Baryon Number Violation Overview

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INT Workshop

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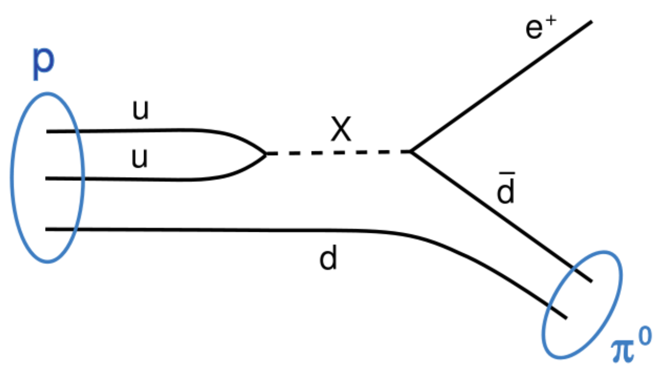
Outline

- Baryon Number Violation: Motivations
- Nucleon decay: Theory and Models $(|\Delta B| = 1)$
- Neutron–antineutron oscillations $(|\Delta B| = 2)$
- Connection with Baryogenesis at high scale

Stability of Matter

Proton stability is not guaranteed by any fundamental symmetry (unlike electron, lightest neutrino or photon)

$$p \longrightarrow e^{+}\pi^{0}$$



Hermann Weyl postulated in 1929 Baryon Number symmetry

Intensely studied by many experiments

$$\tau(p \rightarrow e^{+}\pi^{0}) > 1.3 \times 10^{34} \text{ yrs}$$

SuperKamiokande

Why do we expect proton would decay?

Baryon Number postulated as a symmetry of Nature to stabilize matter

Weyl (1929), Stueckelberg (1939), Wigner (1949)

Unlike electric charge, which guarantees stability of electron, B is not a "fundamental" symmetry

Weak interactions violate B non-perturbatively

't Hooft (1977)

Quantum gravity suspected to violate all global symmetries such as B

Baryon number violation essential for creation of matter asymmetry of the Universe

Sakharov (1967)

Most extensions of Standard Model, notably Grand Unified Theories, lead to baryon number violation

Unification of Forces and Matter

Electromagnetic, weak and strong forces share identical structure: all belong to gauge theories with unitary symmetry

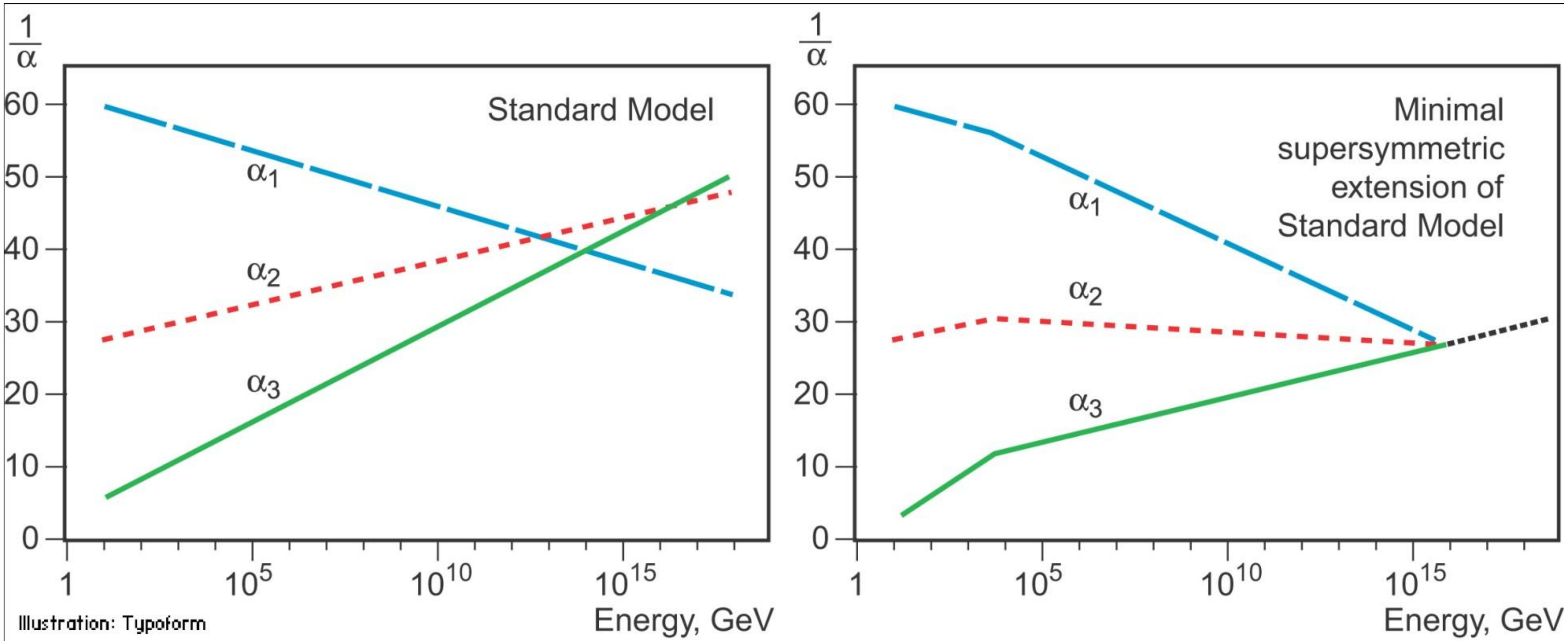
The high energy behavior of these theories support unification idea

Ordinary matter – quarks and leptons – fit neatly within multiplets of the unified symmetry

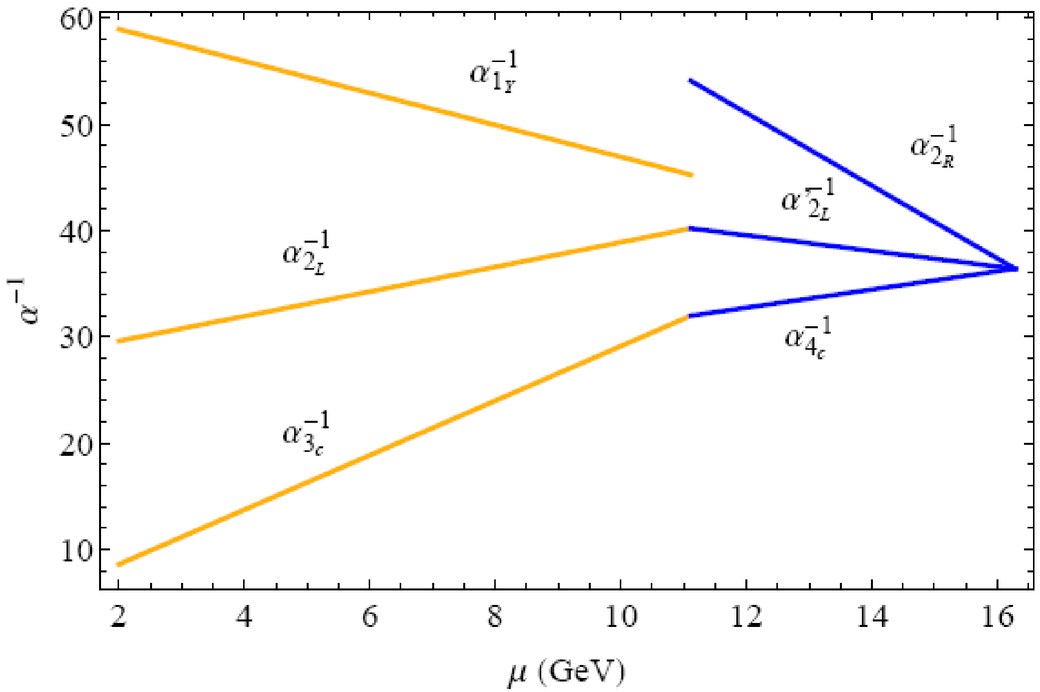
Unified theories are more predictive, many predictions agree with observations

Nucleon decay is the missing link; its discovery would be monumental

Evolution of gauge couplings with energy



Gauge coupling unification without supersymmetry



Intermediate Pati-Salam symmetry: $SU(2)_L \times SU(2)_R \times SU(4)_c$

May be identified as the Peccei-Quinn symmetry breaking scale

From Altarelli, Meloni (2013)

More Hints in favor of Unification

- Electric charge quantization
  - $Q_p = -Q_e$ to better than 1 part in 10^{21}
- Miraculous cancellation of anomalies
- Quantum numbers of quarks and leptons
- Existence of ν_R and thus neutrino mass
- Unification of gauge couplings with low energy SUSY
- $b - \tau$ unification
- Baryon asymmetry of the universe

Unifying Forces and Matter

First successful attempt by Pati and Salam (1973)

Based on $SU(4)_c \times SU(2)_L \times SU(2)_R$ gauge symmetry

$$\psi_L = \begin{pmatrix} u_1 & u_2 & u_3 & e \\ d_1 & d_2 & d_3 & \nu \end{pmatrix}_L , \quad \psi_R = \begin{pmatrix} u_1 & u_2 & u_3 & e \\ d_1 & d_2 & d_3 & \nu \end{pmatrix}_R$$

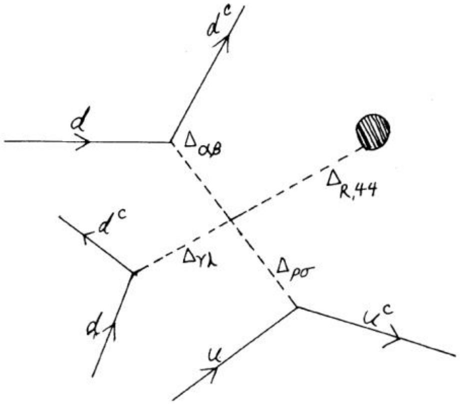
Lepton number identified as fourth color

Scale of $SU(4)_c$ breaking > 2300 TeV $(K_L \to \mu e)$

Baryon number violation occurs via scalar exchange with $|\Delta B| = 2$ selection rule

$n - \overline{n}$ oscillation occurs without proton decay

$$\mathcal{L}_{\rm eff} = \frac{uddudd}{\Lambda^5}$$



Marshak, Mohapatra (1980)

Unification in SU(5)

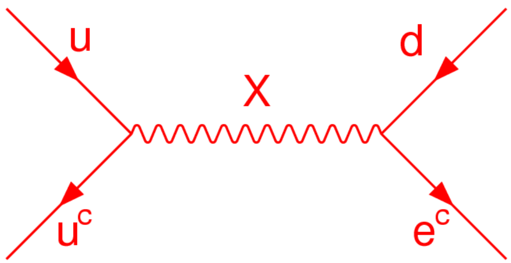
More complete unification of forces and matter discovered in SU(5) by Georgi and Glashow (1974)

$$\mathbf{10}: \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix} \qquad \bar{\mathbf{5}}: (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

Quarks and leptons are unified

Particles unify with antiparticles $\Rightarrow$ Matter is unstable

$p \rightarrow e^+\pi^0$ decay



Matter Fields in SO(10)

$SO(10)$ theories contain Pati-Salam and $SU(5)$ features

All particles and antiparticles are unified in **16**

SO(10)

$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u^c_r : \{+--\ ++\}$	$d^c_r : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u^c_b : \{-+-\ ++\}$	$d^c_b : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u^c_g : \{--+\ ++\}$	$d^c_g : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

Frist 3 spins refer to color, last 2 are weak spins

$$Y = \tfrac{1}{3}\Sigma(C) - \tfrac{1}{2}\Sigma(W)$$

ν^c state crucial for neutrino mass generation

Standard Model

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \tfrac{1}{6})$$

$$u^c = (\, u^c_1 \quad u^c_2 \quad u^c_3 \,) \sim (\bar{3}, 1, -\tfrac{2}{3})$$

$$d^c = (\, d^c_1 \quad d^c_2 \quad d^c_3 \,) \sim (\bar{3}, 1, \tfrac{1}{3})$$

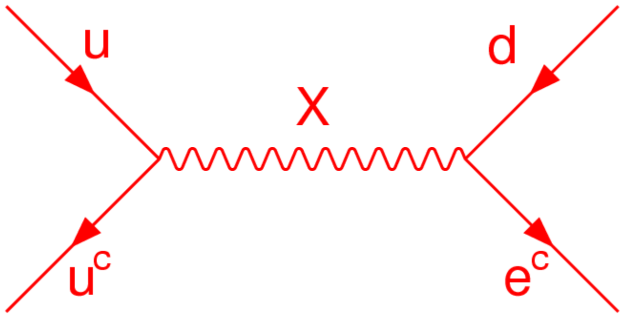
$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, -\tfrac{1}{2})$$

$$e^c \sim (1, 1, +1)$$

$$\nu^c \sim (1, 1, 0)$$

Nucleon Decay in SUSY GUTs

Gauge boson exchange



$$\Gamma^{-1}(p \to e^+\pi^0) = (2.0 \times 10^{35}\ \mathrm{yr}) \times \left(\frac{\alpha_H}{0.01\ \mathrm{GeV}^3}\right)^{-2} \left(\frac{\alpha_G}{1/25}\right)^{-2} \left(\frac{A_R}{2.5}\right)^{-2} \left(\frac{M_X}{10^{16}\ \mathrm{GeV}}\right)^{4}$$

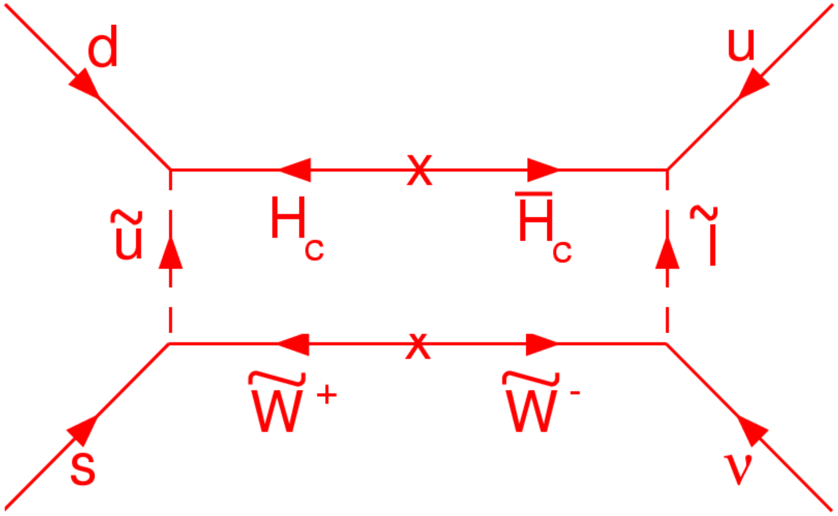
$$(-2\alpha_3^{-1} - 3\alpha_2^{-1} + 3\alpha_Y^{-1})(M_Z) = \frac{1}{2\pi}\left\{36 \ln\left(\frac{M_X}{M_Z}\left(\frac{M_\Sigma}{M_X}\right)^{1/3}\right) + 8 \ln\left(\frac{M_{\mathrm{SUSY}}}{M_Z}\right)\right\}$$

M_Σ: Heavy color octet mass, uncertain: Threshold effect

$\frac{M_\Sigma}{M_X} \leq 1.8$ (perturbation theory)

Hisano, Murayama, Yanagida (1993)
Nath, Perez, Phys. Rept. (2007)

Supersymmetric nucleon decay mode



Sakai, Yanagida (1982)

Weinberg (1982)

$$p \rightarrow \bar{\nu} K^+$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c} M_{SUSY}}]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

Murayama, Pierce (2002)

Proton decay problem in minimal SUSY SU(5)

$SU(5)$ symmetry broken by an adjoint **24** Higgs

Electroweak Higgs doublets are contained in $\mathbf{5} + \bar{\mathbf{5}}$

$\mathbf{5} + \bar{\mathbf{5}}$ Higgs contain color triplet components which mediate proton decay

Gauge coupling unification prefers color triplet mass $< 7 \times 10^{14}$ GeV

$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^+) \simeq 1.2 \cdot 10^{31}\,\text{yrs} \times \left(\frac{0.012\,\text{GeV}^3}{\beta_H}\right)^2 \left(\frac{7}{\bar{A}_S^\alpha}\right)^2 \left(\frac{1.25}{R_L}\right)^2 \times \left(\frac{M_T}{2 \cdot 10^{16}\,\text{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.5\,\text{TeV}}\right)^4 \left(\frac{190\,\text{GeV}}{M_{\tilde{W}}}\right)^2$$

β_H: Hadronic matrix element

Super-Kamiokande Limit: $\tau > 5.9 \times 10^{33}$ yr.

Proton Lifetime in Realistic SUSY SU(5)

Minimal $SU(5)$ mass relations $m_b^0 = m_\tau^0,\ m_s^0 = m_\mu^0,\ m_d^0 = m_e^0$ (at GUT scale) are not consistent with data

Simplest way to fix is to add $5 + \overline{5}$ fermion at GUT scale

$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^+)$	$4 \cdot 10^{33}$ yrs.
$\Gamma^{-1}_{d=5}(n \to \bar{\nu} K^0)$	$2 \cdot 10^{33}$ yrs.
$\Gamma^{-1}_{d=5}(p \to \mu^+ K^0)$	$1.0 \cdot 10^{34}$ yrs.
$\Gamma^{-1}_{d=5}(p \to \mu^+ \pi^0)$	$1.8 \cdot 10^{34}$ yrs.
$\Gamma^{-1}_{d=5}(p \to \bar{\nu} \pi^+)$	$7.3 \cdot 10^{33}$ yrs.
$\Gamma^{-1}_{d=5}(n \to \bar{\nu} \pi^0)$	$1.5 \cdot 10^{34}$ yrs.

Babu, Bajc, Tavartkiladze (2012)

Nucleon lifetime cannot exceed 2×10^{34} yrs in this realistic SUSY $SU(5)$ model if SUSY masses are < 3 TeV

Non-supersymmetric SO(10)

Unlike $SU(5)$, $SO(10)$ allows an intermediate symmetry

$$SO(10) \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times P$$ **Pati-Salam symmetry**

Unification of gauge couplings consistent with data

Intermediate scale may be identified as Peccei-Quinn scale to solve strong CP problem

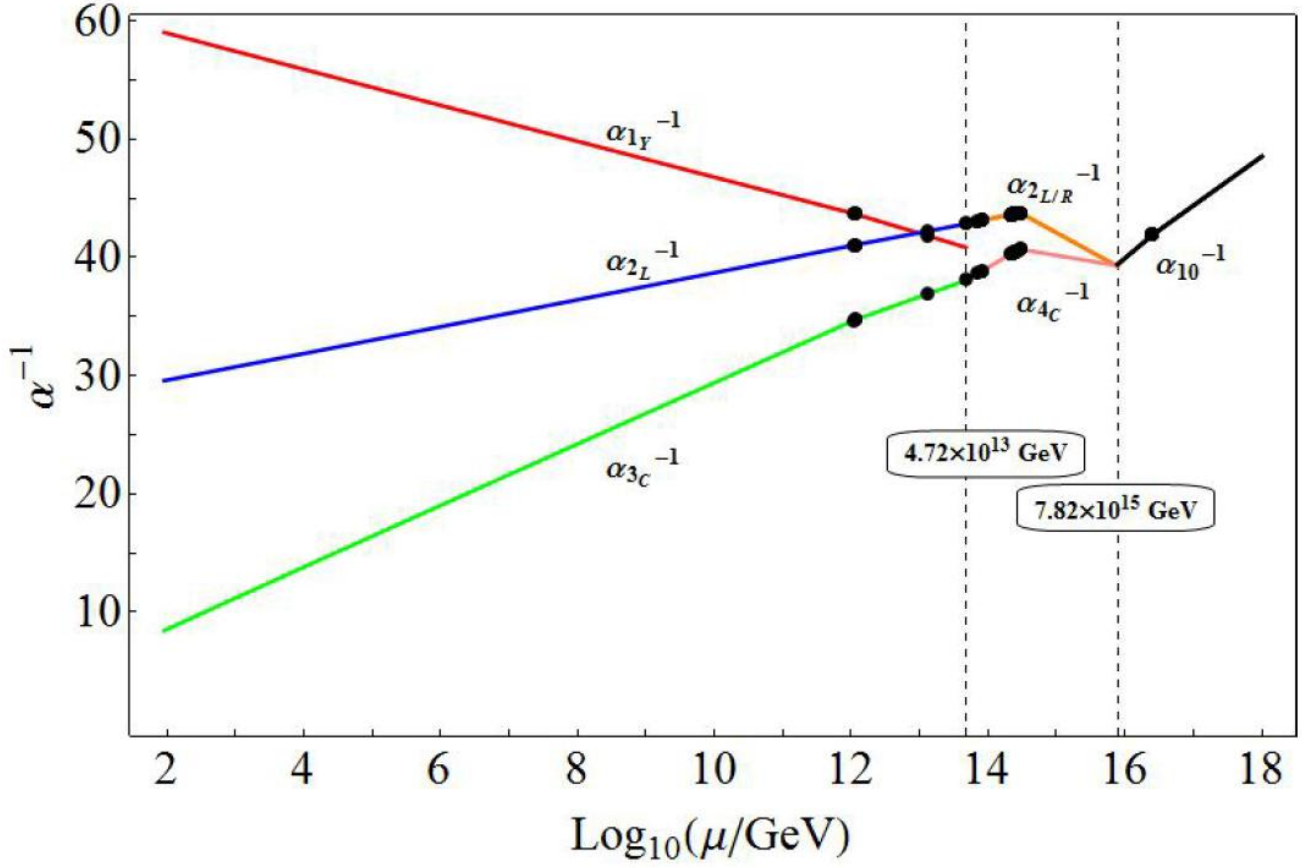
Proton decays via exchange of $SO(10)$ gauge bosons

Lifetime within reach of currently envisioned experiments

Rich literature:

Rizzo, Senjanovic (1980)
Mohapatra, Parida (1993)
Deshpande, Keith, Pal (1995)
Lee, Mohapatra, Parida, Rani (1995)
Bertolini, Luzio, Malinsky (2012)
Altarelli, Meloni (2013)
Babu, Khan (2013)

Gauge coupling evolution in non-SUSY SO(10)



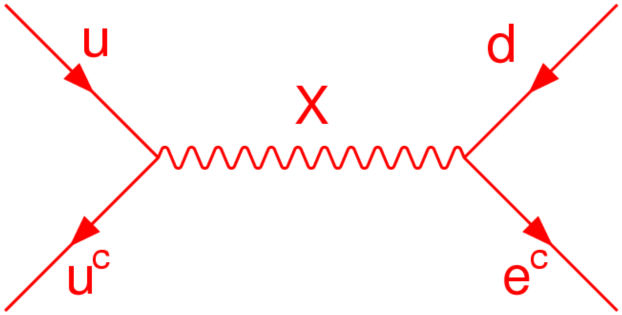
Intermediate symmetry: $SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R \times P$

S. Khan, KSB (2015)

Nucleon Decay in non-SUSY SO(10)

$SO(10)$ breaks to an intermediate Pati–Salam symmetry
$SU(2)_L \times SU(2)_R \times SU(4)_c \times P$

Proton decays to $e^+\pi^0$ via GUT scale X, Y gauge boson exchange

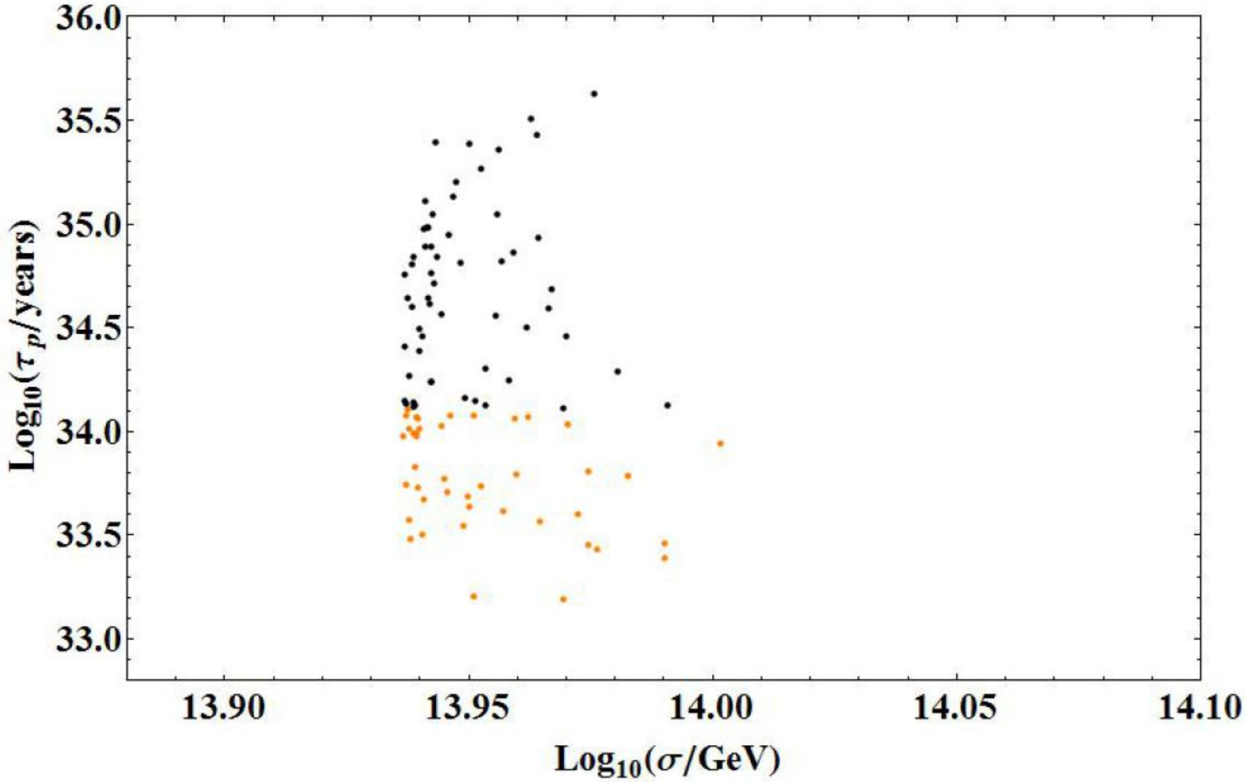


$$\Gamma^{-1}(p \to e^+\pi^0) \approx (8.2 \times 10^{34}\ \text{yr}) \times \left(\frac{\alpha_H}{0.0122\ \text{GeV}^3}\right)^{-2} \left(\frac{\alpha_G}{1/34.7}\right)^{-2} \left(\frac{A_R}{3.35}\right)^{-2} \left(\frac{M_X}{10^{16}\ \text{GeV}}\right)^4$$

Threshold corrections play important role

GUT symmetry breaking via **126** and **54**

Proton lifetime versus intermediate scale



$\tau_p < 3 \times 10^{35}$ yrs.

SUSY SO(10) with Natural Doublet-Triplet Splitting

Babu, Pati, Tavartkiladze (2010)

Higgs sector: $\{45_H + 10_H + 16_H + \overline{16}_H + 16'_H + \overline{16}'_H\}$

$$W_{D-T} = \lambda(\mathbf{10}_H\mathbf{45}_H\mathbf{10}'_H) + M'\mathbf{10}'_H\mathbf{10}'_H$$

$$\langle 45_H \rangle = \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes i\tau_2 \quad \propto B-L$$

Adjoint VEV along $B-L$ gives mass only to color triplets and not to doublets

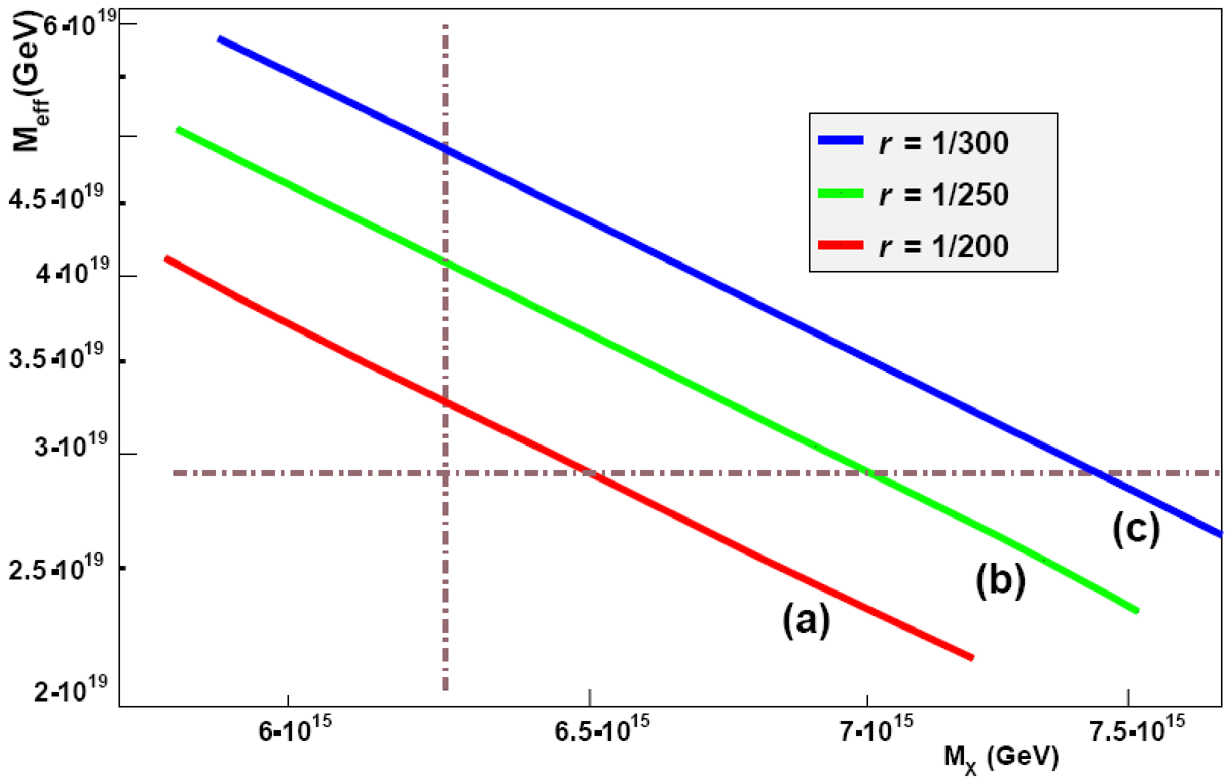
(Dimopoulos-Wilczek mechanism)

Dimopoulos, Wilczek (1981)
Babu, Barr (1993)
Barr, Raby (2000)

Correlation between proton decay modes

$$M_{\rm eff} \simeq 10^{19}\text{GeV}\cdot\left(\frac{10^{16}\text{GeV}}{M_X}\right)^3\left(\frac{3}{\tan\beta}\right)\left(\frac{1/100}{r}\right)\frac{\exp[2\pi(\Delta^{(2)}_{2,w}-\Delta^{(2)}_{3,w}-\delta\alpha_3^{-1})]}{2.54\cdot 10^{-2}}\,.$$

$$r \equiv \frac{M_\Sigma}{M_X} \approx \left(\frac{1}{100} - \frac{1}{300}\right)$$



Proton Lifetime Predictions

$$\Gamma^{-1}_{d=6}(p \to e^+\pi^0) \simeq 1.0\times10^{34}\ \text{yrs}\left(\frac{0.012\text{GeV}^3}{\alpha_H}\right)^2\left(\frac{2.78}{A_R}\right)^2\left(\frac{5.12}{f(p)}\right)\left(\frac{1/20}{\alpha_G(M_X)}\right)^2\left(\frac{M_X}{6.24\times10^{15}\text{GeV}}\right)^4$$

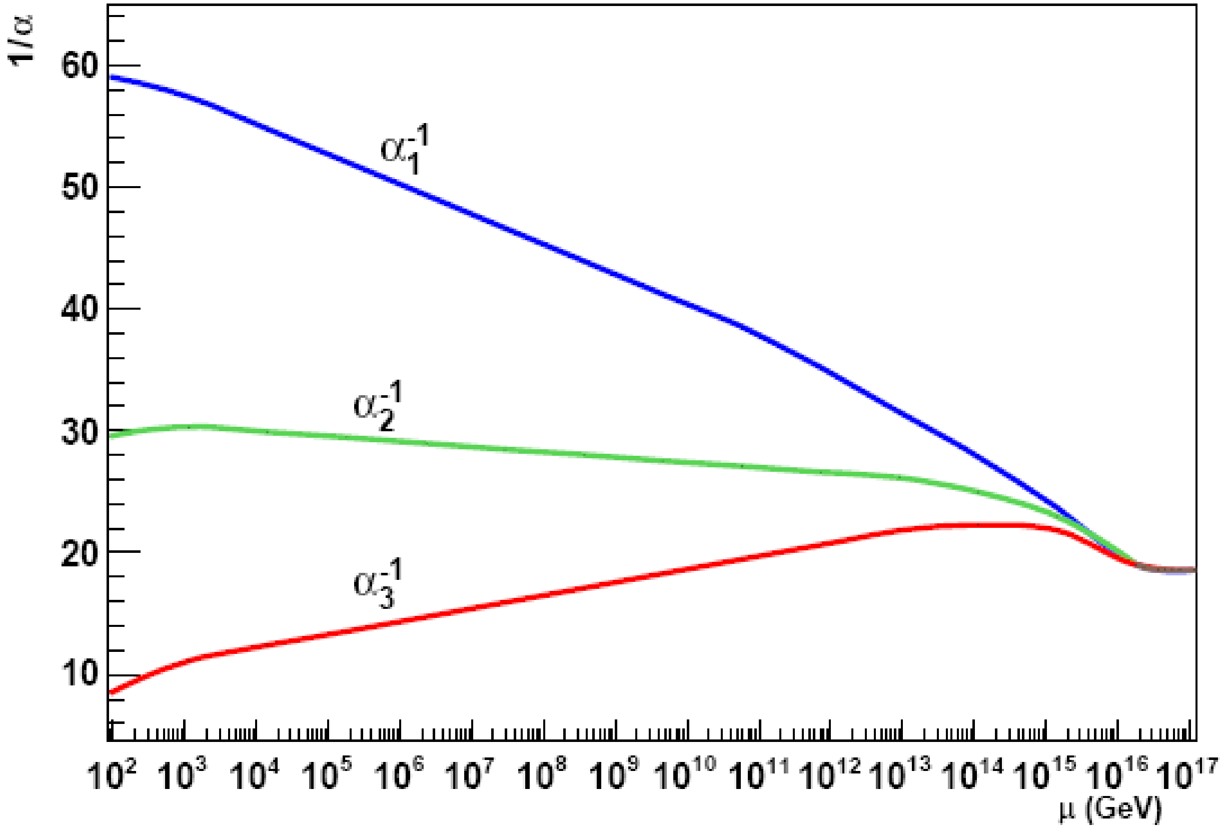
$$\Gamma^{-1}_{d=5}(p \to \bar{\nu}K^+) = 3.5\times10^{33}\text{yrs}\left(\frac{0.012\text{GeV}^3}{|\beta_H|}\right)^2\left(\frac{6.91}{\bar{A}^\alpha_S}\right)^2\left(\frac{1.25}{R_L}\right)^2\left(\frac{M_{\text{eff}}}{3.38\times10^{19}\text{GeV}}\right)^2\times$$
$$\times\left(\frac{m_{\tilde{q}}}{1.5\text{TeV}}\right)^4\left(\frac{130}{m_{\tilde{W}}}\right)^2\left(\frac{3.1}{K^\nu_{d=5}}\right).$$

Imposing the correlation equation we obtain the predictions:

$$\Gamma^{-1}_{d=6}(p \to e^+\pi^0) \lesssim 5.3\times10^{34}\ \text{yrs}$$
$$\Gamma^{-1}(p \to \bar{\nu}K^+) \lesssim (3.1\times10^{34}\ \text{yrs})\times\left(\frac{m_{\tilde{q}}}{1.5\ \text{TeV}}\right)^4\left(\frac{130\ \text{GeV}}{m_{\tilde{W}}}\right)^2 (3/\tan\beta)^2$$

Both modes should be within reach of experiments!

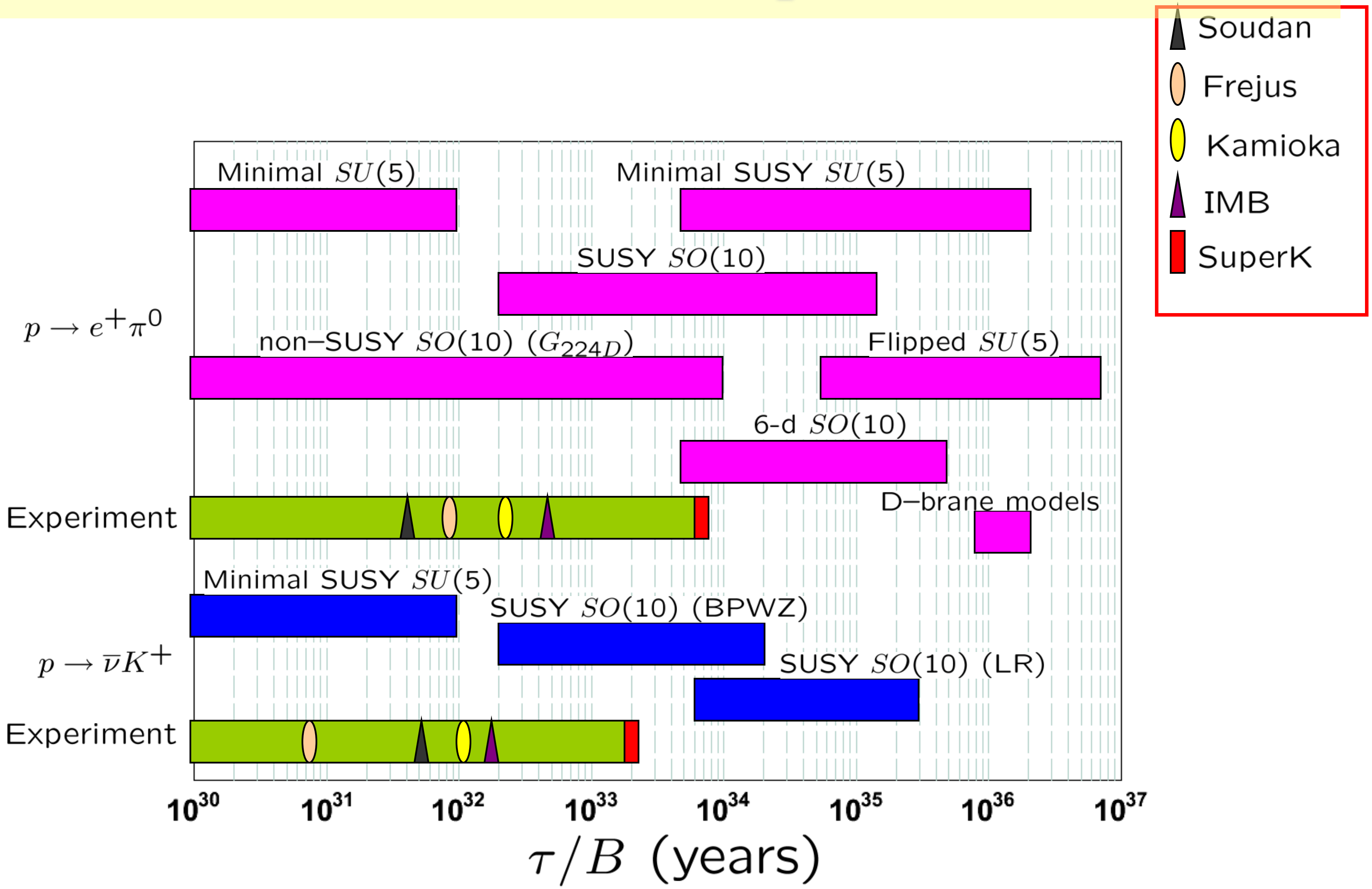
Unification of gauge couplings in SUSY SO(10)



Gauge coupling evolution in explicit SO(10) model

K.S. Babu, J.C. Pati, and Z. Tavartkiladze, JHEP 1006: 084, 2010

Proton Lifetime Expectations



Neutron-antineutron Oscillations

In presence of baryon number violation neutron can spontaneously convert into antineutron

Kuzmin (1970), Glashow (1979), Marshak, Mohapatra (1980)

Violates B by two units: Likely analog of L violation by two units in neutrino Majorana mass

Generated by effective dimension 9 operatos such as

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^5}(u_R d_R u_R d_R d_R d_R)$$

Experiments sensitive to cut-off scale $\Lambda \sim 10^5 - 10^6$ GeV

Probes a different sector of B violation compared to proton decay which tests $|\Delta B| = 1$

$n - \overline{n}$ oscillations may be the source of baryon asymmetry of the Universe

Discovery of $n - \overline{n}$ oscillations would likely rule out high scale models of baryon asymmetry generation (e.g: leptogenesis)

Neutron-antineutron Oscillations Phenomenology

Time evolution of a neutron in presence of $\Delta B = 2$ interactions:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} n \\ \bar{n} \end{pmatrix} = \begin{pmatrix} E_n & \delta m \\ \delta m & E_{\bar{n}} \end{pmatrix} \begin{pmatrix} n \\ \bar{n} \end{pmatrix}$$

Probablility of n oscillating into $\bar{n}$ at time t is

$$P_{n \to \bar{n}}(t) = \frac{(\delta m)^2}{\Delta E_n^2 + (\delta m)^2} \sin^2\{\sqrt{\Delta E_n^2 + 4(\delta m)^2}\, t\}$$

$$E_n = m_n - \vec{\mu}_n.\vec{B} + V_n, \qquad E_{\bar{n}} = m_n + \vec{\mu}_n.\vec{B} + V_{\bar{n}}$$

$$\Delta E_n = E_n - E_{\bar{n}} = -2\vec{\mu}_n.\vec{B} + V_n - V_{\bar{n}}$$

For $\delta m \ll \Delta E_n$, this expression reduces to:

$$P_{\bar{n}} \simeq \left(\frac{\delta m}{\Delta E_n}\right)^2 \sin^2(\Delta E_n t)$$

Case (i): $\Delta E_n t \ll 1$: In this case

$$P_{n\to\bar{n}}(t) \simeq (\delta m \cdot t)^2 \equiv \left(\frac{t}{\tau_{n-\bar{n}}}\right)^2$$

This case corresponds to free neutron oscillation in vacuum

Case (ii): $\Delta E_n \cdot t \gg 1$:

$$P_{n\to\bar{n}}(t) \simeq \frac{1}{2}\left(\frac{\delta m}{\Delta E_n}\right)^2$$

This is realized when bound neutrons inside nucleus "oscillates" to antineutrons

$$\tau_{Nuc.} = R(\tau_{n-\bar{n}})^2 \qquad R \simeq 0.3 \times 10^{24} \text{ sec.}^{-1}$$

Friedman, Gal (2008)

Expt	source of neutrons	$\tau_{Nucl.}(yrs)$	$\tau_{osc.}$(sec)
Soudan	^{56}Fe	0.72×10^{32}	1.3×10^8
Frejus	^{56}Fe	0.65×10^{32}	1.2×10^8
Kamiokande	^{16}O	0.43×10^{32}	1.2×10^8
Super-K	^{16}O	1.77×10^{32}	2.3×10^8

Free neutrons: $\tau_{n-\bar{n}} > 0.86 \times 10^8$ sec. (ILL, 1994)

Some Recent Developments in n-nbar Oscillations

Spin dependence in oscillations has been incorporated

Gardner, Jafari (2014)

Neutron–antinutron oscillation signals can be used to constrain Lorentz invariance violation of order 10^{-23} GeV

Babu, Mohapatra (2015)

It has been realized that free neutron–antineutron oscilaltion would also imply violation of CP symmetry (assuming CPT and Lorentz symmetry)

Berezhiani, Vainshtein (2015)

Baryogenesis and n-nbar oscillations in SO(10)

In $SU(5)$ decays of heavy gauge bosons of color triplet scalars can generate baryon asymmetry

This asymmetry is however washed out by electroweak sphalerons since $(B-L)$ is preserved

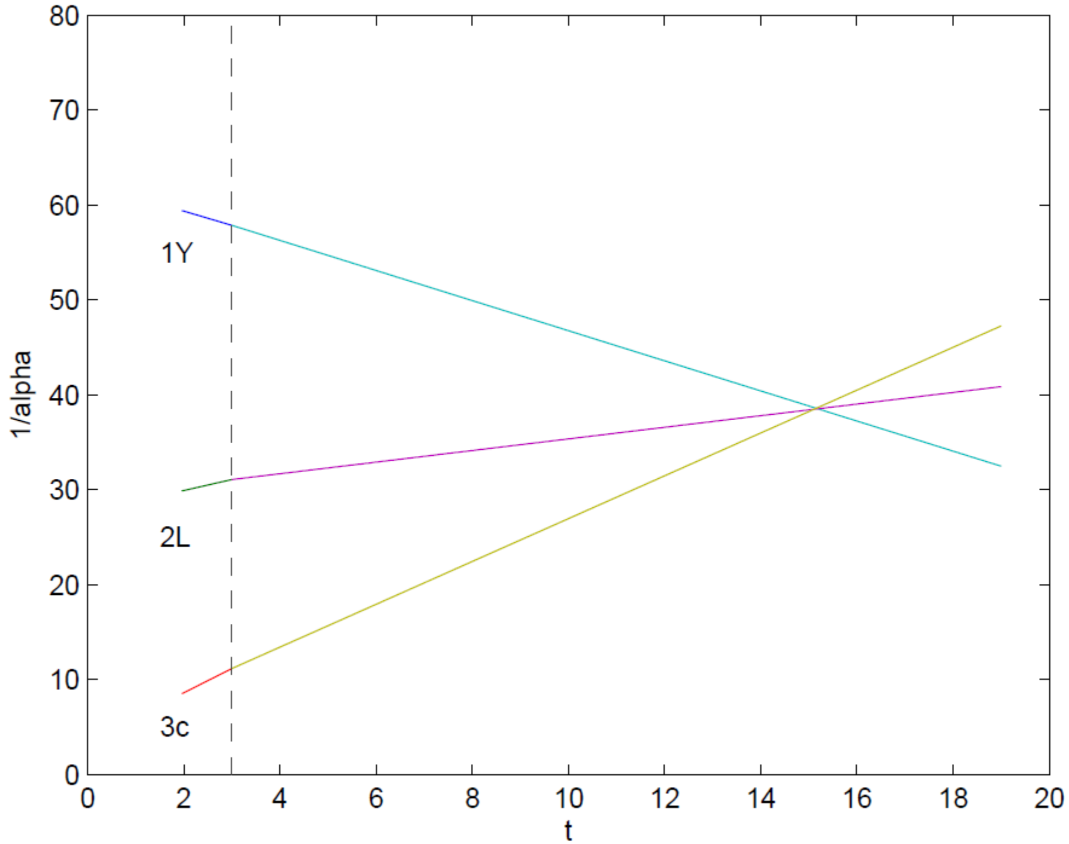
In $SO(10)$, color sextet scalar decays can lead to sphaleron-proof baryon asymmetry K.S. Babu, R.N. Mohpatra (2012)

Recently it has been noted that non-SUSY SO(10) provides a natural dark matter candidate due to $B-L$ symmetry

Mambrini, Nagata, Olive, Quivillon, Zheng (2015)

Minimal SO(10) can provide dark matter, n-nbar oscillations, proton decay, and LHC signals for colored scalars!

TeV-scale Color Sextet scalar and unification

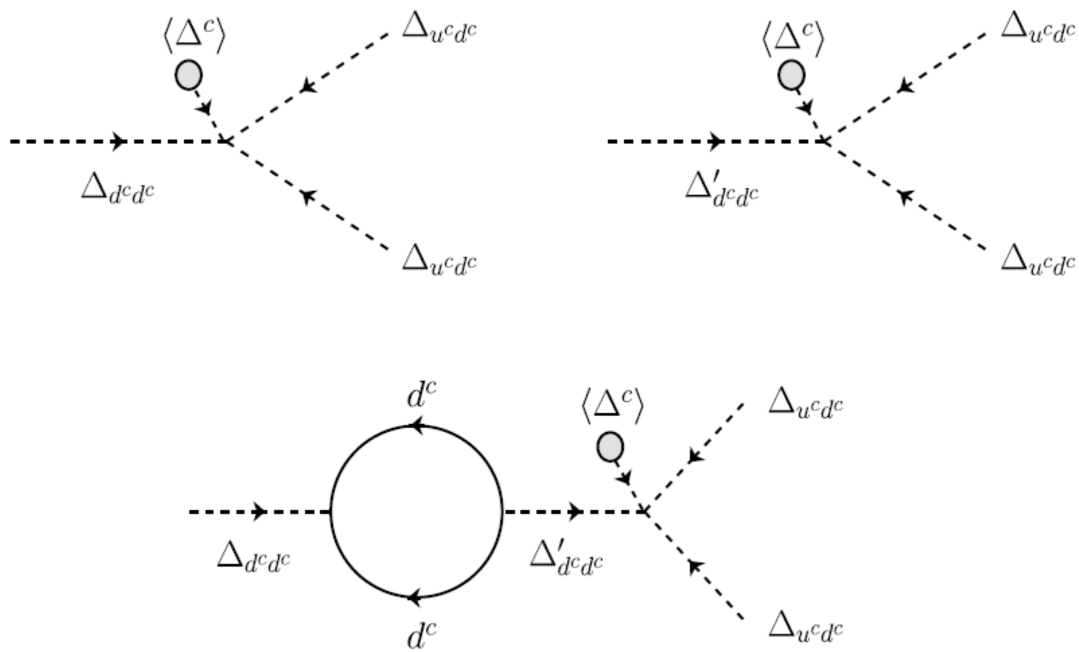


$\Delta_{u^c d^c}(6^*, 1, 1/3)$ scalar and a $(1, 3, 0)$ fermion at 1 TeV

$(1, 3, 0)$ fermion stable dark matter with mass $2.7 - 3$ TeV

K.S. Babu, R.N. Mohpatra (2012)

Baryongenesis via Color sextet decay



$\Delta_{d^c d^c}$: GUT mass; $\Delta_{u^c d^c}$: TeV mass

Minimal $SO(10)$ models have two $\Delta_{d^c d^c}$ (from **126** and **54**) and one $\Delta_{u^c d^c}$

$(B-L)$ asymmetry of the right order induced

Baryongenesis via Color sextet decay (cont.)

$(B-L)$ asymmetry parameter ϵ_{B-L}:

$$\epsilon_{B-L} = (r - \bar{r})(B_1 - B_2)$$

r is branching ratio for $\Delta_{d^c d^c} \to \Delta^*_{u^c d^c} \Delta^*_{u^c d^c}$
$\bar{r}$ is branching ratio for $\Delta^*_{d^c d^c} \to \Delta_{u^c d^c} \Delta_{u^c d^c}$

$B_1 = -4/3, \quad B_2 = 4/3$ ($B-L$ of two final states)

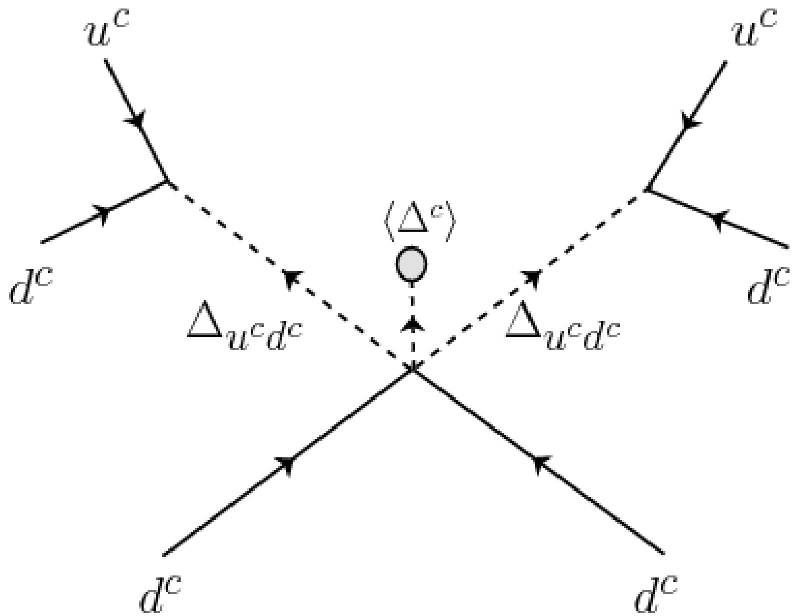
$$\eta = \frac{n_B}{s} \simeq \frac{\epsilon_{B-L}}{g^*} d$$

$$\epsilon_{B-L} \simeq \frac{2}{\pi |\lambda v_R|^2} \mathrm{Tr}(f^\dagger f) \mathrm{Im}[(\lambda v_R)(\lambda' v_R)^*] F \cdot \mathrm{Br}$$

$$F = \frac{M^2_{d^c d^c}}{M^2_{d^c d^c} - M'^2_{d^c d^c}}$$

$\eta_B \approx 10^{-10}$ naturally generated

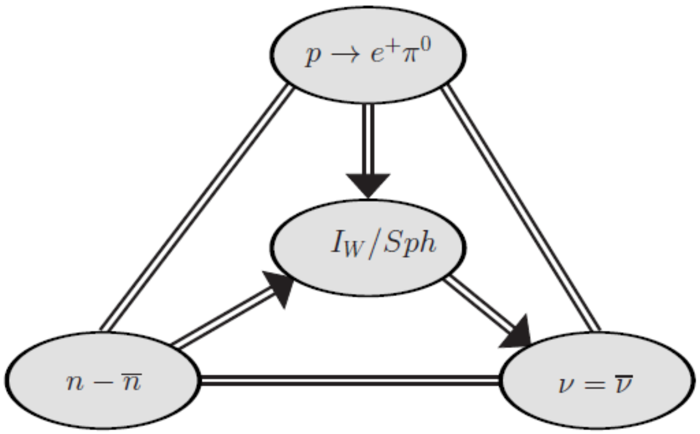
Baryogenesis and n-nbar oscillations



$$M_{\delta_{u^c d^c}} = 1 \text{ TeV}, \; M_{\Delta_{d^c d^c}} = 10^{14} \text{ GeV}, \; f_{11} = 10^{-3}$$

$$\Longrightarrow \qquad \tau_{n-\overline{n}} = (10^9 - 10^{11}) \text{ sec.}$$

A (B-L) Triangle



Electroweak sphaleron/instanton induced effective operator:

$$QQQQQQQQQLLL = [uddudd]\,[uude]\,[\nu\nu]$$

$[uddudd]$: $n - \overline{n}$ oscillations $\qquad$ $[uude]$: $p \to e^+\pi^0$

$[\nu\nu]$: $\beta\beta_{0\nu}$

Discovery of $n - \overline{n}$ transition and $p \to e^+\pi^0$ would imply neutrinos are Majorana fermions!

Summary and Conclusions

- Grand Unification idea very appealing and promising
- Proton decay discovery will be a monumental and maybe within experimental reach soon
- Both $p \to e^+\pi^0$ and $p \to \overline{\nu}K^+$ may be observable
- Large undergound detectors (HyperK, DUNE) essential to test theories
- Neutron–antineutron oscillations test a different sector of theory space
- Observable $n - \overline{n}$ oscillations may be linked to baryon asymmetry of the universe

Proton decay problem in minimal SUSY SU(5)

$$\ln\frac{M_T}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,w})\right)-\ln\kappa^{5/2}$$

$$\ln\frac{(M_X^2M_\Sigma)^{1/3}}{M_Z}=\frac{\pi}{18}\left(5(\alpha_1^{-1}+\Delta^{(2)}_{1,w})-3(\alpha_2^{-1}+\Delta^{(2)}_{2,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,w}-\frac{1}{4\pi})\right)+\frac{1}{6}\ln\kappa$$

$\Delta^{(2)}_{i,w}$: SUSY threshold and 2-loop RGE efects $\kappa\equiv\frac{M(8,1,0)}{M(1,3,0)}=1$

If $\kappa=1$, $M_T<7\times10^{14}$ GeV is needed for $\alpha_3(M_Z)<0.12$

$$\Gamma^{-1}_{d=5}(p\to\bar{\nu}K^+)\simeq1.2\cdot10^{31}\,\text{yrs}\times\left(\frac{0.012\,\text{GeV}^3}{\beta_H}\right)^2\left(\frac{7}{\bar{A}_S^\alpha}\right)^2\left(\frac{1.25}{R_L}\right)^2\times\left(\frac{M_T}{2\cdot10^{16}\text{GeV}}\right)^2\left(\frac{m_{\tilde{q}}}{1.5\text{TeV}}\right)^4\left(\frac{190\,\text{GeV}}{M_{\tilde{W}}}\right)^2$$

Super-Kamiokande Limit: $\tau>5.9\times10^{33}$ yr.

Proton Lifetime in Realistic SUSY SU(5)

Minimal SUSY $SU(5)$ provides a benchmark point for proton lifetime

Minimal $SU(5)$ predicts some wrong mass relations

$$m_b^0 = m_\tau^0, \;\; m_s^0 = m_\mu^0, \;\; m_d^0 = m_e^0 \text{ (at GUT scale)}$$

$$\Rightarrow \frac{m_d}{m_s} = \frac{m_e}{m_\mu}$$ Off by an order of magnitude

This problem should be fixed before discussing $p \to \overline{\nu} K^+$ rate

Simplest way is to add $5 + \overline{5}$ fermion at GUT scale

Mixing of $5 + \overline{5}$ fermions with normal fermions correct wrong mass relations, and improves proton lifetime

Babu, Bajc, Tavartkiladze (2012)