ChPT

Extensions for lattice

Many LECs?

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Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

# CHIRAL PERTURBATION THEORY FOR PRECISION PHYSICS AND BSM

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Vetenskapsrådet

Intersections of BSM Phenomenology and QCD for New Physics Searches, 28/9-3/10/2015 INT Seattle

# Overview


1. Chiral Perturbation Theory
2. Extensions for lattice
3. Many LECs?
4. ChPT program framework
5. Determination of LECs in the continuum
6. Charged Pion Polarizabilities
7. Finite volume
8. Beyond QCD or BSM
9. Conclusions

# Chiral Perturbation Theory

Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques

Derivation from QCD:
H. Leutwyler,
*On The Foundations Of Chiral Perturbation Theory*,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For references to lectures see:
http://www.thep.lu.se/~bijnens/chpt/

# Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about $M_\rho$.



ChPT
Extensions for lattice
Many LECs?
ChPT program framework
Determination of LECs in the continuum
Charged Pion Polarizabilities
Finite volume
Beyond QCD or BSM
Conclusions

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ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

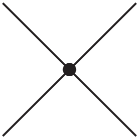
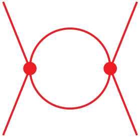
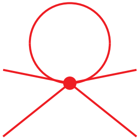
# Goldstone Bosons



Spontaneous breakdown

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum
- $\pi^0, \pi^+ \pi^-, K^0, \overline{K^0}, K^+, K^-, \eta$

## Power counting in momenta: Meson loops, Weinberg powercounting

| rules | | one loop example | |
| --- | --- | --- | --- |
|  | $p^2$ |  | $(p^2)^2\,(1/p^2)^2\,p^4 = p^4$ |
|  | $1/p^2$ |  | $(p^2)\,(1/p^2)\,p^4 = p^4$ |
| $\int d^4p$ | $p^4$ | | |

# Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft $K, \eta$) some version of ChPT exists



ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

# Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix} .$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,,$

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$ ,
left and right external currents: $r(l)_\mu = v_\mu + (-) a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A\rangle = Tr_F\,(A)$

# Lagrangians: Lagrangian structure (mesons, strong)

| | 2 flavour | | 3 flavour | | PQChPT/$N_f$ flavour | |
|---|---|---|---|---|---|---|
| $p^2$ | $F, B$ | 2 | $F_0, B_0$ | 2 | $F_0, B_0$ | 2 |
| $p^4$ | $l_i^r, h_i^r$ | 7+3 | $L_i^r, H_i^r$ | 10+2 | $\hat{L}_i^r, \hat{H}_i^r$ | 11+2 |
| $p^6$ | $c_i^r$ | 52+4 | $C_i^r$ | 90+4 | $K_i^r$ | 112+3 |

$p^2$: Weinberg 1966

$p^4$: Gasser, Leutwyler 84,85

$p^6$: JB, Colangelo, Ecker 99,00

- ➠ $L_i$ LEC = Low Energy Constants = ChPT parameters
- ➠ $H_i$: contact terms: value depends on definition of currents/densities
- ➠ Finite volume: no new LECs
- ➠ Other effects: (many) new LECs

# Mesons: which Lagrangians are known ($n_f = 3$)

| Loops | $\mathcal{L}_{\text{order}}$ | LECs | effects included |
|---|---|---|---|
| $L = 0$ | $\mathcal{L}_{p^2}$ | 2 | strong (+ external $W, \gamma$) |
| | $\mathcal{L}_{e^2 p^0}$ | 1 | internal $\gamma$ |
| | $\mathcal{L}^{\Delta S=1}_{G_F p^2}$ | 2 | nonleptonic weak |
| | $\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$ | 1 | nonleptonic weak+internal $\gamma$ |
| | $\mathcal{L}^{\text{odd}}_{p^4}$ | 0 | WZW, anomaly |
| $L \le 1$ | $\mathcal{L}_{p^4}$ | 10 | strong (+ external $W, \gamma$) |
| | $\mathcal{L}_{e^2 p^2}$ | 13 | internal $\gamma$ |
| | $\mathcal{L}^{\Delta S=1}_{G_8 F p^4}$ | 22 | nonleptonic weak |
| | $\mathcal{L}^{\Delta S=1}_{G_{27} p^4}$ | 28 | nonleptonic weak |
| | $\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$ | 14 | nonleptonic weak+internal $\gamma$ |
| | $\mathcal{L}^{\text{odd}}_{p^6}$ | 23 | WZW, anomaly |
| | $\mathcal{L}^{\text{leptons}}_{e^2 p^2}$ | 5 | leptons, internal $\gamma$ |
| $L \le 2$ | $\mathcal{L}_{p^6}$ | 90 | strong (+ external $W, \gamma$) |

# Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars/axial currents
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)
- Unitarity included perturbatively

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$$M^2 = 2B\hat{m}$$

# LECs and $\mu$



ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

is independent of the scale $\mu$.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

Choice of $\mu$ :

- $m_\pi$, $m_K$: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$
  what about large $N_c$ arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

# Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exist
- Express orders in terms of physical masses and quantities ($F_\pi$, $F_K$)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in $\pi K$ scattering
- Note: remaining $\mu$ dependence can occur at a given order
- Can make quite some difference in the expansion

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating
- but sometimes too many masses so very ambiguous

# Extensions for the lattice

- No new parameters:
  - Finite temperature
  - Finite volume (including $\epsilon$ regime)
  - Twisted mass
  - Boundary conditions: twisted,...
- A few new parameters
  - Partially quenched ($2 \to 2$,$10 \to 11$, $90 \to 112$)
- Many new parameters
  - Wilson ChPT ($2 \to 3$,$10 \to 18$)
  - Staggered ChPT ($2 \to 10$,$10 \to 126$ (but dependencies))
  - Mixed actions
- Other operators
  - Local object with well defined chiral properties: include via spurion techniques
  - Examples: tensor current, energy momentum tensor,...



ChPT

Extensions for lattice

Many LECs?

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Determination of LECs in the continuum

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Finite volume

Beyond QCD or BSM

Conclusions

# Many LECs

- Is this too many parameters to do something?
- But if analytic in quark masses added in the fit not much extra
- Example: meson masses at NNLO have only the possible analytic quark mass dependence and the NLO meson-meson scattering parameters as input

# Program availability

Making the programs more accessible for others to use:

- Two-loop results have very long expressions
- Many not published but available from http://www.thep.lu.se/~bijnens/chpt/
- Many programs available on request from the authors
- Idea: make a more general framework in C++
- CHIRON:

JB,
"CHIRON: a package for ChPT numerical results at two loops,"
Eur. Phys. J. C **75** (2015) 27 [arXiv:1412.0887]
http://www.thep.lu.se/~bijnens/chiron/

# Program availability: CHIRON

- Present version: 0.54
- Classes to deal with $L_i, C_i,\ L_i^{(n)}, K_i$, standardized in/output, changing the scale,...
- Loop integrals: one-loop and sunsetintegrals, also finite volume
- Included so far (at two-loop order):
  - Masses, decay constants and $\langle \bar{q}q \rangle$ for the three flavour case
  - Masses and decay constants at finite volume, partially quenched and partially quenched at finite volume in the three flavour case
  - Masses, decay constants and $\langle \bar{q}q \rangle$ for QCDlike theories including finite volume, partially quenched and both (simplest mass case only)
- A large number of example programs is included
- Manual has already reached 94 pages
- I am continually adding results from my earlier work

# LEC determination: (Partial) History/References

- Original determination at $p^4$: Gasser, Leutwyler, Annals Phys.158 (1984) 142, Nucl. Phys. B250 (1985) 465
- $p^6$ 3 flavour: Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [ hep-ph/0101127]
- Review article two-loops: JB, Prog. Part. Nucl. Phys. 58 (2007) 521 [hep-ph/0604043]
- Update of fits + new input: JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945]
- Recent review with more $p^6$ input: JB, Ecker, Ann. Rev. Nucl. Part. Sci. **64** (2014) 149 [arXiv:1405.6488]
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles, Rev.Mod.Phys. 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:
  Colangelo et al., Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]
  Aoki et al., Eur. Phys. J. C **74** (2014) 9, 2890 [arXiv:1310.8555]

# Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from $p^6$ Lagrangian often significant
- Reliance on estimates of the $C_i$ much larger
- Typically: $C_i^r$: (terms with)
  kinematical dependence $\equiv$ measurable
  quark mass dependence $\equiv$ impossible (without lattice)
  100% correlated with $L_i^r$
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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- Are we really testing ChPT or just doing a phenomenological fit?

# Testing if ChPT works: relations

Yes: JB, Jemos, Eur.Phys.J. C64 (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the $C_i^r$

Included:

- $m_M^2$ and $F_M$ for $\pi, K, \eta$.
- 11 $\pi\pi$ threshold parameters
- 14 $\pi K$ threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations



ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

# Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), $\pi K$ (5) and $K_{\ell 4}$ (1)
  13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving $a_3^-$ significantly did not work well
- $\pi K$: relation involving $a_3^-$ not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that ChPT has trouble with curvature in $K_{\ell 4}$
- Conclusion: Three flavour ChPT "sort of" works

# Fits: inputs

Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [ hep-ph/0101127] (ABT01)

JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945] (BJ12)

JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sci .64 (2014) 149-174 (BE14)

- $M_\pi, M_K, M_\eta, F_\pi, F_K/F_\pi$
- $\langle r^2\rangle^\pi_S$, $c^\pi_S$ slope and curvature of $F_S$
- $\pi\pi$ and $\pi K$ scattering lengths $a^0_0$, $a^2_0$, $a_0^{1/2}$ and $a_0^{3/2}$.
- Value and slope of $F$ and $G$ in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \ldots, \bar{l}_4$
- more variation with $C^r_i$, a penalty for a large $p^6$ contribution to the masses
- 17+3 inputs and 8 $L^r_i$+34 $C^r_i$ to fit

# Main fit

| | ABT01 | BJ12 | $L_4^r$ free | BE14 |
|---|---|---|---|---|
| | old data | | | |
| $10^3 L_1^r$ | 0.39(12) | 0.88(09) | 0.64(06) | 0.53(06) |
| $10^3 L_2^r$ | 0.73(12) | 0.61(20) | 0.59(04) | 0.81(04) |
| $10^3 L_3^r$ | −2.34(37) | −3.04(43) | −2.80(20) | −3.07(20) |
| $10^3 L_4^r$ | $\equiv 0$ | 0.75(75) | 0.76(18) | $\equiv 0.3$ |
| $10^3 L_5^r$ | 0.97(11) | 0.58(13) | 0.50(07) | 1.01(06) |
| $10^3 L_6^r$ | $\equiv 0$ | 0.29(8) | 0.49(25) | 0.14(05) |
| $10^3 L_7^r$ | −0.30(15 | −0.11(15) | −0.19(08) | −0.34(09) |
| $10^3 L_8^r$ | 0.60(20) | 0.18(18) | 0.17(11) | 0.47(10) |
| $\chi^2$ | 0.26 | 1.28 | 0.48 | 1.04 |
| dof | 1 | 4 | ? | ? |
| $F_0$ [MeV] | 87 | 65 | 64 | 71 |

$? = (17+3) - (8+34)$

# Main fit: Comments

- All values of the $C_i^r$ we settled on are "reasonable"
- Leaving $L_4^r$ free ends up with $L_4^r \approx 0.76$
- keeping $L_4^r$ small: also $L_6^r$ and $2L_1^r - L_2^r$ small (large $N_c$ relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for $L_i^r$ and $C_i^r$, including from the scalars
- decent convergence (but enforced for masses)
- Many prejudices went in: large $N_c$, resonance model, quark model estimates,. . .

# Some results of this fit

Mass:

$$\begin{aligned} m^2_\pi/m^2_{\pi phys} &= 1.055(p^2) - 0.005(p^4) - 0.050(p^6)\,,\\ m^2_K/m^2_{K phys} &= 1.112(p^2) - 0.069(p^4) - 0.043(p^6)\,,\\ m^2_\eta/m^2_{\eta phys} &= 1.197(p^2) - 0.214(p^4) + 0.017(p^6)\,,\end{aligned}$$

Decay constants:

$$\begin{aligned} F_\pi/F_0 &= 1.000(p^2) + 0.208(p^4) + 0.088(p^6)\,,\\ F_K/F_\pi &= 1.000(p^2) + 0.176(p^4) + 0.023(p^6)\,.\end{aligned}$$

Scattering:

$$\begin{aligned} a^0_0 &= 0.160(p^2) + 0.044(p^4) + 0.012(p^6)\,,\\ a^{1/2}_0 &= 0.142(p^2) + 0.031(p^4) + 0.051(p^6)\,.\end{aligned}$$

# Update on $K_{\ell 3}$



ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

- Take Bijnens-Talavera 2003 result but update for BE14 parameters
- $f_+^{K^0\pi^-}(0) = 1 - 0.02276 - 0.00754 = 0.970 \pm 0.008$
- in good agreement with the latest lattice numbers:

  talk Jüttner lattice 2015, FLAG preliminary

| | |
|---|---|
| $N_f = 2+1+1$ | 0.9704(32) |
| $N_f = 2+1$ | 0.9677(37) |
| $N_f = 2$ | 0.9560(84) |

# Charged pion polarizabilities: experiment

An example where ChPT triumphed

Review: Holstein, Scherer, Ann. Rev. Nucl. Part. Sci. **64** (2014) 51 [1401.0140]

- Expand $\gamma\pi^\pm \to \gamma\pi^\pm$ near threshold: $(z_\pm = 1 \pm \cos\theta_{\rm cm})$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d_\Omega}_{\rm Born} - \frac{\alpha m_\pi^3 \left((s-m_\pi^2\right)^2}{4s^2\left(sz_+ + m_\pi^2 z_-\right)}\left(z_-^2(\alpha-\beta) + \frac{s^2}{m_\pi^4} z_+^2(\alpha+\beta)\right)$$

- Three ways to measure: (all assume $\alpha + \beta = 0$)
  - $\pi\gamma \to \pi\gamma$ (Primakoff, high energy pion beam)
    Dubna (1985) $\alpha = (6.8 \pm 1.4)\ 10^{-4}\ \text{fm}^3$
    Compass (CERN, 2015) $\alpha = (2.0 \pm 0.6 \pm 0.7)\ 10^{-4}\ \text{fm}^3$
  - $\gamma\pi \to \pi\gamma$ (via one-pion exchange)
    Lebedev (1986) $\alpha = (20 \pm 12)\ 10^{-4}\ \text{fm}^3$
    Mainz (2005) $\alpha = (5.8 \pm 0.75 \pm 1.5 \pm 0.25)\ 10^{-4}\ \text{fm}^3$
  - $\gamma\gamma \to \pi\pi$ (in $e^+e^- \to e^+e^-\pi^+\pi^-$)
    MarkII data analyzed (1992) $\alpha = (2.2 \pm 1.1)\ 10^{-4}\ \text{fm}^3$
- Extrapolation and subtraction: difficult experiments




ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

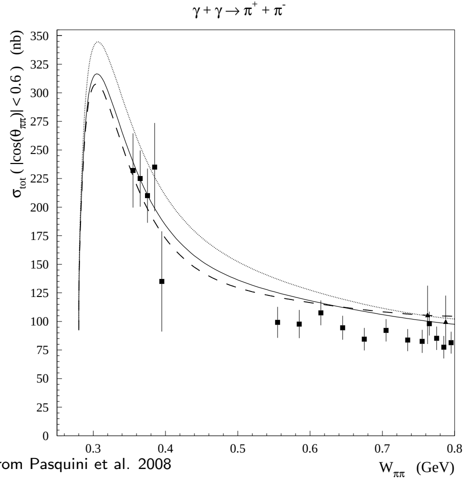
Charged Pion Polarizabilities

Finite volume

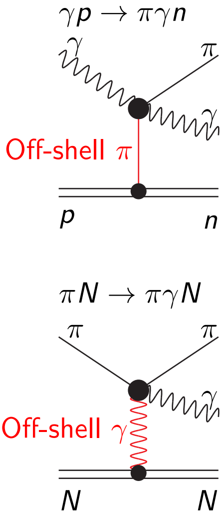
Beyond QCD or BSM

Conclusions

# Polarizabilities: extrapolations needed




from Pasquini et al. 2008



# Charged pion polarizabilities: theory

- ChPT:
  - One-loop JB, Cornet, 1986, Donoghue-Holstein 1989
    $\alpha + \beta = 0$, $\alpha = (2.8 \pm 0.2)\ 10^{-4}\ \text{fm}^3$
    input $\pi \to e\nu\gamma$ (error only from this)
  - Two-loop Bürgi, 1996, Gasser, Ivanov, Sainio 2006
    $\alpha + \beta = 0.16\ 10^{-4}\ \text{fm}^3$,$\alpha = (2.8 \pm 0.5)\ 10^{-4}\ \text{fm}^3$
- Dispersive analysis from $\gamma\gamma \to \pi\pi$:
  - Fil'kov-Kashevarov, 2005 $(\alpha_1 - \beta_1) = (13.0^{+2.6}_{-1.9}) \cdot 10^{-4}\text{fm}^3$
  - Critized by Pasquini-Drechsel-Scherer, 2008
    "Large model dependence in their extraction"
    "Our calculations... are in reasonable agreement with ChPT for charged pions"
    $(\alpha_1 - \beta_1) = (5.7) \cdot 10^{-4}\text{fm}^3$ perfectly possible

# Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions,...
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83, Nucl. Phys. B 307 (1988) 763
  $M_\pi$, $F_\pi$, $\langle \bar{q} q \rangle$ one-loop equal mass case
- I will stay with ChPT and the $p$ regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

# Finite volume: selection of ChPT results

- masses and decay constants for $\pi, K, \eta$ one-loop
  Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- $M_\pi$ at 2-loops (2-flavour)
  Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
  JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
  Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
  Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
  - Twisted boundary conditions and some funny effects: precision formfactors
  - Some results on masses 3-flavours at two loop order

# Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better Bedaque,. . .
- But:
  - Box: Rotation invariance $\rightarrow$ cubic invariance
  - Twisting: reduces symmetry further

  Consequences:
  - $m^2(\vec{p}) = E^2 - \vec{p}^2$ is *not* constant
  - There are typically more form-factors
  - In general: quantities depend on more (all) components of the momenta
  - Charge conjugation involves a change in momentum

# Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better Bedaque,...
- But:
  - Box: Rotation invariance $\to$ cubic invariance
  - Twisting: reduces symmetry further

  Consequences:
  - $m^2(\vec{p}) = E^2 - \vec{p}^2$ is *not* constant
  - There are typically more form-factors
  - In general: quantities depend on more (all) components of the momenta
  - Charge conjugation involves a change in momentum

# Twisted boundary conditions: Two-point function

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u \rangle \neq 0$
- $j_\mu^{\pi^+} = \bar{d}\gamma_\mu u$
  satisfies $\partial^\mu \left\langle T(j_\mu^{\pi^+}(x) j_\nu^{\pi^-}(0)) \right\rangle = \delta^{(4)}(x) \left\langle \bar{d}\gamma_\nu d - \bar{u}\gamma_\nu u \right\rangle$
- $\Pi_{\mu\nu}^a(q) \equiv i \int d^4x e^{iq\cdot x} \left\langle T(j_\mu^a(x) j_\nu^{a\dagger}(0)) \right\rangle$
  Satisfies WT identity. $q^\mu \Pi_{\mu\nu}^{\pi^+} = \left\langle \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \right\rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]

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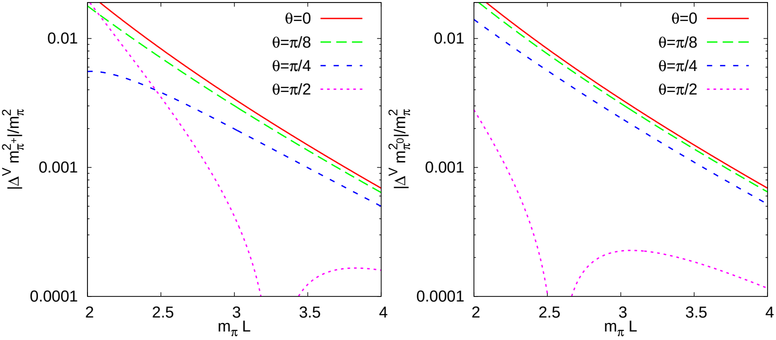
# Twisted boundary conditions: volume correction masses

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$m_\pi L = 2$, $\vec{\theta}_u = (\theta, 0, 0)$, $\vec{\theta}_d = \vec{\theta}_s = 0$

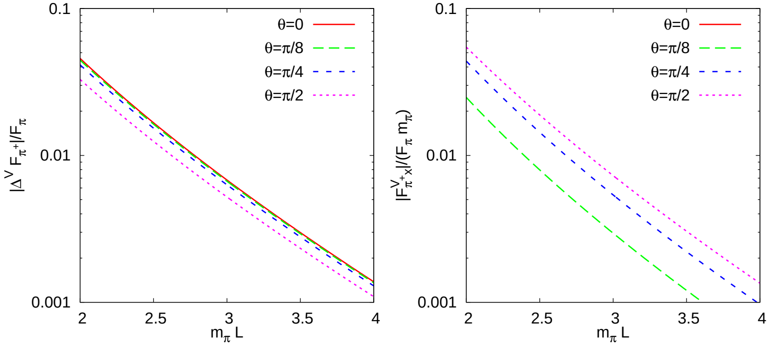



Charged pion mass Neutral pion mass

$\Delta^V X = X^V - X^\infty$ (dip is going through zero)

# Volume correction decay constants: $F_{\pi^+}$

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
- $\langle 0|A^M_\mu|M(p)\rangle = i\sqrt{2}F_M p_\mu + i\sqrt{2}F^V_{M\mu}$
- Extra terms are needed for Ward identities




relative for $F_\pi$ Extra for $\mu = x$

# Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
  earlier two-flavour work:
  Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]
- $\langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
  - $q^2$ for fixed $\vec{p}$ and $\vec{p}'$ has corrections
    small effect
  - This also affects the ward identities, e.g.
    $q^\mu f_\mu = (p^2 - p'^2)f_+ + q^2 f_- + q^\mu h_\mu = 0$
    is satisfied but all effects should be considered



ChPT

Extensions for lattice

Many LECs?

ChPT program framework

Determination of LECs in the continuum

Charged Pion Polarizabilities

Finite volume

Beyond QCD or BSM

Conclusions

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    $q^\mu f_\mu = (p^2 - p'^2)f_+ + q^2 f_- + q^\mu h_\mu = 0$
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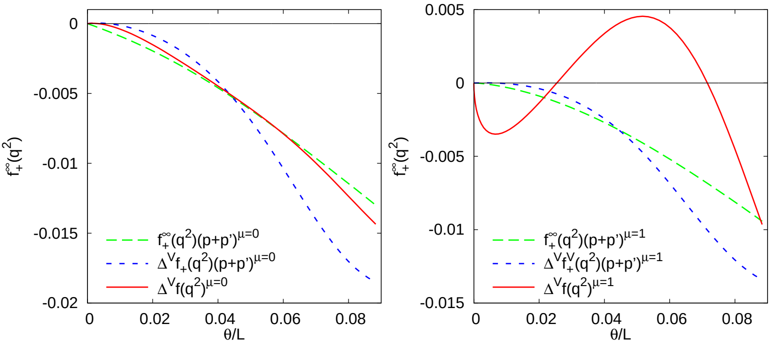
# Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}}\langle\pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$
  $= \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p+p'), \Delta^V f_+(p+p')$ and $\Delta^V f_\mu$




$\mu = t$ $\qquad\qquad$ $\mu = x$

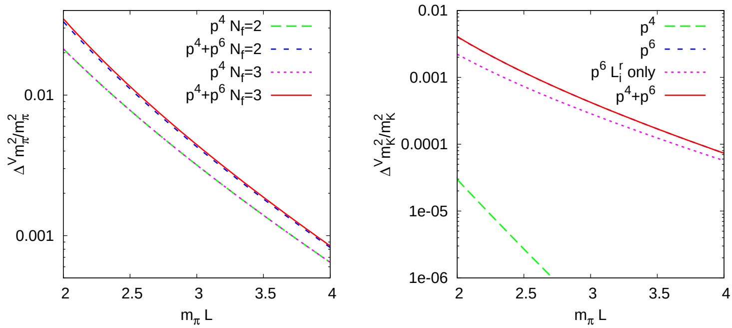
Finite volume corrections large, different for different $\mu$

# Masses at two-loop order

- Sunset integrals at finite volume done

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]
- Loop calculations:

  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]




- Agreement for $N_f = 2, 3$ for pion
- $K$ has no pion loop at LO

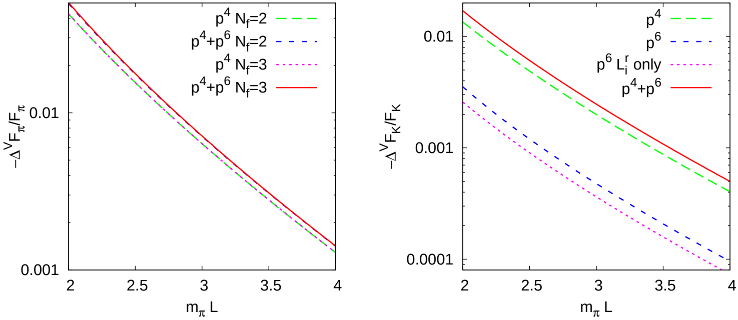
# Decay constants at two-loop order

- Sunset integrals at finite volume done

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]

- Loop calculations:

  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]




- Agreement for $N_f = 2, 3$ for pion
- $K$ now has a pion loop at LO

# QCDlike and/or technicolor theories



- One can also have different symmetry breaking patterns from underlying fermions
- Three generic cases
  - $SU(N) \times SU(N)/SU(N)$
  - $SU(2N)/SO(2N)$ (Dirac) or $SU(N)/SO(N)$ (Majorana)
  - $SU(2N)/Sp(2N)$
- Many one-loop results existed especially for the first case (several discovered only after we published our work)
- Equal mass case pushed to two loops JB, Lu, 2009-11
- Majorana, Finite Volume and partially quenched added JB, Rössler, arXiv:1509.04082

# $N_F$ fermions in a representation of the gauge group

- complex (QCD):
  - $q^T = (q_1 \; q_2 \ldots \; q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R$
    $q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij} \Longrightarrow$ conserved $H = SU(N_F)_V$:
- Real (e.g. adjoint): $\hat{q}^T = (q_{R1} \; \ldots \; q_{RN_F} \; \tilde{q}_{R1} \; \ldots \; \tilde{q}_{RN_F})$
  - $\tilde{q}_{Ri} \equiv C\overline{q}_{Li}^T$ goes under gauge group as $q_{Ri}$
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij}$ $J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_S g^T = J_S \Longrightarrow H = SO(2N_F)$
- Real with $N_F$ Majorana fermions
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - Majorana condensate is $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto \delta_{ij} = I_{ij}$
  - Conserved $g I g^T = I$

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# $N_F$ fermions in a representation of the gauge group



ChPT

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- complex (QCD): $q^T = (q_1\ q_2 \dots\ q_{N_F})$
  - Global $G = SU(N_F)_L \times SU(N_F)_R \quad q_L \to g_L q_L$ and $g_R \to g_R q_R$
  - Vacuum condensate $\Sigma_{ij} = \langle \overline{q}_j q_i \rangle \propto \delta_{ij}$
  - Conserved $H = SU(N_F)_V$: $g_L = g_R$ then $\Sigma_{ij} \to \Sigma_{ij}$
- Pseudoreal (e.g. two-colours):
  $\hat{q}^T = (q_{R1}\ \dots\ q_{RN_F}\ \tilde{q}_{R1}\ \dots\ \tilde{q}_{RN_F})$
  - $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha\beta} C \bar{q}^T_{L\beta i}$ goes under gauge group as $q_{R\alpha i}$
  - some Goldstone bosons have baryonnumber
  - Global $G = SU(2N_F)$ and $\hat{q} \to g\hat{q}$
  - $\langle \overline{q}_j q_i \rangle$ is really $\epsilon_{\alpha\beta}\langle (\hat{q}_{\alpha j})^T C \hat{q}_{\beta i}\rangle \propto J_{Aij}$ $J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
  - Conserved if $g J_A g^T = J_A \Longrightarrow H = Sp(2N_F)$

# Lagrangians

JB, Lu, arXiv:0910.5424, JB Rössler 1509.04082:

4 cases similar with $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices $X^a$ are:

- Complex or $SU(N) \times SU(N)/SU(N)$:
  all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$:
  $SU(2N)$ generators with $X^a J_S = J_S X^{aT}$
- Pseudoreal or $SU(2N)/Sp(2N)$:
  $SU(2N)$ generators with $X^a J_A = J_A X^{aT}$
- Real Majorana or $SU(N)/SO(N)$:
  $SU(N)$ generators with $X^a = X^{aT}$
- $SO(2N)$: not usual way of parametrizing $SO(2N)$ matrices
  - the two are related by a $U(2N)$ transformation:
  - same ChPT except for anomalous sector



ChPT

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# The main useful formulae

Calculating for equal mass case goes through using:

$$\begin{aligned}
\text{Complex :} \quad & \langle X^a A X^a B \rangle = \langle A \rangle \langle B \rangle - \frac{1}{N_F} \langle AB \rangle \,, \\
& \langle X^a A \rangle \langle X^a B \rangle = \langle AB \rangle - \frac{1}{N_F} \langle A \rangle \langle B \rangle \,. \\
\text{Real :} \quad & \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \left\langle A J_S B^T J_S \right\rangle - \frac{1}{2N_F} \langle AB \rangle \,, \\
& \langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle + \frac{1}{2} \left\langle A J_S B^T J_S \right\rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle \,. \\
\text{Pseudoreal :} \quad & \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \left\langle A J_A B^T J_A \right\rangle - \frac{1}{2N_F} \langle AB \rangle \,, \\
& \langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle - \frac{1}{2} \left\langle A J_A B^T J_A \right\rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle
\end{aligned}$$

So can do the calculations for all cases

For the partially quenched case: extension needed

JB, Rössler, arXiv:1509.04082 (done with quark-flow method)

# $\phi\phi \to \phi\phi$

- $\pi\pi$ scattering
  - Amplitude in terms of $A(s,t,u)$

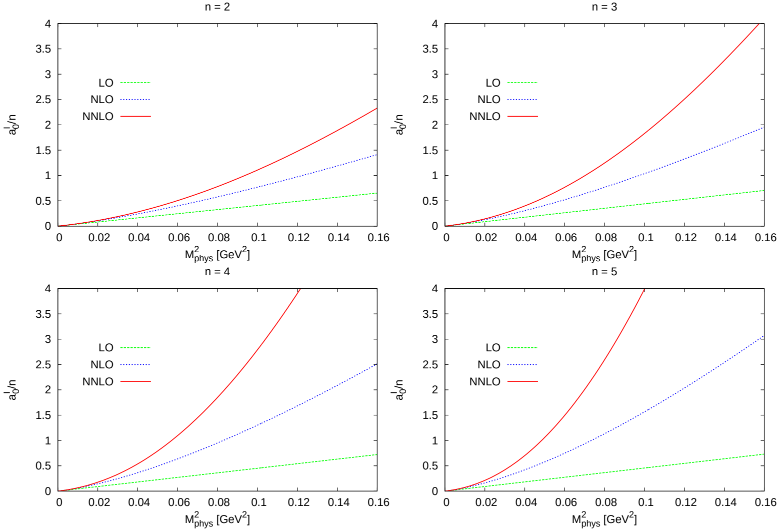
    $M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)\,.$
  - Three intermediate states $I = 0,1,2$
- Our three cases
  - Two amplitudes needed $B(s,t,u)$ and $C(s,t,u)$

$$
\begin{aligned}
M(s,t,u) \quad = \quad & \left[\left\langle X^aX^bX^cX^d\right\rangle + \left\langle X^aX^dX^cX^b\right\rangle\right] B(s,t,u) \\
& + \left[\left\langle X^aX^cX^dX^b\right\rangle + \left\langle X^aX^bX^dX^c\right\rangle\right] B(t,u,s) \\
& + \left[\left\langle X^aX^dX^bX^c\right\rangle + \left\langle X^aX^cX^bX^d\right\rangle\right] B(u,s,t) \\
& + \delta^{ab}\delta^{cd}C(s,t,u) + \delta^{ac}\delta^{bd}C(t,u,s) + \delta^{ad}\delta^{bc}C(u,s,t)\,.
\end{aligned}
$$

$B(s,t,u) = B(u,t,s) \qquad C(s,t,u) = C(s,u,t)\,.$

  - 7, 6 and 6 possible intermediate states
- All formulas similar length to $\pi\pi$ cases but there are so many of them

# $\phi\phi \to \phi\phi$: $a_0^I/n$



# Conclusions for "Beyond QCD"

Calculations done:

- $\langle \bar{q}q \rangle_{\rm phys}$
- $M^2_{\rm phys}$
- $F_{\rm phys}$
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Two-point functions relevant for $S$-parameter
- First three also partially quenched and finite volume

To remember:

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae



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# Other stuff I work on/want to do

- Twisted (thus finite volume) and partially quenched: $K_{\ell 3}$
- Twisted, finite volume, partially quenched vector-two point function: relevant for lattice HVP of $a_\mu$.
- Leading logarithms: another talk
- Get our quark mass isospin breaking at NNLO calculations in an updated shape + combine with em
- Any more suggestions?



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# Conclusions

- ChPT and all the extensions I talked about can be applied (and have often been) to baryons, heavy mesons,. . .
- Gave you some examples of the uses of ChPT
- Future:
  - A tool for studying lattice artefacts, finite volume,. . .
  - Combine with other methods, dispersion relations already heavily done