Classical field initial stages of a heavy ion collision

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Thermalization program, INT 2015

Outline

- CGC, Glasma, JIMWLK evolution
- Initial conditions for CYM
  T.L., [arXiv:1105.5511], PLB 2011
- Wilson loop in glasma
  Dumitru, T.L., Nara [arXiv:1401.4124], PLB 2014
- Debye mass in nonequilibrium classical Yang-Mills
  Work in progress with J. Peuron

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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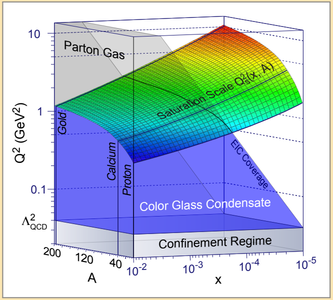
▼

$p_T \sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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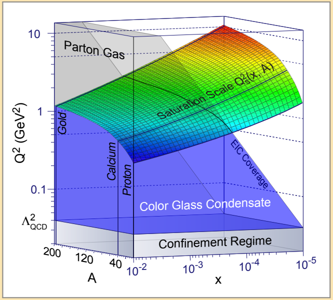

CGC: Effective theory for wavefunction of nucleus

- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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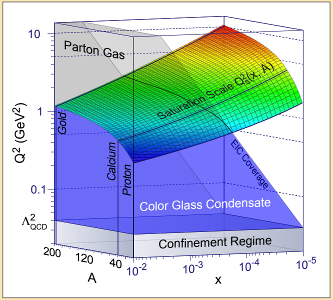

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- Small x = classical gluon field A_μ + quantum flucts.

Glasma: field configuration of two colliding sheets of CGC.
JIMWLK: y-dependence of $W_y[\rho]$; Langevin implementation

Wilson line

Classical color field described as Wilson line

$$U(\mathbf{x}_T) = P\exp\left\{ig\int dx^- A^+_{\rm cov}(\mathbf{x}_T, x^-)\right\} \quad \in \quad SU(3)$$

Color charge ρ : $\quad \nabla_T{}^2 A^+_{\rm cov}(\mathbf{x}_T, x^-) = -g\rho(\mathbf{x}_T, x^-)$

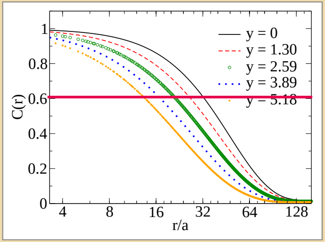
($x^\pm = \frac{1}{\sqrt{2}}(t \pm z)$; $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^z)$; $\mathbf{x}_T$ 2d transverse)

Q_s is characteristic momentum/distance scale

Precise definition here is:

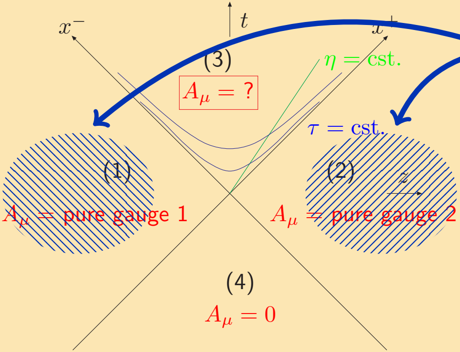
$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, U^\dagger(\mathbf{0}_T)U(\mathbf{x}_T)\right\rangle = e^{-\frac{1}{2}}$$

$$\Longleftrightarrow \mathbf{x}_T^2 = \frac{2}{Q_s^2}$$



Gluon fields in AA collision

Classical Yang-Mills



2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}_T)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

$$U_{(1,2)}(\mathbf{x}_T) = P e^{ig\int dx^- \frac{\rho(\mathbf{x}_T,x^-)}{\nabla_T^2}}$$

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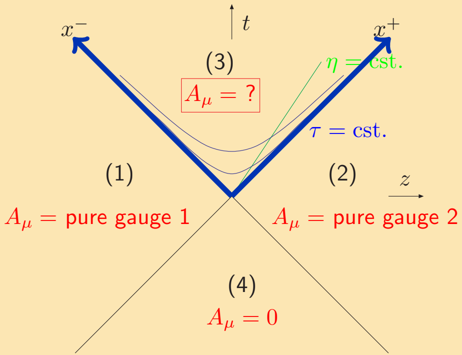

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At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

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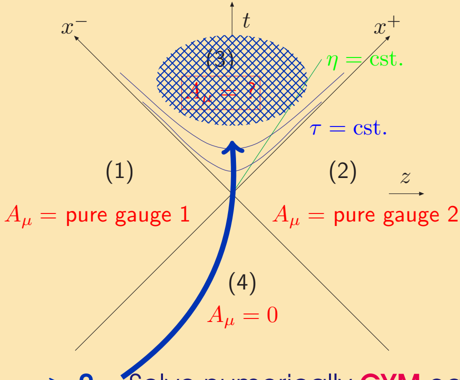

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$\tau > 0$ Solve numerically **CYM** equations $[D_\mu, F^{\mu\nu}] = 0$.
This is the **glasma** field $\Longrightarrow$ Then average over initial U's.

Fix gauge, Fourier-decompose: gluon spectrum

Gluons with $p_T \sim Q_s$ — strings of size $R \sim 1/Q_s$

Gluon spectrum in the glasma

T.L., *Phys.Lett.* **B703** (2011) 325

Q_s is only dominant scale

Parametrically gluon spectrum $$\frac{dN_g}{dy\, d^2\mathbf{x}_T\, d^2\mathbf{p}_T} = \frac{1}{\alpha_s} f\left(\frac{p_T}{Q_s}\right)$$

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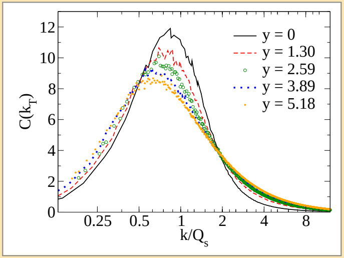

$$C(\mathbf{k}_T) = \frac{{k_T}^2}{N_c} \mathrm{Tr} \left\langle U(\mathbf{k}_T) U^\dagger(\mathbf{k}_T) \right\rangle$$

harder with JIMWLK

(Power law at high k_T) .

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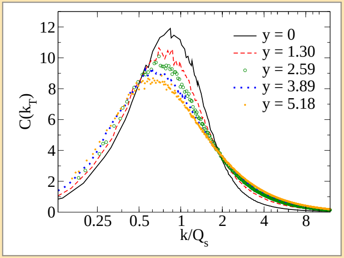

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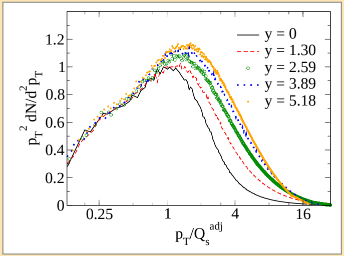
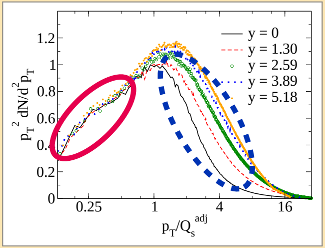


Produced gluon spectrum: harder at higher $\sqrt{s}$

(Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Universality in the IR spectrum?



- Gluon spectrum in the UV depends on initial condition
- IR seems to **scale**, close to

$$\frac{dN}{d^2\mathbf{p}_T} \sim \frac{1}{p_T}$$

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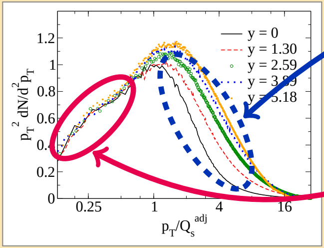

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How to probe $p_T \lesssim Q_s$?

Gauge invariant **Wilson loop**

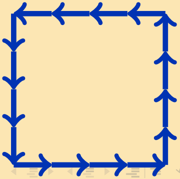
$$W(A) = \frac{1}{N_c}\,\mathrm{Tr}\,\mathbb{P}\exp\left\{ig\oint_A \mathrm{d}\mathbf{x}_T \cdot \mathbf{A}_T\right\}$$

A = area inside loop

2d lattice: links:

$$\uparrow = U_i(\mathbf{x}_T) = \exp\{igaA_i\}$$

$$W(A) = \frac{1}{N_c}\,\mathrm{Tr}$$



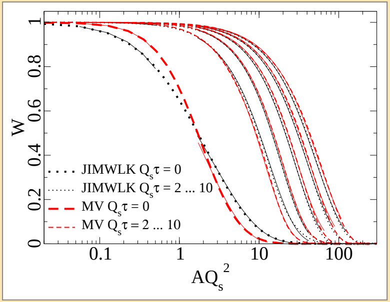
Measure Wilson loops

Dumitru, Nara, Petreska [arXiv:1302.2064], PRD 2013
& Dumitru, T.L., Nara [arXiv:1401.4124]

Calculation is simple:

- Construct initial glasma fields at $\tau = 0$ using e.g.
  - MV model
  - rcJIMWLK
  - fcJIMWLK

  (Try to have same $Q_s a$ to minimize lattice effects)
- Evolve forward in τ
- Measure $W(A)$



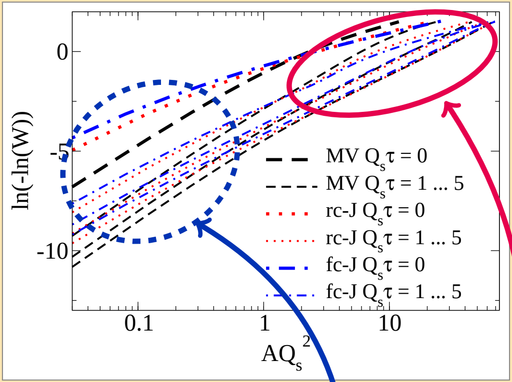
Parametrize both UV ($AQ_s^2 \lesssim 1$) and IR ($AQ_s^2 \gtrsim 1$) as

$$W = \exp\left\{-(\sigma A)^\gamma\right\}$$

Fit is quite good: solid lines in figure.

Fit to Wilson loop area dependence

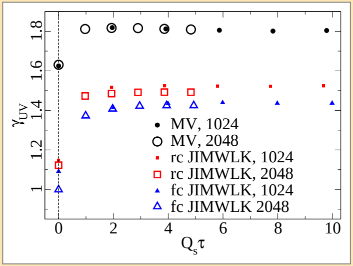
$$W = \exp\{-(\sigma A)^\gamma\} \Longleftrightarrow \ln(-\ln W) = \gamma \ln(A Q_s^2) + \gamma \ln(\sigma / Q_s^2)$$



Main observations

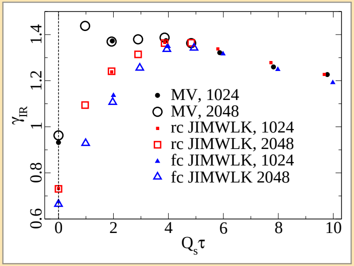
- UV (small loop): initial slope γ stays
- IR (big loop): collapse to universal behavior

Wilson loop scaling exponents



UV ($e^{-3.5} < AQ_s^2 < e^{-0.5}$)

Remembers initial condition



IR ($e^{0.5} < AQ_s^2 < e^{5}$)

Initial conditions collapse to $\gamma_{IR} \approx 1.2$, decreasing slowly with τ

Magnetic field correlator

Wilson loop measures magnetic flux:

$$W(A) = \frac{1}{N_c} \mathrm{Tr}\, \mathbb{P} \exp \left\{ ig \oint_A d\mathbf{x}_T \cdot \mathbf{A}_T \right\} = \frac{1}{N_c} \mathrm{Tr} \exp \left\{ ig \int d^2\mathbf{x}_T B_z(\mathbf{x}_T) \right\}$$

If magnetic field consists of **uncorrelated Gaussian domains**:

$$\langle W(A) \rangle = \exp \left\{ -\frac{1}{2} \frac{1}{N_c} \mathrm{Tr} \left\langle \left[\int d^2\mathbf{x}_T g B_z(\mathbf{x}_T) \right]^2 \right\rangle \right\}$$

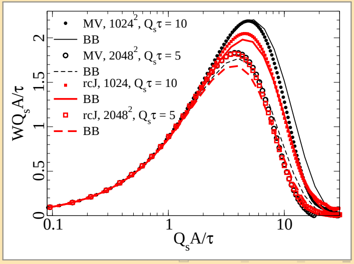
$\Longrightarrow$ $W(A)$ **related to** $\langle B(\mathbf{x}_T) B(\mathbf{y}_T) \rangle$

(No gauge fixing, but connect $B(\mathbf{x}_T)$ and $B(\mathbf{y}_T)$ with gauge link)

Check: compare

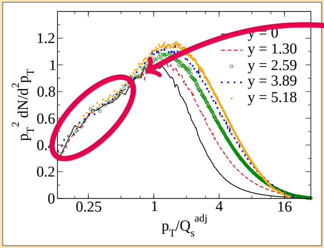
- Direct measurement of $W(A)$
- Reconstruction from *BB*-correlator

good agreement.



The IR region: quasiparticle view

Work in progress with J. Peuron



- Perturbatively $\frac{dN}{d^2\mathbf{p}_T} \sim \frac{1}{p_T^2}$
- Numerically $\frac{dN}{d^2\mathbf{p}_T} \sim \frac{1}{p_T}$
  $\Longrightarrow$ 2d thermal spectrum
- How to understand this in a quasiparticle picture?
- How does this change with isotropization: 2d $\rightarrow$ 3d ?

The numerical calculation here uses

$$\frac{dN}{d^2\mathbf{k}_T} = \frac{1}{2}\left[\frac{E_a^i(\mathbf{k}_T)E_a^i(-\mathbf{k}_T)}{|\mathbf{k}_T|} + |\mathbf{k}_T| A_a^i(\mathbf{k}_T)A_a^i(-\mathbf{k}_T)\right]$$

(Plus appropriate explicit powers of τ in expanding coordinates)

This assumes a linear dispersion relation $\omega(\mathbf{k}_T) = |\mathbf{k}_T|$

Can one go beyond this? Measure $\omega(|\mathbf{k}_T|)$?

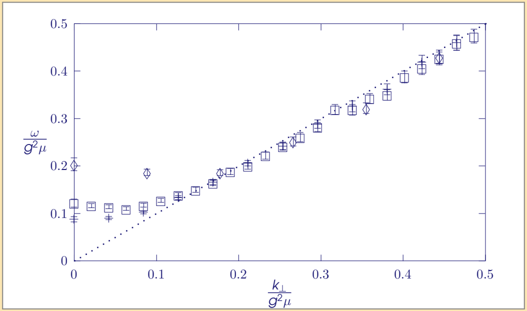
Dispersion relation in Glasma

Idea: $E^i_a(\mathbf{x}_T) \equiv \partial_t A^i_a(\mathbf{x}_T)$ $\Longrightarrow$ define gauge fixed

$$\omega^2(\mathbf{k}_T) = \frac{\left\langle E^i_a(\mathbf{k}_T) E^i_a(-\mathbf{k}_T)\right\rangle_{\text{Coul}}}{\left\langle A^i_a(\mathbf{k}_T) A^i_a(-\mathbf{k}_T)\right\rangle_{\text{Coul}}}$$

(+ appropriate powers of τ in expanding coordinates)

Krasnitz, Venugopalan, `hep-ph/0007108`:



See clear mass gap

$$m^2 \sim \frac{g^2\mu}{\tau} \qquad g^2\mu \sim Q_s$$

Guidelines from finite-T perturbation theory

Straightforward assumption:
mass gap related to Debye/plasmon mass scale.
How to measure it in CYM?

1. In a thermal plasma, the 1-loop plasmon mass is

$$\omega_{\mathrm{pl}}^2 = \frac{4g^2 N_{\mathrm{c}}}{3} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^2} \frac{f(\boldsymbol{k})}{|\boldsymbol{k}|}$$

   (One-loop gluon propagator, f: particle in loop)
   Here classical field $f(\boldsymbol{k}) \sim 1/g^2$, coupling drops out.
2. A homogenous color-E field oscillates at frequency ω_{pl}

Isotropic 3d classical system

- Goal: understand plasmon mass in
  - purely 2d system
  - anisotropic 3d
- First check: 3d isotropic nonexpanding classical case

Prepare initial gauge fields

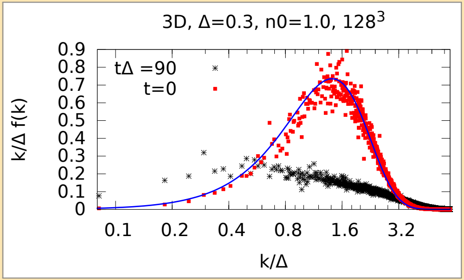
$$\left\langle A_i^a(\boldsymbol{p})A_j^b(\boldsymbol{q})\right\rangle = \frac{n_0}{Vg^2}\delta^{ij}\delta_{ab}(2\pi)^3\delta^{(3)}(\boldsymbol{p}+\boldsymbol{q})\exp\left[-\frac{\boldsymbol{p}^2}{2\Delta^2}\right] \qquad E_i^a = 0$$

This corresponds to:

$$f(\boldsymbol{k}) = n_0\frac{|\boldsymbol{k}|}{\Delta}\exp\left[-\frac{\boldsymbol{k}^2}{2\Delta^2}\right]$$

Idea: clear mass scale Δ

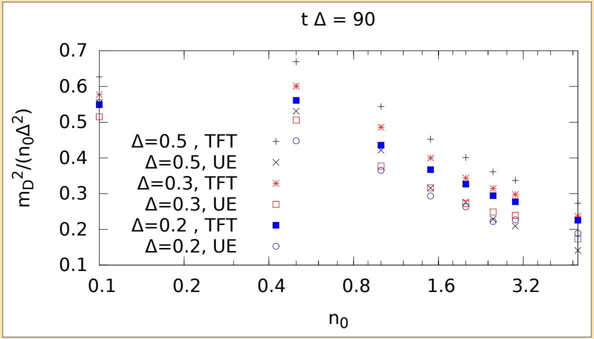
Follow system to large t, but not to full equilibrium



Plasmon mass in isotropic 3d classical system

Extract numerically the plasmon mass with the

- TFT formula $\omega_{\mathrm{pl}}^2 \sim \int_{\boldsymbol{k}} f(\boldsymbol{k})/|\boldsymbol{k}|$ (but assume $\omega(\boldsymbol{k}) = |\boldsymbol{k}|$ to define f)
- Insert homogenous E -field & measure oscillations in t

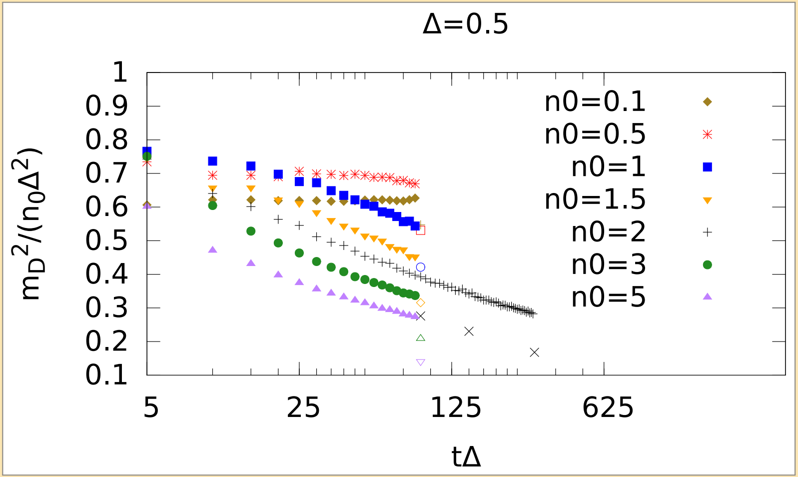


$(m_D^2 = 3\omega_{\mathrm{pl}}^2)$

OK up to factor ~ 1.5

$\Longrightarrow$ need to project out longitudinal part of E-field

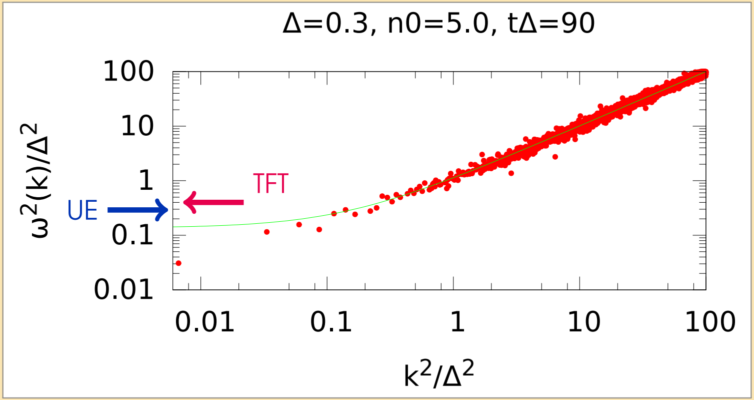
Time dependence



- For dilute system $n_0 \lesssim 1$: very little time dependence
- Dense system: decrease due to UV cascade
  (Nonexpanding system $\Longrightarrow$ thermalizes)

Dispersion relation in nonexpanding 3d

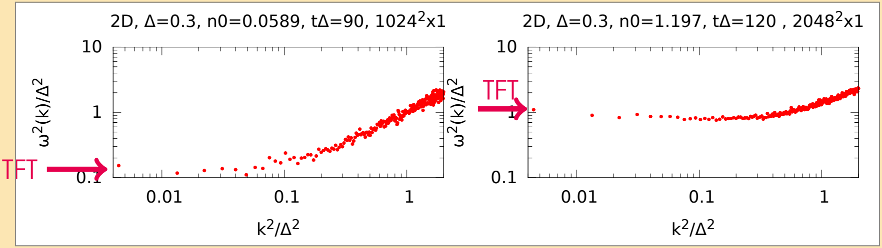
3d system: possible gap in $\omega(\boldsymbol{k})$ seems not to agree with ω_{pl}



Conclusion: in 3d cannot use E^2/A^2 to get mass gap

Dispersion relation in nonexpanding 2d

For 2d system agreement is much better:



Note, for 2d thermal

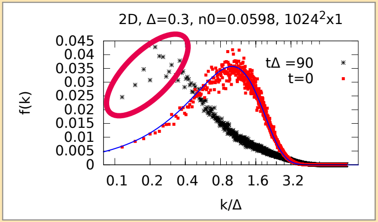
$$\omega_{\text{pl}}^2 \sim \int_{\boldsymbol{k}} f(\boldsymbol{k})/|\boldsymbol{k}|$$

is log IR divergent for

$$f(\boldsymbol{k}) \sim \delta(k_z)/k_T$$

$\Longrightarrow$ TFT formula shaky in 2d

Here IR not yet thermalized



Conclusions

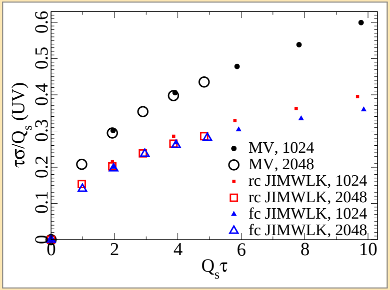
- CYM initial state for AA collision
- Universal behavior in the for $p_T \ll Q_s$ seen in gluon spectrum
- Same universality seen in spatial Wilson loop
- Physics of Debye mass scale is very different in 2d and 3d classical gauge theory.
  - There is no real Debye screening in 2d thermal gauge theory (KT transition)
  - But numerically one observes a nice mass gap

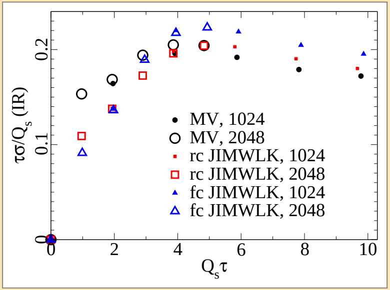
"String tension" coefficients

In expanding system fields naturally decrease as

$$\tau \gg 1/Q_s \implies A_\mu \sim 1/\sqrt{\tau} \implies \sigma/Q_s^2 \sim 1/(Q_s\tau)$$

Plot "string tension" σ as scaling variable $\sigma\tau/Q_s$





UV: initial conditions differ

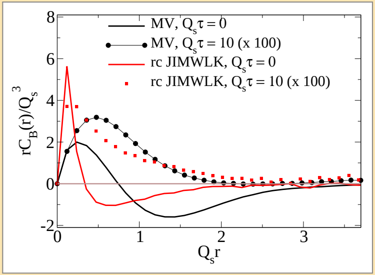
IR: σ universal within $\sim$ 10%

(Note: numerical value of σ/Q_s^2 depends on the convention used to define Q_s)

At $\tau = 0$: $\sigma/Q_s^2 \approx 0.55 \ldots 0.6$ (UV) and $\sigma/Q_s^2 \approx 0.35 \ldots 0.45$ (IR)

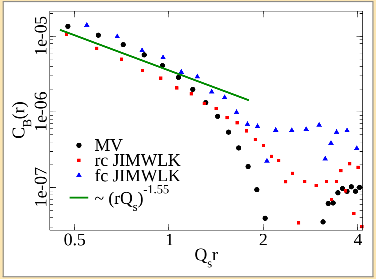
Magnetic field correlator

However: no obvious scaling seen in *BB*-correlator



Same on log plot

$$C(|\mathbf{x}_T - \mathbf{y}_T|) \equiv \mathrm{Tr}\left\langle [B(\mathbf{x}_T)B(\mathbf{y}_T)]_{\text{gauge link}} \right\rangle$$



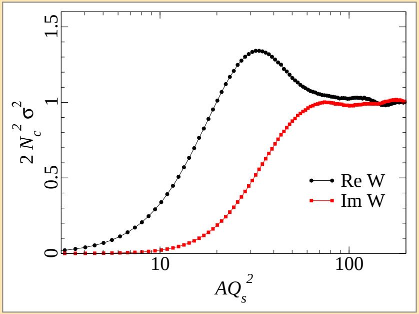
Straight line: $\sim (rQ_s)^{-1.55}$.

(For $C(r) \sim (rQ_s)^{-\alpha}$ one would get $\gamma = 2 - \alpha/2 \Longleftrightarrow \alpha = 4 - 2\gamma$;
from $W(A)$ measured $\gamma = 1.22$)

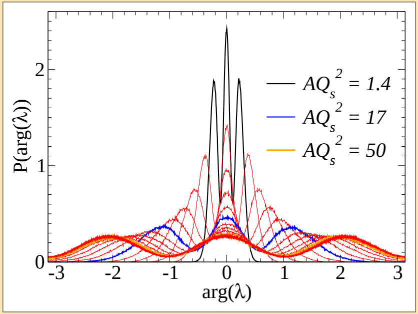
Fluctuations and eigenvalue distributions

How are the Wilson lines distributed in SU(3)?

Fluctuations of ReW and ImW



Eigenvalue phase distribution:



For large areas A both look like random SU(3) matrices:

$$\sigma^2(\mathrm{Re}W) = \sigma^2(\mathrm{Im}W) = \frac{1}{2{N_c}^2} \qquad P(\varphi \equiv \arg(\lambda)) = \frac{1}{2\pi}\left(1 + \frac{2}{3}\cos 3\varphi\right)$$