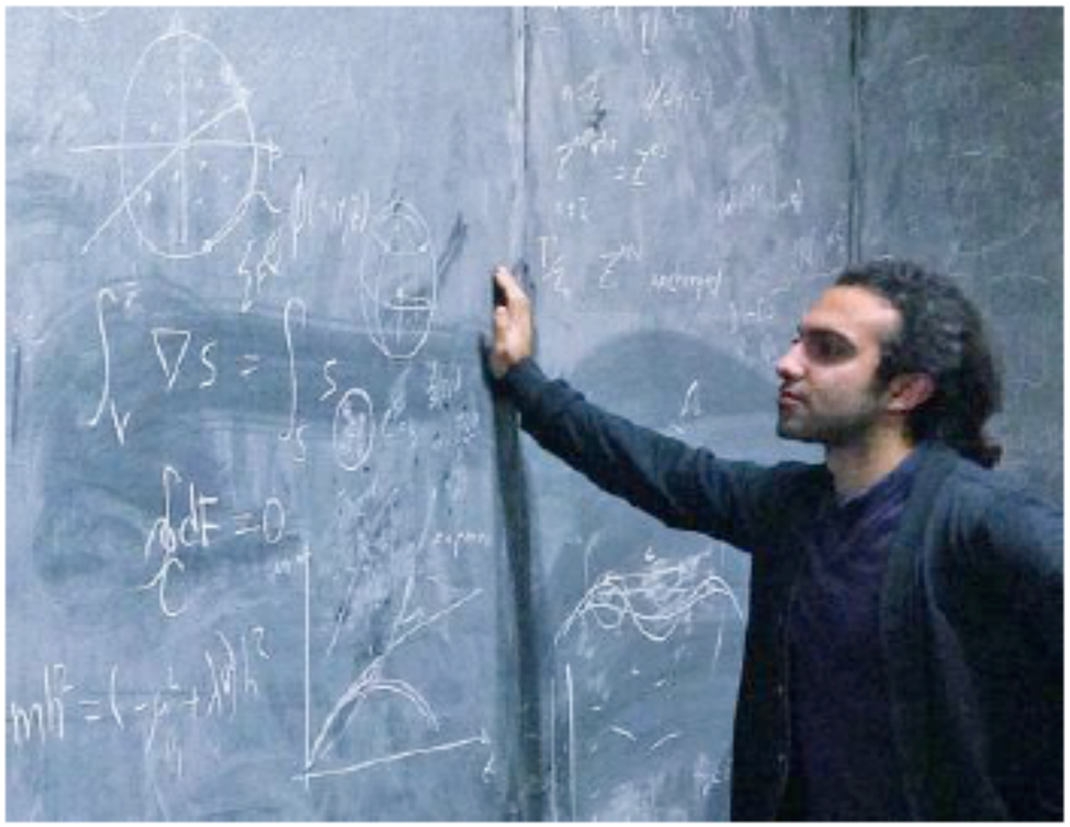
By: Henrik Bendix



Together with two other researchers, PhD fellow Matin Mojaza has developed a method that makes it easier to calculate the result of collisions in particle accelerators. (Photo: Matin Mojaza)

Nature can be rather unpredictable at times. It can for instance be incredibly hard to predict what will happen when nature's tiniest particles collide.

But this has become a bit easier now, as three physicists from three different continents have presented a new mathematical technique which can help theoretical physicists predict the result of experiments in which quarks – the constituents of nuclei – collide.

The new method was developed by Matin Mojaza, a PhD fellow at the Centre for Cosmology and Particle Physics Phenomenology at the University of Southern Denmark, and Stanley Brodsky of Stanford University, US, and Xing-Gang Wu of the Chongqing University in China. The method is described in an article in the journal *Physical Review Letters*.

The three researchers are hoping the new technique can be used to identify new elementary particles that have never been observed before.

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\text{MS}}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

M. Mojaza, Xing-Gang Wu, sjb

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\text{MS}}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

M. Mojaza, Xing-Gang Wu, sjb

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu_{\overline{\mathrm{MS}}}^2 \ \exp(\ln 4\pi - \gamma_E) \ , \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \ \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n!\beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a)\frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

M. Mojaza, Xing-Gang Wu, sjb

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5)\end{aligned}$$

PMC scales thus satisfy

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_3)^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

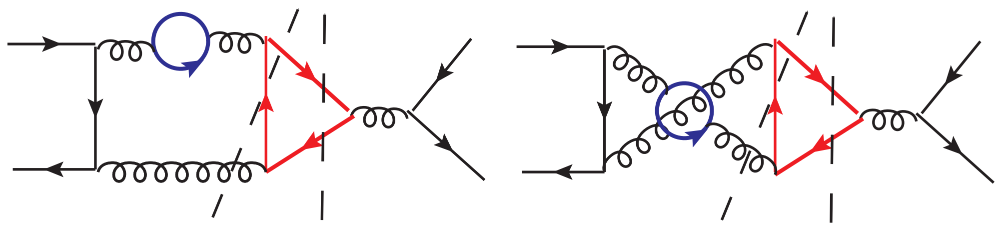
$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

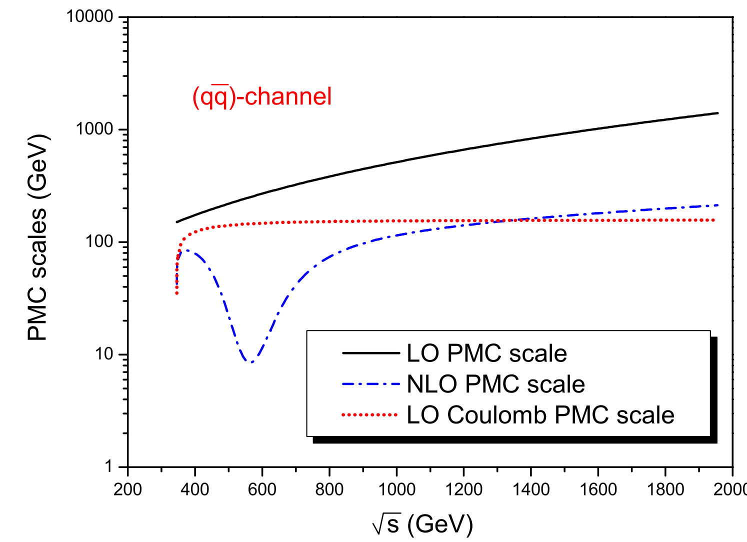
Important Example: Top-Quark FB Asymmetry

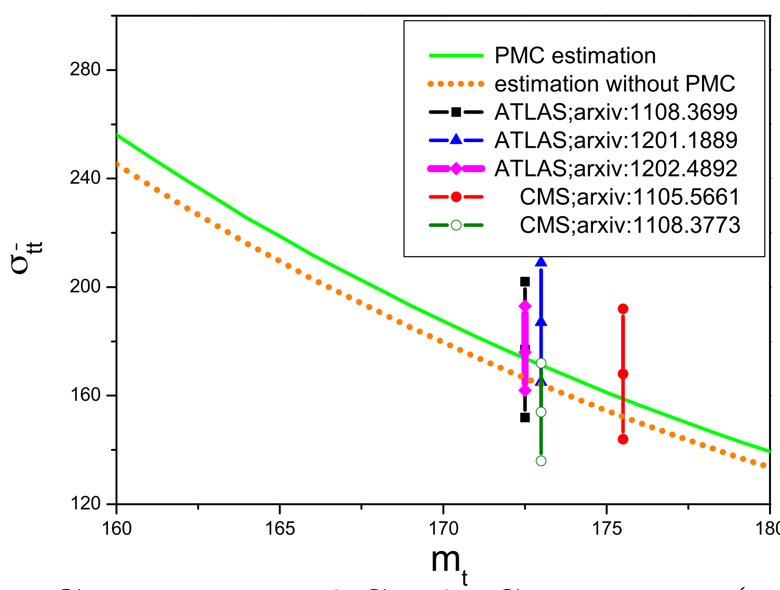
Brodsky, Wu, **Phys.Rev.Lett. 109**, [arXiv:1203.5312]

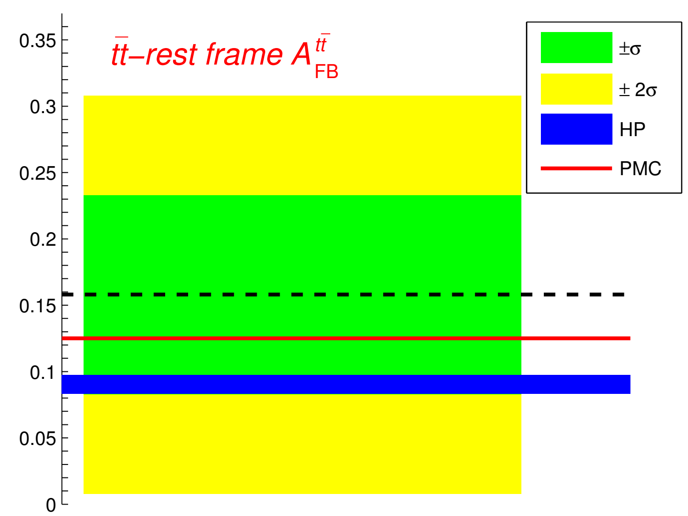
$$A_{FB}^{t\bar{t}} = \frac{\sigma(y_t^{t\bar{t}} > 0) - \sigma(y_t^{t\bar{t}} < 0)}{\sigma(y_t^{t\bar{t}} > 0) + \sigma(y_t^{t\bar{t}} < 0)}$$

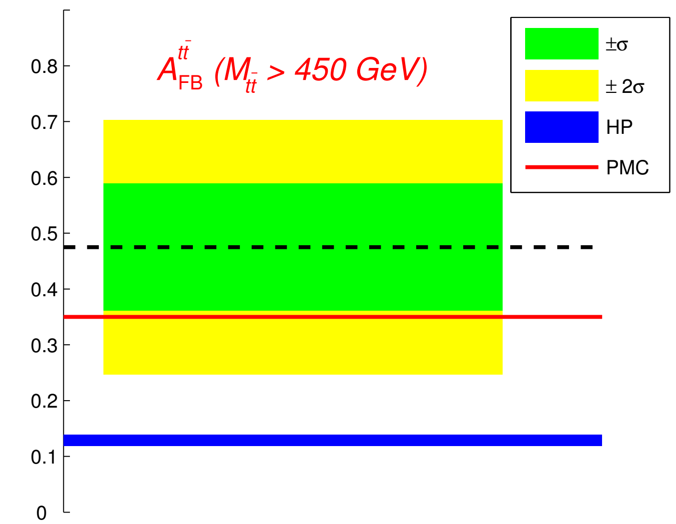


$$\mu_r \neq \mu_f \quad (!)$$









Conventional Scale Setting: $\alpha(\mu) = \alpha_{\overline{MS}}(\mu)$ and $\mu = [\frac{1}{2}Q, 2Q]$

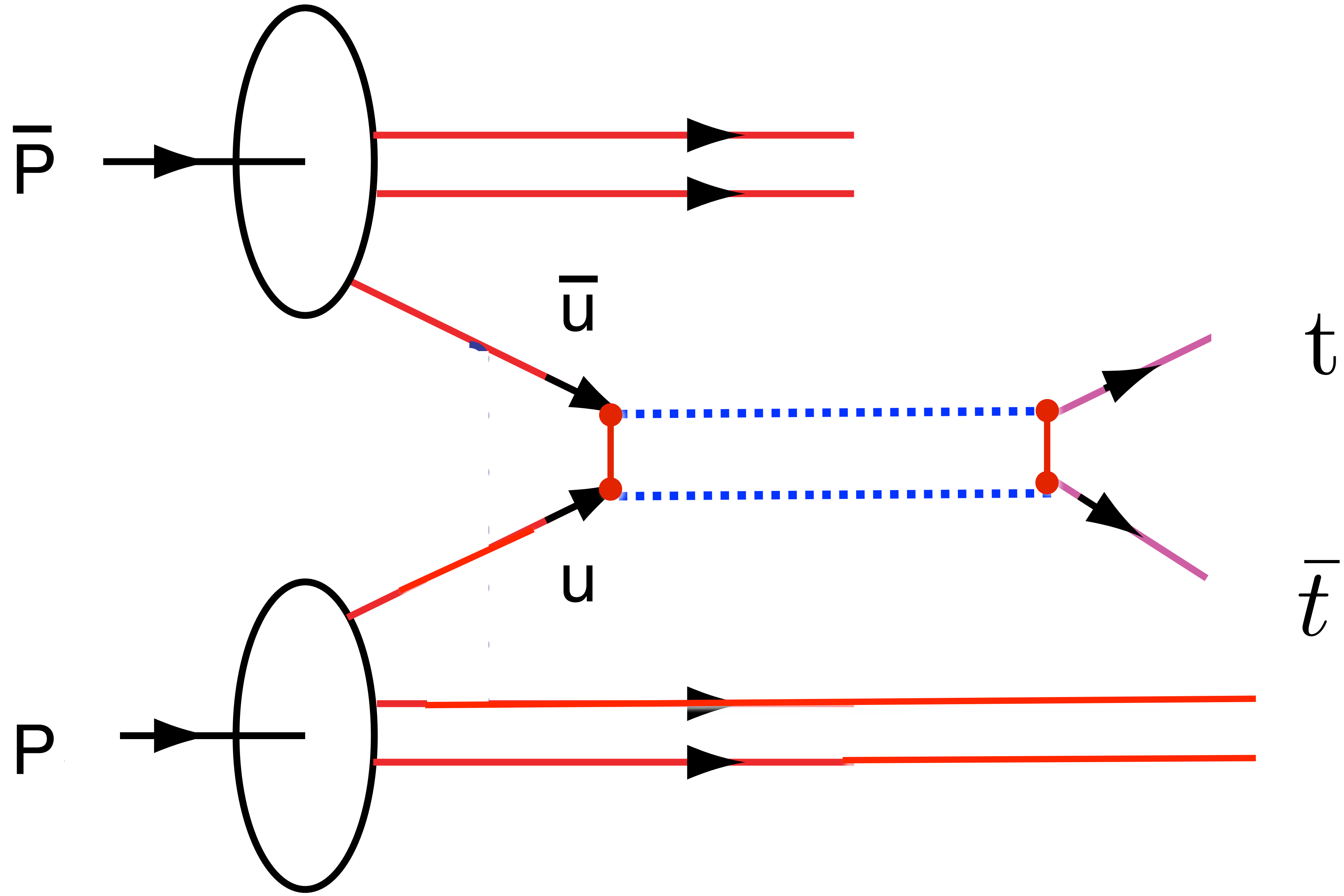
HP: Hollik, Pagani, **Phys.Rev. D84**(2011)

Conventional 'uncertainty estimate' can be misleading

(see also Blumlein & van Neerven, Phys.Lett. B450, 417[1999])

Improving pQCD precision important for exposing new physics correctly!

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron

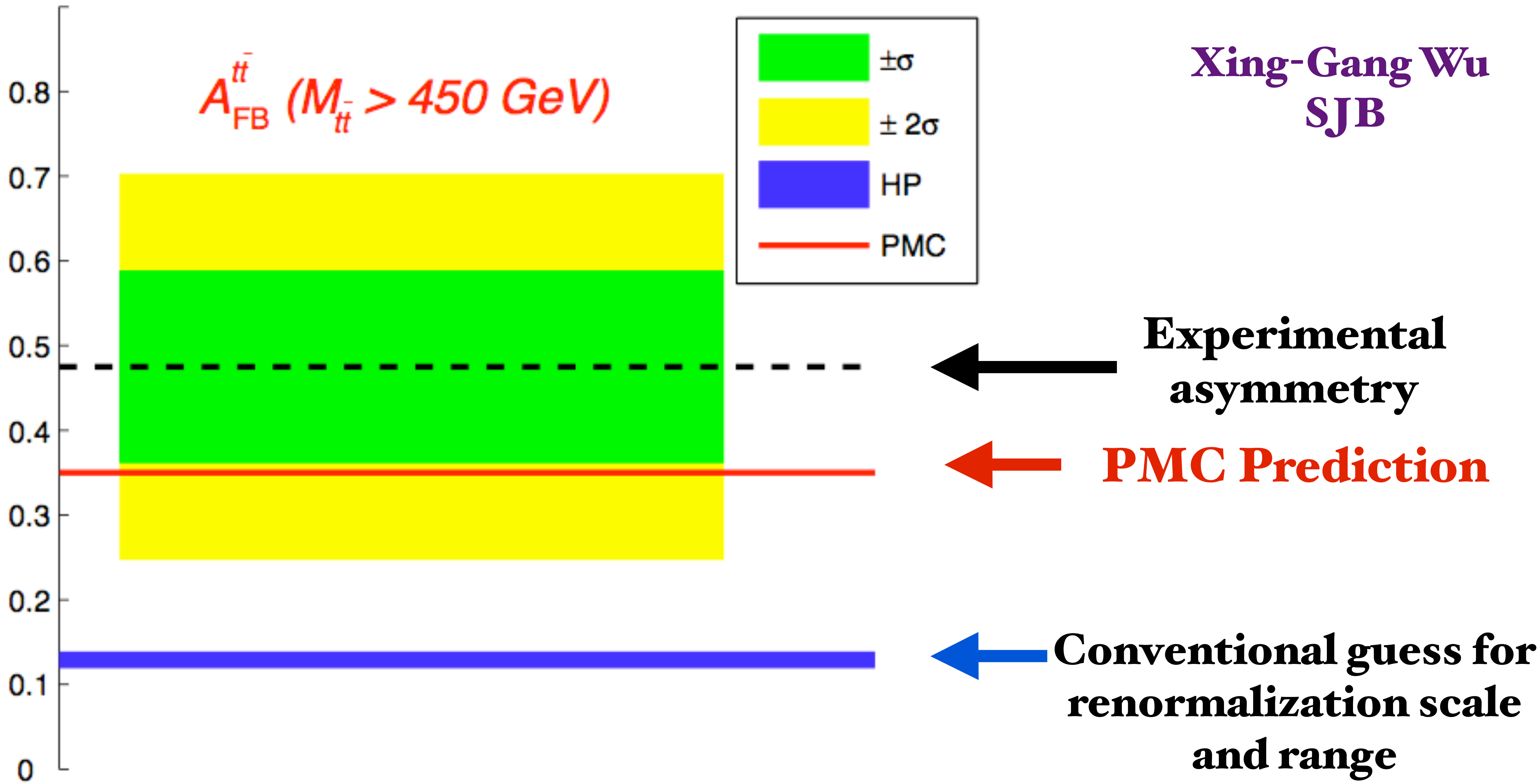


Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

Xing-Gang Wu, sjb

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



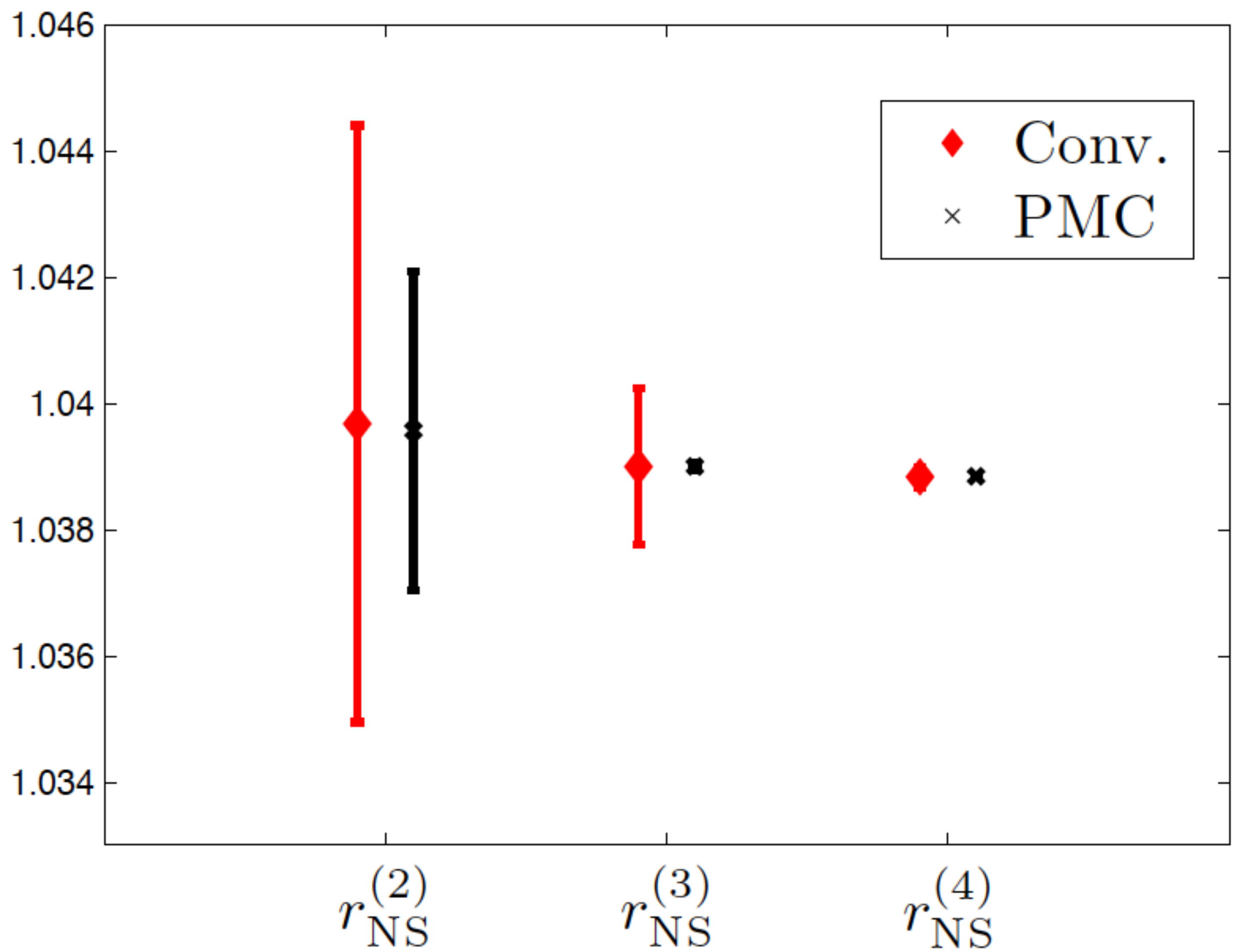
Xing-Gang Wu
SJB

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

What is PMC ?

Principle of Maximum Conformality

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

Identify $\{\beta_i^R\} - terms$ *using* $n_f - terms$

through the $PMC - BLM$ *correspondence principle*

order-by-order ↓

Shift scale of α_s *to* μ_R^{PMC} *to eliminate* $\{\beta_i^R\} - terms$

↓

Conformal Series

Result is independent of μ_R^{init} *and scheme at fixed order*

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

PMC–BLM – one

Phys. Rev. Lett. **109**, 042002 (2012)

R_δ–scheme – two

Phys. Rev. Lett. **110**, 192001 (2013)

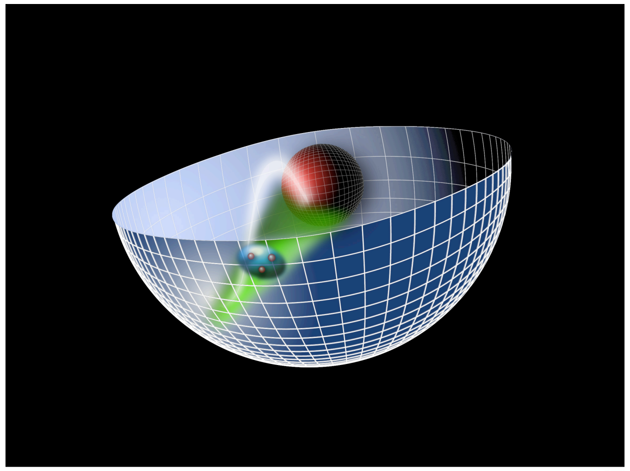
Eliminate β–terms

n_f dependence of pQCD series does not uniquely identify the β terms

Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (*Kataev, Lu, Rathsman, sjb*)***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Eliminates unnecessary theory error***

de Tèramond, Dosch, sjb



AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

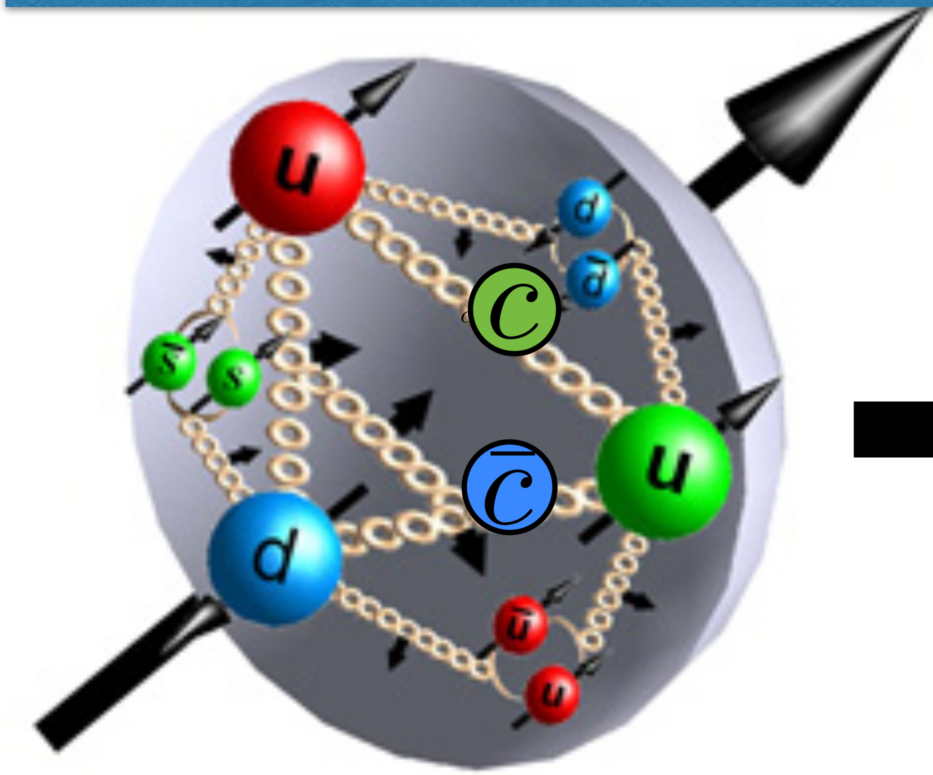
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

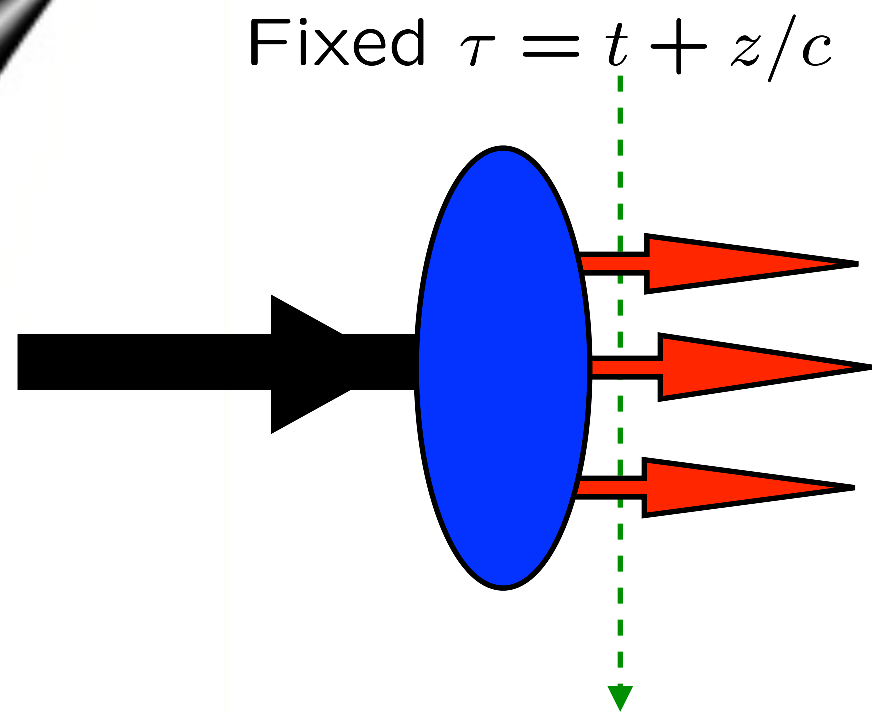
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

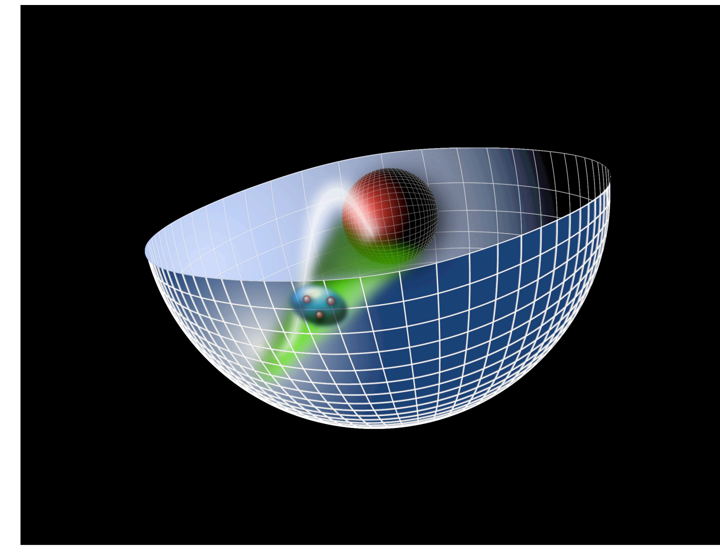
QCD Myths

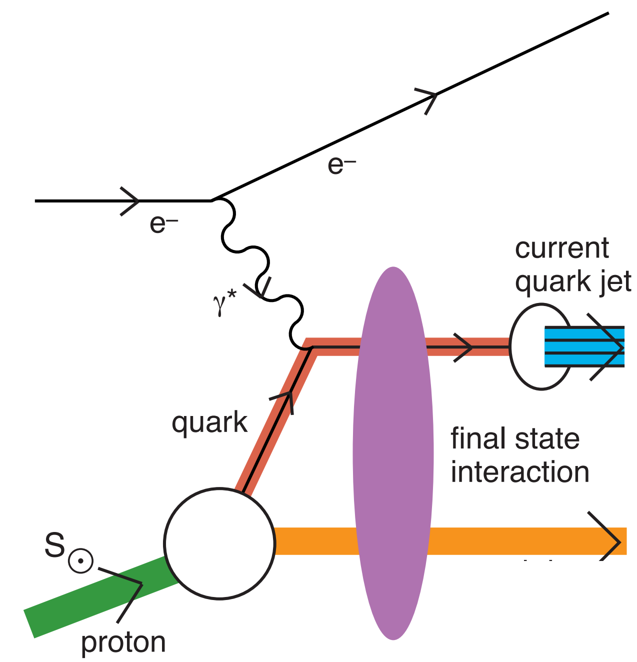
- **ISI and FSI are higher twist effects - only a phase**
- **Momentum and Spin Sum Rules valid for nuclei - in fact not proven!**
- **Anti-Shadowing is Universal - In fact, anti-shadowing is Flavor Dependent!**
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **Heavy quarks arise only from gluon splitting — Intrinsic Strange, Charm, and Bottom**
- **Renormalization scale cannot be fixed — PMC**
- **QCD condensates are vacuum effects**
- **QCD gives 10^{42} to the cosmological constant**

Breakdown of Factorization, Sum Rules, and Insights for QCD from Light-Front Holography









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Intersections of BSM Phenomenology and QCD for New Physics Searches (INT-15-3)

October 15, 2015, INT, University of Washington