Numerical Holography and Heavy Ion Collisions

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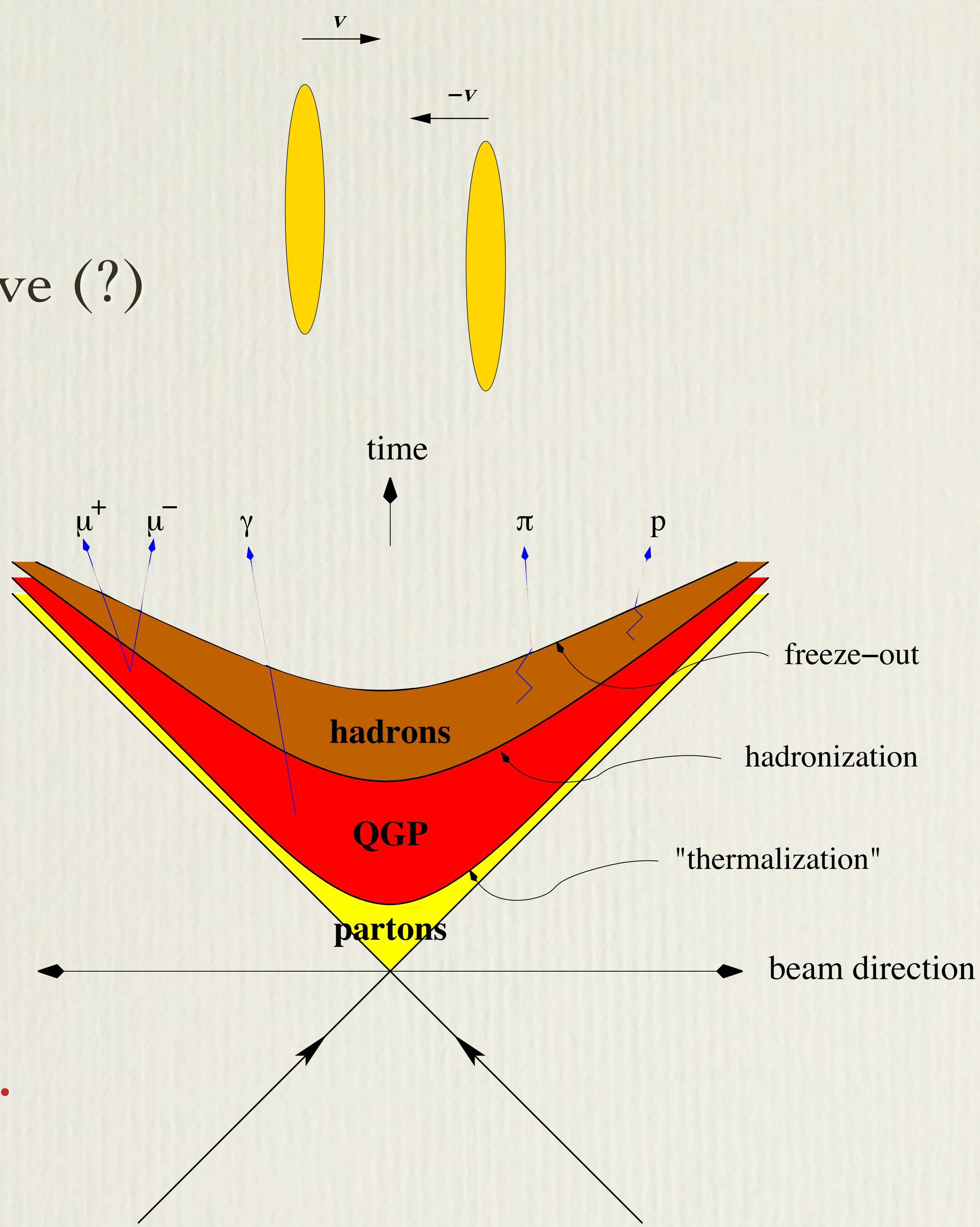
based on work with Paul Chesler

Equilibration in Weakly and Strongly Coupled QFT, INT, August 20, 2015

relativistic heavy ion collisions



Relevant dynamics:

- Very early: partonic, marginally perturbative (?)
- Plasma phase: strongly coupled
  - Evidence: screening lengths, viscosity, ...

Many questions:

- How fast do produced partons isotropize?
- Initial conditions for hydrodynamics?
- Signatures of strongly coupled dynamics?

No fully controlled theoretical methods.

idealize

QCD	➡	$\mathcal{N}$=4 SYM
# colors $N_c = 3$	➡	$N_c = \infty$
't Hooft coupling $\lambda \approx 10$	➡	$\lambda \gg 1$
highly boosted nuclei	➡	lightlike projectiles

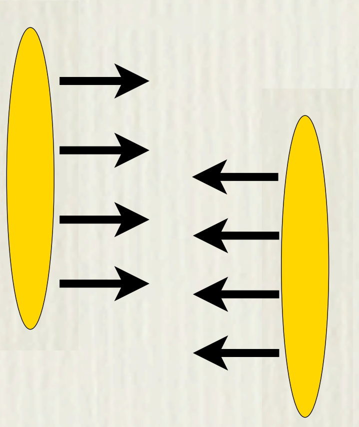
- ✓ non-Abelian plasma
- ✓ hydrodynamic response
- ✘ no hadronization
- ✓✘ conformal
- ✓ dual holographic description

holography

- strongly coupled, large N QFT = classical (super)gravity in higher dimension
  - valid description on all scales
  - gravitational fluctuations: $1/N^2$ suppressed
  - QFT state ↔ asymptotically AdS geometry
  - $O(N^2)$ entropy ↔ gravitational (black brane) horizon
  - thermalization ↔ gravitational infall, horizon formation & equilibration
  - non-equilibrium QFT dynamics ↔ classical gravitational initial value problem

holographic collisions

- scattering ➡ Poincaré patch AdS asymptotics
- warm-up steps:
  - planar shocks: 2+1D PDEs
    no transverse dynamics
  - boost invariant: 1+1D (no radial flow) or 2+1D PDEs
    unrealistic longitudinal dynamics
- recent work:
  - finite "nuclei": 4+1D PDEs (off-center)
    transverse and longitudinal dynamics



initial projectiles

- Exact analytic solution for stable null "projectile"

$$ds^2 = \frac{L^2}{s^2}\left[-dt^2 + d\boldsymbol{x}_\perp^2 + dz^2 + ds^2 + h_\pm(\boldsymbol{x}_\perp, z_\mp, s)\, dz_\mp^2\right]$$

Fefferman-Graham (FG) coordinates $\boldsymbol{x}_\perp \equiv \{x, y\}$ $\quad z_\mp \equiv z \mp t$

- metric deformation function

$$\left(\partial_s^2 - \tfrac{3}{s}\,\partial_s + \nabla_\perp^2\right) h_\pm = 0$$

arbitrary

$$h_\pm(\boldsymbol{x}_\perp, z_\mp, s) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\, e^{i\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\, \widetilde{H}_\pm(\mathbf{k}_\perp, z_\mp)\, 8(s^2/k_\perp^2)\, I_2(k_\perp s)$$

- stress-energy: $\langle T^{00}\rangle = \langle T^{zz}\rangle = \pm\langle T^{0z}\rangle = \kappa\, H_\pm(\boldsymbol{x}_\perp, z_\mp)$

- choose Gaussian profile for simplicity:

$$H_\pm(\boldsymbol{x}_\perp, z_\mp) = \frac{\mathcal{A}}{\sqrt{2\pi w^2}}\, \exp\left(-\tfrac{1}{2} z_\mp^2/w^2\right)\, \exp\left[-\tfrac{1}{2}(\boldsymbol{x}_\perp \mp \boldsymbol{b}/2)^2/R^2\right]$$

$\mathcal{A} = 1 \qquad w = \frac{1}{2} \qquad R = 4 \qquad \boldsymbol{b} = \frac{3}{4} R\,\hat{\boldsymbol{x}}$

initial data

- Superpose left & right-moving shocks
- Transform to infalling Eddington-Finkelstein (EF):

$$ds^2 = \frac{r^2}{L^2}\, g_{\mu\nu}(x,r)\, dx^\mu dx^\nu + 2\, dr\, dt$$

EF coordinates $X \equiv \{x^\mu, r\}$

boundary coordinates (x^μ) — affine parameter (r)

  - compute infalling radial null geodesic congruence

FG coordinates $Y \equiv \{y^\mu, s\}$

geodesic congruence $Y(X)$

transformed metric $G_{MN}(X) = \dfrac{\partial Y^A}{\partial X^M} \dfrac{\partial Y^B}{\partial X^N}\, \widetilde{G}_{AB}(Y(X))$

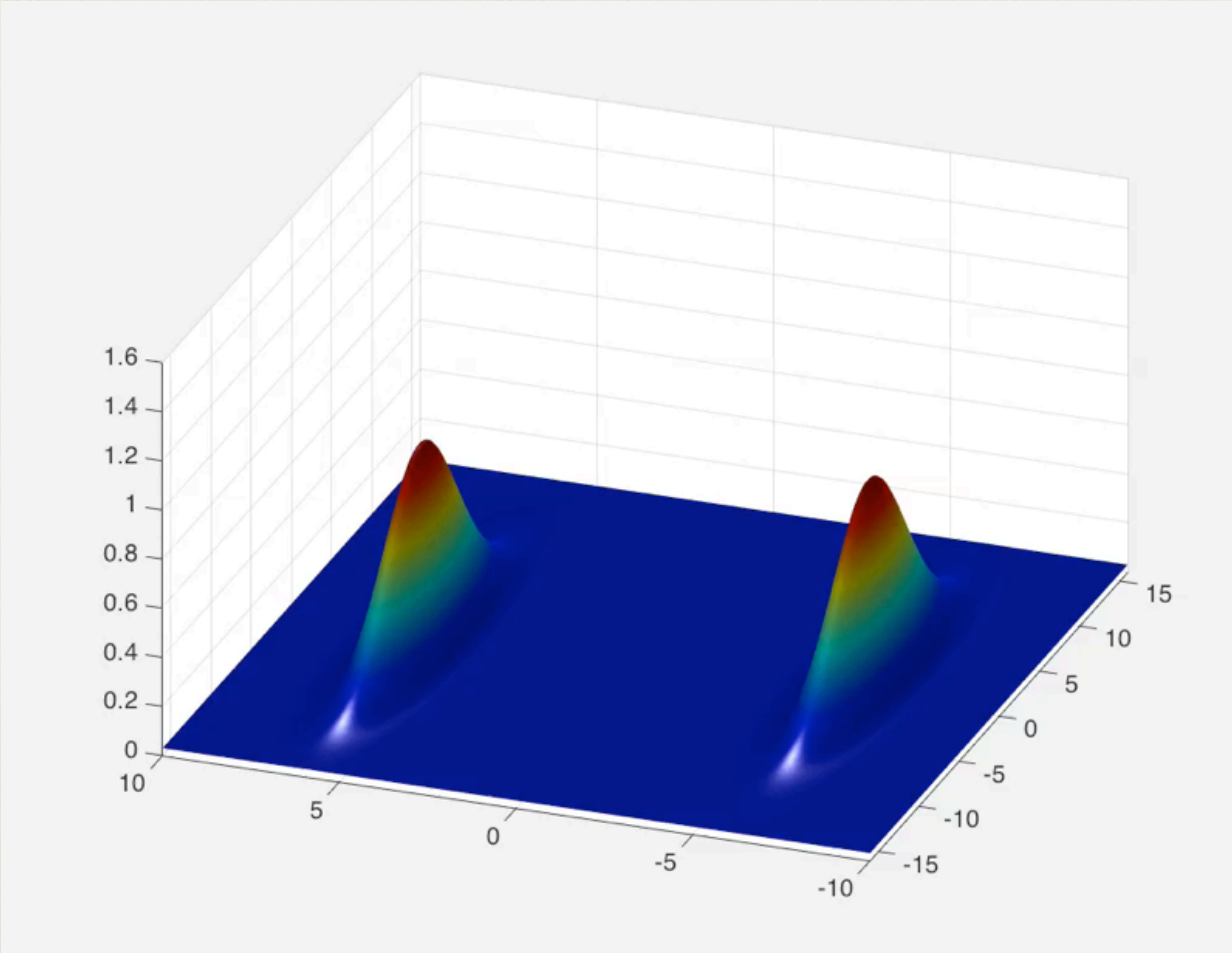
numerical techniques

- characteristic formulation of Einstein equations
- spectral methods w. domain decomposition
- residual diffeo freedom ➨ fix apparent horizon
- periodic spatial compactification
- Matlab implementation (shared memory, multicore)

Off-center collisions

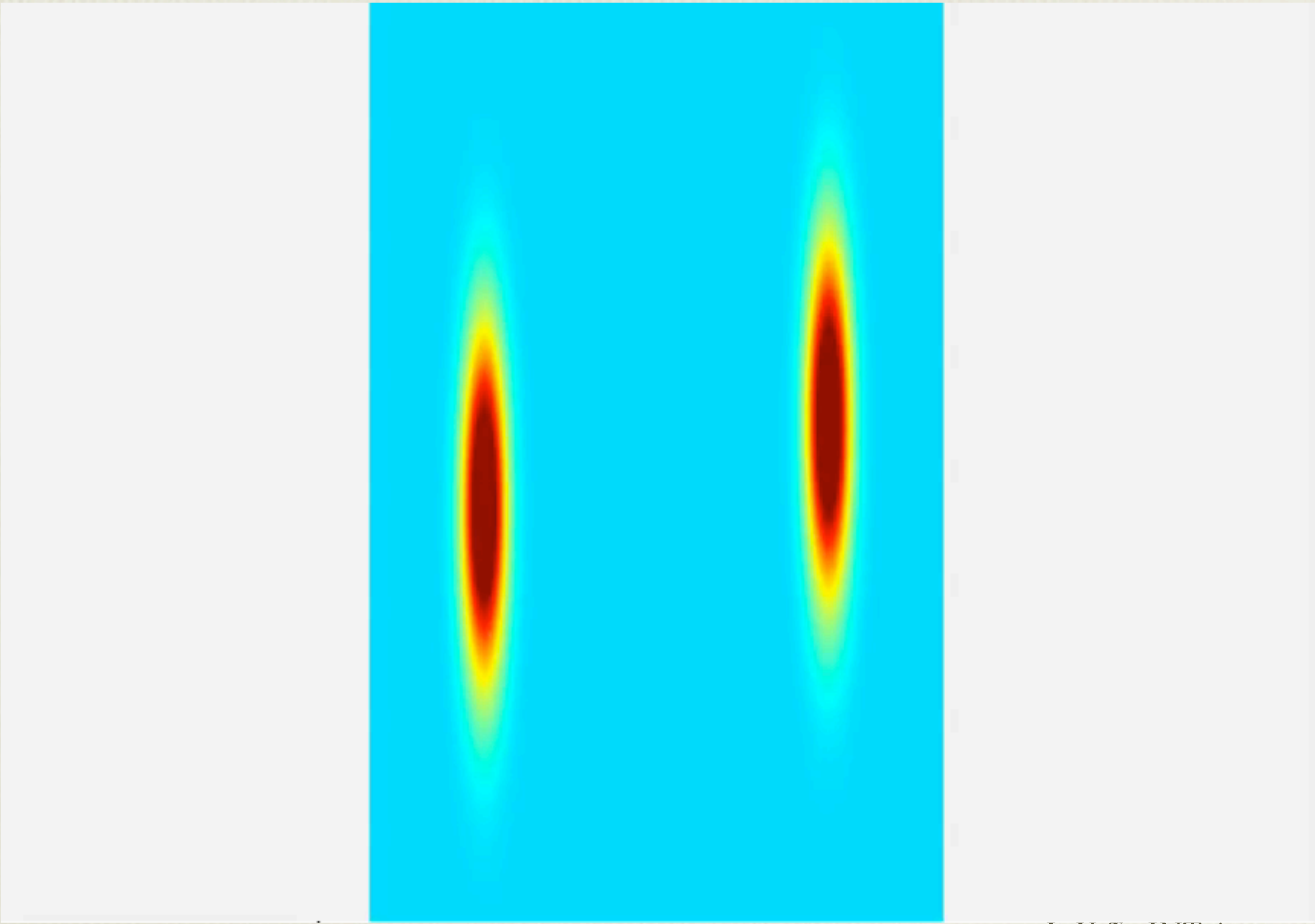
energy density

energy density



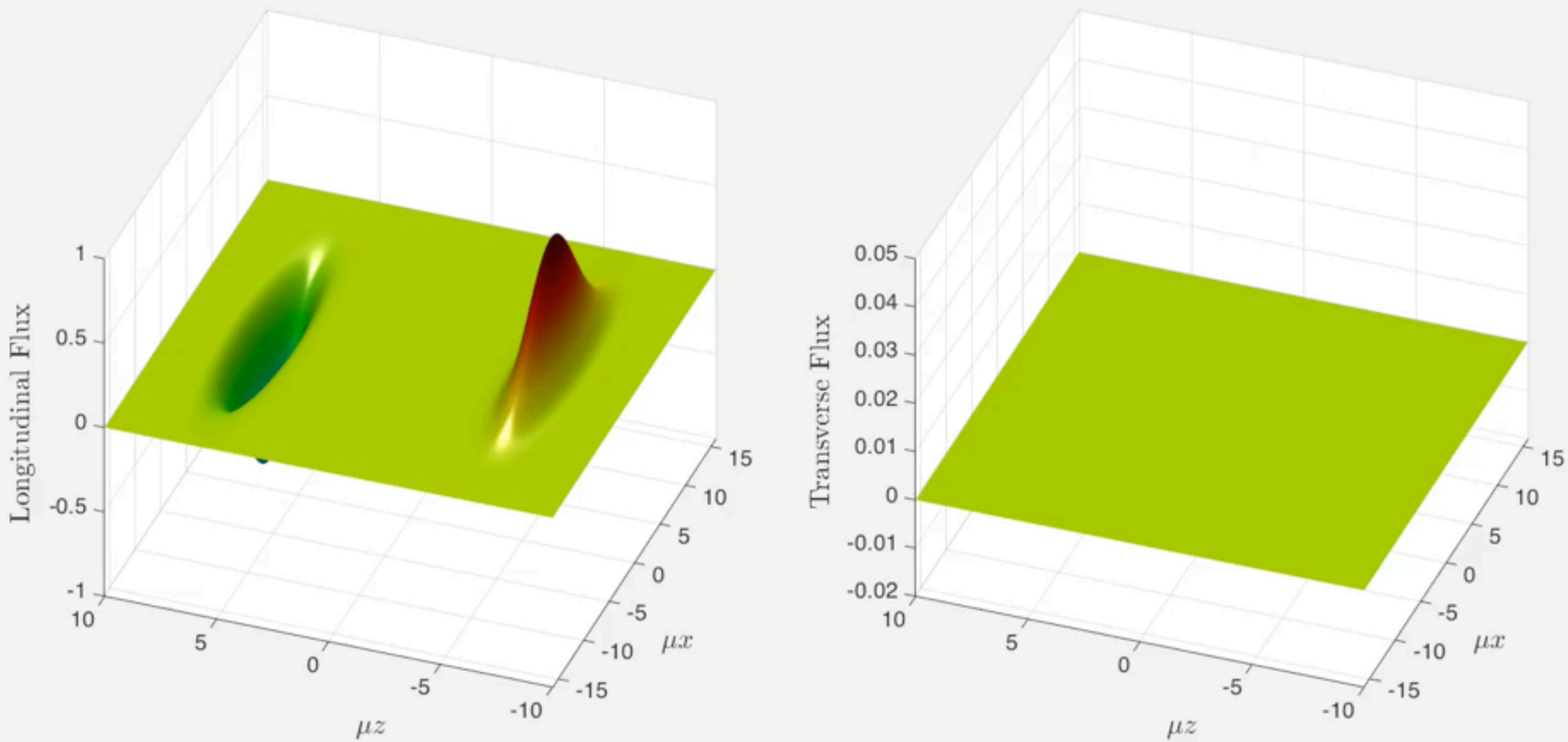
energy density

energy density

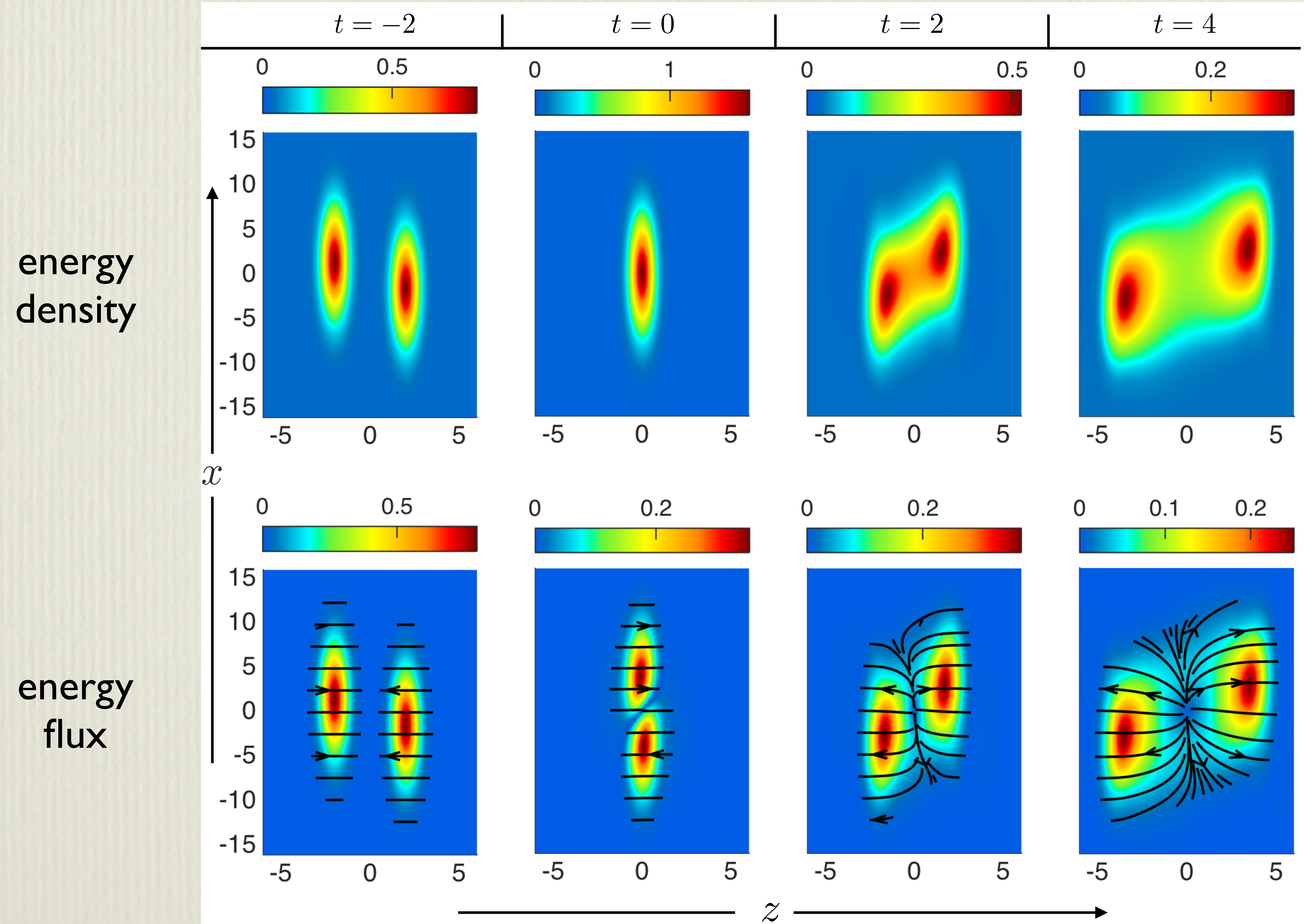


energy flux

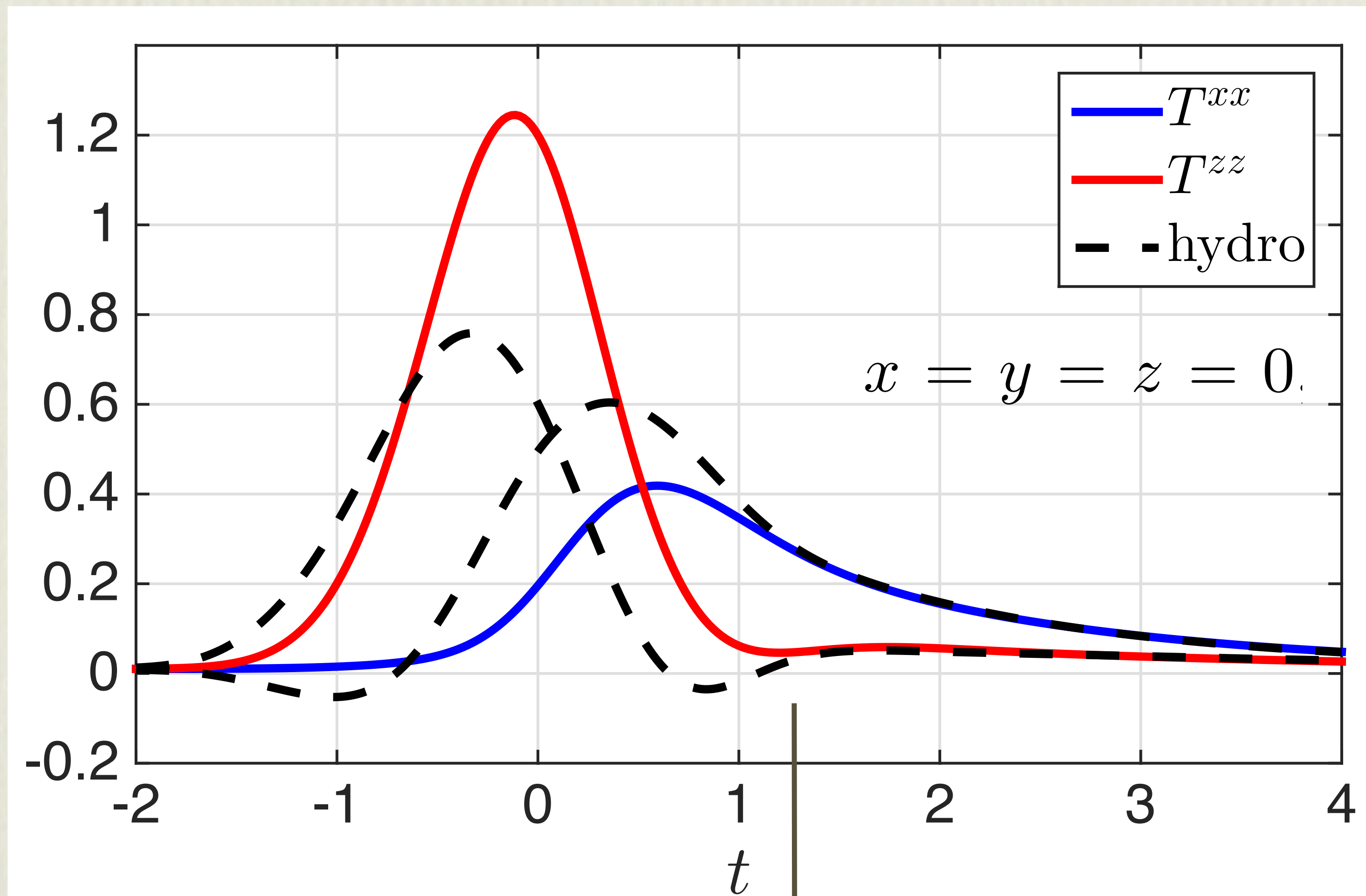
energy flux


Longitudinal Flux
1
0.5
0
-0.5
-1
10
5
0
-5
-10
μz
15
10
5
0
-5
-10
-15
μx
Transverse Flux
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02

snapshots



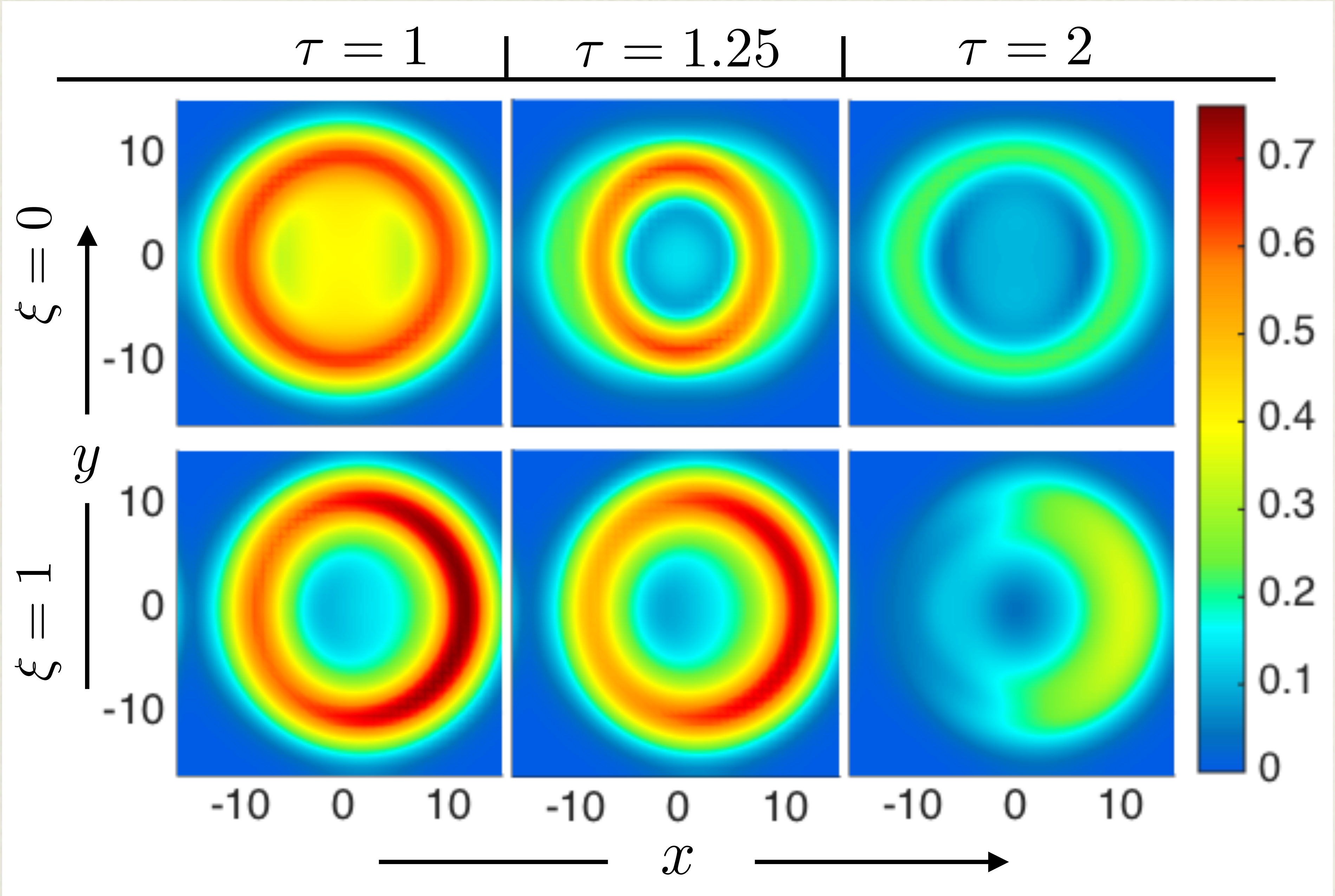
transverse & longitudinal pressure



$t_{\text{hydro}} \approx 1.25$

hydro onset ≈ 30% faster than for planar shocks

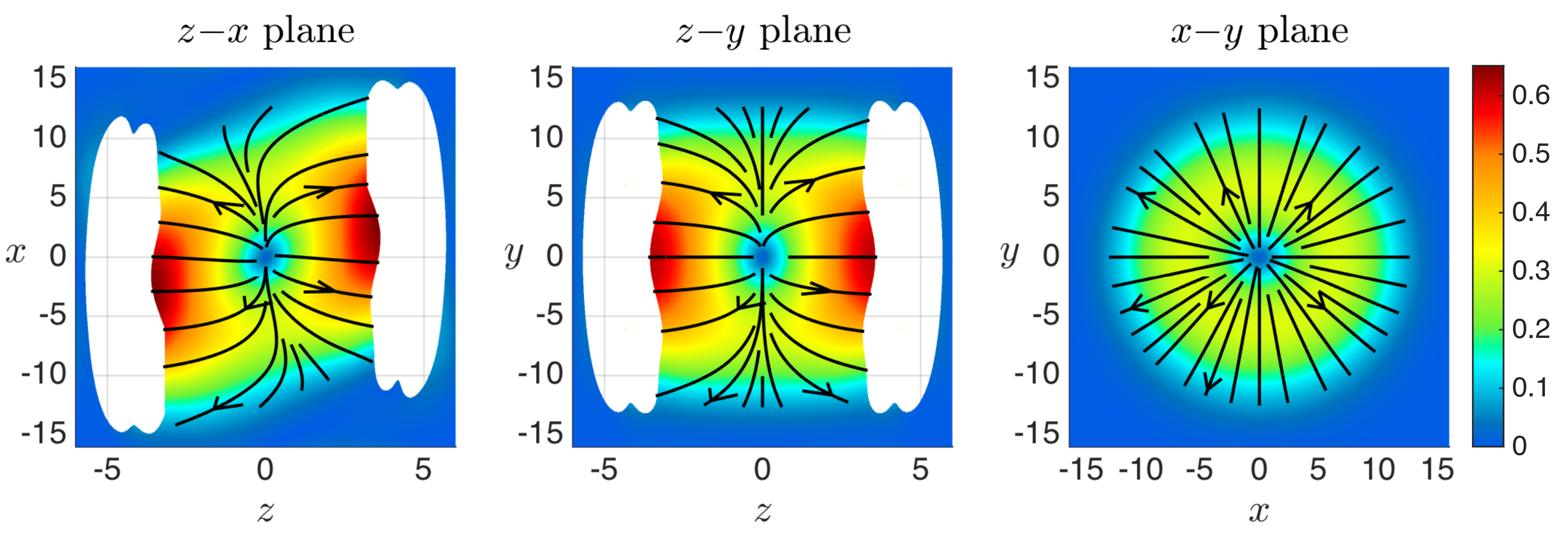
hydrodynamic residual



$\Delta \equiv (1/\bar{p})\sqrt{\Delta T_{\mu\nu}\Delta T^{\mu\nu}}$, $\quad \Delta T^{\mu\nu} \equiv T^{\mu\nu} - T^{\mu\nu}_{\text{hydro}} \quad \bar{p} \equiv \epsilon/3$

flow velocity

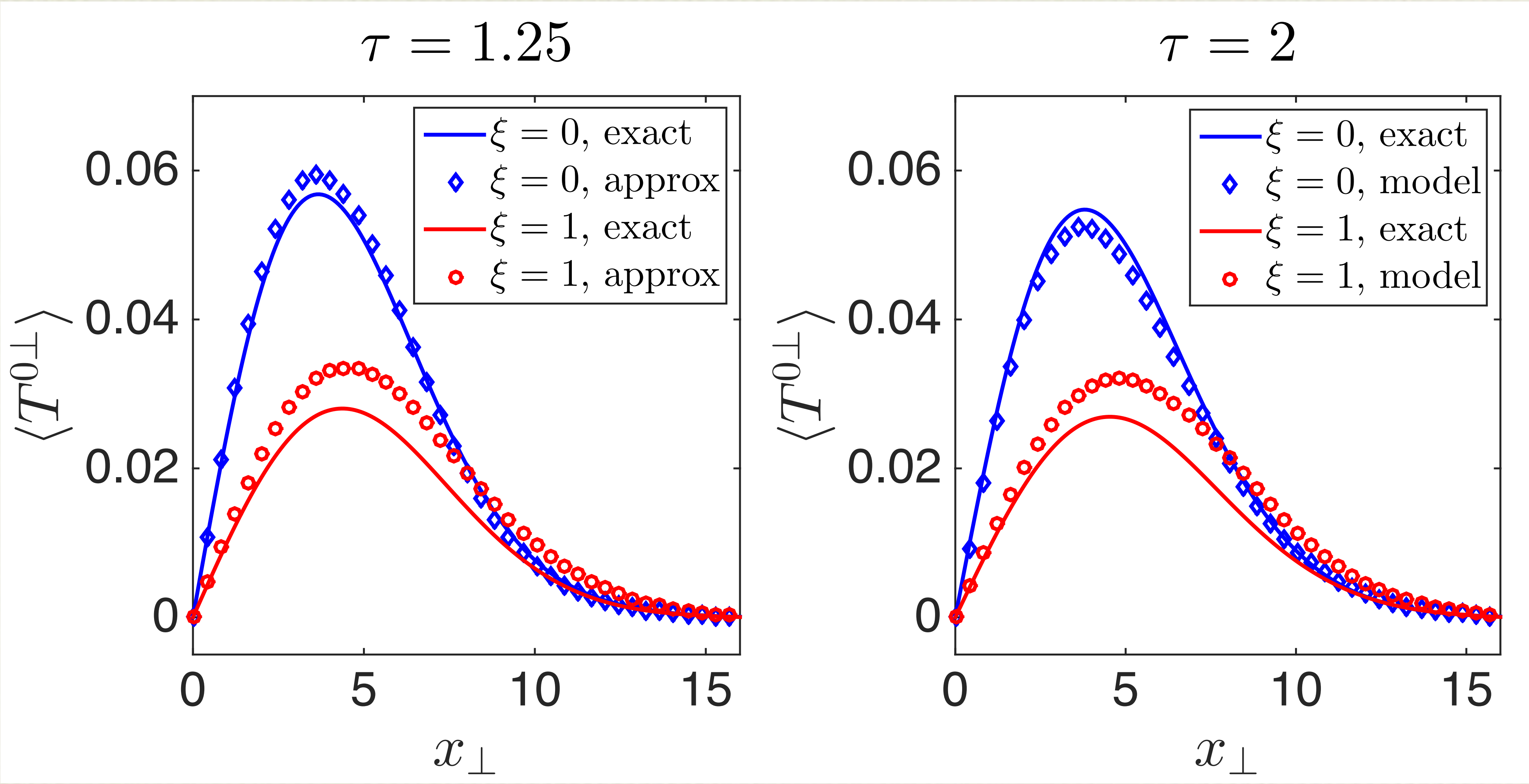
$t = 4$ non-hydro regions excised



substantial radial flow: $v_\perp(x_\perp = 5) \approx 0.3$

$$v_\parallel^{\max} \approx 0.64$$

radial flow



Vredevoogd & Pratt: "universal flow" model (assumes boost invariance & transverse rotational symmetry):

$$T^{0x} = -\tfrac{t}{2}\,\partial_x \epsilon$$
$$T^{0y} = -\tfrac{t}{2}\,\partial_y \epsilon$$

elliptic flow?

- no evident "almond" shape to fluid droplet
- transverse flow nearly symmetric
- negligible transverse pressure anisotropy: $\frac{|T_{xx} - T_{yy}|}{\frac{1}{2}(T_{xx} + T_{yy})} < 1\%$
- because...

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- no evident "almond" shape to fluid droplet
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- because...
    - Gaussian choice of initial energy density profile
    - overlap function: $$\begin{aligned} \varepsilon_+(\vec{x})\,\varepsilon_-(\vec{x}) &\propto e^{-\frac{1}{2}(\mathbf{x}_\perp - \mathbf{b}/2)^2} e^{-\frac{1}{2}(\mathbf{x}_\perp + \mathbf{b}/2)^2} \\ &= e^{-(\mathbf{x}_\perp^2 + (\mathbf{b}/2)^2)} \end{aligned}$$

lessons

- successful proof-of-principle: holographic calculation of colliding "nuclei" without (over)simplifying symmetry assumptions
- numerical solution of 5D gravitational initial value problems feasible with desktop computing resources (and good methods)
- substantial radial flow develops very early
- faster hydro onset in non-planar collisions
- much more to do:
  - variation w. impact parameter, longitudinal thickness, transverse size
  - more realistic non-Gaussian energy density profile
  - fluctuations in initial profile