Thermalization process in far-from equilibrium systems

Soeren Schlichting
Brookhaven National Lab

Based on work in collaboration with
Berges, Boguslavski & Venugopalan

INT Workshop, Seattle August 2015

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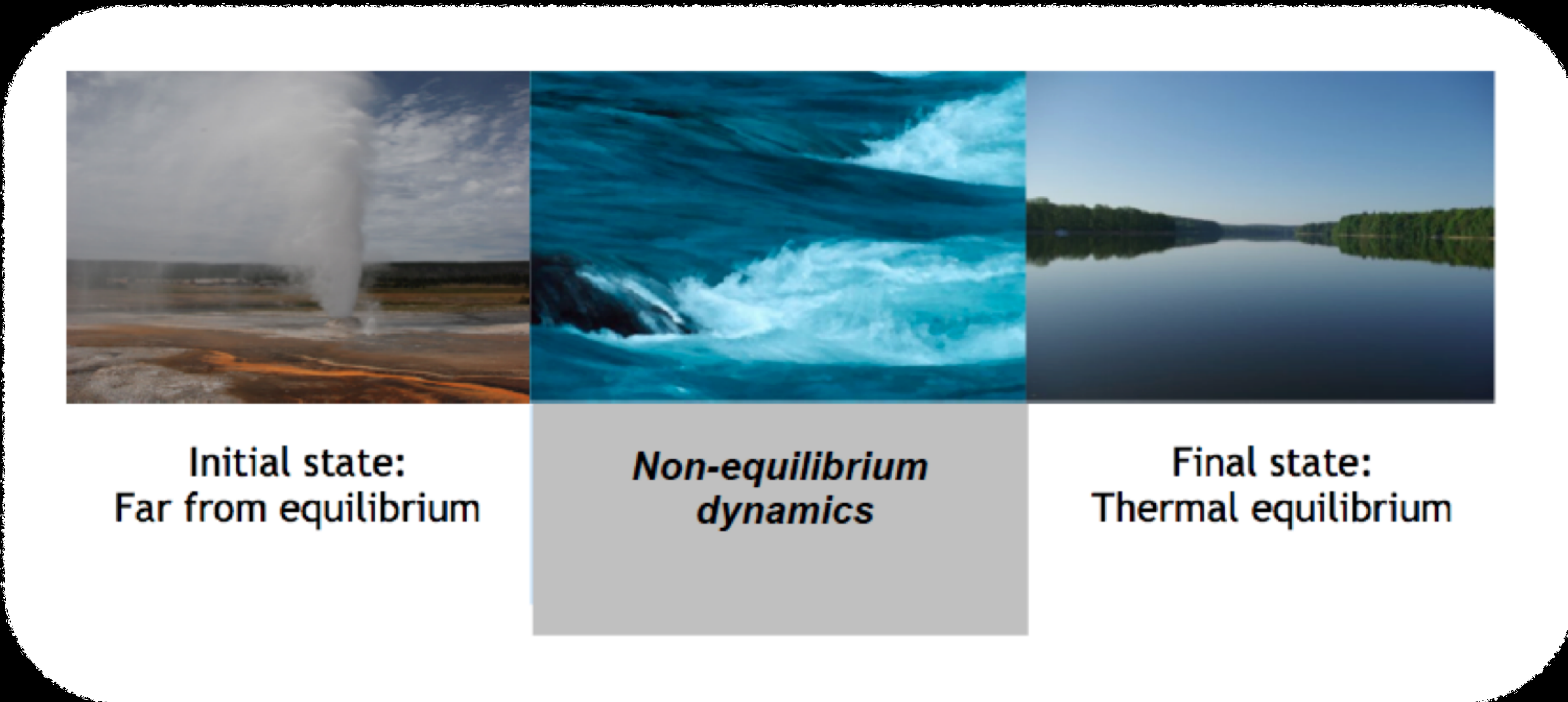


What is the microscopic dynamics underlying the thermalization process?

How well can different theoretical methods describe the dynamics of thermalization?

Outline

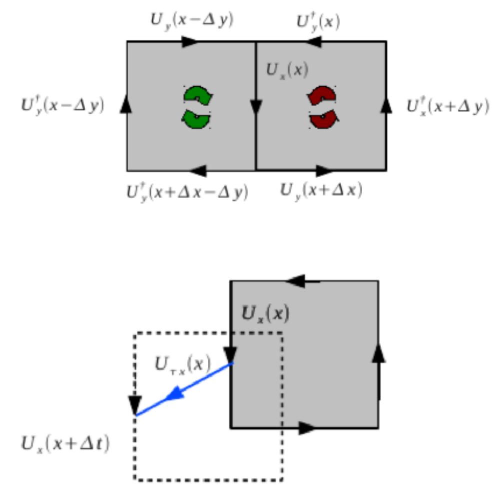
- Theoretical methods to describe non-equilibrium dynamics at weak coupling
- Thermalization process in homogenous & isotropic systems
  - *Similarities and differences between scalar & gauge theories*
    (Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 114007; JHEP 1405 (2014) 054)
  - *Comparison between different weak-coupling methods*
- Early stages of high-energy heavy-ion collisions
  - *Effect of momentum anisotropy & longitudinal expansion*
    (Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011 & 114007)
  - *Universal behavior in scalar & gauge theories far from equilibrium*
    (Berges, Boguslavski,SS, Venugopalan PRL114 (2015) 061601; work in preparation)
- Summary & Conclusions

Theoretical methods

Classical-statistical field theory

Whenever the phase-space occupancy f(t,p) of the characteristic degrees is large, their dynamics can be accurately described in terms of classical field theory

$$D_\mu F^{\mu\nu} = J^\nu$$



Classical field theory can be solved numerically from first-principles using standard lattice gauge theory techniques

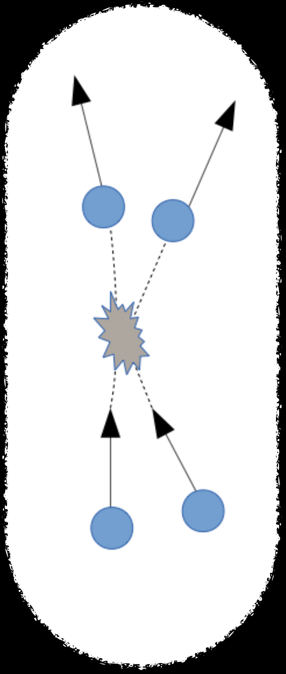
-> Powerful tool to study non-equilibrium dynamics in high occupancy regime f(t,p) >> 1.

Theoretical methods

Kinetic theory

Description of the dynamics in terms of Boltzmann equation for weakly interacting quasi-particles

$$\partial_t f(t,p) = C[f](t,p)$$

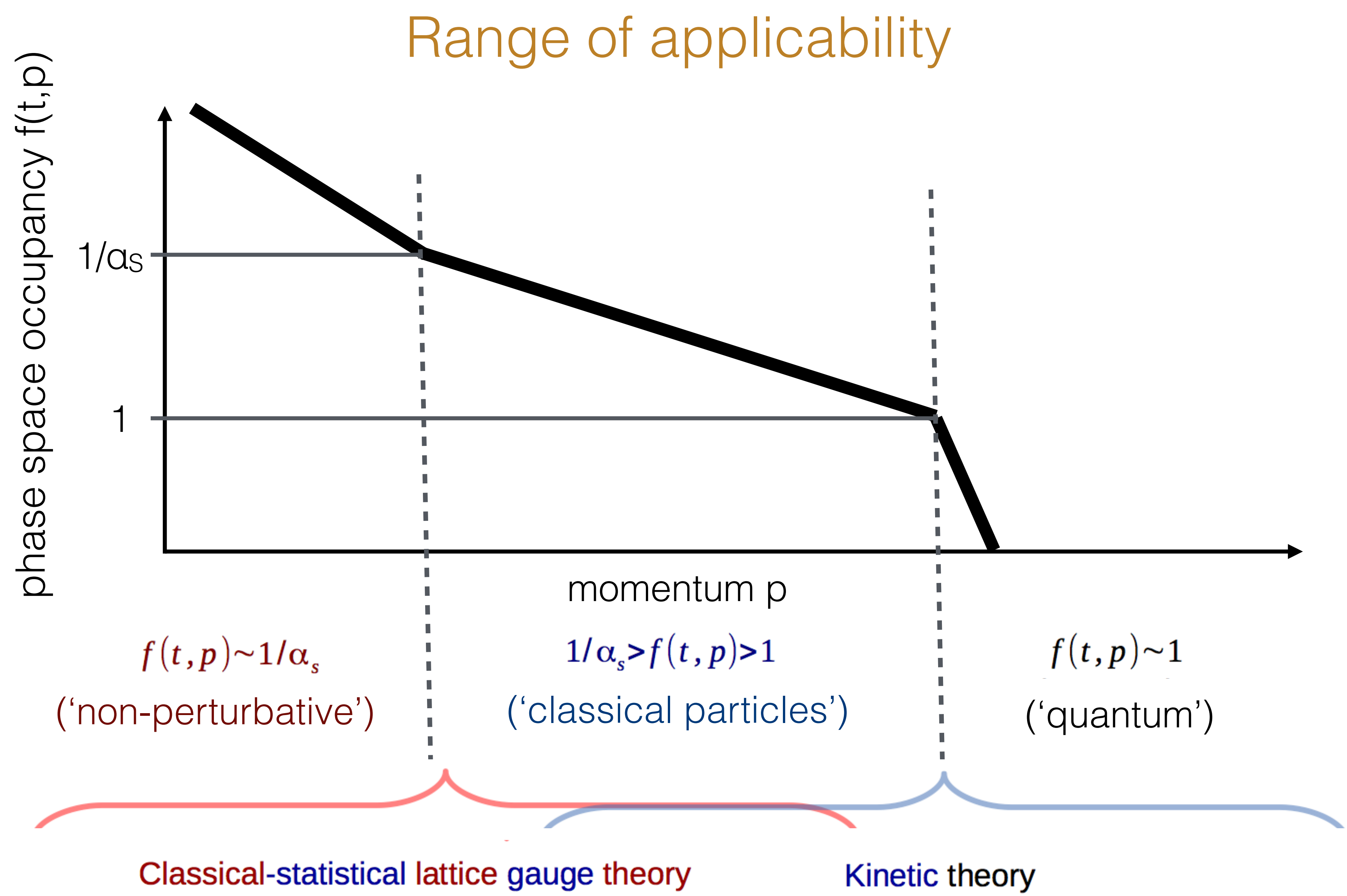


Generally quasi-particles are well defined and weakly interacting as long as

- coupling constant small ($\alpha_S << 1$)
- phase space occupancy is perturbative ($f(t,p) << 1/\alpha_S$)
- limited to description of high momentum modes ($p >> m_D$)

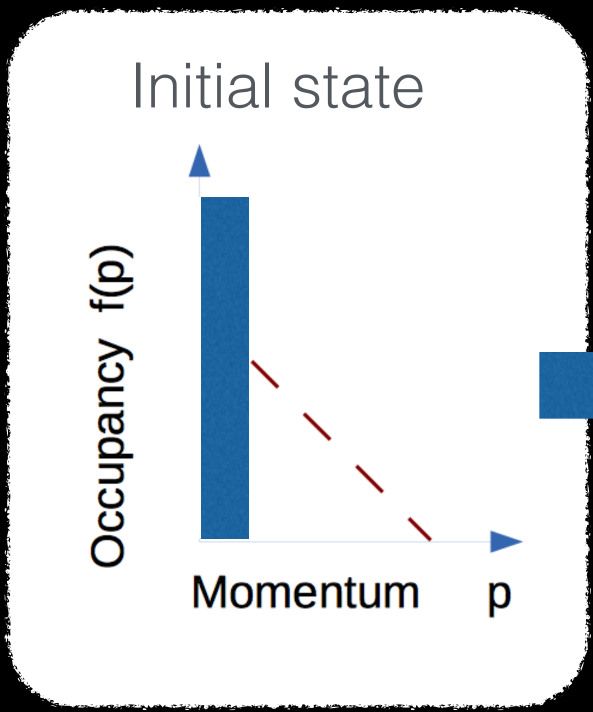
-> Efficient tool for both numerical studies and analytic considerations; straightforward to account for quantum statistics

Theoretical methods



Thermalization process in homogenous & isotropic systems

- Since energy conserved during the thermalization process, the thermal equilibrium state is known
- While initially energy is carried by low momentum excitations, in thermal equilibrium energy is dominated by modes with p~T



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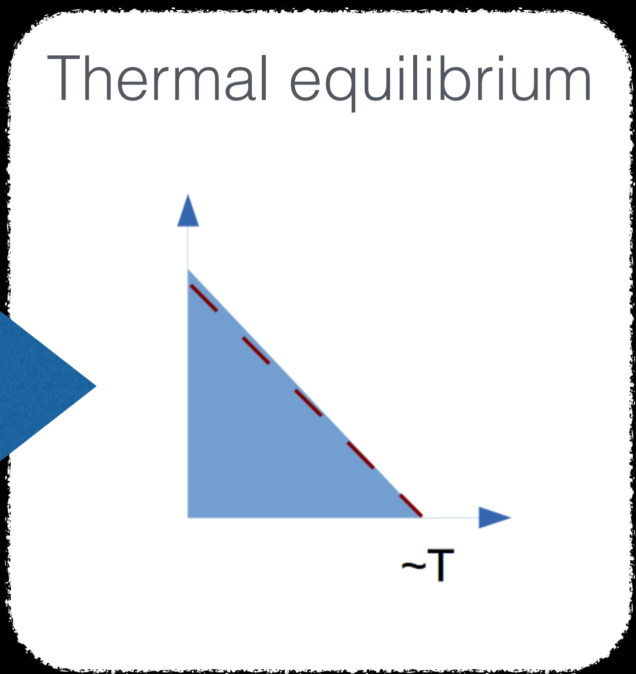
Energy transport towards the UV



Thermalization process — Yang-Mills theory

Classical-statistical simulations

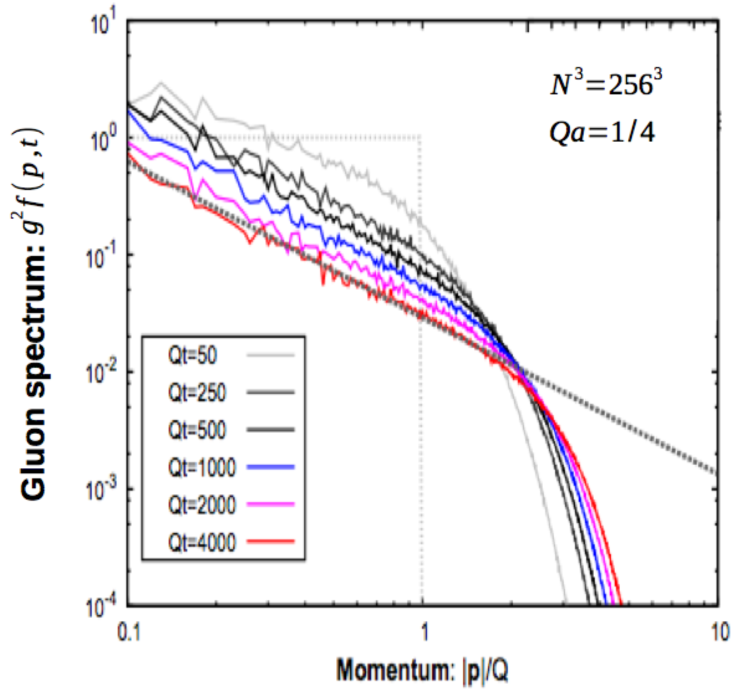
- Choose initial conditions to mimic quasi-particle picture

$$A^a_\mu(t_0, x) = \int \frac{d^3k}{(2\pi)^3} \sqrt{f(k,t_0)} \times [c^k_{\lambda,a} \xi^{(\lambda)k}_\mu(t_0) e^{ikx} + c.c]$$

- Evolve the system according to Hamilton's equations of motion on the lattice

- Extract time evolution of the gluon distribution from correlation functions

$$f(p,t) = \langle \left| \xi^{(\lambda)k}_\mu(t) \overleftrightarrow{\partial}_t A^\mu_a(t,p) \right|^2 \rangle$$ (Coulomb gauge)



-> Infrared power-law

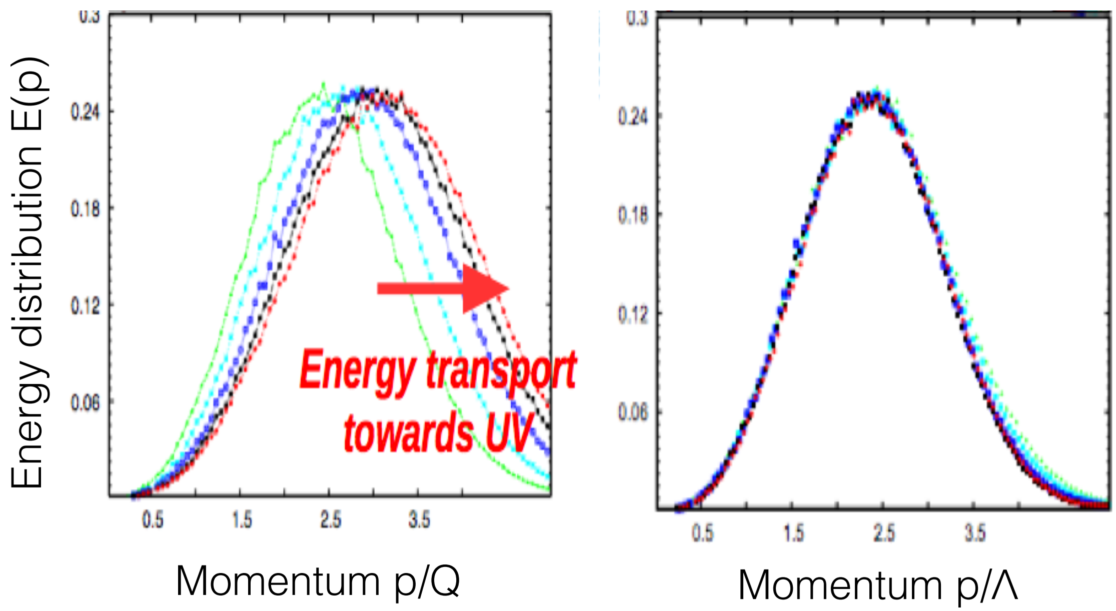
-> Characteristic momentum scale Λ increases with time

(e.g. Kurkela, Moore PRD 86, (2012) 056008; Berges, Boguslavski,SS, Venugopalan PRD 89 (2014) 114007)

Self-similar scaling

Evolution at late times proceeds via a self-similar ultra-violet cascade
-> same process repeats itself over and over again

Dynamics can be entirely described in terms of scaling exponents *α=-4/7, β=-1/7,* and scaling function *f_S which can be* extracted from simulations



SS PRD 86 (2012) 065008

$$f(p,t) = t^{\alpha} \; f_S(t^{\beta} p)$$

Closely related to the phenomenon of wave turbulence and has been named

"turbulent thermalization"

Kinetic theory — qualitative description

- Once the characteristic occupancies become $1 \ll f \ll 1/\alpha$ the dynamics of hard excitations can also be described by kinetic theory

$$\frac{\partial f(t,\mathbf{p})}{\partial t} = C[f](t,\mathbf{p})$$

Search for self-similar scaling solution

$$f(p,t) = t^{\alpha}\; f_S(t^{\beta} p)$$

Scaling behavior of the collision integral

$$\xrightarrow[(f \gg 1)]{scale \;\; invariance} C[f](p,t) = t^{\mu} C[f_S](t^{\beta} p)$$

-> Boltzmann equation can be decomposed into

$$[\alpha + \beta\, \mathbf{p} \cdot \nabla_{\mathbf{p}}]\, f_S(\mathbf{p}) = C[f_S](1,\mathbf{p}),$$

time independent fixed-point condition

$$\alpha - 1 = \mu(\alpha, \beta)$$

scaling relation

Kinetic theory — qualitative description

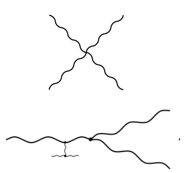
- Dynamical scaling exponents *α, β* are uniquely determined by

Scaling of the collision integral **+** ***Energy conservation***

$$\alpha - 1 = \mu(\alpha, \beta)$$

$$\alpha - 4\beta = 0$$

Independent of microscopic parameters (e.g. coupling constant, number of colors,…)

	Interaction		***Hard scale evolution (Exponent α)***	***Occupancy evolution (Exponent β)***
gauge theory		***2<->2 & eff. 2<->1***	***-1/7***	***-4/7***

(Micha, Tkachev, Kurkela, Moore, Berges, SS, Venugopalan, …)

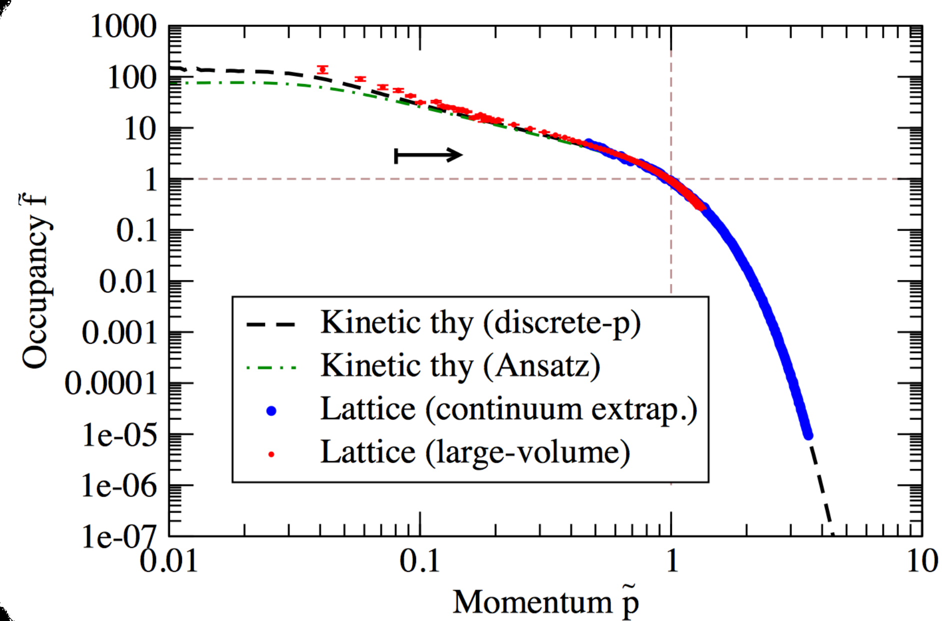
Kinetic theory — quantitative description

Numerical solutions of effective kinetic theory (AMY)

$$\partial_t f(p,t) = -\mathcal{C}_{2\leftrightarrow 2}[f](p) - \mathcal{C}_{1\leftrightarrow 2}[f](p).$$

compared to classical Yang-Mills simulations.

- Dynamics of hard excitations accurately described by eff. kinetic theory.
- Quantitative agreement between the two approaches for momenta above m_D.



(York, Kurkela, Lu, Moore, PRD 89 (2014) 074036; Kurkela,Lu PRL 113 (2014) 18)

Thermalization process — scalar theory

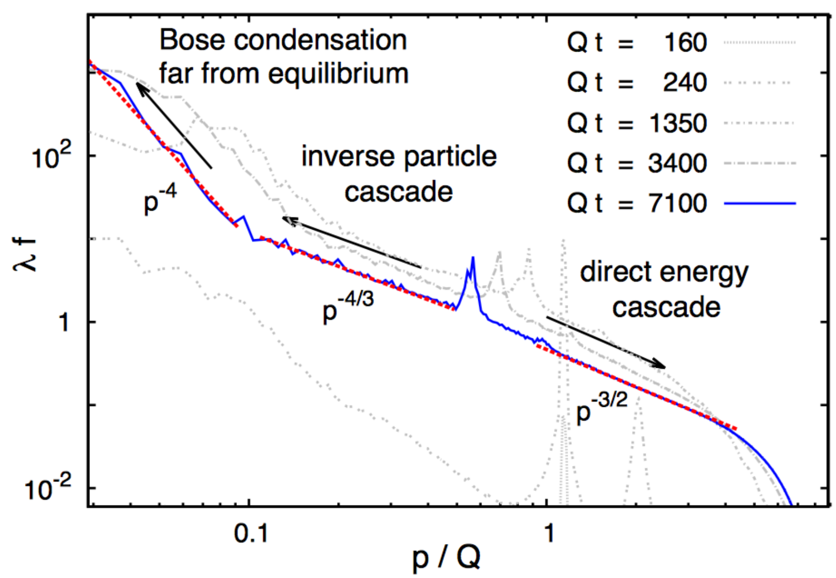
- Cruicial difference to gauge theory is that inelastic (particle number changing) processes are highly suppressed in the scalar theory.

-> In addition to energy conservation, particle number may be effectively conserved over a large time scale.

- Classical statistical simulations show that the scalar theory accommodates for this by the emergence of an inverse particle cascade.

-> Energy is transported to the UV

-> Particle number is transported to the IR and results in the formation of a Bose Condensate

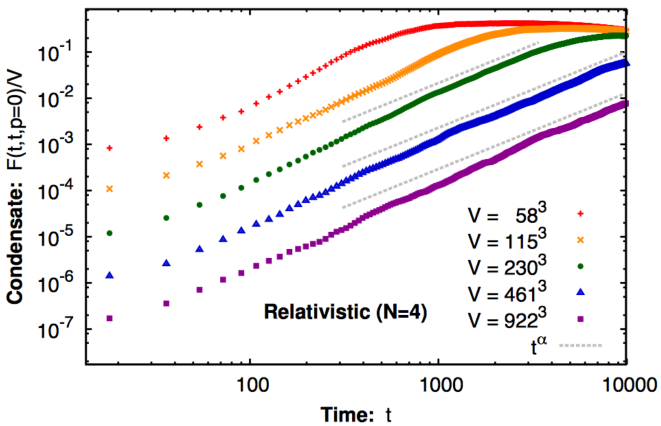


Berges, Boguslavski, SS, Venugopalan
JHEP 1405 (2014) 054

Challenge for kinetic description

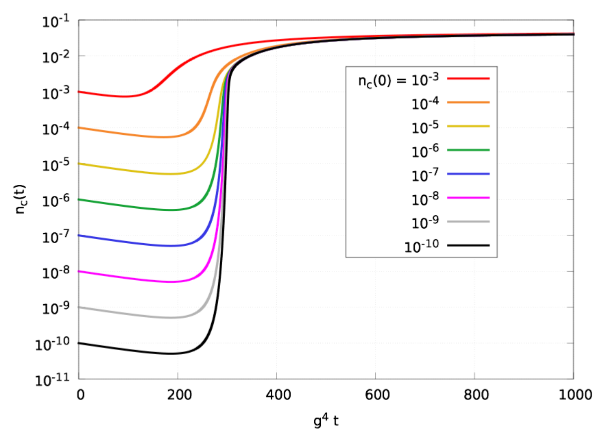
- Description of infrared dynamics of (inverse) particle number cascade involves low momenta and non-perturbatively large occupancies.

Classical lattice simulation



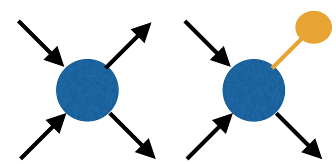
Berges,Boguslavski, Pinero-Orioli
arXiv:1503.02498

Kinetic theory



Epelbaum,Gelis,Tanji,Wu
Phys.Rev. D90 (2014) 12, 125032

- Naive kinetic description in terms of 2<->2 and 2<->1+condensate, fails to describe condensation dynamics correctly



Challenge for kinetic description

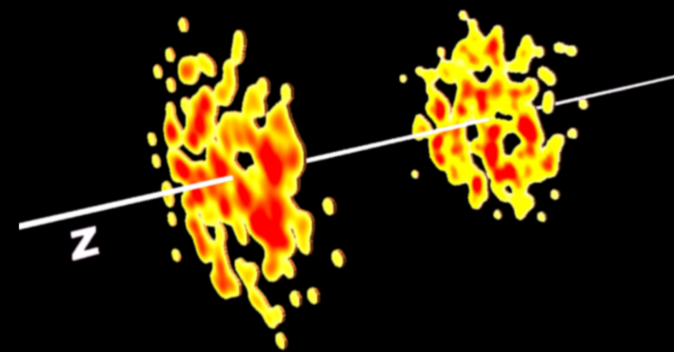
- Description of infrared dynamics of (inverse) particle number cascade involves low momenta and non-perturbatively large occupancies.
- Can not simply neglect the infrared dynamics in this case because it affects the dynamics of hard particles through non-local interactions (hard+hard <-> hard + soft), which dominate UV cascade.

-> Clearly interplay between IR and UV sector poses a challenge to kinetic description

Solution for scalars suggested in terms of (1/N) vertex-resummation to extend range of validity towards infrared regime (talk K. Boguslavski)

Heavy-ion collisions in the weak-coupling picture

- High-energy nuclei feature a large number of small-x gluons with typical momentum $Q_s(s)$



-> At high energies $Q_s(s) \gg \Lambda_{QCD}$ such that $\alpha_s(Q_s) \ll 1$ is small

- Collision of high-energy nuclei leads to a far-from equilibrium state 'Glasma' characterized by a large phase space occupancy of gluons

$$f(p \sim Q_s) \sim 1/\alpha_s$$

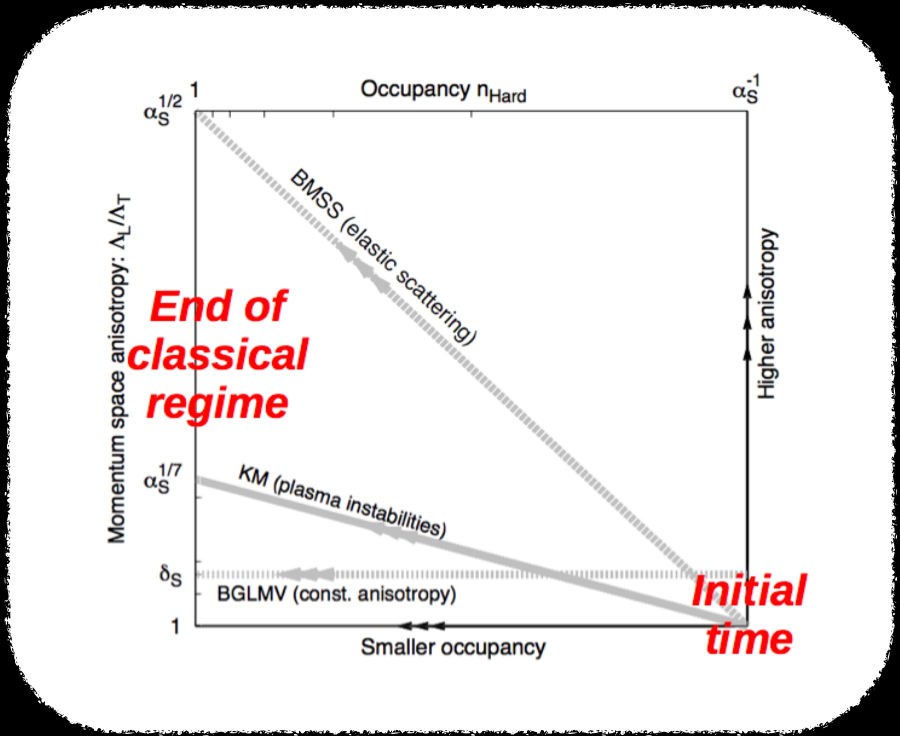
High initial gluon density allows for an effective classical description of the early stages

Effect of long. expansion

- Dilution of the system & red-shift of longitudinal momenta
  -> System can be anisotropic on large time scales

- Different scenarios developed based on kinetic theory
  - Baier et al. (BMSS),
    PLB 502 (2001) 51-58
  - Kurkela, Moore (KM),
    JHEP 1111 (2011) 120
  - Blaizot et al. (BGLMV),
    Nucl. Phys. A 873 (2012) 68-80

  . . .

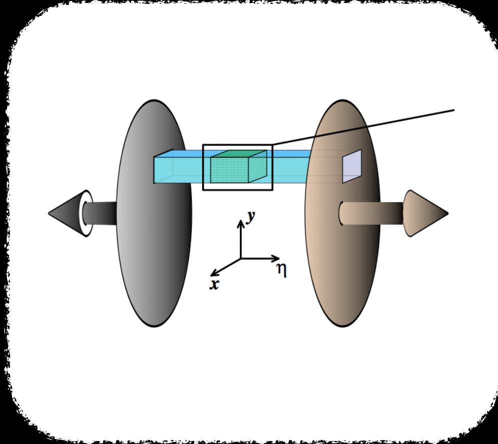


- Difference arises due to treatment of soft (non-perturbative) physics

-> Need first-principles simulations to decide which scenario is realized

Qualitative description of early stages

- We will neglect the transverse expansion of the system and consider a system which is only expanding in the longitudinal direction

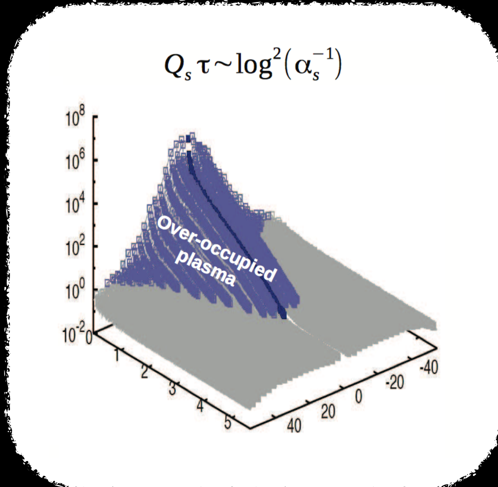


- Characterize the initial state at $\tau_0 \sim 1/Q_s$ in terms of an initial gluon distribution

initial occupancy

$$f(p_T, p_z, \tau_0) = \frac{n_0}{\alpha_s} \Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

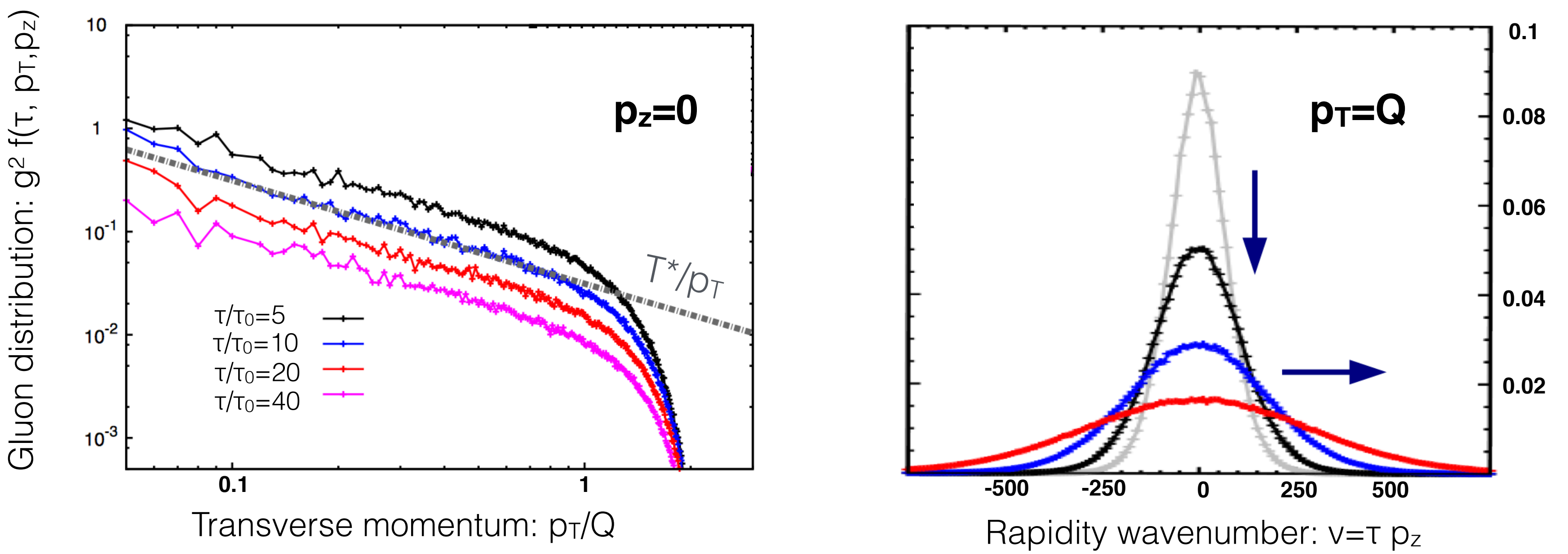
initial anisotropy



-> Captures the possible dependencies on initial occupancy and momentum space anisotropy

Evolution in classical regime

Single particle spectrum

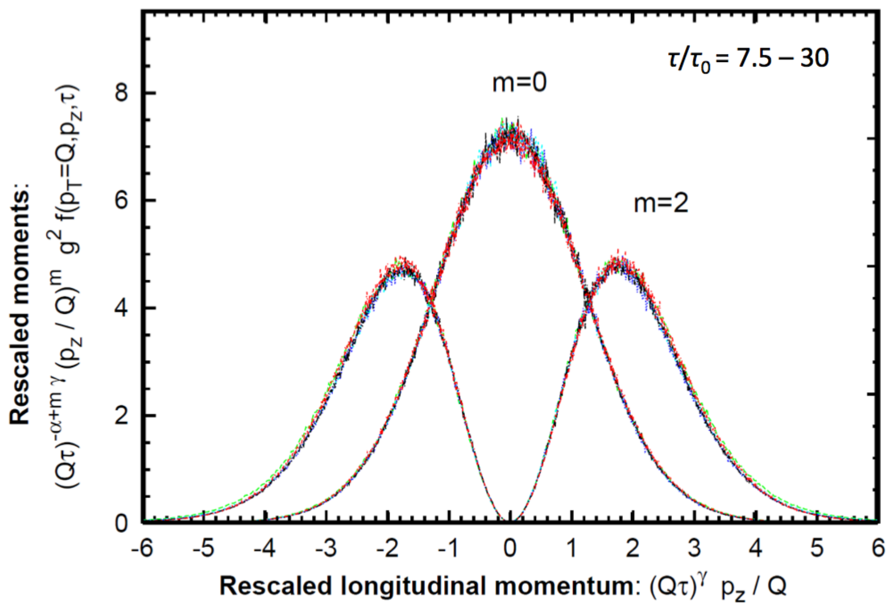


Transverse spectrum shows thermal-like $1/p_T$ behavior up to Q_S.

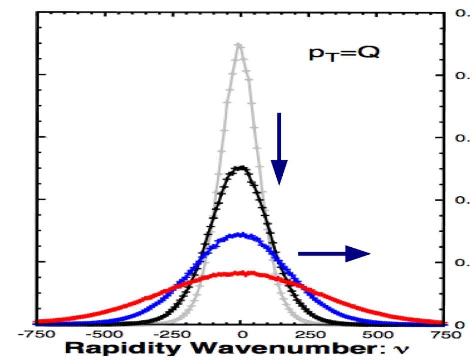
Dynamics in the scaling regime consists of *longitudinal momentum broadening* — not strong enough to completely compensate for red-shift due to longitudinal expansion

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

Self-similarity



rescaling



Dynamics can be entirely described in terms of universal scaling exponents *α=-2/3, β=0, γ=1/3* and scaling function f_S extracted from simulations

$$f(\mathrm{p_T}, \mathrm{p_z}, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta}\mathrm{p_T}, (Q\tau)^{\gamma}\mathrm{p_z}\Big)$$

(Berges,Boguslavski,SS,Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)

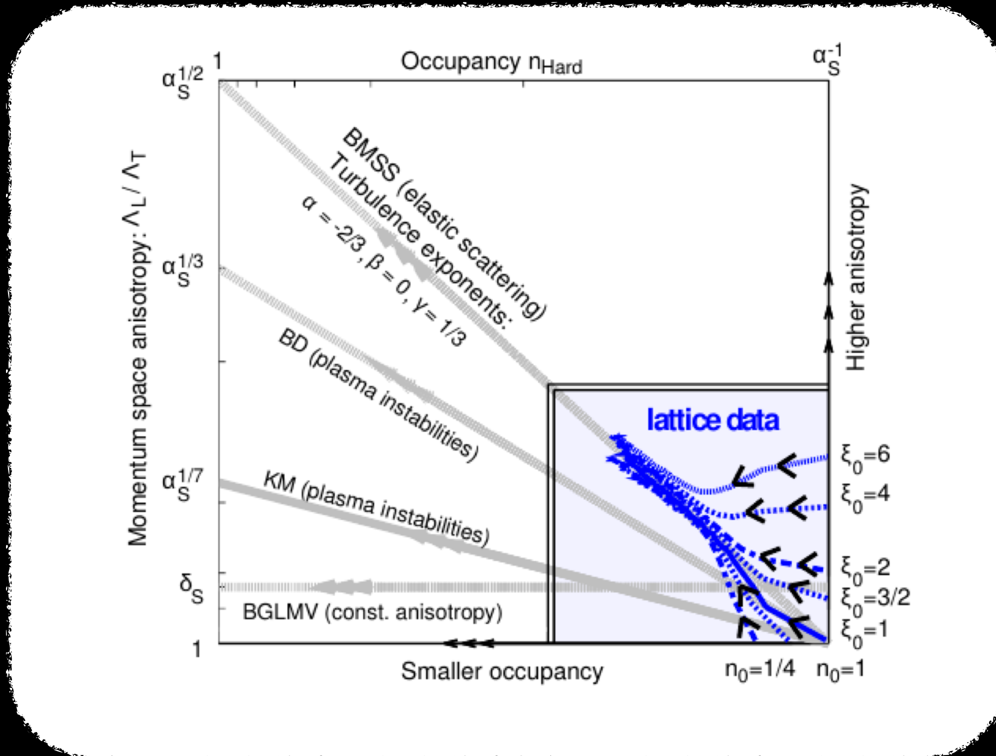
Kinetic theory comparison

- Universal scaling behavior independent of the initial conditions

- Surprising agreement with "bottom up" scenario (2<->2 and 2<->1)

(Baier,Mueller,Schiff, Son, PLB 502, 51 (2001))

- No sign of plasma instabilities affecting the late time evolution of hard excitations



Could be that the "pre-factor" is just small and instabilities only become important when the system is extremely anisotropic?

Could be that we do not understand something about screening in anisotropic plasmas / interplay of soft and hard excitations?

Scaling analysis

Consider the Boltzmann equation (c.f. Baier et al. PLB 502 (2001) 51-58)

$$[\partial_\tau - \frac{p_Z}{\tau}\partial_{p_z}] f(p_T, p_Z, \tau) = C[f](p_T, p_Z, \tau)$$

with a self-similar evolution

$$f(p_T, p_Z, \tau) = (Q\tau)^\alpha f_S((Q\tau)^\beta p_T, (Q\tau)^\gamma p_Z)$$

→ ***Non-thermal fixed point solution*** $(f \gg 1)$

$$[\alpha + \beta p_T \partial_{p_T} + (\gamma - 1) p_Z \partial_{p_z}] f_S(p_T, p_Z) = Q^{-1} C[f_S](p_T, p_Z)$$

→ ***Scaling exponents determined by scaling relations for***

- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$

Scaling exponents are independent of detailed microscopic dynamics
-> Could be any process with the same parametric dependencies
-> Chance to observe identical behavior in different physical systems

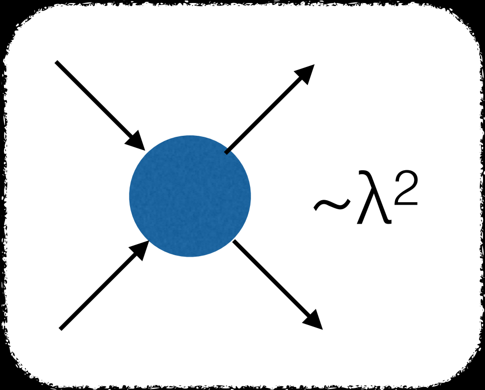
Expanding scalar field theory

Consider massless N-component scalar field theory with quartic self-interaction

$$S[\varphi] = \int d^4x \sqrt{-g(x)} \left(\frac{1}{2}(\partial_\mu \varphi_a) g^{\mu\nu} (\partial_\nu \varphi_a) - \frac{\lambda}{24N} (\varphi^2)^2 \right)$$

in a longitudinally expanding setup.

Even though perturbatively there is no preference for small angle scattering, one at least expects elastic processes to dominate



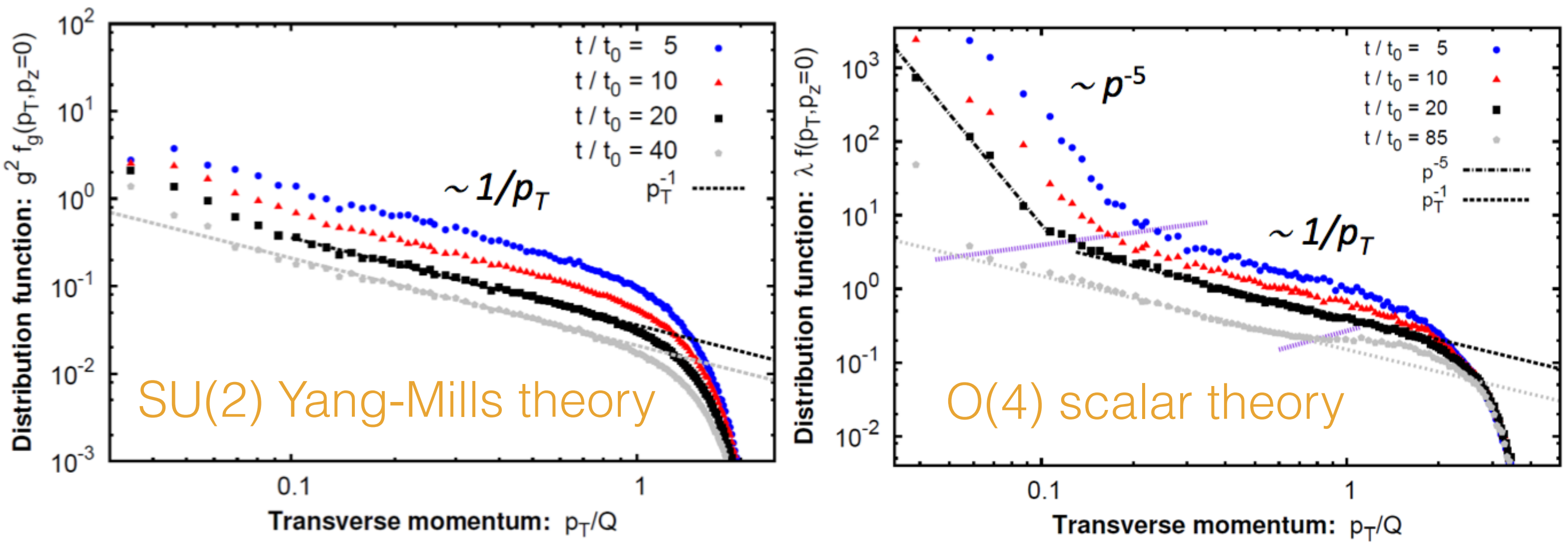
-> Energy conservation
& Particle number conservation

(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Comparison of simulations

Comparison of longitudinally expanding Yang-Mills and scalar field theory in the classical regime of high occupancy

Evolution of the single particle spectrum



Scalar theory shows three distinct scaling regimes at soft ($\sim p^{-5}$), intermediate ($\sim 1/p_T$) and hard momenta ($\sim$const)

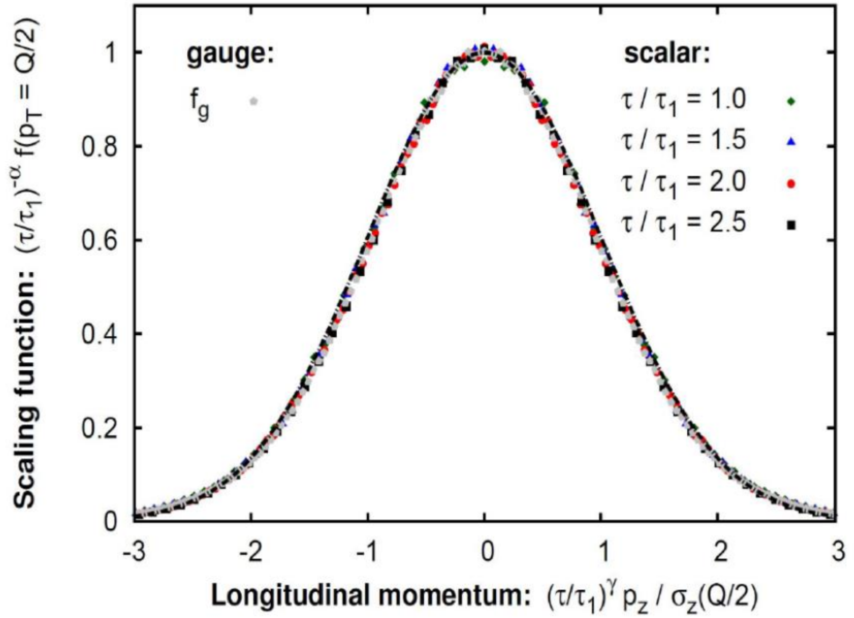
->Common $\sim 1/p_T$ scaling regime

(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Universality far from equilibrium

- Scaling exponents and scaling functions agree in the inertial range of momenta, where both theories show $1/p_T$ behavior

Normalized fixed-point distribution



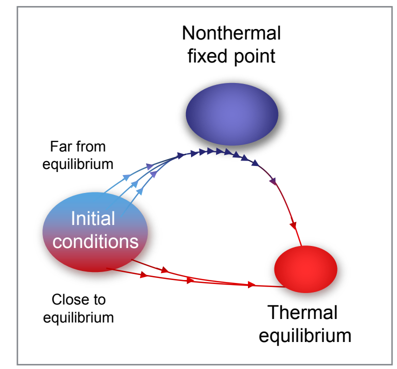
Kinetic interpretation in expanding scalar theory still unclear —

Points to the fact that "bottom-up" solution is more general than small angle scattering

(Berges,Boguslavski,SS,Venugopalan PRL114 (2015) 061601)

Summary & Conclusions

- Generally the thermalization process in far-from equilibrium system proceeds via “non-thermal” fixed points, which correspond to self-similar attractor solutions.



- We observe for the first time qualitative agreement between kinetic theory predictions and classical lattice simulations in some systems

Clearly problems remain concerning in all cases where infrared dynamics is important (plasma instabilities, condensation in scalars)

- Striking universality observed between expanding scalar and gauge theory in the early time classical regime.

-> Even though precise origin is still unclear there is an exciting possibility to learn about the dynamics of thermalization process from different physical systems.