Progress on computing the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD(+QED)

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Outline I

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) = -\vec{\mu} \cdot \vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin ($c = \hbar = 1$)

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}$$

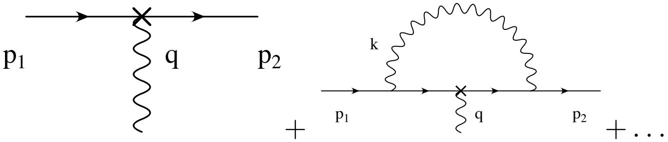
The Landé g-factor is predicted from the free Dirac eq. to be

$$g = 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



$$\langle \mu(p')|J^\mu|\mu(p)\rangle = \bar{u}(p')\left(\gamma^\mu F_1(q^2) + i\,\frac{[\gamma^\mu,\gamma^\nu]\,q^\nu}{2}\,\frac{F_2(q^2)}{2m}\right)u(p)$$

which results from Lorentz invariance and charge conservation when the muon is on-mass-shell and where $q = p' - p$

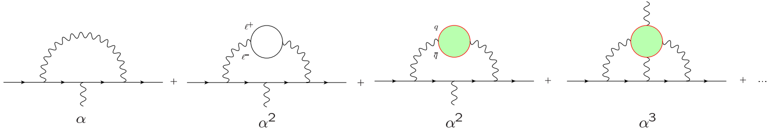
$$F_2(0) = \frac{g-2}{2} \equiv a_\mu \qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$



Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\dots$

hadronic contributions $\sim 6 \times 10^{-5}$ smaller, dominate theory error.

Experiment - Standard Model Theory = difference

SM Contribution	Value $\pm$ Error $(\times 10^{11})$	Ref
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]
HVP LO	6923 ± 42	[Davier et al., 2011]
	6949 ± 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]
HLbL	105 ± 26	[Prades et al., 2009]
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014b]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592089 ± 63	[Bennett et al., 2006]
Diff (Exp $-$ SM)	287 ± 80	[Davier et al., 2011]
	261 ± 78	[Hagiwara et al., 2011]
	249 ± 87	[Aoyama et al., 2012]

New experiments+new theory=new physics

- Fermilab E989, begins in early 2017, aims for 0.14 ppm
- J-PARC E34, "late 2010's", aims for 0.1 ppm
- Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\rightarrow \ \sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow \ \sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow \ \sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations important to trust theory errors

Outline I

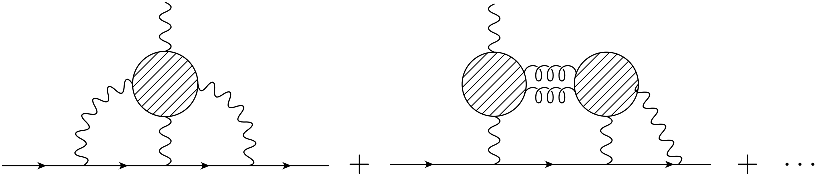
1. Introduction
   - Nature - Standard Model

2. HLbL
   - non-perturbative QED
   - Perturbative QED in configuration space
   - next steps

3. Summary/Outlook

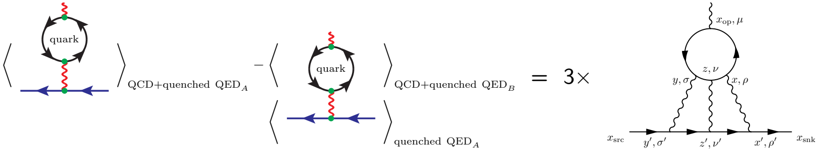
4. References

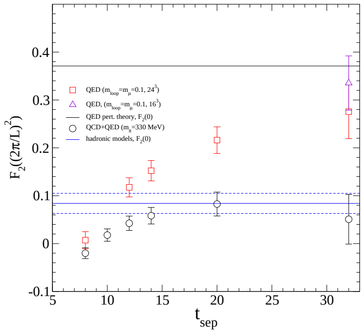
Hadronic light-by-light (HLbL) scattering



- Models: $(105 \pm 26) \times 10^{-11}$ [Prades et al., 2009, Benayoun et al., 2014]
  $(116 \pm 40) \times 10^{-11}$ [Jegerlehner and Nyffeler, 2009]
  systematic errors difficult to quantify
- Dispersive approach difficult, but progress is being made [Colangelo et al., 2014c, Colangelo et al., 2014a, Pauk and Vanderhaeghen, 2014b, Pauk and Vanderhaeghen, 2014a, Colangelo et al., 2015]
- First non-PT QED+QCD calculation [Blum et al., 2015a]
- Very rapid progress with pQED+QCD (L. Jin) [Blum et al., 2015b]
- New HLbL scattering calculation by Mainz group [Green et al., 2015]

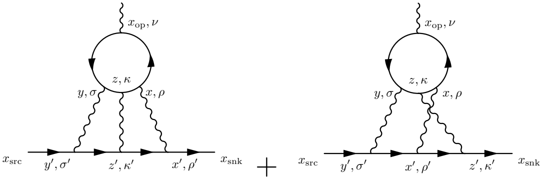
Non-perturbative QED method [Blum et al., 2015a]





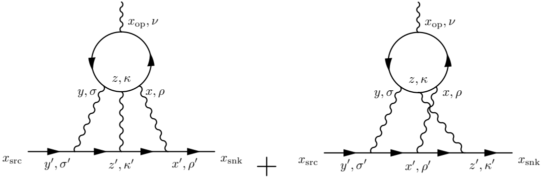
- quark-connected part of HLbL
- $a^{-1} = 1.7848$ GeV, $(2.7 \text{ fm})^3$
- $m_\pi = 330$ MeV, $m_\mu = 190$ MeV
- Consistent with model expectations (J. Bijnens)
- Agreement with models accidental
- $O(\alpha^2)$ noise, $O(\alpha^4)$ corrections

Point source method in pQED (Luchang Jin) [Blum et al., 2015b]

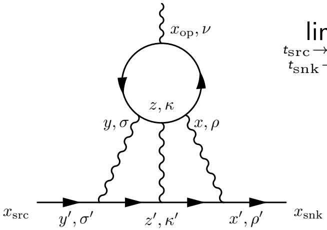


+ 4 more

- Compute quark loop non-perturbatively
- Photons, muon on lattice, but use (exact) tree-level propagators
- Work in configuration space
- Do QED two-loop, quark-loop integrals stochastically
- Key insight: quark loop exponentially suppressed with x and y separation. Concentrate on "short distance" (π Compton λ)
- Chiral (DW) fermions at finite lattice spacing: UV properties like in continuum, modified by $O(a^2)$



$$
\begin{aligned}
\mathcal{F}_\nu(x, y, z, x_{\rm op}, x_{\rm snk}, x_{\rm src}) \quad = \\
-(-ie)^3 \sum_{q=u,d,s} (ie_q)^4 \Big\langle \mathrm{tr}\big[\gamma_\nu S_q\,(x_{\rm op}, x)\,\gamma_\rho S_q(x, z)\gamma_\kappa S_q(z, y)\gamma_\sigma S_q\,(y, x_{\rm op})\big] \Big\rangle_{\rm QCD} \\
\cdot \sum_{x',y',z'} G_{\rho\rho'}(x, x') G_{\sigma\sigma'}(y, y') G_{\kappa\kappa'}(z, z') \\
\cdot \Big[S_\mu\,(x_{\sf snk}, x')\,\gamma_{\rho'} S_\mu(x', z')\gamma_{\kappa'} S_\mu(z', y')\gamma_{\sigma'} S_\mu\,(y', x_{\sf src}) \\
+ S_\mu\,(x_{\sf snk}, z')\,\gamma_{\kappa'} S_\mu(z', x')\gamma_{\rho'} S_\mu(x', y')\gamma_{\sigma'} S_\mu\,(y', x_{\sf src}) \\
+4 \text{ other permutations} \Big].
\end{aligned}
$$



FT muon source, sink $\mathcal{F}_\nu(\vec{q}, x, y, z, x_{\rm op}) =$

$$\lim_{\substack{t_{\rm src}\to-\infty \\ t_{\rm snk}\to\infty}} e^{E_{q/2}(t_{\rm snk}-t_{\rm src})} \sum_{\vec{x}_{\rm snk},\vec{x}_{\rm src}} e^{-i\frac{\vec{q}}{2}\cdot(\vec{x}_{\rm snk}+\vec{x}_{\rm src})} e^{i\vec{q}\cdot\vec{x}_{\rm op}} \mathcal{F}_\nu(x, y, z, x_{\rm op}, x_{\rm snk}, x_{\rm src})$$

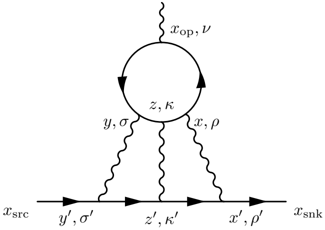
with mom. transfer $\vec{q} = 2\pi\vec{z}/L$, and use translational invariance to shift origin:

$$\begin{aligned}
\mathcal{M}_\nu(\vec{q}) &= \sum_{x,y,z} \mathcal{F}_\nu(\vec{q}, \frac{x-y}{2}, -\frac{x-y}{2}, z-w, x_{\rm op}-w) \\
&= \sum_r \left\{ \sum_{z',x'_{\rm op}} \mathcal{F}_\nu(\vec{q}, r, -r, z', x'_{\rm op}) \right\} \\
&= \left(\frac{\not{q}^+ + m_\mu}{2E_{q/2}}\right) \left(F_1(q^2)\gamma_\nu + \frac{F_2(q^2)}{2m}\frac{i}{2}[\gamma_\nu, \gamma_\beta](q_\beta)\right) \left(\frac{\not{q}^- + m_\mu}{2E_{q/2}}\right)
\end{aligned}$$

$$w = \frac{x+y}{2}, \quad r = \frac{x-y}{2}, \quad z' = z - w \quad \text{and} \quad x'_{\rm op} = x_{\rm op} - w$$

Sum over r and w stochastically, do $x'_{\rm op}$ and z' sums exactly

$$G(x,x')_{\rho\rho'} \quad = \quad \sum_k \frac{1}{(2\sin k/2)^2} e^{ik(x-x')}$$



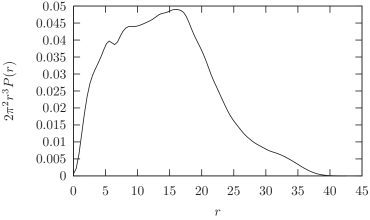
- QED_L [Hayakawa and Uno, 2008]
- Muon propagators FV (analytic), tree-level DWF with $L_s = \infty$
- Rand'ly choose quark loop location w
- Compute 2 point source props in QCD at x, y, connect sink points at $x'_{\rm op}$ and z', do the latter sums exactly
- $t_{\rm src}$, $t_{\rm snk} = w^0 \pm T/2$ for each w
- Do sums over r, w (x, y) stochastically, average over QCD configurations then yields $\mathcal{M}_\nu(\vec{q})$

- Use importance sampling to do sum over r efficiently (sample $|r| \lesssim 1$ fm most frequently). Empirical choice:

$$p(|x_i - w|) \propto \begin{cases} 1 & (|x_i - w| < R) \\ 1/|x_i - w|^{3.5} & (|x_i - w| \geqslant R) \end{cases},$$

The distribution of the relative distance $|r|$ between any two points drawn from this set is:

$$P(r) = \sum_x p(|x - r|)p(|x|)$$



- 2+1f DWF+I-DSDR ensemble RBC/UKQCD
- 171 MeV pion mass
- $R = 4$, so do *all* points with $r = 3$ or less in this case

Point source method, initial results

Label	size	$m_\pi L$	m_π/GeV	#qcdtraj	t_{sep}	$\frac{F_2 \pm \mathrm{Err}}{(\alpha/\pi)^3}$	$\frac{\mathrm{Cost}}{\mathrm{BG/Q\ rack\ days}}$
16I	$16^3 \times 32$	3.87	0.423	16	16	0.1235 ± 0.0026	0.63
24I	$24^3 \times 64$	5.81	0.423	17	32	0.2186 ± 0.0083	3.0
24IL	$24^3 \times 64$	4.57	0.333	18	32	0.1570 ± 0.0069	3.2
32ID	$32^3 \times 64$	4.00	0.171	47	32	0.0693 ± 0.0218	10

Table 2. Central values and errors. $a^{-1} = 1.747\mathrm{GeV}$ except for 32ID where $a^{-1} = 1.371\mathrm{GeV}$. Muon mass and pion mass ratio is fixed at physical value. For comparison, at physical point, model estimation is 0.08 ± 0.02.

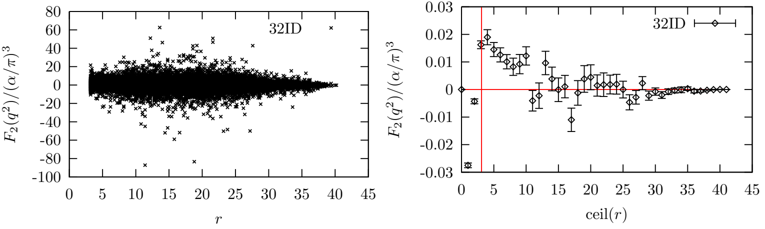


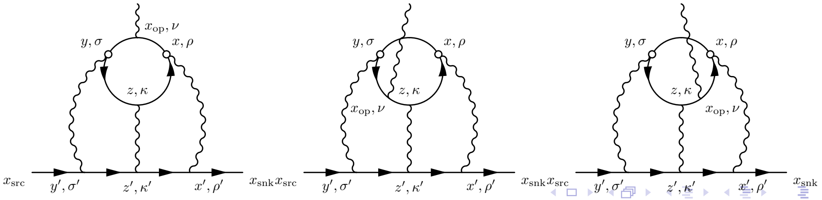
Figure 13. $32^3 \times 64$ lattice, with $a^{-1} = 1.371\mathrm{GeV}$, $m_\pi = 171\mathrm{MeV}$, $m_\mu = 134\mathrm{MeV}$.

Current conservation

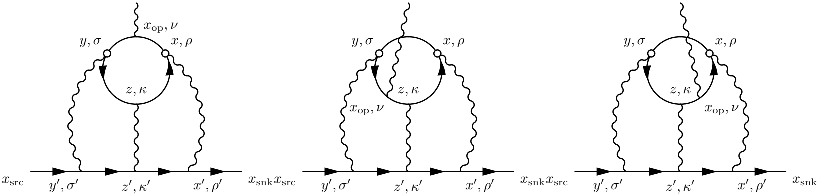
- At least one (lattice) conserved current to have convergent amplitude in continuum limit. Choose ext. photon, $J_\mu(x_{\rm op})$
- $\mathcal{M}_\mu \sim F_1(q)\gamma^\mu + i\gamma^\mu\gamma^\nu q^\nu F_2(q)/2m$ relies on WI
- To maintain constant signal-to-noise as $q \to 0$, Ward identity must be exact on each gauge configuration

$$\partial_\mu \langle j^\mu(x_{\rm op})\bar\psi(x)\gamma^\rho\psi(x)\cdots\rangle = i\delta(x_{\rm op}-x)\langle\bar\psi(x)\gamma_\nu\psi(x)\cdots\rangle - i\delta(x_{\rm op}-x)\langle\bar\psi(x)\gamma_\nu\psi(x)\cdots\rangle + \cdots$$

$$\langle j^\mu(x_{\rm op})\bar\psi(x)\gamma^\rho\psi(x)\bar\psi(z)\gamma^\nu\psi(z)\bar\psi(y)\gamma^\sigma\psi(y)\rangle =$$



Current conservation



- Compute all 3 diagrams so WI exact on each configuration
- signal *and* error vanish as $q \to 0$. Error on $F_2(q^2) \sim$ constant
- new diagrams require (6) sequential source props
- One more trick: restrict sum over z,

$$\sum_{x,y,z} \mathcal{F}_\mu(\mathbf{q}; x, y; z, x_{\rm op}) = \sum_{\substack{x,y,z \\ |x-y| < \min(|x-z|,|y-z|)}} 3\mathcal{F}_\mu(\mathbf{q}; x, y; z, x_{\rm op})$$

- Skews distribution towards small r where noise is smaller, signal larger

Moment method for $F_2(0)$

- Can do calculation directly at zero momentum for large L

$$
\begin{aligned}
\mathcal{M}_\mu(q) &= \gamma_0\left(\frac{\not{q}^+ + m_\mu}{2E_{q/2}}\right)\left(\frac{F_2(q^2)}{2m}\frac{i}{2}[\gamma_\nu,\gamma_\beta](-q_\beta)\right)\left(\frac{\not{q}^- + m_\mu}{2E_{q/2}}\right)\gamma_0 \\
&= \sum_r \sum_{z,x_{\rm op}} \mathcal{F}_\mu(\vec{q}; -\frac{r}{2}, +\frac{r}{2}; z, x_{\rm op}) \\
&= \sum_{x_{\rm op}} \exp\left(iq\cdot x_{\rm op}\right)\mathcal{F}'_\mu(q, x_{\rm op}) \\
&\approx \sum_{x_{\rm op}} \left(1 + iq\cdot x_{\rm op}\right)\mathcal{F}'_\mu(q, x_{\rm op}) \\
&\approx \sum_{x_{\rm op}} iq\cdot x_{\rm op}\mathcal{F}'_\mu(q, x_{\rm op}) \\
\frac{\partial}{\partial q_i}\mathcal{M}_\nu(\vec{q})|_{\vec{q}=0} &= i\sum_{r,z,x_{\rm op}} \mathcal{F}'_\nu\left(\vec{q}=0, r, -r, z, x_{\rm op}\right)(x_{\rm op})_i \sim F_2(0)
\end{aligned}
$$

The "1" term vanishes in ∞ volume, exponentially small in FV

Excited state contamination

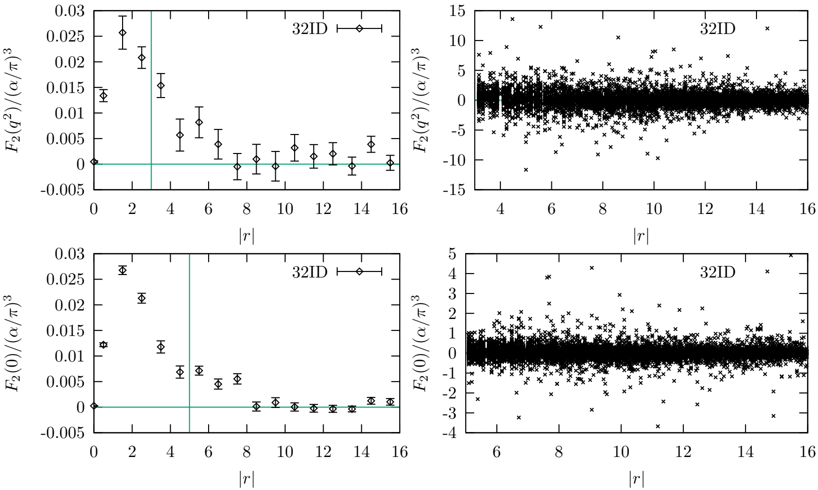
Usual method:

- (hadronic) external states "interpolated" far from operator insertion point x_{op}
- excited states exp. suppressed relative to ground state

Our method:

- Sum over x_{op}
- Includes points where $t_{\text{op}} = t_{src}$ or t_{snk} or is nearby
- Origin of quark loop $x + y$ in middle of t_{src} and t_{snk}, so these contributions are exponentially suppressed.
- usual choice: $t_{snk} - t_{\text{src}} = T/2$, but check for contamination with shorter separations

Point source method with all improvements



- Conserved current (upper), moment (lower) methods
- 171 MeV Pion, $m_\pi L \gtrsim 4$
- AMA used for quark propagators (1000 evecs, CG: 100 iters)

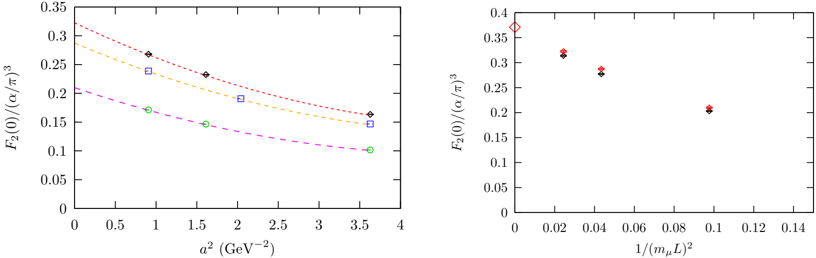
Point source method with all improvements

Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$	r_{max}	SD	LD	ind-pair
Exact	0.0693(218)	47	$58 + 8 \times 16$	2.04	3	−0.0152(17)	0.0845(218)	0.0186
Conserved	0.1022(137)	13	$(58 + 8 \times 16) \times 7$	1.78	3	0.0637(34)	0.0385(114)	0.0093
Mom. (approx)	0.0994(29)	23	$(217 + 512) \times 2 \times 4$	1.08	5	0.0791(18)	0.0203(26)	0.0028
Mom. (corr)	0.0060(43)	23	$(10 + 48) \times 2 \times 4$	0.44	2	0.0024(6)	0.0036(44)	0.0045
Mom. (tot)	0.1054(54)	23						

5% statistical error for nearly physical pion mass!

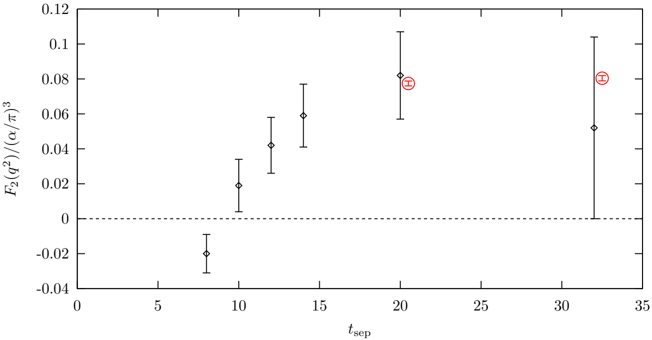
Cost: 13.2 BG/Q Rack-days (Rack = 1024 nodes = 16384 cores)

Continuum and ∞ volume limits in QED



- Using all improvements
- a set using physical muon mass
- QED systematics large, $O(a^4)$, $O(1/L^2)$, but under control
- Limits quite consistent with PT result

- Including all improvements, statistical errors reduced by 10×

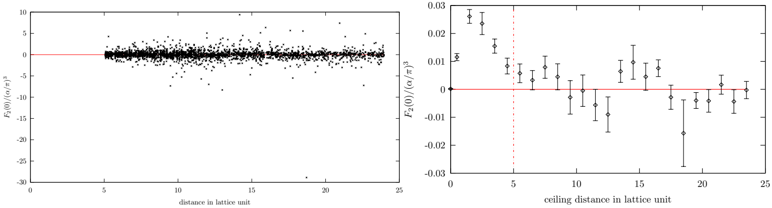


- quark-connected part of HLbL, $q = 2\pi/L$, 0
- $a^{-1} = 1.7848$ GeV, $(2.7\ \text{fm})^3$
- $m_\pi = 330$ MeV, $m_\mu = 190$ MeV
- Strong check on method(s)

On-going calculation: physical point ($m_\pi = 140$ MeV)

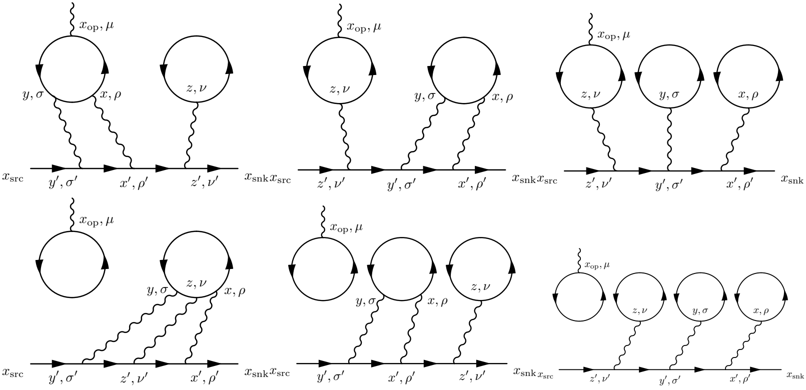
ALCC award on MIRA at ANL ALCF,

- Applying improved point source method to physical light quark mass 2+1f Möbius DWF ensemble (RBC/UKQCD)
- $(5.5\ \text{fm})^3$ QCD box, $a = 0.114$ fm ($a^{-1} = 1.7848$ GeV)
- Use AMA with 2000 low-modes, $\sim$ 4500 sloppy props per configuration



on track to beat goal of 20% statistical error

M. Hayakawa's talk at Lattice 2015



- SU(3) Flavor (only 1 survives), Zweig suppressed
- Requires explicit HVP subtraction when any quark loop with two photons is not connected to others by gluons
- Use same importance sampling as for connected

Solving QED FV effects

- Integrand exponentially suppressed with distance between any pair of points on the quark loop. FV effect is small.
- Amplitude *not* suppressed with distance between points on muon line and loop. FV effect is large.
- Put QED in larger, perhaps ∞, box, QCD unchanged
- use ∞ volume photon on finite box (Lehner, Lattice 2015)
- Can compute average QCD loop and do muon line once, offline, so free to experiment with size of QED box

Outline I

Summary/Outlook

- First calculations for connected part very promising– calculation with controlled errors clearly within reach of lattice methods.
- 5% stat. errors already for near physical pions
- FV effects large but controllable. ∞ volume limit consistent with PT. Put QCD and QED in different boxes
- Applying improved point source method to physical quark mass 2+1f Möbius DWF ensemble RBC/UKQCD
- Disconnected part challenging, new ideas under investigation
- Lattice important to compare (SM) with experiment

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- Lattice computations done on
  - Ds cluster at FNAL (USQCD)
  - USQCD BQ/Q at BNL

Outline I

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