Thermalization in strongly coupled 2D CFT

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Outline

- Vacuum and thermal correlator in CFT
- Holographic description of thermalization in CFT
- Recipe of bulk-bulk correlator
- Boundary correlator in coordinate space
- Integrated boundary correlator
- Summary

Probe of thermalization in strongly coupled theory

No quasi-particle picture, we look at thermalization from correlation functions

One point function: VEV of operators
used in hydrodynamic evolution

Two point function: spectral densities
Includes fluctuations of hydrodynamic modes
Includes more generic out-of-equilibrium physics, e.g. photon production in history of QGP

Properties of CFT correlator in vacuum and thermal state

$$iG^{>}(2|1) \equiv \langle O(t_2,x_2)O(t_1,x_1)\rangle$$

$$t_{21} = t_2 - t_1$$
$$x_{21} = x_2 - x_1$$

O scalar operator with mass dimension d

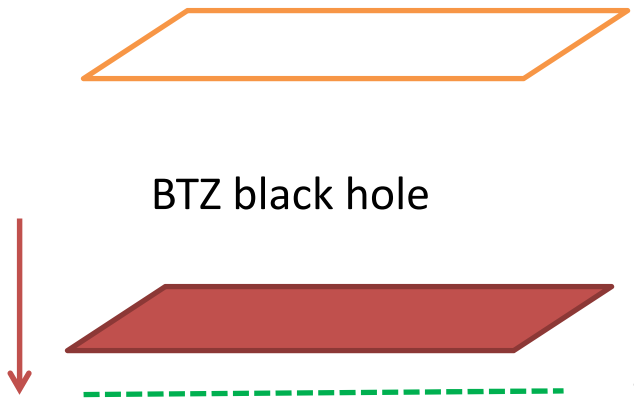
$$iG_0{}^{>}(2|1) = \frac{2}{\pi}\frac{1}{(-(t_{21} - i\epsilon)^2 + x_{21}{}^2)^d}$$

vacuum: long range correlation in space and time (power law)

$$iG_{th}{}^{>}(2|1) = \frac{1}{2\pi}\frac{(2\pi T)^{2d}}{(-\cosh(2\pi T t_{21} - i\epsilon) + \cosh(2\pi T x_{21}))^d}$$

thermal: screening in space and time (exponential)

Gravity dual of thermalization process



$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + dx^2 + \frac{dz^2}{f(z)}\right)$$
$$f(z) = 1 - m\,z^2$$

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx^2 + dz^2)$$

Shell starts falling from the boundary z=0 at t=0, dual to:

$t < 0$, vacuum
$t = 0$, injection of energy density m
$t > 0$, thermalizing state. $t \to \infty$, thermal state

Features of our thermalizing state

$$T^{tt}(t,x) = T^{xx}(t,x) = 2m\theta(t)$$
$$O(t) = 0$$

$O \propto TrG^2$, mass dimension 2

One point function thermalizes instantaneously after energy injection

Two point function takes longer to thermalize

In this work, focus on two point function of O:

Wightman correlator for thermalizing state $\langle O(t,x)O(t',x')\rangle$

Retarded correlator is state independent
Can be calculated from response to external source.
Wightman correlator is state dependent
Needs to know initial density matrix of state

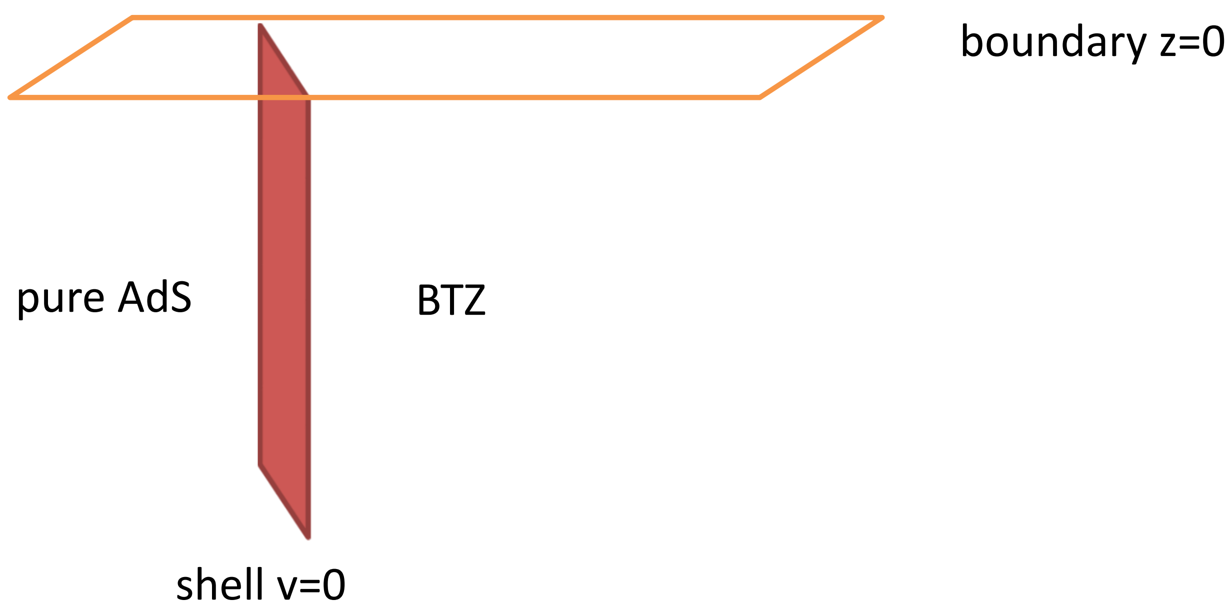
Caron-Huot, Chesler, Teaney, PRD 2011

AdS-Vaidya metric

Eddington-Finkelstein Coordinate $dv = dt - \int_0^z \frac{dz'}{f(z')}$ v=t on the boundary

$$ds^2 = \frac{-f dv^2 - 2dvdz + dx^2}{z^2} \qquad f = 1 - mz^2\theta(v)$$

Set m=1 from now ⟺ set $T = \frac{1}{2\pi}$



Dictionary

Wightman correlator for thermalizing state $\langle O(t,x)O(t',x')\rangle$

Extrapolated dictionary (good for out-of-equilibrium)

$$\langle \mathcal{O}(x_1)\ldots\mathcal{O}(x_n)\rangle_{CFT} = C_n \lim_{z\to 0} z^{-n\Delta}\langle\phi(x_1,z)\ldots\phi(x_n,z)\rangle_{bulk}$$

Skenderis, van Rees, JHEP 2010
Keranen, Kleinert, JHEP 2015

ϕ bulk field dual to boundary operator O
mass dimension $\Delta = 2$

Representation of bulk correlator

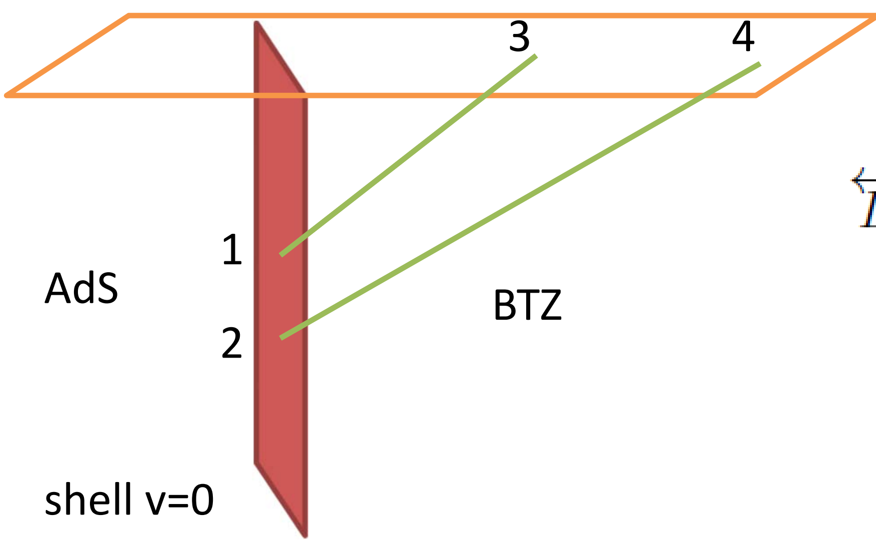
$$G^{>}(4|3) = \int dz_1 dx_1 dz_2 dx_2 G_0^{>}(2|1) \overleftrightarrow{D}^{v1} \overleftrightarrow{D}^{v2} G^R_{\rm th}(3|1) G^R_{\rm th}(4|2)$$

Caron-Huot, Chesler, Teaney, PRD 2011

Initial data $$iG_0^{>}(2|1) = \langle \hat{\phi}(v_2, x_2, z_2)\hat{\phi}(v_1, x_1, z_1)\rangle$$

Propagators
$$iG^R_{\rm th}(3|1) = \langle [\hat{\phi}(v_3, x_3, z_3), \hat{\phi}(v_1, x_1, z_1)]\rangle \theta(t_3 - t_1)$$
$$iG^R_{\rm th}(4|2) = \langle [\hat{\phi}(v_4, x_4, z_4), \hat{\phi}(v_2, x_2, z_2)]\rangle \theta(t_4 - t_2)$$

$$iG^{>}(4|3) = \langle \hat{\phi}(v_4, x_4, z_4)\hat{\phi}(v_3, x_3, z_3)\rangle$$



boundary z=0

$$\overleftrightarrow{D}^{v} = \sqrt{-g} g^{vz} \overleftrightarrow{\partial}_z$$

Propagator

$$iG^R_{\rm th}(3|1) = \langle[\hat\phi(v_3,x_3,z_3),\hat\phi(v_1,x_1,z_1)]\rangle\theta(t_3 - t_1)$$

⟨ ⟩ in BTZ

BTZ is equivalent to AdS locally

$$ds^2_{\rm BTZ} = \frac{1}{z^2}\left(-(1-z^2)dt^2 + \frac{dz^2}{1-z^2} + dx^2\right)$$

$$ds^2_{\rm AdS} = \frac{1}{\bar z^2}\left(-d\bar t^2 + d\bar z^2 + d\bar x^2\right).$$

$$\bar x = \sqrt{1-z^2}e^x\cosh t,$$

$$\bar t = \sqrt{1-z^2}e^x\sinh t,$$

$$\bar z = ze^x.$$

Euclidean AdS bulk-bulk correlator

$$G_E(2|1) = \frac{1}{4\pi}\left(\frac{1}{\sqrt{1-\xi^2}} - 1\right)$$

$$iG^R_{\rm th}(3|1) = i\left(G^>_{\rm th}(3|1) - G^<_{\rm th}(3|1)\right)\theta(t_3-t_1)$$

$$= \Big[G_E(\xi = \frac{z_3z_1}{\cosh(x_{31}) - \sqrt{1-z_3^2}\sqrt{1-z_1^2}\cosh(v_3-v_1+y_3-y_1 - i\epsilon)^2})$$

$$- G_E(\xi = \frac{z_3z_1}{\cosh(x_{31}) - \sqrt{1-z_3^2}\sqrt{1-z_1^2}\cosh(v_3-v_1+y_3-y_1 + i\epsilon)^2})\Big]\theta(t_3-t_1)$$

Non-vanishing only on the lightcone

Initial data

$$iG_0^>(2|1) = \langle \hat{\phi}(v_2, x_2, z_2)\hat{\phi}(v_1, x_1, z_1)\rangle$$

$\langle\,\rangle$ in AdS

$$iG_0^>(2|1) = G_E(\xi = \frac{2z_2 z_1}{z_2^2 + z_1^2 - (v_2 - v_1 + z_2 - z_1 - i\epsilon)^2 + (x_{21})^2})$$

Euclidean AdS bulk-bulk correlator $G_E(2|1) = \frac{1}{4\pi}\left(\frac{1}{\sqrt{1-\xi^2}} - 1\right)$

Singular as $v_1, v_2 \to 0$ and $x_{21} \to 0$ ($\xi \to 1$, lightcone singularity)!
Regularize by subtracting BTZ counterpart

$$iG_{\rm th}^>(2|1) = G_E(\xi = \frac{z_2 z_1}{\cosh(x_{21}) - \sqrt{1-z_2^2}\sqrt{1-z_1^2}\cosh(v_2 - v_1 + y_2 - y_1 - i\epsilon)})$$

$$y_i = -\frac{1}{2}\ln\frac{1-z_i}{1+z_i}$$

$$\Delta G^>(2|1) = G_0^>(2|1) - G_{\rm th}^>(2|1)$$

free of singularity

Regularized CFT correlator

$$\Delta G^{>}(2|1) = G_0^{>}(2|1) - G_{\rm th}^{>}(2|1) \Longrightarrow \Delta G^{>}(4|3) = G^{>}(4|3) - G_{\rm th}^{>}(4|3)$$

$$\Delta G^{>}(4|3) = \int dz_1 dx_1 dz_2 dx_2 \Delta G^{>}(2|1) \overleftrightarrow{D}^{v1} \overleftrightarrow{D}^{v2} G_{\rm th}^{R}(3|1) G_{\rm th}^{R}(4|2)$$

$$\langle G^{>}(4|3)\rangle \to z_4^2 z_3^2 \langle O(v_4, x_4) O(v_3, x_3)\rangle$$

$\Delta\langle O(v_4, x_4) O(v_3, x_3)\rangle$ Boundary correlator in thermalizing state – thermal correlator

As $v_3, v_4 \to 0$,

$\Delta\langle O(v_4, x_4) O(v_3, x_3)\rangle \to$ vacuum correlator – thermal correlator

As $v_3, v_4 \to \infty$,

Not quite true

$\Delta\langle O(v_4, x_4) O(v_3, x_3)\rangle \to 0$

Correlator in coordinate space

$$\Delta\langle O(v_4, x_4) O(v_3, x_3)\rangle$$

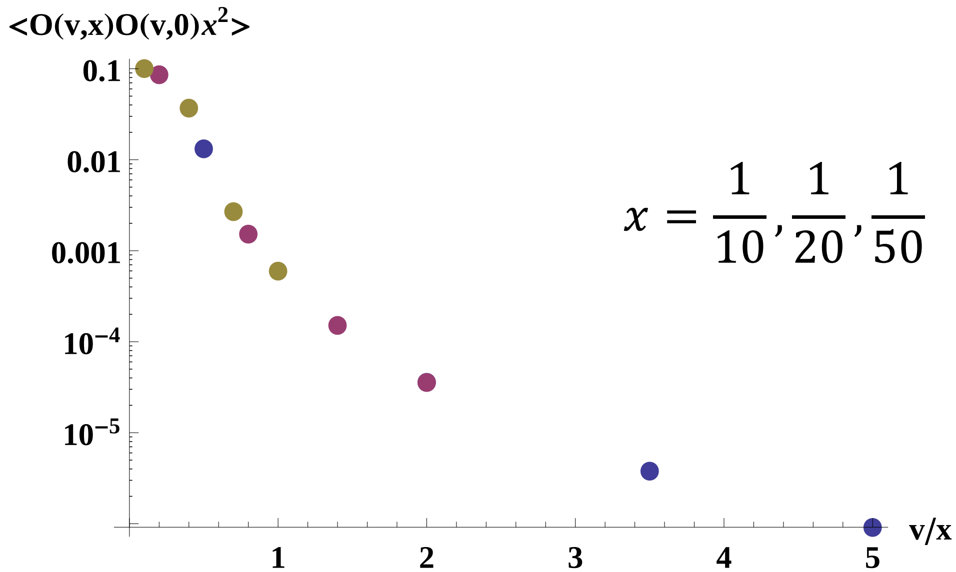
- Equal time correlator $v_3 = v_4, x_{43} \neq 0$
  measure of spatial decorrelation
- Equal space correlator $v_3 \neq v_4, x_{43} = 0$
  measure of temporal decorrelation

Correlator in momentum space

Keranen, Kleinert, JHEP 2015

Equal-time correlator $\langle \Delta O(v,x) O(v,0) \rangle$

$x, v \ll 1, \qquad \langle \Delta O(v,x) O(v,0) \rangle = \frac{1}{x^2} f(\frac{v}{x})$



$x = \frac{1}{10}, \frac{1}{20}, \frac{1}{50}$

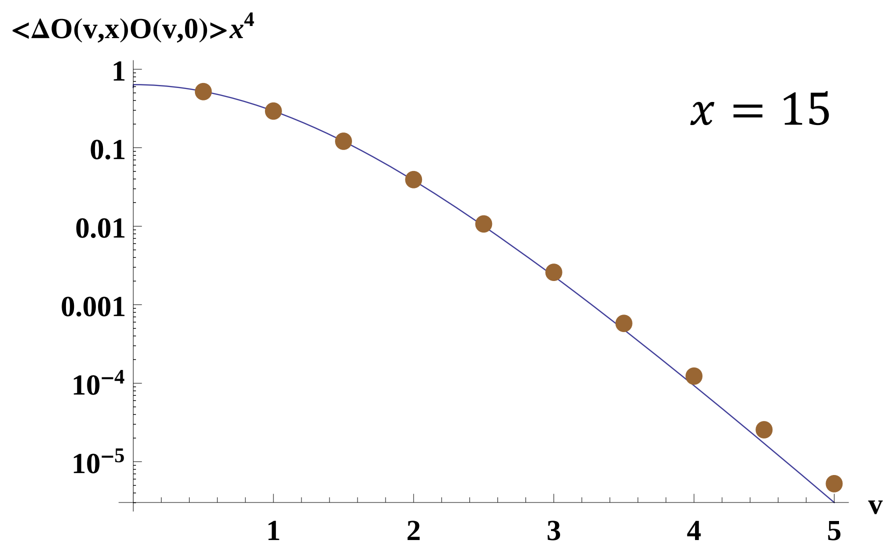
$v \to 0, \qquad \langle \Delta O(v,x) O(v,0) \rangle \to \frac{2}{\pi}\frac{1}{x^4} - \frac{1}{2\pi}\frac{1}{(-1+\cosh x)^2} \longrightarrow \frac{1}{x^2}, \qquad x \ll 1$

vacuum

thermal

Equal-time correlator $\langle \Delta O(v,x) O(v,0) \rangle$

$x \gtrsim 2v, \qquad \langle \Delta O(v,x) O(v,0) \rangle = \frac{1}{x^4} \tilde{f}(v)$



$v \to 0, \qquad \langle \Delta O(v,x) O(v,0) \rangle \to \frac{2}{\pi} \frac{1}{x^4} - \frac{1}{2\pi} \frac{1}{(-1 + \cosh x)^2} \longrightarrow \frac{1}{x^4}, \qquad x \gg 1$

vacuum

thermal

Thermalization of distance x

$$O(v2,x)O(v1,0) \sim \begin{cases} e^{-\pi(v2+v1)/\tau} & x > v2+v1 \\ e^{-2\pi x/\tau} & |v2-v1| < x < v2+v1 \end{cases}$$

τ, screening length in initial state

Calabrese, Cardy, PRL 2006

In AdS/CFT, sees sharp transition between different regimes: artifact of geodesic approximation

Aparicio, Lopez, JHEP 2009
Balasubramanian et al, PRL&PRD 2010

Smooth transition occurs when $|x - v2 - v1| \lesssim \tau$

Equal-time correlator $\langle \Delta O(v4, x) O(v3,0) \rangle$

$$\langle O(v,x)O(v,0)\rangle = \frac{18}{\pi x^4}\frac{v4\coth v4 - 1}{sinh^2 v4}\frac{v3\coth v3 - 1}{sinh^2 v3} \qquad x \gg v4, v3$$

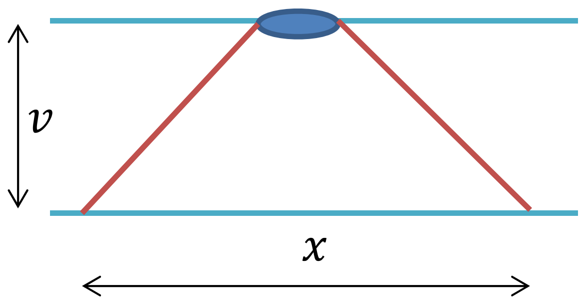
reminiscent of long range correlation
in initial state

$v4, v3 \to 0, \quad \langle O(v4,x)O(v3,0)\rangle \to$ vacuum correlator

$v4, v3 \gg 1, \qquad \langle O(v4,x)O(v3,0)\rangle \sim \frac{1}{x^4} v4\ v3\ e^{-2v4-2v3}$

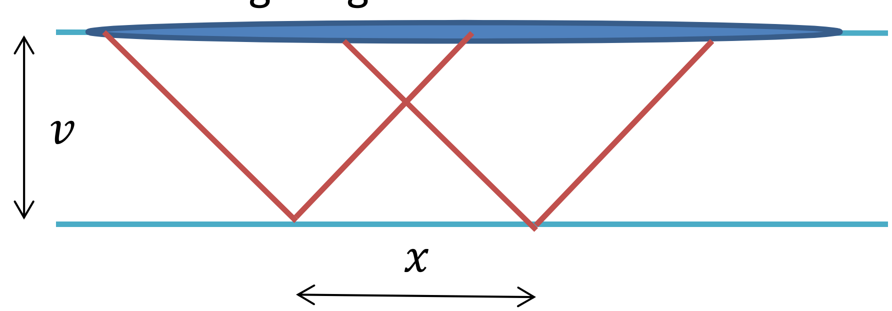
Enhanced correlation

Initial state w/wo long range correlation



Thermalization time $v \sim x/2$

Calabrese, Cardy, PRL 2006



$$\langle O(v,x)O(v,0)\rangle \sim \frac{1}{x^4} v^2 \, e^{-4v}$$

Thermalization reached

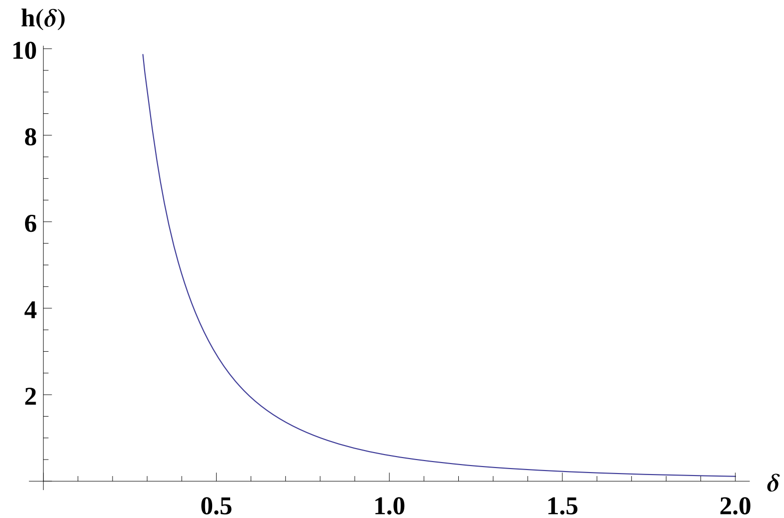
$\langle O(v,x)O(v,0)\rangle \sim$ thermal $\sim e^{-2x}$

$\ln x$ correction to thermalization time?

Transition between $x > v4 + v3$& $x < v4 + v3$

$\langle O(v4, x = v4 + \delta) O(v3,0) \rangle \sim e^{-2v4} h(\delta)$ $\qquad v3 \ll 1, v4 \gg 1, \delta \sim 1$

$h(\delta)$ analytically known, monotonous decaying function



$\langle O(v4, x = v4 + \delta) O(v3,0) \rangle \sim e^{-2v4} h(\delta) \sim$ thermal $\sim e^{-2x}$

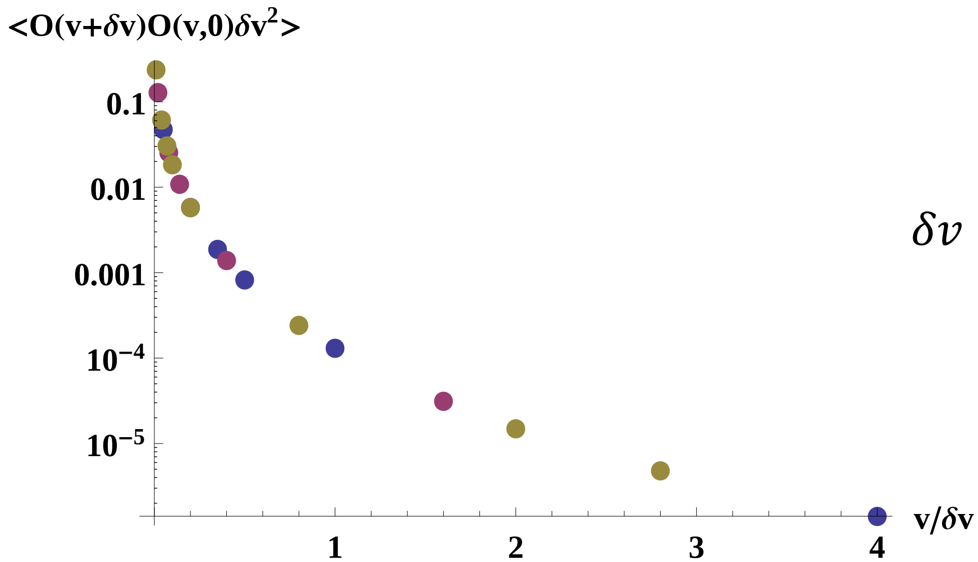
Thermalization time: $v4 + v3 + O(1)$

No $\ln x$ correction! in thermalization time

Also confirmed numerically for $v4 = v3$.

Equal-space correlator $\langle \Delta O(v+\delta v,0)O(v,0)\rangle$

$\delta v \ll 1,\qquad \langle \Delta O(v+\delta v,0)O(v,0)\rangle = \frac{1}{\delta v^2}\, g(\frac{v}{\delta v})$



$\delta v = \frac{1}{10}, \frac{1}{20}, \frac{1}{50}$

$v \to 0,\qquad \langle \Delta O(v+\delta v,0)O(v,0)\rangle \nrightarrow$ vacuum - thermal correlator

$$\frac{2}{\pi}\frac{1}{\delta v^4} - \frac{1}{2\pi}\frac{1}{(-1+\cosh\delta v)^2}$$

$\delta v \to 0,\quad \langle \Delta O(v+\delta v,0)O(v,0)\rangle \to$ finite

Equal-space correlator $\langle \Delta O(v + \delta v, 0) O(v, 0) \rangle$

$\delta v \gg 1, v \lesssim 1, \qquad \langle \Delta O(v + \delta v, 0) O(v, 0) \rangle \sim v\, \delta v\, e^{-2(2v + \delta v)}$

$\langle \Delta O(v + \delta v, 0) O(v, 0) \rangle \sim$ thermal $\sim e^{-2\delta v}$

Thermalization time $v \lesssim 1$

$O(v + \delta v, 0) O(v, 0) \sim e^{-\pi \delta v / \tau} \qquad v \gtrsim 1$ Calabrese, Cardy, PRL 2006

Thermalization time ~ 1, independent of interval δv.
In contrast to equal-time case.

Spatially integrated correlator (k=0 mode)

k=0 mode $\int dx \, \langle \Delta O(v4, x) O(v3,0) \rangle$

Two analytic limits:

$v_3, v_4 \ll 1$ $$\int dx \, \langle \Delta O(v4, x) O(v3,0) \rangle \simeq \frac{3\pi}{512\sqrt{v4 \, v3}}$$

$v_3 \ll 1, v_4 \gg 1$ $$\int dx \, \langle \Delta O(v4, x) O(v3,0) \rangle \simeq \frac{3\pi}{128\sqrt{v3}} v4 e^{-2v4}$$

k=0 mode ≠ long distance?

$$\int dx\, \langle \Delta O(v,x) O(v,0) \rangle \sim \frac{1}{v}$$ when $v \ll 1$

$x, v \ll 1, \qquad \langle \Delta O(v,x) O(v,0) \rangle = \frac{1}{x^2} f(\frac{v}{x})$

$$\int dx\, \langle \Delta O(v,x) O(v,0) \rangle \sim \int dx \frac{1}{x^2} f\left(\frac{v}{x}\right) \sim \frac{1}{v}$$

At early time, dorminant contribution from short distance!

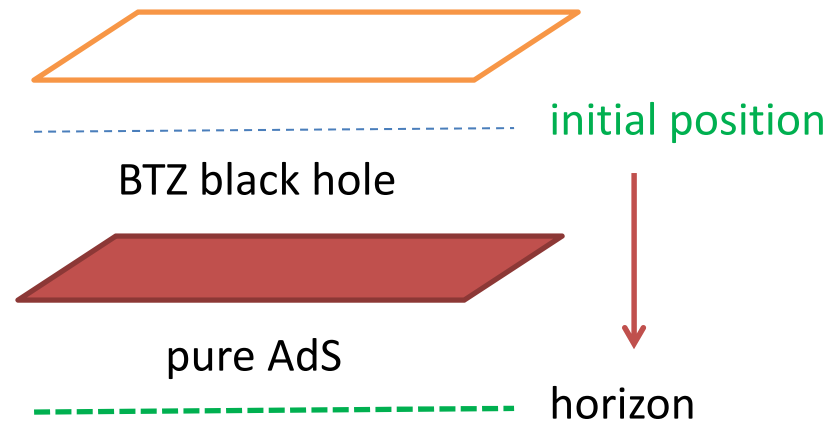
Summary

Thermalization in 1+1D CFT with long range correlation in initial state:

- Enhanced correlation, however does not lead to $ln x$ correction in thermalization time as compared to the case without initial long range correlation
- Qualitatively different thermalization scenarios in thermalization of equal-time correlator and equal-space correlator
- Should distinguish low momentum and long distance. At early time, low momentum correlator can be dorminated by short distance contribution.

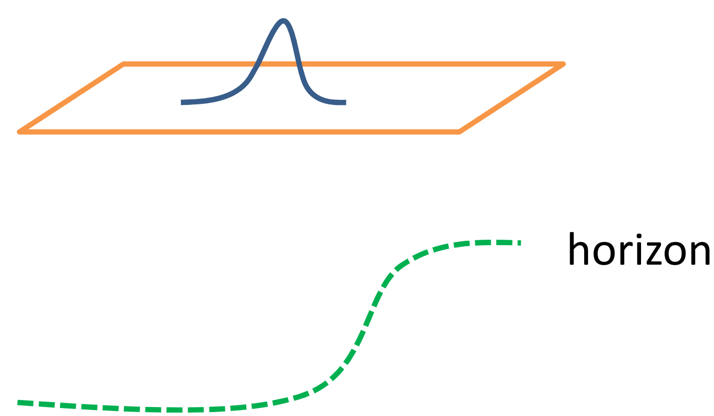
Compare with similar models

Gravitational collapse of massive shell:
Initial state introduces additional scale



SL, Shuryak, PRD 2008

Gravitational collapse of gravitational pulse:
deformation of boundary metric
Initial state with tiny temperature



Chesler, Yaffe, PRL 2009

Frequency modes with finite resolution

Gabor transform

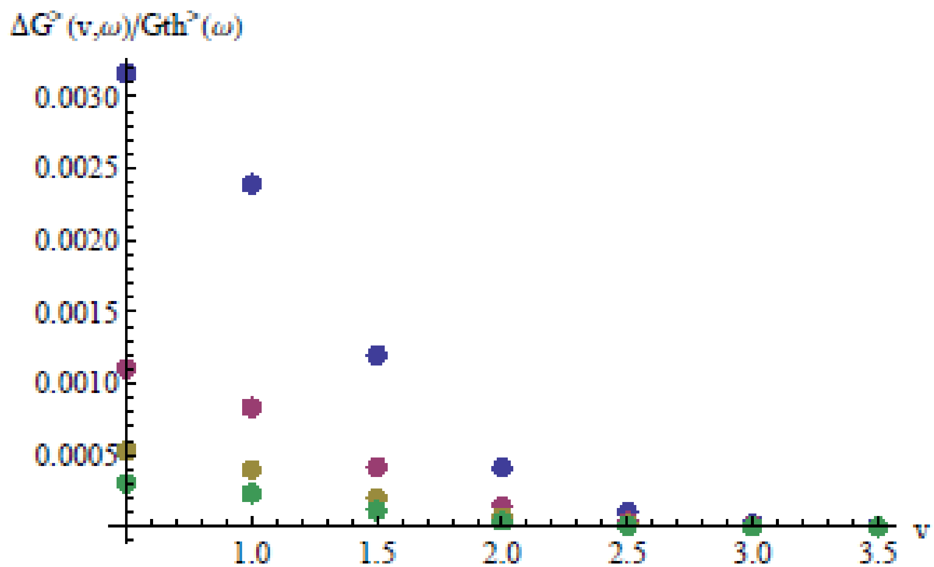
$$i\Delta G^{>}(\bar{v},\omega) = \frac{1}{\sqrt{\pi\sigma}} \int dv_3 dv_4 e^{-\frac{(v_3-\bar{v})^2}{2\sigma^2}} e^{-\frac{(v_4-\bar{v})^2}{2\sigma^2}} e^{i\omega(v_4-v_3)} i\Delta G^{>}(v_4,v_3)$$

Chesler, Teaney 2011

Time resolution $\sim\sigma$, requires $\omega \gtrsim 1/\sigma$

$$iG^{>}_{\rm th}(\omega) = \frac{1}{\sqrt{\pi\sigma}} \int dv_3 dv_4 e^{-\frac{(v_3-\bar{v})^2}{2\sigma^2}} e^{-\frac{(v_4-\bar{v})^2}{2\sigma^2}} e^{i\omega(v_4-v_3)} \int dx_{43} iG^{>}_{\rm th}(v_4,v_3,x_{43})$$

Decoupling of frequency modes



choose $\sigma = 1$

$\omega = 1$, $\omega = 2$, $\omega = 3$ and $\omega = 4$

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Early decoupling of high frequency modes