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Numerical simulations of acoustically generated gravitational waves

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arXiv:1504.03291

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INT-15-2c Week 3, 20 August 2015

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2. Helsinki Institute of Physics and University of Helsinki, Finland
3. University of Sussex, United Kingdom

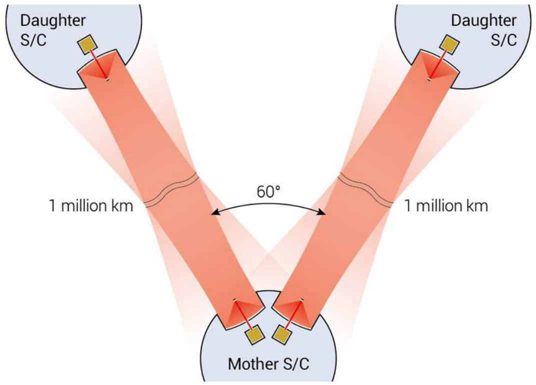
Invitation

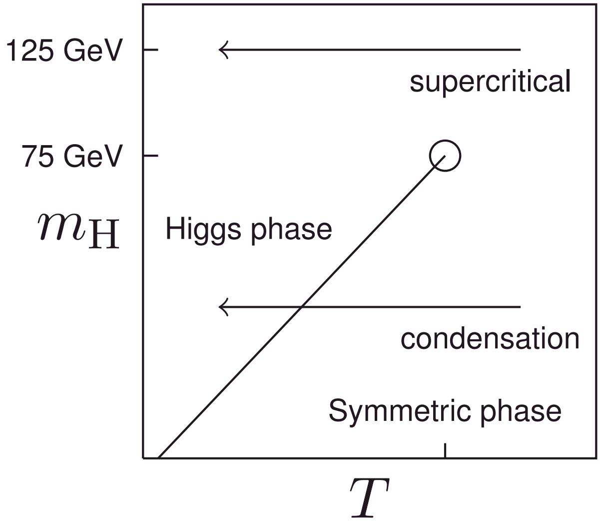
- We are hosting the second eLISA Cosmology Working Group workshop in Stavanger, 22-25 September
- Topics include first order phase transitions and GWs, so if you want to get involved this is an excellent opportunity to join in
- . . . maybe see some of you there?
- We're also hosting SEWM 2016, so even if GWs aren't your thing, perhaps we'll see you in Norway soon. . . ?

Motivation and context

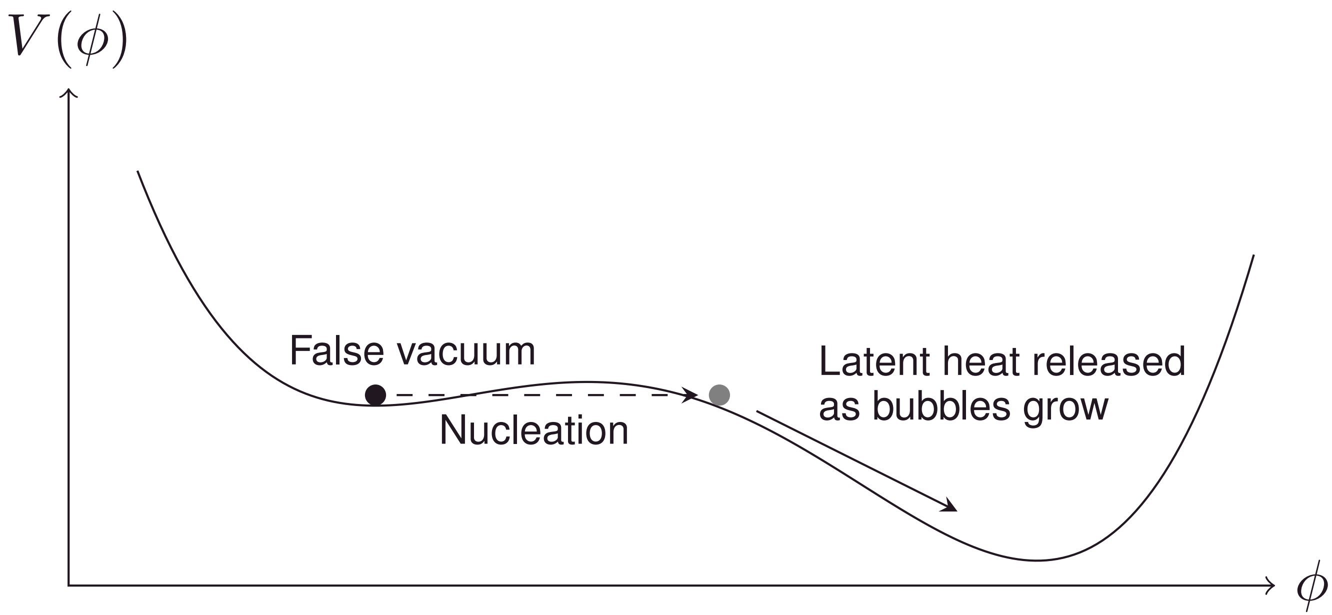
- GWs are a unique and promising test of high energy physics (advanced LIGO and VIRGO restarting; KAGRA; eLISA scheduled for 2034)
- Sources of GWs in the early universe include inflation, defects and bubble collisions at first order PTs
- Standard Model EW PT is a crossover, but first order common in extensions (singlet, 2HDM, etc.)

  Andersen, Laine *et al.*, Kozaczuk *et al.*, Kamada and Yamada, Carena *et al.*, Bödeker *et al.*. . .
- A first-order phase transition around the EW scale *could* give the right conditions for baryogenesis
- What physics can we extract from the GW power spectrum at EW scales?





First order phase transitions



Very familiar:

Water melting

Water boiling

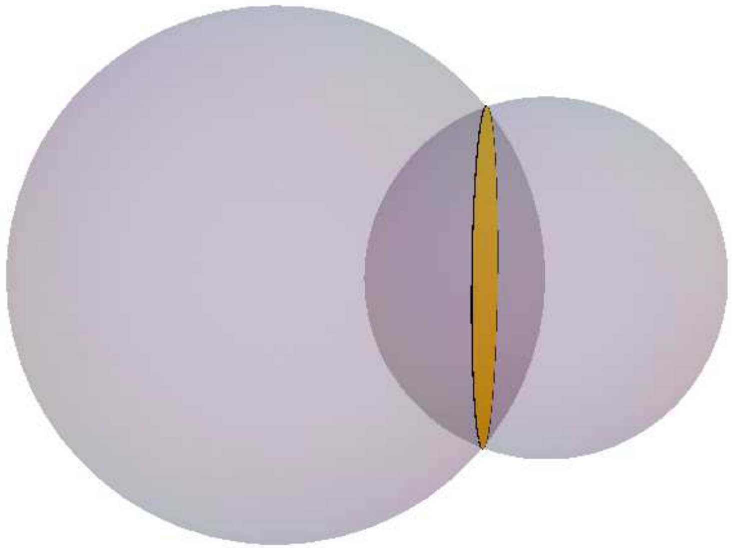
Something pretty

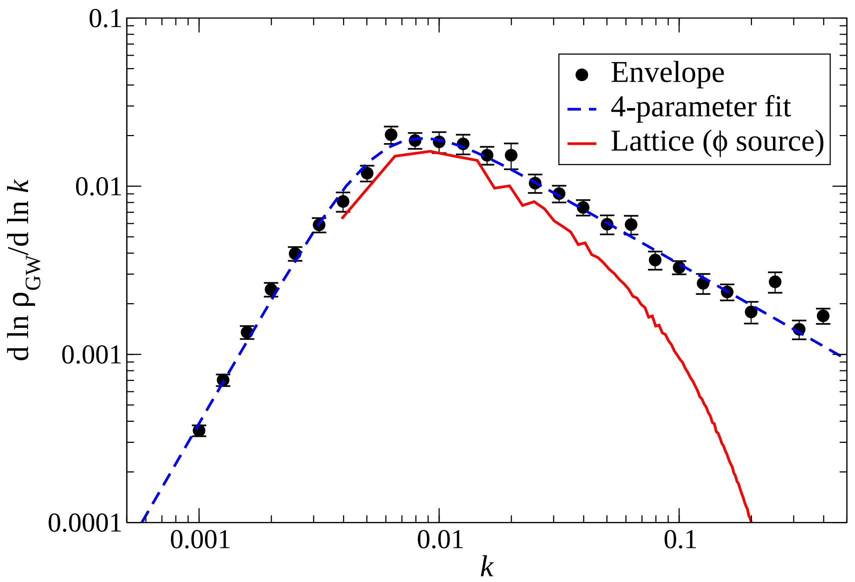
Envelope approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kamionkowsky and Turner

- Thin-walled bubbles, no fluid
- Bubbles expand with velocity v_{w}
- Stress-energy tensor $\propto R^3$ on wall
- Overlapping bubbles $\rightarrow$ GWs
- Keep track of solid angle
- Collided portions of bubbles source gravitational waves
- Resulting power spectrum is simple
  - One scale (R_*)
  - Two power laws (k^3, k^{-1})
  - Amplitude

$\Rightarrow$ 4 numbers define spectral form



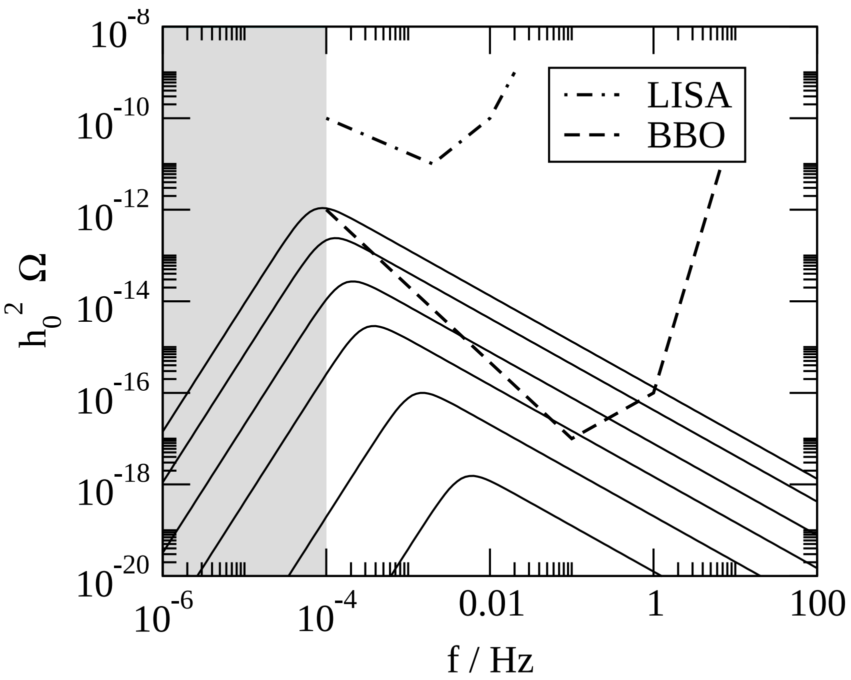


The envelope approximation makes predictions

Espinosa, Konstandin, No and Servant; Huber and Konstandin

4-5 numbers parametrise the transition:

- α, vacuum energy fraction
- $v_{\rm w}$, bubble wall speed
- κ, conversion efficiency to fluid KE
- Transition rate:
  - H_*, Hubble rate at transition
  - β, bubble nucleation rate

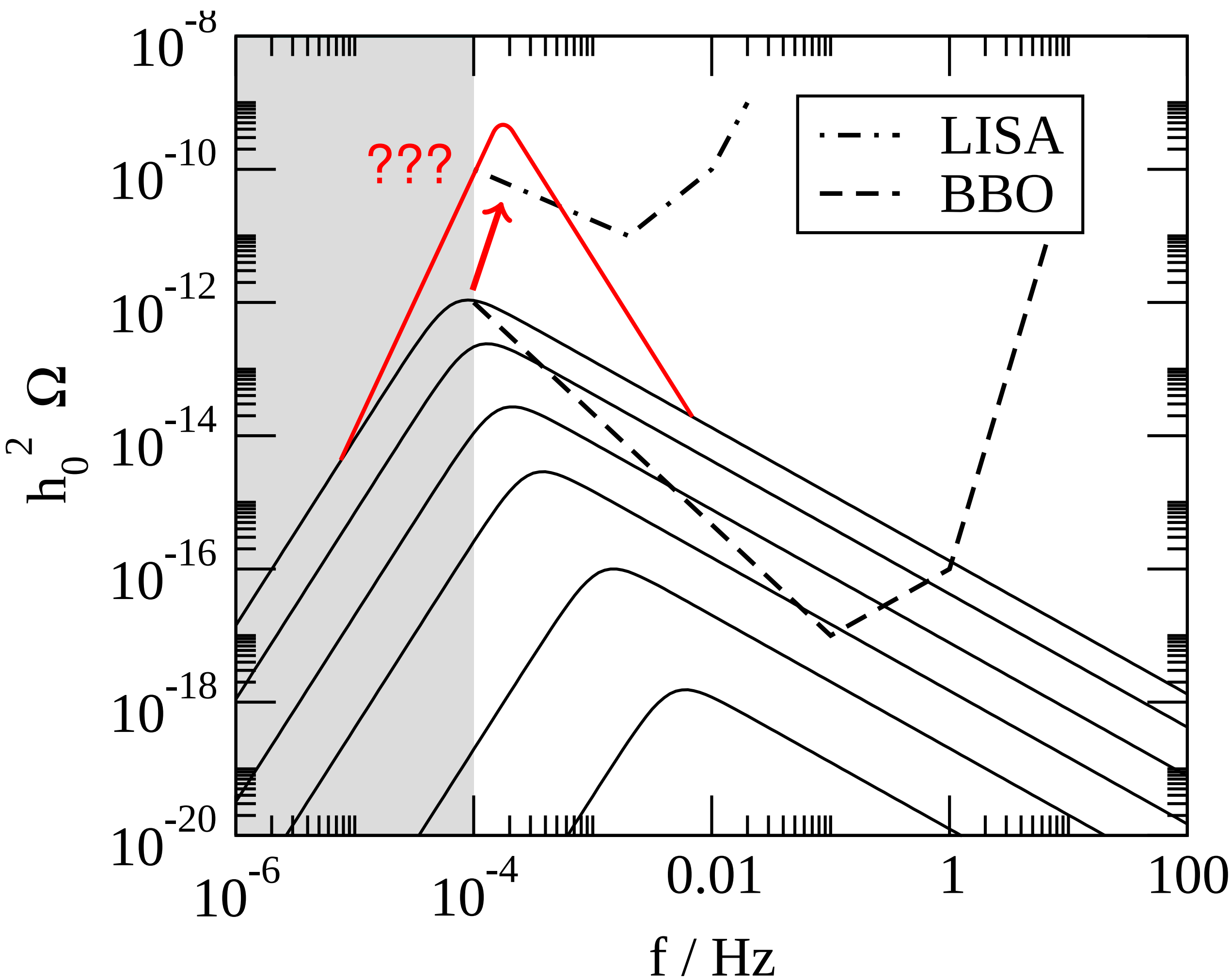


From Konstandin and Huber

Energy in GWs ($\Omega_{\rm GW} = \rho_{\rm GW}/\rho_{\rm Tot}$):

$$\Omega_{\rm GW}^{\text{envelope}} \approx \frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^2} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2\alpha^2}{(\alpha+1)^2}$$

The envelope approximation makes predictions. . . but are they too conservative?



From Konstandin and Huber

The shock waves set up by the expanding Higgs field are neglected: need to model the light fields as a relativisic plasma. Does this change things?

Our approach: field+fluid system

- Scalar ϕ + ideal fluid u^μ
  - Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits
    Ignatius, Kajantie, Kurki-Suonio and Laine

$$\partial_\mu T^{\mu\nu} = \partial_\mu (T^{\mu\nu}_{\text{field}} + T^{\mu\nu}_{\text{fluid}}) = 0$$

  - Parameter η sets the scale of friction due to plasma

$$\partial_\mu T^{\mu\nu}_{\text{field}} = \eta u^\mu \partial_\mu \phi \partial^\nu \phi \qquad \partial_\mu T^{\mu\nu}_{\text{fluid}} = -\eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

- Effective potential $V(\phi, T)$ can be kept simple

$$V(\phi, T) = \tfrac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \tfrac{1}{3}AT\phi^3 + \tfrac{1}{4}\lambda\phi^4$$

  - γ, T_0, A, λ chosen to match scenario of interest

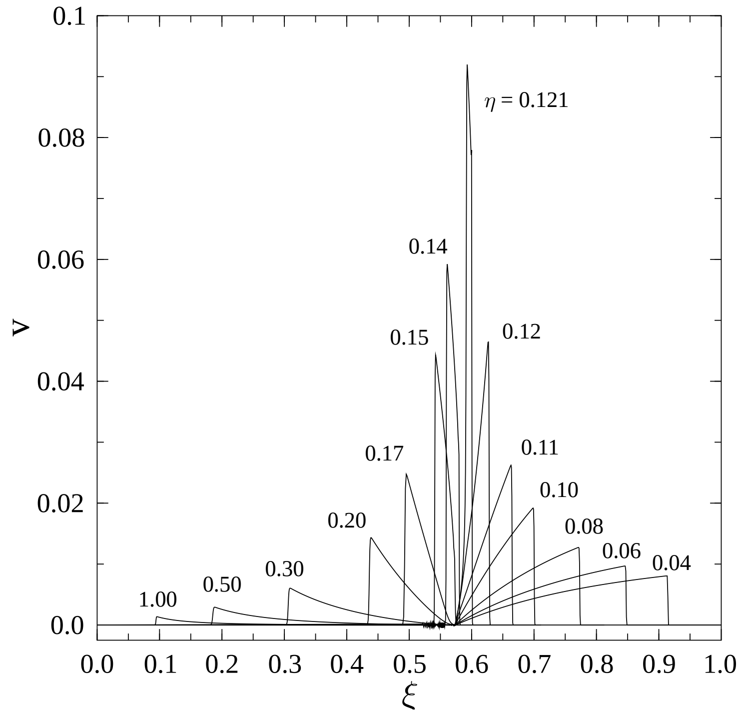
- Equations of motion (+ continuity equation)

$$\partial_\mu \partial^\mu \phi + \frac{\partial V(\phi,T)}{\partial \phi} = -\eta u^\mu \partial_\mu \phi$$

$$\partial_\mu \left\{ [\epsilon + p]\, u^\mu u^\nu - g^{\mu\nu}[p - V(\phi, T)] \right\} = \left(\eta u^\mu \partial_\mu \phi + \frac{\partial V(\phi,T)}{\partial \phi} \right) \partial^\nu \phi$$

The η parameter

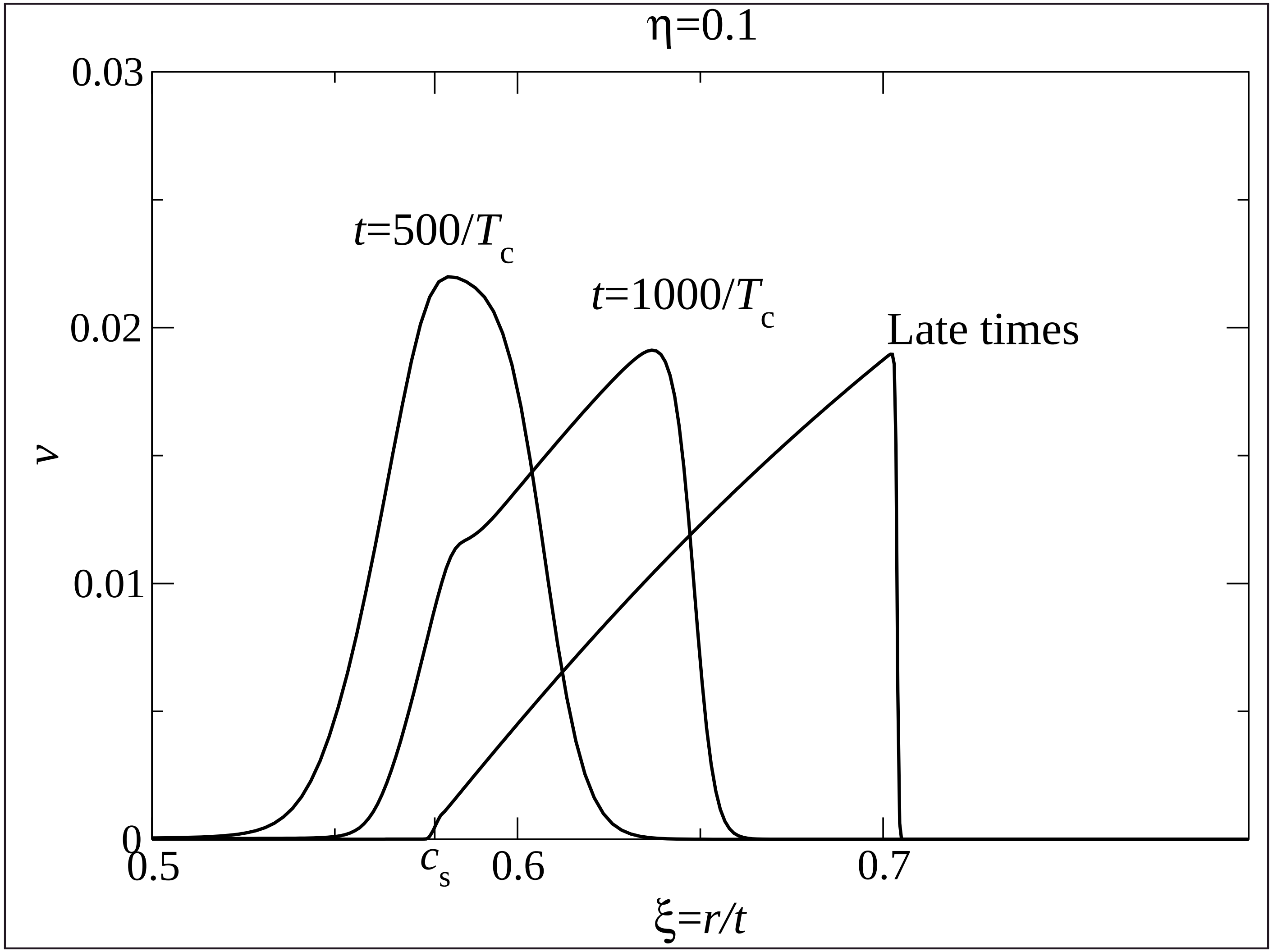
- The value of η sets the velocity of bubble wall v_w Kurki-Suonio and Laine



- Distinguish between:
  - Detonations ($v_\mathrm{w} > c_\mathrm{s}$, rarefaction wave behind wall)
  - Jouguet case ($v_\mathrm{w} \approx c_\mathrm{s}$, subsonic compared to fluid in front; supersonic compared to fluid behind)
  - Deflagrations ($v_\mathrm{w} < c_\mathrm{s}$, shock front leads wall)

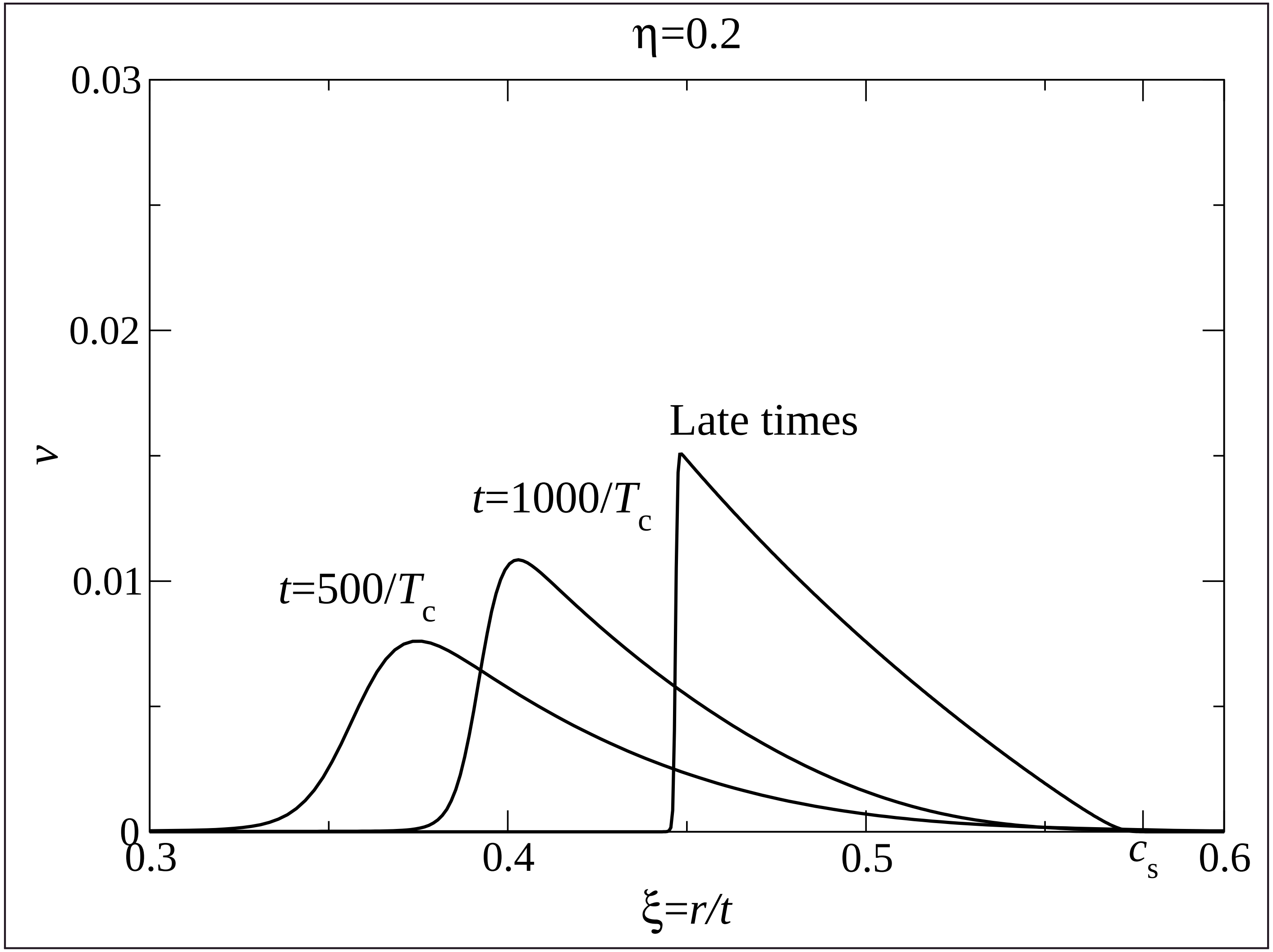
Velocity profile development - detonation [optional movie]

Here, $\eta = 0.1$ (detonation)



Velocity profile development - deflagration [optional movie]

Here, $\eta = 0.2$ (deflagration)



Gravitational waves Weinberg

- Weak field approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- after some algebra, and assuming a harmonic coordinate system, we get

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

- Consider only terms at lowest order in the perturbation h_{ij}

$$T_{ij}^{\mathrm{f}} = W^2(\epsilon + p)V_i V_j \qquad T_{ij}^{\phi} = \partial_i\phi\partial_j\phi$$

– these are our two sources (fluid and field).

Gravitational waves from simulations of the early universe

- As we have seen, metric perturbations evolve as

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

with transverse-traceless (TT) projection in momentum space,

$$T_{ij}^{\mathrm{TT}}(\mathbf{k}) = \Lambda_{ij,lm}(\hat{\mathbf{k}}) T_{lm}(\mathbf{k})$$

– costly! Lots of FFTs...

- Fortunately, can use Garcia-Bellido and Figueroa; Easther, Giblin and Lim

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi T_{ij}^{\text{Traceless}}$$

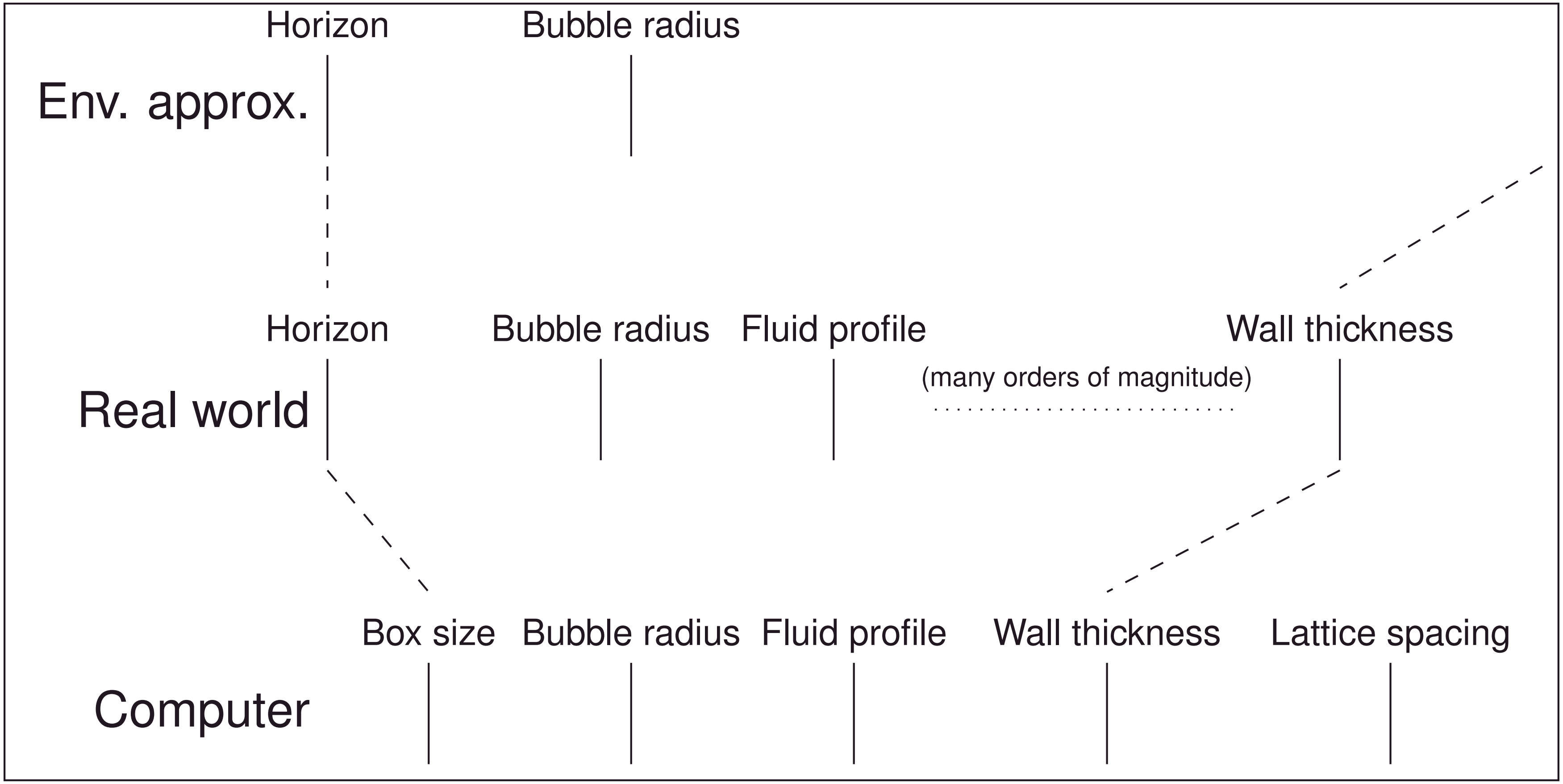
and project $h_{ij}(k) = \Lambda_{ij,lm}(k) u_{ij}(k)$ later

- Power $\rho_{\text{GW}} = T_{00}^{\text{grav}}$ per logarithmic interval,

$$\frac{d\rho_{\text{GW}}}{d\ln k} = \frac{1}{32\pi G V} \frac{k^3}{(2\pi)^3} \int d\Omega \; \Lambda_{ij,lm}(\hat{\mathbf{k}}) \dot{u}_{ij}(t,\mathbf{k}) \dot{u}_{lm}^*(t,\mathbf{k})$$

Dynamic range issues

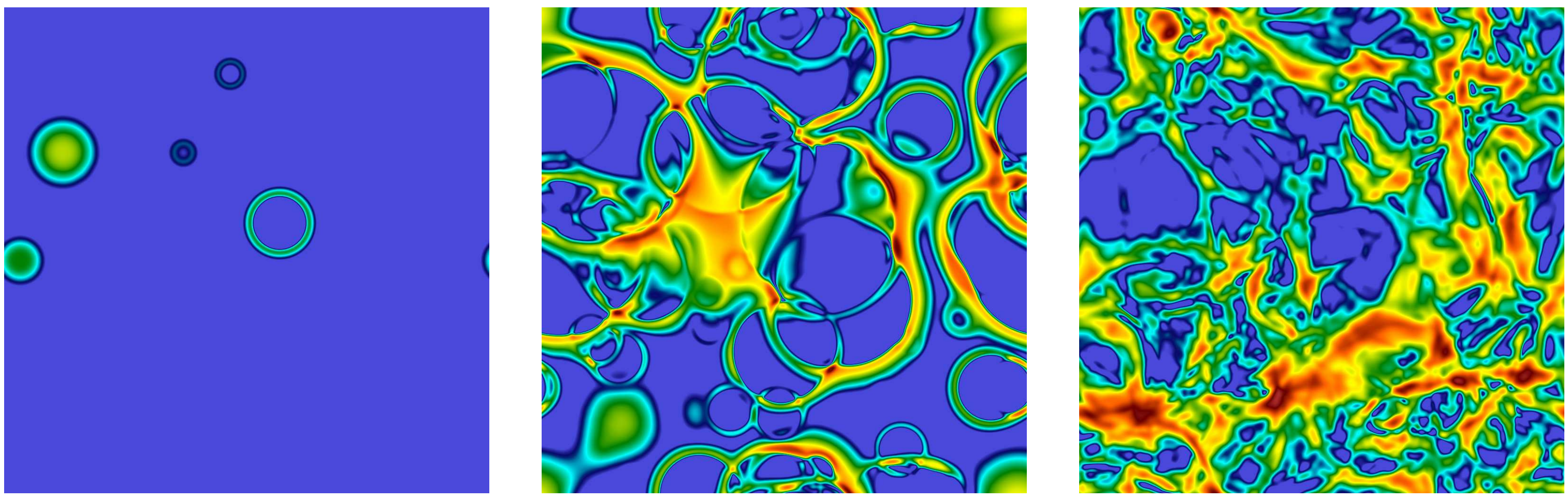
- Most realtime lattice simulations in the early universe have a single [nontrivial] length scale
- Here, many length scales important



- Simulations in arXiv:1504.03291 are with 2400^3 lattice, $\delta x = 2/T_c$
  $\rightarrow$ approx 200k CPU hours each ($\sim$ 3M total)

Simulation slice example [optional movie]

Simulations at 1024^3, deflagration, fluid kinetic energy density, $\sim$250 bubbles

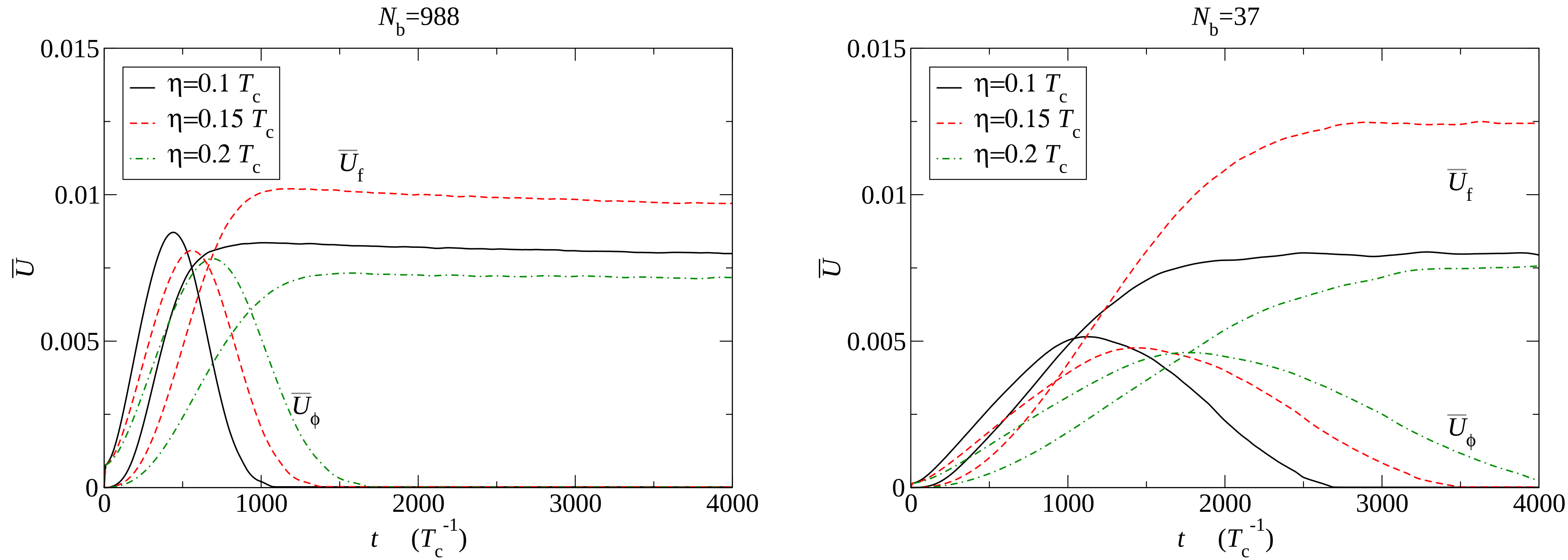


$t = 500\ T_{\rm c}^{-1}$ $t = 750\ T_{\rm c}^{-1}$ $t = 1000\ T_{\rm c}^{-1}$

How the sources behave over time

- $\overline{U}_{\mathrm{f}}$ is the rms fluid velocity; $\overline{U}_{\phi}$ the analogous field quantity
- Constructed from T^{f}_{ii} and T^{ϕ}_{ii}, they indicate how strong each source is

$$(\bar\epsilon+\bar p)\overline{U}_{\mathrm{f}}^2 = \frac{1}{V}\int d^3x\, \underbrace{W^2(\epsilon+p)}_{(T^{\mathrm{f}}_{ii})^2} \qquad (\bar\epsilon+\bar p)\overline{U}_{\phi}^2 = \frac{1}{V}\int d^3x\, \underbrace{(\partial_i\phi)^2}_{(T^{\phi}_{ii})^2}$$

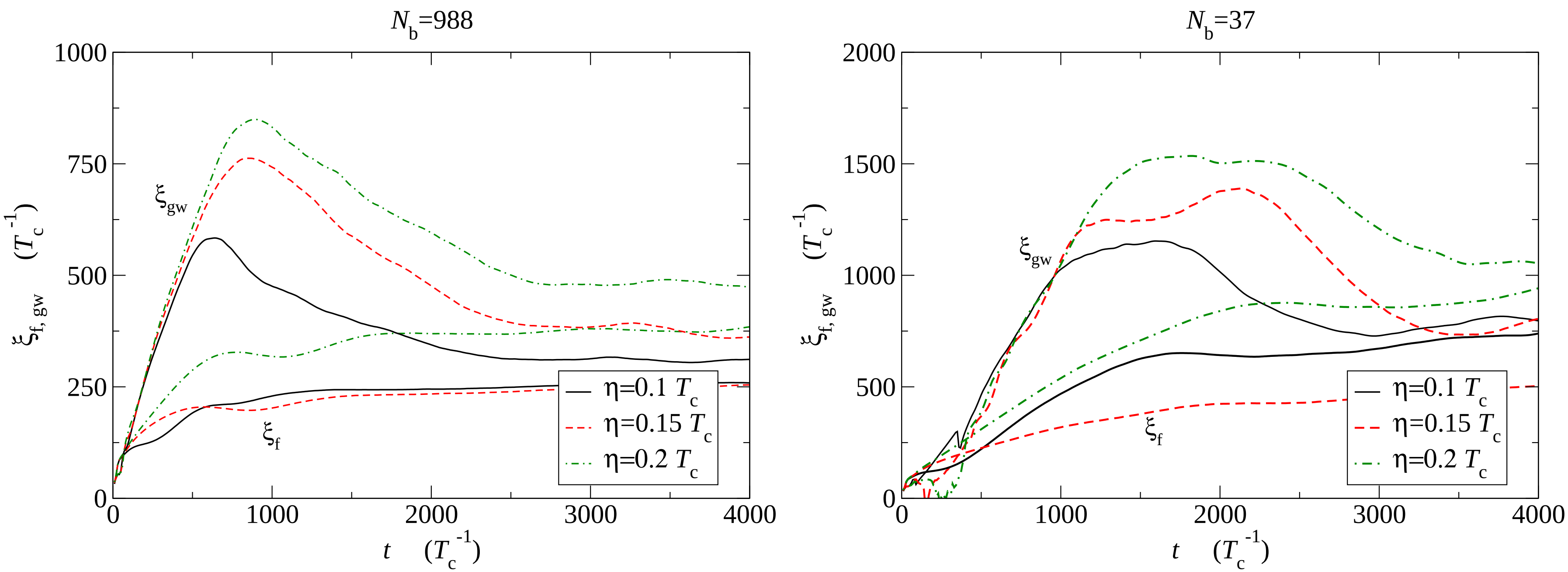


Fluid characteristic length scale is imprinted in GW power spectrum

Define the fluid integral scale

$$\xi_{\mathrm{f}} = \frac{1}{\langle V^2 \rangle} \int \frac{d^3 k}{(2\pi)^3} |k|^{-1} P_V(k)$$

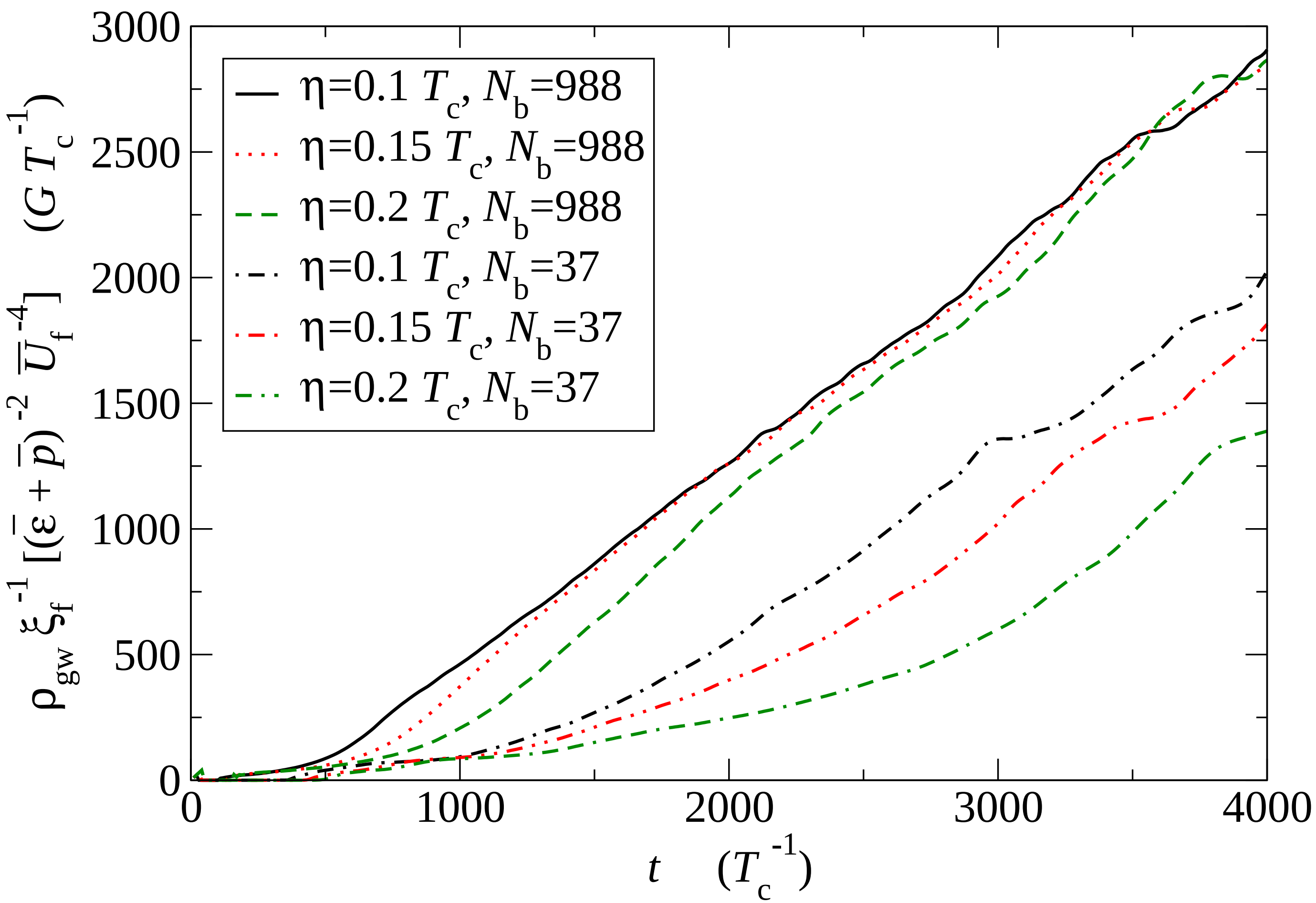
and the analogous quantity ξ_{GW} for the gravitational wave power spectrum.



This length scale is what sets the peak of the fluid power spectrum.

Acoustic waves source linear growth of gravitational waves

- Sourced by T^{f}_{ij} only (T^{ϕ}_{ij} source is small constant shift)



- Source generically scales as $\rho_{\mathrm{GW}} \propto t[G\xi_{\mathrm{f}}(\bar{\epsilon}+\bar{p})^2\overline{U}_f^4]$

Lifetime of sound waves and increase in GW power

- Does the acoustic source matter?
  - Sound is damped by (bulk and) shear viscosity Arnold, Dogan and Moore; Arnold, Moore and Yaffe

$$\left(\frac{4}{3}\eta_{\rm s} + \zeta\right)\nabla^2 V^i_{\parallel} + \ldots \Rightarrow \tau_\eta(R) \sim \frac{R^2\epsilon}{\eta_{\rm s}}$$

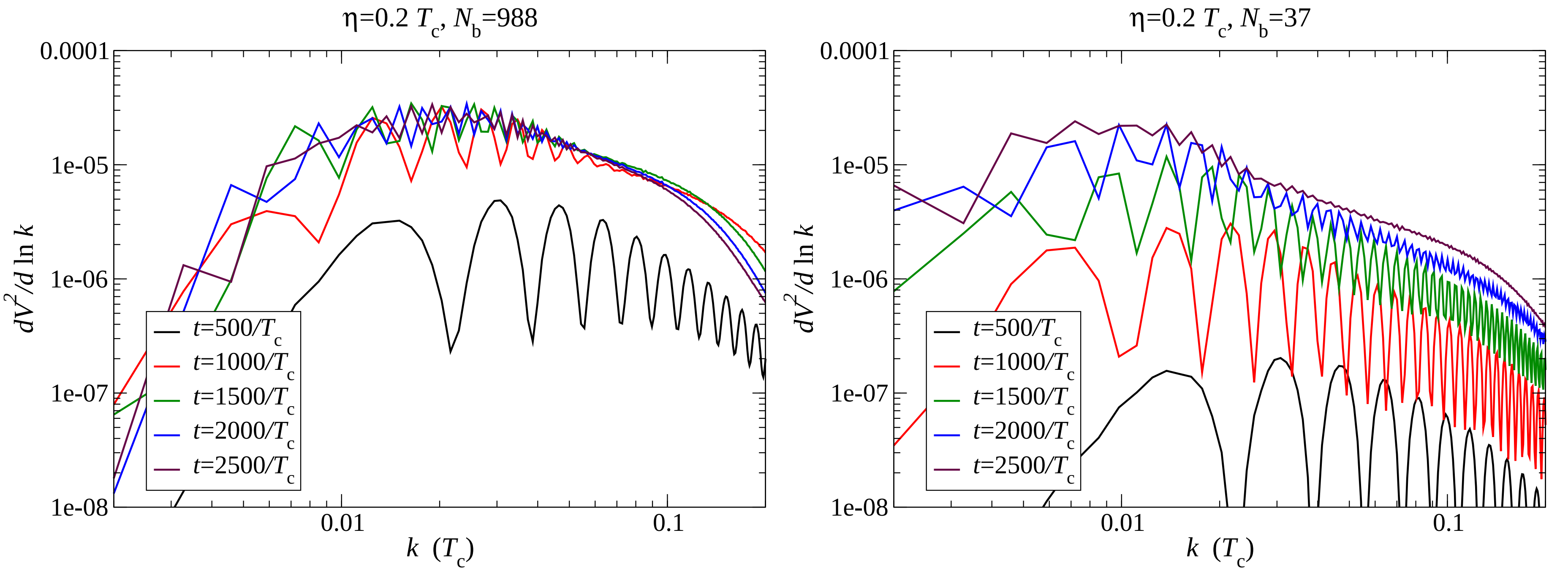
  - Compared to $\tau_{H_*} \sim H_*^{-1}$, on length scales

$$R^2 \gg \frac{1}{H_*}\frac{\eta_{\rm s}}{\epsilon} \sim 10^{-11}\frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100\,{\rm GeV}}\right)$$

    the Hubble damping is faster than shear viscosity damping.
- Does the acoustic source enhance GWs?
  - Yes, we have

$$\Omega_{\rm GW} \approx \left(\frac{\kappa\alpha}{\alpha+1}\right)^2 (H_*\tau_{H_*})(H_*\xi_{\rm f}) \Rightarrow \frac{\Omega_{\rm GW}}{\Omega_{GW}^{\text{envelope}}} \gtrsim 60\frac{\beta}{H_*}.$$

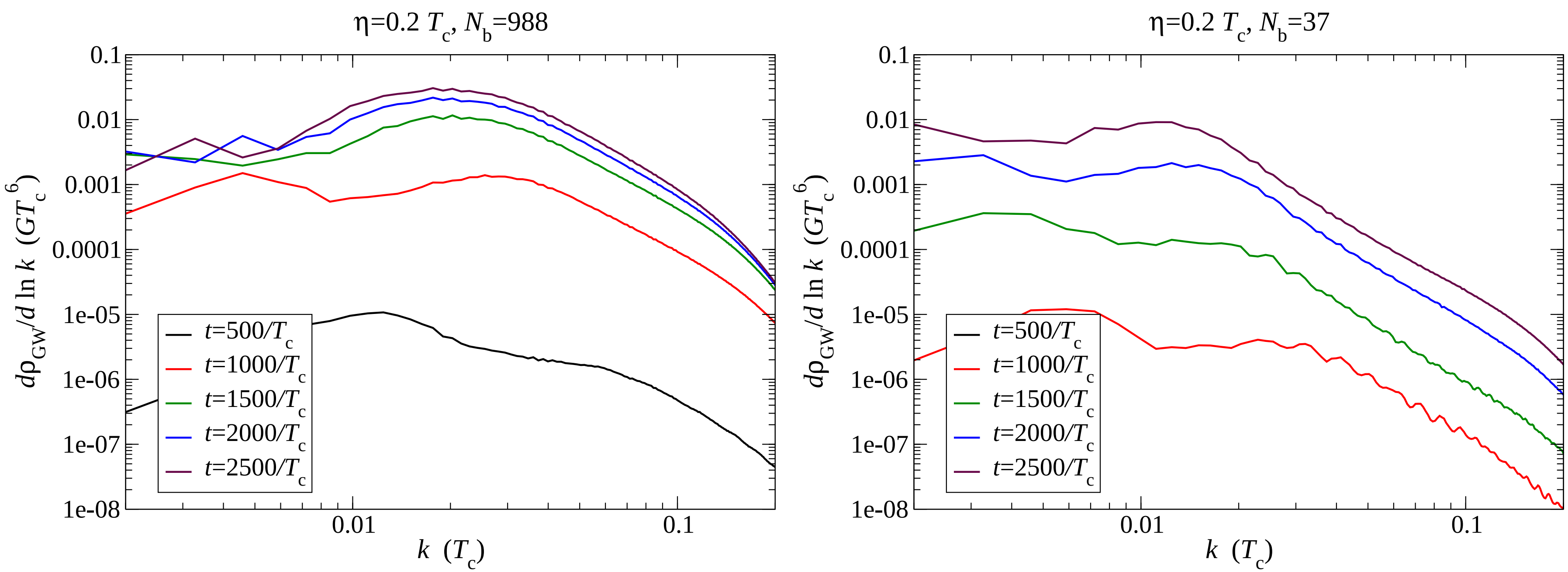
Velocity power spectra and power laws



- Weak transition: $\alpha_{T_\mathrm{N}} = 0.01$, $v_\mathrm{w} = 0.44$
- Power law behaviour above peak is k^{-1}, approximately
- “Ringing” due to simultaneous bubble nucleation, not physically important
- Power is in the longitudinal modes – acoustic waves, not turbulence
- If we know $\mathrm{d}V^2/\mathrm{d}\ln k$, can work out $\dot\rho_\mathrm{GW}/\mathrm{d}\ln k\ldots$?

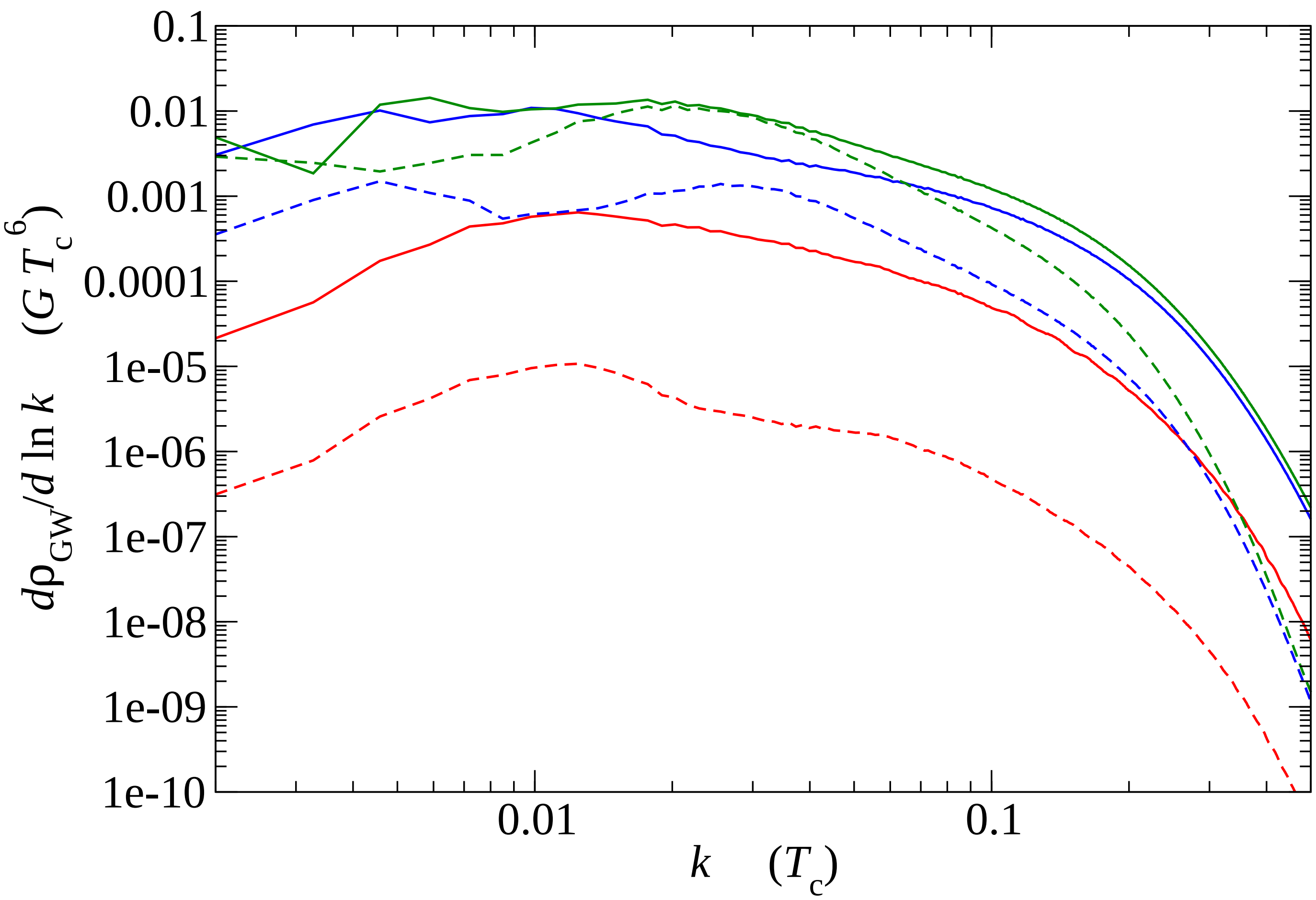
GW power spectra and power laws

- Sourced by T^{f}_{ij} only



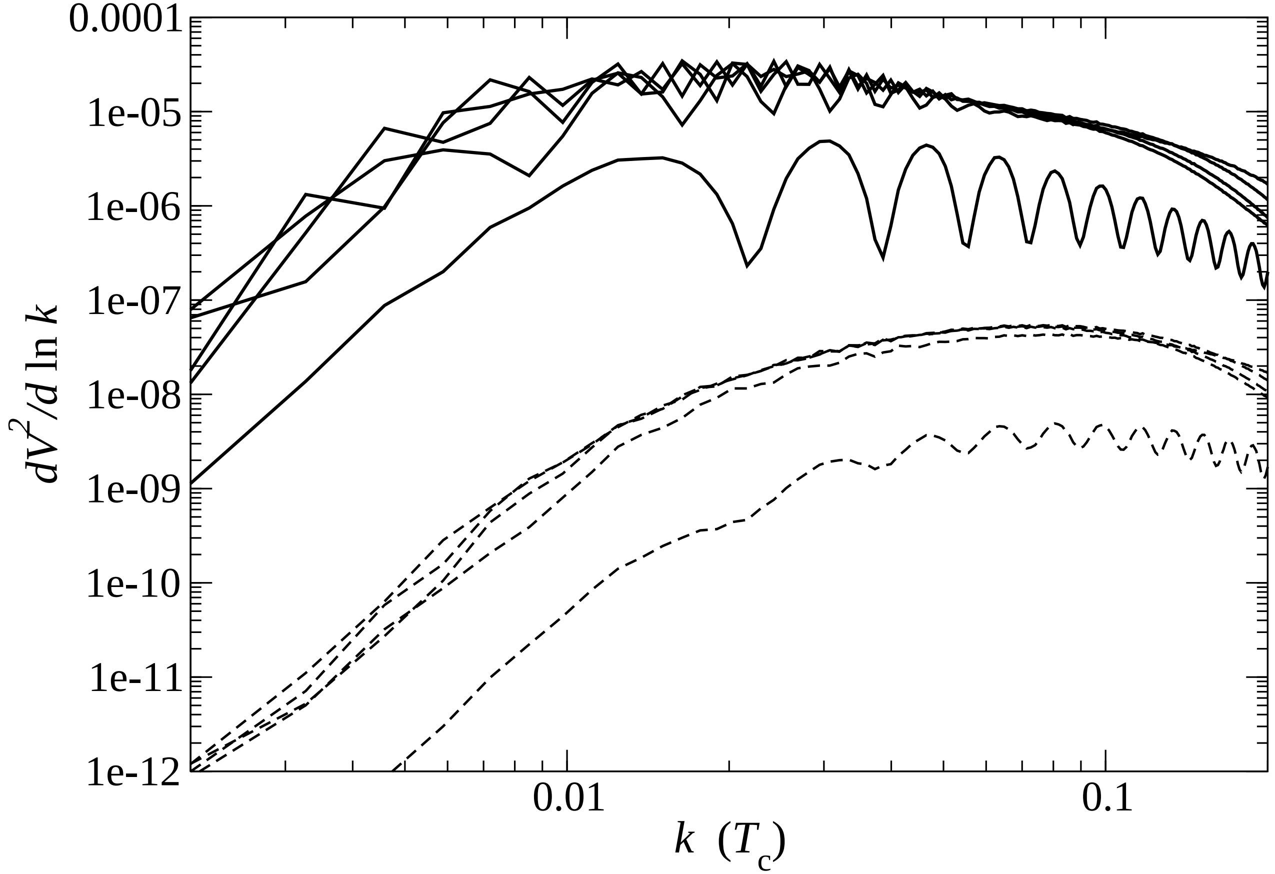
- Approximate k^{-3} power spectrum
- Finite size of box means that we choose not to probe behaviour below peak k

GW power spectra – field and fluid sources



- By late times, fluid source dominates at all length scales
- $500/T_\mathrm{c}$, $1000/T_\mathrm{c}$, $1500/T_\mathrm{c}$ (‘before’, ‘during’, ‘after’ collision)
- Fluid source shown by dashed lines, total power solid lines

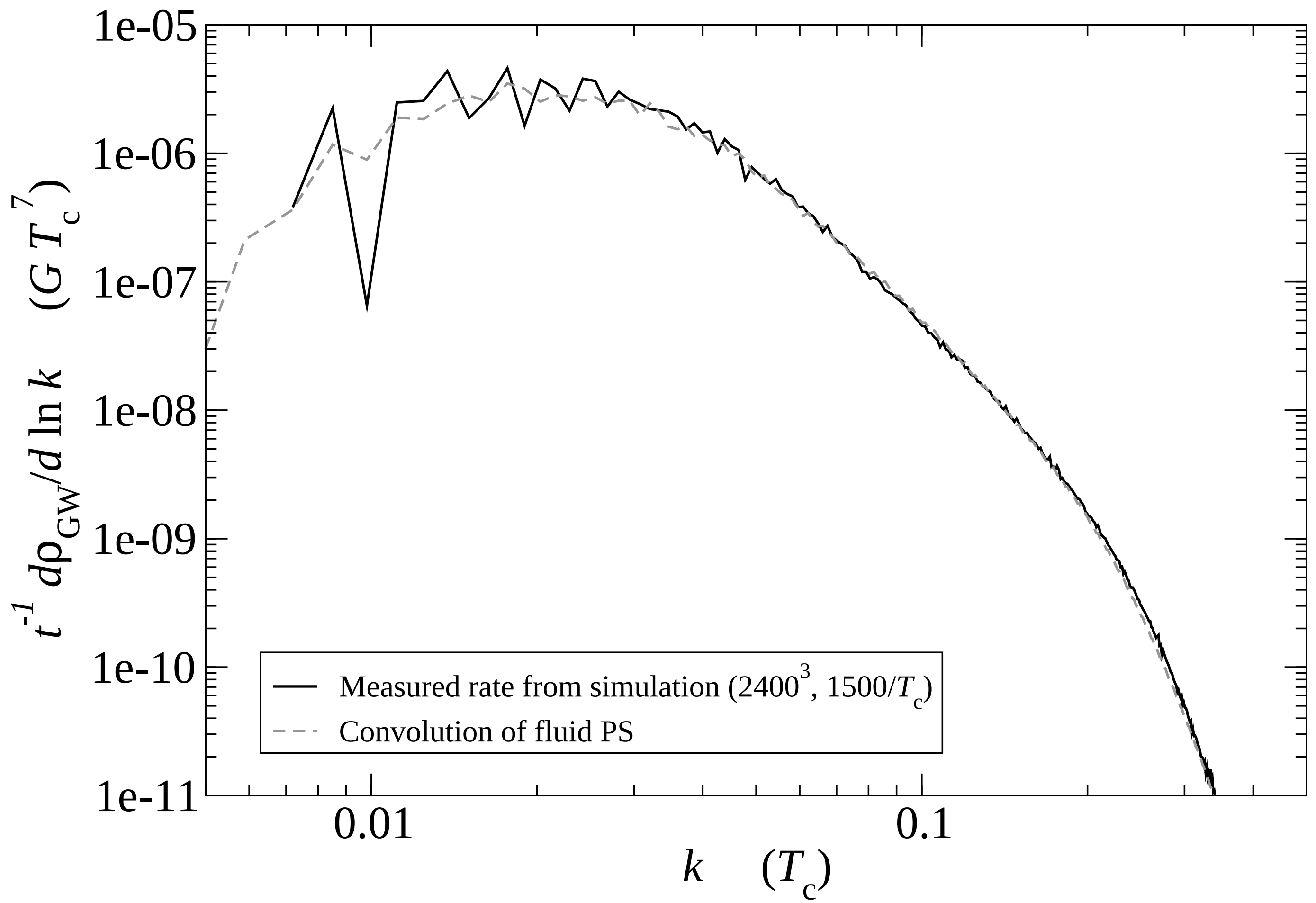
Transverse versus longitudinal modes – turbulence?



- Most power is in the longitudinal modes – acoustic waves, not turbulence
- System is quite linear. Reynolds number is ~ 100.

Going from the profile to fluid power to GW power

Going from a fluid power spectrum to the GW power spectrum is easy:



where the dashed curve is obtained by performing a numerical convolution of the fluid power spectrum.

Summary and outlook

- Today
  - New source of GWs: sound waves from colliding bubble droplets
  - Rate of GW energy production is **generically** $\rho_{\mathrm{GW}} \propto t[G\xi_{\mathrm{f}}(\bar{\epsilon}+\bar{p})^2\overline{U}_f^4]$
  - $\mathrm{O}(10^2)$ enhancement over envelope approximation at EW scale
    $\rightarrow$ good news for models that do not produce strongly first-order PTs
  - Power laws different from envelope approximation
  - Still four parameters – power spectrum remains simple to parametrise
  - Need larger simulations – 18M CPU hours awarded by PRACE
- Soon
  - Instabilities Megevand, Membiela and Sanchez
  - Turbulence
  - Strong transitions ($\alpha_{T_{\mathrm{N}}} \sim 1$)
  - ‘Inverse acoustic cascade’ Kalaydzhyan, Shuryak
  - Runaway transitions
- Building a science case for eLISA
- Implications for DECIGO, BBO