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Holographic entanglement entropy in anisotropic systems

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Motivation

- Use entanglement entropy to study time evolution of strongly coupled systems

Why use entanglement entropy?

- Might show new features in the thermalization process which is not captured by other observables
- Relatively easy to compute via the gauge gravity duality

- Most therm. studies have quantum quenches ala Cardy and Calabrese in mind

Our goal

- Study the evolution of entanglement entropy in holographic models that are used to study heavy ion collisions

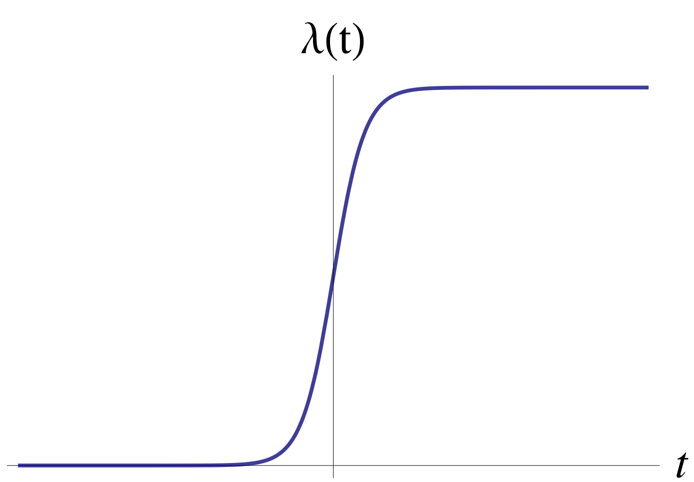
Outline

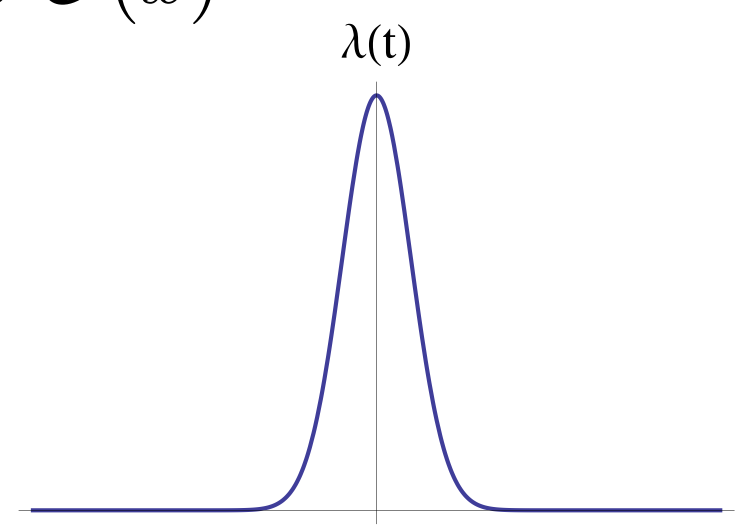
- Start with quenches and collapsing shell models
- Entanglement entropy in an anisotropic system
- Entanglement entropy in shock wave collisions

Quenches

- Take QFT and prepare it in its vacuum state
- Excite system by injecting energy into the system
- E.g. time dependent coupling

$$H = H_0 + \lambda(t) \int dx\, \mathcal{O}(x)$$





- Use **entanglement entropy** to study the dynamics of the system

Entanglement entropy

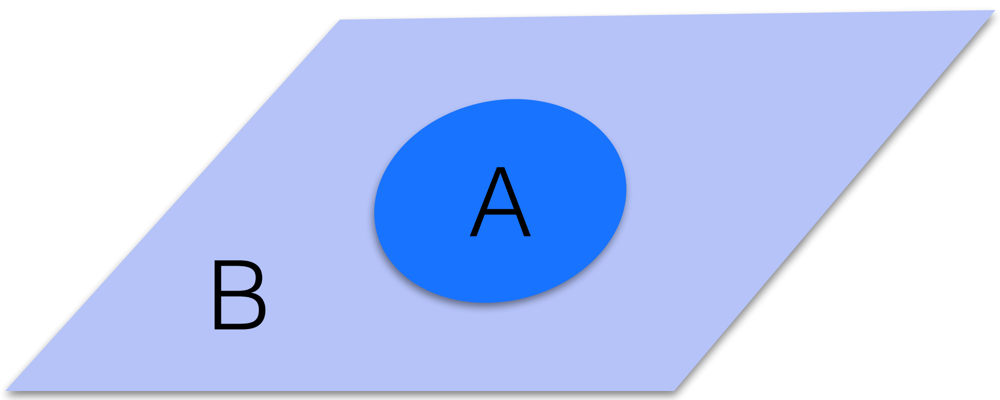
Definition

- Split a system into two parts, a subsystem of interest A and the rest B
- Observables in A are determined by the reduced density matrix

$$\rho_A = \mathrm{Tr}_B \rho$$

- Von Neumann entropy of subsystem A

$$S_{EE}(\rho) = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$



Properties

- Nonlocal quantity
- Serves as an order parameter in condensed matter systems
- Prop. to the degrees of freedom: $S(l) = \frac{c}{3} \log \frac{l}{\epsilon}$
- Prop. to the area of the entangling surface

Quench of 2 dim CFT's

Cardy, Calabrese (2005)

- Prepare system in pure state $|\Psi_0\rangle$
- at t=0 quench from $\lambda_0 \to \lambda$
- System evolves unitarily according to $H(\lambda)$

$$S_A(t) \sim \begin{cases} \dfrac{\pi c t}{6\epsilon} & (t < \ell/2) \, , \\ \dfrac{\pi c \, \ell}{12\epsilon} & (t > \ell/2) \, , \end{cases}$$

- Linear increase with time
- Saturation at l/2
- Can be understood in terms of quasiparticle pairs created by the quench

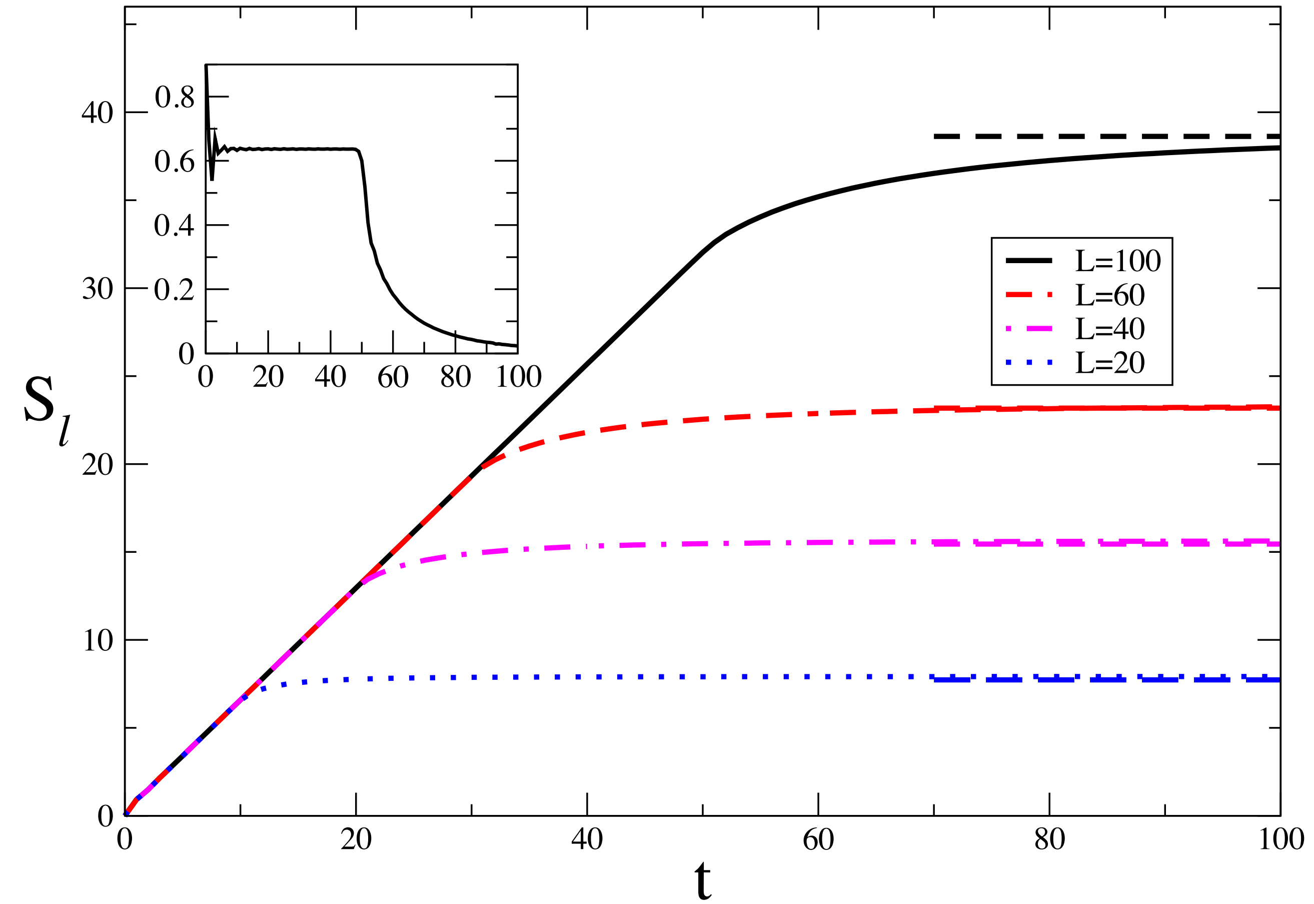
Quench of the Ising model

Cardy, Calabrese (2005)

- Quantum Ising chain in transverse magnetic field

$$H_I(h) = -\frac{1}{2}\sum_j \left[\sigma_j^x \sigma_{j+1}^x + h\sigma_j^z\right]$$

- Quench system at t=0 from infinite h to h =1 (from uncorrelated state to critical point)
- Linear scaling of EE



The gravity story

Correlators

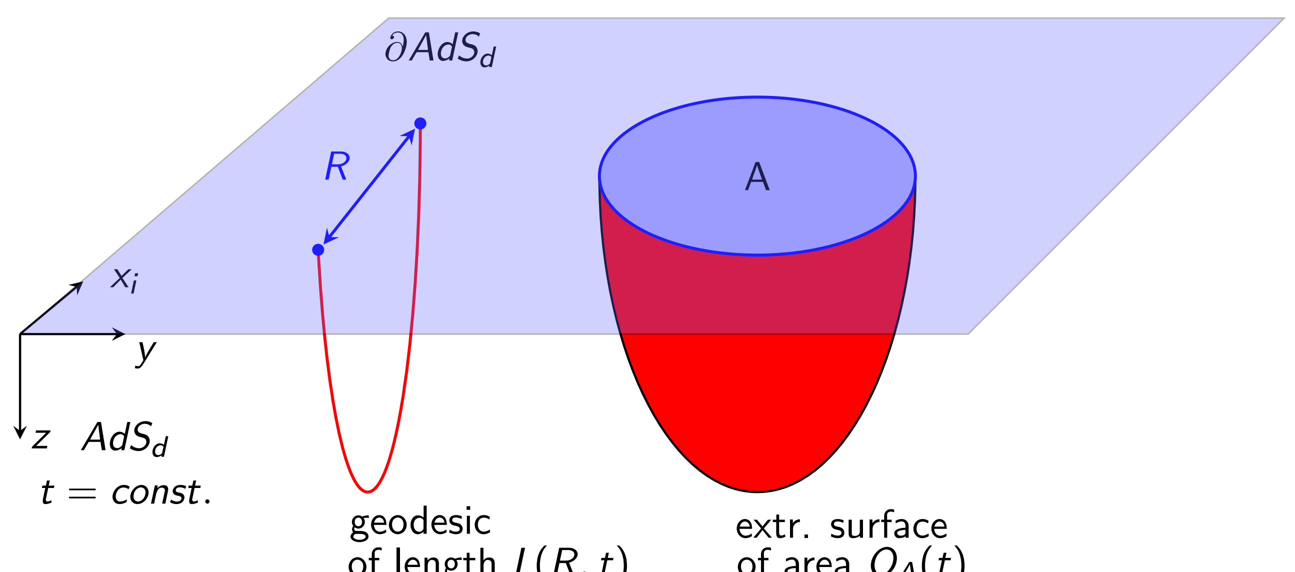
- Equal time two point function for operators of large conformal weight

$$\langle \mathcal{O}(t,\vec{x})\mathcal{O}(t,\vec{x}')\rangle \sim e^{-\Delta L}$$

Entanglement entropy

- The EE is conjectured to be given by the area of extremal surfaces

$$S_{EE} = \frac{A_{ext}}{4G_N}$$

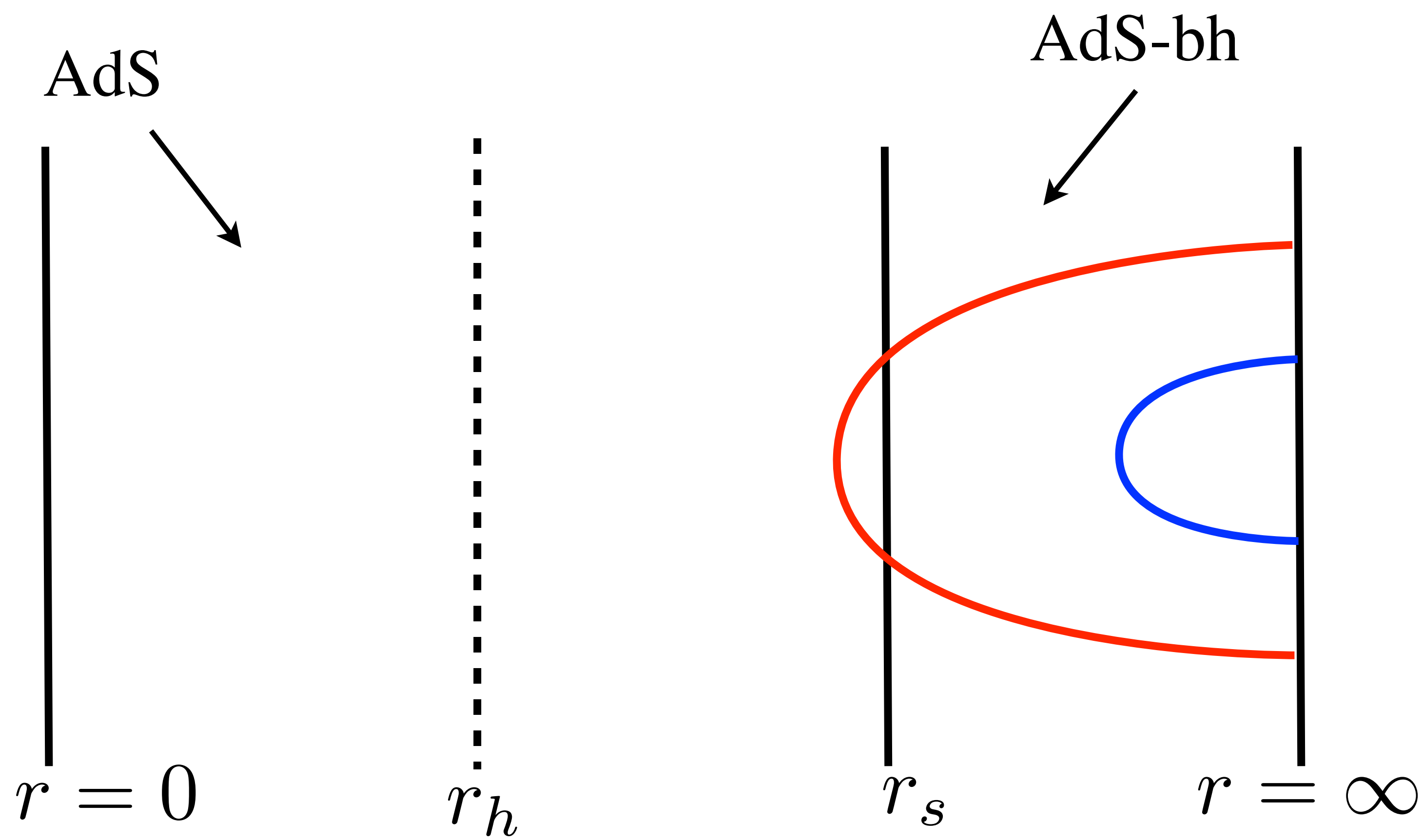


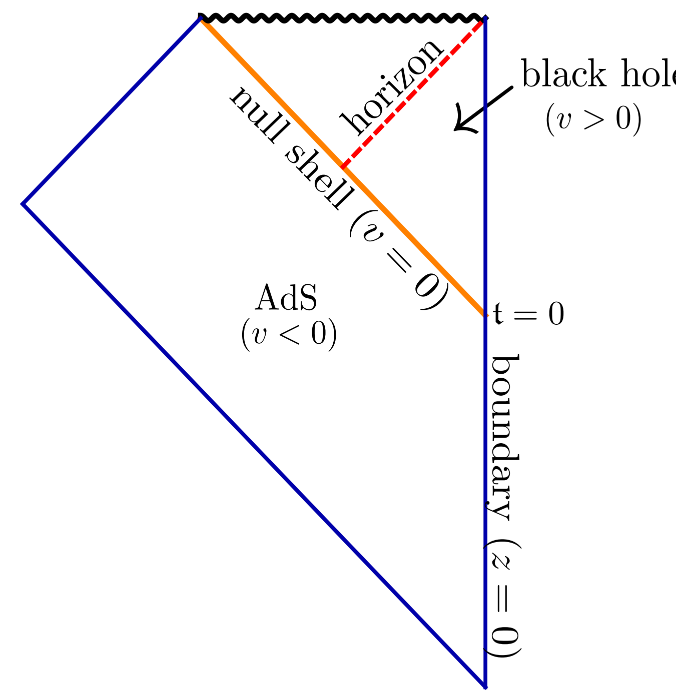
S. Ryu, T. Takayanagi, hep-th/0603001

Far from equilibrium dynamics

The falling shell setup

Danielsson, Keski-Vakkuri, Kruczenski (1999); Lin, Shuryak (2008)





$$ds^2 = \frac{1}{z^2}\left[-(1 - m(v)z^d)dv^2 - 2dvdz + d\vec{x}^2\right]$$

Thermalization from geometric probes:

- Top down thermalization: High energetic modes approach equilibrium value first

EE in the Vaidya space time

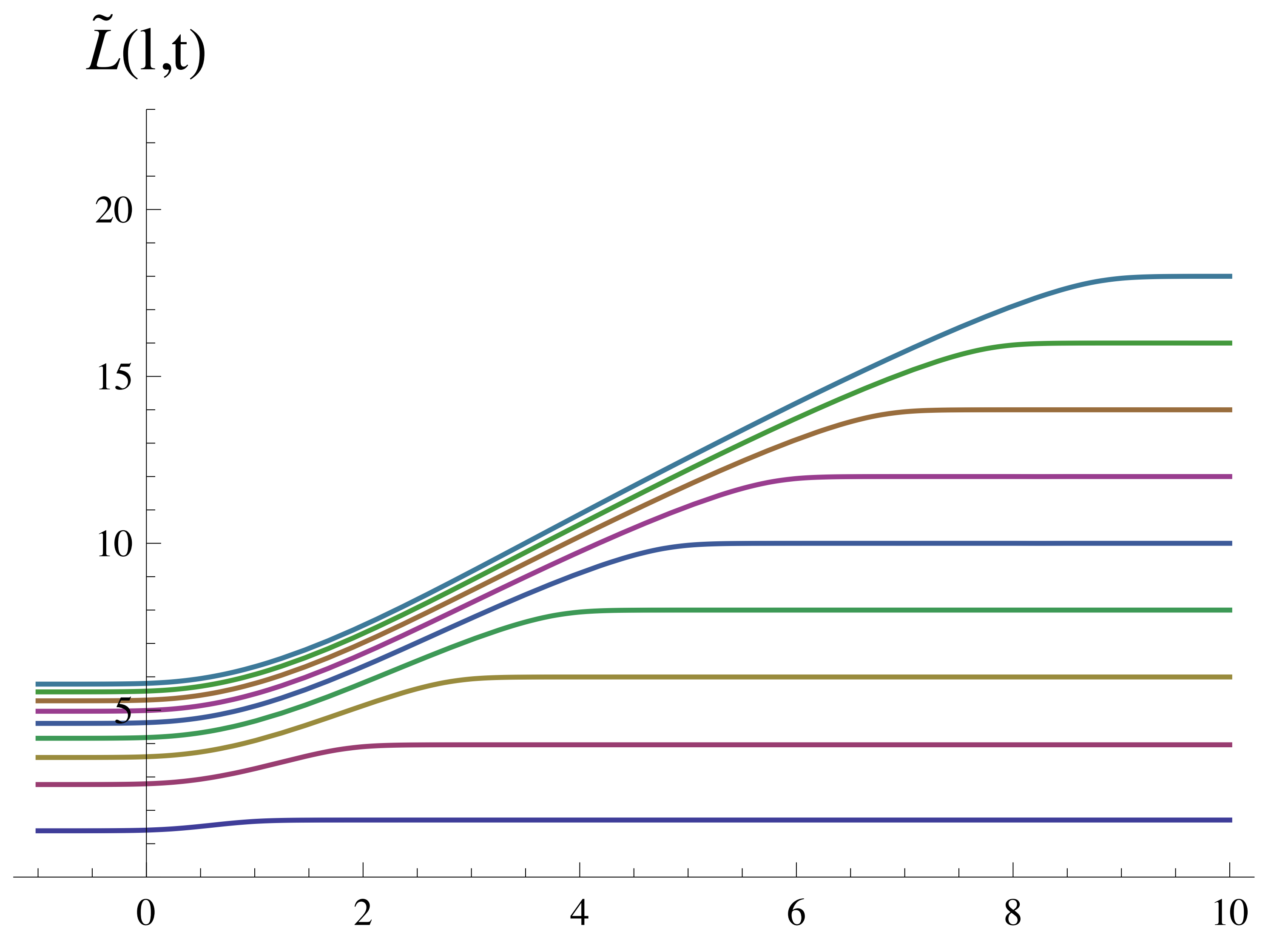
Abajo-Arrastia, Apricio, Lopez (2010)

EE in 2-dim field theory from holography

- Linear growth
- saturation happens at

$$t \sim \frac{l}{2} + a$$

constant shift from mass profile



Stages of the time evolution

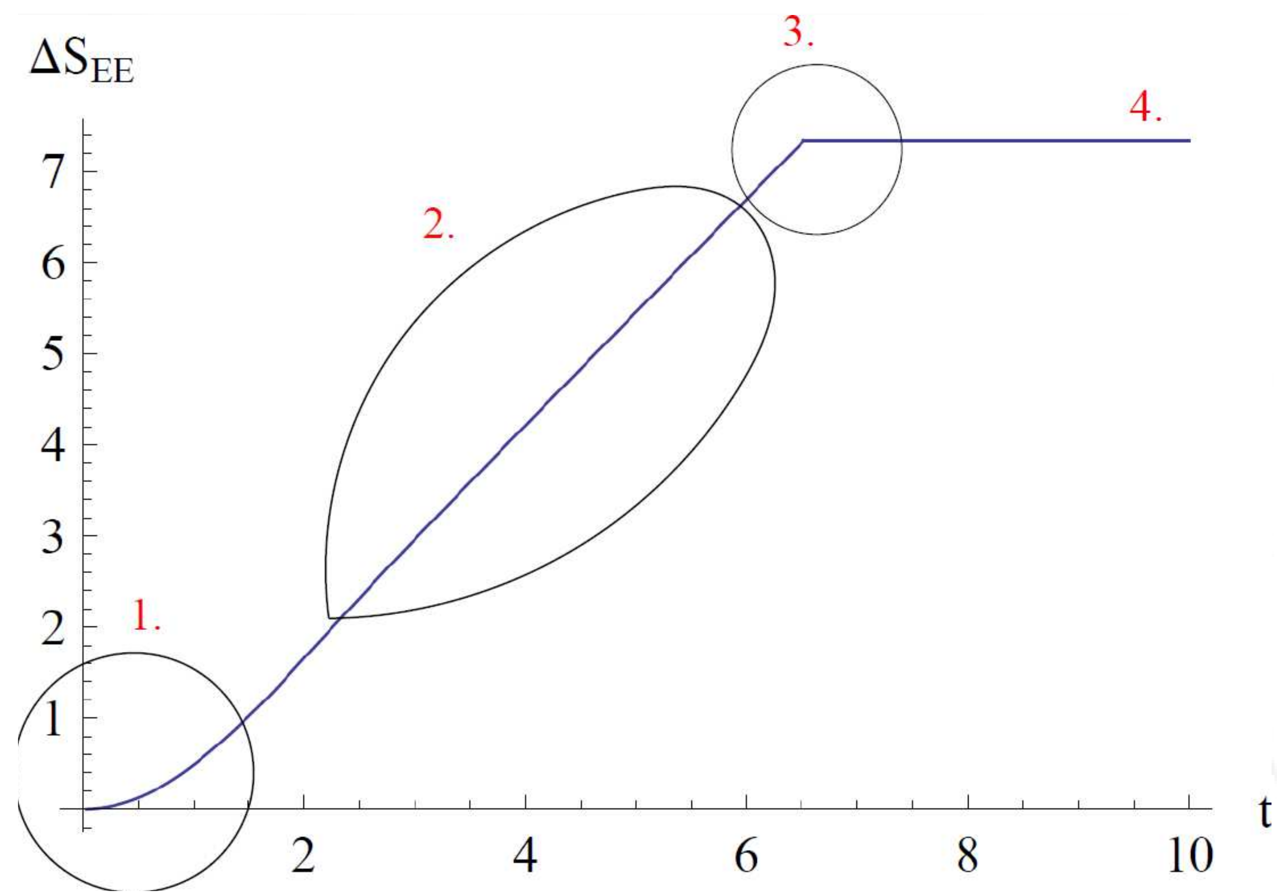
Liu and Suh (2013)

1. Quadratic part:

$$\Delta S_{EE} = \epsilon \, A \, t^2$$

2. Linear part

$$\Delta S_{EE} = A \, s_{eqn} \, v_E \, t$$



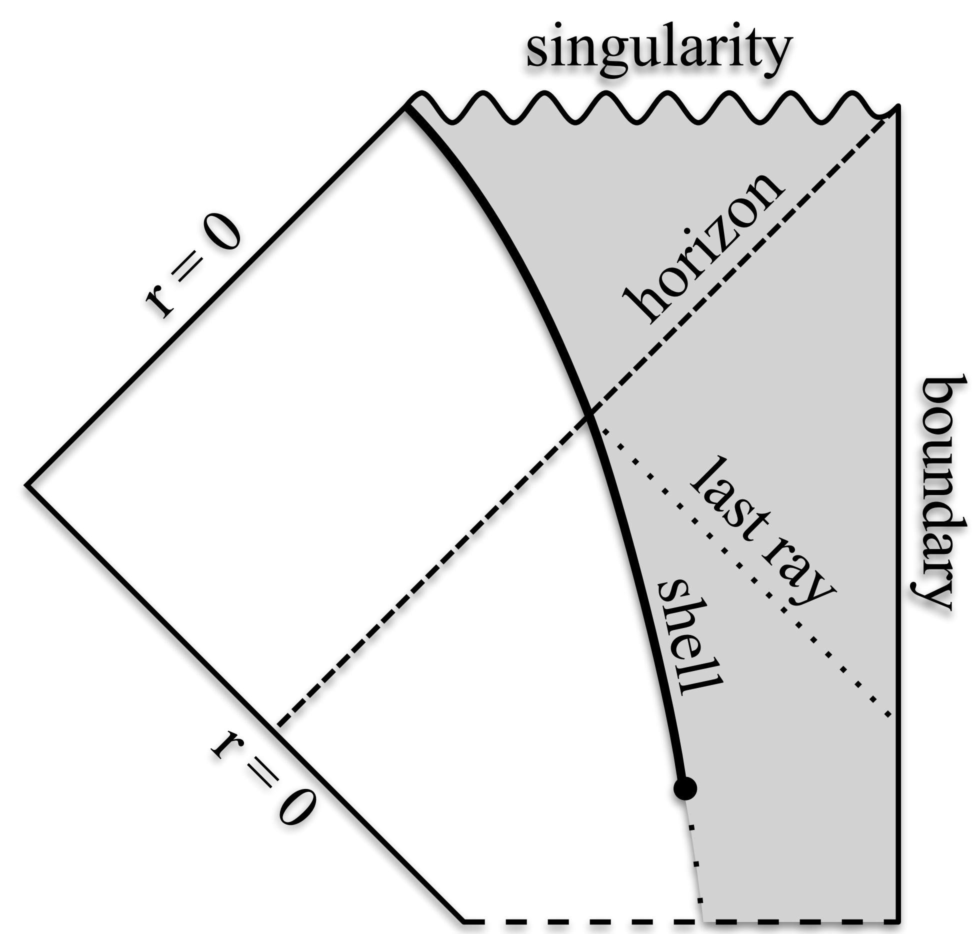
- Both parts 1 & 2 seem universal
- The coefficient v_E characterizes how fast A is getting entangled

$$v_E = \frac{(\eta - 1)^{\frac{\eta-1}{2}}}{\eta^{\eta/2}} \qquad \eta = \frac{2(d-1)}{d}$$

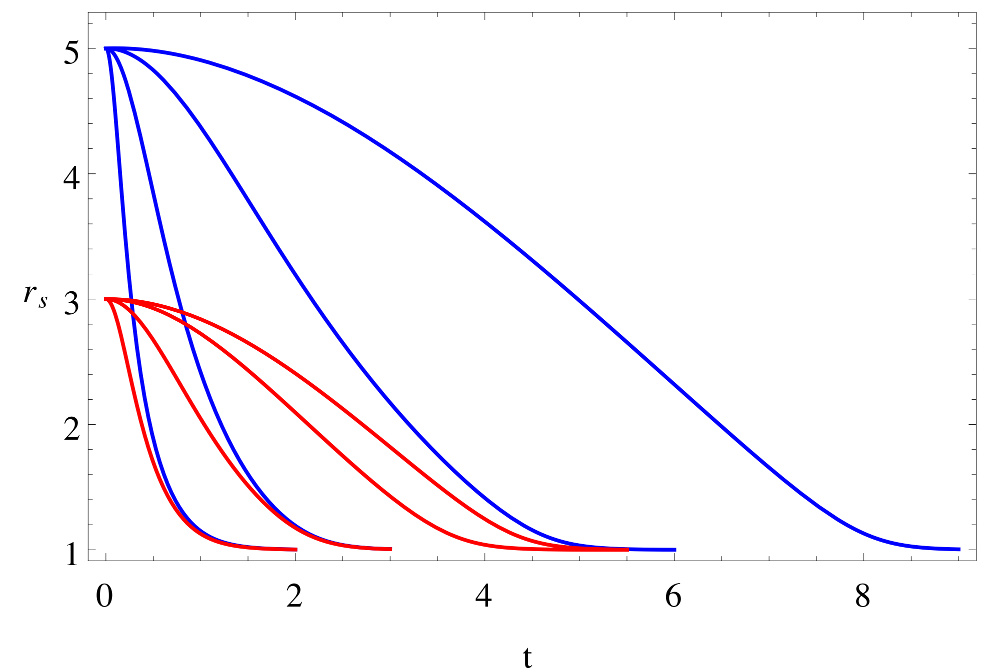
- Conjecture: Collapse to Schwarzschild black hole maximizes v_E
- Linear scaling comes from critical surfaces behind the EH

Slower collapse

- Release shell from rest at a certain position in the bulk
- Equations of motion follow from Einstein equations for different equations of state: p=c E
- In the dual field theory this is a state that starts out thermal at short length scales



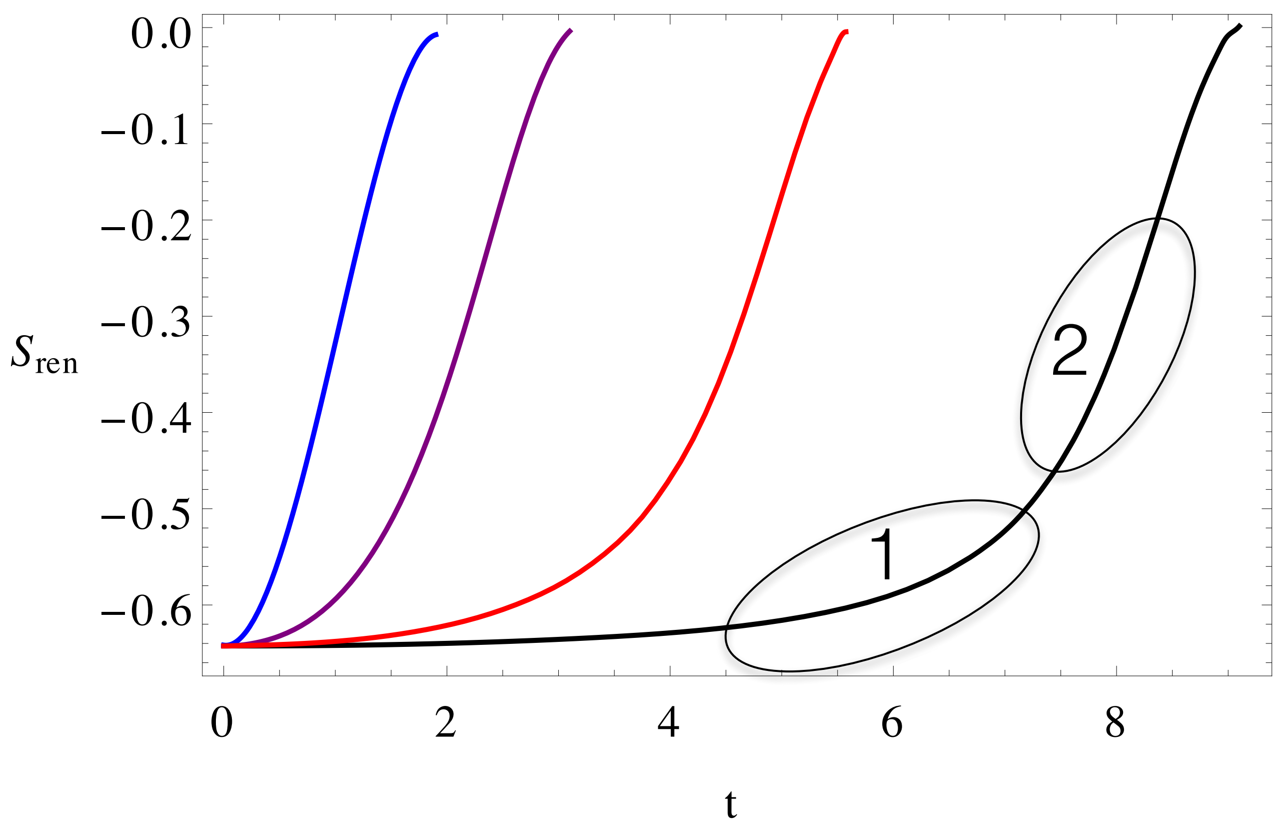
Keranen, Nishimura, SS, Taanila, Vuorinen (2014)



shell trajectories for different EoS

EE in the collapsing shell setup

EE for different equations of state p = c E



Keranen, Nishimura, SS, Taanila, Vuorinen (2014)

1. Quadratic part: depends on the acceleration (depends on c) of the shell:

$$S_{EE} = A\,F(z_0)\,a\,t^2$$

2. Linear part same as before: $S_{ren} = A s_{eq} v_E t$

- Linear scaling only depends on equilibrium state and originates from geometry behind the horizon

- Linear scaling seems quite generic in falling shell models. Also appears in geometries with Lifshitz scaling and hyper scaling violation
- Next we move on to geometries that are more relevant for heavy ion collisions

The anisotropic geometry

Chessler, Yaffe (2009); Heller, Mateos, van der Schee, Trancanelli (2012)

- Anisotropic asymptotically AdS_5 spacetime

$$ds^2 = -A(r,v)dv^2 + 2drdv + \Sigma^2(r,v)\Big(e^{-2B(r,v)}dx_\parallel^2 + e^{B(r,v)}d\vec{x}_\perp^2\Big)$$

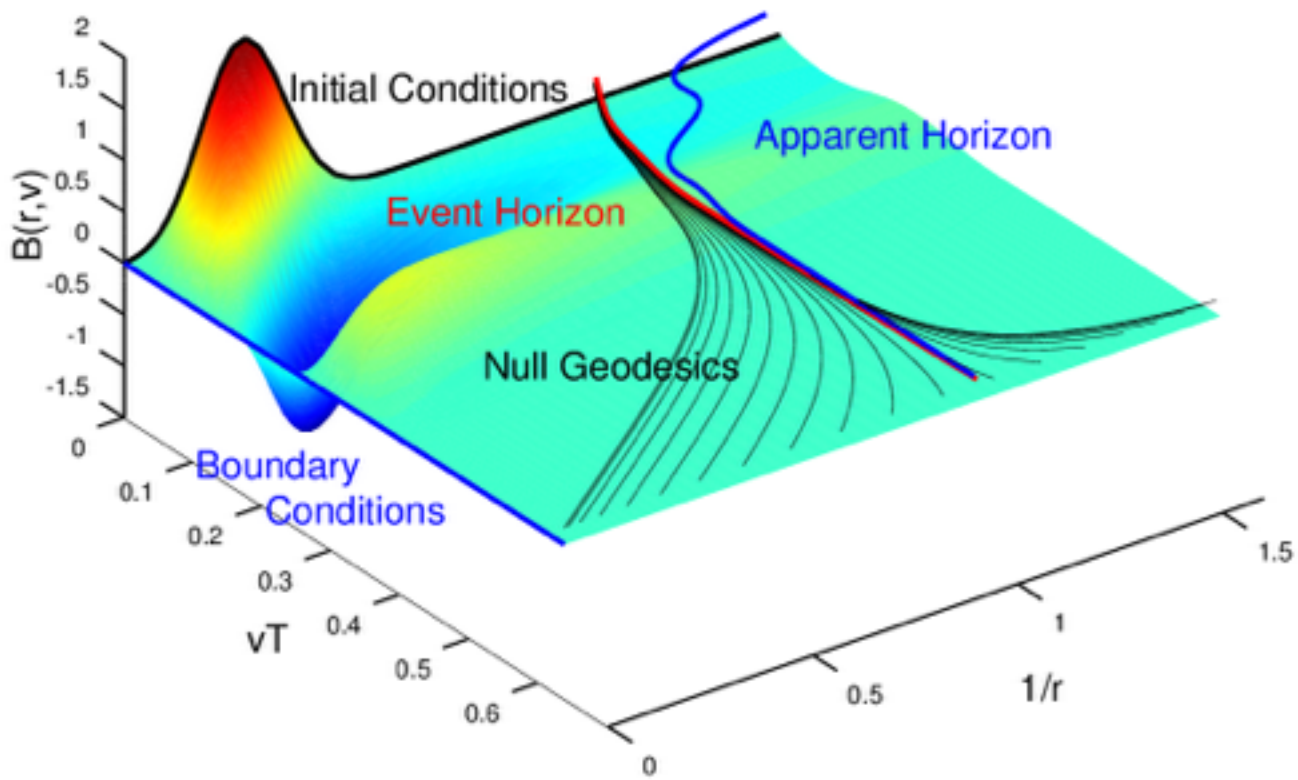
- Introduces anisotropy between long. and transverse directions
- Energy momentum tensor

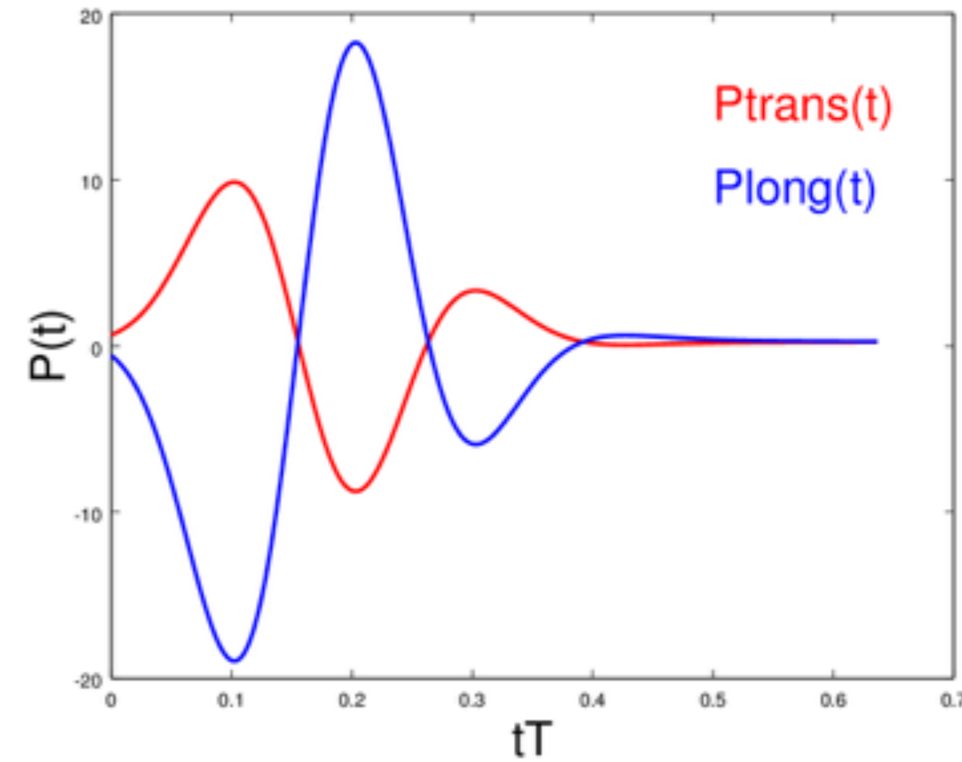
$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\,\mathrm{diag}\left(\mathcal{E},\ P_\parallel(t),\ P_\perp(t),\ P_\perp(t)\right)$$

- Create far from equilibrium state by choosing anisotropy function on the initial time slice

$$B(r,v_0) = \frac{\beta}{r^4}\exp\left[-\left(\frac{1}{r}-\frac{1}{r_0}\right)^2/\omega^2\right]$$

The geometry





- System evolves towards static Schwarzschild black brane solution
- Approach to equilibrium shows exponential damped oscillations

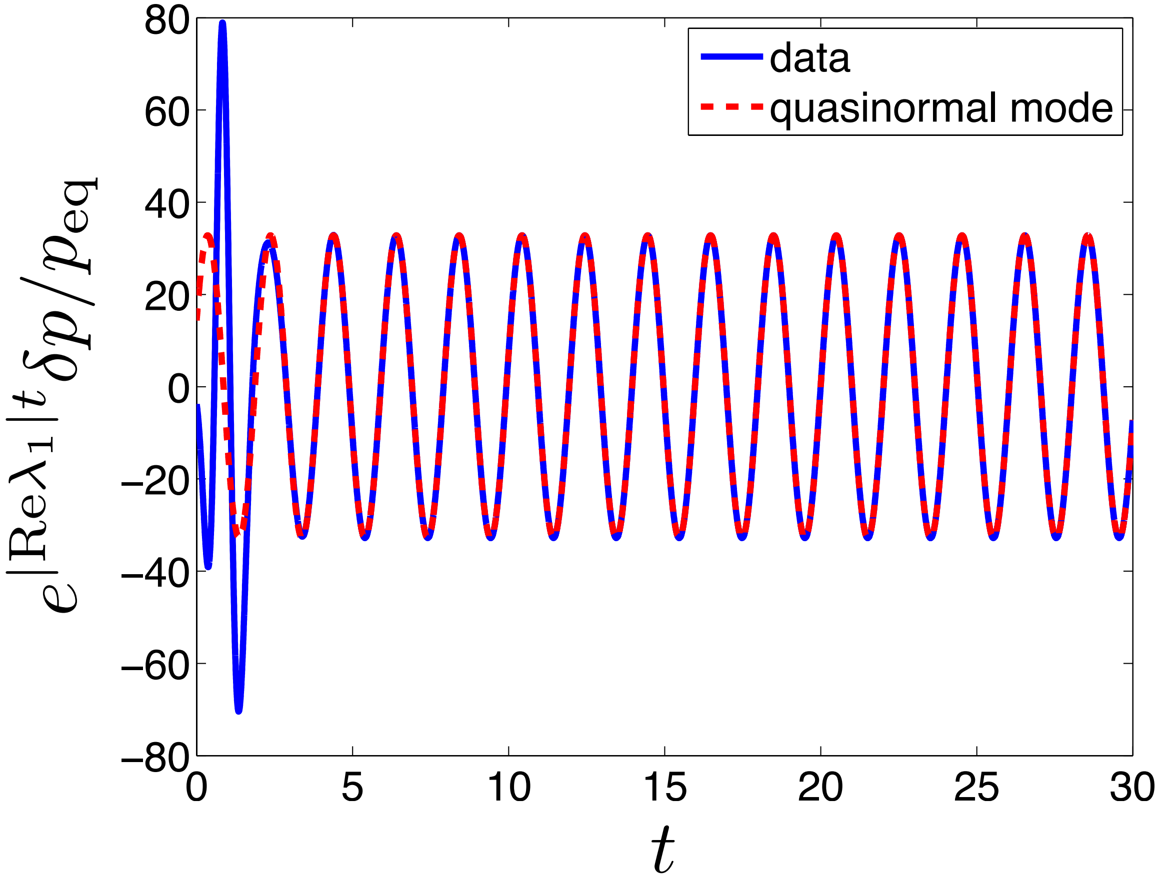
The geometry: late times

Quasinormal modes

- At sufficiently late times: linearised regime
- Approach to equilibrium accurately described by lowest QNM
- QNM from spin two symmetry channel of grav. fluctuations
- Response of the system

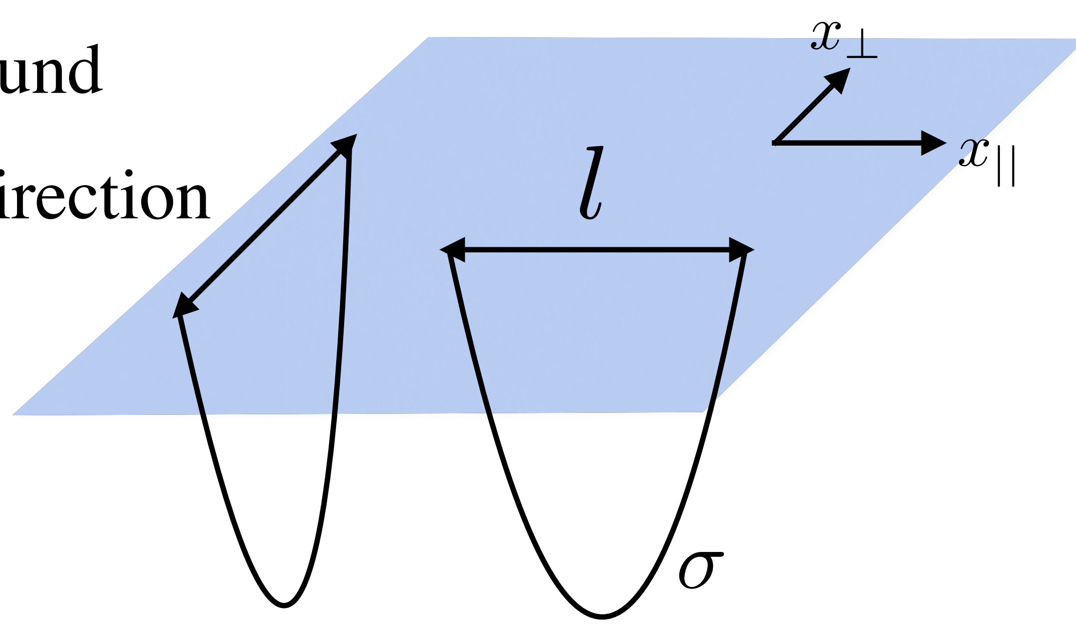
$$\delta p(t) \sim \mathrm{Re}\left(c_1 e^{-i\,\omega_1 t}\right)$$

$$\frac{\omega_1}{\pi T} = \pm 3.119452 - 2.746676\,i$$



Chessler, Yaffe (2013)

Correlators in the anisotropic geometry



- Calculate geodesic length in anisotropic background
- Separate them in the longitudinal or transverse direction

- To obtain geodesic length we have to solve the geodesic equation in the two subspaces

$$ds_\perp^2 = -Adv^2 - \frac{2}{z^2}dzdv + \Sigma^2 e^B dx_\perp^2$$

$$ds_\parallel^2 = -Adv^2 - \frac{2}{z^2}dzdv + \Sigma^2 e^{-2B} dx_\parallel^2$$

- The length is given by

$$L_\parallel = \int_{-\sigma_m}^{\sigma_m} d\sigma \sqrt{-A(v')^2 - \frac{2}{z^2}z'v' + \Sigma^2 e^{-2B}(x'_\perp)^2}$$

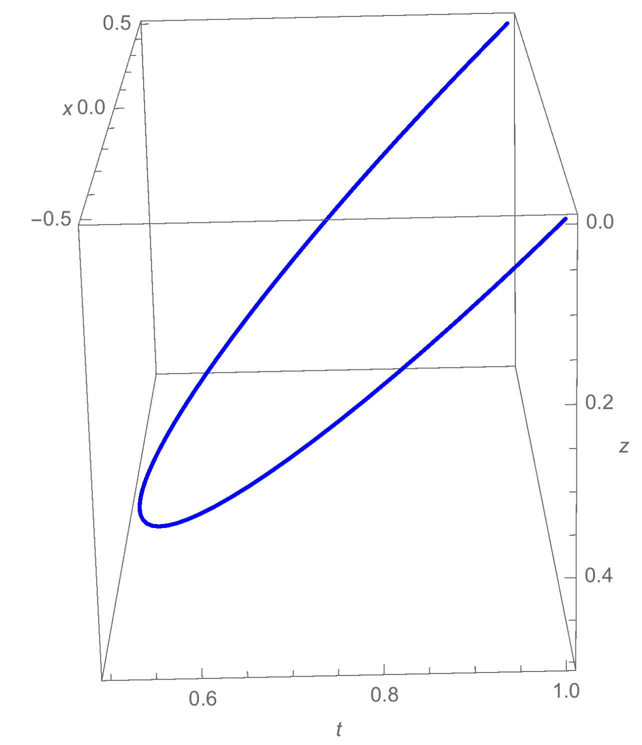
$$L_\perp = \int_{-\sigma_m}^{\sigma_m} d\sigma \sqrt{-A(v')^2 - \frac{2}{z^2}z'v' + \Sigma^2 e^{B}(x'_\perp)^2}$$

Numerical implementation

Relaxation method

- Start with initial guess
- Iteratively relax to the true solution
- Start with pure AdS solution

$$x_\pm(z) = \pm\sqrt{\frac{l^2}{4} - z^2} \qquad v(z) = v_0 - z$$



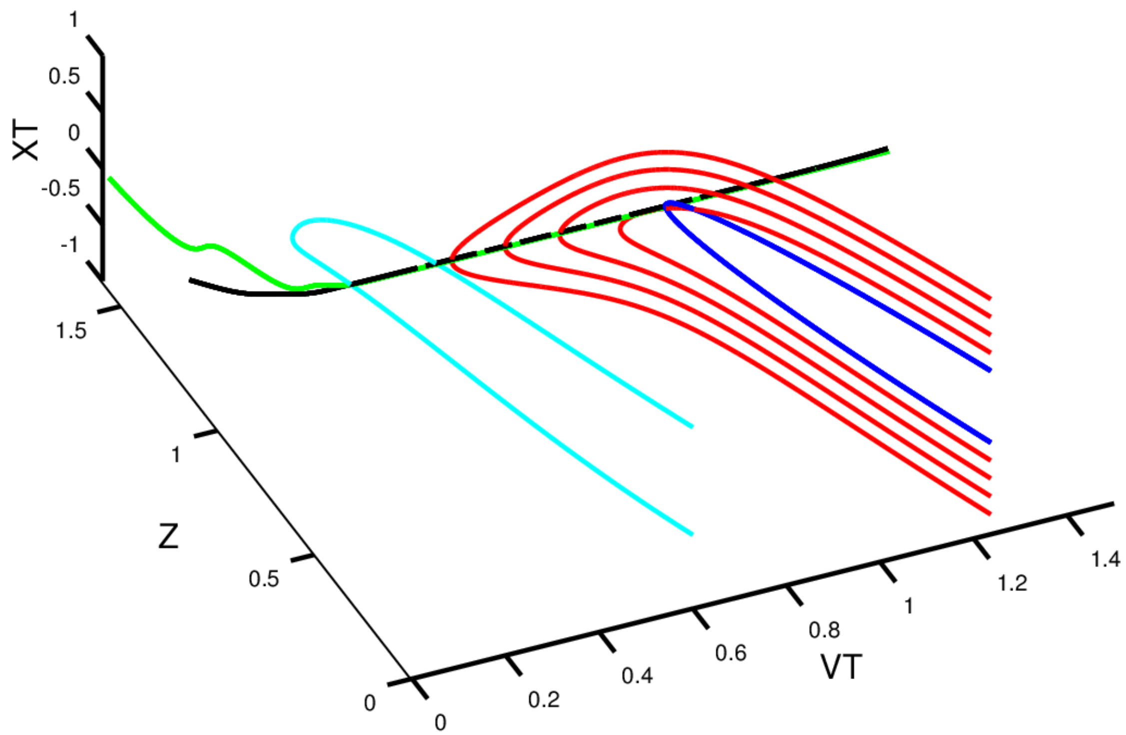
- Geodesics bend back in time: limits time domain
- Use non affine parametrization that covers both branches

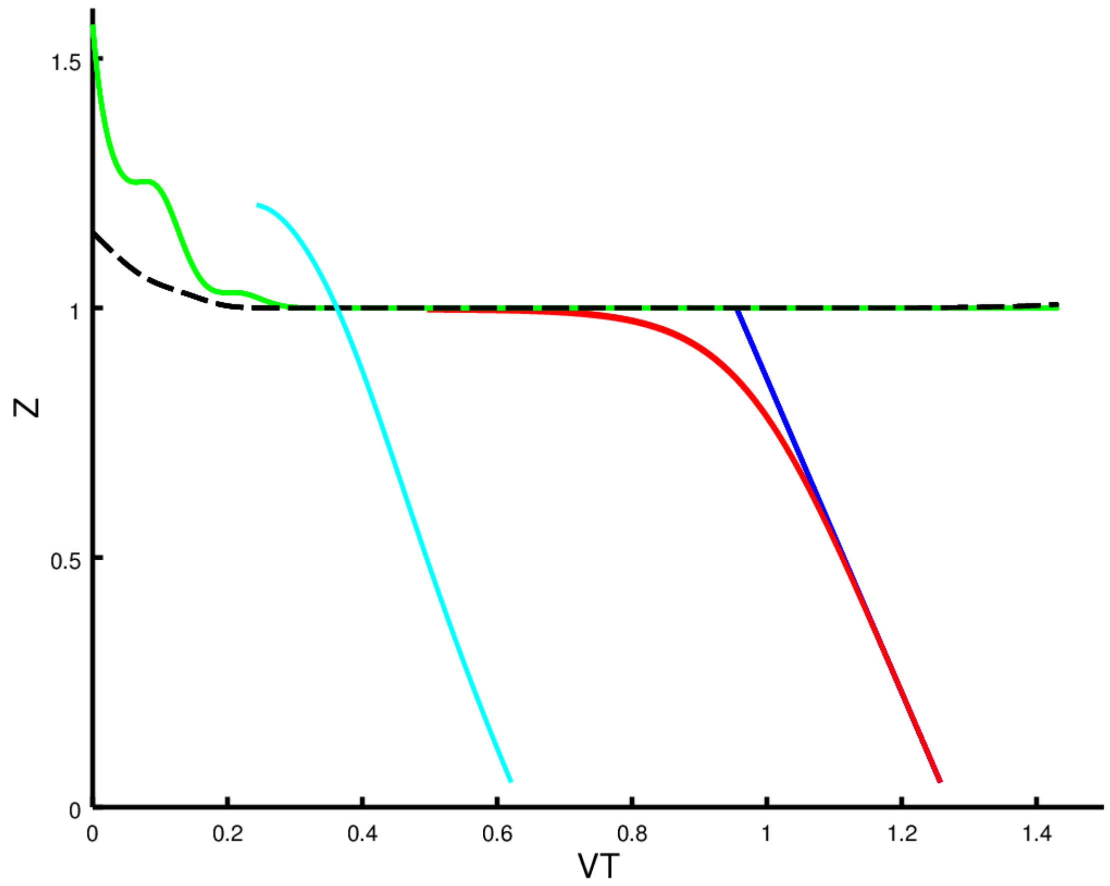
$$z(\sigma) = \frac{l}{2}(1-\sigma^2) \qquad x(\sigma) = \frac{l}{2}\left(\sigma\sqrt{2-\sigma^2}\right) \qquad v(\sigma) = v_0 - z(\sigma)$$

Geodesics

Profile of the geodesics

- At late times geodesics approach the apparent horizon without crossing it
- At early times and far from equilibrium geodesics can cross the horizon

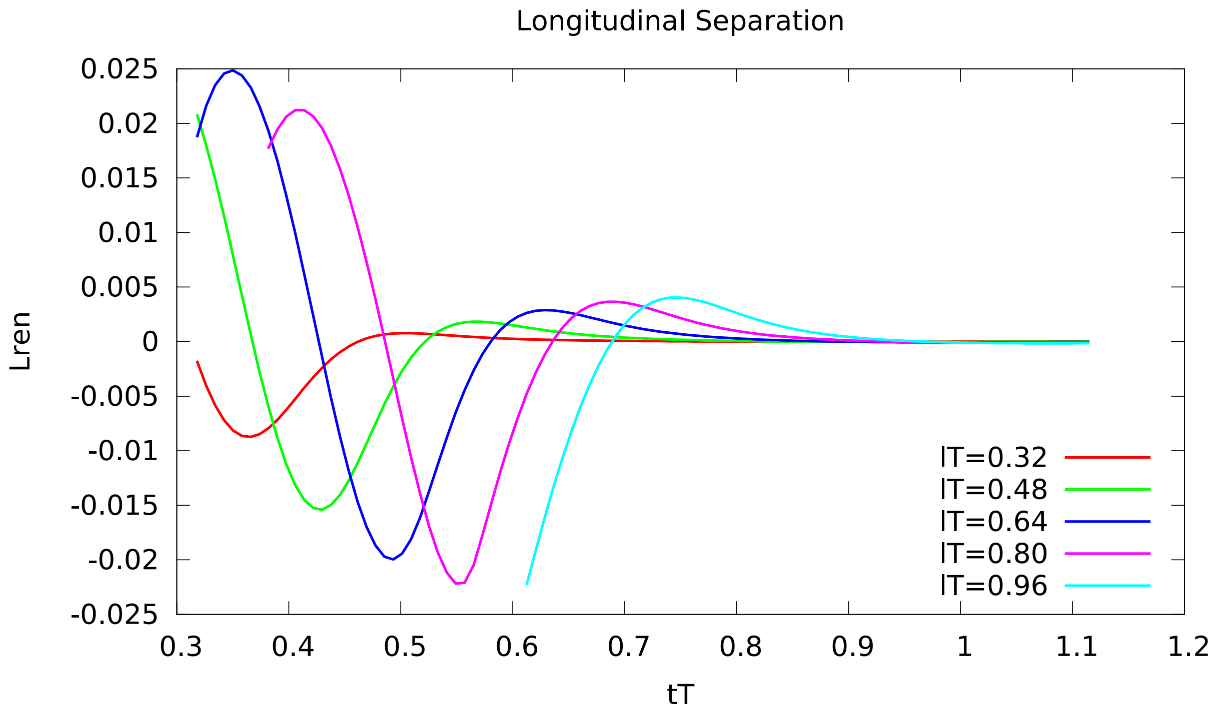


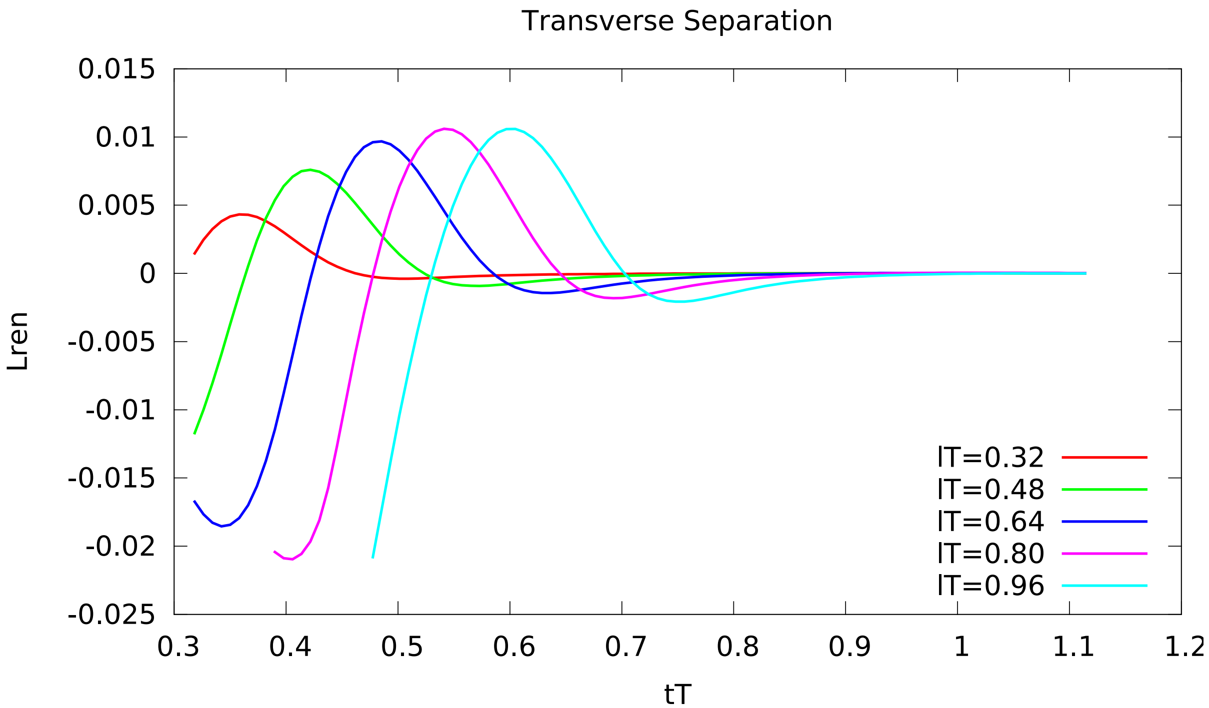


Geodesic length

- To make approach to thermal equilibrium most transparent we normalise the geodesic length

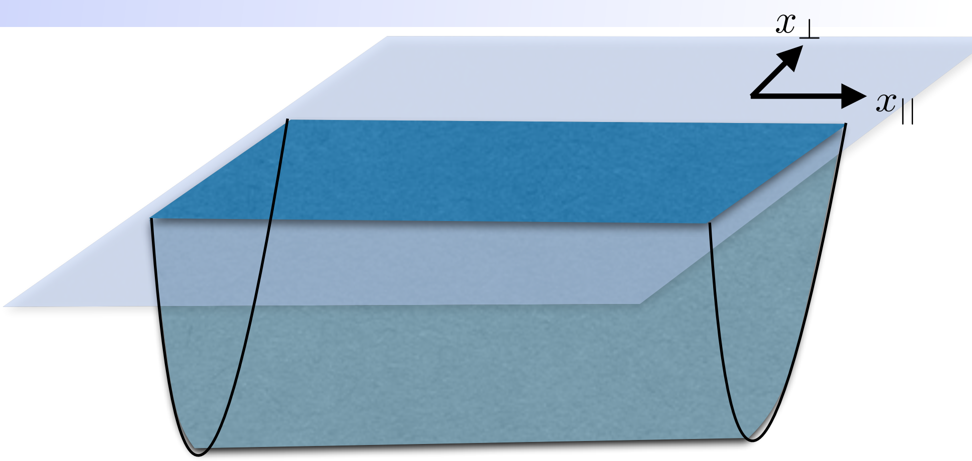
$$L_{\mathrm{ren}} = \frac{L - L_{\mathrm{th}}}{L_{\mathrm{th}}}$$





- Transverse and longitudinal directions oscillate out of phase
- Thermalization time increases as separation increases

Holographic entanglement entropy



- Extremize the 3-surface functional

$$\mathcal{A} = \int d^3\sigma \sqrt{\det\left(\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} g_{\mu\nu}\right)}$$

- In the case of a strip entangling region with finite extend in the transverse or longitudinal direction the problem reduces to finding geodesics in an auxiliary spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = h_{\alpha\beta}dx^\alpha dx^\beta + \phi_1^2 dx_2^2 + \phi_2^2 dx_3^2$$

- The area functional becomes

$$\mathcal{A} = \int dx_3 \int dx_2 \int d\sigma \sqrt{\phi_1^2\phi_2^2 h_{\alpha\beta}\frac{\partial x^\alpha}{\partial \sigma}\frac{\partial x^\beta}{\partial \sigma}}\,.$$

- Finding extremal surfaces reduces to finding geodesics in the conformal metric

$$d\tilde{s} = \tilde{h}_{\alpha\beta}dx^\alpha dx^\beta = \phi_1^2\phi_2^2 h_{\alpha\beta}dx^\alpha dx^\beta$$

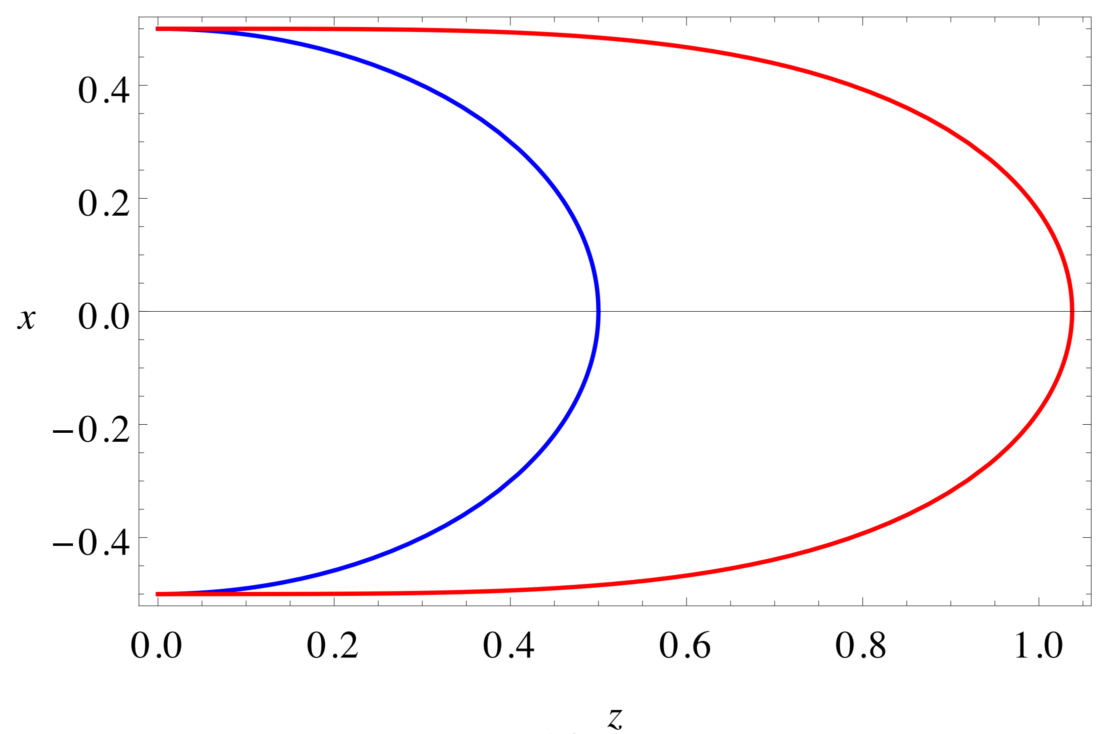
Holographic entanglement entropy

- In the case at hand the conformal metrics are

$$d\tilde{s}_{\perp}^2 = \Sigma^4 e^{-B}\big(- A dv^2 + 2 dr dv + \Sigma^2 e^{B} dx_{\perp}^2 \big)$$
$$d\tilde{s}_{\parallel}^2 = \Sigma^4 e^{2B}\big(- A dv^2 + 2 dr dv + \Sigma^2 e^{-2B} dx_{\parallel}^2 \big)\,.$$

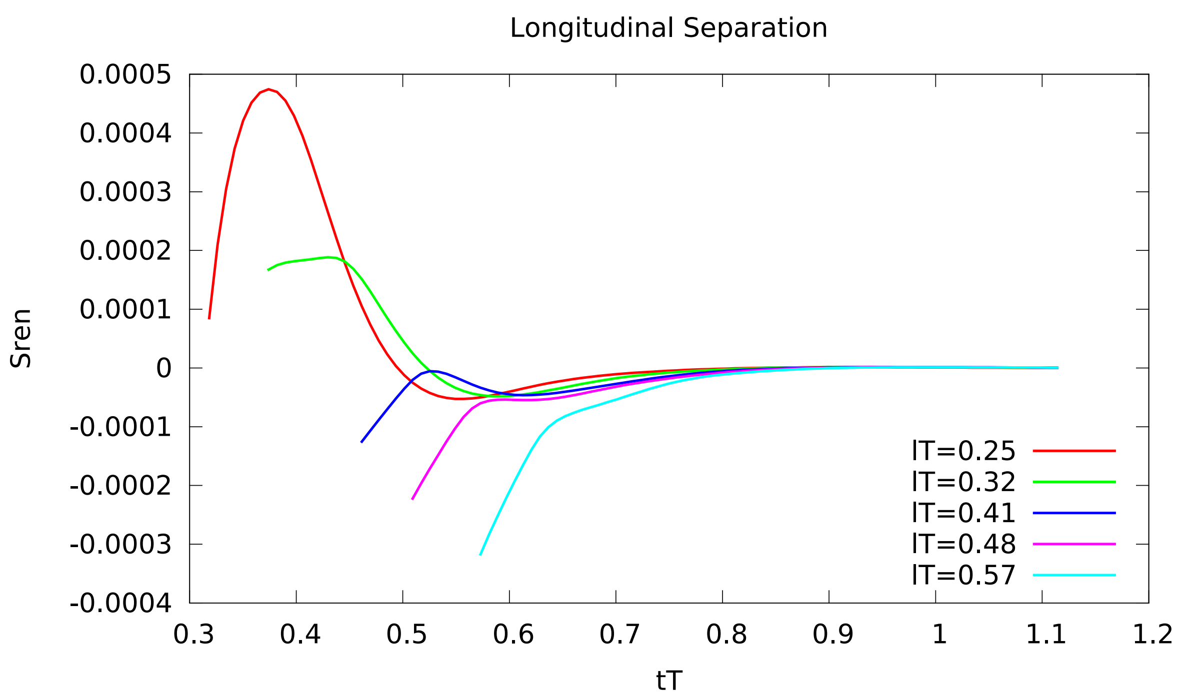
- Initial guess from conformal metrics

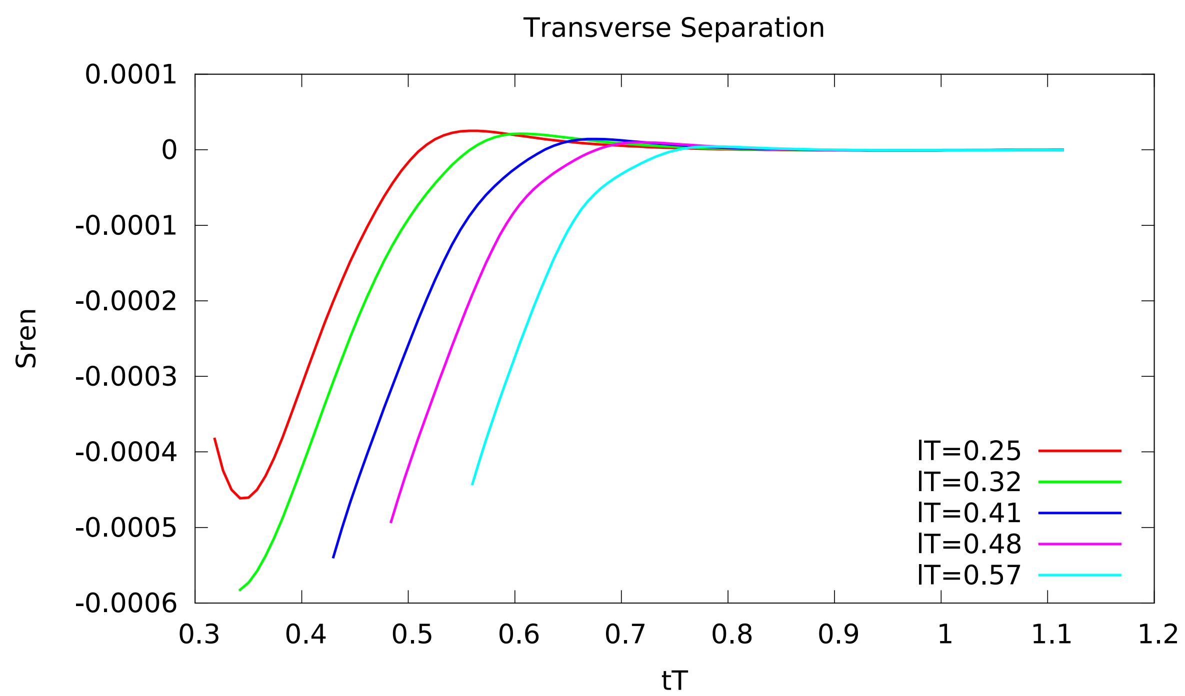
$$ds^2 = \frac{1}{z^6}\left(-dv^2 - 2dzdv + dx^2\right) \qquad x_{\pm} = \mp\frac{l}{2} \pm \frac{Lz^4}{4}\,{}_2F_1\left[\tfrac{1}{2}, \tfrac{2}{3}, \tfrac{5}{3}; L^2 z^6\right]$$



Holographic entanglement entropy

Results

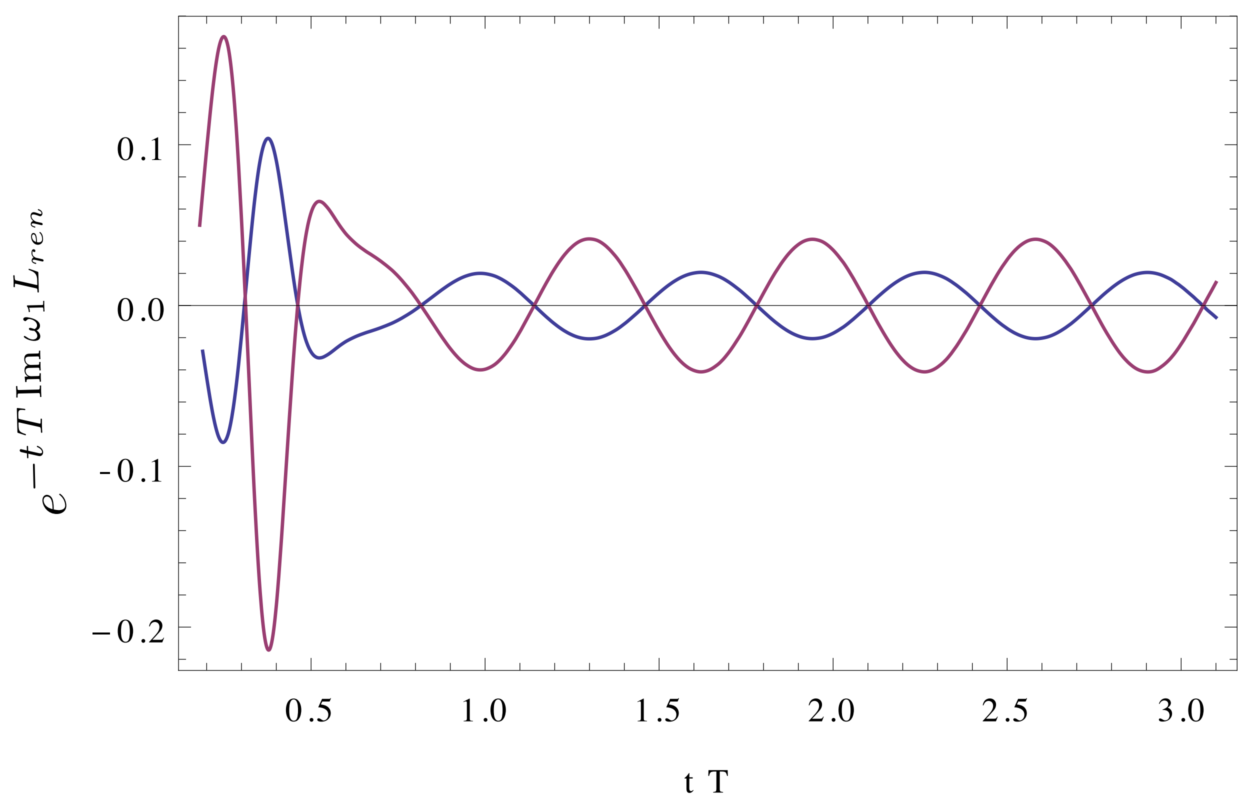


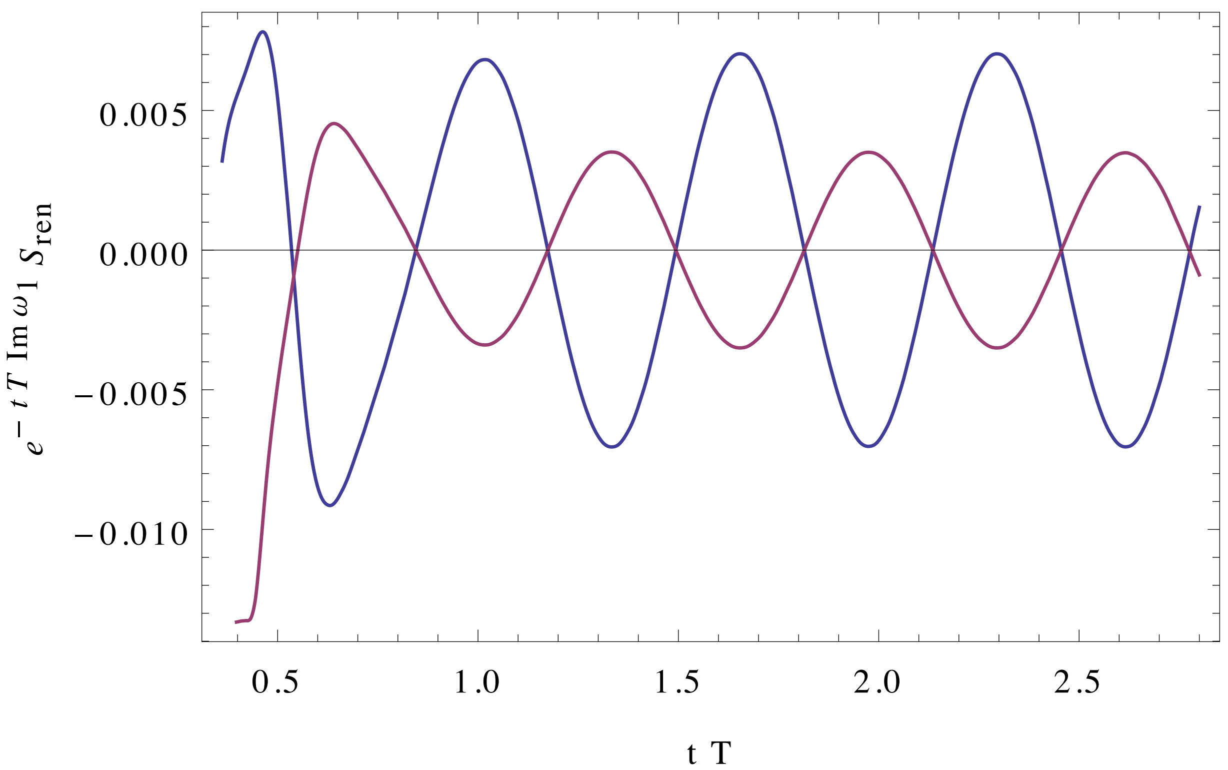


- Similar behaviour as for the geodesics
- At early times extremal surfaces can extend beyond apparent horizon
- Extend much further into the bulk as geodesics

Late time behaviour

Results

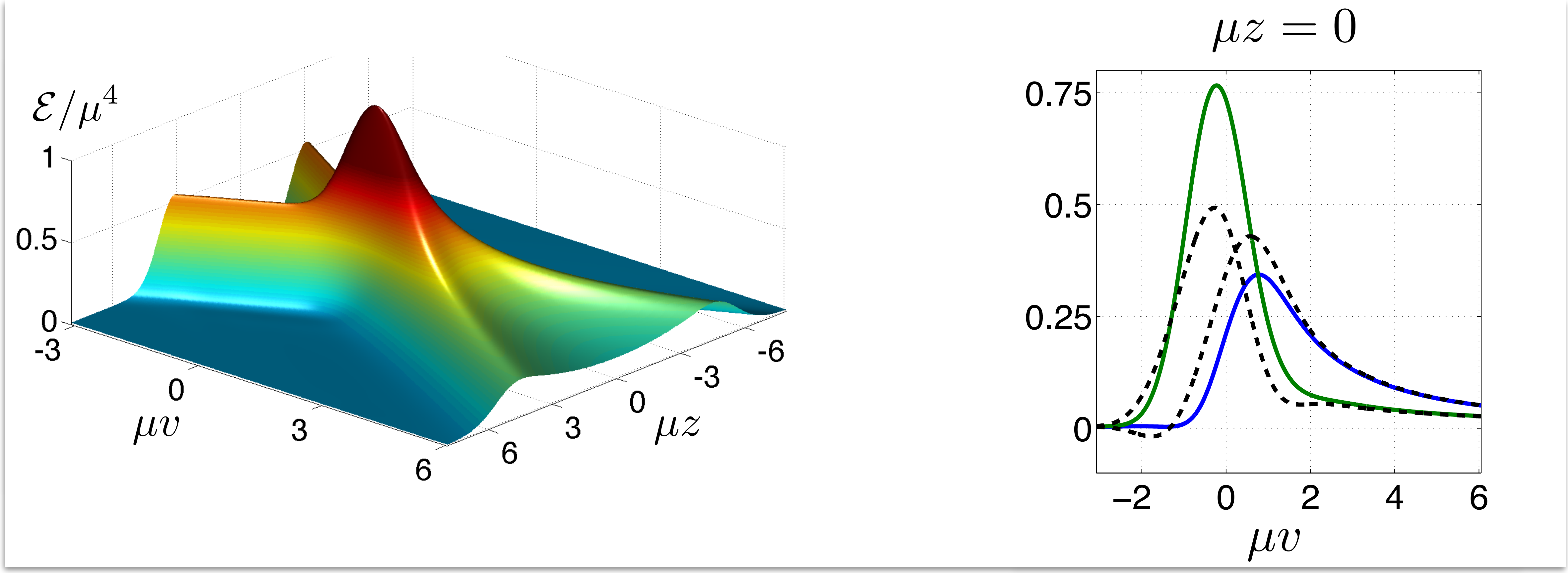




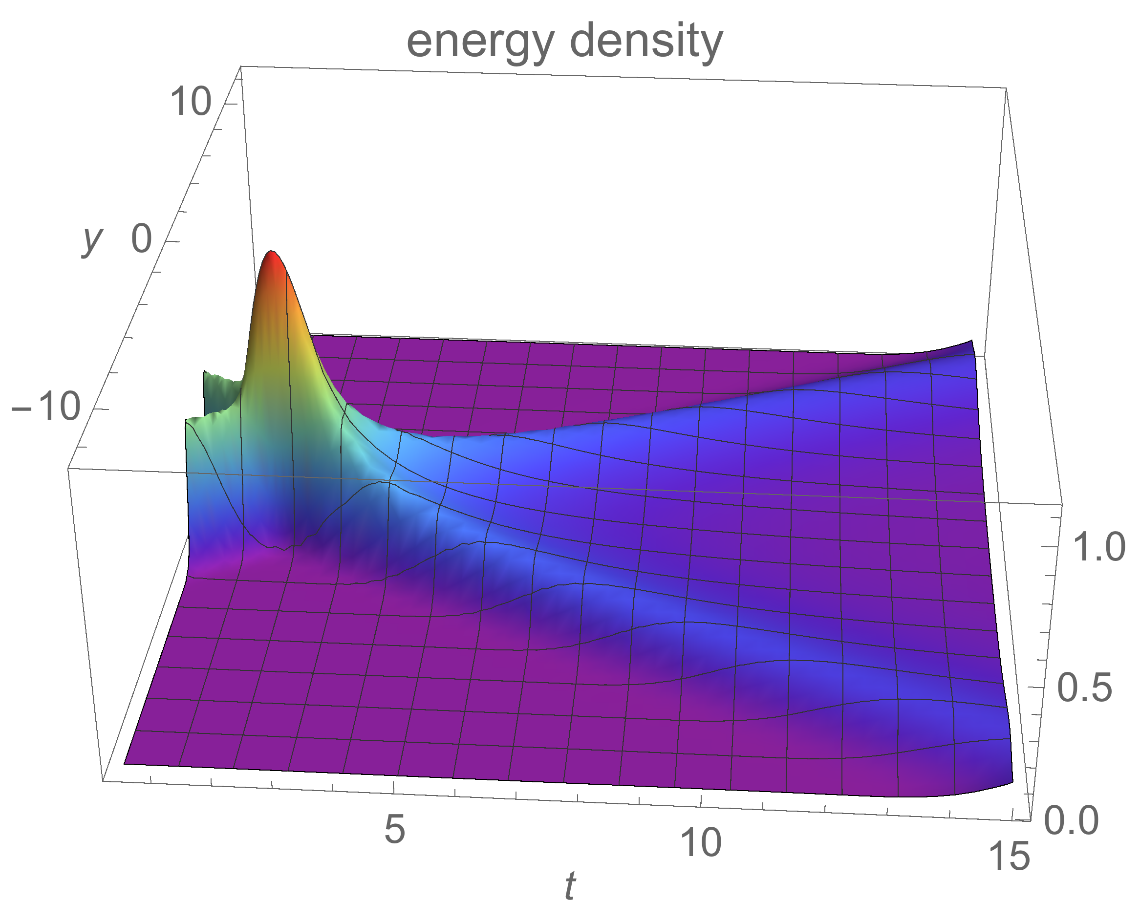
- Geodesic length and EE follows quasinormal ringing at late times
- At early times extremal surfaces can extend beyond apparent horizon
- Extend much further into the bulk as geodesics

Shock wave collisions

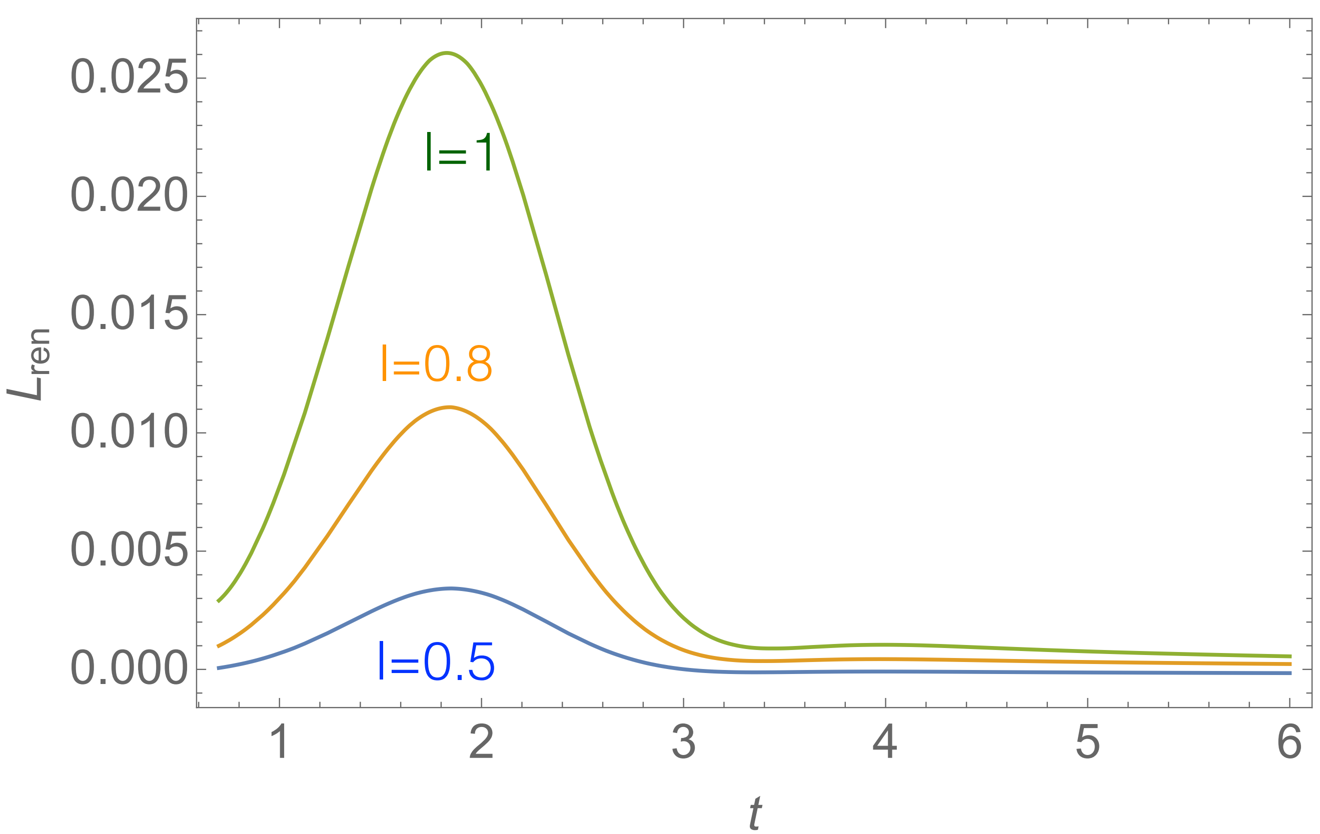
- HIC is modelled by two colliding sheets of energy with infinite extend in transverse direction and a Gaussian distribution in the long direction
- Hydrodynamics applies although system is still anisotropic



Shock wave collisions

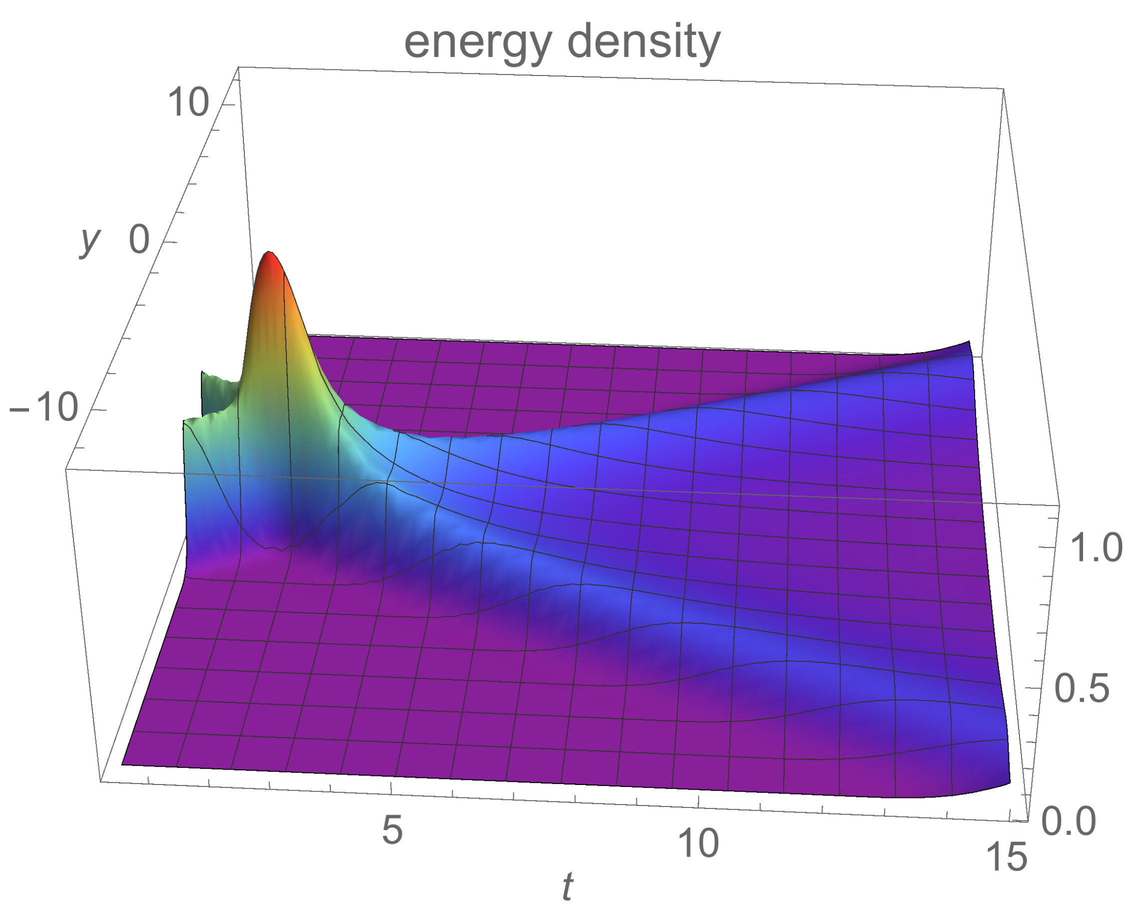


Geodesic length

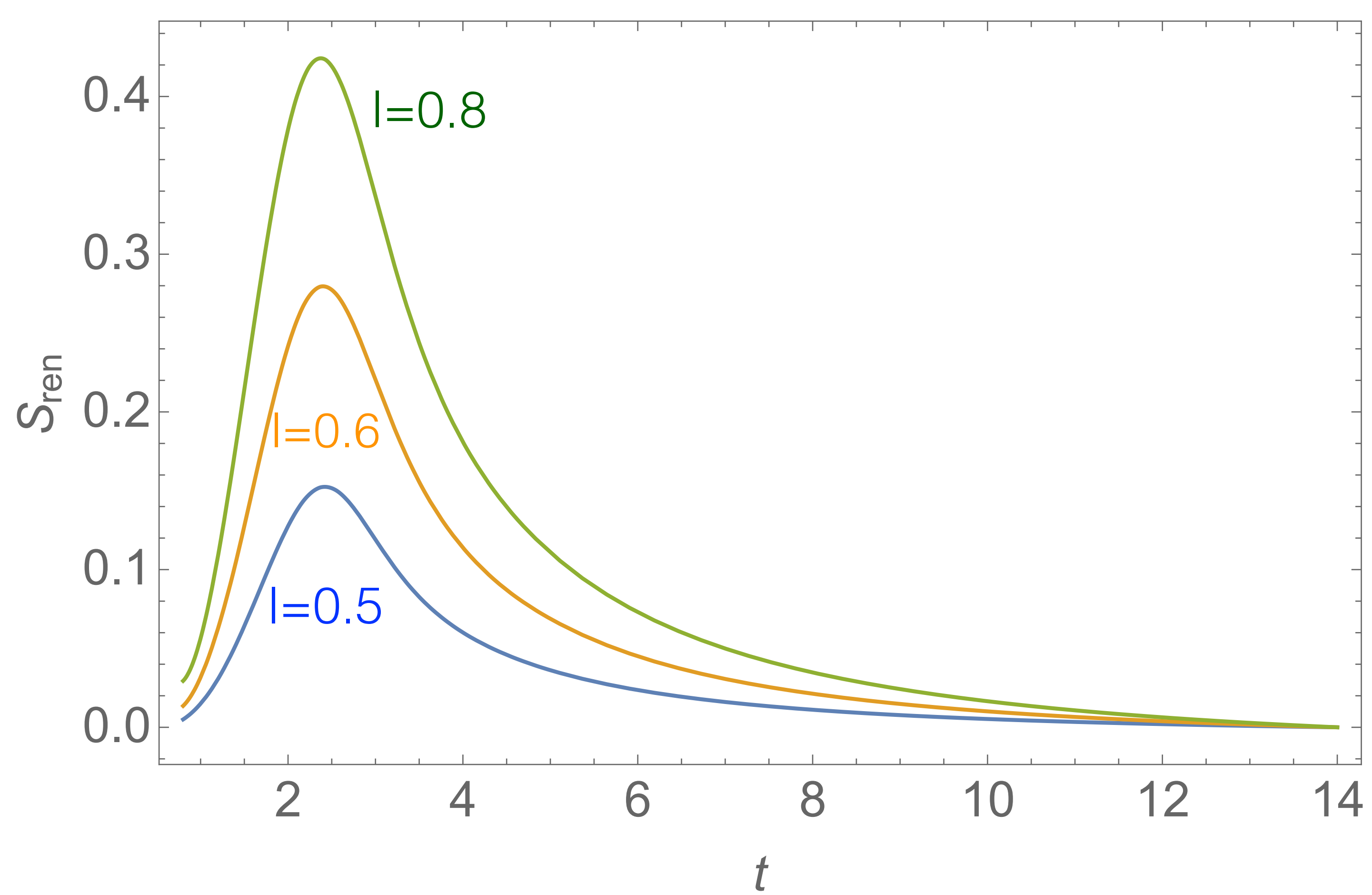


- Linear rise and fall off before and after the collision

Shock wave collisions



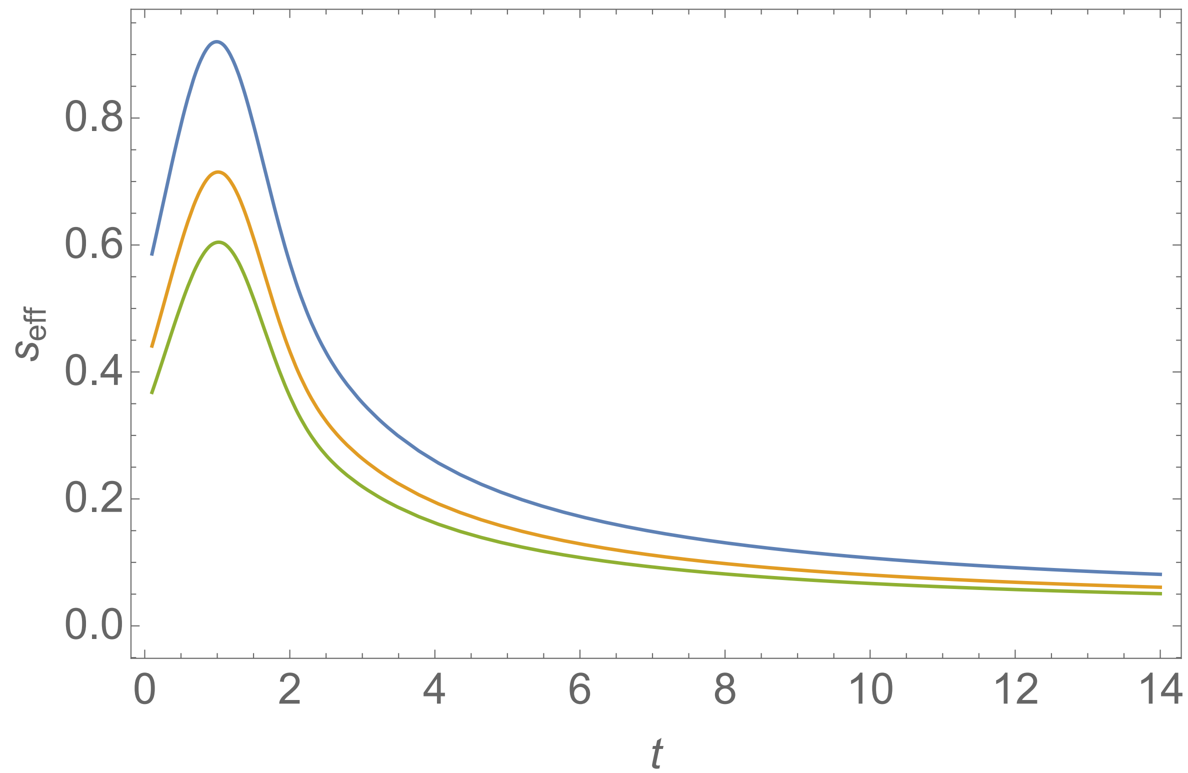
Entanglement entropy

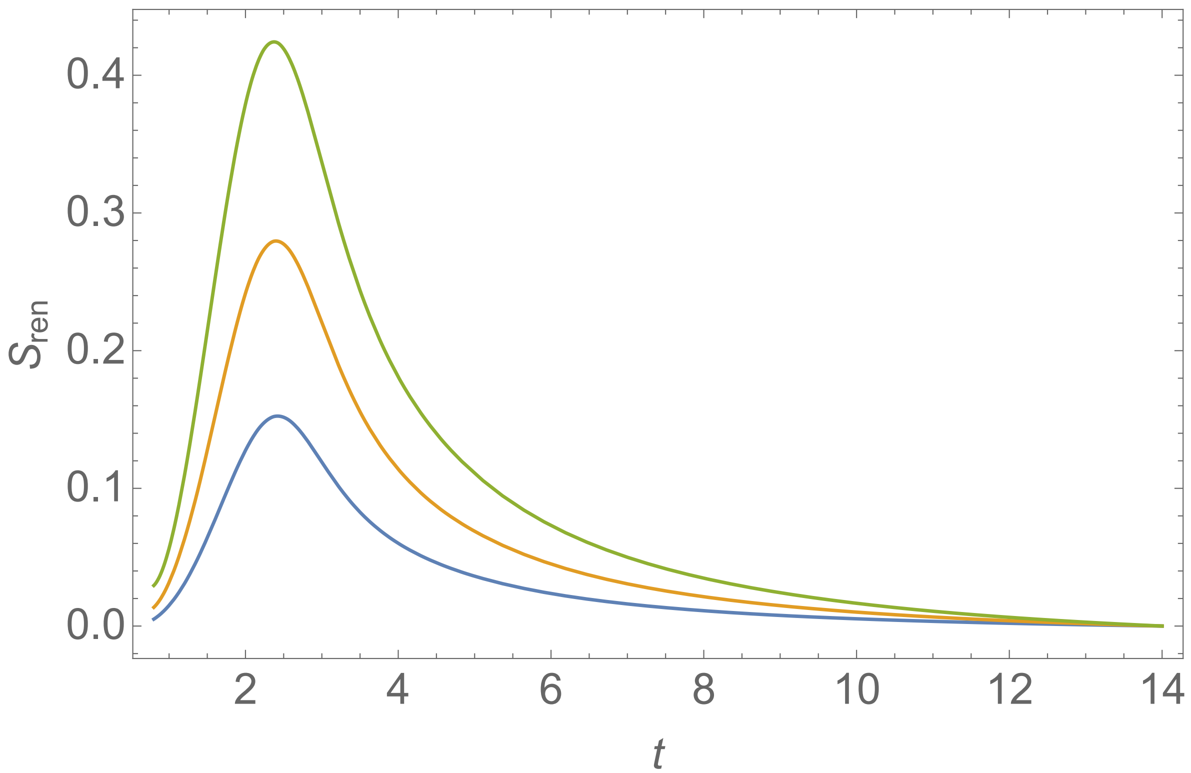


- Linear rise before the collisions
- Power law fall off after the collision

Fall off behaviour

Comparison with entropy density





- Define effective entropy density: $s_{eff} \sim A_{ah}$
- Entropy density: $s_{eff} \sim \dfrac{1}{t}$
- Entanglement entropy: $S_{ren} \sim \dfrac{1}{t^2}$

Conclusions

- Entanglement entropy is a useful theoretical probe to study the time evolution of strongly coupled systems
- Allows one to extract information from behind the horizon
- EE in collapsing shell models shows universal linear scaling in time
- In the anisotropic case EE shows oscillations around thermal value with QNM ringdown at late times
- Preliminary results suggest that entropy density and EE show different fall off behaviour