Chiral Drag Force

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As it was argued a long time ago[1] one could calculate drag force for a heavy quark moving through the strongly coupled holographic plasma by considering a probe string, described by NG action

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-det\ g}\ ,\ g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

where the corresponding bulk AdS-BH metric for the plasma is

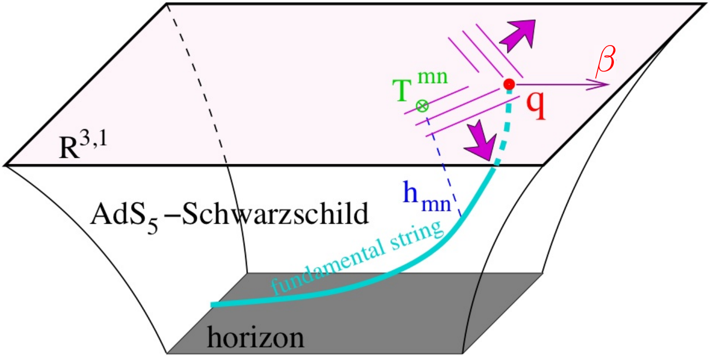
$$ds^2 = H^{-1/2}(-f(r)dt^2 + dx^2) + H^{1/2}\left(\frac{dr^2}{f(r)} + d\Omega_5^2\right)$$

and $f(r) = 1 - \frac{M}{r^4}\ ,\ H(r) = 1 + \frac{1}{r^4}$.

Then EOMs are

$$\nabla_\alpha P^\alpha_\mu = 0\ \ ,\ \ P^\alpha_\mu = -\frac{1}{2\pi\alpha'}G_{\mu\nu}\partial^\alpha X^\nu$$

[1] L. Yaffe et al, JHEP 0607, 013 (2006); S. Gubser, Phys.Rev.D 74, 126005(2006)


T^{mn}
q
β
$R^{3,1}$
h_{mn}
AdS_5 –Schwarzschild
fundamental string
horizon

For the trailing string the action is reduced in the static gauge to

$$S = -\frac{1}{2\pi\alpha'}\int \, dtdr\sqrt{1+\frac{f(r)}{H(r)}x'^2-\frac{\dot{x}^2}{f(r)}}$$

and the corresponding ansatz for the string profile is

$$x(t,r) = vt + \xi(r) + o(t)$$

where v is the late-time velocity of the quark and for the solution we have

$$\xi' = \pm\pi_\xi \frac{H(r)}{f(r)}\sqrt{\frac{f(r)-v^2}{f(r)-\pi_\xi^2 H(r)}} \ \ , \ \ \pi_\xi = \frac{v \ r_h^2}{\sqrt{1-v^2}}$$

So finally

$$\xi = -\frac{v}{2r_h}\left(\tan^{-1}\frac{r}{r_h}+\log\sqrt{\frac{r+r_h}{r-r_h}}\right) \ \ , \ \ \frac{dp_x}{dt} = \sqrt{-g}P_x^r = -\frac{r_h^2}{2\pi\alpha'}\frac{p_x}{m}$$

To conserve energy momentum tensor one has to exert an external force upon the heavy quark to keep constant its velocity:

$$\partial_\nu T^{\nu\mu} = -f^\mu(t)\delta^{(3)}(\vec{x} - \vec{v}t)\ ,\ \ f^\mu(t) = \lim_{r\to\infty} n_M \int d^3x\sqrt{-g}\,T^{M\mu}$$

where $f^\mu(t)$ is the drag force on the quark (the endpoint of the string) and n_M is the unit normal to the boundary.

$$f^\mu(t) = -\frac{dp^\mu}{dt}(t) = -\lim_{r\to\infty} \eta^{\mu\nu}\pi^r_\nu(t,r)$$

where

$$\pi^r_\mu(t,r) = -\frac{\sqrt{\lambda}}{2\pi} G_{\mu N}\frac{1}{\sqrt{-g}}\left(g_{tr}\partial_t X^N - g_{tt}\partial_r X^N\right)$$

Finally, from the string solution the drag force is

$$f^\mu_{(0)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi^2 T^2}{\gamma}(sw^\mu + u^\mu)$$

where $w^\mu = \gamma(1,\vec{v})$ and $s = u\cdot w = -\gamma$

Let's turn to the simplest generalization - one can include hydro perturbations to the theory[2]. Then the starting point is

$$S = -\frac{1}{16\pi G_5}\int \left(\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right) + \frac{\kappa}{3}\epsilon^{MNOPQ}A_M F_{NO} F_{PQ}\right) d^5x$$

and it was shown that the theory at the boundary could be described by relativistic hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0\ ,\ \ T^{\mu\nu} = w u^\mu u^\nu + P g^{\mu\nu} + \tau^{(1)\mu\nu}$$
$$\partial_\mu J^\mu = 0 \quad , \quad J^\mu = n u^\mu + \nu^{(1)\mu},$$

where $\nu^{(1)}$ and $\tau^{(1)\mu\nu}$ are corrections of the first order in gradients.

[2] J. Erdmenger et al, JHEP 0901, 055 (2009)

Solving Einstein-Maxwell equations by the gradient expansion we find the bulk metric as a perturbative series:

$$ds^2 = r^2 k(r) u_\mu u_\nu dx^m u dx^\nu + r^2 h(r) P_{\mu\nu} dx^\mu dx^\nu + r^2 \pi_{\mu\nu}(r) dx^\mu dx^\nu$$
$$+ r^2 j_\sigma(r) \left(P^\sigma_\mu u_\nu + P^\sigma_\nu u_\mu \right) dx^\mu dx^\nu - 2S(r) u_\mu dx^\mu dr$$

and up to the first order in gradients it is (in $h(r) = 1$ gauge)

$$S(r) = 1 \ , \ k(r) = -f(r) + \frac{2}{3r} \partial \cdot u \ , \ \pi_{\mu\nu}(r) = F(r) \sigma_{\mu\nu}$$
$$j_\sigma = -\frac{1}{r}(u \cdot \partial) u_\sigma + \frac{3\sqrt{3} Q^3 \kappa}{2\sqrt{2} M r^6} l_\sigma + J(r) \partial_\sigma \frac{\mu}{T}$$

where $f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6}$.

The same gradient expansion procedure can be used to find corrections to the drag force[3] and at $\mu = 0$

$$\vec{x}(t,r) = \vec{x}_0(t,r) + \vec{x}_1(t,r) \quad , \quad \vec{x}_0(t,r) = \vec{v}\left(t - \frac{1}{\pi T}\left(\tan^{-1}\frac{r}{\pi T} - \frac{\pi}{2}\right)\right)$$

One can check that $\vec{x}^{(1)} = t\ D_t \vec{x}_0(t,r)|_{t=0} + \vec{g}(r)$ solves EOMs and after some algebra the correction to the drag force reads

$$f^{(1)}_\mu = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi T}{\gamma}\Big(c_1(s)(u_\mu(w\cdot\partial)s - s\partial_\mu s - s(su_\alpha + w_\alpha)\partial^\alpha U_\mu)$$
$$+c_2(s)U_\mu(\partial\cdot u) - \sqrt{-s}(u\cdot\partial)U_\mu\Big)$$

where $c_1(s) = \pi/2 - \tan^{-1}\left(\sqrt{-s}\right) - \pi TF(\pi T\sqrt{-s})$ and $c_2(s) = \frac{1}{3}(\sqrt{-s} + (1+s^2)c_1(s))$

[3] M. Lekaveckas et al, JHEP 1402, 068 (2014)

We can now turn on the chemical potential expanding in powers of $\frac{\mu}{T}$. The first non-zero contribution appears at the second order and reads

$$f^{(0)}_\mu = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi^2 T^2}{\gamma}(sw^\mu + u^\mu)\left(1 + \frac{(1+3s)}{6\pi^2 s}\left(\frac{\mu}{T}\right)^2\right)$$

$$f^{(1,2)}_\mu = \frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(\frac{2c_5(s)}{s}\left((w\partial)\log\frac{\mu}{T}\right)u_\mu + c_3(s)\left(u_\mu(w\partial)s + s\partial_\mu s\right)\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}U_\mu\left(c_6(s)(w\partial)\log\frac{\mu}{T} - c_4(s)(\partial u) + c_7(s)(su^\alpha + w^\alpha)\partial_\alpha s + c_{10}(s)(w\partial)s\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(c_8(s)(u\partial)U_\mu + c_9(s)(w\partial)U_\mu + 2c_5(s)\partial_\mu\log\frac{\mu}{T}\right)$$

where $c_i(s)$ describe kinematic properties and we won't bring them here.

Apart from the usual hydro it was shown that in the holographic (and pure hydrodynamic) description there are novel contributions into the current[4] missed before

$$J_\mu = \xi B_\mu + \xi_\omega \omega_\mu .$$

which actually can be obtained also on the field theoretical side and in P-even theory:

$$\vec{J}_5(x) = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2+\mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\vec{\Omega}$$
$$\vec{J}(x) = \frac{\mu_5}{2\pi^2}\vec{B} + \frac{\mu\mu_5}{\pi^2}\vec{\Omega}.$$

These transport phenomena are closely tied with the anomaly and called chiral effects. Note that they also have counterparts in the entropy and momentum currents.

[4] J. Erdmenger et al, JHEP, D. T. Son and P. Surowka PRL

Chiral effects were firstly obtained about 80s in papers of A. Vilenkin. It was shown that they coincide in the linear response theory[5]:

$$J_\mu(x) = \int dx' \Pi^R_{\mu\nu} A^\nu(x')$$

$$\sigma_B = \lim_{k\to 0} \sum_{ij} \frac{i}{2k_j} \epsilon_{ijl} \Pi_{ij}|_{\omega=0} = \frac{\mu_5}{2\pi^2},$$

and in the limit of strong fields (as sum of LLs)

$$\vec{J}_{R(L)} = \sum_{n p_y p_z} \langle \psi_n | \gamma^i | \psi_n \rangle f(\epsilon_{n p_z} - \mu_{R(L)}) = \pm \frac{\mu_{R(L)}}{4\pi^2} B$$

$$\vec{J} = \vec{J}_R + \vec{J}_L = \frac{\mu_R - \mu_L}{4\pi^2} B = \frac{\mu_5}{2\pi^2} B$$

[5] see e.g. Phys. Rev. D 22, 3080

Among other properties of anomalous transport it should be emphasized that it might be non-dissipative:

$$\vec{J} = \sigma_B \vec{B} \quad , \quad \vec{J} = \sigma_E \vec{E}$$

and for time time reversal quantities we have

$$\vec{J}^T = -\vec{J} \ , \quad \vec{E}^T = +\vec{E} \ , \quad \vec{B}^T = -\vec{B}$$

it means that effectively

$$\sigma_B^T = \sigma_B \quad , \quad \sigma_E^T = -\sigma_E$$

while it is obvious that σ_E must be positive.

So CME could be of a dissipation-free nature as a Hall current which is also time reversal $J \sim E \times B$.

The bad part of the story is that chiral effects appear to be highly dependent on the IR physics. Large radiative corrections[6]:

$$\delta J_5 \;=\; -\frac{\alpha_{el} e B \mu}{2\pi^3}\left(\ln\frac{2\mu}{m_f} + \ln\frac{m_\gamma^2}{m_f^2} + \frac{4}{3}\right),$$

Back reaction of a medium and instabilities[7]:

$$\mathbf{curl} B = \sigma_B B \quad\Rightarrow\quad (\Delta + \sigma^2) B = 0$$

Moreover partial sum of the diagram series results in modification of CME:

$$\vec{J}_{CME} = \lim_{k\to 0}\frac{\sigma_B k^2}{k^2 - \sigma_B^2}\vec{B} = 0$$

[6] I. Shovkovy et al Phys.Rev. D88 (2013) 2, 025025

[7] N. Yamamoto et al Phys.Rev.Lett. 111 (2013) 052002

Since there is no dissipation involved one won't expect anomalously driven contributions into f_μ. Also it is a four-vector and the only allowed contribution must be proportional to the vector component of the g:

$$f_\mu \sim j_\mu(r_{wsh})$$

which in turn has anomalous contributions in the presence of CS (manifested by κ):

$$j_\sigma = ... + \frac{\kappa}{\pi^2 T^2}\left(\frac{\mu}{T}\right)^2 C_B(r) B_\sigma + \frac{2\sqrt{3}\kappa Q^3}{Mr^6} l_\sigma$$

and by the direct check we can show that coefficient functions are non-zero at the world-sheet horizon.

Thus for a heavy quark running through the thermal holographic plasma **there is a contribution to the drag force caused by the anomaly (!)**

$$f_\mu = -\frac{\sqrt{\lambda}}{2\pi^3\gamma}\left(\frac{\mu}{T}\right)^2 s^2\kappa\ C_B(\pi T\sqrt{-s})\,(B_\mu + (B\cdot w)w_\mu) - \\ -\frac{\kappa\sqrt{\lambda}\mu^3}{3\gamma\pi^3 T^2}\frac{l_\mu + (l\cdot w)w_\mu}{s}.$$

This result is difficult to predict since the axial anomaly can't directly contribute to the physics of heavy particles while this force is non-zero even for a quark at rest.

More anomalies bring more effects and adding gravitational CS term to our consideration

$$S_{gCS} = -\frac{1}{16\pi G_5}\int \sqrt{-g}d^5x\ \kappa_g \epsilon^{MNPQR} A_M R^A_{BNP} R^B_{AQR}$$

we expect to gain T^2 term in the axial current along vorticity and similarly one finds contribution to the vortical part of the chiral drag force

$$\vec{f} = -\frac{\kappa_g \sqrt{\lambda}\mu}{\gamma\pi^3} C_V(s)\vec{l}.$$

Where $\frac{\kappa}{\kappa_g} = 24$ in the case of a single $U(1)_L$ chiral fermion theory. Note that this effect is less suppressed from the phenomenological point of view. However we will mostly drop the gravitational anomaly in what follows.

In the presence of external B and Ω the drag force for $\vec{w} = \vec{u} = 0$ has form

$$\vec{f} = -\frac{\kappa\sqrt{\lambda}}{2\pi^3}\frac{\mu^2}{T^2}\vec{B} - \frac{2\kappa\sqrt{\lambda}}{3\pi^3}\frac{\mu^3}{T^2}\vec{\Omega}$$

and comparing with the usual drag force

$$\vec{f} = \frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{v}$$

we get

$$\vec{v}_{terminal} = \kappa\frac{\mu^2\vec{B}}{(\pi T)^4} + \frac{4\kappa}{3}\frac{\mu^3\vec{\Omega}}{(\pi T)^4}$$

Thus anomalous transports gain corrections by **heavy** quarks (at finite heavy quarks density) and it is surprising!

But we worked in the Landau rest frame and chiral effects do have corrections there as well as the entropy current does:

$$J_\mu = nu_\mu + C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\varepsilon + P}\right)B_\mu + \frac{1}{2}C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\varepsilon + P}\right)\ell_\mu\,,$$

$$s_\mu = su_\mu - \frac{1}{2}\frac{C\mu^2 s}{\varepsilon + P}B_\mu - \frac{1}{3}\frac{C\mu^3 s}{\varepsilon + P}\ell_\mu\,,$$

By a direct calculation one may check that the boost to the entropy rest frame is

$$\vec{v}_{boost} = -\frac{1}{2}\frac{C}{\epsilon + P}\left(\mu^2\vec{B} + \frac{4}{3}\mu^3\vec{\Omega}\right)$$

and we can readily see considerable similarity

For SYM $\mathcal{N} = 4$ we have

$$C = -\frac{N_c^2 \kappa}{\pi^2} = \frac{N_c^2}{4\pi^2\sqrt{3}}\ , \ \epsilon + P = \frac{N_c^2 \pi^2 T^4}{2}$$

and hence $\vec{v}_{boost} = \vec{v}_{terminal}$, this statement is shown to be correct to all orders in powers of μ/T.

- In the entropy rest frame the chiral drag force for a heavy quark at rest is zero (in the absence of gCS)
- In the entropy rest frame there are non-zero charge and momentum transports caused by chiral effects so we may conclude that they do not dissipate.

A heavy ($m \to \infty$) quark may be considered as a defect in the fluid flow and the absence of the drag force indicates non-dissipativity of the anomalous transport. It can be shown that the absence of the drag force holds in the presence of other gradients.

Let's consider a quark at rest in the local fluid rest frame it feels

$$\vec{f}^B = -\kappa\sqrt{\lambda}\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2}{2\pi^3 T^2}\vec{B}$$

and it accelerates up to the terminal velocity

$$\vec{v}_{\rm terminal} = \kappa\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2}{(\pi T)^4}\vec{B}$$

on the timescale of order $m|\vec{v}_{\rm terminal}|/|\vec{f}^B| \sim 2\pi m/(\sqrt{\lambda}(\pi T)^2) \sim 1fm/c$. Then the characteristic momentum gained by quarks is

$$\vec{p}_{\rm terminal} = m_b\,\kappa\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2\vec{B}}{(\pi T)^4} = -0.449\,m_b\frac{\mu_V\mu_A\vec{B}}{(\pi T)^4}$$

$$\simeq -3\ {\rm MeV}\,\frac{m_b}{4.2\ {\rm GeV}}\,\frac{\mu_V}{0.1\ {\rm GeV}}\,\frac{\mu_A}{0.1\ {\rm GeV}}\,\frac{\vec{B}}{(0.1\ {\rm GeV})^2}\left(\frac{0.5\ {\rm GeV}}{\pi T}\right)^4$$

Thus the chiral magnetic drag force on b(c)-quarks and antiquarks gives them a common momentum of order ~ 3 MeV (~ 1 MeV).

The other possible (though more suppressed) phenomenological consequence is a correction to the screening length of a $q\bar{q}$ dipole (the hot wind story[8]). Let suppose that the velocity of the dipole is transverse to it and parallel to the magnetic field then

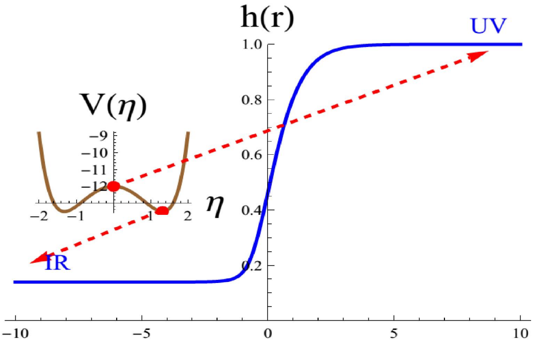
$$L_s = \max_H \left(L_0(H; v + v_A) + 2v \int_{r_c^0}^{\infty} \frac{r^4 \left(v_A(r) - v_A(r_c^0)\right) dr}{(r^4 f(r))^{1/2}(r^4(f(r) - v^2) - H^2)^{3/2}} \right)$$

where we've introduced a characteristic anomalous velocity $v_A = -j_z(r_c^0)$. In the same manner one may try to look for further consequences of the chiral drag force in related topics.

[8] K. Rajagopal et al, JHEP 0703 (2007) 066

The next question is how far CEs are from the superfluidity/conductivity. In the later case there is a limiting velocity which comes from the LC on the QFT side and on the holographic side it corresponds to the absence of a non-trivial probe string profile[9]:

$$f_\mu \sim v\theta(v - v_{IR})$$



- New contribution into the drag force with anomalous nature tied with CME and CVE
- The same sign of the effect for quarks and antiquarks despite of their charge
- The effect of the chiral drag force is in principle observable despite of smallness
- The picture can be turned around to show that a heavy defect doesn't dissipate the momentum flow of chiral effects
- Further theoretical and phenomenological discussions are open: limiting velocity, contribution into diffusion, anomalous wind, etc.