Electric Dipole Moments on the lattice with higher-dimensional operators

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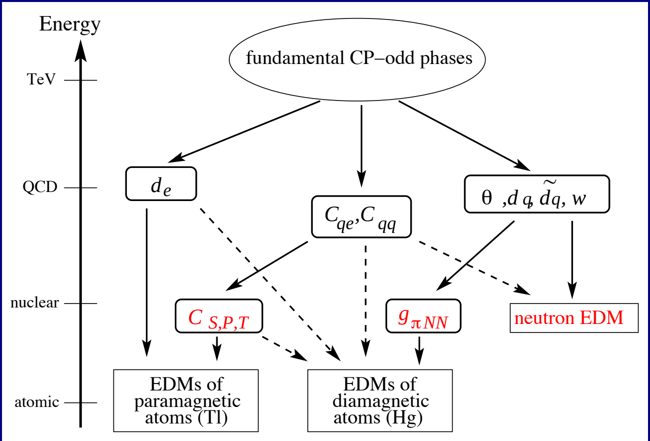
Oct 1, 2015

Introduction
Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Anomalously small strong CP violation from dim 3 and 4.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\rm EW}/M^2_{\rm BSM}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo-electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\rm BSM}$:
  - Weinberg operator (Gluon chromo-electric dipole moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
Dimensions 3 and 4

Consider the chiral and CP violating parts of the action $\mathcal{L} \supset d_i^\alpha O_i^\alpha$, where i is flavor and α is operator index.
Consider only one chiral symmetric CP violating term: $\Theta G\tilde{G}$
Convert to polar basis

$$d_i \equiv |d_i| e^{i\phi_i} \equiv \frac{\sum_\alpha d_i^\alpha \langle\Omega|\,\mathcal{I}m\, O_i^\alpha|\pi\rangle}{\sum_\alpha \langle\Omega|\,\mathcal{I}m\, O_i^\alpha|\pi\rangle}$$

Then CP violation is proportional to:

$$\bar{d}\bar{\Theta}\,\mathcal{R}e\,\frac{d_i^\alpha}{d_i} - |d_i|\,\mathcal{I}m\,\frac{d_i^\alpha}{d_i} \quad \text{with} \quad \frac{1}{\bar{d}} \equiv \sum_i \frac{1}{d_i} \quad \bar{\Theta} \equiv \Theta - \sum_i \phi_i$$

CP violation depends on $\bar{\Theta}$ and on a *mismatch* of phases between d_i^α and d_i.

Lattice
Lattice QCD

Lattice is a nonperturbative formulation of QCD.

Lattice uses a hard regulator:

$$\bar\psi \not{p} \psi \to \bar\psi \not{W}(p) \psi \, ,$$

where $W(p)$ is a periodic function:

$$W(p) = W(p + 2\pi a^{-1}) \, .$$

Hard regulators introduce a scale and allow mixing with lower dimensional operators.

Hard regulators are unambiguous: no renormalon problem.

Ultraviolet divergence regulated by the periodicity:

$$\int_{-\infty}^{\infty} dp = \sum_{m=-\infty}^{\infty} \int_{\pi(m-1)/a}^{\pi(m+1)/a} dp \to \int_{-\pi/a}^{\pi/a} dp$$

Infrared controlled by calculating in a finite universe.

$$\int dp f(p) \to \sum_n (\frac{2\pi}{L}) f(\frac{2\pi n}{L} + p_0)$$

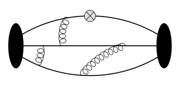
Real world reached by

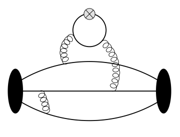
$$\lim_{\substack{L\to\infty \\ a\to 0}} .$$

Current calculations $a \sim 0.05–0.15\,\mathrm{fm}$ and $L \sim 3–5\,\mathrm{fm}$.

Lattice
Euclidean Space

Equal-time vacuum matrix elements of Weyl-ordered operators.
To extract $\langle n|O|n\rangle$:





$$
\begin{aligned}
&\mathrm{Tr}\, e^{-\beta H}\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger \\
&\quad = \quad e^{-\beta E_s}\langle s|\hat{n}e^{-HT_f}Oe^{-HT_i}\hat{n}^\dagger|s\rangle \\
&\quad \underset{\beta\to\infty}{\longrightarrow} \quad \langle\Omega|\hat{n}|n_f\rangle e^{-M_fT_f}\langle n_j|O|n_i\rangle e^{-M_iT_i}\langle n_i|\hat{n}^\dagger|\Omega\rangle \\
&\quad \underset{T_i,T_f\to\infty}{\longrightarrow} \quad \langle n|O|n\rangle e^{-M_0(T_i+T_f)}
\end{aligned}
$$

Lattice
Lattice Basics

We can extract nEDM in two ways.

- As the difference of the energies of spin-aligned and anti-aligned neutron states:

$$d_n = \frac{1}{2}\left(M_{n\downarrow} - M_{n\uparrow}\right)\big|_{E=E\uparrow}$$

- By extracting the CP violating form factor of the electromagnetic current.

$$\begin{aligned}\langle n|J_\mu^{\mathrm{EM}}|n\rangle &\sim \frac{F_3(q^2)}{2M_n}\bar{n}\, q_\nu \sigma^{\mu\nu}\gamma_5\, n \\ d_n &= \lim_{q^2\to 0}\frac{F_3(q^2)}{2M_n}\end{aligned}$$

Difficult to perform simulations with complex $\not{\text{CP}}$ action

Expand and calculate correlators of the $\not{\text{CP}}$ operator:

$$\begin{aligned}
\langle\, C^{\not{\text{CP}}}(x,y,\ldots)\,\rangle_{\text{CP}+\not{\text{CP}}} &= \int [\mathcal{DA}] \exp\left[-\int d^4x(\mathcal{L}^{\text{CP}} + \mathcal{L}^{\not{\text{CP}}})\right] \\
&\qquad \times C^{\not{\text{CP}}}(x,y,\ldots) \\
&\approx \int [\mathcal{DA}] \exp\left[-\int d^4x \mathcal{L}^{\text{CP}}\right] \\
&\qquad \times \left(1 - \int d^4x \mathcal{L}^{\not{\text{CP}}}\right) C^{\not{\text{CP}}}(x,y,\ldots) \\
&= \langle\, C^{\not{\text{CP}}}(x,y,\ldots)\, \mathcal{L}^{\not{\text{CP}}}(p_\mu = 0)\,\rangle_{\text{CP}}
\end{aligned}$$

Lattice
Systematics summary

- *Number of quarks* 2+1
  Isospin breaking beyond current calculations.
  Charm is sometimes included.
- *Quark mass* $M_{\pi,\mathrm{min}} < 200$ MeV.
  May be possible to work at the physical point.
  At least, χPT from $M_\pi < 400$ MeV.
- *Discretization* $a < 0.1$ fm (2/3 points)
  Discretization errors differ in different schemes.
  May be problematic if all points $a > 0.1$ fm.
- *Volume* $M_\pi L > 4$
  At least $M_\pi L > 3$. OK if $\exp(-M_\pi L)$ at 3 masses.
- *Renormalization* Nonpeturbative matching
  At least improved 1-loop perturbation theory.
- *Excited states* $t_{\mathrm{sep,max}} > 1.5$ fm
  At least $t_{\mathrm{sep}} > 1.2$ fm. Extrap from 3 t_{sep} OK.
- *Disconnected diagrams*

Quark EDM

Quark EDM

$$\mathcal{L} \supset -\frac{i}{2} \sum d_q \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}$$

Note that $\sigma_{\mu\nu}\gamma_5 \propto \epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$.

$$d_N = \sum d_q \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle \equiv d_q g_T^q$$

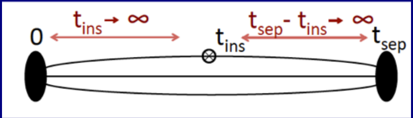
g_T calculated on the lattice:

Bhattacharya, Cirigliano, Gupta, Lin, Yoon, **arXiv:1506.04196 [hep-lat]**

Bhattacharya, Cirigliano, Cohen, Gupta, Joseph, Lin, Yoon **arXiv:1506.06411 [hep-lat]**

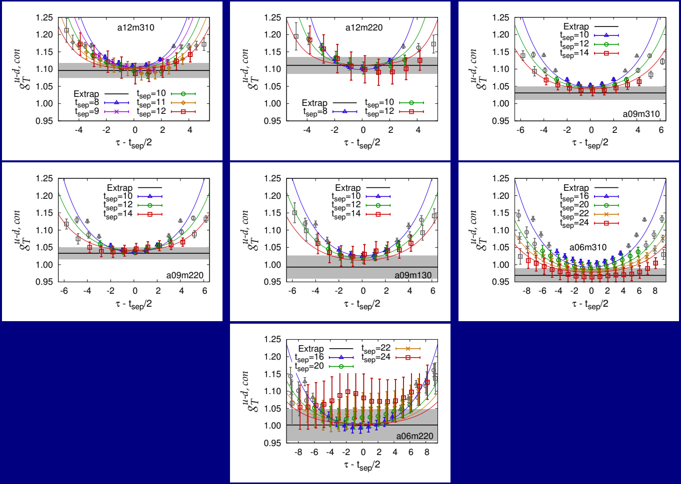
$$a \in [0.06, 0.12]\ \mathrm{fm}, \quad m_\pi \in [130, 310]\ \mathrm{MeV}, \quad m_\pi L \in [3.3, 5.5]$$

Quark EDM
Excited states



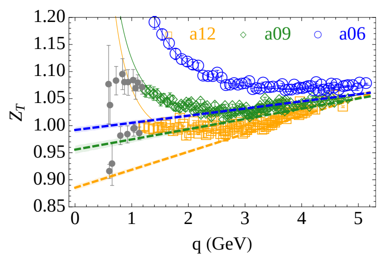
$$\begin{aligned} C^{3pt} &= |\mathcal{A}_0|^2\langle 0|T|0\rangle e^{-M_0 t_{sep}} + |\mathcal{A}_1|^2\langle 1|T|1\rangle e^{-M_1 t_{sep}} + \\ &\quad \mathcal{A}_0^*\mathcal{A}_1\langle 0|T|1\rangle e^{-M_0 t_{ins} - M_1(t_{sep}-t_{ins})} + \\ &\quad \mathcal{A}_1^*\mathcal{A}_0\langle 1|T|0\rangle e^{-M_1 t_{ins} - M_0(t_{sep}-t_{ins})} \end{aligned}$$

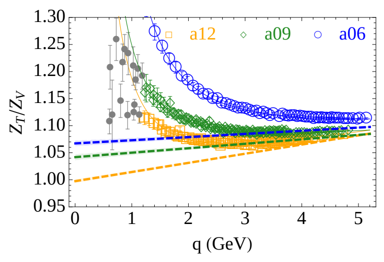
Top line has no dependence on t_{ins}: need multiple t_{sep}.
$\mathcal{A}$ and M can be obtained from 2-pt functions.


a12m310
a12m220
a09m310
a09m220
a09m130
a06m310
a06m220
Extrap
t_{sep}=8
t_{sep}=9
t_{sep}=10
t_{sep}=11
t_{sep}=12
t_{sep}=14
t_{sep}=16
t_{sep}=20
t_{sep}=22
t_{sep}=24
$g_T^{u-d, con}$
τ - t_{sep}/2

Quark EDM
Renormalization

RI-SMom scheme: Nonexceptional symmetric momentum matrix element has tree-level value.

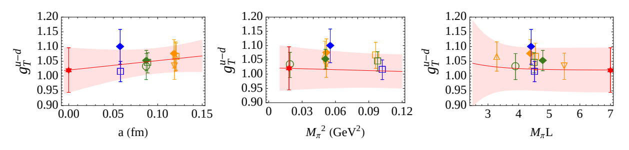




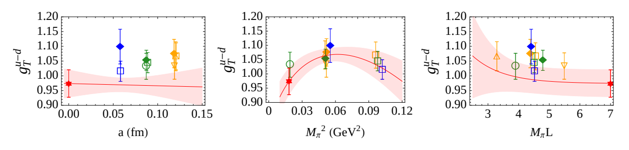
Quark EDM

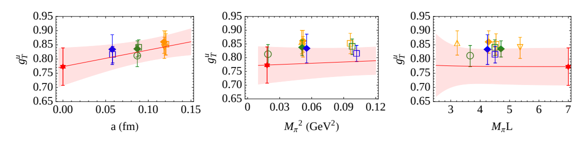
Extrapolation

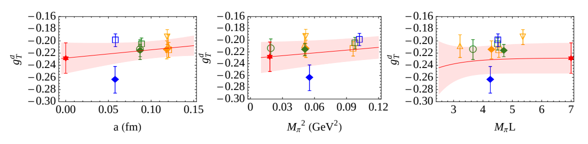
$$g_T = c_1 + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$

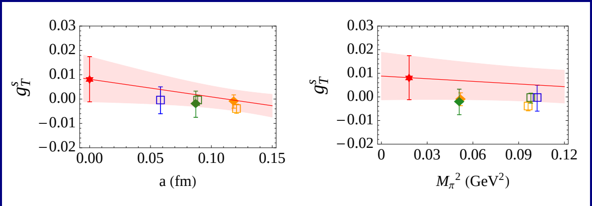


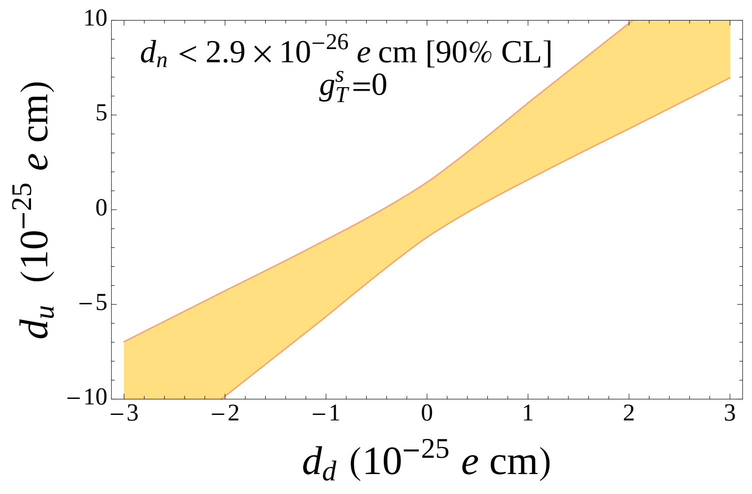
$$g_T = c_1 \left(1 + \frac{M_\pi^2(1+4g_A^2)}{2(4\pi F_\pi)^2} \ln \frac{M_\rho^2}{M_\pi^2}\right) + c_2 a + c_3 M_\pi^2 + c_4 e^{-M_\pi L}$$



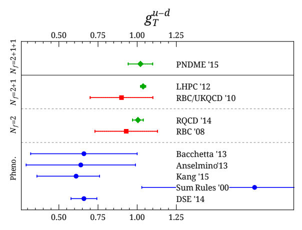

0.95
0.90
0.85
0.80
0.75
0.70
0.65
g_T^u
0.00
0.05
0.10
0.15
a (fm)
0
0.03
0.06
0.09
0.12
M_π^2 (GeV^2)
3
4
5
6
7
M_πL

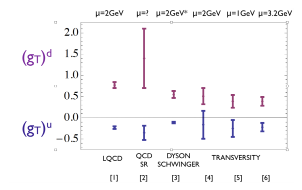

−0.16
−0.18
−0.20
−0.22
−0.24
−0.26
−0.28
−0.30
g_T^d
0.00
0.05
0.10
0.15
a (fm)
0
0.03
0.06
0.09
0.12
M_π^2 (GeV^2)
3
4
5
6
7
M_πL


0.03
0.02
0.01
0.00
−0.01
−0.02
g_T^s
0.00
0.05
0.10
0.15
a (fm)
0
0.03
0.06
0.09
0.12
M_π^2 (GeV^2)


$d_n < 2.9 \times 10^{-26}$ e cm [90% CL]
g_T^s=0
d_u (10^{-25} e cm)
d_d (10^{-25} e cm)
10
5
0
−5
−10
−3
−2
−1
0
1
2
3

Quark EDM
Comparison





[1] Bhattacharya *et al.* 2015
[2] Pospelov-Ritz 2000
[3] Pitschmann *et al.* 2014
[4] Bacchetta *et al.* 2013
[5] Anselmino *et al.* 2013
[6] Kang *et al.* 2015

Quark CEDM
Renormalization and Mixing

Operator basis:

$ig\bar\psi\tilde\sigma^{\mu\nu}G_{\mu\nu}t^a\psi$ $\qquad \partial^2\left(\bar\psi i\gamma_5 t^a\psi\right)$

$\frac{ie}{2}\bar\psi\tilde\sigma^{\mu\nu}F_{\mu\nu}\left\{Q,t^a\right\}\psi$

$\mathrm{Tr}\left[MQ^2t^a\right]\frac{1}{2}\tilde F_{\mu\nu}F^{\mu\nu}$ $\qquad \mathrm{Tr}\left[Mt^a\right]\frac{1}{2}\tilde G^a_{\mu\nu}G^{\mu\nu a}$

$\mathrm{Tr}\left[Mt^a\right]\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\psi\right)$ $\qquad \frac{1}{2}\partial_\mu\left(\bar\psi\gamma^\mu\gamma_5\left\{M,t^a\right\}\psi\right)\Big|_{\mathrm{traceless}}$

$\frac{1}{2}\bar\psi i\gamma_5\left\{M^2,t^a\right\}\psi$ $\qquad \mathrm{Tr}\left[M^2\right]\bar\psi i\gamma_5 t^a\psi$

$\mathrm{Tr}\left[Mt^a\right]\bar\psi i\gamma_5 M\psi$

$i\bar\psi_E\gamma_5 t^a\psi_E$ $\qquad \mathrm{Re}\,\partial_\mu\left[\bar\psi_E\gamma^\mu\gamma_5 t^a\psi\right]$

$\mathrm{Re}\,\bar\psi\gamma_5\not{\partial}t^a\psi_E$ $\qquad \mathrm{Re}\,\frac{ie}{2}\bar\psi\left\{Q,t^a\right\}\not{A}^{(\gamma)}\gamma_5\psi_E$

RI-$\tilde{\mathrm{S}}$Mom Conditions:

$$\begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{ren}} = \begin{pmatrix} Z_O & Z_{ON} \\ 0 & Z_N \end{pmatrix} \begin{pmatrix} O \\ N \end{pmatrix}_{\mathrm{bare}}$$

O: Gauge-invariant operators, does not vanish by equation of motion.

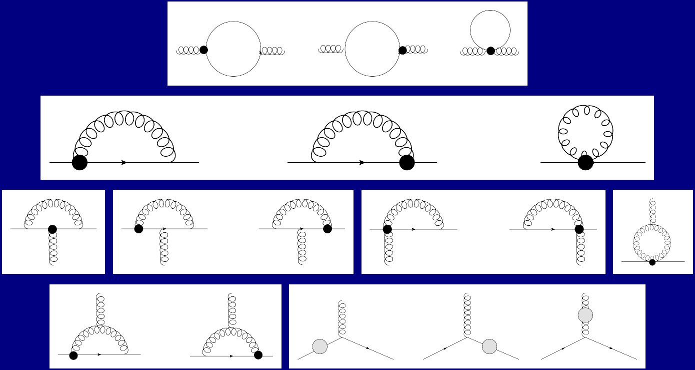
N: Gauge-dependent operators, restricted by BRST, vanish by equation of motion.

Impose conditions on matrix elements of quarks and gluons:

- Use $\overline{\mathrm{MS}}$ quark masses in the expansion.
- Three point functions at $p^2 = p'^2 = q^2 = -\Lambda^2 \ll 0$ (RI-SMOM).
- Four point functions at $p^2 = p'^2 = k^2 = q^2 = s = u = t/2 = -\Lambda^2$.

This choice eliminates most non-1PI contributions.

(See arXiv:1502.07325 [hep-ph]).



Quark CEDM

Schwinger source method

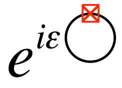
The quark chromo-EDM operator is a quark bilinear.
Add it to the Dirac operator in the propagator inversion routine:

$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

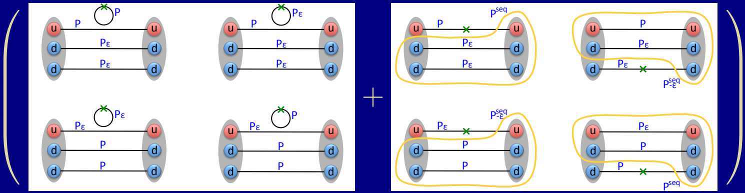
The fermion determinant gives a ‘reweighting factor’

$$\begin{aligned} &\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu}))}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})} \\ &= \exp \mathrm{Tr} \ln \left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] \\ &\approx \exp\left[i\epsilon\,\mathrm{Tr}\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] . \end{aligned}$$

3-point function



$\times$



The chromoEDM operator is dimension 5.
Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{\rm QCD} \sim 1$.
Need to check linearity.

Quark CEDM
Lattice implementation

Using BiCGStab in Chroma (Clover on HISQ $a \approx 0.12$fm, $m_\pi \approx 310$MeV)

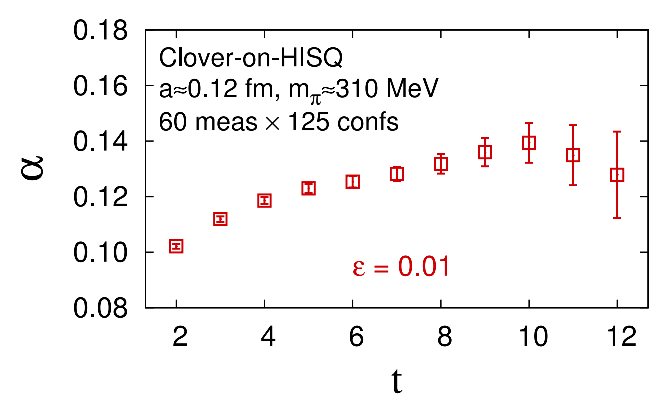
- Cost of $\not{D}$ increases by about 7%.
- Condition number changes by less than 5%.
- Can use $\epsilon = 0$ as initial guess.

Each extra inversion less than the cost of the $\epsilon = 0$ inversion.

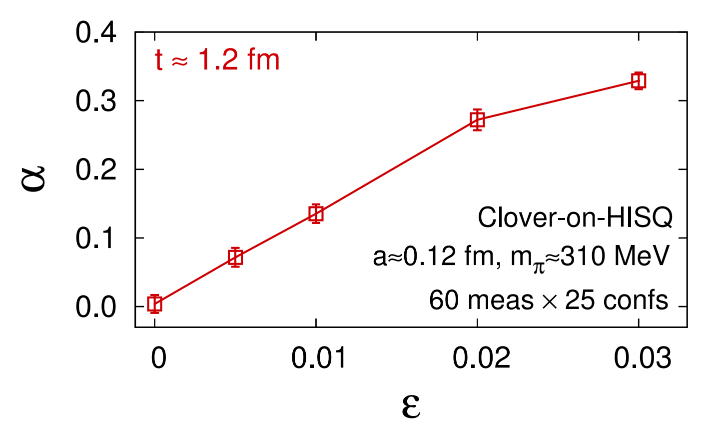
Accuracy	$\epsilon = 0.005$	$\epsilon = 0.01$
10^{-8}	85%	86%
10^{-3}	51%	66%
5×10^{-3}	28%	45%

Calculation of connected EDM measurement on each configuration is about 1.5 times the cost of V/A form factors measurements.

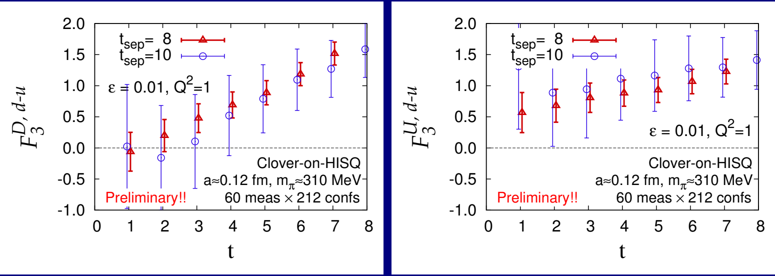
Quark CEDM
Numerical tests



Propagator $= \exp(-i\alpha\gamma_5)(\not p + m)exp(-i\alpha\gamma_5)$; Preliminary; Connected Diagrams Only



Preliminary; Connected Diagrams Only



Preliminary; Connected Diagrams Only

- Connected F_3 does not get contribution from $\dim \mathcal{O} < 5$.
- Observed F_3 from CQEDM, QEDM, or will vanish on extrapolation.
- Perturbative subtraction of QEDM contribution possible, and determination of proportionality possible.

Conclusions

Summary

- QEDM contributions from u, d, and s quarks under control.
- Methods developed for QCEDM.
- Study of systematics for QCEDM needed.
- Most divergent mixing with $\frac{\alpha_s}{a^2}\bar{\psi}\gamma_5\psi$.
  nEDM due to this same as due to $\frac{\alpha_s}{ma^2}G\cdot\tilde{G}$.
  Current estimates of nEDM due to
  - $\text{CEDM}^{\overline{\text{MS}}} \Rightarrow O(1)$
  - $\frac{\alpha_s}{ma^2}\Theta G\cdot\tilde{G} \Rightarrow \frac{O(0.1)}{5\text{MeV}a^2}O(10^{-3})$e-fm $= O(1)$
    at $a \approx 0.1\text{fm}$.

  Expect O(1–10) cancellation. Important for disconnected diagrams.
- Chiral symmetry does not remove this mixing.