Anomalies and Non-Equilibrium

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Motivation

- Hydrodynamics is a very universal effective field theory used to describe heavy-ion collisions and condensed matter systems
- Inclusion of anomalies of discrete symmetries
- Applications to quark-gluon plasma, non-equilibrium modelling of chiral phenomena
- Classical manifestations anomalies
- Anomalies in turbulence
- Relation between partition functions and entropy current constraints

COINCIDENCES

	Analogies between turbulence and quantum field theory	
	Turbulence	QFT
Strong coupling	high-*Re* Navier-Stokes	QCD at low energies
Exactly soluble	Burgers Model	Schwinger Model
Numerics	Integration of Navier-Stokes	Lattice gauge simulations
Large N	Kraichnan DIA	't Hooft planar limit
Heuristics	Vortex-stretching	Flux tubes
Non-equilibrium	Energy cascade	Out-of-equilibrium QGP
Anomalies	Dissipation anomaly	Chiral anomalies
Sociology	Navier-Stokes regularity	Yang-Mills mass gap

Vortex-Streching

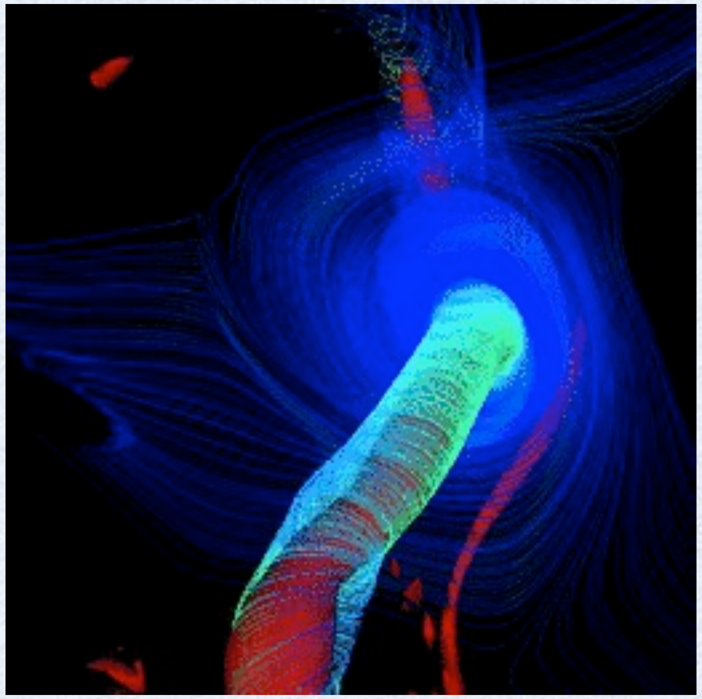
In the long-wavelength, non-relativistic limit the fluid equations are:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \triangle \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0.$$

Experiments suggest that energy dissipation

$$\varepsilon = \nu |\nabla \mathbf{u}|^2$$

does not vanish in the limit of vanishing viscosity, for a variety, of turbulent flows. This fact was a basic assumption in the 1941 theory of turbulence due to A. Kolmogorov.



source University of Münster, Institut for Physics

Onsager noticed that if we divide the above equation by viscosity and take the zero-viscosity limit the velocity becomes non-differentiable.

We can also rewrite the RHS of the energy dissipation as proportional to enstrophy. Thus turbulence is a mechanism of enstrophy generation, which is know as vortex-streching.

Effective Equations

Consider a locally space-averaged velocity

$$\overline{\mathbf{u}}_\ell(\mathbf{x}) = \int \mathrm{d}^d r\, G_\ell(\mathbf{r})\mathbf{u}(\mathbf{x}+\mathbf{r})$$

where $G_\ell(\mathbf{r}) = \ell^{-d} G(\mathbf{r}/\ell)$ is an averaging kernel that is non-negative, smooth and rapidly decaying.

We can averege out the Navier-Stokes equations

$$\partial_t \overline{\mathbf{u}}_\ell + \nabla \cdot [\overline{\mathbf{u}}_\ell \overline{\mathbf{u}}_\ell + \boldsymbol{\tau}_\ell] = -\nabla \overline{p}_\ell + \nu \triangle \overline{\mathbf{u}}_\ell, \quad \nabla \cdot \overline{\mathbf{u}}_\ell = 0$$

where we introduced subscale stress-tensor

$$\boldsymbol{\tau}_\ell = \overline{(\mathbf{u} \otimes \mathbf{u})}_\ell - \overline{\mathbf{u}}_\ell \otimes \overline{\mathbf{u}}_\ell$$

This is analogous to the Wilson-Kadanoff RG approach. The viscous term can be show to be irrelevant in terms of RG analysis. This results in a simplification in the inertial range of scales

$$\partial_t \overline{\mathbf{u}}_\ell + \nabla \cdot [\overline{\mathbf{u}}_\ell \overline{\mathbf{u}}_\ell + \boldsymbol{\tau}_\ell] = -\nabla \overline{p}_\ell, \quad \nabla \cdot \overline{\mathbf{u}}_\ell = 0$$

Dissipative Anomaly

The effective equations are equivalent to the ones obtained by coarse-graining procedure of incompressible Euler equations. However, the equations are not well-defined for the singular velocity fields and only meaningful in a sense of distributions.

As realized by Onsager, the Euler equations in this generalized sense do not guarantee the conservation of energy. It can be shown that the generalized energy balance equation has the form

$$\partial_t \left(\frac{1}{2}|\mathbf{u}|^2\right) + \nabla \cdot \left[\left(\frac{1}{2}|\mathbf{u}|^2 + p\right)\mathbf{u}\right] = -D(\mathbf{u}),$$

where $D(\mathbf{u})$ is a non-vanishing distribution (Duchon, Robert)

$$D(\mathbf{u}) = \lim_{\ell\to 0} \frac{1}{4\ell} \int \mathrm{d}^d r \, (\nabla G)_\ell(\mathbf{r}) \cdot \left[\delta\mathbf{u}(\mathbf{r})|\delta\mathbf{u}(\mathbf{r})|^2\right]. \quad \delta\mathbf{u}(\mathbf{r};\mathbf{x}) \equiv \mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x}).$$

This is called a dissipative anomaly. Polyakov pointed out that the non-conservation of the symmetries of Euler equation is analogous to the anomalous non-conservation of symmetries in QFT.

Khokhlov saw-tooth

Let us consider the so-called Burgers equation. It is a 1d model, in which we can se the dissipative anomaly.

$$\partial_t u + u\partial_x u = \nu \partial_x^2 u.$$

This equation has a simple solution known as Khokhlov saw-tooth.

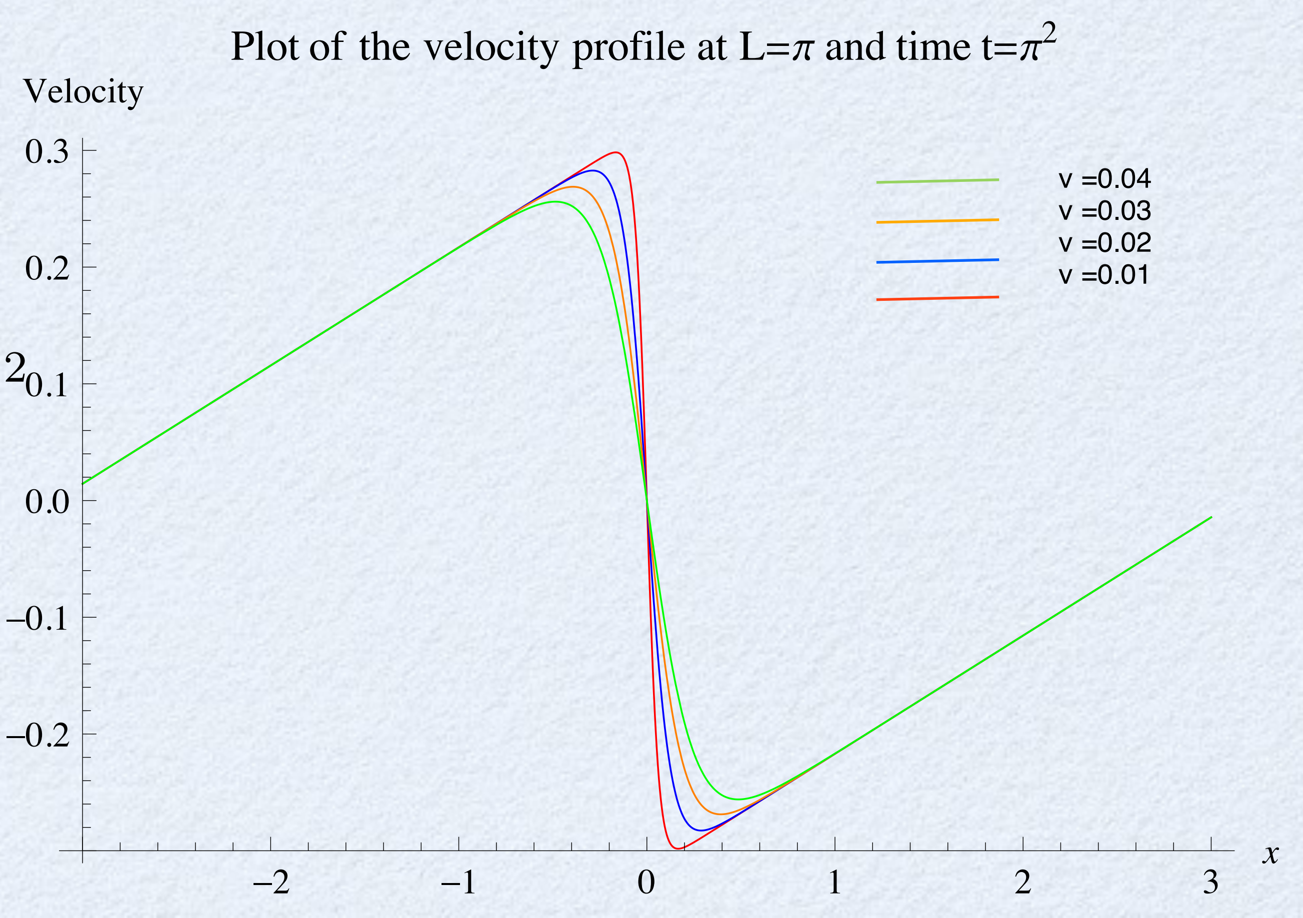
$$u(x,t) = \tfrac{1}{t}[x - L\tanh(\tfrac{Lx}{2\nu t})].$$

Using the explict formula we have

$$\begin{aligned}\frac{1}{2}\langle u^2(t)\rangle &= \frac{1}{2L}\int_{-L}^{L}\frac{1}{2}u^2(x,t)dx \\ &= \frac{1}{2L}\int_0^L\left(\frac{x-L}{t}\right)^2 dx = \frac{1}{6}\left(\frac{L}{t}\right)^2\end{aligned}$$

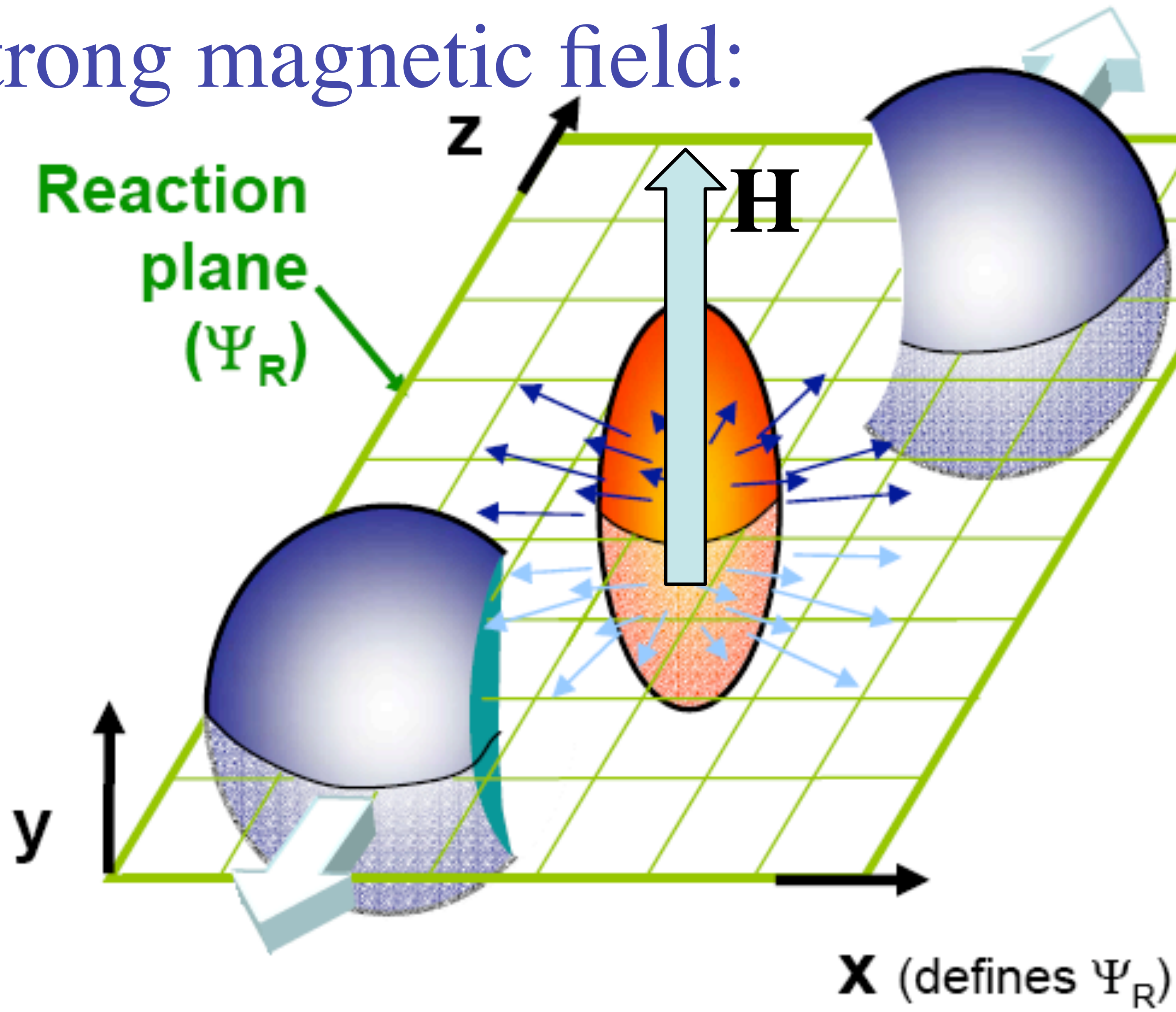
The energy dissipation is positive

$$\langle\varepsilon(t)\rangle = -\frac{d}{dt}\frac{1}{2}\langle u^2(t)\rangle = \frac{1}{3}\frac{L^2}{t^3}$$



Nucleus-Nucleus collision

Relativistic ions create a strong magnetic field:



Chiral vortical effect

$$J^\mu = \xi \omega^\mu$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$$

$$\xi = C_{anom}\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + p}\right)$$

Chiral magnetic effect

$$J^\mu = \xi_B B^\mu$$

$$B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu F_{\lambda\rho}$$

$$\xi_B = C_{anom}\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + p}\right)$$

PARITY-ODD HYDRO

Relativistic fluid with one conserved charge described by conservation laws

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

plus equations that express $T^{\mu\nu}$ and J^μ in terms of local temperature T, chemical potential μ, and fluid velocity u^μ:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$J^\mu = n u^\mu + \nu^\mu$$

The definition of velocity is ambiguous beyond leading order. We fix it by imposing (Landau frame)

$$u_\mu \tau^{\mu\nu} = 0$$

Vorticity

$$\partial_\mu \left(s u^\mu - \frac{\mu}{T} \nu^\mu \right) = -\partial_\mu \left(\frac{\mu}{T} \right) \nu^\mu - \frac{1}{T} \partial_\mu u_\nu \tau^{\mu\nu}$$

The left hand side is then interpreted as the divergence of the entropy current $\partial_\mu J_s^\mu$. When the current is chiral, or when the fundamental theory does not preserve parity, it is possible to construct one additional Lorentz structure that may appear in the current J_μ

$$\omega^\mu \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

The new term is consistent with Lorentz symmetry, but its divergence is now:

$$\partial_\mu J_s^\mu = \ldots - \xi(T,\mu) \partial_\mu \left(\frac{\mu}{T} \right) \omega^\mu$$

One has to revisit the entropy current argument

$$J_s^\mu = \ldots + D(T,\mu) \omega^\mu$$

Hydrodynamics with anomalies

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

$$\partial_\mu J^\mu = C_{anom} E^\mu B_\mu$$

There are only two new terms consistent with symmetry that can be added to the entropy current

$$J_s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu$$

Requiring that contributions with undetermined signs cancel on both side we find $\bar{\mu} = \mu/T$

$$D(\bar{\mu}) = T^2 \frac{1}{3} C_{anom} \bar{\mu}^3; \qquad D_B(\bar{\mu}) = T^2 \frac{1}{2} C_{anom} \bar{\mu}^2$$

We have new transport coefficients (vortical and magnetic conductivities) up to an integration constant

$$\xi = C_{anom}\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right); \qquad \xi_B = C_{anom}\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right)$$

Gravitational anomalies

In the previous calculation we had two integration constants which we cannot constrain by hydrodynamic reasoning. However, we can calculate them using linear response theory in weakly coupled field theory. It turns out these constants emerge as a consequence of gravitational anomalies

$$\xi_M = \lim_{k_n \to 0} \sum_{ij} \epsilon_{ijk} \frac{-i}{2k_n} \langle J^i_M T^{0j} \rangle |_{\omega=0}$$

$$\xi^B_{MN} = \lim_{k_n \to 0} \sum_{ij} \epsilon_{ijk} \frac{-i}{2k_n} \left\langle J^i_M J^j_N \right\rangle |_{\omega=0}$$

In the case of vortical conductivity we get T^2 correction

$$\xi_M = \frac{1}{8\pi^2} \sum_{f=1}^{N} T^f_{M\ f} \left((\mu^f)^2 + \frac{\pi^2}{3} T^2 \right)$$

Kinetic theory

Kinetic theory treats the evolution of the one-particle distribution function, which can be associated with the number of on-shell particles per unit phase space

$$f(\vec{p}, \vec{x}; t) = \frac{dN}{d^3p d^3x}$$

If collisions between particles can be neglected and there is no Berry phase effects, the evolution of $f(\vec{p}, \vec{x}; t)$ follows from Liouville's theorem

Given this interpretation the particle number density should be proportional to

$$\int d^3p f(\vec{p}, \vec{x}; t)$$

Summing instead with a weight of particle energy, one expects a result proportional to the product of number density and energy, or energy density, which is a part of the energy-momentum tensor.

Hydro ⇔ Kinetic Theory

We can derive hydrodynamic quantities from kinetic theory e.g.

$$T^{\mu\nu} \equiv \int \frac{d^4p}{(2\pi)^3} p^\mu p^\nu \delta(p^\mu p_\mu - m^2) 2\theta(p^0) f(p,x)$$

If we take the distribution function in equilibrium we recover energy-momentum tensor of a perfect fluid. One can derive the correspondence between kinetic theory out of equilibrium and viscous hydrodynamics by considering small departures from equilibrium where

$$f(p^\mu, x^\mu) = f_{\mathrm{eq}}\left(\frac{p^\mu u_\mu}{T}\right)\left[1 + \delta f(p^\mu, x^\mu)\right]$$

This procedure allows one to study dissipative effects (first order in the derivatives of fields). Performing the integral one gets perfect fluid contribution plus shear tensor

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \int \frac{d^4p}{(2\pi)^3} p^\mu p^\nu f_{\mathrm{eq}} \delta f = T^{\mu\nu}_{(0)} + \pi^{\mu\nu}$$

Anomalous part

Solving the Weyl equation we obtain

$$\psi = \int_0^\infty \frac{dE_p}{2\pi}\frac{1}{\sqrt{2E_p}}\left[a_p e^{ip.x} + b_p^\dagger e^{-ip.x}\right]_{p^\mu = E_p\left[u^\mu + \chi_{d=2}\epsilon^{\mu\nu}u_\nu\right]}$$

Populating these states leads to anomalous correction to hydrodynamics

$$T^{\mu\nu} = \sum_{species}\int_0^\infty \frac{dE_p}{2\pi}(f_q + f_{-q})E_p\left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]\left[u^\nu + \chi_{d=2}\epsilon^{\nu\lambda}u_\lambda\right]$$
$$= \varepsilon u^\mu u^\nu + p(g^{\mu\nu} + u^\mu u^\nu) + q^\mu_{anom}u^\nu + q^\nu_{anom}u^\mu$$

$$J^\mu = \sum_{species}\int_0^\infty \frac{dE_p}{2\pi}(qf_q - qf_{-q})\left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]$$
$$= nu^\mu + J^\mu_{anom}$$

helicity current

$$J^\mu_S = -\sum_{species}\int_0^\infty \frac{dE_p}{2\pi}(\mathcal{H}_q + \mathcal{H}_{-q})\left[u^\mu + \chi_{d=2}\epsilon^{\mu\alpha}u_\alpha\right]$$
$$= su^\mu + J^\mu_{S,anom}$$

Gibbs Current

The above anomalous quantities can be generated from

$$\bar{\mathcal{G}}_{anom} = \sum_F \int_0^\infty \frac{dE_p}{2\pi} g_q \ \chi_{d=2} u$$
$$\bar{J}_{anom} = -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial \mu} \quad , \quad \bar{J}_{S,anom} = -\frac{\partial \bar{\mathcal{G}}_{anom}}{\partial T}$$
$$\bar{q}_{anom} = \bar{\mathcal{G}}_{anom} + T\bar{J}_{S,anom} + \mu \bar{J}_{anom}$$

where $g_q \equiv -\frac{1}{\beta} \ln\left[1 + e^{-\beta(E_p - q\mu)}\right]$ and we used Hodge duals for simplicity.

We have to evaluate one thermal integral to get

$$\bar{\mathcal{G}}_{anom} = -2\pi \left[\frac{\mu^2}{2!(2\pi)^2} \left(\sum_{species} \chi_{d=2} q^2 \right) + \frac{T^2}{4!} \left(\sum_{species} \chi_{d=2} \right) \right] u$$

Crucial observation : the anomalous contribution is completely proportional to the U(1) anomaly coefficient $\sum_{species} \chi_{d=2} q^2$ and the Lorentz anomaly coefficient $\sum_{species} \chi_{d=2}$

Anomaly Polynomials

The anomaly coefficients of a system are summarised by a polynomial in gauge field strength and space-time curvature:

$$\mathcal{P}_{anom}(F,\mathfrak{R}) \equiv -2\pi \left[\frac{F^2}{2!(2\pi)^2} \left(\sum_{species} \chi_{d=2} q^2 \right) - \frac{p_1(\mathfrak{R})}{4!} \left(\sum_{species} \chi_{d=2} \right) \right]_{2d}$$

Using this we can write a rule to get from the anomaly polynomial to the anomaly induced Gibbs current

$$\bar{\mathcal{G}}_{anom} = u\, \mathcal{P}_{anom} \left(F \mapsto \mu \ ,\ p_1(\mathfrak{R}) \mapsto -T^2 \right)$$

Motivated by this result and Berry phase calculation we can generalise the Gibbs current to higher dimensions introducing concept of chiral spectral current, repeat the analysis and match to hydrodynamics

$$\mathcal{G}^{\mu}_{anom} = \sum_F \int_0^\infty dE_p \mathcal{J}^{\mu}_q \ g_q$$

Using adiabaticity

$$\bar{\mathcal{J}}_q = \frac{\chi_{d=2n}}{2\pi} \left(\frac{qB + 2\omega E_p}{2\pi} \right)^{n-1} \wedge \frac{u}{(n-1)!}$$

Examples

The structure of Gibbs functionals in higher dimensions

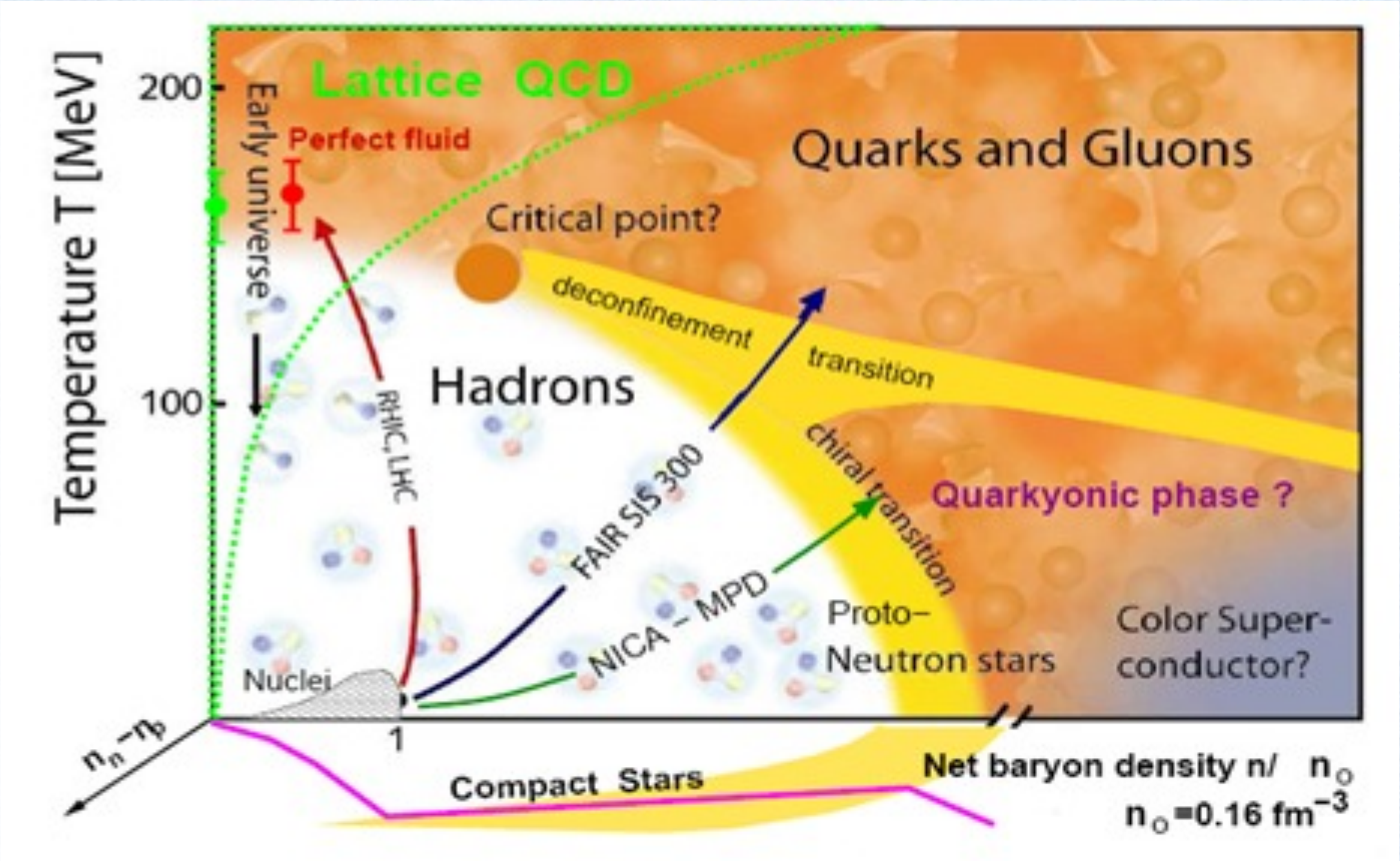
$$(\bar{\mathcal{G}}_{anom})_{d=2} = -2\pi \sum_{species} \chi_{d=2} \left[\frac{1}{2!}\left(\frac{q\mu}{2\pi}\right)^2 + \frac{T^2}{4!}\right] u$$

Cardy entropy formula + first law of thermodynamics

$$(\bar{\mathcal{G}}_{anom})_{d=4} = -2\pi \sum_{species} \chi_{d=4} \left[\frac{1}{3!}\left(\frac{q\mu}{2\pi}\right)^3 + \left(\frac{q\mu}{2\pi}\right)\frac{T^2}{4!}\right] (2\omega)\wedge u$$
$$- 2\pi \sum_{species} \chi_{d=4} \left[\frac{1}{2!}\left(\frac{q\mu}{2\pi}\right)^2 + \frac{T^2}{4!}\right] (qB)\wedge u$$

$$(\bar{\mathcal{G}}_{anom})_{d=6} = -2\pi \sum_{species} \chi_{d=6} \left[\frac{1}{4!}\left(\frac{q\mu}{2\pi}\right)^4 + \frac{1}{2!}\left(\frac{q\mu}{2\pi}\right)^2\frac{T^2}{4!} + \frac{7}{8}\frac{T^4}{6!}\right] (2\omega)^2\wedge u$$
$$- 2\pi \sum_{species} \chi_{d=6} \left[\frac{1}{3!}\left(\frac{q\mu}{2\pi}\right)^3 + \left(\frac{q\mu}{2\pi}\right)\frac{T^2}{4!}\right] (2\omega)\wedge (qB)\wedge u$$
$$- 2\pi \sum_{species} \chi_{d=6} \left[\frac{1}{2!}\left(\frac{q\mu}{2\pi}\right)^2 + \frac{T^2}{4!}\right] \frac{(qB)^2}{2!}\wedge u$$

QCD Phase Structure



source Shanghai Jiao Tong University, Institut of Nuclear and Particle Physics, Astronomy and Cosmology (INPAC)

Usually, "the phase diagram of QCD" is drawn in the plane spanned by the temperature T and the baryon chemical potential μ. But various additional directions, i.e., higher-dimensional versions of the phase diagram, are of interest as well, for example chiral chemical potential.

INFORMATION GEOMETRY

The field of information geometry was developed in order to study the phase space of statistical systems using geometry. A given statistical ensemble is represented as a point on a Riemannian manifold. This manifold is endowed with a metric which is precisely the Fisher-Rao information metric. The system is characterised by a set of thermodynamic parameters β which include inverse temperature and generalized chemical potentials for the conserved quantities. One can write down a Gibbs measure for this system

$$p(x|\beta) = \exp\left(-\sum_i \beta^i H_i(x) - \ln \mathcal{Z}(\beta)\right),$$

where $H_i(x)$ includes hamiltonian and conserverd currents.

We define the Fisher information matrix

$$G_{ij}(\beta) = -\left\langle \frac{\partial^2 \ln p(x|\beta)}{\partial\beta_i \partial\beta_j} \right\rangle.$$

It is a metric and can be proven to be unique.

Ricci Scalar

This manifold is endowed with a metric which is precisely the Fisher-Rao information metric. In such a geometrization a scalar curvature plays a central role and contains the information about phase transitions.

Example: 1d Ising model in magnetic field

$$G_{ij} = \frac{1}{N}\partial_i\partial_j\{N\beta + \ln[(\cosh h + \eta)^N + (\cosh h - \eta)^N]\},$$

where $\eta = \sqrt{\sinh^2 h + \mathrm{e}^{-4\beta}}$.

The thermodynamic curvature reads: $\mathcal{R} = 1 + \eta^{-1}\cosh h$

The presence of this divergence may be understood by taking a Legendre transform of the Fisher–Rao metric, which is given by the Hessian matrix of the entropy. One observes that the nondegeneracy condition for this metric is precisely the concavity condition for the entropy, and thus its breakdown, where the curvature diverges, does indeed signal a phase transition point.

Chiral Vortical Effect

To make a direct connection between fluids with anomalies and information geometry in 1 + 1 and 3 + 1 dimension it is convenient to take fluid configurations on $R \times S^1$ and $R \times S^3$.
It provides a natural cut-off and permits to calculate the partition effective action exactly

$$
\begin{aligned}
\mathcal{W} &= \beta\left[p\frac{2\pi^2 R^3}{(1-R^2\Omega_1)(1-R^2\Omega_2)}\right] \\
&\quad -\beta\left[\mathfrak{F}^{\omega}_{anom}\frac{2\pi R^2\Omega_1}{1-R^2\Omega_1}\frac{2\pi R^2\Omega_2}{1-R^2\Omega_2}\right].
\end{aligned}
$$

$$
\mathfrak{F}^{\omega}_{anom} = \tilde{c}_A\mu^3 + c_m\frac{\mu}{\beta^2}, \qquad \tilde{c}_A = -\frac{1}{3!(2\pi)^2}\sum \chi_i q_i^3, \qquad c_m = -\frac{1}{4!}\sum \chi_i q_i
$$

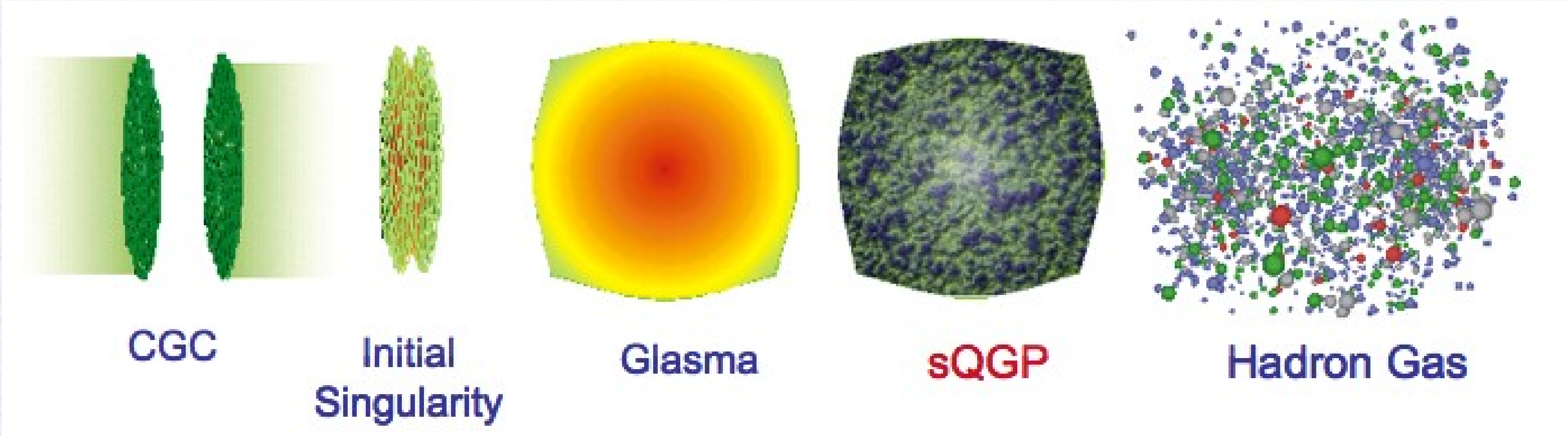
We have two parts in the effective action. One is universal and fixed in terms of data due to anomalies, the other is proportional to pressure, which is specific to microscopic details. It can be fixed in holography but we are interested in the universal part that dominates for large radius.

Critical points

Manifold $\mathcal{M}$	Critical point
$\mathcal{M}^{\beta,\mu}$	$T_c = \left(\frac{3(3-2\sqrt{2})\tilde{c}_A}{c_m}\right)^{1/2}\mu\ ;\qquad T_c = \left(\frac{3(3+2\sqrt{2})\tilde{c}_A}{c_m}\right)^{1/2}\mu$
$\mathcal{M}^{\beta,\Omega_i}$	$T_c = \left(\frac{(2\sqrt{3}-3)\tilde{c}_A}{3c_m}\right)^{1/2}\mu$
$\mathcal{M}^{\mu,\Omega_i}$	$\mu_c = \left(\frac{(2\sqrt{3}-3)c_m}{3\tilde{c}_A}\right)^{1/2}T$
$\mathcal{M}^{\Omega_1,\Omega_2}$	no critical point
$\mathcal{M}^{\beta,\mu,\Omega_i}$	$T_c = \left(\frac{\tilde{c}_A}{c_m}\right)^{1/2}\mu;\qquad T_c = \left(\frac{(3+2\sqrt{3})\tilde{c}_A}{3c_m}\right)^{1/2}\mu$
$\mathcal{M}^{\beta,\Omega_1,\Omega_2}$	$T_c = \left(\frac{(\sqrt{33}-5)\tilde{c}_A}{4c_m}\right)^{1/2}\mu$
$\mathcal{M}^{\mu,\Omega_1,\Omega_2}$	$\mu_c = \left(\frac{c_m}{3\tilde{c}_A}\right)^{1/2}T$
$\mathcal{M}^{\beta,\mu,\Omega_1,\Omega_2}$	no critical point

We obtain a set of critical points fixed in terms of anomaly coefficients. Perhaps on could check on the lattice.

GLASMA



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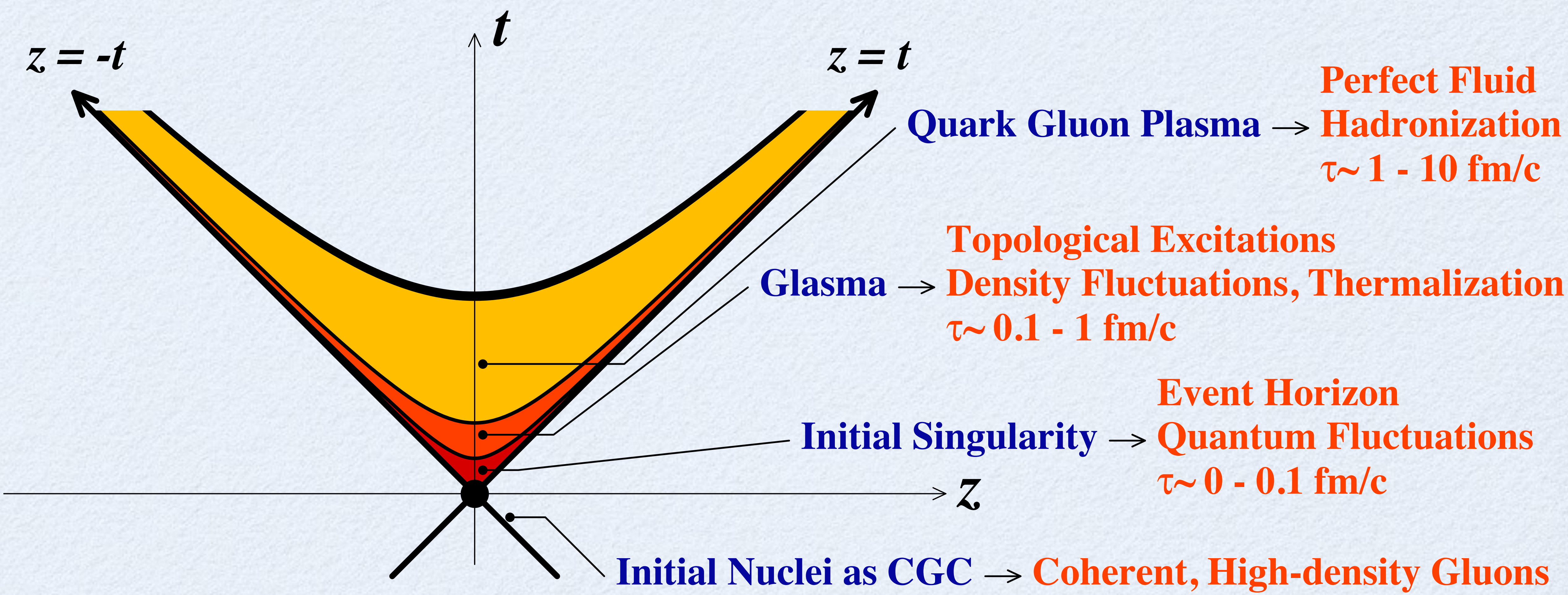

Information Geometry of Glasma

The next step is to construct the information geometry for Color Glass Condensate and Glasma. Proposal of Peschanski

$$\Sigma^{Y_1 \to Y_2} = \kappa \left\{ \left(R_1^2/R_2^2 - 1 \right) - \log \left(R_1^2/R_2^2 \right) \right\}$$

$$\Sigma^{Y_1 \to Y_2}_{glasma} \sim \kappa_{gl} \; Q_2^2/Q_1^2$$

where $R_i = 1/Q_S(Y_i)$ and Q_S denotes saturation momentum.

Relative entropy can be used to define Fisher metric

$$D_{KL}(p||q) \equiv \Sigma = \int d\mu(z) p(z) \log \frac{p(z)}{q(z)}$$

$$D_{KL}(p||q) \simeq G^{\text{Fisher}}_{MN} \; \delta\xi^M \delta\xi^N$$

$$G^{\text{Fisher}}_{MN} \equiv \int d\mu(z) \;\; q(z) \frac{\partial \log q}{\partial \xi^M} \frac{\partial \log q}{\partial \xi^N} .$$

Summary and goals

- Partition functions are very useful in the analysis of anomalies. Many questions are left unanswered
- Close anologs between QFT and turbulence
- Role of parity anomalies
- Phase transitions from information geometry
- Information geometry of CGC and glasma
- Role of magnetic field