Anisotropic hydrodynamics for conformal Gubser flow

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Primary reference: M. Nopoush, R. Ryblewski, and MS, PRD 91, 045007 (2015)

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Outline

- Gubser flow (flash review)
- aHydro subject to Gubser flow
- Comparison with exact RTA solution
- Conclusions I

And if time permits…

- aHydro for central AA and pA collisions
- Conclusions II

Motivation

- An exact solution of the RTA Boltzmann equation subject to Gubser flow was obtained recently [see talk by M. Martinez earlier this week]
- The solution allows for arbitrary η/s → can cover ideal hydrodynamics to free streaming within kinetic framework
- Solutions show that the system is highly anisotropic at early <u>and</u> late times (large radii) and standard hydro treatments breakdown
- We would like to know how well anisotropic hydrodynamics works to describe this new exact solution
- Along the way we will generate an exact solution specific to aHydro that can be used to test aHydro codes

Gubser Flow

[S. Gubser, 1006.0006;
S. Gubser and Y.Yarom, 1012.1314]

Gubser flow is a cylindrically-symmetric and boost-invariant flow that possesses a high degree of symmetry when mapped to Weyl-rescaled deSitter space

$$SO(3)_q \times SO(1,1) \times Z_2$$

$SO(3)_q$	$SO(1,1)$	Z_2
rotational symmetry around beam axis + conformal symmetry	boost invariance	reflection symmetry around the collision plane

See talk early this week by M. Martinez for more details

The parameter q above is an arbitrary energy scale that sets the radial extent of the system at a given proper time.

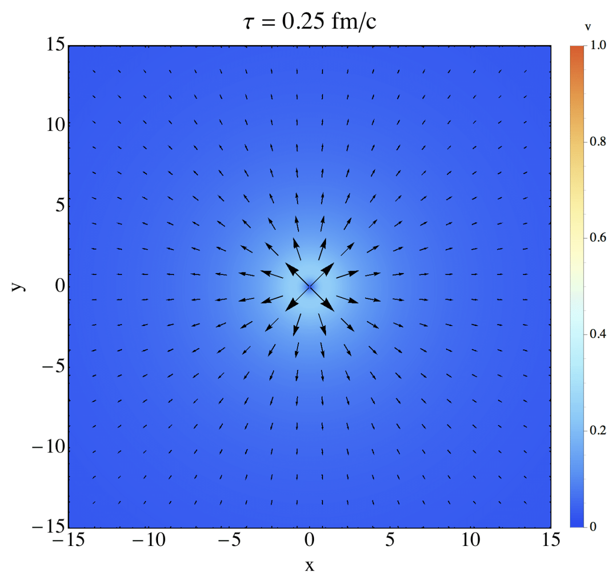
Polar Milne components

$$\tilde{u}^\tau = \cosh(\theta_\perp)$$
$$\tilde{u}^r = \sinh(\theta_\perp)$$
$$\tilde{u}^\phi = 0$$
$$\tilde{u}^\varsigma = 0$$

Transverse rapidity

$$\theta_\perp = \tanh^{-1}\left(\frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}\right)$$

This flow is quite strong: The de Sitter space velocity gradients grow exponentially $e^{|\rho|}$



Weyl-rescaled de Sitter Coordinates

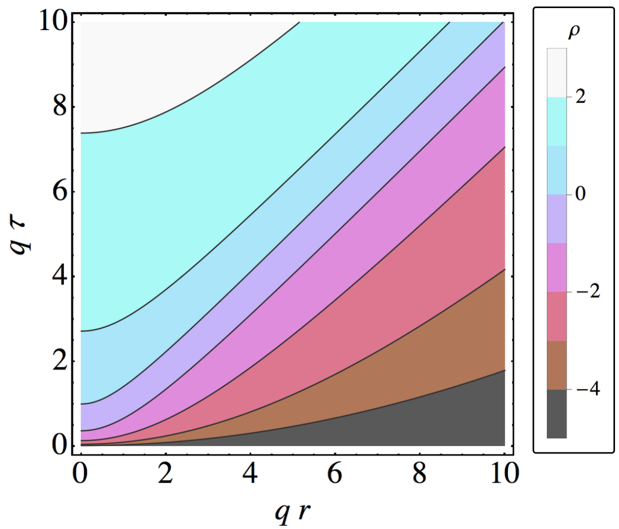
$$dS_3 \times \mathbf{R} \quad \hat{g}_{\mu\nu} = \frac{1}{\tau^2}\frac{\partial x^\alpha}{\partial \hat{x}^\mu}\frac{\partial x^\beta}{\partial \hat{x}^\nu} g_{\alpha\beta} \quad \mathbf{R}^{3,1}$$

$$\hat{g}_{\mu\nu} = \mathrm{diag}(-1,\, \cosh^2\!\rho,\, \cosh^2\!\rho\,\sin^2\!\theta,\, 1)$$

$$d\hat{s}^2 = -d\rho^2 + \cosh^2\rho \underbrace{\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{SO(3)_q} + d\varsigma^2$$

$$\sinh\rho = -\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}$$

$$\tan\theta = \frac{2qr}{1 + q^2\tau^2 - q^2 r^2}$$



Polar Milne components

$$\tilde{u}^\tau = \cosh(\theta_\perp)$$
$$\tilde{u}^r = \sinh(\theta_\perp)$$
$$\tilde{u}^\phi = 0$$
$$\tilde{u}^\varsigma = 0$$

After Weyl rescaling and coordinate transformation the Gubser flow four-velocity is static!

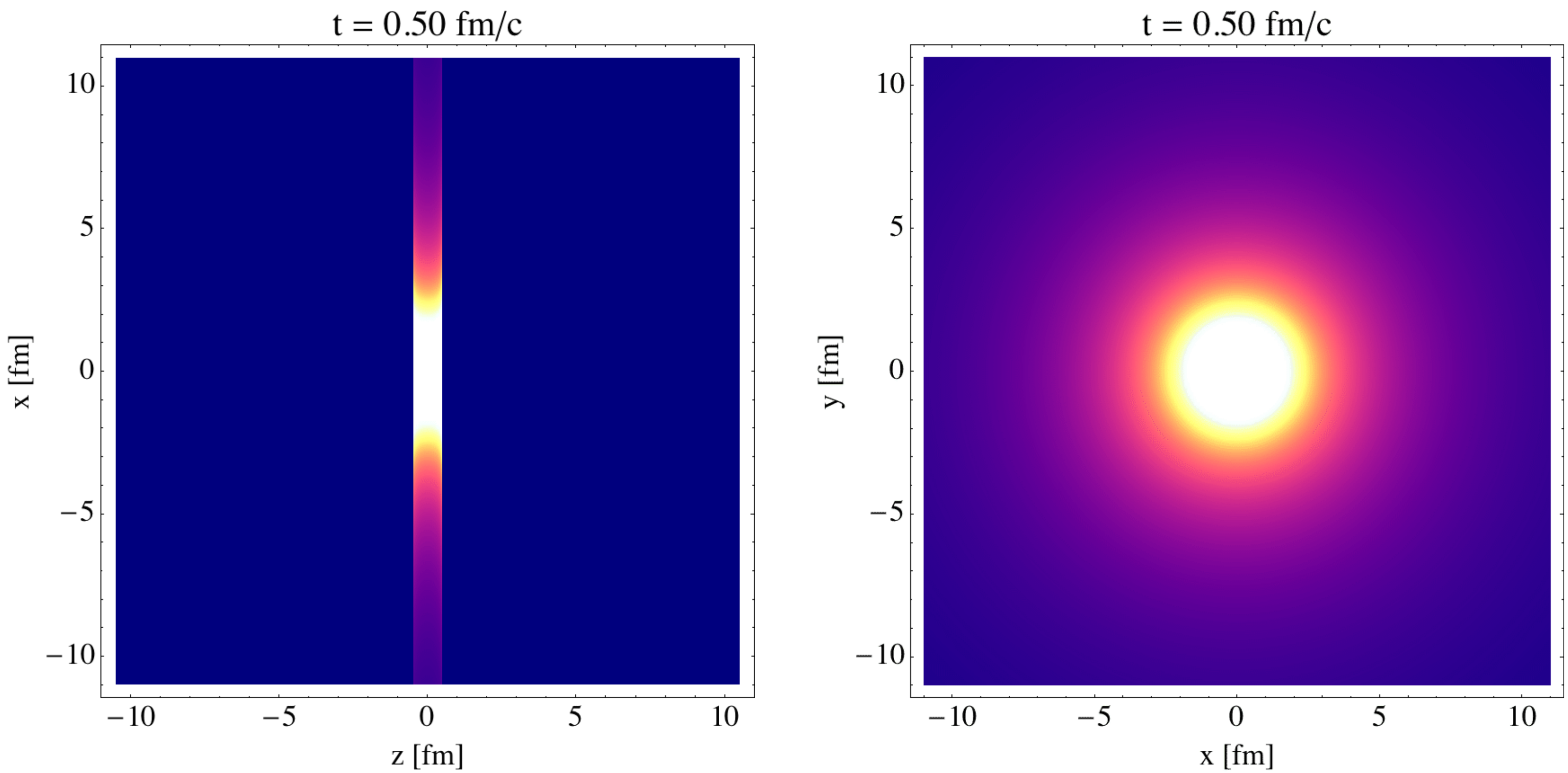
$$\longrightarrow \quad \hat{u}^\mu = \tau \frac{\partial \hat{x}^\mu}{\partial x^\nu} u^\nu \quad \longrightarrow \quad \hat{u}^\mu = (1,\, 0,\, 0,\, 0)$$

de Sitter space flow velocity

[S. Gubser, 1006.0006;
S. Gubser and Y.Yarom, 1012.1314]

Results of the 1+1d kinetic solution

[G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]



Visualization of the effective temperature

Metric and basis four-vectors

Minkowski basis vectors

$$u^\mu_{LRF} \equiv (1,0,0,0)$$
$$\mathcal{X}^\mu_{LRF} \equiv (0,1,0,0)$$
$$\mathcal{Y}^\mu_{LRF} \equiv (0,0,1,0)$$
$$\mathcal{Z}^\mu_{LRF} \equiv (0,0,0,1)$$

$$g^{\mu\nu} = -u^\mu u^\nu + \mathcal{X}^\mu \mathcal{X}^\nu + \mathcal{Y}^\mu \mathcal{Y}^\nu + \mathcal{Z}^\mu \mathcal{Z}^\nu$$

Weyl-rescaling + coord transform

$$\hat{u}^\mu = \tau \frac{\partial \hat{x}^\mu}{\partial x^\nu} u^\nu$$
$$\hat{\Theta}^\mu = \tau \frac{\partial \hat{x}^\mu}{\partial x^\nu} \mathcal{X}^\nu$$
$$\hat{\Phi}^\mu = \tau \frac{\partial \hat{x}^\mu}{\partial x^\nu} \mathcal{Y}^\nu$$
$$\hat{\varsigma}^\mu = \tau \frac{\partial \hat{x}^\mu}{\partial x^\nu} \mathcal{Z}^\nu$$

Weyl-rescaled de Sitter basis vectors

$$\hat{u}^\mu = (1,\, 0,\, 0,\, 0)$$
$$\hat{\Theta}^\mu = (0,\, (\cosh\rho)^{-1},\, 0,\, 0)$$
$$\hat{\Phi}^\mu = (0,\, 0,\, (\cosh\rho\sin\theta)^{-1},\, 0)$$
$$\hat{\varsigma}^\mu = (0,\, 0,\, 0,\, 1)$$

Orthonormality relations

$$\hat{u}\cdot\hat{u} \equiv \hat{u}^\mu \hat{u}_\mu = -1$$
$$\hat{\Theta}\cdot\hat{\Theta} \equiv \hat{\Theta}^\mu \hat{\Theta}_\mu = 1$$
$$\hat{\Phi}\cdot\hat{\Phi} \equiv \hat{\Phi}^\mu \hat{\Phi}_\mu = 1$$
$$\hat{\varsigma}\cdot\hat{\varsigma} \equiv \hat{\varsigma}^\mu \hat{\varsigma}_\mu = 1$$

The Weyl-rescaled de Sitter metric can be expressed in terms of the basis four-vectors

$$\hat{g}_{\mu\nu} = -\hat{u}_\mu \hat{u}_\nu + \hat{\Theta}_\mu \hat{\Theta}_\nu + \hat{\Phi}_\mu \hat{\Phi}_\nu + \hat{\varsigma}_\mu \hat{\varsigma}_\nu$$

$$\hat{g}_{\mu\nu} = \mathrm{diag}(-1,\, \cosh^2\!\rho,\, \cosh^2\!\rho\,\sin^2\!\theta,\, 1)$$

$$\hat{g} \equiv \det \hat{g}_{\mu\nu} = -\cosh^4\!\rho\,\sin^2\!\theta$$

Anisotropic hydrodynamics beginning

M. Martinez and MS, 1007.0889
W. Florkowski and R. Ryblewski, 1007.0130

Viscous Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = \underbrace{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))}_{\text{Isotropic in momentum space}} + \delta f$$

Anisotropic Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\text{anisotropy}}\,) + \delta\tilde{f}$$

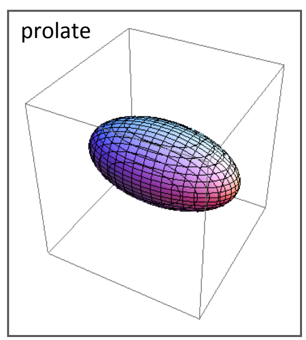
Treat this term "perturbatively"

[D. Bazow, U. Heinz, and MS, 1311.6720; D. Bazow, U. Heinz, and M. Martinez, 1503.07443]
→ "vaHydro"

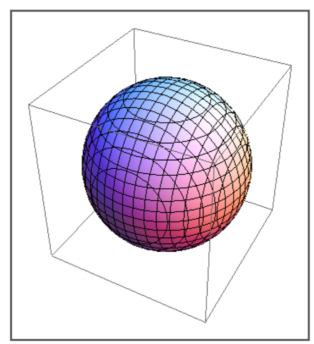
→ "Romatschke-Strickland" form in LRF

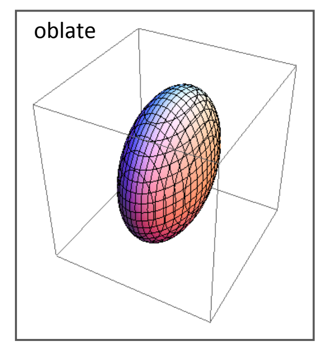
$$f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

$$\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1$$



$-1 < \xi < 0$


$\xi = 0$



$\xi > 0$

A slightly more general version

[L. Tinti and W. Florkowski, 1312.6614 (massless);
M. Nopoush, R. Ryblewski, and MS, 1405.1355 (massive)]

In aHydro we assume that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{traceless anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}} \underbrace{\Phi}_{\text{“Bulk”}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- For a (massless) conformal system one must take Φ → 0
- In “leading-order” aHydro we assume that the most important anisotropies are the diagonal ones and we ignore the $\delta\tilde{f}(x,p)$; in “vaHydro” one includes these using moments-based expansion such as Grad-14

Momentum-space ellipticities

[M. Nopoush, R. Ryblewski, and MS, 1405.1355]

Instead of writing equations in terms of the anisotropy parameters ξ_x, ξ_y, and ξ_z it is convenient to use $\alpha_i \equiv (1 + \xi_i + \Phi)^{-1/2}$

$$f(x,p) = f_{\rm eq}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right) = f_{\rm eq}\left(\frac{1}{\lambda}\sqrt{\sum_i \frac{p_i^2}{\alpha_i^2} + m^2}\right)$$

$$\Phi = \frac{1}{3}\sum_i \alpha_i^{-2} - 1$$

In conformal limit (m = 0, Φ = 0)

$$f(x,p) = f_{\rm eq}\left(\frac{1}{\lambda}\sqrt{\sum_i \frac{p_i^2}{\alpha_i^2}}\right)$$

$$\frac{1}{3}\sum_i \alpha_i^{-2} = 1 \longrightarrow$$ Two independent ellipticities, e.g. α_x and α_z

de Sitter space + SO(3)$_q$ invariance

Conformal form

$$\hat{\Xi}^{\mu\nu} = \hat{u}^\mu \hat{u}^\nu + \hat{\xi}^{\mu\nu}$$

Diagonal anisotropy tensor

$$\hat{\xi}^{\mu\nu} = \hat{\xi}_\theta \hat{\Theta}^\mu \hat{\Theta}^\nu + \hat{\xi}_\phi \hat{\Phi}^\mu \hat{\Phi}^\nu + \hat{\xi}_\varsigma \hat{\varsigma}^\mu \hat{\varsigma}^\nu$$

SO(3)$_q$ invariance requires that distribution function can only depend on

$$\hat{p}_\Omega^2 \equiv \hat{p}_\theta^2 + \hat{p}_\phi^2 / \sin^2\theta$$

Therefore,

$$\hat{\xi}_\theta = \hat{\xi}_\phi \quad (\hat{\alpha}_\theta = \hat{\alpha}_\phi)$$

→ Symmetries require spheroidal form

Tracelessness of anisotropy tensor requires

$$\hat{\xi}_\theta + \hat{\xi}_\phi + \hat{\xi}_\varsigma = 0$$

As a result, the anisotropy tensor only has one independent component, which we choose to be the rapidity component.

NB: A similar reduction occurs in viscous hydro subject to Gubser flow

Energy-Momentum Tensor

Ellipsoidal form

$$\hat{T}^{\mu\nu} = \hat{\varepsilon}\hat{u}^\mu\hat{u}^\nu + \hat{P}_\theta\hat{\Theta}^\mu\hat{\Theta}^\nu + \hat{P}_\phi\hat{\Phi}^\mu\hat{\Phi}^\nu + \hat{P}_\varsigma\hat{\varsigma}^\mu\hat{\varsigma}^\nu$$

Kinetic energy-momentum tensor (2nd-moment)

$$\hat{T}^{\mu\nu} \equiv \frac{1}{(2\pi)^3}\int \frac{d^3\hat{p}}{\sqrt{-\hat{g}}\,\hat{p}^0}\hat{p}^\mu\hat{p}^\nu f(\hat{x},\hat{p})$$

$$\hat{\varepsilon} = \hat{u}_\mu\hat{T}^{\mu\nu}\hat{u}_\nu$$
$$\hat{P}_\theta = \hat{\Theta}_\mu\hat{T}^{\mu\nu}\hat{\Theta}_\nu$$
$$\hat{P}_\phi = \hat{\Phi}_\mu\hat{T}^{\mu\nu}\hat{\Phi}_\nu$$
$$\hat{P}_\varsigma = \hat{\varsigma}_\mu\hat{T}^{\mu\nu}\hat{\varsigma}_\nu$$

$\longrightarrow$

$$\hat{\varepsilon} = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{2\pi^2}H_2(\bar{y})$$
$$\hat{P}_\theta = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{4\pi^2}H_{2T}(\bar{y})$$
$$\hat{P}_\phi = \hat{P}_\theta$$
$$\hat{P}_\varsigma = \frac{3\,\hat{\alpha}_\theta^4\hat{\lambda}^4}{2\pi^2}H_{2L}(\bar{y})$$

- $\bar{y} = \sqrt{\frac{3\hat{\alpha}_\varsigma^2 - 1}{2}}$
- $SO(3)_q$ symmetry → $P_\theta = P_\phi$
- H-functions are relatively simple analytic functions

Energy-momentum conservation

$$\boxed{D_\mu \hat{T}^{\mu\nu} = 0}$$

D_μ is the geometrical covariant derivative which obeys, e.g. when acting on a vector

$$D_\mu \hat{T}^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, \hat{T}^{\mu\nu} \right) + \Gamma^\nu_{\lambda\mu} \hat{T}^{\lambda\mu}$$

gives

$$\partial_\rho \hat{\varepsilon} + 2 \tanh\rho \, (\hat{\varepsilon} + \hat{P}_\theta) = 0$$

$$\partial_\theta \hat{P}_\theta = \partial_\phi \hat{P}_\phi = \partial_\varsigma \hat{P}_\varsigma = 0$$

→ " $\partial_\tau \varepsilon = -\dfrac{\varepsilon + P_L}{\tau}$ "

→ $SO(3)_q$ invariance + boost invariance

$$4 \frac{d \log \hat{\lambda}}{d\rho} + \frac{3\hat{\alpha}_\varsigma^2 \left(\frac{H_{2L}(\bar{y})}{H_2(\bar{y})} + 1 \right) - 4}{3\hat{\alpha}_\varsigma^2 - 1} \frac{d \log \hat{\alpha}_\varsigma}{d\rho} + \tanh\rho \left(\frac{H_{2T}(\bar{y})}{H_2(\bar{y})} + 2 \right) = 0$$

$$\bar{y} = \sqrt{\frac{3\hat{\alpha}_\varsigma^2 - 1}{2}}$$

2^{nd} moment of the RTA Boltzmann EQ

- Next consider the second moment of the RTA Boltzmann equation
- Symmetries and m=0 reduce DOFs to only 2!

$$\hat{\mathcal{I}}^{\lambda\mu\nu} = \int \frac{d^3\hat{p}}{\sqrt{-\hat{g}}\,\hat{p}^0}\, \hat{p}^\lambda \hat{p}^\mu \hat{p}^\nu f(\hat{x},\hat{p})$$

$$D_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu} = -\frac{1}{\hat{\tau}_{\rm eq}}\left(\hat{u}_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu} - \hat{u}_\lambda \hat{\mathcal{I}}^{\lambda\mu\nu}_{\rm eq}\right)$$

$$\begin{aligned}\hat{\mathcal{I}} = \hat{\mathcal{I}}_\rho &\Big[\hat{u}\otimes\hat{u}\otimes\hat{u}\Big] \\ &+ \hat{\mathcal{I}}_\theta\Big[\hat{u}\otimes\hat{\Theta}\otimes\hat{\Theta} + \hat{\Theta}\otimes\hat{u}\otimes\hat{\Theta} + \hat{\Theta}\otimes\hat{\Theta}\otimes\hat{u}\Big] \\ &+ \hat{\mathcal{I}}_\phi\Big[\hat{u}\otimes\hat{\Phi}\otimes\hat{\Phi} + \hat{\Phi}\otimes\hat{u}\otimes\hat{\Phi} + \hat{\Phi}\otimes\hat{\Phi}\otimes\hat{u}\Big] \\ &+ \hat{\mathcal{I}}_\varsigma\Big[\hat{u}\otimes\hat{\varsigma}\otimes\hat{\varsigma} + \hat{\varsigma}\otimes\hat{u}\otimes\hat{\varsigma} + \hat{\varsigma}\otimes\hat{\varsigma}\otimes\hat{u}\Big]\end{aligned}$$

$$\downarrow$$

$$\begin{aligned}\partial_\rho \hat{\mathcal{I}}_\theta + 4\tanh\rho\,\hat{\mathcal{I}}_\theta &= \frac{1}{\hat{\tau}_{\rm eq}}\Big[\hat{\mathcal{I}}_{\theta,\rm iso} - \hat{\mathcal{I}}_\theta\Big] \\ \partial_\rho \hat{\mathcal{I}}_\varsigma + 2\tanh\rho\,\hat{\mathcal{I}}_\varsigma &= \frac{1}{\hat{\tau}_{\rm eq}}\Big[\hat{\mathcal{I}}_{\varsigma,\rm iso} - \hat{\mathcal{I}}_\varsigma\Big] \\ \partial_\theta \hat{\mathcal{I}}_\theta = \partial_\phi \hat{\mathcal{I}}_\phi &= \partial_\varsigma \hat{\mathcal{I}}_\varsigma = 0 \\ \hat{\mathcal{I}}_\rho &= 2\hat{\mathcal{I}}_\theta + \hat{\mathcal{I}}_\varsigma\end{aligned}$$

$$\hat{\mathcal{I}}_\theta = \hat{\mathcal{I}}_\phi$$

$$\hat{\mathcal{I}}_\rho = \sum_{i=\theta,\phi,\varsigma} \hat{\mathcal{I}}_i$$

Final aHydro Eqs

- The result is two ordinary differential equations that describe the de Sitter time evolution of the scale λ and a single anisotropy parameter α_ζ
- We need initial values for these at some value ρ_0; in practice, we take a "large" negative $\rho_0 = -10$

Energy conservation

$$4\frac{d\log\hat{\lambda}}{d\rho} + \frac{3\hat{\alpha}_\varsigma^2\left(\frac{H_{2L}(\bar{y})}{H_2(\bar{y})}+1\right)-4}{3\hat{\alpha}_\varsigma^2-1}\frac{d\log\hat{\alpha}_\varsigma}{d\rho} + \tanh\rho\left(\frac{H_{2T}(\bar{y})}{H_2(\bar{y})}+2\right) = 0$$

Second Moment

$$\frac{6\hat{\alpha}_\varsigma}{1-3\hat{\alpha}_\varsigma^2}\frac{d\hat{\alpha}_\varsigma}{d\rho} - \frac{3\left(3\hat{\alpha}_\varsigma^4-4\hat{\alpha}_\varsigma^2+1\right)}{4\hat{\tau}_{\rm eq}\hat{\alpha}_\varsigma^5}\left(\frac{\hat{T}}{\hat{\lambda}}\right)^5 + 2\tanh\rho = 0$$

$$\hat{T} = \frac{\hat{\alpha}_\varsigma}{\bar{y}}\left(\frac{H_2(\bar{y})}{2}\right)^{1/4}\hat{\lambda} \qquad \bar{y} = \sqrt{\frac{3\hat{\alpha}_\varsigma^2-1}{2}}$$

Landau matching

Limits

- In the limit $t_{eq} \rightarrow 0$ we obtain the original <u>ideal hydro</u> solution obtained by Gubser and Yarom

$$\hat{T}(\rho) = \hat{T}_0 \left(\frac{\cosh \rho_0}{\cosh \rho} \right)^{2/3}$$

- In the limit $t_{eq} \rightarrow \infty$ we obtain the <u>free streaming</u> solution obtained in G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.7048

$$\hat{\varepsilon}_{\rm FS} = \frac{3\hat{\lambda}_0^4 \hat{\alpha}_{\varsigma,0}^4}{\pi^2} \mathcal{H}_\varepsilon(\mathcal{C}_{\rho_0,\rho})$$

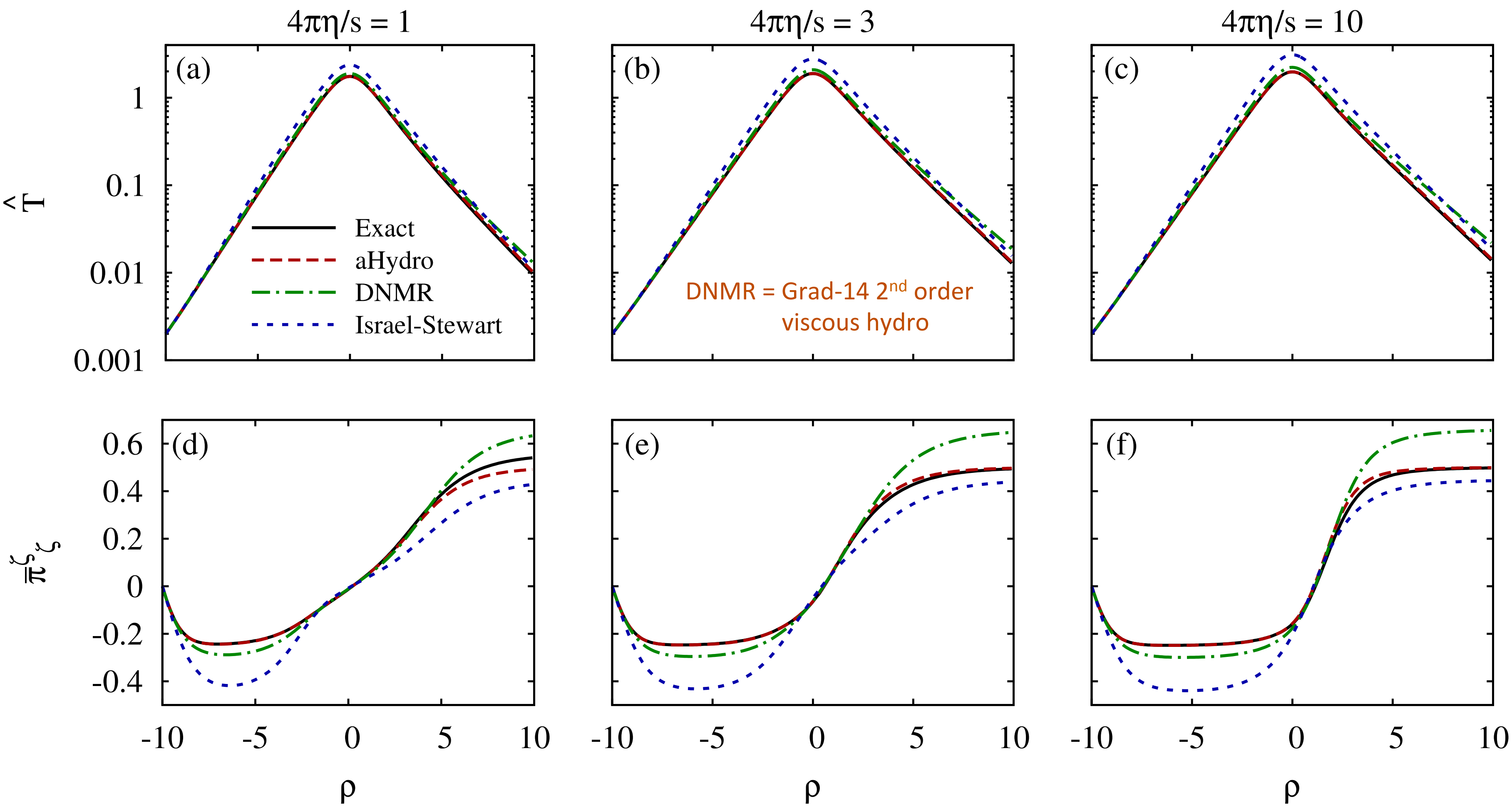
$$(\hat{\pi}_\varsigma^\varsigma)_{\rm FS} = \frac{\hat{\lambda}_0^4 \hat{\alpha}_{\varsigma,0}^4}{\pi^2} \mathcal{H}_\pi(\mathcal{C}_{\rho_0,\rho}^{-1})$$

$$\mathcal{C}_{\rho_0,\rho} = \frac{\hat{\alpha}_{\theta,0} \cosh \rho_0}{\hat{\alpha}_{\varsigma,0} \cosh \rho}$$

Comparison with exact solution

[M. Nopoush, R. Ryblewski, and MS, 1410.6790]

Exact Solution: [G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]



Isotropic initial conditions

Comparison with exact solution

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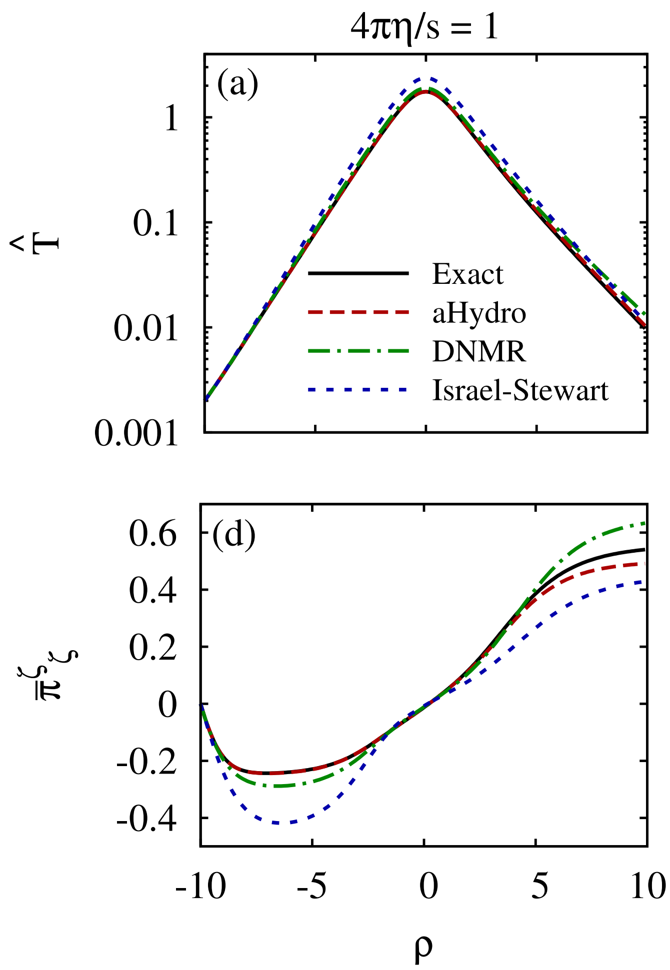

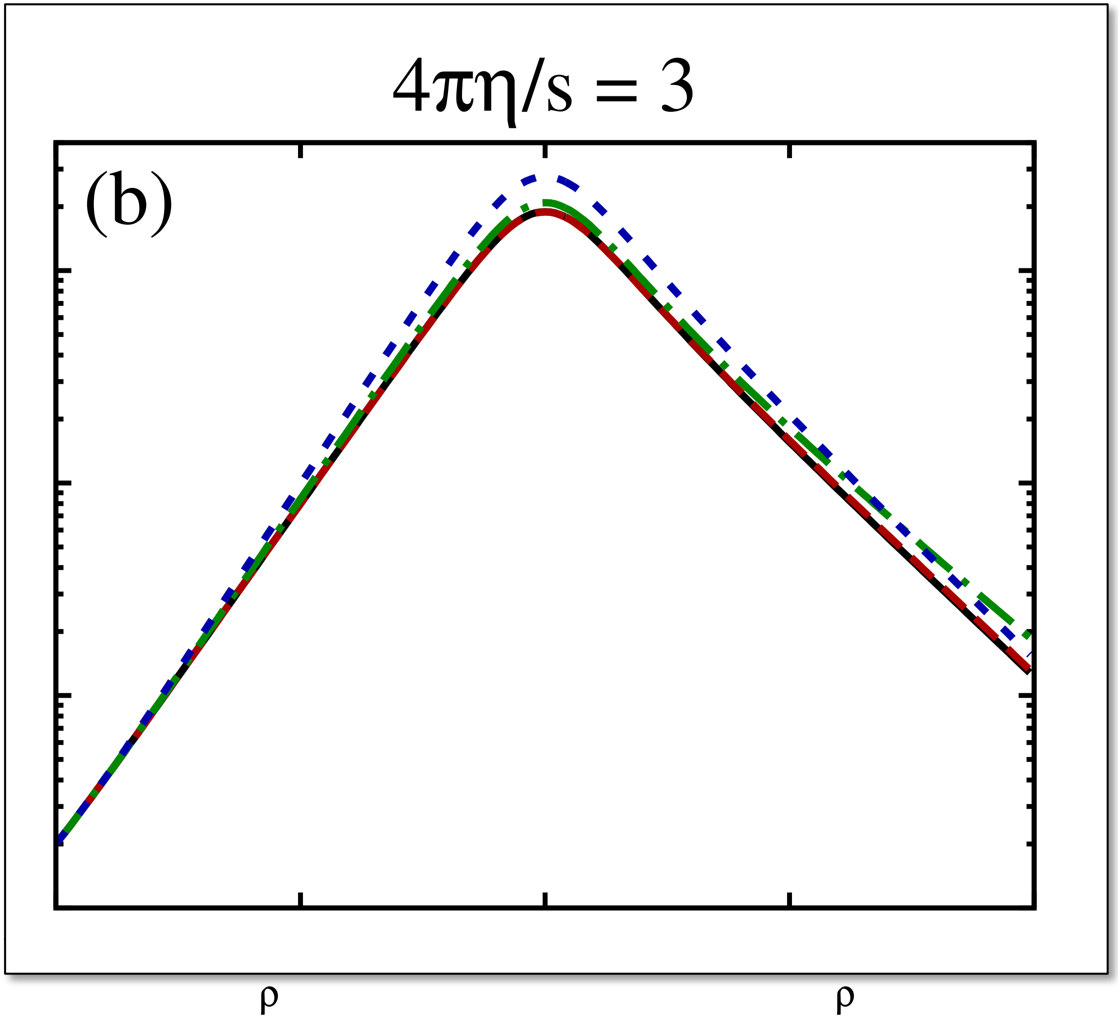

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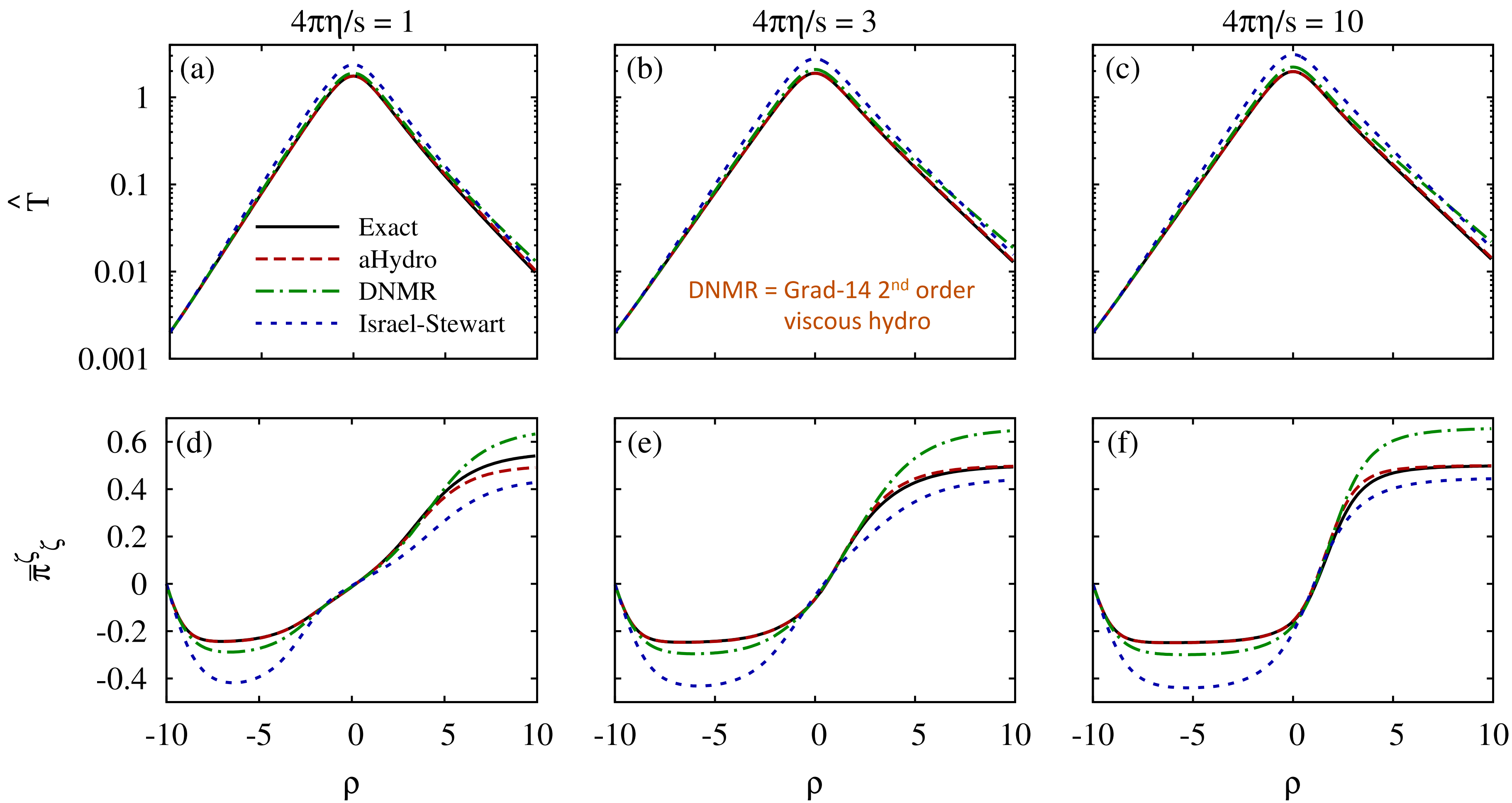

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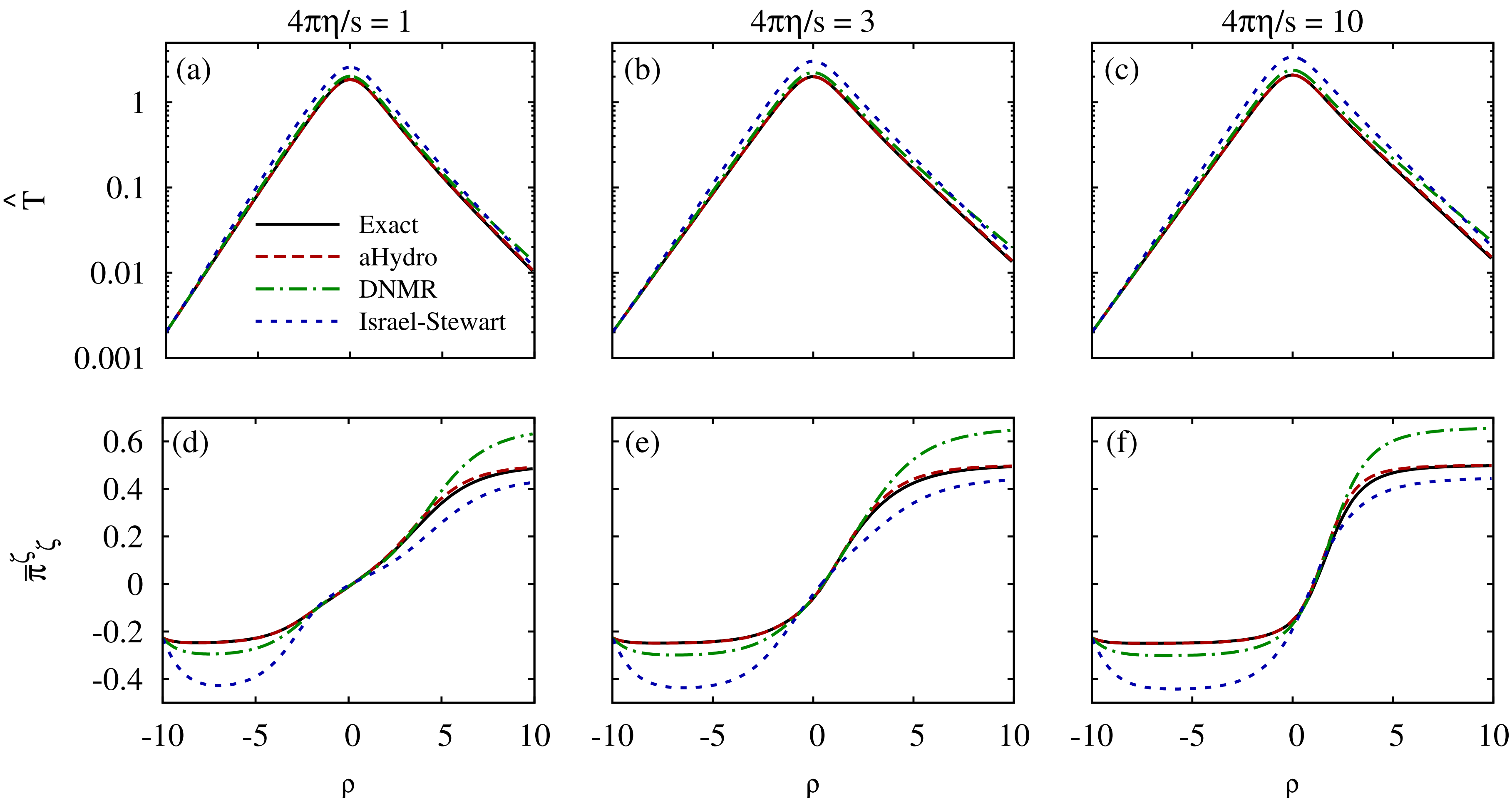
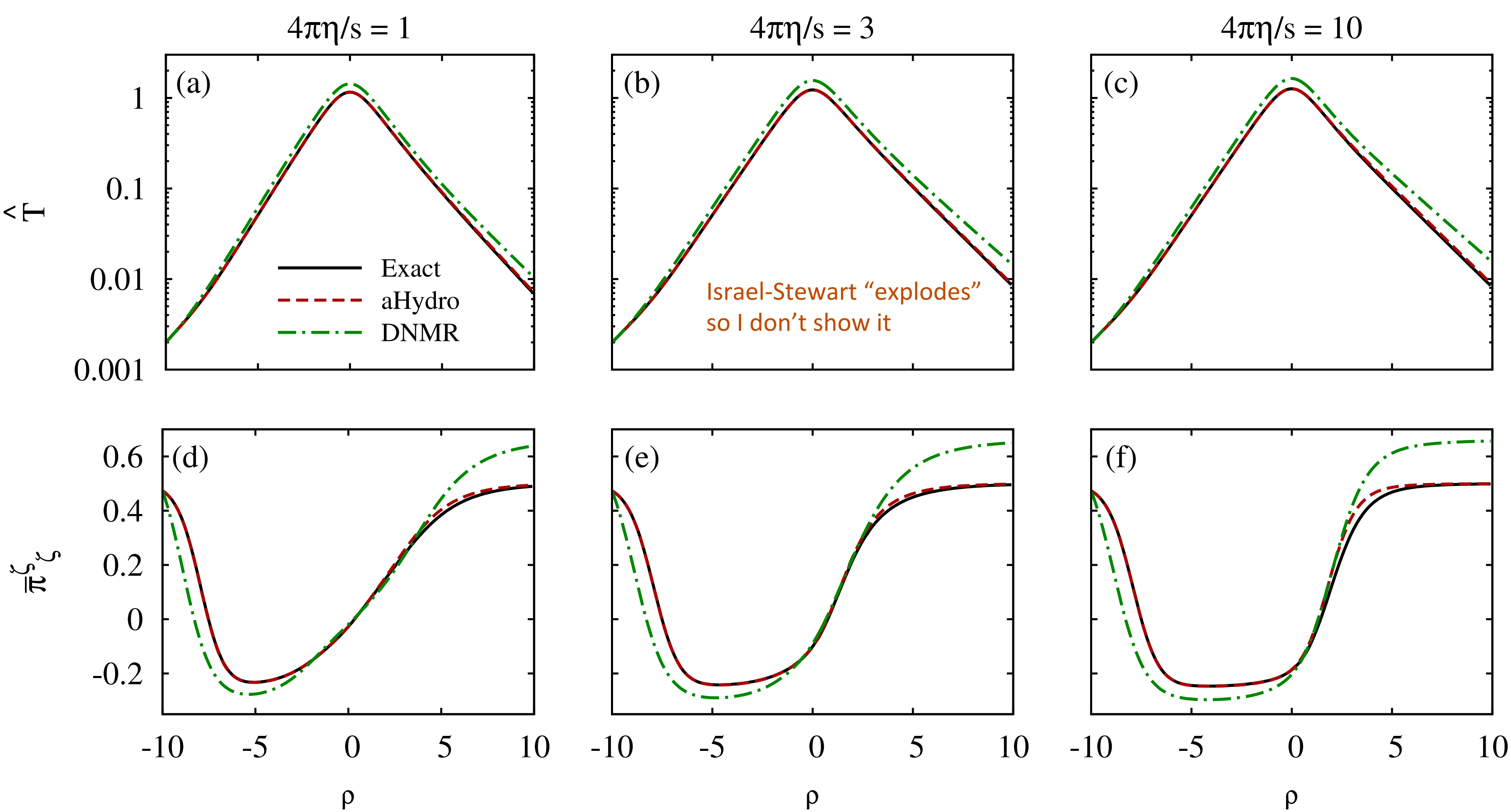


Oblate ($P_{L,0}$ / $P_{T,0}$ << 1) initial conditions

Comparison with exact solution

[M. Nopoush, R. Ryblewski, and MS, 1410.6790]

Exact Solution: [G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]



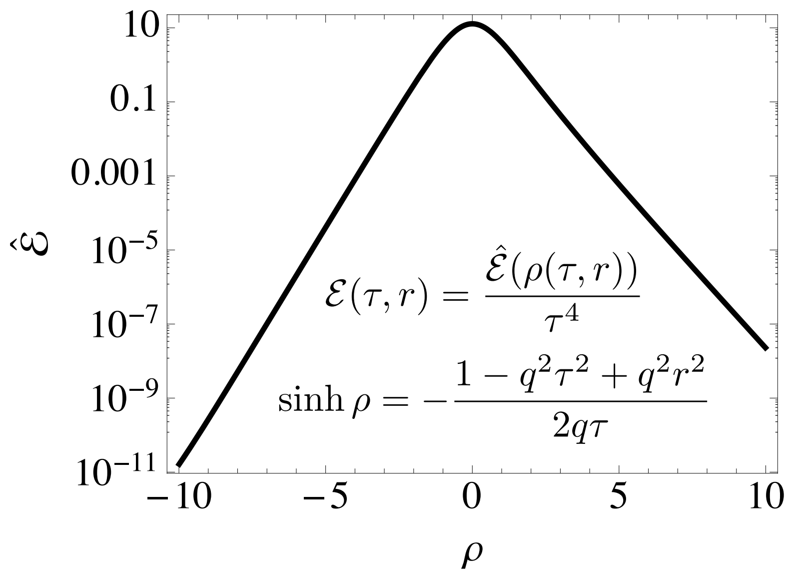
Prolate ($P_{L,0}$ / $P_{T,0}$ >> 1) initial conditions

Comparison with exact solution

[M. Nopoush, R. Ryblewski, and MS, 1410.6790]

Exact Solution: [G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]

- Results are not very easy to interpret intuitively, so let's map them back to Minkowski space by reversing the Weyl-rescaling and coordinate transformation



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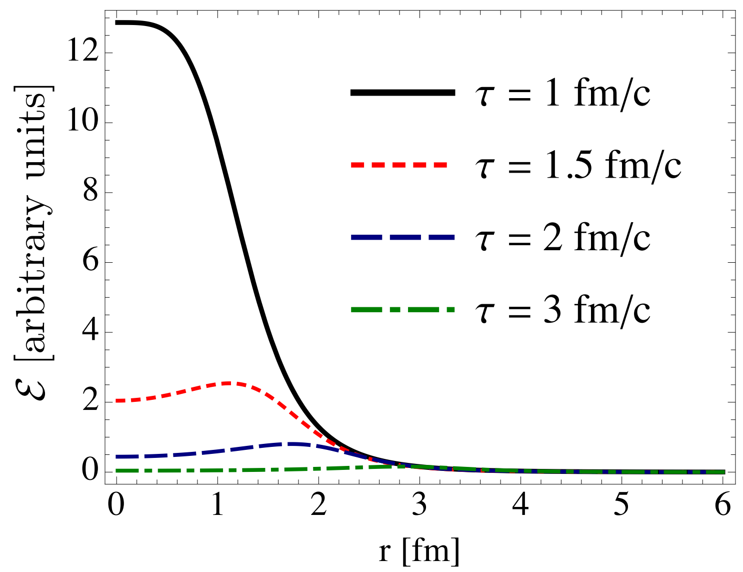

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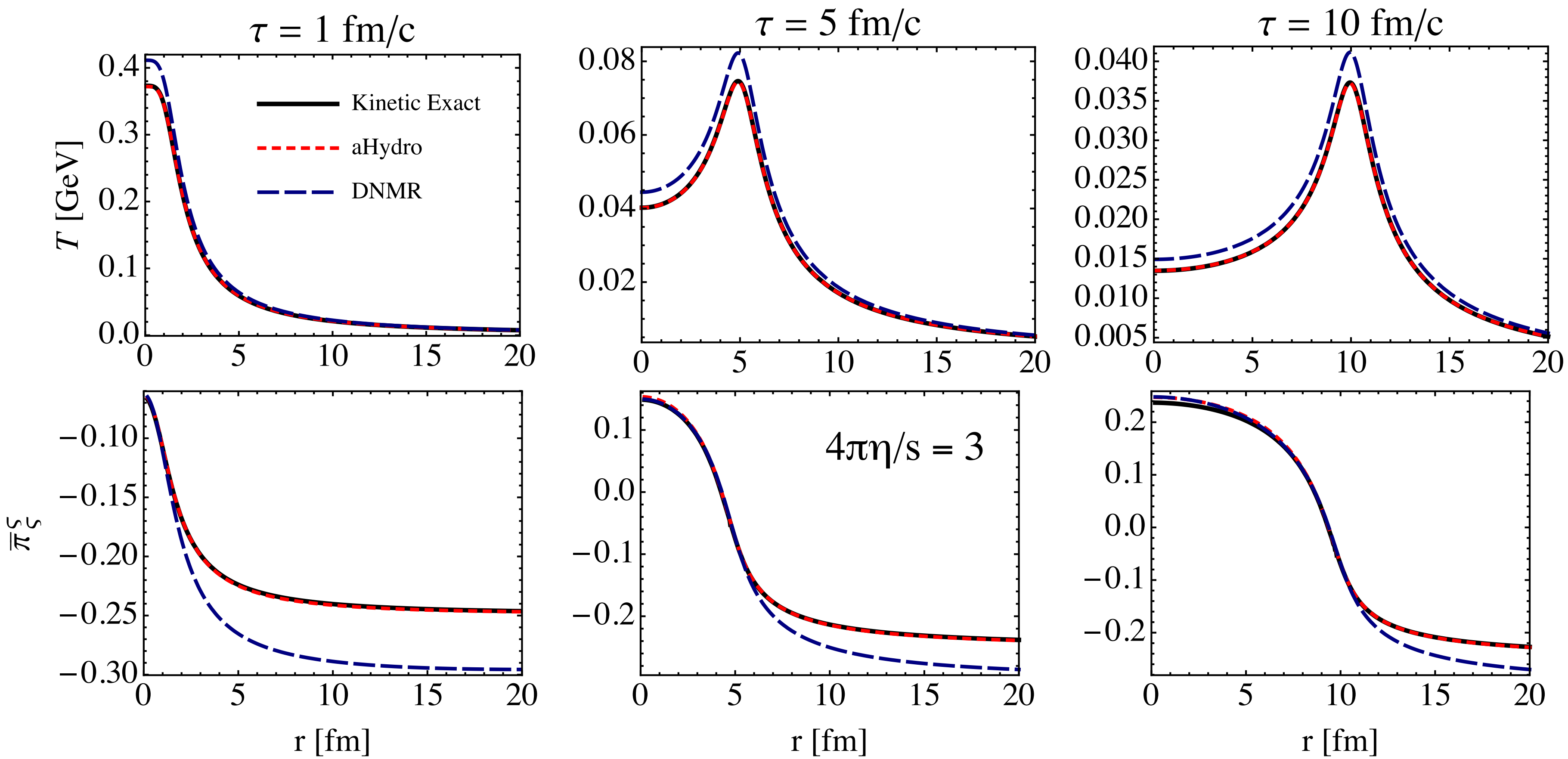


Conclusions I

- Brief run through of aHydro + Gubser flow
- Final result is two ordinary differential equations which describe the de Sitter time evolution of a single scale and anisotropy parameter
- They (analytically) reduce to Gubser's ideal hydro result when the relaxation time goes to zero and the exact free streaming result in the limit of infinite relaxation time
- Result of numerically integrating the coupled nonlinear diff eqs seems to agree quite well with the exact solution; better than standard 2nd-order viscous hydro

Making aHydro ready for primetime

[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS, 1506.05278]

In a recent paper, we showed how to

1. Implement a realistic lattice-based equation of state
2. Implement anisotropic Cooper-Frye freezeout

For 1+1D boost-invariant and cylindrically-symmetric expansion (central collision), we then compare LO aHydro with second-order viscous hydrodynamics.

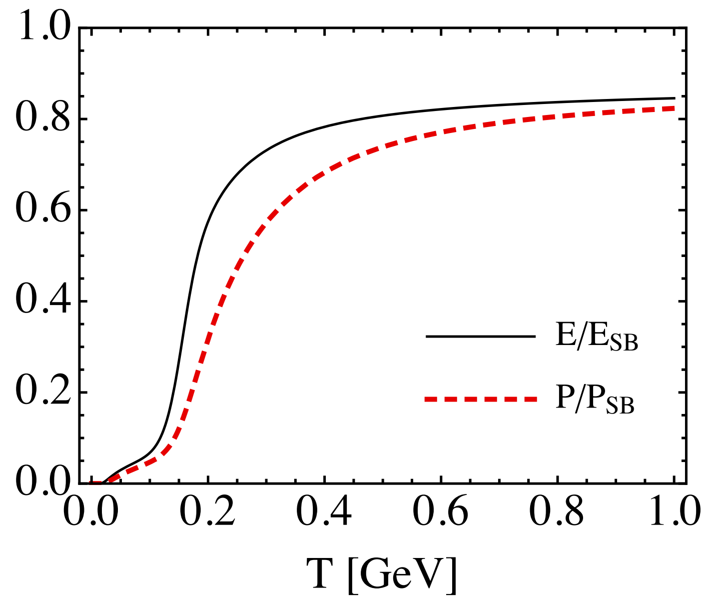
Realistic equation of state

$$n(\Lambda,\xi) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\rm aniso} = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda,\xi) = T^{\tau\tau} = \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_\perp(\Lambda,\xi) = \frac{1}{2}\left(T^{xx}+T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_L(\Lambda,\xi) = -T^\varsigma_\varsigma = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$



Anisotropic Freezeout

[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

We use the ellipsoidal form for the distribution function for both the dynamical equations and also for "anisotropic freezeout".

Use energy density (scalar) to determine the freezeout hypersurface Σ → e.g. $T_{\rm FO}$ = 150 MeV

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} + \Phi$$

isotropic | anisotropy tensor | bulk correction

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

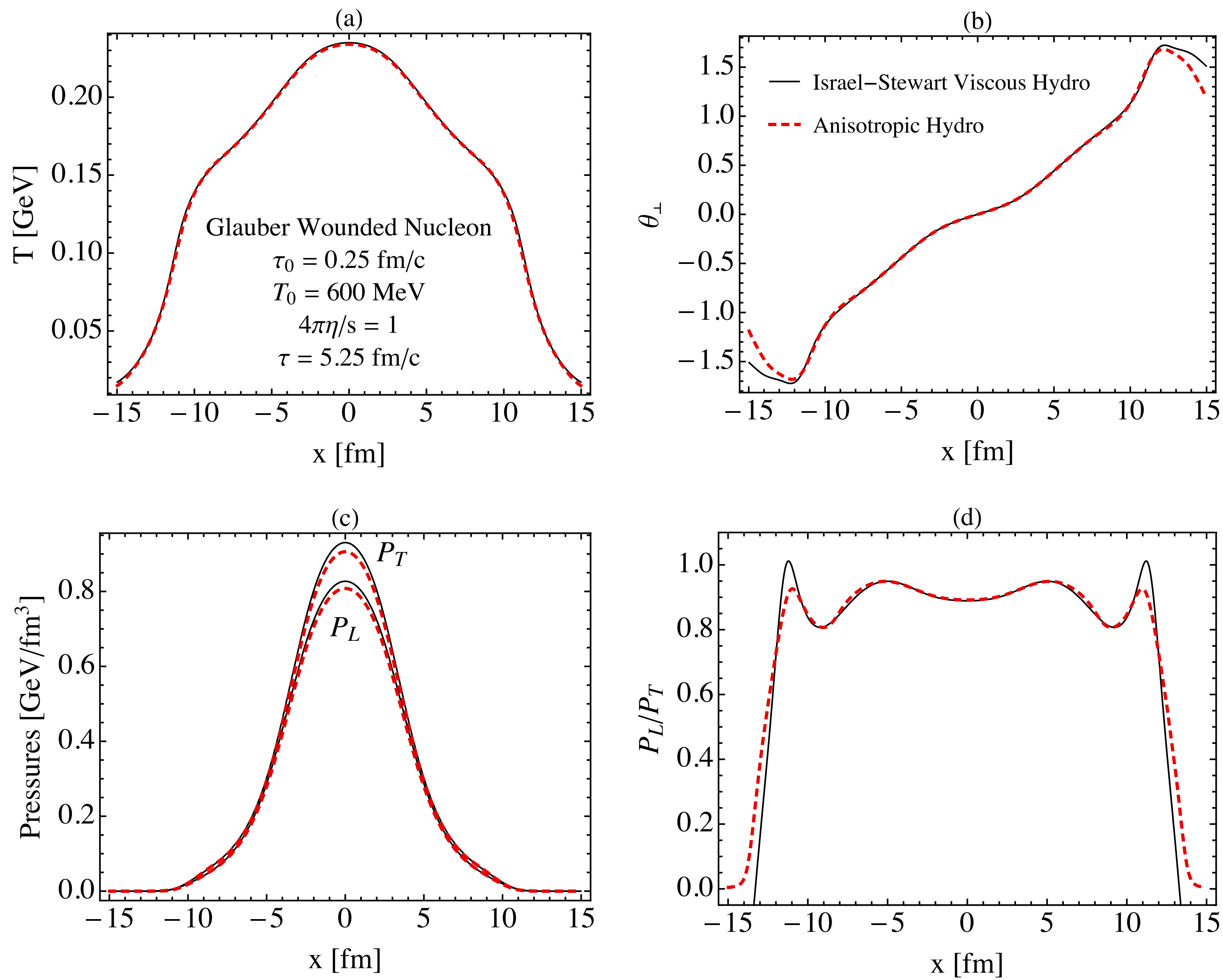
NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq}) \frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p \cdot u/T) + a] \qquad a = \text{-}1,\ +1,\ \text{or } 0$$

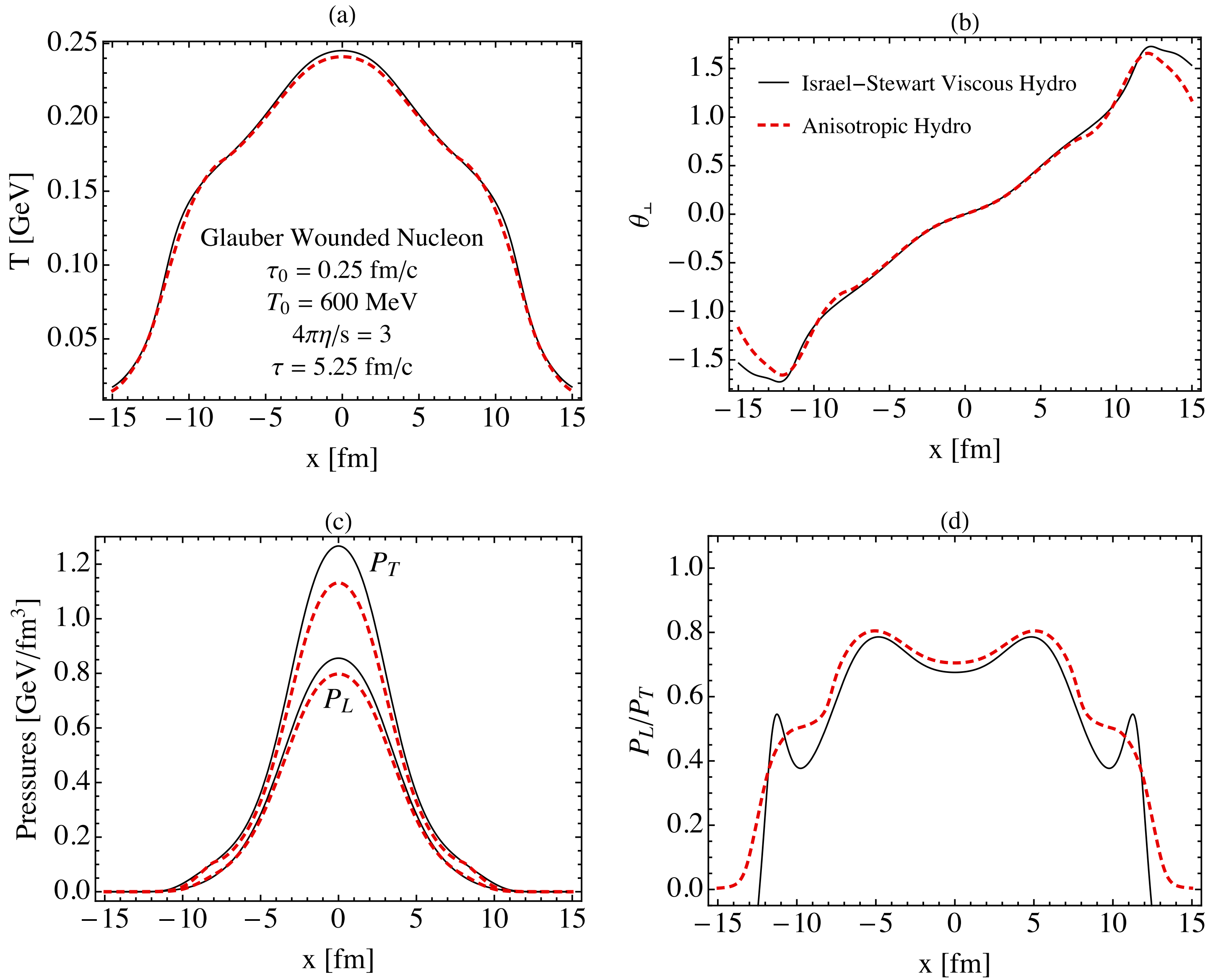
This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical but unclear how important this is in the end

1+1d aHydro and vHydro results



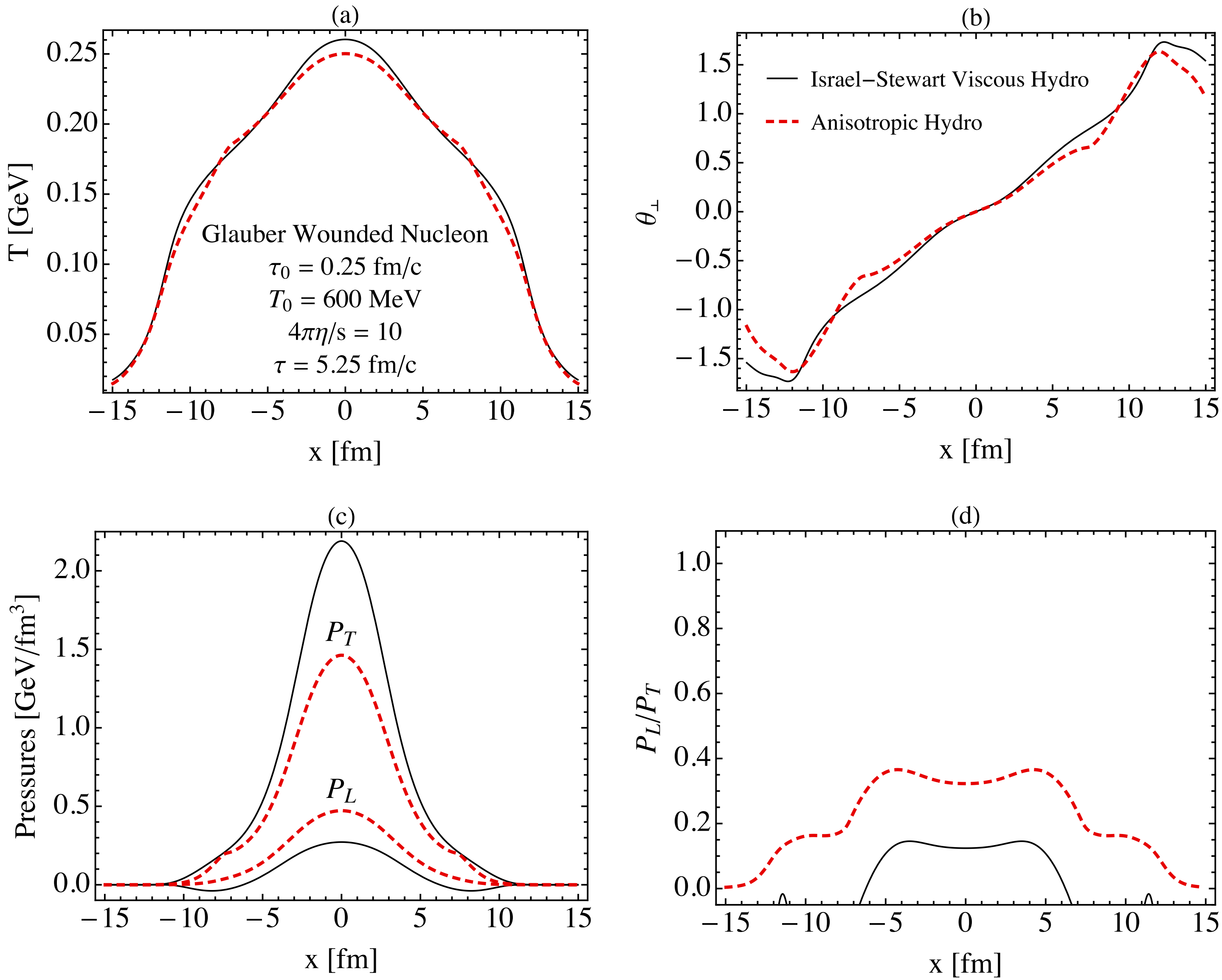
[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

1+1d aHydro and vHydro results

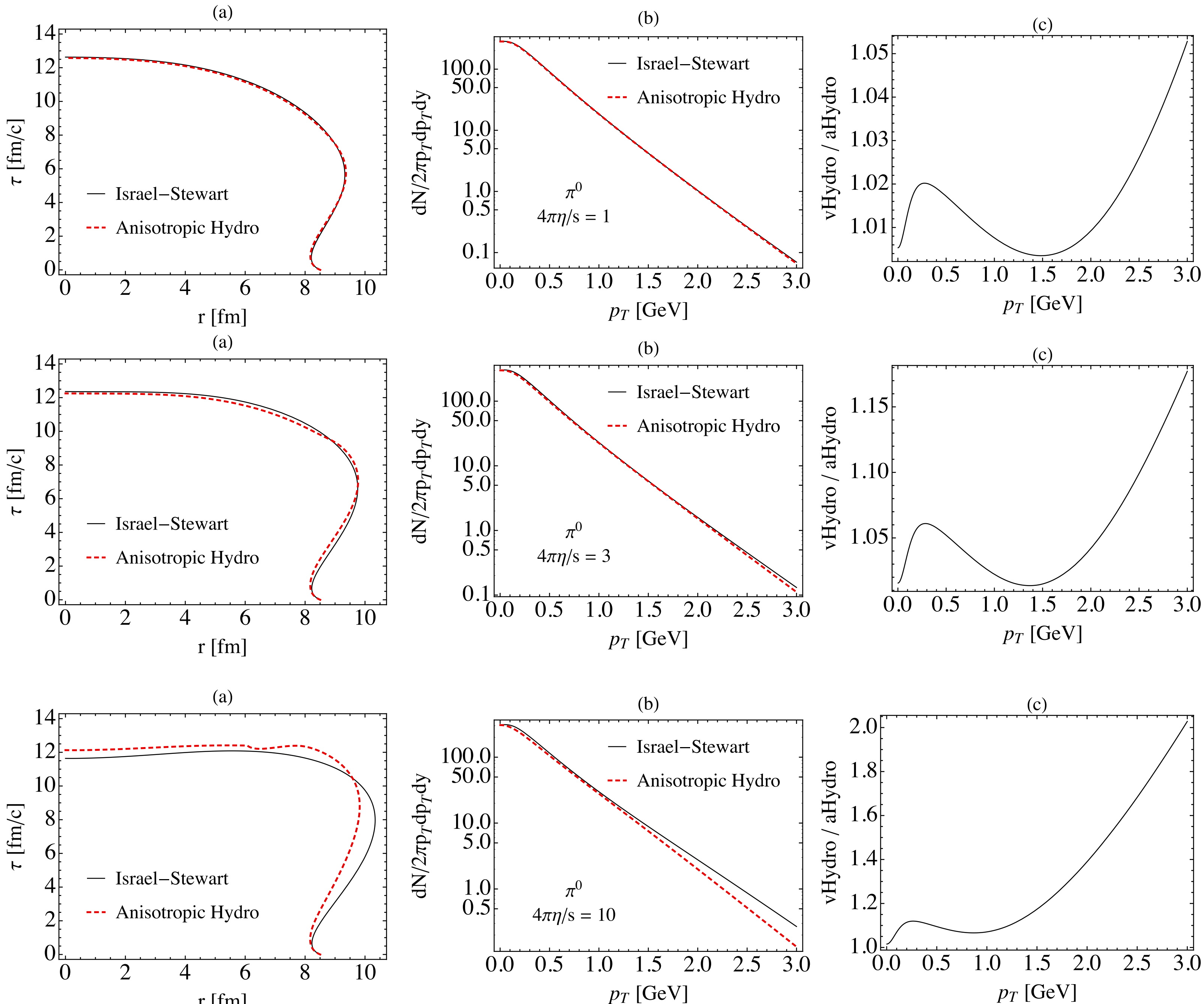


[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

1+1d aHydro and vHydro results



 [Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]



[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

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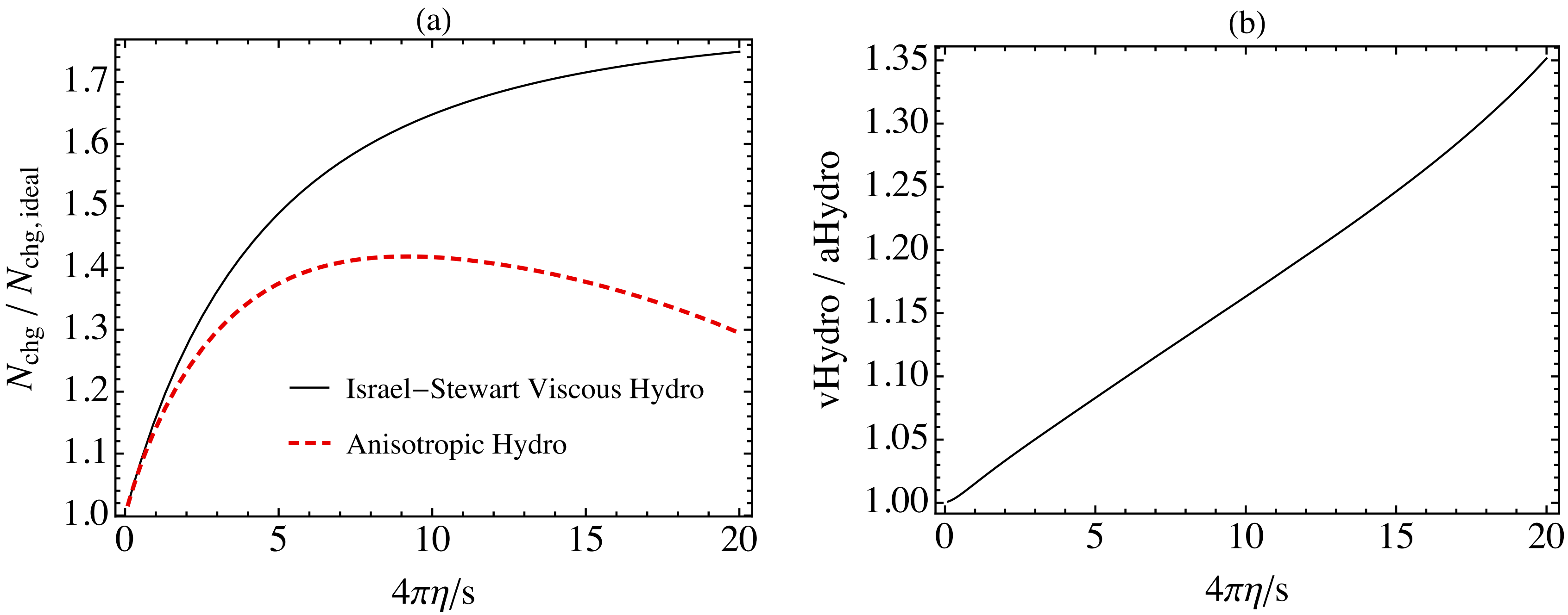
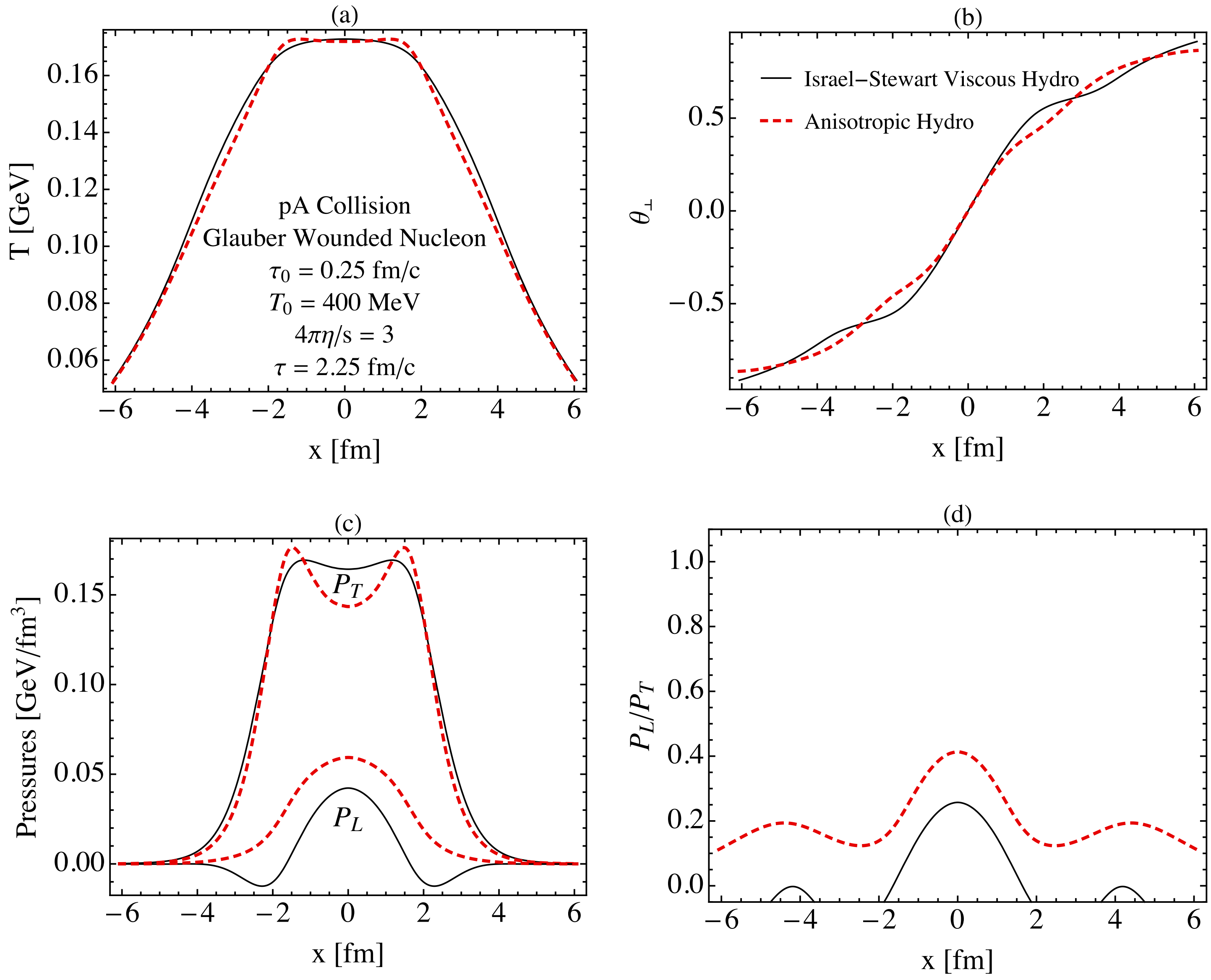


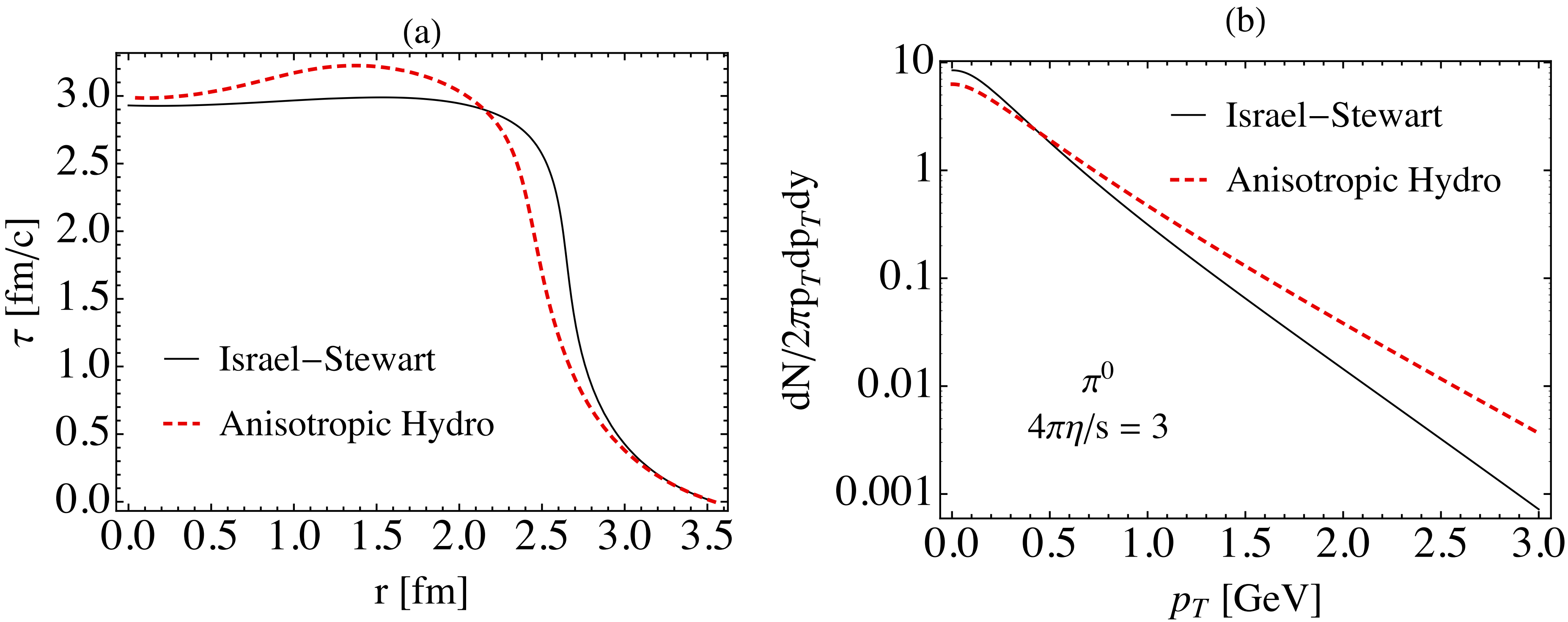
[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

1+1d aHydro and vHydro results



[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

1+1d aHydro and vHydro results



[Bazow, Heinz, Martinez, Nopoush, Ryblewski, MS,1506.05278]

Conclusions II

- For Pb-Pb collisions, in the limit of small shear viscosities aHydro agrees well with 2nd order viscous hydro for the effective temperature profile etc

- Of course, differences grow larger as eta/s is increased

- There is less viscous particle production in aHydro than 2nd order viscous hydro

- For pA collisions there are large anisotropies in the pressures throughout the QGP lifetime and consequently there are large anisotropic (viscous) corrections to the freezeout distribution

BACKUP

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the ideal hydro limit when $\xi=0$ ($\Lambda \rightarrow T$)

- For longitudinal (0+1d) free streaming, the LRF distribution function is of spheroidal form; limit emerges automatically in aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in viscous hydro)

Sketch of the 1+1d kinetic solution

[G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]

- Start with the RTA Boltzmann equation subject to Gubser flow
- Make a Weyl-rescaling (homogeneous transformation of RTA Boltzmann eq.) + coord. transformation of the kinetic equation
- Use the fact that the distribution function can only depend on $SO(3)_q$ x $SO(1,1)$ x Z_2 invariants

$SO(3)_q$ invariance $\longrightarrow$ $\hat{p}_\Omega^2 = \hat{p}_\theta^2 + \dfrac{\hat{p}_\phi^2}{\sin^2\theta}$

$SO(1,1)$ invariance $\longrightarrow$ $\hat{p}_\varsigma$ (related to the w variable from 0+1d solution)

Z_2 $\longrightarrow$ $\varsigma \to -\varsigma$ Reflection symmetry

$$f(\hat{x}^\mu, \hat{p}_i) \longrightarrow f(\rho; \hat{p}_\Omega^2, \hat{p}^\varsigma)$$

Sketch of the 1+1d kinetic solution

[G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]

- For a conformal system the relaxation time must be proportional to the inverse temperature (no other scale)

$$\tau_{\rm eq} = \frac{c}{T}$$

For RTA kernel $c = 5\eta/\mathcal{S}$

- This gives

$$\boxed{\frac{\partial}{\partial\rho} f(\rho;\hat{p}_\Omega^2,\hat{p}_\varsigma) = -\frac{\hat{T}(\rho)}{c}\left[f(\rho;\hat{p}_\Omega^2,\hat{p}_\varsigma) - f_{\rm eq}\Big(\hat{p}^\rho/\hat{T}(\rho)\Big)\right]}$$

with $\hat{p}^\rho = \sqrt{\dfrac{\hat{p}_\Omega^2}{\cosh^2\rho} + \hat{p}_\varsigma^2}$ (mass shell constraint)

- This looks exactly like the Bjorken-flow problem solved previously!

Sketch of the 1+1d kinetic solution

[G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]
[M. Nopoush, R. Ryblewski, and MS, 1410.6790]

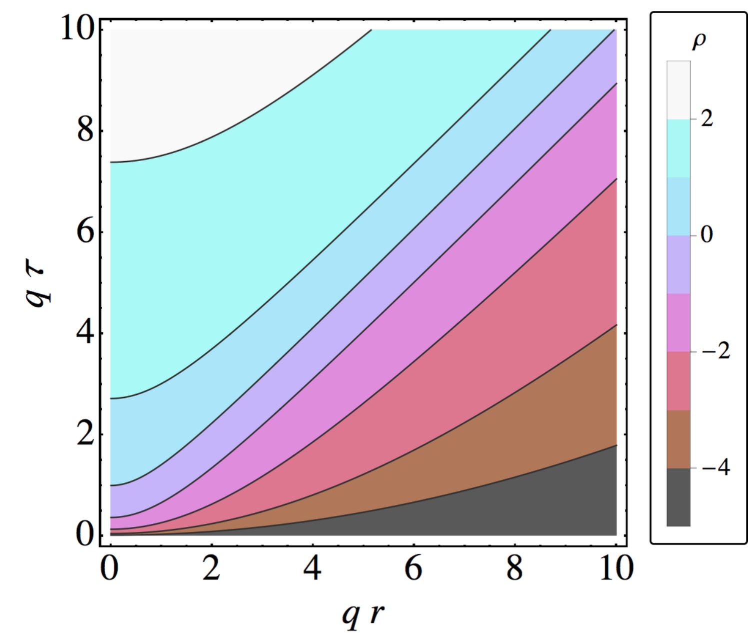
- As before, we can turn this into a 1d integral equation for the energy density and, once that it is solved, we can determine all components of the energy-momentum tensor and the full distribution function

$$\hat{\varepsilon}(\rho) = D(\rho,\rho_0)\hat{\varepsilon}_{\rm FS} + \frac{3}{\pi^2 c}\int_{\rho_0}^{\rho} d\rho' D(\rho,\rho')\mathcal{H}_\varepsilon\left(\frac{\cosh\rho'}{\cosh\rho}\right)\hat{T}^5(\rho')$$

$$\mathcal{H}_\varepsilon(x) \equiv \frac{x^2}{2} + \frac{x^4}{2}\frac{\tanh^{-1}\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

- For heavy ion application, the initial value for the de Sitter space energy density should be provided at $\rho_0 \rightarrow -\infty$ which maps to $\tau_0 \rightarrow 0^+$

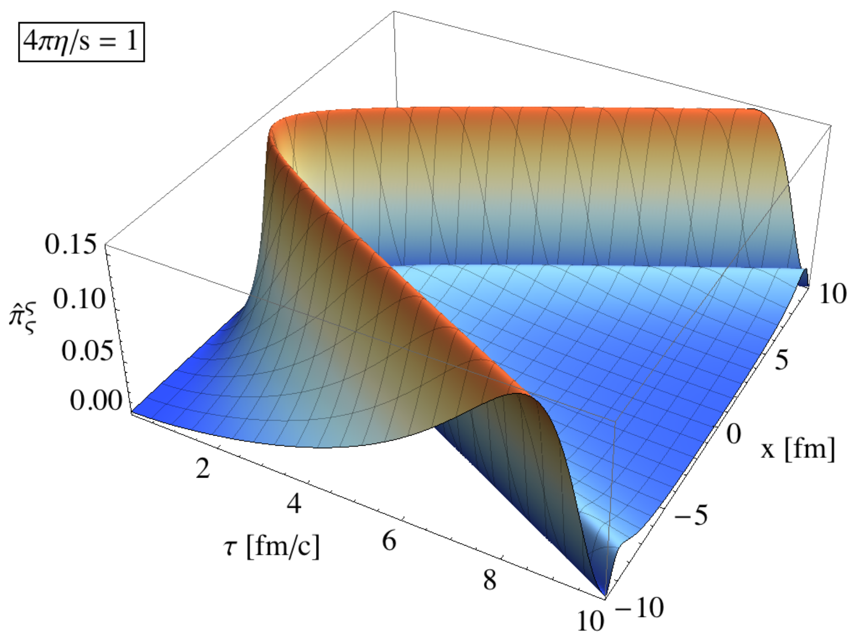
- I will show results for $\rho_0 = -10$ which, for $q = 1$, maps to $\tau_0 < 5 \times 10^{-4}$ fm/c

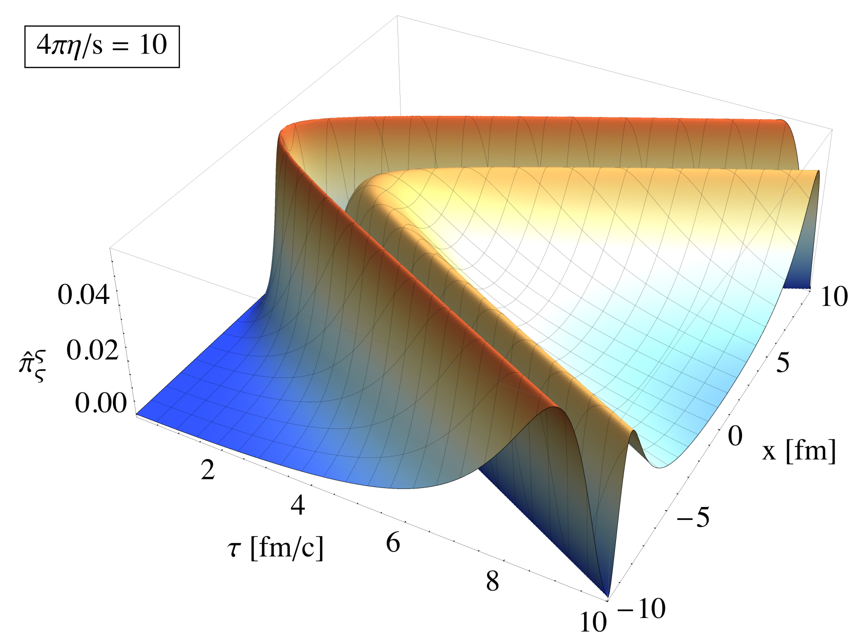


Results of the 1+1d kinetic solution

[G. Denicol, U.W. Heinz, M. Martinez, J. Noronha, and MS, 1408.5646 and 1408.7048]

Gives exact solution in the forward light cone.
Below I show the solution for the scaled shear correction.

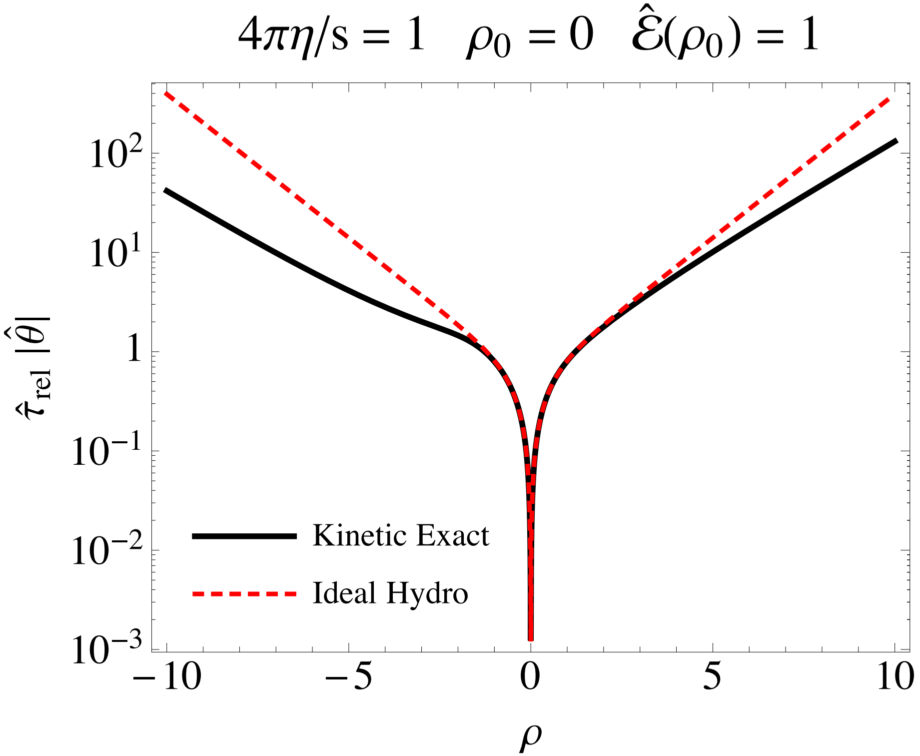




Why is this nontrivial?

Knudsen number in de Sitter coordinates

$$\mathrm{Kn} = \hat{\tau}_{\mathrm{micro}}/\hat{\tau}_{\mathrm{macro}} = \hat{\tau}_{\mathrm{rel}}\,|\hat{\theta}| \equiv \underbrace{\hat{\tau}_{\mathrm{rel}}}_{c/\hat{T}}\,\underbrace{|\hat{\nabla}\cdot\hat{u}|}_{2\tanh(\rho)}$$



Exponentially large gradients at early and late de Sitter times!