Towards a semi-holographic model for heavy-ion collisions

Anton Rebhan
with: E. Iancu, A. Mukhopadhyay, F. Preis, S. Stricker

Institute for Theoretical Physics
TU Wien, Vienna, Austria

INT-15-2c August 17, 2015

Semi-holographic models

Semi-holography: dynamical boundary theory coupled to a strongly coupled conformal sector with gravity dual

oxymoron coined by Faulkner & Polchinski, JHEP 1106 (2011) 012 [arXiv:1001.5049] in study of holographic non-Fermi-liquid models

- retains only the universal low energy properties, which are most likely to be relevant to the realistic systems
- allows more flexible model-building

further developed for NFLs in: A. Mukhopadhyay, G. Policastro, PRL 111 (2013) 221602 [arXiv:1306.3941]

Semi-holographic models

Semi-holography: dynamical boundary theory coupled to a strongly coupled conformal sector with gravity dual

oxymoron coined by Faulkner & Polchinski, JHEP 1106 (2011) 012 [arXiv:1001.5049] in study of holographic non-Fermi-liquid models

- retains only the universal low energy properties, which are most likely to be relevant to the realistic systems
- allows more flexible model-building

further developed for NFLs in: A. Mukhopadhyay, G. Policastro, PRL 111 (2013) 221602 [arXiv:1306.3941]

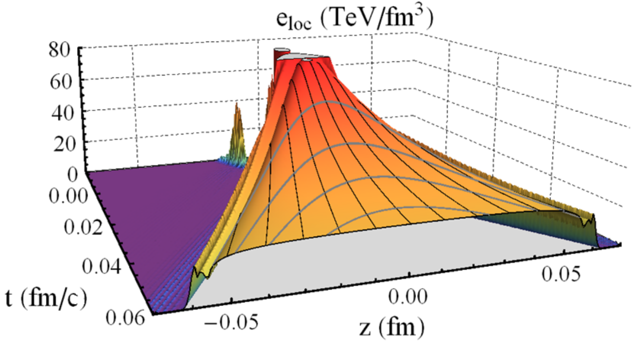
recently proposed for combining pQCD (CGC) as boundary theory with AdS/CFT description of thermalization:
E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003 [arXiv:1410.6448]

see also: hybrid strong/weak coupling model for jet quenching by J. Casalderrey-Solana et al., JHEP 1410 (2014) 19 and arXiv:1508.00815

Gravity dual of heavy-ion collisions

pioneered and developed in particular by P. Chesler & L. Yaffe [JHEP 1407 (2014) 086]

most recent attempt towards quantitative analysis along these lines:
Wilke van der Schee, Björn Schenke, 1507.08195

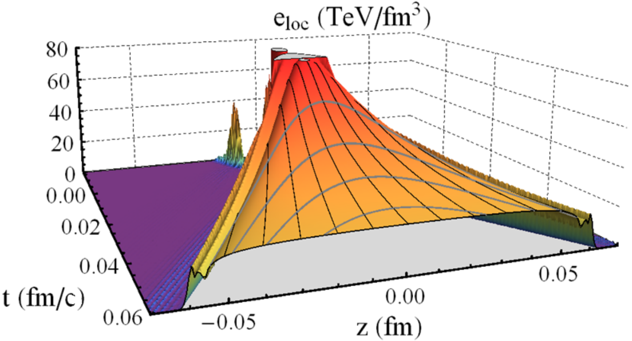


had to scale down energy density by a factor of 20 (6) for the top LHC (RHIC) energies

Gravity dual of heavy-ion collisions

pioneered and developed in particular by P. Chesler & L. Yaffe [JHEP 1407 (2014) 086]

most recent attempt towards quantitative analysis along these lines:
Wilke van der Schee, Björn Schenke, 1507.08195



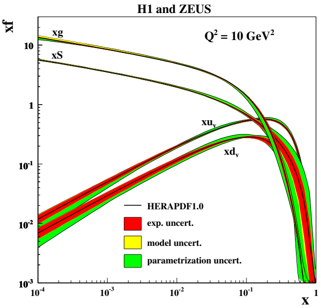
had to scale down energy density by a factor of 20 (6) for the top LHC (RHIC) energies

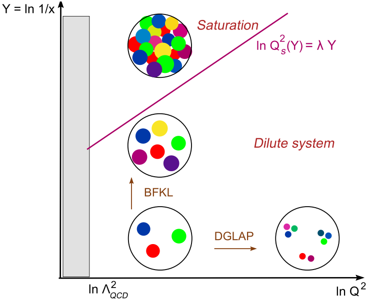
perhaps improved by involving pQCD for (semi-)hard degrees of freedom?

pQCD and Color-Glass-Condensate framework

recap: [e.g. F. Gelis et al., arXiv:1002.033]

gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing longitudinal momentum fraction x at large, fixed Q^2





HIC: high gluon density $\sim \alpha_s^{-1}$ at "semi-hard" scale Q_s ($\sim$ few GeV)

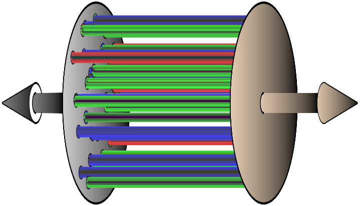
weak coupling $\alpha_s(Q_s) \ll 1$ but highly nonlinear because of large occupation numbers

description in terms of classical YM fields as long as gluon density nonperturbatively high

Color-Glass-Condensate evolution of HIC at LO

effective degrees of freedom in this framework:

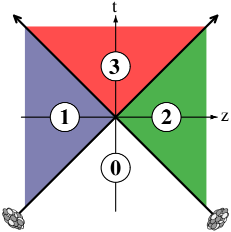
1. color sources ρ at large x (frozen on the natural time scales of the strong interactions and distributed randomly from event to event)
2. gauge fields A^μ at small x
   (saturated gluons with large occupation numbers $\sim 1/\alpha_s$, with typical momenta peaked about $k_\perp \sim Q_s$)



glasma: non-equilibrium matter, with high occupation numbers $\sim 1/\alpha_s$

initially longitudinal chromo-electric and chromo-magnetic fields that are screened at distances $1/Q_s$ in the transverse plane of the collision

Color-Glass-Condensate evolution of HIC at LO



colliding nuclei as shock waves with frozen color distribution

classical YM field equations

$$D_\mu F^{\mu\nu}(x) = \delta^{\nu +}\rho_{(1)}(x^-, \mathbf{x}_\perp) \,+\, \delta^{\nu -}\rho_{(2)}(x^+, \mathbf{x}_\perp)$$

in Schwinger gauge $A^\tau = (x^+ A^- + x^- A^+)/\tau = 0$
with ρ from random distribution (varying event-by-event)

outside the forward light-cone (3):
(causally disconnected from the collision)
pure-gauge configurations

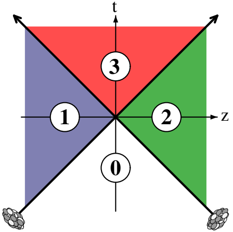
$$A^+ \,=\, A^- \,=\, 0$$
$$A^i(x) = \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp)$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{\mathrm{i}}{g}\, U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp)$$
$$U_{(1,2)}(\mathbf{x}_\perp) = \mathrm{P}\,\exp\left(-\mathrm{i}g \int \mathrm{d}x^\mp \frac{1}{\nabla^2_\perp}\rho_{(1,2)}(x^\mp, \mathbf{x}_\perp)\right)$$

inside forward light-cone:
numerical solution with initial conditions at $\tau = 0$:

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \frac{\mathrm{i}g}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

Color-Glass-Condensate evolution of HIC at LO

colliding nuclei as shock waves with frozen color distribution



classical YM field equations

$$D_\mu F^{\mu\nu}(x) = \delta^{\nu+}\rho_{(1)}(x^-, \mathbf{x}_\perp) + \delta^{\nu-}\rho_{(2)}(x^+, \mathbf{x}_\perp)$$

in Schwinger gauge $A^\tau = (x^+ A^- + x^- A^+)/\tau = 0$
with ρ from random distribution (varying event-by-event)

outside the forward light-cone (3):
(causally disconnected from the collision)
pure-gauge configurations

$$\begin{aligned}
A^+ &= A^- = 0 \\
A^i(x) &= \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp) \\
A^i_{(1,2)}(\mathbf{x}_\perp) &= \tfrac{\mathrm{i}}{g}\, U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp) \\
U_{(1,2)}(\mathbf{x}_\perp) &= \mathrm{P}\ \exp\left(-\mathrm{i}g \int \mathrm{d}x^\mp \tfrac{1}{\nabla^2_\perp}\rho_{(1,2)}(x^\mp, \mathbf{x}_\perp)\right)
\end{aligned}$$

inside forward light-cone:
numerical solution with initial conditions at $\tau = 0$:

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \tfrac{\mathrm{i}g}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

Aim of semi-holographic model: include bottom-up thermalization from relatively soft gluons with higher α_s and their backreaction when they build up thermal bath

Semi-holographic glasma evolution

[E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003]

IR-CFT=effective theory of strongly coupled soft glasma (gluon) modes $k \ll Q_s$, deformed by coupling to dynamics of semi-hard glasma modes $\sim Q_s$

minimalistic coupling through gauge-invariant dimension-4 operators

$$S_{\text{glasma}} = -\int \mathrm{d}^4x \, \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta} F^{\alpha\beta}) - \frac{\gamma}{Q_s^4} \int \mathrm{d}^4x \, t_{\mu\nu} \mathcal{T}^{\mu\nu} - \frac{\beta}{Q_s^4} \int \mathrm{d}^4x \, h \, \mathcal{H}$$

IR-CFT energy-momentum tensor $\mathcal{T}^{\mu\nu}$ coupled to energy-momentum tensor $t_{\mu\nu}$ of YM (glasma) fields

$$t_{\mu\nu}(x) = \frac{1}{N_c} \mathrm{Tr}\Big(F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} \eta_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \Big) \, ,$$

IR-CFT scalar operator $\mathcal{H}$ (dual to dilaton) coupled to "glueball" operator of glasma fields = YM action

$$h(x) \; = \; \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta} F^{\alpha\beta})$$

β, γ dimensionless and $O(1/N_c^2)$; other couplings suppressed by powers of Q_s^{-1}

Semi-holographic glasma evolution

[E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003]

IR-CFT=effective theory of strongly coupled soft glasma (gluon) modes $k \ll Q_s$, deformed by coupling to dynamics of semi-hard glasma modes $\sim Q_s$

minimalistic coupling through gauge-invariant dimension-4 operators

$$S_{\text{glasma}} = -\int \mathrm{d}^4x \, \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta}) - \frac{\gamma}{Q_s^4} \int \mathrm{d}^4x \, t_{\mu\nu} \mathcal{T}^{\mu\nu} - \frac{\beta}{Q_s^4} \int \mathrm{d}^4x \, h \, \mathcal{H}$$

IR-CFT energy-momentum tensor $\mathcal{T}^{\mu\nu}$ coupled to energy-momentum tensor $t_{\mu\nu}$ of YM (glasma) fields

$$t_{\mu\nu}(x) = \frac{1}{N_c} \mathsf{Tr}\Big(F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\Big),$$

IR-CFT scalar operator $\mathcal{H}$ (dual to dilaton) coupled to "glueball" operator of glasma fields = YM action

$$h(x) \; = \; \frac{1}{4N_c} \mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})$$

β, γ dimensionless and $O(1/N_c^2)$; other couplings suppressed by powers of Q_s^{-1}

modified YM (glasma) field equations (inside forward light-cone)

$$D_\mu F^{\mu\nu} = -\frac{\beta}{Q_s^4} D_\mu(F^{\mu\nu}\mathcal{H}) + \frac{\gamma}{Q_s^4} D_\mu(F^{\mu\nu}\mathcal{T}^{\alpha\beta}\eta_{\alpha\beta}) - \frac{2\gamma}{Q_s^4} D_\mu(\mathcal{T}^{\mu\alpha}F_\alpha{}^\nu + F^\mu{}_\alpha \mathcal{T}^{\alpha\nu})$$

Semi-holographic glasma evolution

IR-CFT: marginally deformed AdS/CFT

in Fefferman-Graham coordinates:

$$
\begin{aligned}
\Phi(z,x) &= \frac{\beta}{Q_s^4} h(x) + \cdots + z^4 \frac{4\pi G_5}{l^3} \mathcal{H}(x) + \mathcal{O}(z^6), \\
G_{rr}(z,x) &= \frac{l^2}{z^2}, \\
G_{r\mu}(z,x) &= 0, \\
G_{\mu\nu}(z,x) &= \frac{l^2}{z^2} \Big(\underbrace{\eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)}_{g_{(0)\mu\nu}} + \cdots + z^4 \big(\underbrace{\frac{4\pi G_5}{l^3}}_{2\pi^2/N_c^2} \mathcal{T}_{\mu\nu}(x) + P_{\mu\nu}(x) \big) \\
&\qquad + \mathcal{O}\big(z^4 \ln z\big) \Big),
\end{aligned}
$$

with $P_{\mu\nu} = \frac{1}{8} g_{(0)\mu\nu} \Big(\big(\mathrm{Tr}\, g_{(2)}\big)^2 - \mathrm{Tr}\, g_{(2)}^2 \Big) + \frac{1}{2} (g_{(2)}^2)_{\mu\nu} - \frac{1}{4} g_{(2)\mu\nu} \mathrm{Tr}\, g_{(2)}$

[de Haro, Solodukhin, Skenderis, CMP 217 (2001) 595]

Coupling of glasma EFT and IR-CFT

IR-CFT, like glasma EFT, interpreted as living in Minkowski space
instead of covariantly conserved energy-momentum tensor

$$\nabla_\mu \mathcal{T}^{\mu\nu}(x) = \frac{\beta}{Q_s^4} \mathcal{H}(x) \nabla^\nu h(x),$$

with metric $g_{(0)\mu\nu}(x) = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)$

nonconservation in Minkowski space, with driving forces derived from UV $t_{\mu\nu}(x)$

$$\partial_\mu \mathcal{T}^{\mu\nu} = \frac{\beta}{Q_s^4} \mathcal{H}\, g_{(0)}^{\mu\nu}[t]\, \partial_\mu h - \mathcal{T}^{\alpha\nu} \Gamma^\gamma_{\alpha\gamma}[t] - \mathcal{T}^{\alpha\beta} \Gamma^\nu_{\alpha\beta}[t]$$

with $\Gamma^\mu_{\nu\rho}[t] = \frac{\gamma}{2Q_s^4} \left(\partial_\nu t^\mu{}_\rho + \partial_\rho t^\mu{}_\nu - \partial^\mu t_{\nu\rho} \right) + \mathcal{O}\left(t^2\right)$

total conserved energy-momentum tensor

$T^{\mu\nu} = t^{\mu\nu} + \mathcal{T}^{\mu\nu} +$ local and non-local hard-soft interaction terms

Iterative solution

Practical implementation will have to be done presumably in iterative procedure

1. Solve LO glasma evolution with $\gamma, \beta = 0$
2. solve gravity problem with boundary condition provided by resulting $\mathcal{T}^{\mu\nu}(\tau), \mathcal{H}(\tau)$
3. solve glasma evolution with $\gamma, \beta \propto \tanh(\alpha_0 \tau Q_s)$
4. goto 2) until convergence reached

Simple toy model tests

First test with dimensionally reduced (spatially homogeneous) YM fields $A^a_\mu(t)$ which already have nontrivial (chaotic) dynamics

Simplest situation with $\beta = 0$ and $t^{\mu\nu}$ isotropic has closed-form gravity solution (constructed by F. Preis & S. Stricker)

only $\mathcal{E} + \mathcal{P}$ needed in semi-holographic glasma equations: function of $\tilde{p} = \frac{\gamma}{Q_s^4} p$

$$
\begin{aligned}
\frac{\bar{\mathcal{E}} + \bar{\mathcal{P}}}{N_c^2/2\pi^2} \quad = \quad & (1-3\tilde{p})^{-3}(\tilde{p}+1)^{-4}\Big\{ c\left[1 - 27\tilde{p}^5 - 27\tilde{p}^4 + 18\tilde{p}^3 + 10\tilde{p}^2 - 7\tilde{p}\right] \\
& - \frac{9}{64}\tilde{p}\tilde{p}'^4 - \frac{1}{64}\tilde{p}'^4 + \frac{3}{32}\tilde{p}^2\tilde{p}'^2\tilde{p}'' + \frac{1}{16}\tilde{p}\tilde{p}'^2\tilde{p}'' - \frac{1}{32}\tilde{p}'^2\tilde{p}'' \Big\},
\end{aligned}
$$

with c an integration constant that corresponds to having a black hole already at $t = 0$

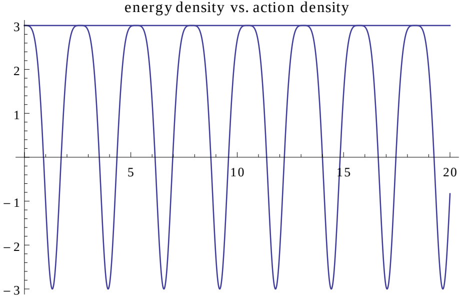
(necessary because with homogeneity and isotropy and no dilaton, there are no local degrees of freedom that could create black hole)

nevertheless nontrivial effects from boundary deformation on $\mathcal{T}^{\mu\nu}$

Simple toy model tests

YM fields:
$\exists$ a solution with homogeneous isotropic energy-momentum tensor ($p = \varepsilon/3$)
by homogeneous color-spin locked oscillations $A_0^a = 0, \quad A_i^a = \delta_i^a f(t)$

$$f(t) = C\,\mathrm{sn}(C(t - t_0)| - 1)$$ (Jacobi elliptic function sn)

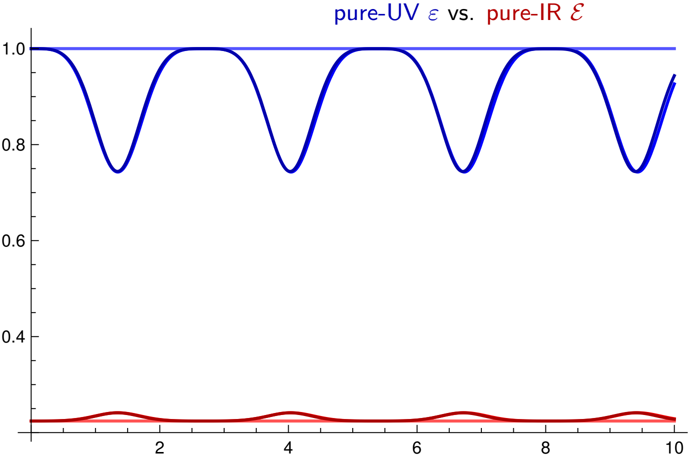


Convergence of iterations

Coupled glasma equation of toy model is 4th order nonlinear ODE
— no reasonable solutions found directly —

Convergence of iterations

Coupled glasma equation of toy model is 4th order nonlinear ODE
— no reasonable solutions found directly —
but iterative solution converges very quickly:



(UV not able to give off energy to IR permanently because of isotropy and homogeneity)

Conclusions and outlook

- Pure gauge-gravity thermalization likely too strong
- Semi-holographic framework of Iancu and Mukhopadhyay proposes to combine LO glasma evolution with thermalization of soft degrees of freedom in AdS/CFT
- First tests suggest that proposed iterative scheme could indeed be numerically stable and convergent
- Next: anisotropic homogeneous toy models
- Eventually: spatially inhomogeneous LO glasma simulations