



Lattice QCD Input to Axion Cosmology

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INT-15-3

Intersections of BSM Phenomenology and QCD for New Physics Searches
Institute for Nuclear Theory

Tuesday, October 13th, 2015

PRD **92** 034507 / arXiv:1505.07455 – E. Berkowitz, M. Buchoff, E. Rinaldi.

Outline

- Introduction
- Whence axions?
- What is the over-closure bound?
- Inputs to the over-closure bound from lattice QCD
- Outlook

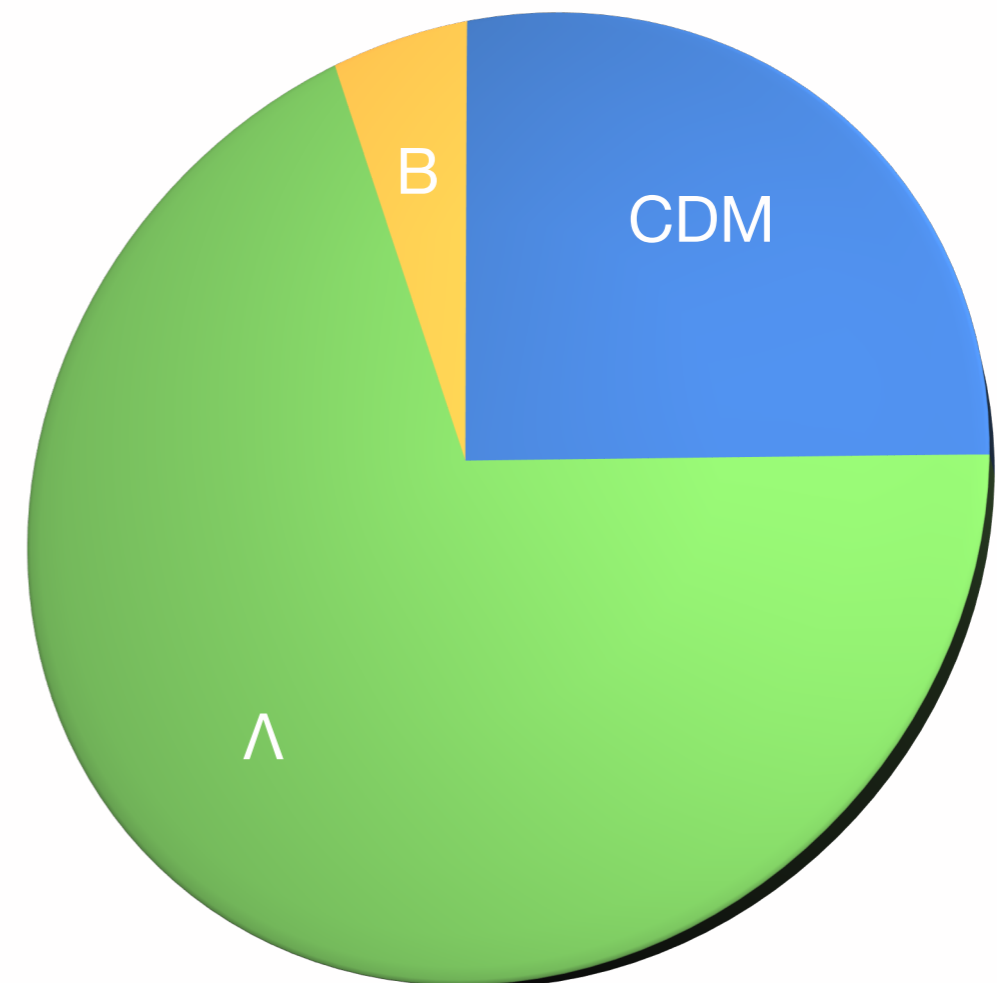
Big Idea

- Axions were originally proposed to deal with the Strong CP Problem, also form a plausible DM candidate.
 - Calculating the axion energy density requires nonperturbative QCD input.
- Being sought in ADMX (LLNL, UW) & CAST (CERN), and (soon) IAXO with large discovery potential in the next few years.
- Requiring $\Omega_a \leq \Omega_{\text{CDM}}$ yields a lower bound on the axion mass today.

Preskill, Wise & Wilczek, Phys Lett B **120** (1983) 127-132



The Economist, 19 Dec 2006



$\Omega_{\text{tot}} = 1.000(7)$
PDG 2014 via

P.A.R. Ade, et al., (Planck Collab. 2013 XVI), arXiv: 1303.5076v1.

QCD Theta Term

- QCD has a parameter, θ .
 - Controls QCD CP violation.
 - Topological.
- θ can take any value in $(-\pi, \pi]$.
- Neutron EDM $\approx 3 \cdot 10^{-26} \text{e}\cdot\text{cm}$
Baker et al., PRL 97, 131801 (2006) / hep-ex/0602020
 - $|\theta| \lesssim 10^{-10}$

$$\mathcal{L}_{\text{QCD}} \ni \theta \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

CP Violating

$$Q = \frac{1}{32\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \in \mathbb{Z}$$
$$e^{iS} \propto e^{iQ\theta}$$

Strong CP Problem:
Why is θ so small?

(Some) Resolutions of the Strong CP Problem

- Just declare CP to be good in the strong sector
 - Why in the strong and not in the weak?

- $m_u = 0 \quad \bar{q} \left(i\not{D} - m e^{i\theta' \gamma_5} \right) q$

't Hooft PRL **37** 8 (1976)

Jackiw & Rebbi, PRL **37** 127 (1976)

Callan, Dashen & Gross PLB **63** 335 (1976)

Kaplan & Manohar PRL **56** 2004 (1986)

- $m_u \neq 0$

Gasser & Leutwyler PhysRept **87** 77-169 (1982)

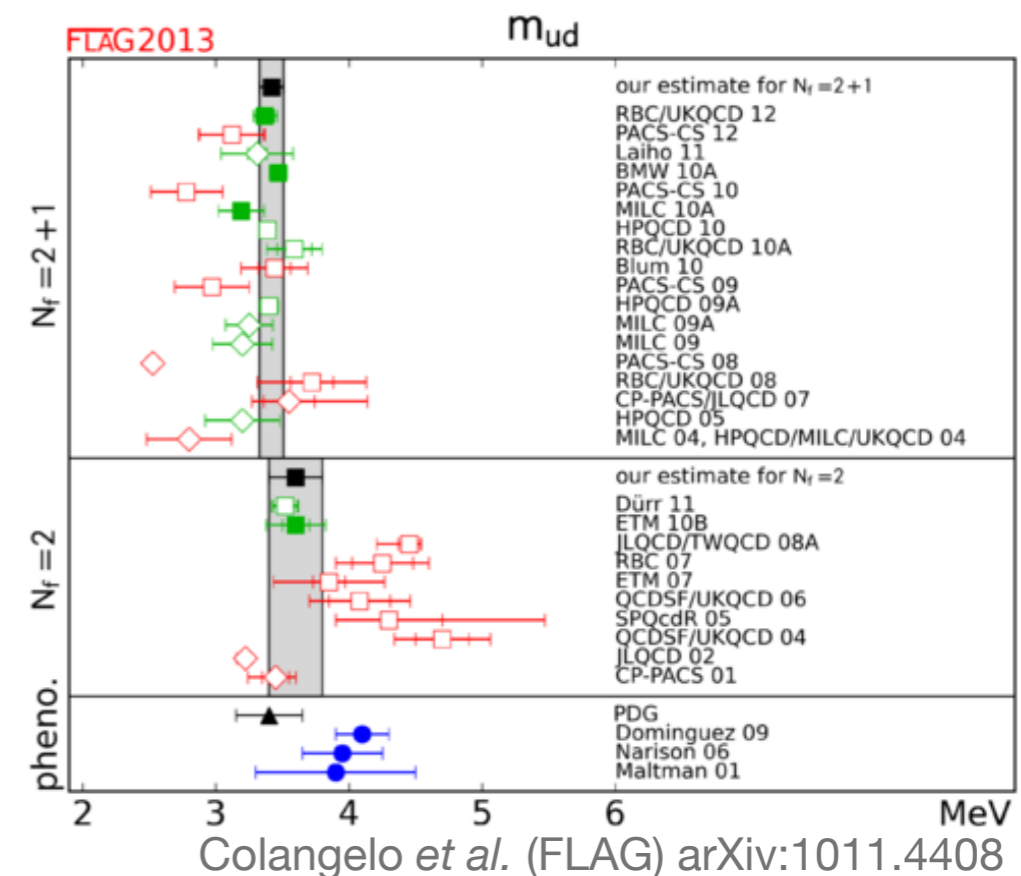
- Additional Peccei-Quinn symmetry & axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Fine tuning problem can be reintroduced via high-dimensional (11^+) operators at Planck scale.

Holman *et al.* arXiv:hep-ph/9203206

Cheung arXiv:1003.0941



Axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Couple to topological charge
- Otherwise have shift symmetry.
- Amenable to effective theory treatment
- PQ symmetry can break before or after inflation.

$$\mathcal{L}_{\text{QCD}} \ni \theta \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

Axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Couple to topological charge

$$\mathcal{L}_{\text{axions}} = \frac{1}{2} (\partial_\mu a)^2 + \left(\frac{a}{f_a} + \theta \right) \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Otherwise have shift symmetry.

$$a \rightarrow a + \alpha$$

- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos \left(\theta + \frac{\langle a \rangle}{f_a} \right)$$

$$m_a^2 f_a^2 = \left. \frac{\partial^2 V}{\partial \theta^2} \right|_{\theta=0}$$

- PQ symmetry can break before or after inflation.

Axions

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- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos \left(\theta + \frac{\langle a \rangle}{f_a} \right)$$

$$m_a^2 f_a^2 = \chi$$

- PQ symmetry can break before or after inflation.

Axions

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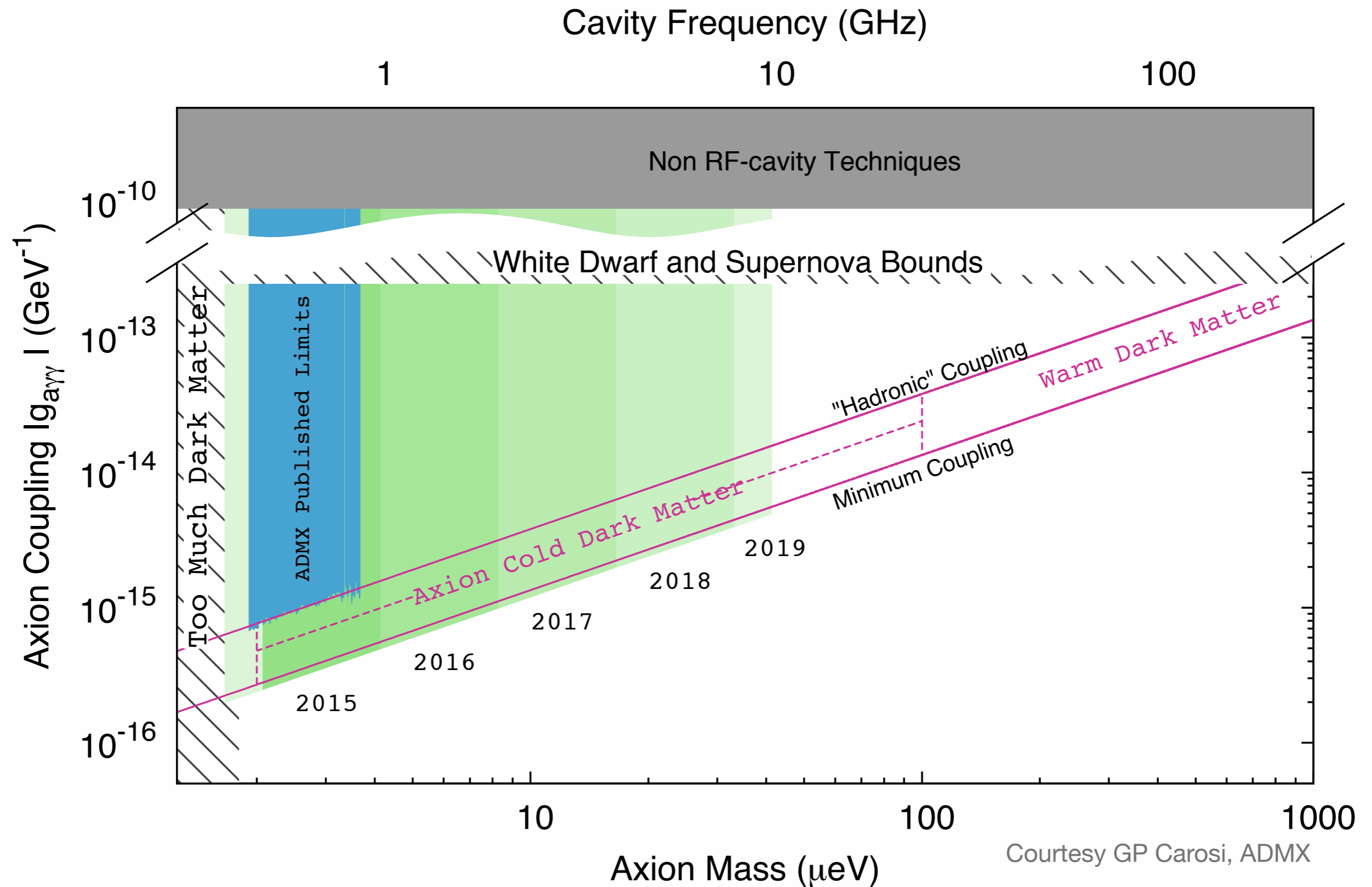
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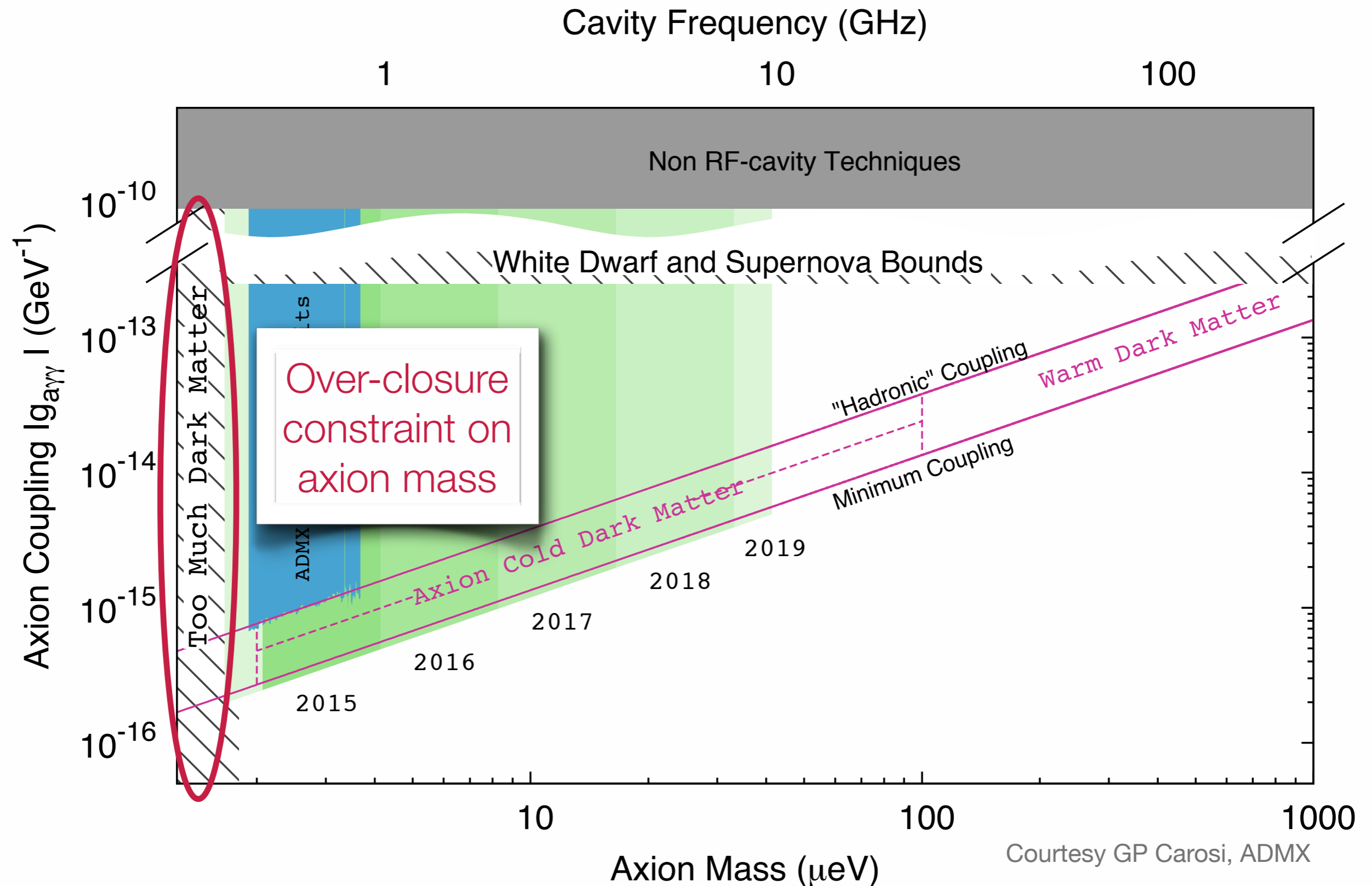
$$m_a^2 f_a^2 = \chi_{\text{QCD}}$$

Axion mass Topological Susceptibility

Current Axion Constraints



Current Axion Constraints



The Over-Closure Bound

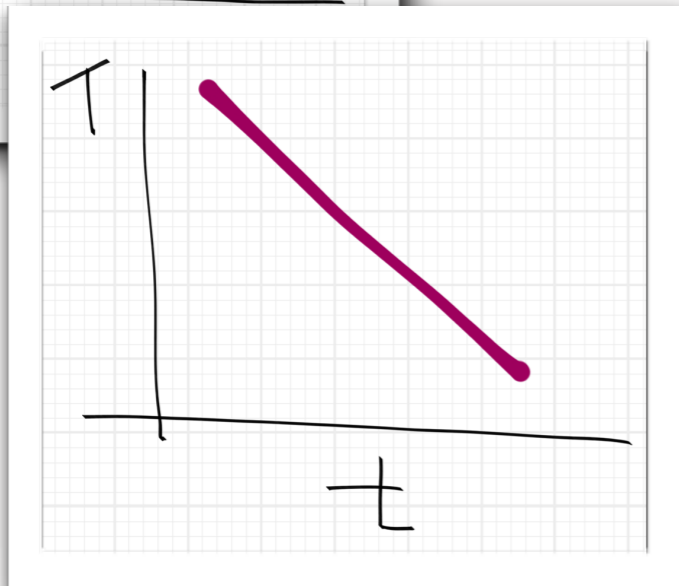


High temperature arguments
imply χ vanishes as $T \rightarrow \infty$

The Over-Closure Bound



High temperature arguments
imply χ vanishes as $T \rightarrow \infty$



Universe cools as it expands

The Over-Closure Bound

High temperature arguments imply χ vanishes as $T \rightarrow \infty$

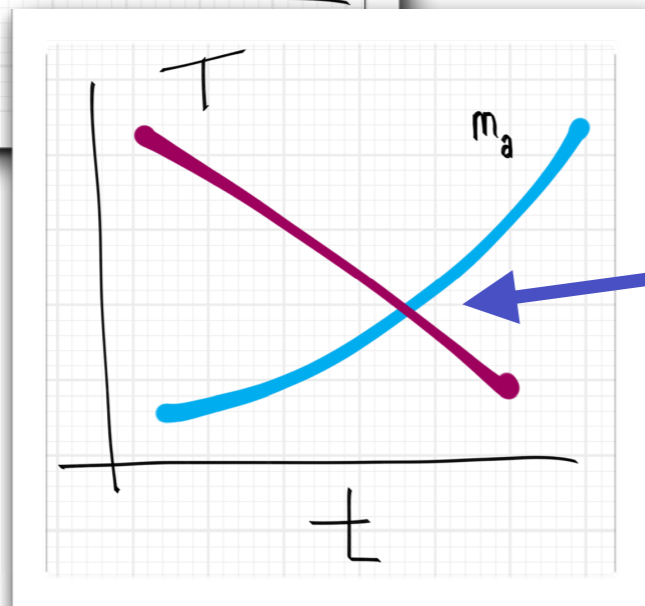
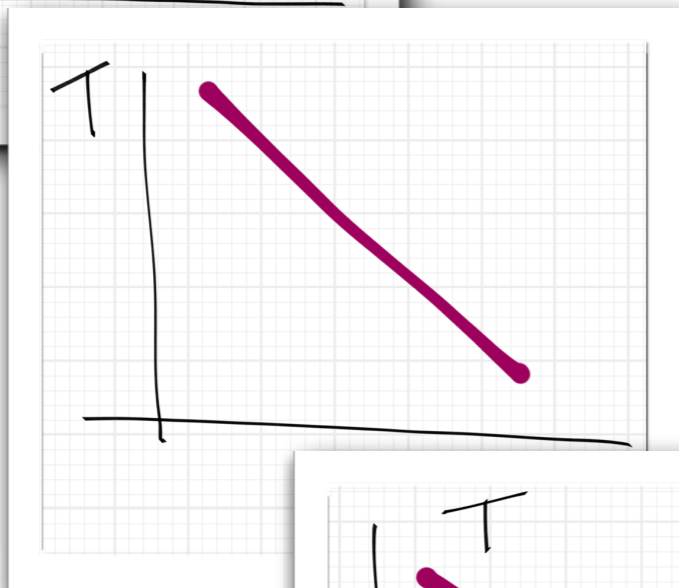
Universe cools as it expands

Axion number fixed when

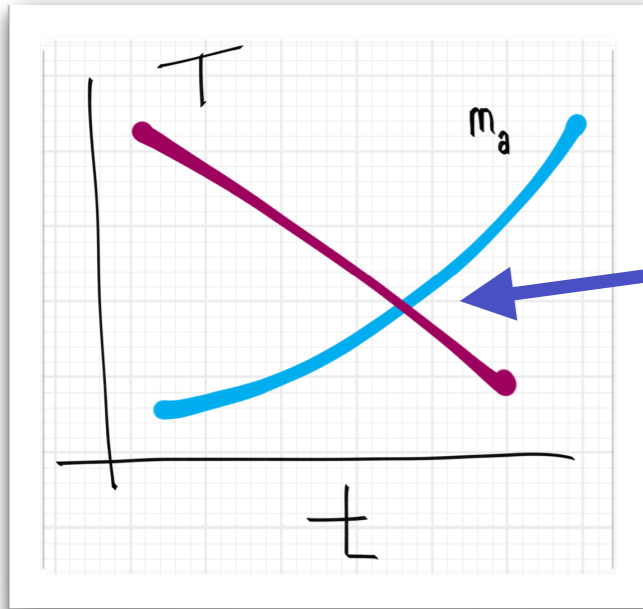
$$3H \sim m_a$$

$$T_1 \approx 5.5 T_c$$

H : Hubble constant



The Over-Closure Bound



Axion number fixed when

$$3H \sim m_a$$

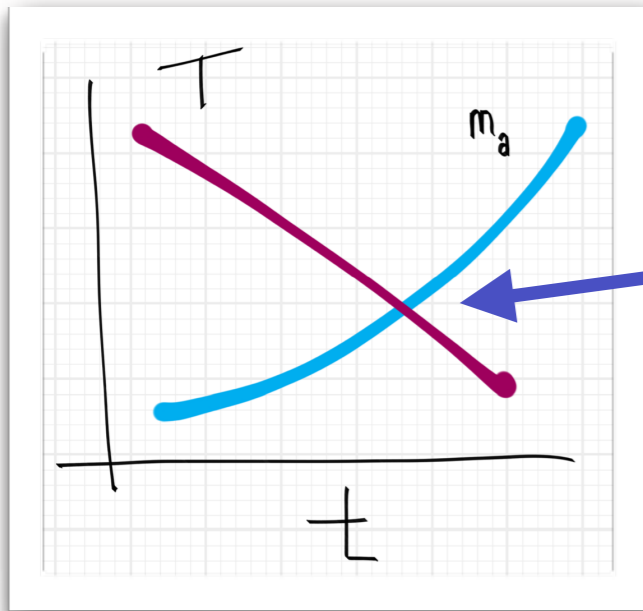
$$T_1 \approx 5.5 T_c$$

$$3H(T_1) \sim m_a(T_1)$$

$$9H^2(T_1) f_a^2 \sim \chi(T_1)$$

$$T_1 = T_1(f_a, \chi(T))$$

The Over-Closure Bound



Axion number fixed when

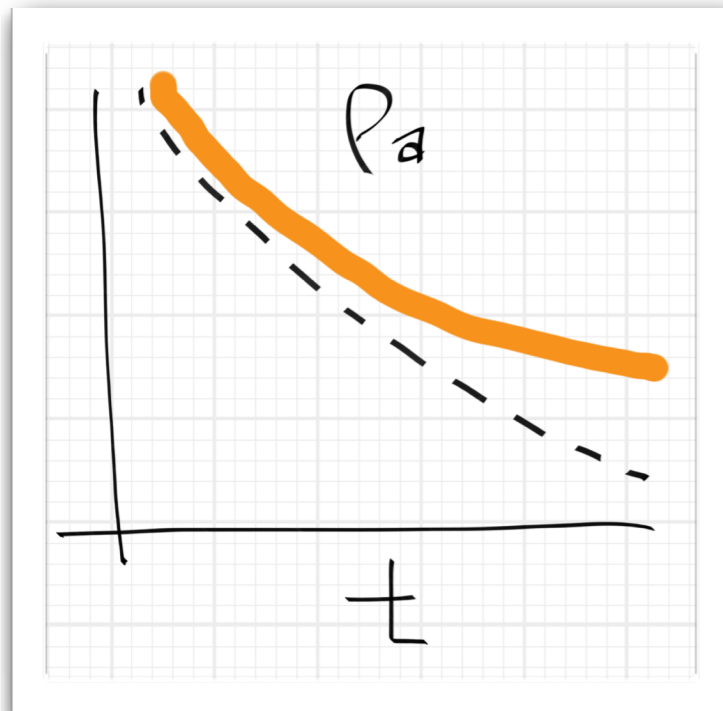
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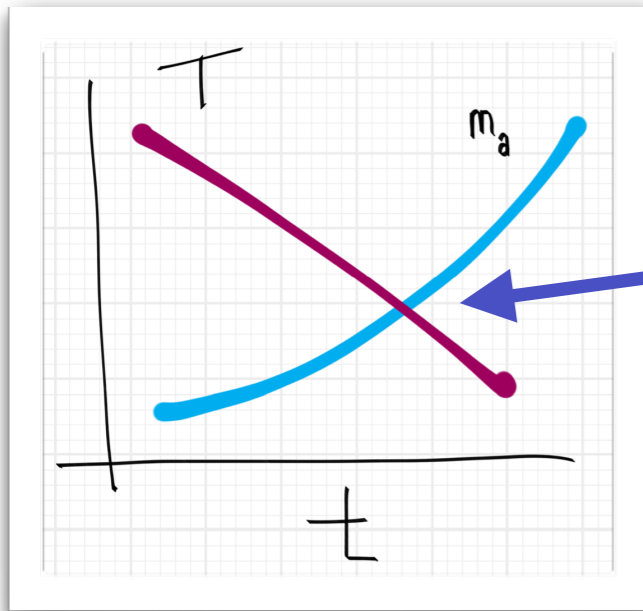
$$T_1 = T_1(f_a, \chi(T))$$



Axions continue to get heavier after production stops!

$$\rho(t) \neq \left(\frac{a(t_1)}{a(t)} \right)^3 \rho(t_1)$$

The Over-Closure Bound



Axion number fixed when

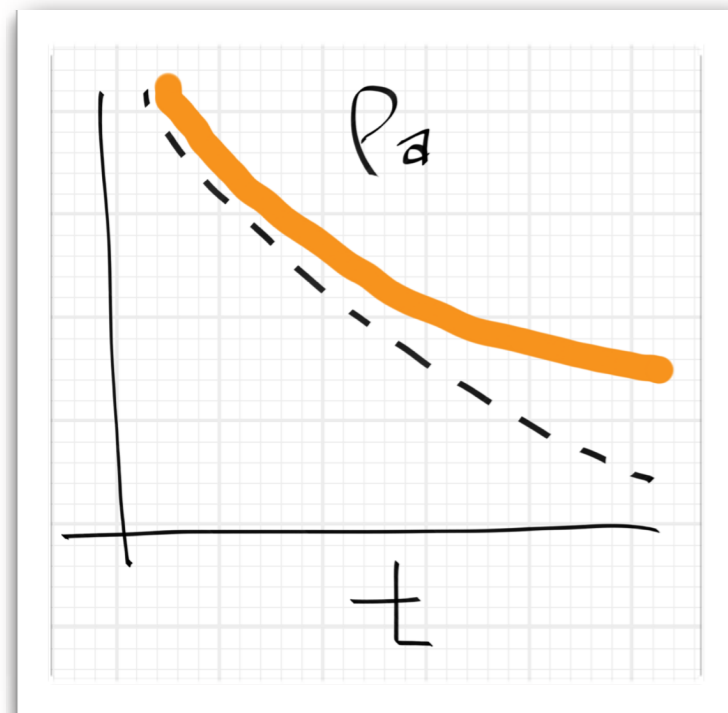
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Axions continue to get heavier after production stops!

$$\frac{\rho(t)R^3}{m_a(t)} = \# \text{ axions in a fixed comoving volume}$$

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

Cosmological EOM: $\rho(T_1) = \frac{1}{2}\theta_1^2 m_a^2 f_a^2 = \frac{1}{2}\theta_1^2 \chi(T_1)$

$$\rho(T_\gamma) = \frac{1}{2}\theta_1^2 \sqrt{\chi(T_\gamma)\chi(T_1)} \left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

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$$\rho(T_\gamma) = \frac{1}{2}\theta_1^2 \sqrt{\chi(T_\gamma)\chi(T_1)} \left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

χ^{PT}

$$f_a^2 m_a^2(T_\gamma) = \frac{m_u m_d}{(m_u + m_d)^2} f_\pi^2 m_\pi^2$$

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

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χ^{PT} (green arrow pointing to $\chi(T_\gamma)$)
 Strong Dynamics (red arrow pointing to $\chi(T_1)$)
 Cosmology (blue arrow pointing to $\left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$)
 $3H \sim m_a$ (blue arrow pointing to $\left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$)

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

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$3H \sim m_a$

χ PT

Strong Dynamics

Cosmology

Initial conditions

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

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Initial conditions

χ PT

Strong Dynamics

Cosmology

$3H \sim m_a$

Late PQ breaking: $\langle \theta_1^2 \rangle = \frac{\pi^2}{3}$
 constraints on m_a, f_a

Early PQ breaking: constraints depend on θ_1

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

$$\rho(T_\gamma) = \frac{1}{2}\theta_1^2 \sqrt{\chi(T_\gamma)\chi(T_1)} \left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

If T_1 goes down, $\rho(T_\gamma)$ goes up...

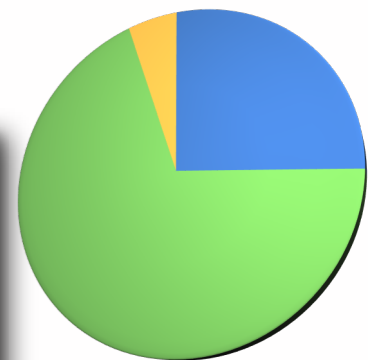
$$9H^2(T_1)f_a^2 \sim \chi(T_1)$$

T_1 goes down as f_a goes up.

If f_a goes up, $\rho(T_\gamma)$ goes up...

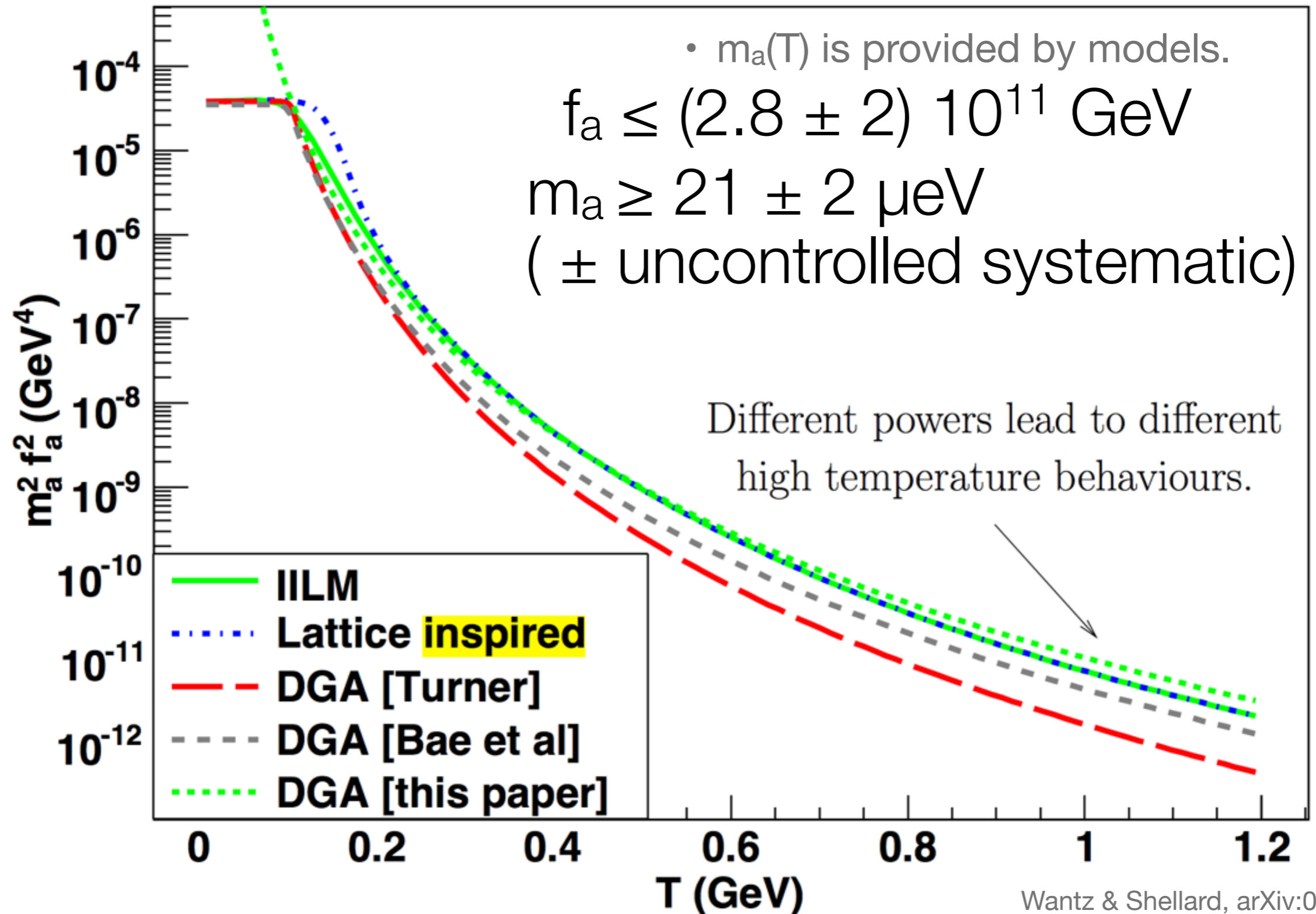
$$m_a^2 f_a^2 = \chi$$

Small m_a are **excluded** by
observed dark matter abundances!

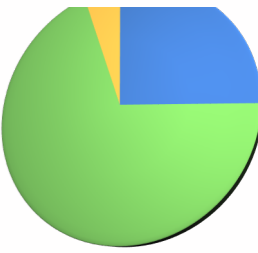


Prior Over-Closure Bound

$$\frac{\rho}{\rho_c} < \Omega_{\text{CDM}} = 0.12$$

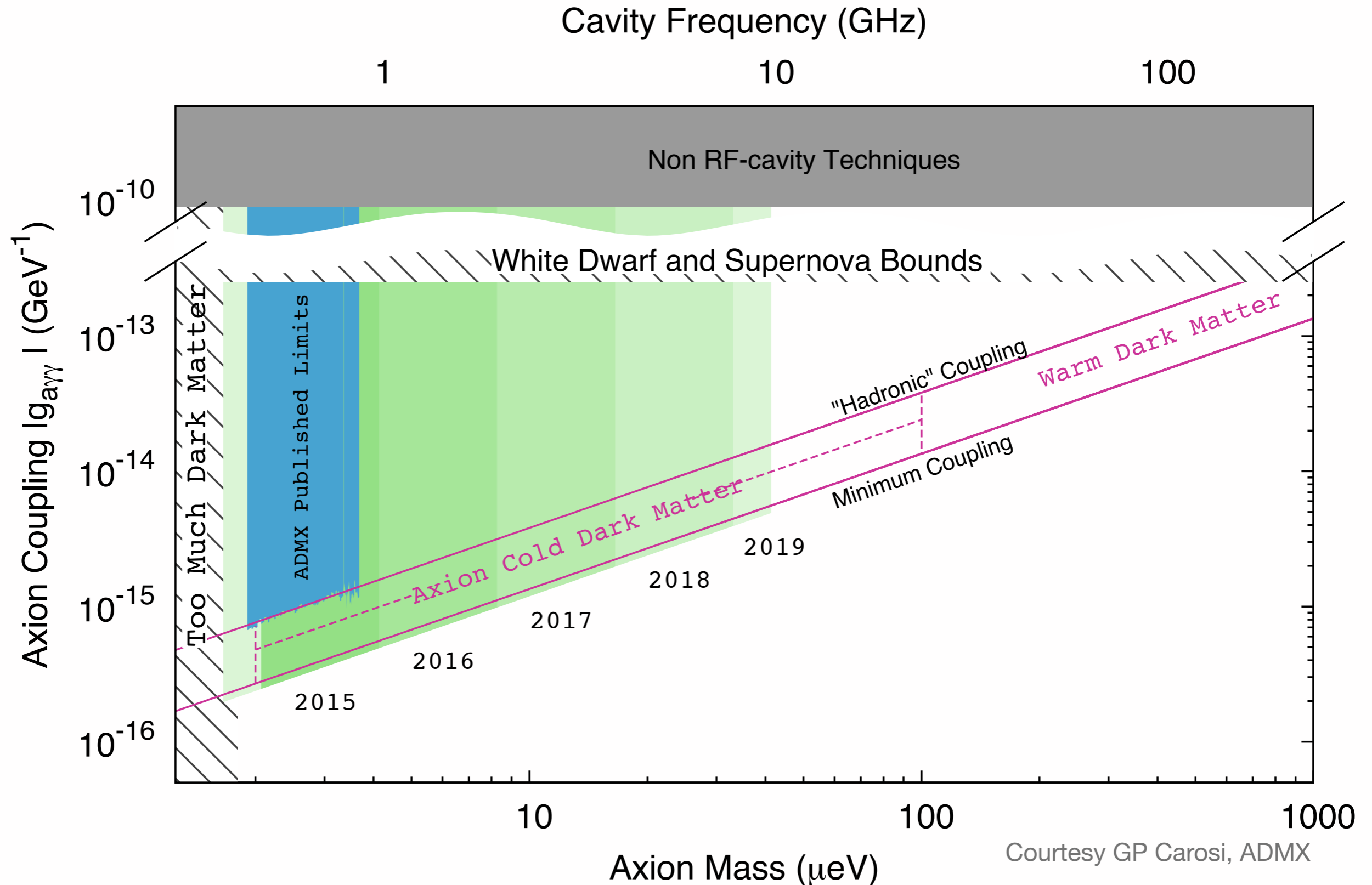


$$\Omega_a \leq \Omega_{\text{CDM}}$$



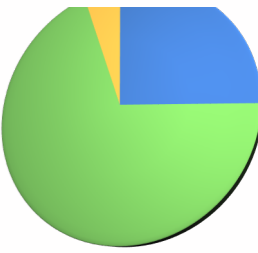
Current Axion Constraints

Wantz & Shellard, arXiv:0910.1066



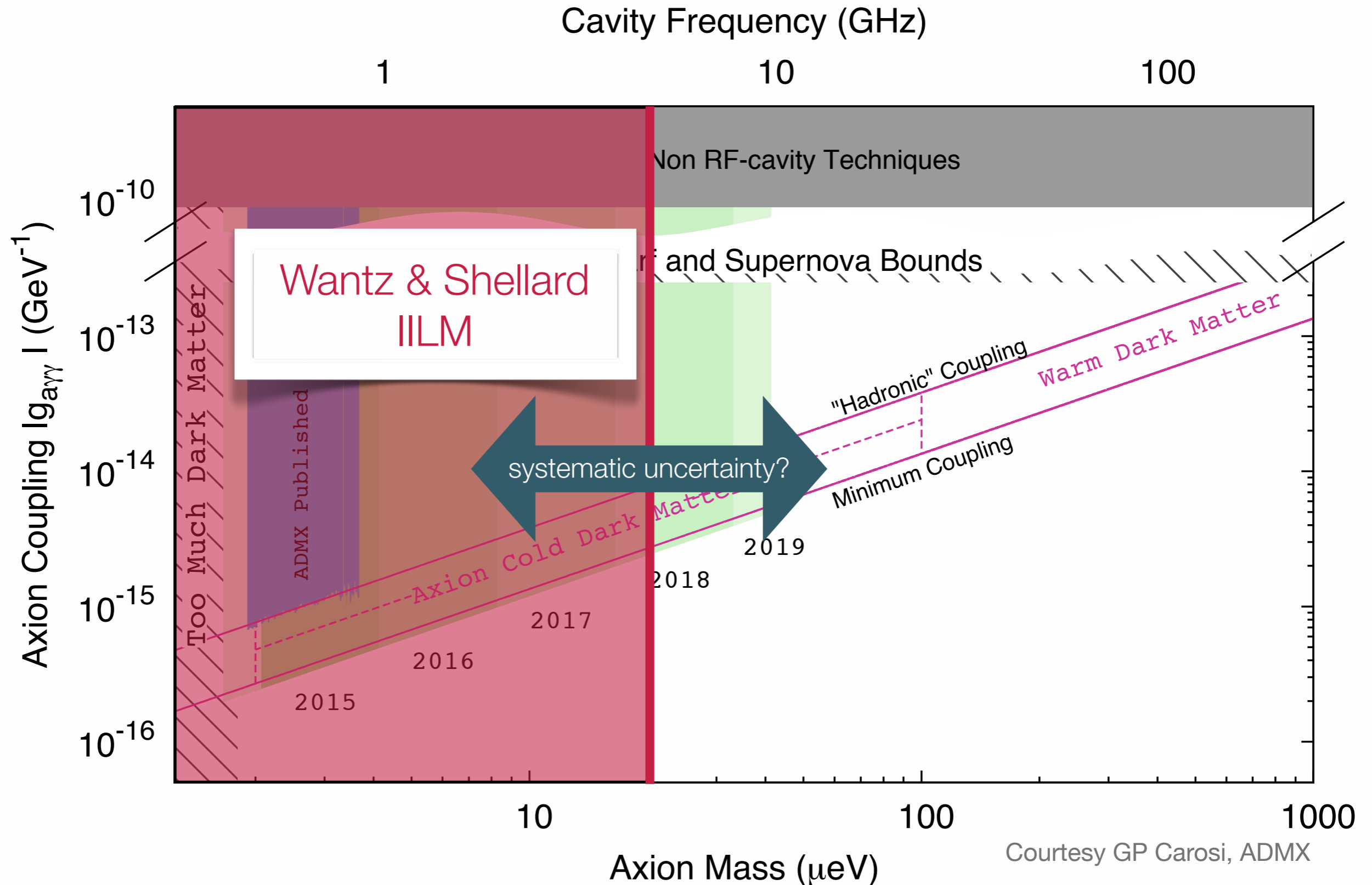
Courtesy GP Carosi, ADMX

$$\Omega_a \leq \Omega_{\text{CDM}}$$

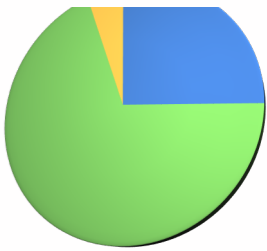


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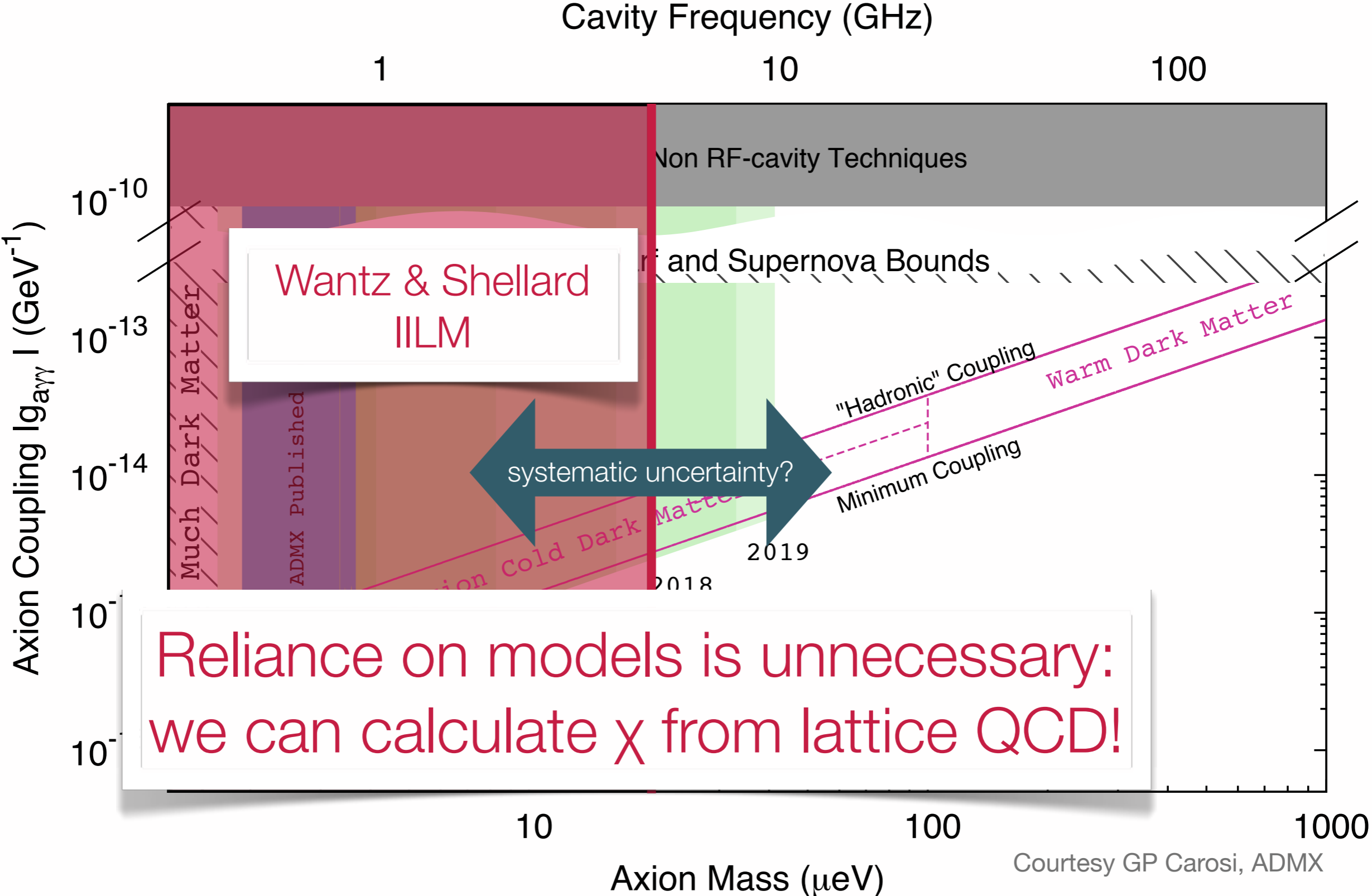


$$\Omega_a \leq \Omega_{\text{CDM}}$$



Current Axion Constraints

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CAVEAT

We study pure Yang-Mills, and not yet full QCD.

- Dramatically more efficient algorithms enable huge statistics and volumes, shorter autocorrelation times.
- T_c is ~ 284 MeV, compared to 154 MeV in QCD.
- High temperature tends to suppress quark loops.
 - What counts as high temperature?
 - Unclear if this holds true for topological observables.

But: for getting a lower bound on m_a it's not so bad!

Overview of Lattice Ensembles & Measurements

Berkowitz, Buchhoff, and Rinaldi, arXiv:1505.07455

- SU(3) YM with Wilson plaquette action
- T between 1.2 and 2.5
- N_σ between 48 and 144 (larger at higher T)
- N_τ either 6 or 8
- Between 14000 and 52000 measurements
 - Combined hot & cold starts
 - Cut of 2000 cfg.s for thermalization
 - 10 compound sweeps of 1 heatbath step and 8 over-relaxation steps

$$\frac{1}{32\pi^2} \sum_x \epsilon^{\mu\nu\rho\sigma} \square_{\mu\nu} \square_{\rho\sigma}$$

$Q_{\mathbb{R}}$

raw measurement

$Q_{\mathbb{Z}}$

naïve rounding

Q_a

artifact corrected

Lucini & Teper, hep-lat/0103027

Q_f

globally fit
del Debbio *et al.*, hep-th/0204125

Q_{OV}

overlap

Q_{WF}

Wilson Flow

Overview of Lattice Ensembles & Measurements

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del Debbio *et al.*, hep-th/0204125

Essentially no
discretization or finite
volume corrections

Overview of Lattice Ensembles & Measurements

Berkowitz, Buchhoff, and Rinaldi, arXiv:1505.07455

T/T_c	β	$a\sqrt{\sigma}$	N_τ	N_σ	N_{meas}	central value $\chi^{1/4}/T_c \pm \delta\chi^{1/4}/T_c$ statistical error for							
						χ_R		χ_Z		χ_a		χ_f	
1.2	6.001	0.2161	6	64	14000	0.3880	0.0012	0.3814	0.0012	0.3871	0.0012	0.4192	0.0013
1.31	6.053	0.1979	6	48	15600	0.3495	0.0009	0.3130	0.0009	0.3392	0.0010	0.3691	0.0011
				64	36000	0.3424	0.0006	0.3358	0.0006	0.3402	0.0007	0.3703	0.0007
				80	14000	0.3426	0.0010	0.3389	0.0010	0.3416	0.0010	0.3735	0.0011
	6.242	0.1484	8	64	33998	0.3634	0.0010	0.3493	0.0010	0.3520	0.0010	0.3687	0.0010
				96	14000	0.3556	0.0015	0.3533	0.0014	0.3537	0.0015	0.3703	0.0015
1.4	6.095	0.1852	6	64	54000	0.3153	0.0005	0.3077	0.0005	0.3095	0.0005	0.3370	0.0005
1.5	6.139	0.1729	6	64	54000	0.2928	0.0005	0.2833	0.0005	0.2814	0.0005	0.3068	0.0005
1.6	6.182	0.1621	6	64	53998	0.2721	0.0005	0.2587	0.0005	0.2568	0.0005	0.2799	0.0005
1.7	6.223	0.1525	6	64	24000	0.2536	0.0008	0.2330	0.0008	0.2369	0.0008	0.2585	0.0008
1.8	6.263	0.1441	6	64	24000	0.2343	0.0008	0.2005	0.0009	0.2178	0.0008	0.2368	0.0008
				80	32000	0.2320	0.0006	0.2262	0.0006	0.2185	0.0006	0.2368	0.0006
				96	14000	0.2306	0.0016	0.2170	0.0017	0.2236	0.0015	0.2312	0.0016
1.9	6.301	0.1365	6	64	24000	0.2175	0.0009	0.1672	0.0011	0.2019	0.0008	0.2190	0.0009
				80	34000	0.2164	0.0006	0.2095	0.0006	0.2026	0.0006	0.2189	0.0006
1.99	6.550	0.0973	8	64	14795	0.2013	0.0034	0.1800	0.0036	0.1986	0.0029	0.2013	0.0034
2.0	6.338	0.1297	6	48	15600	0.2040	0.0018	0.1292	0.0027	0.1898	0.0016	0.2042	0.0018
				64	25598	0.2032	0.0010	0.1390	0.0014	0.1893	0.0009	0.2041	0.0010
				80	26000	0.2014	0.0008	0.1920	0.0008	0.1888	0.0007	0.2030	0.0008
				96	14000	0.2004	0.0008	0.1961	0.0008	0.1900	0.0008	0.2038	0.0009
2.1	6.373	0.1235	6	80	24000	0.1880	0.0009	0.1749	0.0009	0.1774	0.0008	0.1889	0.0009
2.5	6.502	0.1037	6	128	14000	0.1497	0.0010	0.1479	0.0010	0.1494	0.0008	0.1492	0.0010
				144	15797	0.1525	0.0008	0.1513	0.0008	0.1495	0.0006	0.1518	0.0008

Q_R

raw measurement

Q_Z

naïve rounding

Q_a

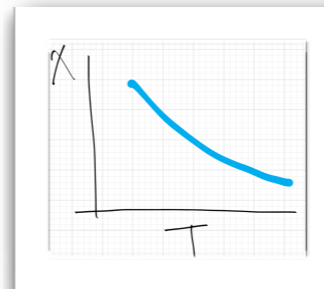
artifact corrected
Lucini & Teper, hep-lat/0

Q_f

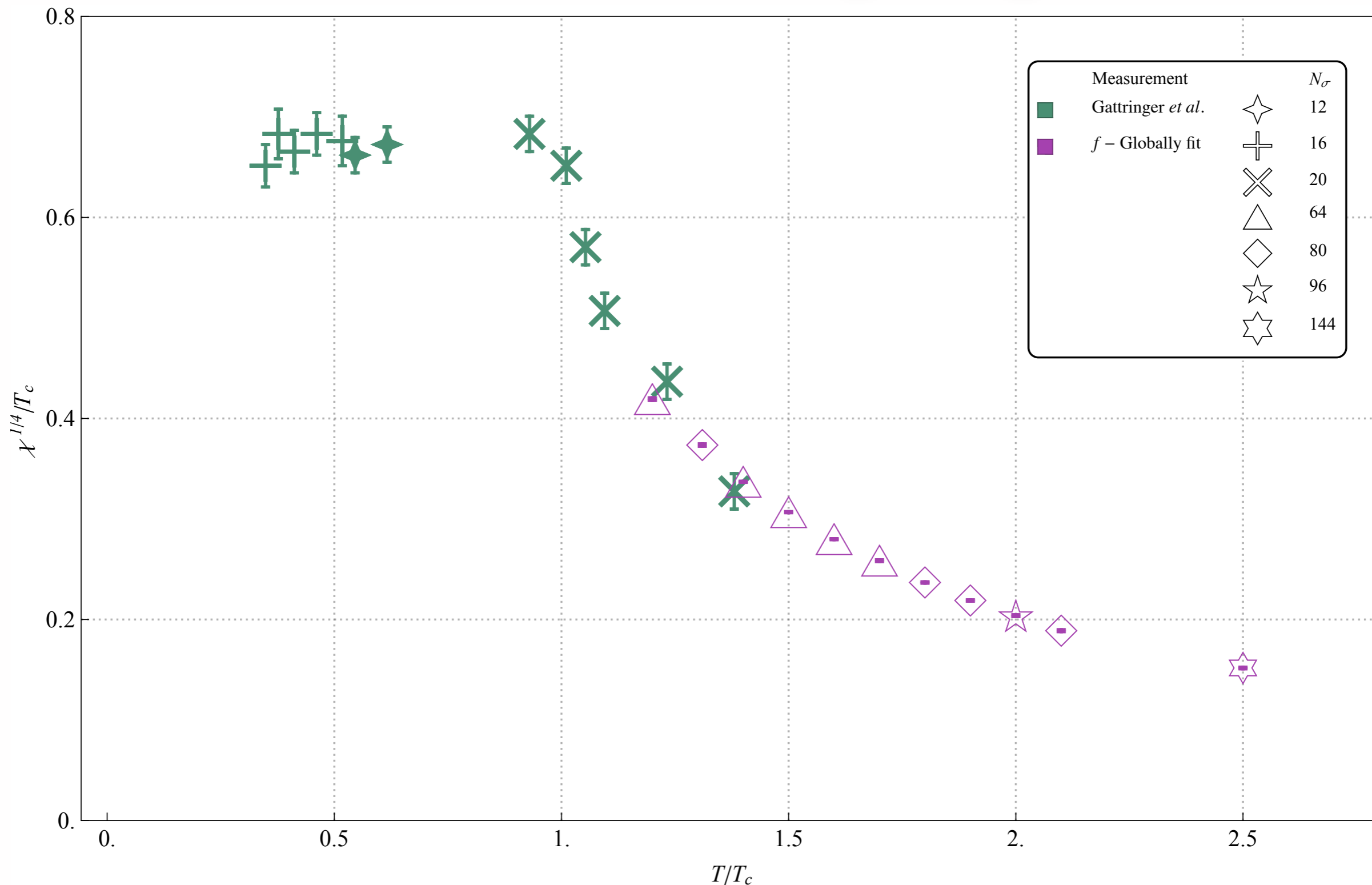
globally fit
del Debbio *et al.*, hep-th/

Best Lattice Results

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455

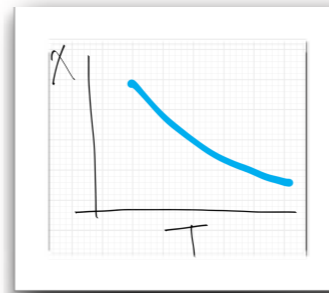


Pure glue measurements show χ vanishes as $T \rightarrow \infty$

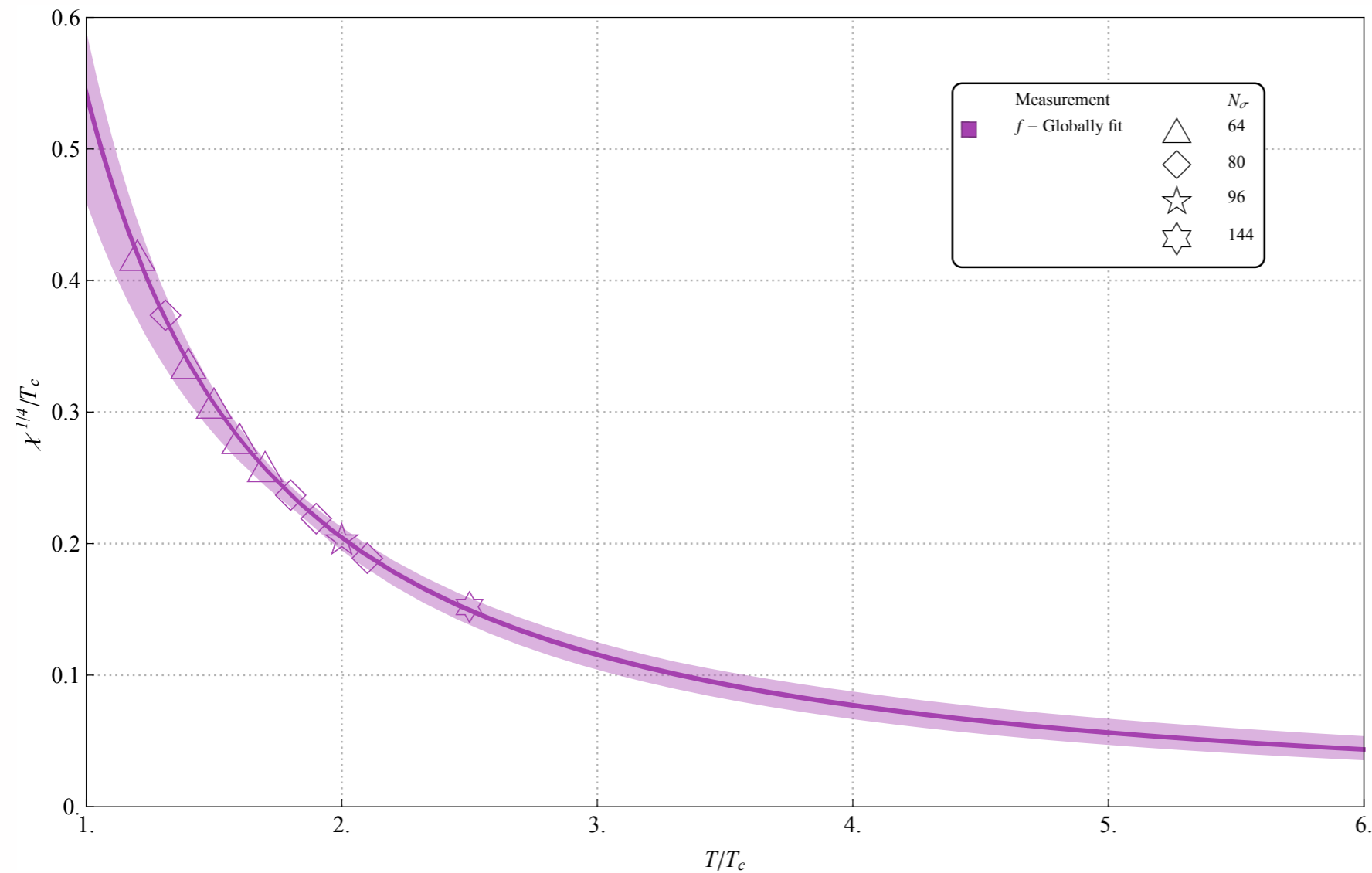


DIGM Best Fit & Extrapolation

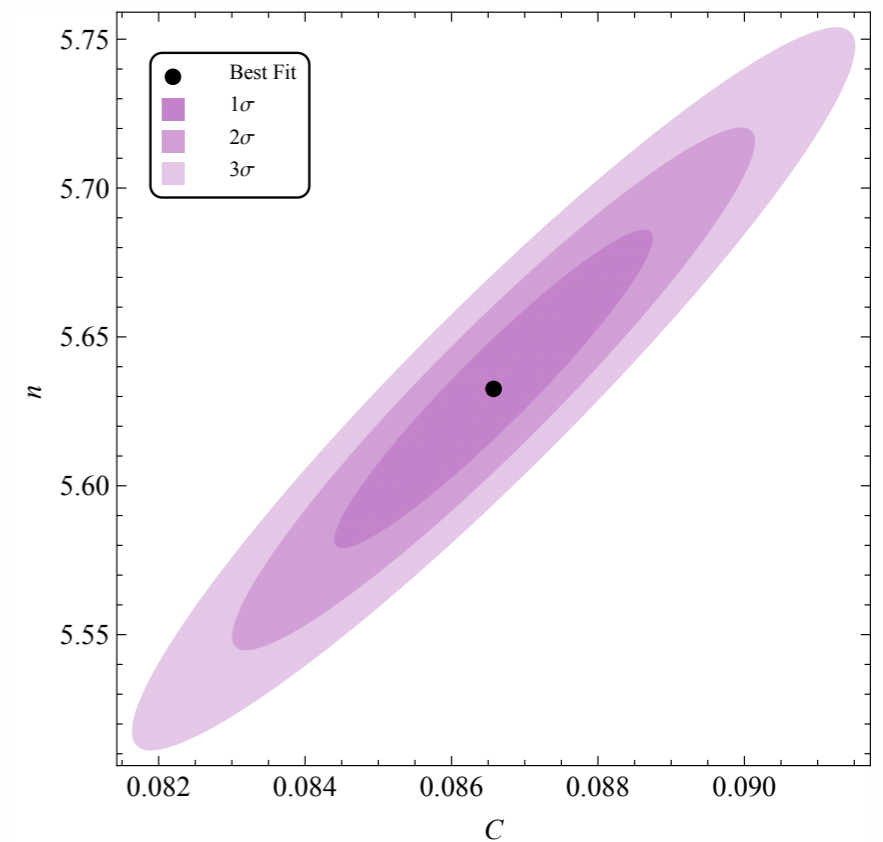
Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



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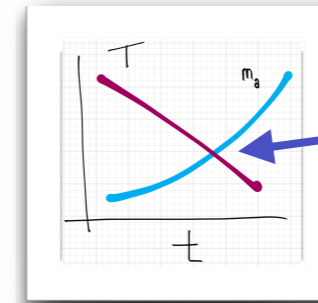
$$\frac{\chi}{T_c^4} = \frac{C}{(T/T_c)^n}$$



	C	n
Best Fit	0.0865	5.63
Covariance Matrix		
C	$2. \times 10^{-6}$	$5. \times 10^{-5}$
n	$5. \times 10^{-5}$	0.0012

Axion Production Ceases

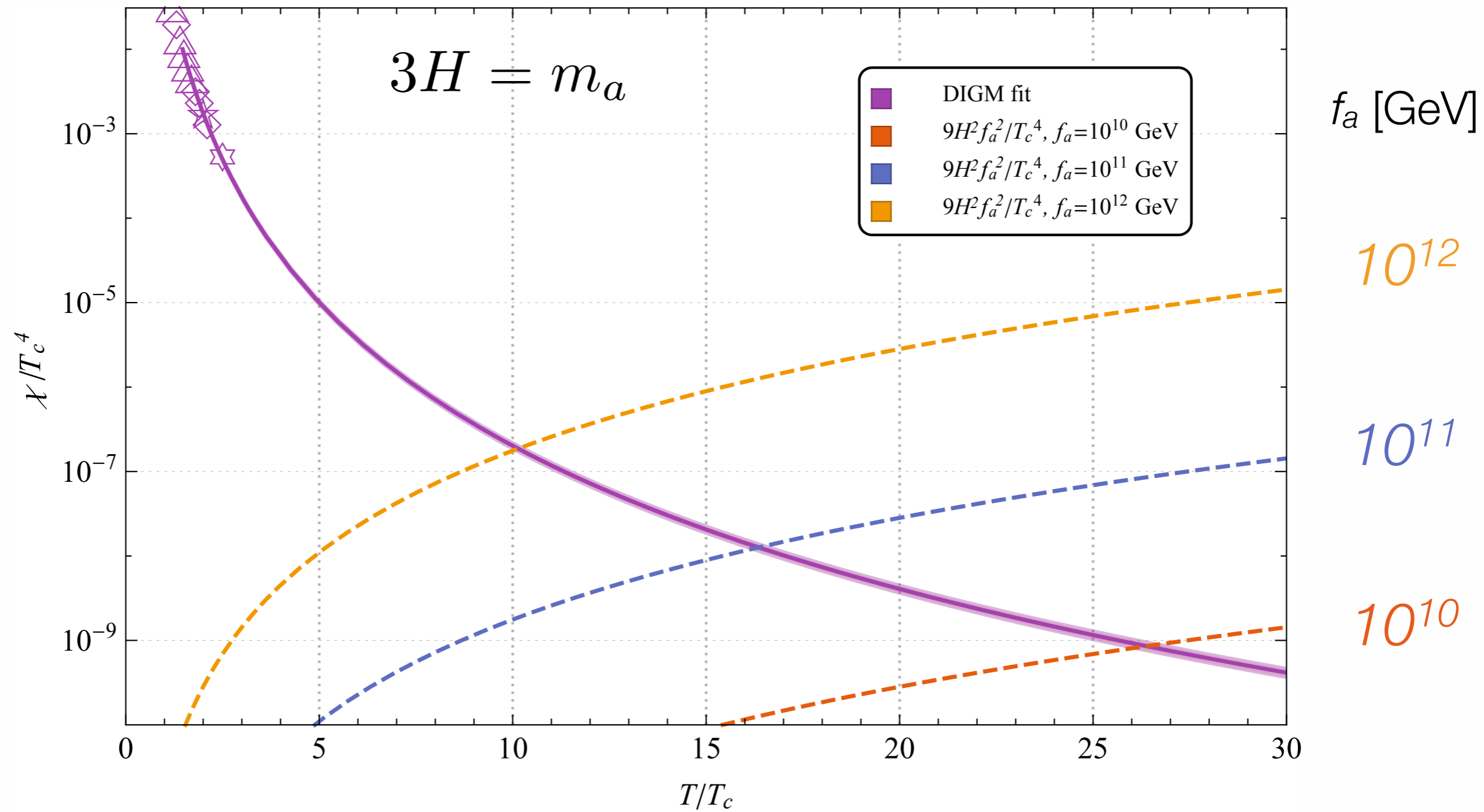
Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



Axions production stops when

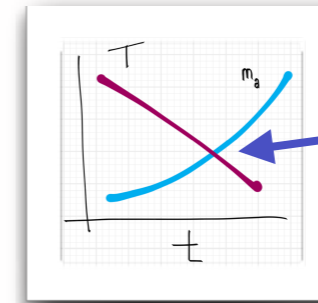
$$3H \sim m_a$$

$T_1 \approx 5.5 T_c$ from models



Axion Production Ceases

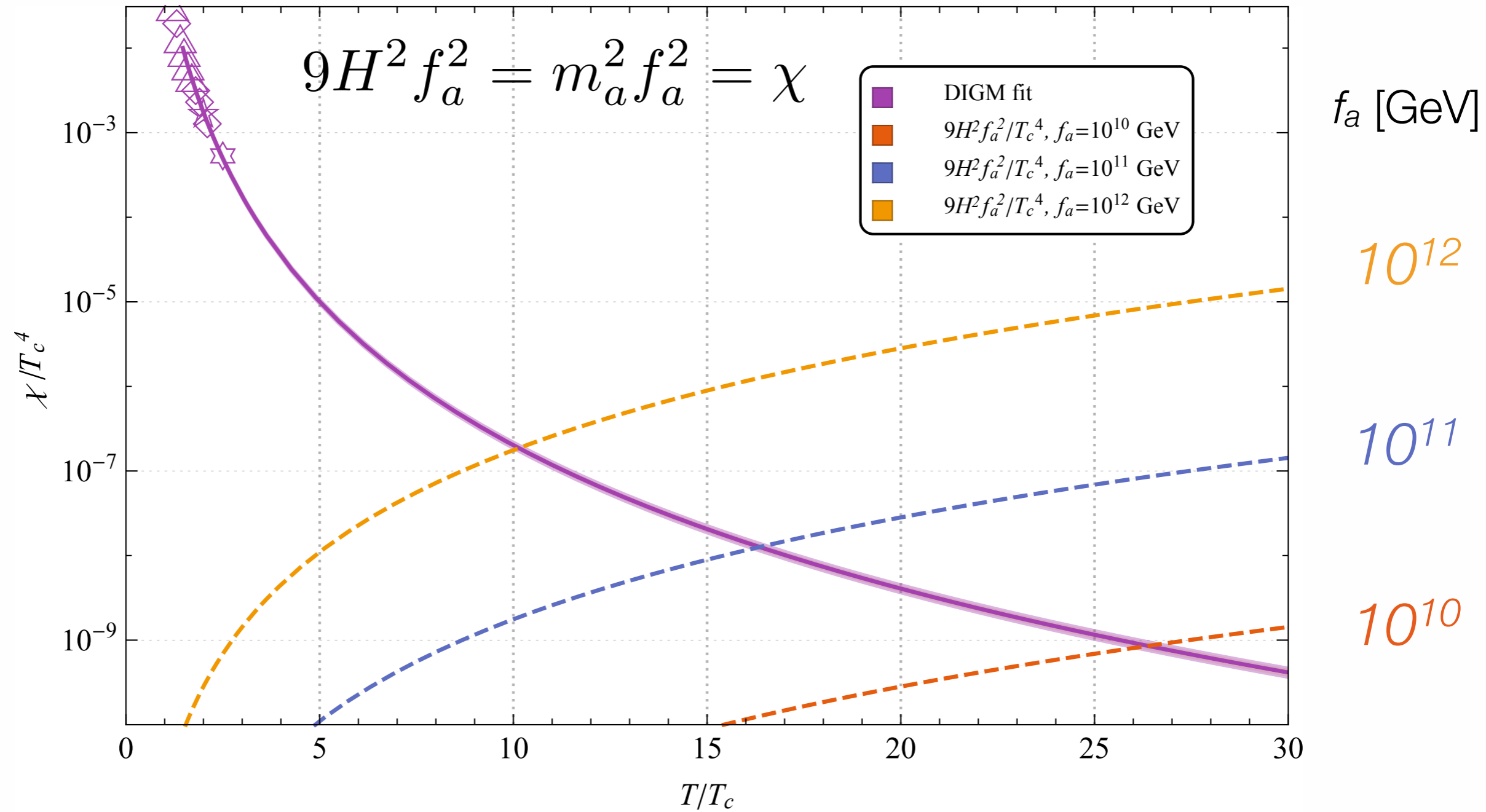
Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



Axions production stops when

$$3H \sim m_a$$

$T_1 \approx 5.5 T_c$ from models



$$H^2 = \frac{\pi^2}{90} \frac{1}{m_P^2} g_{*R}(T) T^4$$

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

Cosmological EOM: $\rho(T_1) = \frac{1}{2}\theta_1^2 m_a^2 f_a^2 = \frac{1}{2}\theta_1^2 \chi(T_1)$

$$\rho(T_\gamma) = \frac{1}{2}\theta_1^2 \sqrt{\chi(T_\gamma)\chi(T_1)} \left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

$3H \sim m_a$

χ PT

Strong Dynamics

Cosmology

Initial conditions

Axion Density $\frac{\rho(t)R^3}{m_a(t)} = \#$ axions in a fixed comoving volume

Cosmological EOM: $\rho(T_1) = \frac{1}{2}\theta_1^2 m_a^2 f_a^2 = \frac{1}{2}\theta_1^2 \chi(T_1)$

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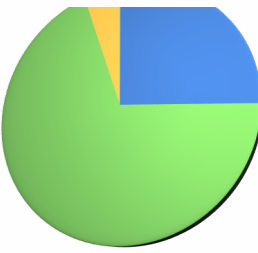
χ PT

Lattice!

Cosmology

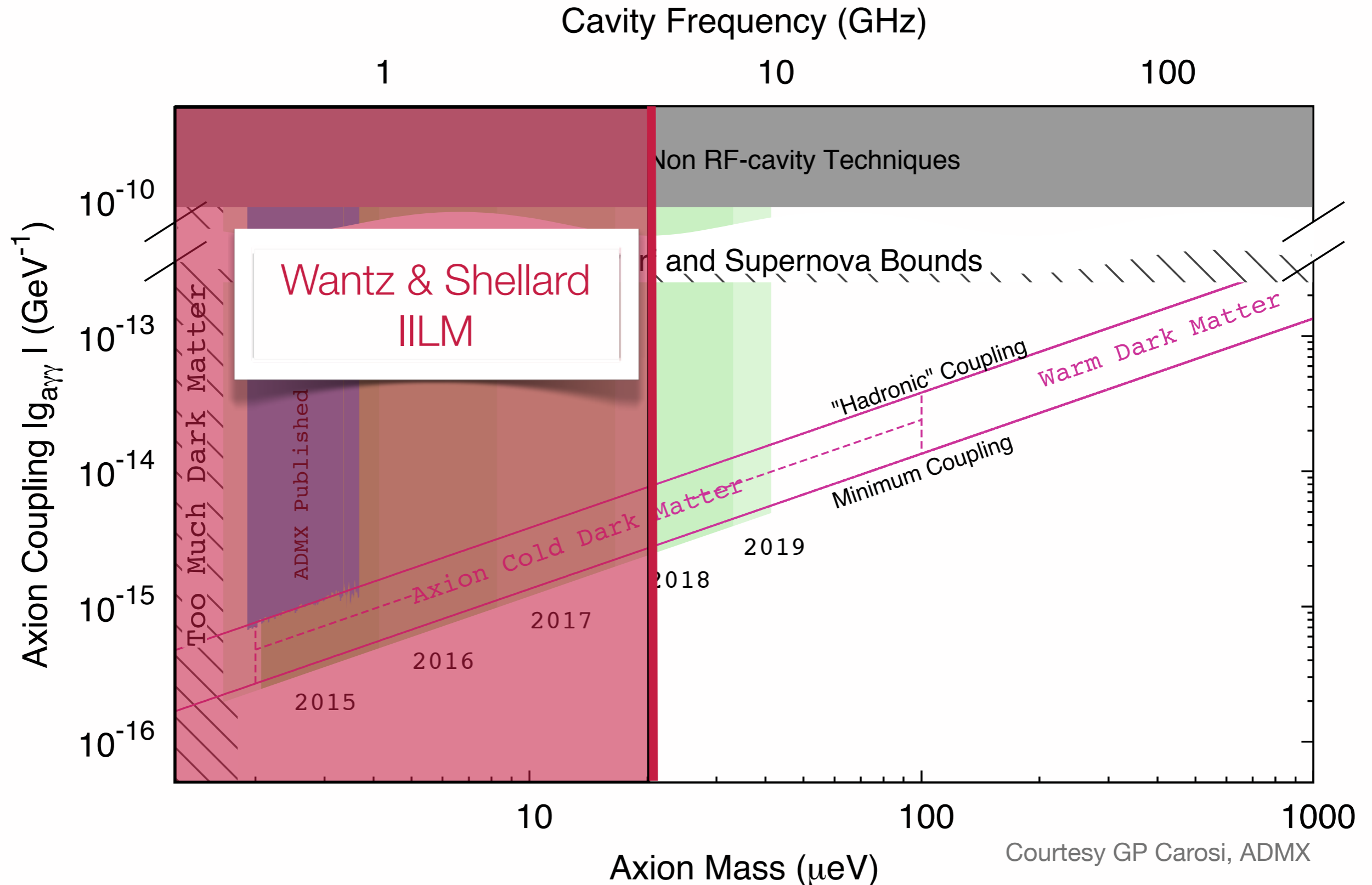
Initial conditions

$$\Omega_a \leq \Omega_{\text{CDM}}$$

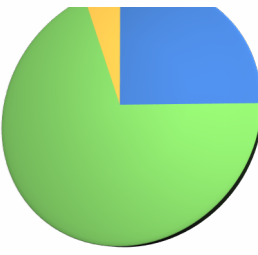


The Over-Closure Bound As It Stands Today

Berkowitz, Buchhoff, and Rinaldi, arXiv:1505.07455

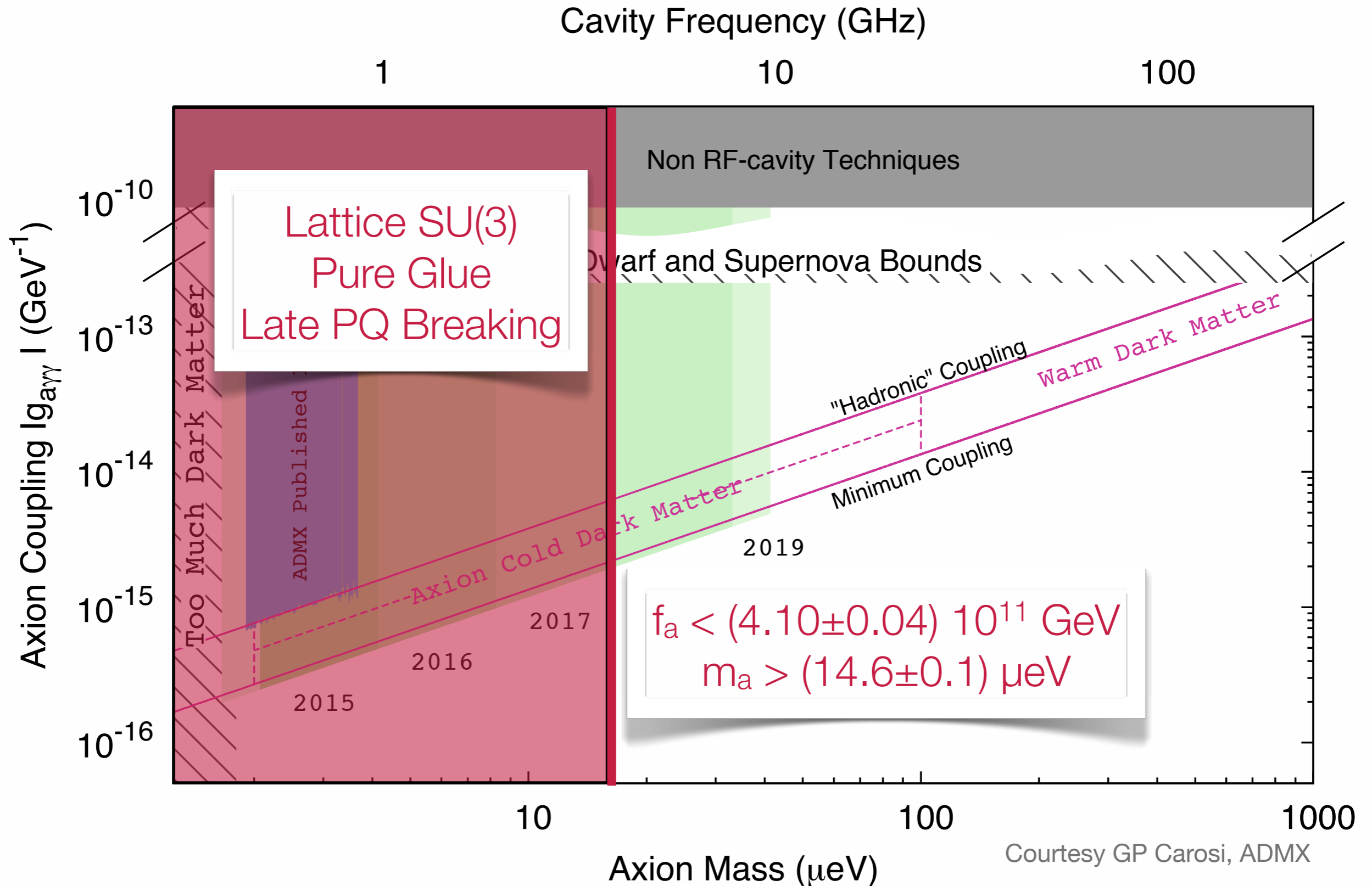


$$\Omega_a \leq \Omega_{\text{CDM}}$$

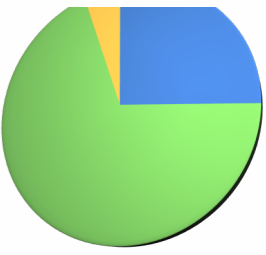


The Over-Closure Bound As It Stands Today

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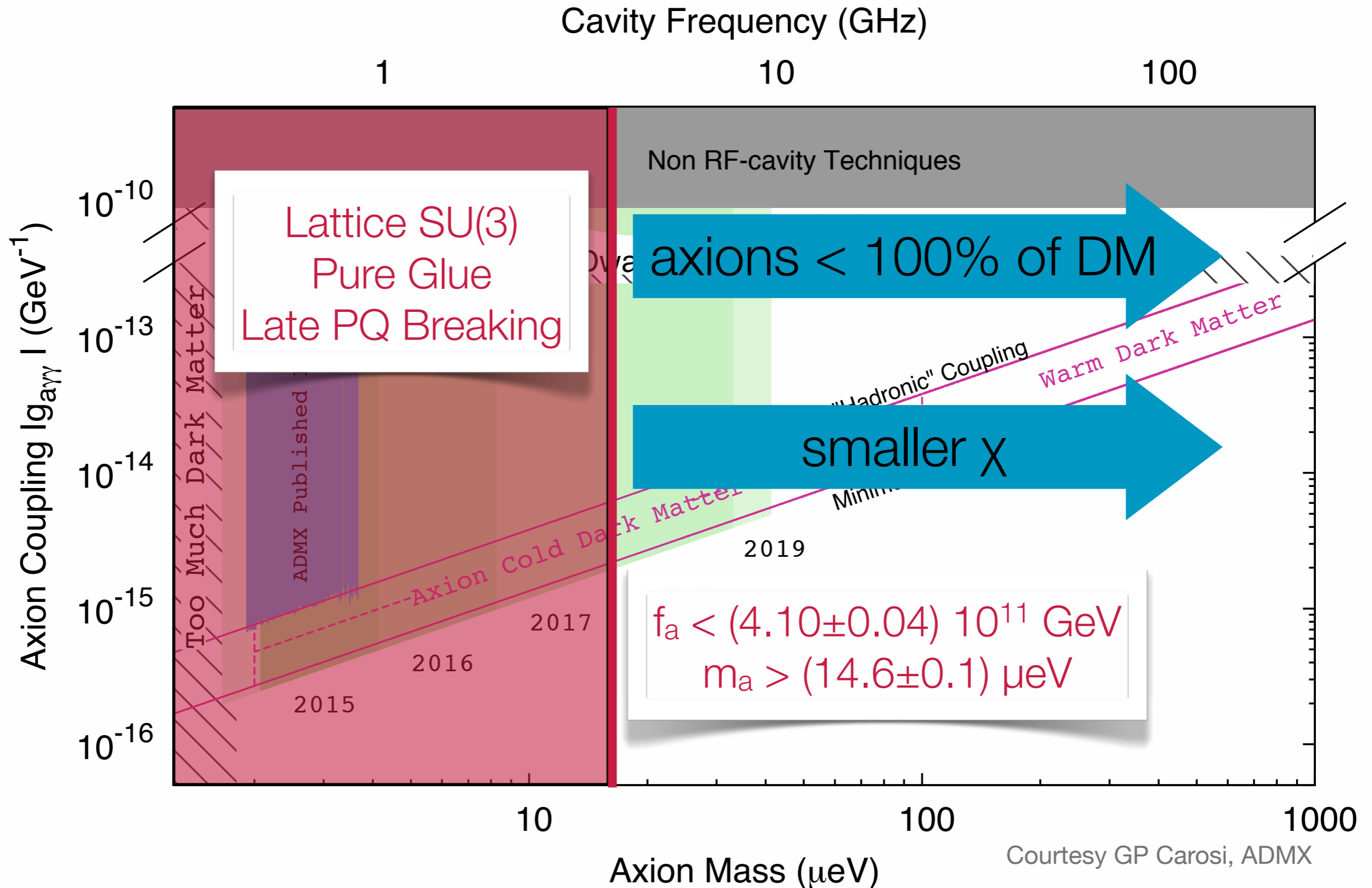


$$\Omega_a \leq \Omega_{\text{CDM}}$$



The Over-Closure Bound As It Stands Today

Berkowitz, Buchhoff, and Rinaldi, arXiv:1505.07455



Conclusions & Outlook

- PQ symmetry:
 - cleans up the Strong CP problem
 - provides a plausible, largely unconstrained DM candidate: the axion.
- Axion searches will search large swaths of interesting parameter space soon.
- Power law (DIGM-inspired) fits outstandingly to pure glue at high temperature.



The Economist, 19 Dec 2006

Lattice QCD can provide important nonperturbative input for calculating Ω_a

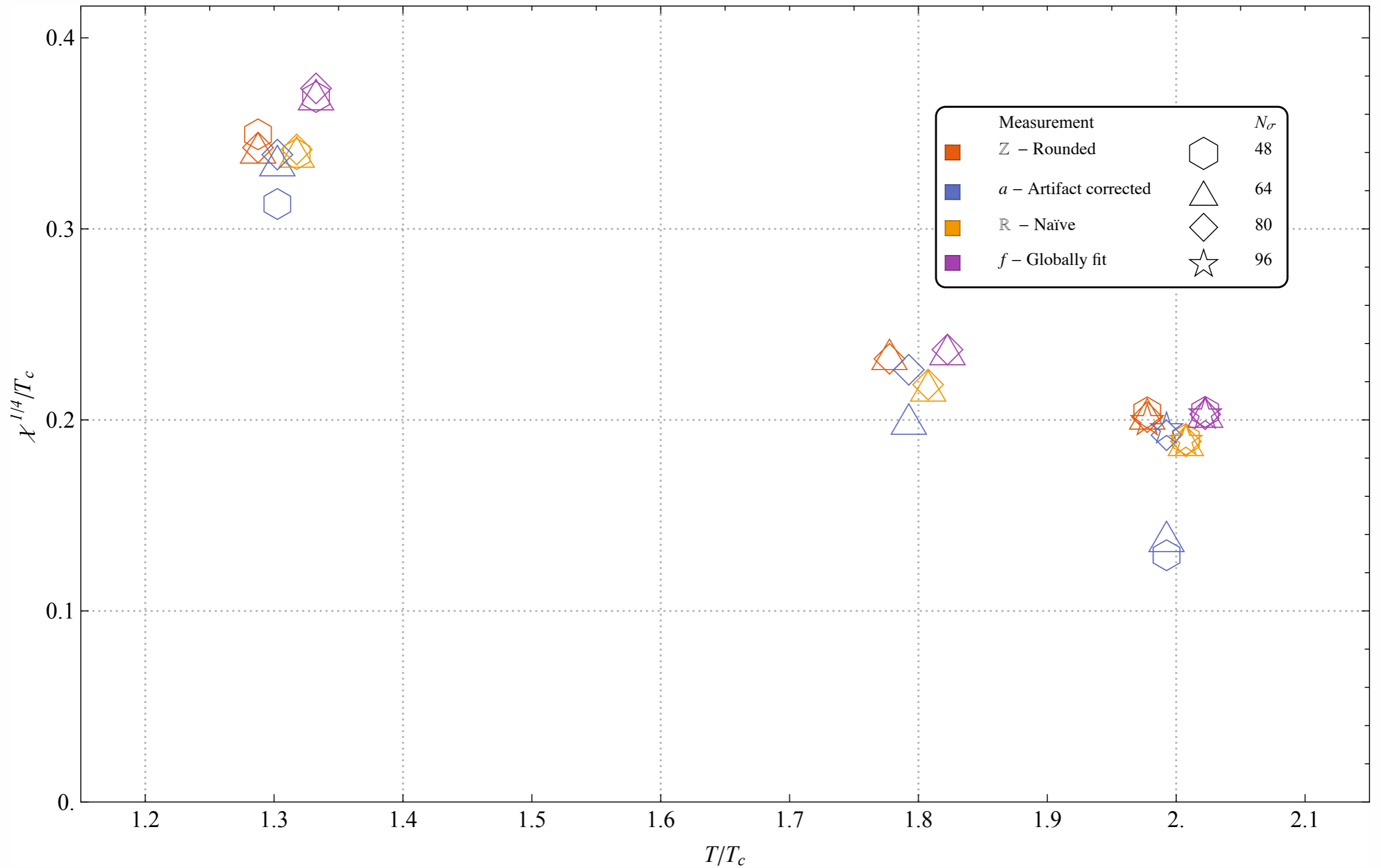
Future Steps

- Measure higher moments? May be able to get χ_4, χ_6 .
T=0: Cé, Consonni, Engle & Giusti, arXiv:1506.06052
- Incorporate quarks
- Move to Wilson Flow definition
- Move to anisotropic lattices to alleviate finite-volume effects at high T.
- Fixed topology methods / open boundary conditions at high T.
Aoki *et al.*, arXiv:0707.0396v2 Lüscher & Schæfer, arXiv:1105.4749
- Finite θ :
 - Imaginary- θ has no sign problem
 - Real, finite θ may be amenable to Langevin methods

Backup Slides

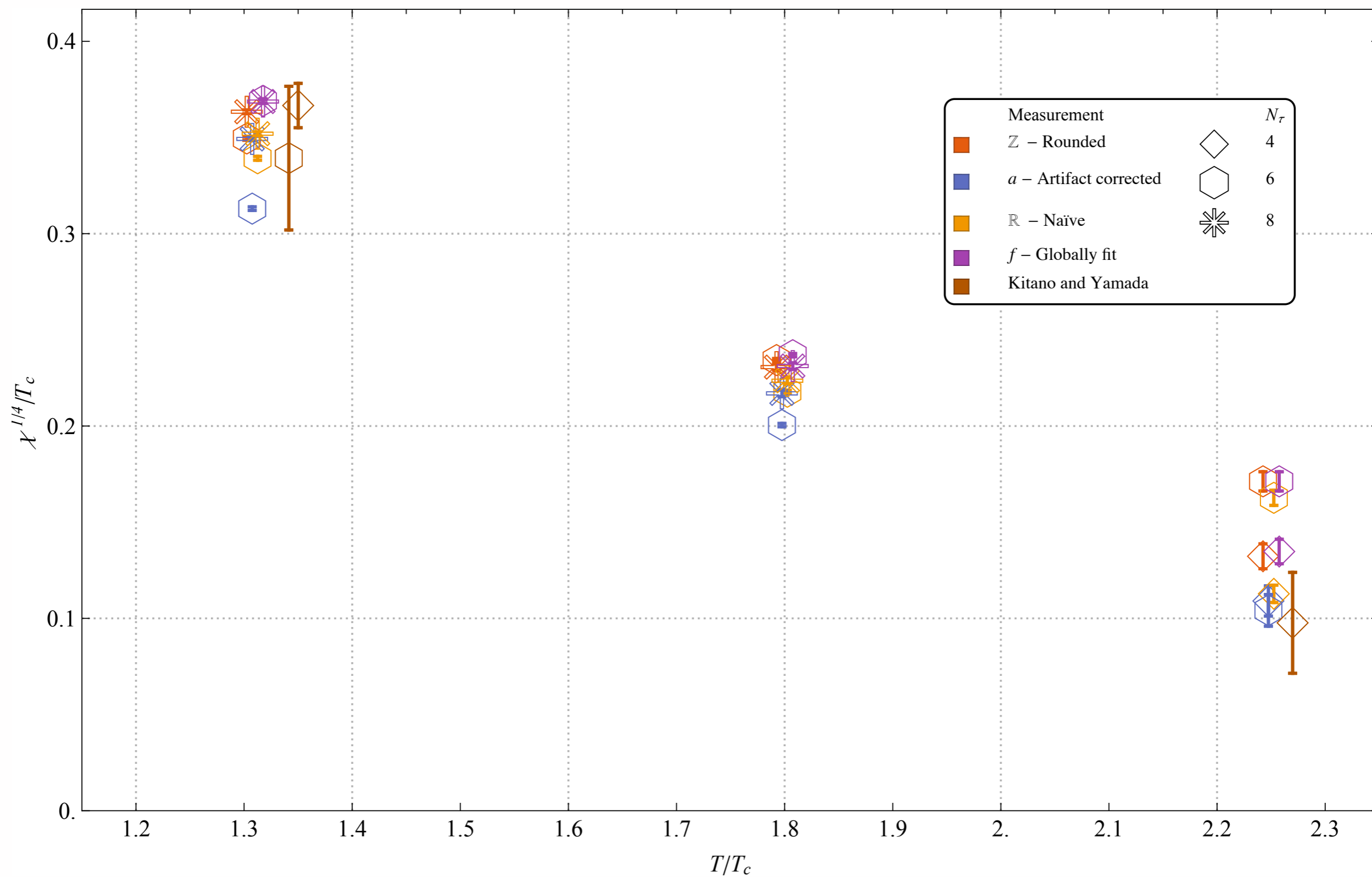
Finite Volume Effects

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455

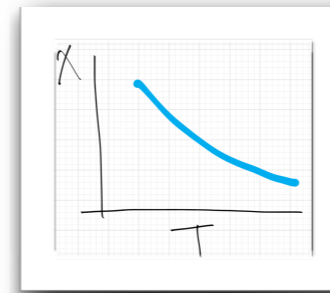


Discretization Effects

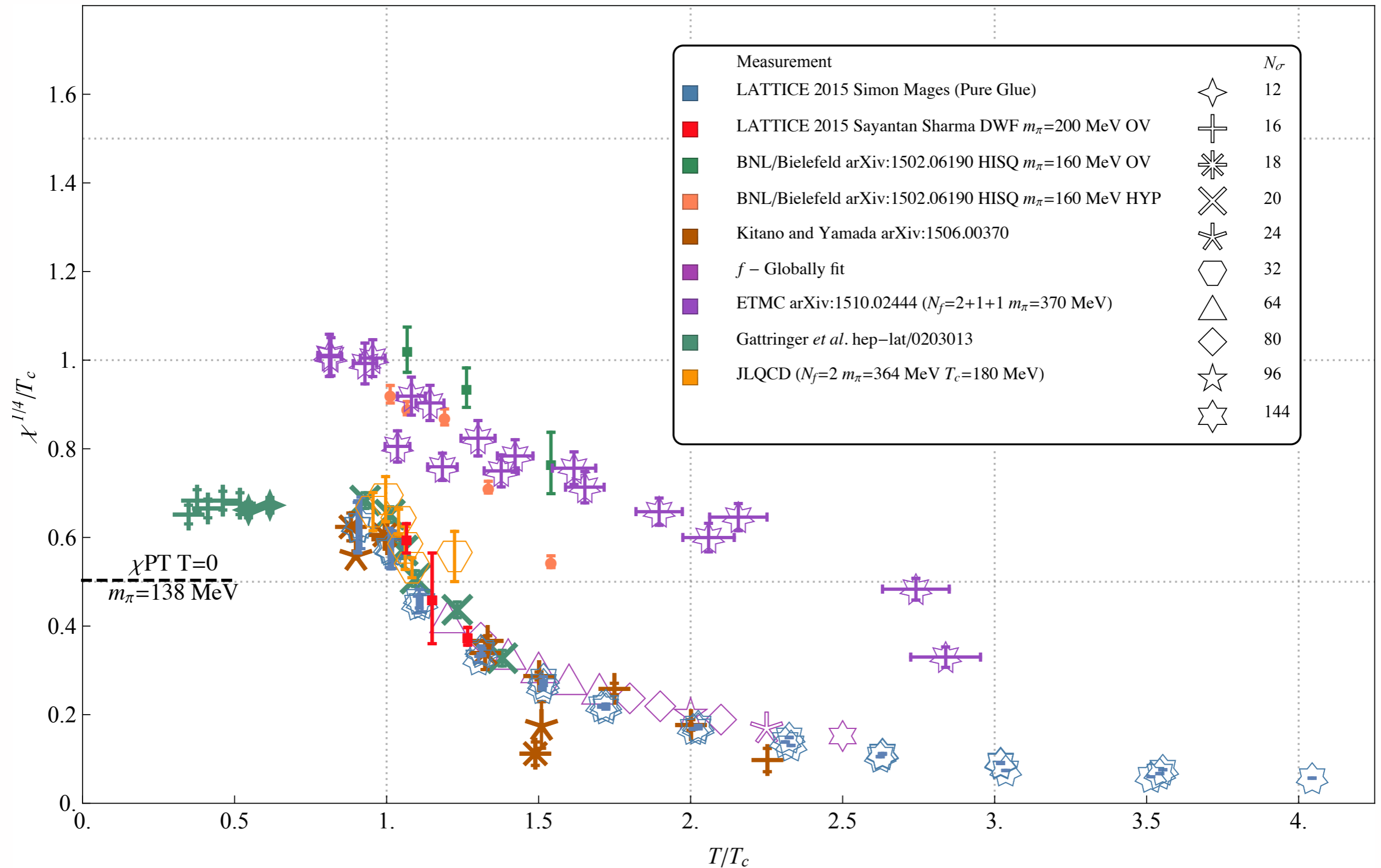
Berkowitz, Buchhoff, and Rinaldi (arXiv:1505.07455), Kitano & Yamada (arXiv:1506.00370)



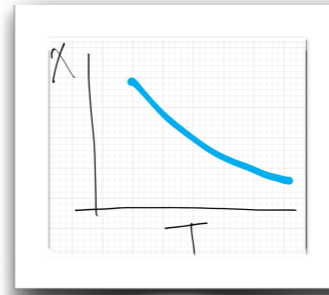
Comparisons



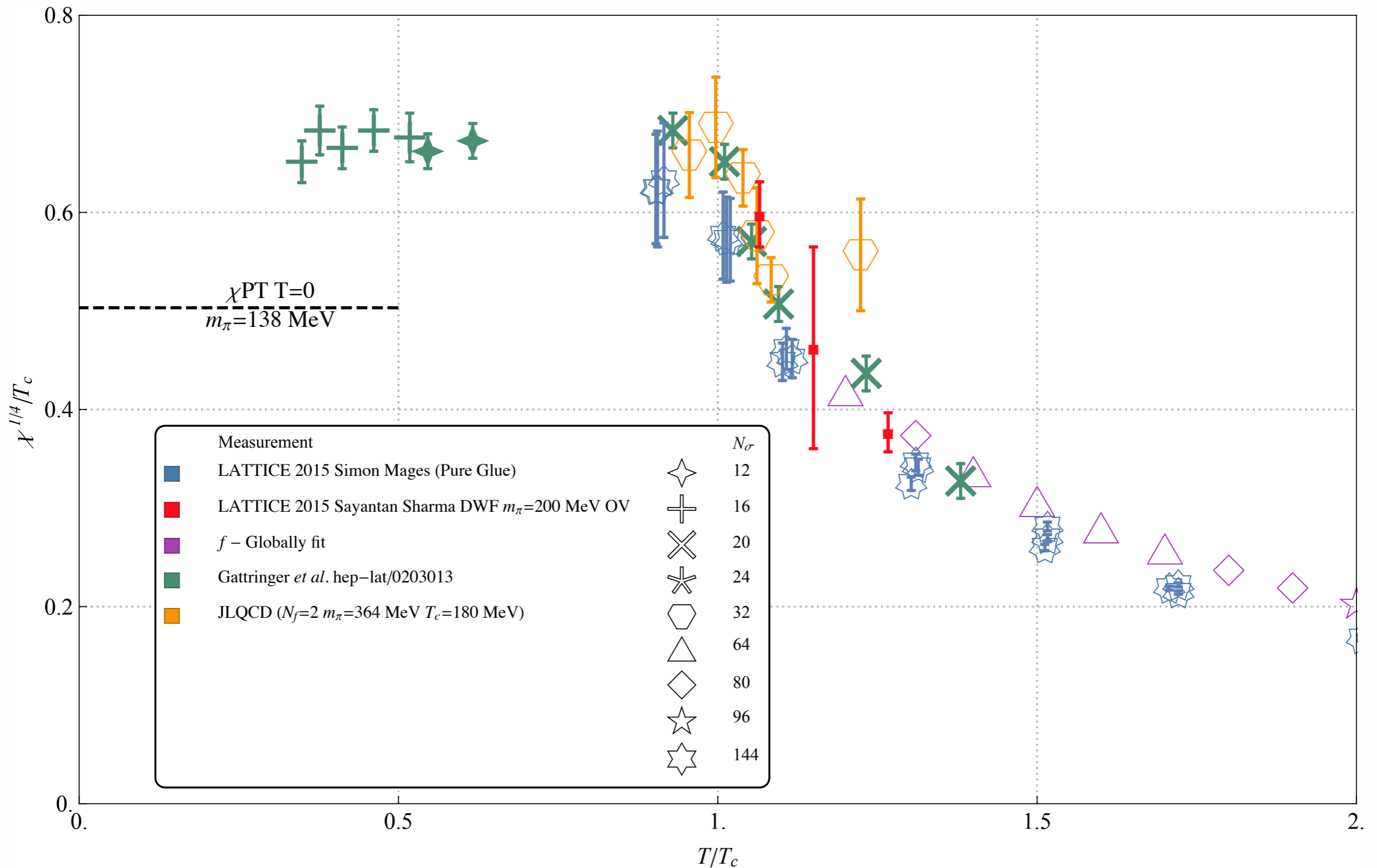
Pure glue measurements show χ vanishes as $T \rightarrow \infty$



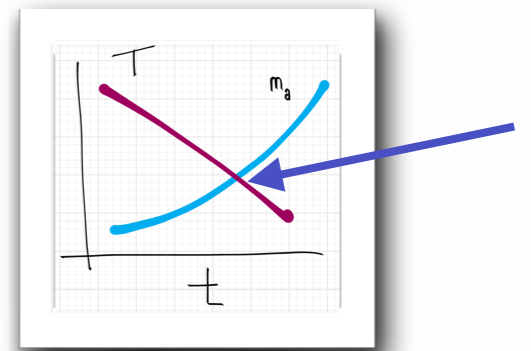
Comparisons



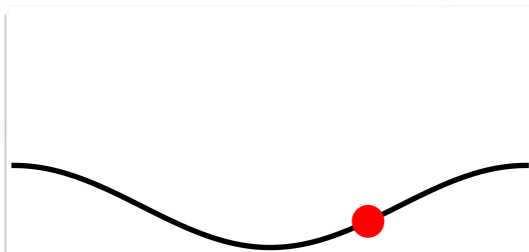
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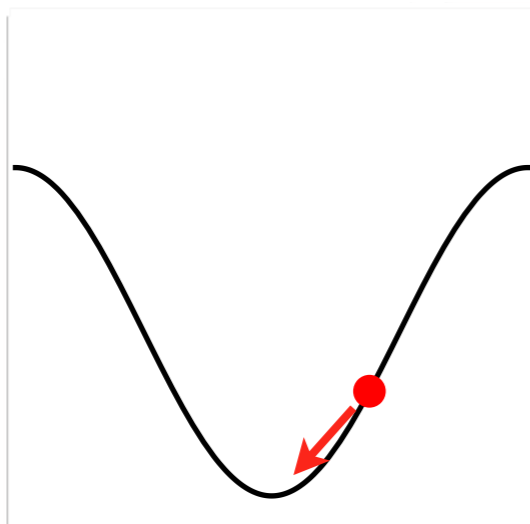
The Misalignment Mechanism



At high temperature, the axion potential is flat.



At $T \sim 1$ GeV, potential develops and axion mass begins to grow.



Axion begins to oscillate when Compton λ fits inside the universe

$$3H \sim m_a$$